

4th ecpr General Conference
Pisa, Italy, 6 - 8 September 2007
Seat bias formulas in PR systems

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Seat Bias Formulas in Proportional Representation Systems

1. General three-factor formula
2. Special two-factor formula

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General three-factor formula

$$\left(q - \frac{1}{2}\right) \cdot \left(\frac{1}{j} + \frac{1}{j+1} + \dots + \frac{1}{\ell} - 1\right) \cdot (1 - \ell t)$$

method • party rank • threshold
Rank j signifies the j th-largest out of ℓ parties.

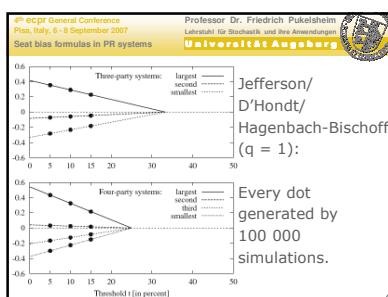
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Threshold contribution is negligible:

Seat apportionment among parties:
 t small, $t = 3\%, 5\% \Rightarrow (1 - \ell t) \approx 1$

Seat apportionment among districts:
 t large, $t \approx 0\% \Rightarrow (1 - \ell t) \approx 1$



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Special two-factor formula

$$\left(q - \frac{1}{2}\right) \cdot \left(\frac{1}{j} + \frac{1}{j+1} + \dots + \frac{1}{\ell} - 1\right)$$

method • party rank

Party rank $j = 1$: largest party
Party rank $j = 2$: second-largest party, etc.

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Divisor methods with stationary rounding are determined by a parameter $q \in [0, 1]$

$q=1$: DivDwn (Jefferson/D'Hondt/Ha-Bi)
 $q=1/2$: DivStd (Webster/Sainte-Laguë/Sch)
 $q=0$: DivUp (Adams)

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Divisor method with rounding down (Jefferson / D'Hondt / Hagenbach-Bischoff; $q=1$)

Seat fractions

Theoretical seat biases for 3 parties

Seat apportionments are biased in favor of larger parties and at the expense of smaller parties.

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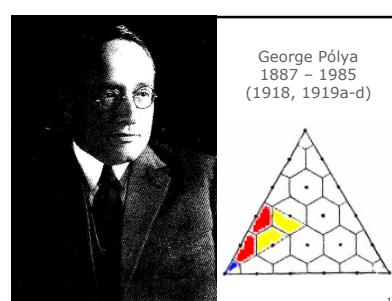
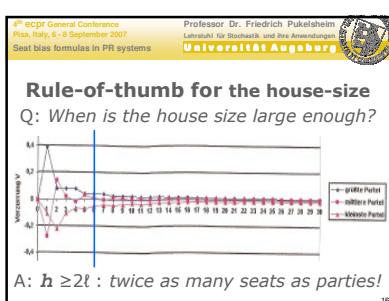
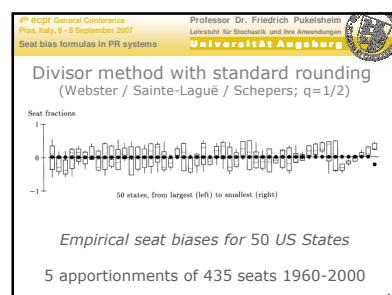
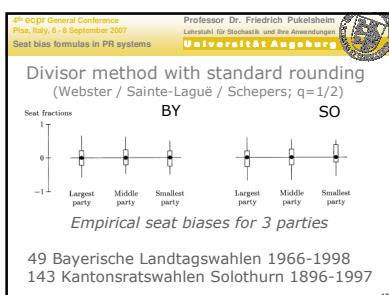
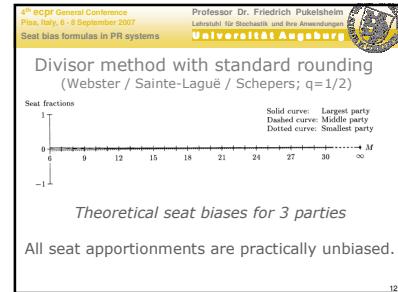
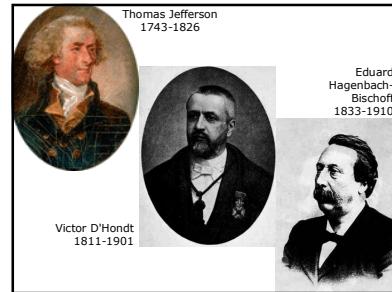
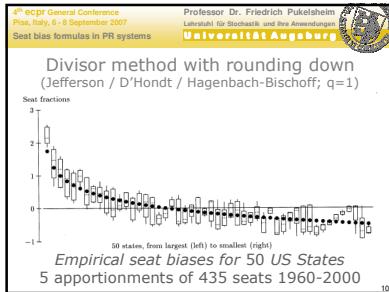
Divisor method with rounding down (Jefferson / D'Hondt / Hagenbach-Bischoff; $q=1$)

BY SO

Empirical seat biases for 3 parties

49 Bayerische Landtagswahlen 1966-1998
143 Kantonsratswahlen Solothurn 1896-1997

Friedrich Pukelsheim Seat bias formulas in proportional representation systems



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... many data sets and apportionment methods:
www.uni-augsburg.de/bazi

Berechnung von
Anzahlen mit
Zuteilungsmethoden
Calculation of
Allocations by
Apportionment methods
in the Internet