

Heiner Zieschang – his life and work  
Toulouse 6<sup>th</sup> September 2007

I am most grateful to the organisers of the conference for inviting me to pay tribute to Heiner Zieschang and his life and work.

I first met Heiner Zieschang in the summer of 1980 shortly before I started a one-year visit to Bochum as a Humboldt Fellow. A preliminary to my visit was a six week German language course in Cologne and I came to Bochum for a day to get acquainted; I had not previously met Heiner.

At that time the Humboldt Foundation saw the promotion of the German language as part of its cultural mission, and was willing to fund such language courses, even although by then most German mathematicians and scientists not only spoke and understood English well but were starting to write their scientific papers in English. Heiner himself was sympathetic to the Humboldt Foundation's aim and, although we spoke English on our first meeting, once I was established in Bochum we would mostly converse in German, sometimes even for mathematics. It is interesting to note, though, that the last research paper that Heiner wrote in German, with Bruno Zimmerman, was published in 1982, though of course he continued to write papers in Russian throughout and there are publications in French when he worked with native French speakers. One of my minor jobs when I was in Bochum was to polish Heiner's written English – we used to have long arguments about commas. In English, there are almost no actual rules for commas – one proceeds by instinct in regard to the sense : this always troubled Heiner who was used to the more than 50 rules for commas in German (now, I believe, reduced to half a dozen or so).

I remember that on the day of my visit we went for a walk – as we would do on many other occasions – and finished our excursion with a visit to Toni's Pizza House near the university. Heiner was well-known as a regular customer and immediately engaged in a lively conversation with the manager with whom he was clearly on friendly terms. This was a typical encounter when one was with Heiner – he was very sociable and always interested in people and their doings.

At that point in 1980, a dozen or so years after his arrival in Bochum, Heiner was a well-established member of the Mathematics Institute in Bochum. Then, and perhaps even today, German university departments were very different from the

British and American model that I was familiar with. Instead of a strong central authority, every 'Lehrstuhlinhaber', i.e. 'full Professor', had his (almost never her) little army of followers. I thought it was a bit like the Holy Roman Empire and its component states, each presided over by a Prince-Bishop or Kurfurst. Heiner's territory was half of the fourth floor in the Mathematics section of the gigantic Gebaeude NA and his troops consisted of several officers (in my time Frank Levin, Ralf Stoecker, Gordon Wassermann and Bruno Zimmermann) plus cavalry and infantry in the shape of Assistenten, Doktoranden and Diplomanden, with his secretary, the redoubtable Frau Marlene Schwarz as chief of staff. I was entertained by one small example of the spirit and independence of a Lehrstuhl - contrary to all rules and practice, the Lehrstuhl members at one point decorated 'their' personal seminar room when it began to look dingy, painting a Moebius band round the walls but leaving a small gap where two little puzzled painters wondered why the other was upside down.

Heiner had a fairly steady routine, rising usually about 6.00am ( I have this information by report only) to eat a good breakfast and sometimes do some mathematics before setting off for the university which was a pleasant 20 minute walk away. Typically he would lecture from 8.00am to 10.00am, transact business from 10.00 to 12.00, then from 12.00 to 2.00 maybe give another lecture or see students and a bit later in the afternoon set off home, perhaps to coffee and cake followed in the evening by Griessbrei and Rote Grutze – the former is a kind of porridge and the latter a chilled fruit stew of redcurrants, blackcurrants and raspberries that is popular in North Germany. Usually one day a week was spent entirely at home.

Fridays were different since, on Friday afternoon, Heiner ran his Topology seminar from 2.00 to 4.00. Each week there was a talk on recent work – sometimes the same person would carry on for two or three weeks. Many were by members of the Lehrstuhl - students reporting on progress in their thesis, often Heiner himself as well as other staff members, and indeed visitors would give talks as well. Beforehand, from 1.30- 2.15 there was the ceremonial drinking of coffee – this was an informal meeting where Heiner would discuss minor matters of business but there was a clear expectation that Lehrstuhl members should attend.

Now let me move on to mathematics. Gerhard Burde has talked about Heiner's life and work through until the early 1980's and I shall carry on from that point. However I would like to set the scene by looking at some statistics concerning Heiner's published work.

Altogether Heiner published about 100 mathematical papers or books. (A precise count is difficult to make since it is not always clear how to count research announcements, translations, multiple editions of books – and I may have missed or misclassified something.

About 80% of Heiner’s papers were research papers and another 15% were survey articles or research announcements. There are six books. Of his published works about one third were ‘single author’ and two-thirds ‘joint author’. It is interesting to note that in the period 1962- 1982 this split is almost reversed - in these first twenty years, about 60% of his papers were ‘single author’ and 40% were ‘joint author’.

It is also interesting to count the number of his coauthors. I have found altogether 39 – which becomes 112 when one counts with multiplicity. In terms of nationality, the tables below give the corresponding distributions.

Nationality of coauthors	American	1
	Brazilian	1
	British	2
	Canadian	2
	Chinese	1
	French	2
	German	22
	Russian	8
Nationality of coauthors, counted with multiplicity	American	5
	Brazilian	10
	British	9
	Canadian	4
	Chinese	6
	French	18
	German	33
	Russian	31

In looking at Heiner’s work from the beginning of the 1980’s I am going to focus on the more extensive collaborations. Overall I think two themes emerge from such an analysis – one is the very explicit and concrete nature of the problems he studied and the other is the interplay between the topology and the algebra.

My own collaboration with Heiner –covering roughly 1981- 1991- began during my Humboldt Fellowship. At Heiner’s suggestion, we looked to see if we could generalise the Higgins- Lyndon argument for Whitehead’s theorem on automorphisms of free groups to the case of free products. It was fairly clear what

we needed to do – namely copy the free group argument with the obvious modifications.

By the end of my time in Bochum, I was able to take home with me a draft version of a paper to write up properly. But when I wrote out all the details there was a gap – and I could produce an example which I thought showed that the gap was impossible to fill. I wrote to Heiner explaining the problem – and almost immediately he solved it. To use a metaphor, he bridged the gap instead of filling it – technically what he did was to define the problem away by extending the basic generating set we were working with beyond the immediate analogue suggested by the free group model. Once we had done this we were able to achieve all our goals. Our final work together was a long survey article in then Encyclopaedia of Mathematics. This had an odd linguistic history – we wrote in English which was translated into Russian for publications and then appeared in the original English when Springer published a translation of the whole Encyclopaedia into English.

My work with Heiner was entirely algebraic so I was not surprised when in 1982 Heiner began his collaboration with Michel Boileau on 3-manifolds. Heiner and Michel met at a conference in Suusex in the UK and found they were interested in very similar questions. They wrote four main papers altogether, three on Heegard decompositions of Seifert manifolds and one on Montesinos links.

The first of these, which Gerhard Burde has already mentioned has perhaps the most striking result and at one point relies quite heavily on Heiner's earlier work on generators of planar discontinuous groups (with Norbert Pecynski and Gerhard Rosenberger). The main result in the paper is a proof that for 'most' closed orientable Seifert 3-manifolds, the Heegard genus and the algebraic rank of the fundamental group coincide but there is also an example where the Heegard genus is three and the algebraic rank is two. Both to show that the exceptions are comparatively rare but also to construct exceptional examples, one has to use the fact that for most Fuchsian groups the geometric and algebraic rank coincide but also the fact that there are exceptions.

The second paper on Montesinos links establishes a similar result, but this time without exceptions – the bridge number of a Montesinos link is always equal to the algebraic rank of its fundamental group. Again Heiner's paper with Pecynski and Rosenberger is an essential ingredient

Their third joint paper (with Markus Rost as additional coauthor) contains a detailed classification of genus two Heegard decompositions of the exteriors of torus knots and also of certain Seifert fibre spaces. The paper is heavily geometric, with many diagrams, but, as in the previous cases, it relies on earlier work of Heiner involving groups, this time results on the number of Nielsen equivalence classes of presentations of the fundamental group of a torus knot.

The fourth joint paper is again a classification result, this time for certain Seifert fibre spaces. The emphasis is geometrical but once again some group theory is needed, this time on conjugacy of elements in triangle groups. On this occasion Heiner had **not** already proved the necessary result himself but he knew that I was familiar with work of Paul Schupp on conjugacy in small cancellation groups – and that this was what was needed - and so, for doing almost no work at all, I was enlisted as joint author. Altogether these four papers provided an outstanding example of the kind of problem Heiner liked.

I might also add that Heiner's energy level was such that he was not easily satisfied by just one, let alone two collaborators – when he was collaborating with both myself and Michel, Heiner wrote 'Knots' with Gerhard Burde (publ. 1987) and 'An introduction to Algebraic Topology' with Ralph Stoecker (publ. 1988), each several hundred pages long and two papers with Anatoly Fomenko, both almost 30 pages, on the topological properties of integrable Hamiltonian systems.

Heiner had what one might, in a different context, call a 'roving eye', though of course Heiner was looking for mathematical collaborators rather than female companions. In the course of his collaboration with Michel, Heiner visited Toulouse and met some of Michel's colleagues including Claude Hayat. They embarked upon a study of degree one mappings between 3-manifolds, writing some six papers with Shicheng Wang and some with other authors. Shicheng, like me, was a Humboldt Fellow in Bochum.

They began with the case of degree one maps into lens spaces and followed this up by an analysis of degree one maps between Seifert manifolds, using a definition of minimality for Seifert manifolds, based on the existence of such maps. The result is an almost complete classification – in a simple list - of all possible minimal manifolds, with only the Poincare homology sphere causing uncertainty. However, a further paper, with Sergei Matveev, produced an explicit and complete expression for the degree of a map from a Seifert manifold to the Poincare homology sphere. As

always there is some algebra underlying the geometry although here there is less emphasis on combinatorial group theory.

There are also two papers with a different emphasis, the cohomology rings of 3-manifolds. The first paper has four authors with Heiner and Claude joined by John Bryden and Peter Zwengrowski. The main content is a method to compute, in the case of an orientable Seifert manifold with orbit surface the 2-sphere, cup products for arbitrary coefficients and the complete structure of the cohomology ring for coefficients in the field of two elements. As such the paper is entirely algebra but by using a previous result of Shastri and Zwengrowski, the paper provides a way to verify whether or not an orientable Seifert manifold with orbit surface the 2-sphere admits a degree one map to real three-dimensional projective space..

The method of proof follows a technique of Heiner's teacher Reidemeister in using the chain complex of the universal cover. I mention this to point out that the calculation of the boundary maps needs the Fox calculus – which was a key tool in Heiner's first published paper in 1962.

The authors describe the calculations they carry out in the last two sections of the paper as 'somewhat arduous'. I can imagine that nonetheless Heiner will have enjoyed these calculations – he possessed both the courage and the stamina to undertake calculations before which lesser mortals might quail. (In his laudatio for Heiner's 60th birthday, Gerhard Burde quotes Reidemeister as saying of Heiner that 'er weiss wie in der Mathematik zuzupacken' - which freely translates as 'he knows how to get things done in mathematics'). There is also a paper with five authors, namely Heiner, Claude and three students from Bochum

It is perhaps worth remarking that cohomology was not Heiner's natural home. The questions that interested him were almost always very explicit. He once observed to a doctoral student that she probably needed some cohomology to make progress in her thesis but, unfortunately she had not been well trained in this area – because Heiner himself, under Reidemeister, had not mastered it.

While all this was going on within the period 1996- 2002, Heiner published another book, namely 'An Introduction to Linear Algebra' which ran to 654pp.

Heiner's final collaboration, covering 1997- 2004, was with Semeon Bogaty, Daciberg Goncalves and Elena Kudryavsteva. In this he was completing a kind of cycle, returning to the subject of surfaces and their connections with equations in free groups - and also to collaborations with Russian mathematicians (but adding a

Brazilian). In total there are fifteen papers, written by Heiner with various combinations of one, two or all three of his collaborators. The setting is that of maps between closed surfaces - mostly orientable - and the study of either coincidence points for pairs of maps, or when one of the two is just a constant map, the 'roots' of a single map. These notions are explored in various ways and the results obtained can be applied to describe the solution sets of certain quadratic equations in free groups. Daciberg Goncalves has, in an earlier talk at the conference, given us a delightful account both of the work itself, which has the typical clarity and precision characteristic of all Heiner's work, and of the collaboration itself and the pleasure that working with Heiner brought to all his coauthors.

This very brief account of Heiner's later work with a range of international collaborators does less than justice to his influence and achievements. In his later years, he also travelled a great deal, spending sabbaticals in France and in the USA. and he went to conferences in Brazil, Canada, the Czech Republic, Italy, Korea, Turkey, United Kingdom – there may be omissions in this list - and of course was a frequent visitor to Russia.

In many ways Heiner was a remarkable man. This was recognised by the award of honours from the University of Moscow and the University of Toulouse. The recognition by the University of Moscow as Honorary Professor is of unusual distinction since there are in total only about 400 such awards, with Goethe an early recipient of this honour . One can also see his influence if one looks at some of the bibliographical detail and the title pages of his works – they reveal, as we have seen, a large number of collaborators with a considerable geographical spread in terms of the origin of his coauthors but also the fact that sometimes the work was actually done in Bochum indicating that not only had Heiner worked with them on the mathematics but that he had arranged funding for them to visit Bochum. He also gave practical and personal help to mathematics institutions and individual mathematicians and their families, particularly in the former Soviet Union and then Russia. His work in Russia was generously acknowledged in a five page article in Russian Mathematical Surveys on the occasion of his sixtieth birthday. I particularly like the emphasis that the authors (A.A. Malt'sev, S.P. Novikov, A.V. Zaruela) placed upon Heiner's ability to translate a topological problem into an algebraic one, solve the algebraic problem and then translate the solution back to the topology and in my remarks above I have sought to mention some of the algebraic techniques Heiner brought to bear on topological problems. (They also say that topologists considered Heiner a topologist while algebraists considered him an algebraist. As one who is primarily an algebraist I agree with the former but not the latter – I am not

questioning his talent for algebra but his interest was almost always grounded in the topology.)

Gerhard Burde has already mentioned how exceptional Heiner was in his interest in the USSR and his determination to learn Russian. Later, when he began to collaborate with native French speakers he applied the same energy and determination to master French. Although he also collaborated with Shicheng Wang, I am not aware that his Chinese was of the same standard as his other languages.

In person Heiner was unassuming and he disliked unnecessary formality and ceremony. He was also playful – not normally a quality associated with a distinguished German professor. Mathematicians are often playful, in my experience – but even by mathematical standards Heiner was playful. Every visitor who came to stay with him, as many did, would be offered a poppyseed roll for breakfast but told by Heiner that this was actually opium. The joke amused him every time. He was a generous host and would devote much of his time to ensure that his visitors saw something beyond just the working day in the Mathematics Institute. He was a keen theatregoer – in my time Bochum was fortunate in having one of the best theatres in Germany – and he also liked to take his visitors to places of historic or scenic interest. I can remember three or four of us going on a trip to the Eifel southwest of Cologne where we eventually reached the border and walked across the unguarded bridge and entered Luxemburg briefly. (I have a feeling that only Heiner possessed a document that actually allowed him to enter Luxemburg.) Once he took myself and Misha Balavadze from Georgia all the way to Holstein in north Germany where he grew up and had friends whom he and Ute had made when they went on holiday there. Heiner also came to London several times when we were in active collaboration but since normally he could only spare a limited number of days he would, humourously of course, complain that all he ever saw was the seminar room in the Queen Mary College mathematics department and the London underground route between my house, Queen Mary and Heathrow airport. Of course these are my own experiences but Heiner would keep all his friends and collaborators in touch with his Christmas letters and I know he was as generous with all his visitors and collaborators as he was with me.

It was a particular pleasure for me to see him, together with Ute and Marlene Schwarz, when they came on holiday to my native country of Scotland. They came as part of a travel group and Heiner enjoyed the uncertainty of the rest of the party in relation to this gentleman travelling with two ladies, explaining to me when we met that he had brought two wives with him, namely 'Hauptfrau und Nebenfrau' – which



translates as 'principal wife and subsidiary wife' but the translation lacks the sharpness and wit of the German. Sadly, it was on this trip that Marlene told me Heiner was ill. All three came back to England the following summer and my wife and I met them in London, but it still came as a shock that he died no more than a few months later. He is greatly missed.

DJC