Semiclassical QCD-Lagrangian for Nuclear Physics

E.R. Hilf, Institut für Kernphysik, T.H. Darmstadt,
D-6100 Darmstadt, FRG

Résumé - La function de QCD-Lagrange semiclassique prouve correctement les limits du Confinement et de la liberté asymptotique. - au contraire de la limit classique du QCD.

Ou discussé application de la methode sur la physique nucleaire en comparison des autres approximation du QCD.

Abstract - The semiclassical QCD-Lagrangian yields the correct limits of Confinement and asymptotic freedom in contrast to the Classical Limit.
The applicability of the method to Nuclear Physics is discussed in comparison to other QCD-approximations.

I - INTRODUCTION

The Quantum-Chromodynamics is the exact theory of strong interaction. The Nucleus is bound by the strong interaction. Thus the QCD should in principle be able to describe nuclear properties. The obstacles are twofold:

The nucleus is a rather complex object of the dynamics of a large number of quarks, loosely substructured to nucleons. Thus in chapter II we will discuss what may be learned from QCD for Nuclear Physics in analogy to what could be learned from QED for Solid State Physics. -

The QCD-Lagrangian could up to now not be treated directly to calculate even the simplest dynamic applications such as a meson or a nucleon. But we will review in chapter III the present attempts to simplify or approximate or even model the full QCD down to concepts or schemes which might be calculable for applications.

In chapter IV we will settle in especial on the semiclassical QCD-Lagrangian approach. This approach is interesting also as compared to other semiclassical approximations in other quantum-theories of Physics (as e.g. the QED) as here

- the full quantum theory could not be solved,
- the classical limit is unphysical (in contrast e.g. to the Maxwell-equations as of QED),
- the lowest order quantum corrections to the classical limit yield a good model of the QCD with no additional parameters,
- already the next term of the formal semiclassical expansion diverges.

Results will be shown in chapter V in that the first order-semiclassical approach yields automatically bag-like structures of the gluon fields in the vicinity of local colour-distributions as may be encountered by quarks in a nucleon.

The vacuum results in being dominated by colour-magnetic fields as known from full QCD-studies.

II - THE NUCLEUS, A BOUND SUPERSTRUCTURE OF QUARKS

The possible range of applicability of the diverse approximations to QCD to Nuclear Physics may be inferred by studying the QED-analogue. Even here although the exact theory is known and in principle treatable, the tightly bound systems of several charged particles, the free atoms, could not be calculated exactly, but in the nonrelativistic limit of QM with relativistic corrections. The same holds for the opposite extreme of completely crunched atoms in an unbound charged plasma. For a very small range of intermediate densities there exists a loosely bound superstructure, the solid, where the binding comes about by a slight polarisation of the leastbound electrons of the atoms due to neighbouring atoms. It is this small range, where semiclassical approximations of QM have been successfully recently, because the inner more bound electrons of each atom can be approximated to be untouched, and the position of the atoms can be treated separately by the QM for mass-points.

As for quarks the analogy is almost perfect: the free nucleon is a tightly bound 3-quark system (to be expected if there exists the perfect colour-symmetry), the assumed to exist quark-plasma being rather dense and unbound.

For the very small density-region at $\rho_o \sim (4\pi/3)^{-1}\, r_o^{-3}$, $r_o = 1.18$ fm, the nuclei are known to exist as a comparatively very loosely bound superstructure of nucleons. This binding should come about by a slight polarisation of the least bound parts of the nucleon assuming that the inner part can be assumed to be untouched, and that the multinucleon dynamics can be coped with by pointlike nucleons (say HF). Thus we expect that it may be properties of this superstructure, which are suitable to be treated by a to be developped semi-classical approximation of QCD.

There are some differences between these two fields of physics: the quarks in the nucleon do not have a massive center (as have the electrons in the atom),

- although tunneling of quarks is possible /1/, it has to obey perfect colour-symmetry, that is free quarks do not exist,
- the vacuum is not empty as in QED but filled with colour-magnetic fields (gluons). Thus the attempt here will be to present and study a semi-classical approach which will treat the slight polarisation of the gluonic cloud around the three nucleonic quarks due to neighbouring nucleons instead of a gluon vacuum, while treating the quarks crudely as a classical but polarizable colour-charge-distributions.

Nuclear properties expected to be calculable by semi-classical QCD may be the strength of nucleon forces with their density-dependence as e.g. used in Hartree-Fock-approaches, but not the precise interior structure or nucleons or the global multinucleon dynamics of the superstructure, the Nucleus.

III - PRESENT APPROXIMATIONS OF QCD

The Quantum-Chromodynamics is assumed to be the exact theory of strong interaction. Its Lagrangian reads

$$\mathcal{L} = \frac{1}{4} F^2 + \psi \not{D} \psi ,$$

analogous to the minimal coupling of QED, however, here the Dirac-fields for the quarks $\psi_{sfc}(x)$ carry in addition to the spinor index s two more quantum numbers, the flavour index $f = 1, \ldots, 6$ (not of interest here) and the colour index $c = 1,2,3$. The gluon vector-potential $A^a_\mu(x)$ with $a = 1, \ldots 8$ (due to the adjoint representation of SU_{3c} with its Group generators $\lambda^a_{cc'}$, the Gell-Mann Matrices) yield the minimal coupling covariant derivative

$$\not{D} = \gamma D = \gamma^\mu_{ss'} \left(i\partial_\mu - g A^a_\mu(x) \cdot \lambda^a_{cc'} \right) .$$

The field strength tensor is then formed by

$$F := [D,D] = \delta^{ac} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right) + g f^{abc} A^b_\mu A^c_\nu$$

demonstrating the selfcoupling term of the gluons (which yield the gluonic vacuum).

The structure constants of SU_{3c},

$$f^{abc} \lambda^c = [\lambda^a, \lambda^b]$$

exhibit that the 8th component decouples from the SU_{2c} subgroup,

$$[\lambda^i, \lambda^8] = 0 \quad , \quad i = 1, 2, 3 .$$

Thus SU_{2c} is often (and will be here) used as a model for the technically more complicated SU_{3c} case.

To calculate the groundstate of multi-quark systems Variational Methods in analogy to the Rayleigh-Ritz method of the QM are being developped. There a Hamiltonian can be gained. Depending on the gauge chosen one can shuffle the seemingly unsurmountable calculational difficulties (due to the selfcoupling) for application even to an isolated meson, either into the Hamiltonian with no b.c. as persued by E. Schütte, Bonn, with the gauge $\partial_\mu A^\mu = 0$, or in temporal gauge $A^a_0 = 0$, have a simple Hamiltonian

$$\underset{\sim}{H} = \frac{1}{2} \Sigma \int d^3x \, (\underset{\sim}{E}^2 + \underset{\sim}{B}^2)$$

for the colour-electromagnetic fields but a seemingly technically unsurmountable b.c., the Gauß law. It seems to be a major breakthrough, that L. Polley, from T.H. Darmstadt could prove that the Gauß law is automatically fullfilled by the groundstate of $\underset{\sim}{H}$ for a given quark system. Thus the mass of heavier hadrons may soon be estimated from QCD.

The lattice-approximation treats the dynamics only on a discrete grid of space-time points. The great technical advantage is that the covariant derivative D is then represented by a unitary transformation, connecting neighbour lattice points,

$$U = \exp \left\{ i\, g \cdot A^a_\mu\, \lambda^a \cdot d \right\}$$

while the field strength $F^a_{\mu\nu}$ is represented by the respective products of U, forming the smallest plaquette of links. Observables can then be formed by starting from at least quadratic expressions in $F^a_{\mu\nu}$ such as the local action $F^a_{\mu\nu}\, F^a_{\mu\nu}$ or the stress-tensor $F^a_{\mu\rho}\, F^a_{\nu\sigma}\, \varepsilon_{\rho\sigma\tau}$ from which the expectation-values of physically interesting quantities are formed by summing over all possible field configurations with the statistical weight of exp $(-\beta S)$, where S is the global action. Due to the necessary Wick-rotation grounstate properties (temperature $T = 1/\beta \rightarrow 0$) are approximated by a large number of lattice points in time-direction, $n_\beta \rightarrow \infty$, which implies very large computing times. Masses of groundstates for the lighter hadrons have been calculated by Schierholz et al. at DESY. Periodic boundary conditions for the gluon fields have been assumed.

For the simple case of a massive colour-point-charge calculating the Wilson loop <L> L. Polley et al. /2/ at Darmstadt proved, that these periodic boundary conditions in the continuum limit yield an empty Hilbert-space, such that the interpretation of the resulting (empty) trace over the state space cannot be interpreted as a free energy. The respective lattice-approximations where periodic b.c. are widely used for simplicity, yield arbitrary (sometimes even negative) results for tr exp $(-\beta F)$) while L. Polley, M. Wendel et al. at Darmstadt and R. Gavai at Bielefeld demonstrated recently that physical b.c. such as $A^a_i = 0$ at the surface are easy to implement in a lattice-calculation and yield always positive values for the trace over exp $(-\beta F)$. By the way the phase transition behaviour to the quark-gluon plasma is pretty much affected by the choice of b.c.

For complex objects (arrangements) of quarks, such as in a hadron or an α-particle, or even more so a small nucleus, lattice-calculation (neglecting the sea-quarks) are sure to come. Because the lattices, chosen for finite computing time, will be only slightly larger than the objects, the correct choice of b.c. will be important to gain a solution and by that a glimpse of what to expect in the limes of infinite lattice size, which would be the correct physics of the strong interaction part of the object. .

For comparison, saving of computing time, predictions, and for more complex arrangements of quarks it may be good for practical use to have available an analytical approximation to QCD.

The simple classical limit to QCD, as ED to QED, replacing the field tensors in the Lagrangian by classical fields and the dynamic charges particles by classical sources is wellknown as the Yang-Mills theory,

$$\mathcal{L} \cong F^2 \quad .$$

As studied by many groups this Lagrangian yields neither confinement nor asymptotic freedom and is thus not useful for applications, although effects of classical selfcoupling terms have been studied.

As a next step there are two quite different semi-classical approaches, the Baker and Zachariasen Lagrangian /3/

$$L \cong F D^2 F$$

which was gained by making use of the Ward-identities taking the zero momentum-transfer limes and rewriting the result in the above covariant manner.

The other approach, the semi-classical Adler-Lagrangian was first developped by Adler at Princeton. As could be shown by Leutwyler et al. /4/, it is the first term of a semi-classical expansion for small coupling, a one loop approximation. By comparison to the respective derivation in QED L. Polley at Darmstadt could show that the correct form is

$$L \triangleq F^2 \ln F^2/e\,\kappa^4 \quad ,$$

where κ is related to the scaling length of QCD. As will be shown in chapter IV and V this Lagrangian does yield the correct qualitative behaviour for confinement and asymptotic freedom, as well as a magnetic dominated vacuum. The loop expansion cannot be improved because already the two loop expansion contains $\ln \ln F^2/\kappa^4$ terms which destroy the qualitatively correct strong coupling limit.

Historically mentioned should be models of QCD which try to simulate the QCD-features of confinement, asymptotic freedom etc. by introducing seemingly suitable Lagrangians with sufficiently many parameters adjustable to experimental data. Some similarities to the infinite number-of-colours expansion of QCD shows the Skyrmion-Model /5/ with the Lagrangian

$$L_{Sk} = \int d^3x\, f_\pi^2 \cdot \mathrm{tr}\, (\partial_\mu U)^2 + a \cdot \mathrm{tr}\, [(\partial_\mu U)U^+,(\partial_\nu U)U^+]^2$$

with f_π and a as fitparameters. U_i is a set of four fields simulating the π^+, π^o, π^- and a soliton.

The soliton-model of Friedberg and Lee, as developped by Wilets and Goldflam neglects the pions and adopts a standard soliton Lagrangian,

$$L = a\,\sigma^2 + b\,\sigma^3 + c\,\sigma^4 + \psi \not{\partial} \psi + \psi\,\sigma\,\psi \quad ,$$

where a gives the soliton mass, c the strength of the selfcoupling and b the soliton feature. The quark mass may be neglected compared to the finite quark mass σ gained due to the coupling to the soliton field, /6/.

The simple limit of assuming an infinite soliton-mass yields the well-known and widely studied bag-models, where the microscopic field-description of the bag-formation is lost, and a bag-radius with b.c. have to be artificially introduced. In contrast to the soliton models they cannot be used to study the polarization of nucleons if embedded in a nucleus.

IV - THE SEMI-CLASSICAL LAGRANGIAN

The vector potential field operators are replaced by their expectation values,

$$A_\mu^a(x) = \langle \underset{\sim}{A}_\mu^a(x) \rangle \quad ,$$

the quarks by classical colour charge currents $j_\mu^a(x)$. We will restrict ourselves here to static sources $\rho^a(x)$. The relation of a Hamiltonian $H[j]$ to the Lagrange-function is such that one first gains the free energy $\mathcal{F}[j] = -\beta^{-1} \log Z$ form the partition function $Z[j] = \mathrm{tr} \exp(-\beta \underset{\sim}{H}[j])$.

A contact transformation from currents to their canonical conjugate, the vector potential yields the global action,

$$\Gamma[A] := \mathcal{F}[j] + \int d^3x \; j \cdot A \; , \qquad (1)$$

where the action is the integral over the Lagrange density,

$$\Gamma[A] = \int d^3x \; \mathcal{L}[A] \quad ,$$

and the field equations are gained by

$$j = \frac{\delta\Gamma[A]}{\delta A} \quad .$$

The Lagrangian used here can be derived by the one loop approximation,

$$\mathcal{L} = F^2 \ln \frac{F^2}{e\,\kappa^4} \quad .$$

For the vacuum, $j = 0$, the action coincides with the free energy $\mathcal{F}$, and for $T \to 0$ the free energy with the energy.

But $\mathcal{L}$ shows a nontrivial minimum for a nonzero value of $F^2 = B^2 - E^2$,

$$\varepsilon := \delta_{F^2} \mathcal{L} = 0 \quad \text{for} \quad F^2 = \kappa^4 \quad .$$

This is a result of the gluonic selfinteraction, in contrast to the photon-case. By the way $\delta\mathcal{L}/\delta F^2$ is in electrodynamics named as dielectric function ε . Thus $\varepsilon = 0$ for the gluonic vacuum, which is dominated by colour-magnetic fields, since $\kappa^4 > 0$ and thus $B^2 > E^2$.

That for nonzero charge distribution $\rho(x)$ at x the local vector fields $-A(x)$ decrease, for groundstate contributions may be inferred already from (1); as j increases a decrease of A may pay off to balance the increase of $\mathcal{F}[j, T\to 0]$ from its gluonic vacuum value, and ε becomes nonzero.

Although for simplicity we use specific gauges and work with gauge fields, the Lagrangian depends on the observable F^2 only and is thus fully gauge-invariant, as all the results for observables.

Because of the fairly simple analytic structure of $\mathcal{L}$ there is hope that it may turn rather useful to calculate complicated quark structures, especially if one is interested in the polarization of hadrons in nuclei, and is not so much interested in their precise interior structure, where the replacement of $\bar{\psi}\psi$ by $\rho^a(x)$ may turn to be too crude.

V – QUANTITATIVE RESULTS FOR L_{SQCD}

For the case of somewhat concentrated colour-charge distribution $\rho^a(x)$, chosen to resemble the quark-distribution within a hadron L. Polley at Darmstadt /7/ presented interesting analytic results evaluating the semi-classical Lagrangian which will be summarized here.

For simplicity SU_{2C} is chosen. The assumption of highest possible spherical symmetry for the vector fields $A^a_\mu(x)$ leads to the Witten-Ansatz, which for lowest topological quantum number yields the ansatz of Wu and Yang,

$$A^a_o = \frac{G(r)}{r} \cdot r_a \qquad ; \quad A^a_i = \frac{(H(r)-1)}{r^2} \cdot r_n \cdot \varepsilon_{ian}$$

with two arbitrary radial functions $G(r)$ and $H(r)$. Thus observable fields such as F^2 or ε are purely radially dependent. While $G(r)$ enters into E^2 only, and thus describes the colour-electric part, $H(r)$ determines $B^2(r)$,

$$E^2 = (G')^2 + \frac{2}{r^2} \cdot G^2 \cdot H^2 \quad ,$$

$$B^2 = \frac{2}{r^2}(H')^2 + \frac{1}{r^4}(H^2-1)^2 \quad .$$

Regularity conditions are $H(0) = 1$, $H'(0) = G(0) = 0$.
The resulting differential equations for G and H are

$$H'' + \frac{1}{r^2} \cdot H \cdot (1-H^2) + G^2 H = -H' \cdot (\log\varepsilon)' \quad ,$$

$$(r^2 \cdot \varepsilon \cdot G')' - \varepsilon G H^2 = \rho(r)$$

with $\varepsilon = \log F^2/\kappa^4$ and $F^2 = B^2 - E^2$.

L. Polley could show that for the simplest assumption of $G \equiv 0$ and a smooth function $\varepsilon(r) \to 0$ for $r \to \infty$ ther exist <u>no</u> solutions of the first differential equation. But for a sharp, unanalytical transition from a region $r < r_o$ where $\rho(r)$ may be nonzero to $\varepsilon \equiv 0$ for $r > r_o$ as the outside vacuum, solutions do exist if and only if

$$r_o < 1/\sqrt{\kappa} \quad .$$

Because of the known scaling length of QCD of about 200 MeV r_o results to $\lesssim 1.2$ fm.

In other words, hadrons in this model do not interact if they are further away then $2 \cdot r_o$. Outside r_o the true gluonic vacuum is found. Thus a macroscopic "bag"-type structure is found with no parameters but with the semi-classical Lagrangian, demonstrating that confinement is a result of quantum-fluctuations and not a classical feature of selfcoupling (since it is not found in the Yang-Mills limit).

The simplification of $G \equiv 0$ is acceptable because the charge distribution of SU_{2C} then is zero, but we can easily go over to SU_{3C} by adding a $\rho^8(r)$ distribution. Because the selfcoupling terms do not couple A^8 with the SU_{2C}-fields, we get for the Gauß-law

$$\rho^8 = \underline{\nabla} (\varepsilon \cdot E^8)$$

and ρ^8 can be chosen appropriately to simulate the quark-structure of the respective hadron. The total colour-charge of the hadron is of course always zero, as may be inferred by integrating over the surface at $r_1 > r_0$

$$Q := \int_{\partial V} \underline{\nabla} (\varepsilon E) = \int_{\partial V} (\varepsilon E) = \int_{\substack{\partial V \\ r_1 > r_0}} (\varepsilon E) = 0 \quad .$$

Thus here only colour-neutral hadrons exist.

Hadrons embedded in nuclear matter one can now easily model by rereplacing the boundary conditions at r_0 by b.c. at $r_1 < r_0$ with

$$\bar{\rho} = \frac{1}{\frac{4\pi}{3} r_1^3} \quad ,$$

where here $\bar{\rho}$ is the average matter density. Such calculations are in progress. The interesting result aimed at is that it may ultimately be possible,

- although not to calculate the nuclear groundstate density,
- to calculate the density-dependence of the semi-empirical forces as used in the Skyrme-density dependent Hartree-Fock calculations.

Hereby QCD would be ultimately connected to the enormous wealth of observed experimental data such as the 1500 nuclear groundstate masses.

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