LETTERS TO THE EDITORS

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Is there an Æther?

IN the last century, the idea of a universal and all-pervading æther was popular as a foundation on which to build the theory of electromagnetic phenomena. The situation was profoundly influenced in 1905 by Einstein's discovery of the principle of relativity, leading to the requirement of a four-dimensional formulation of all natural laws. It was soon found that the existence of an æther could not be fitted in with relativity, and since relativity was well established, the æther was abandoned.

Physical knowledge has advanced very much since 1905, notably by the arrival of quantum mechanics, and the situation has again changed. If one re-examines the question in the light of present-day knowledge, one finds that the æther is no longer ruled out by relativity, and good reasons can now be advanced for postulating an æther.

Let us consider in its simplest form the old argument for showing that the existence of an æther is incompatible with relativity. Take a region of space-time which is a *perfect vacuum*, that is, there is no matter in it and also no fields. According to the principle of relativity, this region must be isotropic in the Lorentz sense—all directions within the light-cone must be equivalent to one another. According to the æther hypothesis, at each point in the region there must be an æther, moving with some velocity, presumably less than the velocity of light. This velocity provides a preferred direction within the light-cone in space-time, which direction should show itself up in suitable experiments. Thus we get a contradiction with the relativistic requirement that all directions within the light-cone are equivalent.

This argument is unassailable from the 1905 point of view, but at the present time it needs modification, because we have to apply quantum mechanics to the æther. The velocity of the æther, like other physical variables, is subject to uncertainty relations. For a particular physical state the velocity of the æther at a certain point of space-time will not usually be a well-defined quantity, but will be distributed over various possible values according to a probability law obtained by taking the square of the modulus of a wave function. We may set up a wave function which makes all values for the velocity of the æther equally probable. Such a wave function may well represent the perfect vacuum state in accordance with the principle of relativity.

One gets an analogous problem by considering the hydrogen atom with neglect of the spins of the electron and proton. From the classical picture it would seem to be impossible for this atom to be in a state of spherical symmetry. We know experimentally that the hydrogen atom can be in a state of spherical symmetry—any spectroscopic S-state is such a state—and the quantum theory provides an explanation by allowing spherically symmetrical wave functions, each of which makes all directions for the line joining electron to proton equally probable.

We thus see that the passage from the classical theory to the quantum theory makes drastic alterations in our ideas of symmetry. A thing which cannot be symmetrical in the classical model may very well be symmetrical after quantization. This provides a means of reconciling the disturbance of Lorentz symmetry in space-time produced by the existence of an æther with the principle of relativity.

There is one respect in which the analogy of the hydrogen atom is imperfect. A state of spherical symmetry of the hydrogen atom is quite a proper state—the wave function representing it can be normalized. This is not so for the state of Lorentz symmetry of the æther.

Let us assume the four components v_μ of the velocity of the æther at any point of space-time commute with one another. Then we can set up a representation with the wave functions involving the v's. The four v's can be pictured as defining a point on a three-dimensional hyperboloid in a four-dimensional space, with the equation:

$$v_0^2 - v_1^2 - v_2^2 - v_3^2 = 1 \qquad v_0 > 0. \quad (1)$$

A wave-function which represents a state for which all æther velocities are equally probable must be independent of the v's, so it is a constant over the hyperboloid (1). If we form the square of the modulus of this wave function and integrate over the three-dimensional surface (1) in a Lorentz-invariant manner, which means attaching equal weights to elements of the surface which can be transformed into one another by a Lorentz transformation, the result will be infinite. Thus this wave function cannot be normalized.

The states corresponding to wave functions that can be normalized are the only states that can be attained in practice. A state corresponding to a wave function which cannot be normalized should be looked upon as a theoretical idealization, which can never be actually realized, although one can approach indefinitely close to it. Such idealized states are very useful in quantum theory, and we could not do without them. For example, any state for which there is a particle with a specified momentum is of this kind—the wave function cannot be normalized because from the uncertainty principle the particle would have to be distributed over the whole universe—and such states are needed in collision problems.

We can now see that we may very well have an æther, subject to quantum mechanics and conforming to relativity, provided we are willing to consider the perfect vacuum as an idealized state, not attainable in practice. From the experimental point of view, there does not seem to be any objection to this. We must make some profound alterations in our theoretical ideas of the vacuum. It is no longer a trivial state, but needs elaborate mathematics for its description.

I have recently[1] put forward a new theory of electrodynamics in which the potentials A_μ are restricted by:

$$A_\mu A_\mu = k^2,$$

where k is a universal constant. From the continuity of A_0 we see that it must always have the same sign and we may take it positive. We can then put

$$k^{-1} A_\mu = v_\mu \quad (2)$$

and get v's satisfying (1). These v's define a velocity. Its physical significance in the theory is that if there is any electric charge it must flow with this velocity, and in regions where there is no charge it is the velocity with which a small charge would have to flow if it were introduced.

We have now the velocity (2) at all points of space-time, playing a fundamental part in electrodynamics. It is natural to regard it as the velocity of some real physical thing. Thus with the new theory of electrodynamics we are rather forced to have an æther.

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St. John's College,
Cambridge. Oct. 9.

[1] *Proc. Roy. Soc.*, [A, **209**, 291 (1951)].

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IN *Nature* of November 24, p. 906, Dirac draws an interesting conclusion from his new electrodynamics ; namely, the necessity of a revival of the æther concept. I have some doubts whether such an inference is necessary. Indeed, if we eliminate the coefficient λ from Dirac's equations, we obtain the following system of equations :

$$f_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} \tag{1}$$

$$f^{\rho\nu}{}_{,\nu} \left\{ \delta_\rho{}^\mu - \frac{A_\rho A^\mu}{k^2} \right\} = 0. \tag{2}$$

Considering (2) from the algebraic point of view, we see that two cases must be considered :

$$\text{Case (1) : det} \left| \delta_\rho{}^\mu - \frac{A_\rho A^\mu}{k^2} \right| \neq 0. \tag{3}$$

In this case, equations (2) lead to $f^{\rho\nu}{}_{,\nu} = 0$, that is, to Maxwell's equations if charged particles are absent.

$$\text{Case (2) : det} \left| \delta_\rho{}^\mu - \frac{A_\rho A^\mu}{k^2} \right| = 0. \tag{4}$$

Equation (4) means precisely $A_\mu A^\mu = k^2$. This condition must be fulfilled only if we are looking for a solution for which

$$k^2\lambda = A_\rho f^{\rho\nu}{}_{,\nu} \neq 0,$$

that is, if charges are present. Thus (4) need not be fulfilled for a vacuum, or for any regions for which

$$k^2\lambda = A_\rho f^{\rho\nu}{}_{,\nu} = 0.$$

As to Dirac's formulation, the replacement of the equation

$$A_\mu A^\mu - k^2 = 0$$

by

$$\lambda(A_\mu A^\mu - k^2) = 0$$

leaves all physical conclusions of his electrodynamics unchanged and seems to remove the necessity of reviving the æther concept.

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Jan. 31.

INFELD has shown how the field equations of my new electrodynamics can be written so as not to require an æther. This is not sufficient to make a complete dynamical theory. It is necessary to set up an action principle and to get a Hamiltonian formulation of the equations suitable for quantization purposes, and for this the æther velocity is required.

The existence of an æther has not been proved, of course, because my new electrodynamics has not yet justified itself. It will probably have to be modified by the introduction of spin variables before a satisfactory quantum theory of electrons can be obtained from it, and only after this has been accomplished will one be able to give a definite answer to the æther question.

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Feb. 16.