DESIGN OPTIMIZATION OF MULTI-LINK SUSPENSION SYSTEM FOR TOTAL VEHICLE HANDLING AND STABILITY*

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ABSTRACT

This paper discusses the design problem of vehicles using multi-link suspension system with the aim of totally optimizing vehicle handling and stability. Since this problem includes many evaluation items, and multi-link suspension system has interconnected behavior, the optimization is so complicated. An efficient and computable model is indispensable for compromising the total optimization. This paper investigates a hierarchical structure of objectives, introduces appropriate simulation models for respective items, and formulates a mathematical optimization model based on them. Further, we apply a genetic algorithm based optimization method to this problem. The genetic algorithm is based on Simple GA and introduces several extensions such as fitness function for constrained multi-objective optimization problems, similarity-based selection, direct crossover within side constraints, etc. The result of optimization calculation shows the validity of the optimization model and the optimization algorithm as mathematical computation based design methods.

1 INTRODUCTION

Suspension used in an automobile is a system mediating the interface between the vehicle and the road, and their functions are related to a wide range of drivability such as handing ability, stability, comfortability and so forth (Dixon, 1996). Since the total optimization of such contents requires much of design freedom, a multi-link suspension system, that is principally a parallel six-bar universal linkage, is getting installed to passenger cars, mainly to high-grade cars. On the other hands, such design freedom makes the design process for determining link geometry, etc. more complicated, and it is not so easy to design the suspension system with promising insights. This leads the necessity of a new generation of design methodology that can realize a potential of the complicated system toward total optimality.

This paper discusses the total design method for both finding optimization possibilities within an established system structure and configuration and optimizing the system attributes that correspond to such possibilities, under the concerns with the total drivability optimization of a vehicle using multi-link suspension systems. For this direction, first we hierarchically structure design items from design variables that represent suspension geometry to evaluation criterion related to practical operation situations, and then organize an optimization problem by selecting a mathematically operational part from the whole design problem. Finally we show that the optimal design solutions can be obtained by means of a genetic algorithm based optimization calculation, since the formulation results to a large-scale multi-objective constrained optimization problem. Besides, a sequence of these procedures must be applicable to other design problems as an effective methodology as well as the design problem of a multi-link suspension system, when considering that various mechanical systems have become complicated to have high levels of functions.

2 MULTI-LINK SUSPENSION SYSTEM AND ITS DESIGN PROBLEM

As aforementioned, the reason why multi-link suspension systems are introduced mainly for high-grade cars is the high degree of design freedom for various function items.

The essential difference between design problems for simple systems and ones for complicated systems is that in the former cases the mapping from design items to function items is relatively independent and it is possible to separately determine respective items, and that in the latter cases the interactions between all items are complicated as well as their structure and configuration and the tradeoff among function items is not straightforward. This tendency seems to have become more obvious under the up-to-date technologies that try to condense more functions into a certain size of a system.

The above context can be found in the design problem of multi-link suspension systems used in automobiles. The fundamental functions of an automobile are to run straight, to turn and stop, and to run on both good and bad roads. That is, they consist of various operation modes. While there are a variety of suspension types (Dixon, 1996), their performance depends on both the selection of their types and the adjustment of their component link sizes. When focusing on a specific operation mode, the suspension geometry of simple types can be relatively easily determined to be ideal, since the relationship between link sizes and the specific function is straightforward. However, it is necessary to introduce complicated suspension types for realizing totally superior performance against all operation modes (Ushio *et al.*, 1991), and the corresponding design problem of suspension geometry is not so easy due to the aforementioned nature of complicated systems design.

Under these points, the multi-link suspension system that this paper is going to discuss is principally a parallel six-bar universal linkage. It is generally impossible to understand the immediate relationships between link sizes and respective function items. So, the conventional design situation requires many times of try-and-errors for finding a superior design solution. If the design problem can be mathematically formulated and the optimization algorithm suitable for its characteristics is organized, such a design method can be effective.

3 STRUCTURE OF MULTI-LINK SUSPENSION DESIGN PROBLEM

First, we reveal the design problem structure of the multi-link suspension under the purpose of making automotive drivability superior such as handling ability, stability and comfortability, and clarify the part where an optimization under mathematical framework is applicable.

3.1 Structure Analysis by ISM Method

For the direction, it is necessary to find an operational part of the whole design problem, as well to clarify design requirements for design activities. Since this is essential not only for design optimization but also for design itself, and it is especially important for complicated systems, the ISM (Interpretive Structural Modeling) method (e.g., Warfield, 1973), that can establish the hierarchical structure of system problems, has been introduced for system planning problems (e.g., Akagi *et al.*, 1984). Under the advantage of ISM method, we analyze the contents of the multi-link suspension design problem.

3.2 Design Contents

The design problem of multi-link suspensions is to totally optimize or compromise all performance under every operational mode of a vehicle by optimally determining suspension geometry. Under a fixed suspension type and configuration, the geometric dimensions of element links, the positions of joints, the coefficients of spring-dampers and the stiffness of stabilizers should be determined. As for the evaluation items, while they can be finally integrated into three items, handling ability, stability and comfortability, the integrated indexes are linked with passenger's feeling. However, the design optimization requires some rationally quantitative measures. The performance indexes that can be measured by driving experiments with physical vehicles, the indexes gotten by computer simulation of driving situations, the characteristic features related to suspension geometry itself can be counted as the candidates for such measures. While these should be selectively used for respective purposes, a set of measures must include all direction of evaluation items. Further, the relationships against cost and preciseness of each measure must be also considered toward structurization of the design problem, since they are based on physical experiments or computer simulations.

3.3 Hierarchicalized Levels

Proceeding from the above point, we enumerate 77 items as design related items, check the relationships between every pair of them, and apply the ISM method for hierarchical structurization. Then, we reorganize the eleven levels that are categorized by the ISM method into another number of levels by considering their physical meanings, that is, how to obtain those indexes. Figure 1

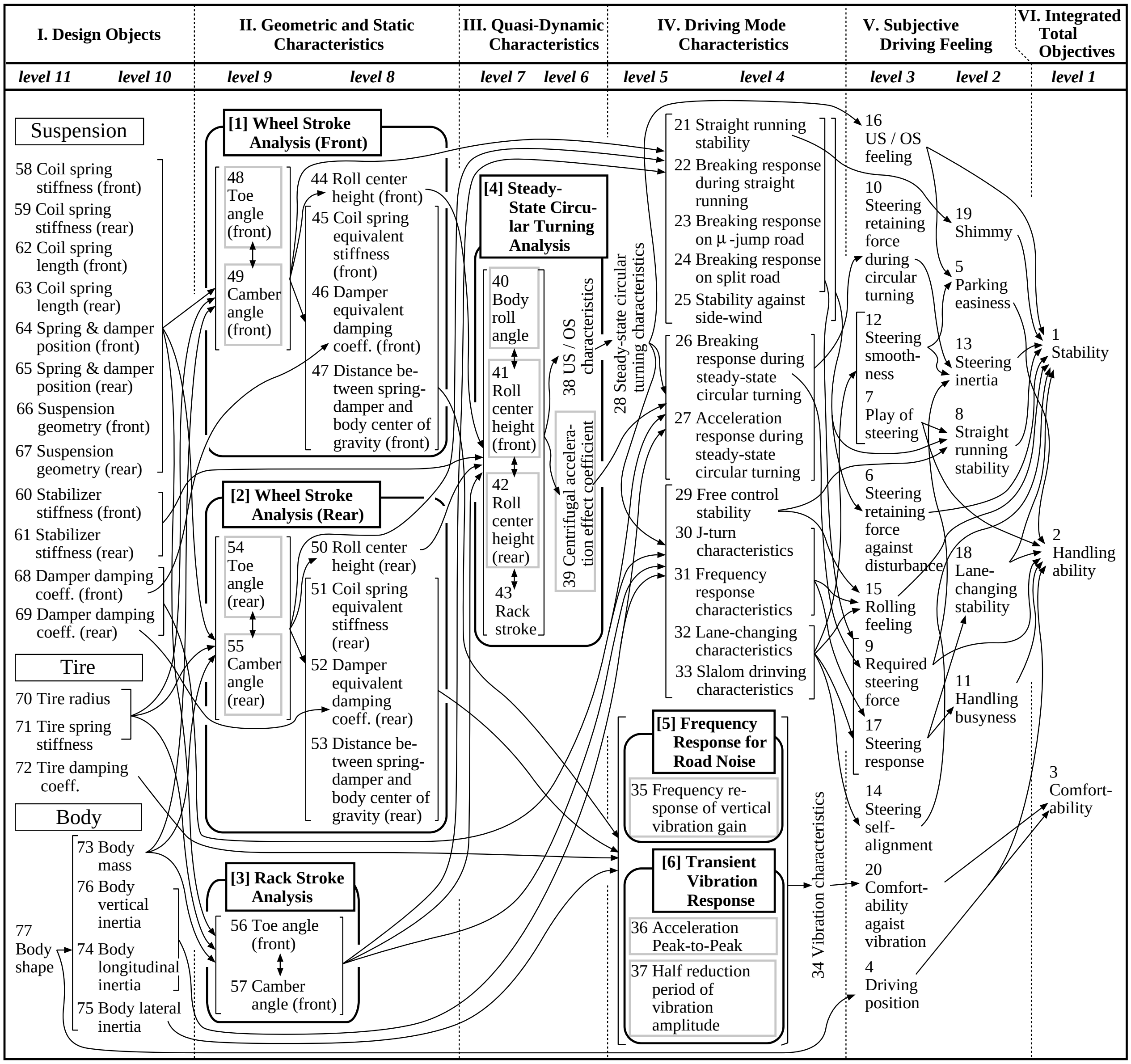

Figure 1 Hierarchical categorization of design items through ISM analysis

shows the resulted hierarchical structure and consequent levels, and the following is the six levels that correspond to concrete contents of respective items:

I. Design variables ··· Attribute variables such as link lengths, joint positions, that represent the design entity.

II. Geometric and static characteristics ··· Features that are calculated from kinematic relationships in some link trajectories and statically balanced attitudes.

III. Quasi-dynamic characteristics ··· Features that are calculated from quasi-dynamically balanced attitudes corresponding to steady driving modes.

IV. Driving mode characteristics ··· Physical features that are calculated from dynamic simulations of several driving modes.

V. Subjective driving feeling ··· Subjective measures against individual physical characteristics.

VI. Integrated total objectives ··· Indexes integrating some subjective measures.

Among these levels, the items from II to IV can be principally defined by mathematical means, but some of the items from III to IV require complicated simulation codes and expensive computation for their evaluation. On the other hands, the items from V to VI are related to so-called KANSEI measures, that may be called as human-oriented sensitivity, and they are not suitable for mathematical operation in general.

3.4 Mathematically Operational Design Model

The consequence of ISM based hierarchical structure reveals the range where the mathematical optimization technique can be applicable.

As for the relationships between individual items shown in Fig. 1 and computer simulations or analysis calculations, each bracketed number in the figure corresponds to the items that can be obtained through a series of analysis with relatively less computational efforts. Such analytic operations are as follows:

[1]: Wheel-stroke analysis for the front suspension.

[2]: Wheel-stroke analysis for the rear suspension.

[3]: Rack-stroke analysis.

[4]: Steady-state circular turning analysis by means of cross-sectional equilibrium analysis of a full vehicle for centrifugal force.

[5]: Frequency response analysis against road noise with a simplified vibration model of a full vehicle.

[6]: Transient vibration analysis against a single bump with a simplified vibration model of a full vehicle.

As shown Fig. 1, some part of these corresponds to the replacement of the items in level IV, except the items related to [5] and [6], while the items in level IV relate to various real vehicle experiments and dynamic computer simulations. The items of [5] and [6] can be evaluated through the simplified vehicle model that is a vibration model with springs and dampers and that does not include suspension geometry. Consequently, the entire set of design items that can be obtained by [1] to [6] is a combination of analysis operations that is appropriate for the design optimization calculation related to all aspects of handling ability, stability and comfortability. Thus, the constraints and objective functions can be formulated based on the above simulation analysis for multi-link suspension optimization, and an appropriate set of design variables should be selected so as to suit for mathematical optimization calculation. These efforts must result in a complete set of object model for the design optimization.

4 FORMULATION OF MULTI-LINK SUSPENSION DESIGN PROBLEM

This section shows the formulation of an optimization problem of a multi-link suspension system. When

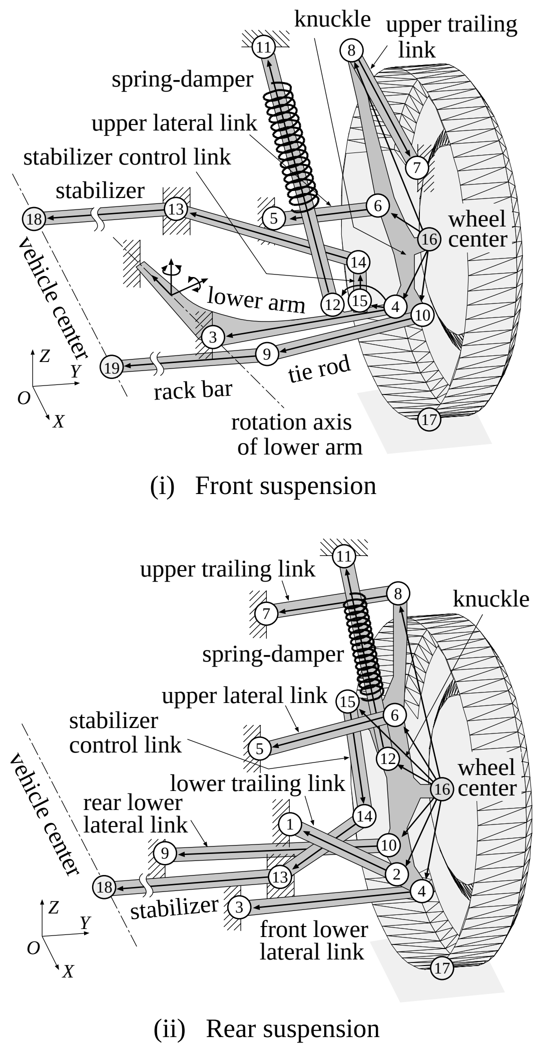

Figure 2 Suspension geometry

applying any design optimization procedure, there are, for instance, some options on the selection of a set of design variables for representing the design object due to the inter-dependency between design variables. However, which design variables are directly manipulated though an optimization algorithm is practically important for efficient optimization calculation, and some cares are necessary for avoiding the inadequate formulation that may fail into missing optimization.

4.1 Multi-Link Suspension System

Figure 2 shows the configuration of front and rear multi-link suspension systems that is optimized in this paper, respectively. As aforementioned, while a multi-link suspension system is a parallel six-bar universal linkage, its practical configuration must be obeyed with stroke motion of a wheel and it includes a stabilizer and a spring-damper system. When considering the realization of suspension functionality, it must be effective to generate superior designs by mathematical means, since the link sizes and joint positions are delicately interconnected on the functions.

4.2 Design Variables

The design variables are introduced from link geometry, spring-damper coefficients and stabilizer stiffness, since these are directly related to suspension functions. Besides, while the cornering characteristics of tires are so important for superior drivability, the tires are not considered as design variables to focus on the suspension systems themselves in this study.

Among the design variables, there are several options on how to represent the link geometry. For instance, we can independently determine the geometries of all links and the positions of all joints. However, in this case there are some cases where they cannot be assembled as a suspension system or where the wheel orientation is not acceptable. For avoiding these situations, we first give the wheel position and orientation under the statically balanced situation as constants, then define the relative positions of characteristic points of the whole suspension system over respective chains of joint points, that are shown with arrows in Fig. 2, from the whole center (Joint ⑯), under the statically balanced situation, and use such relative positions as the design variables. For instance, the body-side joint position of the front spring-damper, i.e., ⑪, is represented as the vector sum of the positions of ④ from ⑯, ⑫ from ④ and ⑪ from ⑫. Besides, as for the body-side joint ③ of the front-wheel lower arm, since the joint is not a universal joint with three degrees of freedom but a revolute joint with one degree of freedom, the orientation of its joint rotation is taken as design variables. As a result, we define 39 design variables for joint positions of ③ to ⑮ and two orientation variables of the lower arm for the front suspension and 45 design variables for joint positions of ① to ⑮ for the rear suspension, i.e., 86 design variables for suspension geometry. Further, we introduce six design variables for spring stiffness coefficients and damping coefficients of spring-dampers for both front and rear suspensions and stabilizer stiffness for both front and rear suspensions. The total number of design variables reaches to 92 under the above definition.

4.3 Objective Functions

The following eleven items are selected as objective functions, that can be calculated through six analytic operations [1] to [6], from the hierarchical structure of design items shown in Fig. 1:

(1) The transitions of toe angle and camber angle under the wheel stroke of both front and rear suspensions are close to zero, (two objectives for the front and two for the rear; calculated by [1] and [2]).

(2) The roll center height under the statically balanced situations are close to the preferable values, that are assigned from marketing viewpoint and vehicle characterization, respectively for the front and rear suspensions, (one objective for each one; calculated by [4]).

(3) The role angle of the body under the steady-state circular turning analysis for centrifugal force at 0.5 G, where G is gravitational acceleration, is close to 3 degree, (one objective; calculated by [4]).

(4) The centrifugal acceleration effect coefficient under steady-state circular turning analysis is close to a preferable value, that is also assigned from marketing viewpoint and vehicle characterization, (one objective; calculated by [4]).

(5) The minimum difference between the vertical vibration gain against stationary road noise from 1 Hz to 30 Hz and ISO measure on 8 hours' comfortability limit is large as possible, (one objective; calculated by [5]).

(6) Under the transitional vibration response corresponding to the projection passing, the maximum peak-to-peak value after the vehicle is passed a projection is small enough, (one objective; calculated by [6]), and the period for 50 percent reduction of vibration amplitude is also short enough, (one objective; calculated by [6]).

4.4 Constraints

The following constraints must be considered so as that the suspensions can physically consist:

(1) The front and rear suspensions must fit within the wheel houses, respectively. That is, every joint position is within the certain upper and lower constraints. These bounds correspond to the wheel houses respectively, (78 constraints for the front and 90 constraints for the rear).

(2) The geometry of the front lower arm must be within a certain range. That is, the joints ⑫ and ⑮ must exist between the joints ③ and ④, (four constraints).

(3) Each suspension must be movable for afore-specified range of both vertical stroke and rack value. That is, there are statically balanced states against all afore-specified stroke ranges of the wheel stroke analyses and rack stroke analysis, (four constraints for the front and two constraints for the rear; calculated by [1], [2] and [3]).

Further, the following constraints must be considered as minimum levels of performance items, since they are not taken as the objective functions:

(1) The kingpin offset and toe angle under statically balanced situations are within the acceptable ranges, respectively, (four constraints for the front and two constraints for the rear).

(2) The orientations of wheels are close to the vertical position against the road surface. That is, the outside wheel has negative camber under the rack stroke situation, (one constraint; calculated by [3]).

(3) The difference between the outside wheel and inside wheel is kept against the maximum rack angle for turnability. That is, the toe angle of the inside wheel is larger than the toe angle of the outside wheel against the maximum rack angle of the rack stroke analysis, (one constraint; calculated by [3]).

(4) The under-steer characteristic must be ensured. That is, the rack value is monotonically increased against the increase of turning velocity under the steady-state circular turning analysis, (one constraint; calculated by [4]).

(5) Steady-state circular turn is possible to afore-specified velocity. That is, the vehicle can have stationary balanced situations within an afore-specified range of centrifugal acceleration under the steady-state circular turning analysis, (one constraint; calculated by [4]).

(6) The period until the transient vibration is settled is within an acceptable range, (one constraint; calculated by [6]).

The total number of constraints, all of which are inequalities, is counted as 189.

4.5 Mathematical Characteristics

The optimization problem that is formulated with the above design variables, objective functions and constraints is a multi-objective nonlinear constrained optimization problem in a real number space. The distinguishing points of this problem are that the number of design variables are so large since the multi-link suspension geometry as shown in Fig. 2 are optimized together, that the number of constraints are also large, and that the constraints are susceptible to the fine changes of design variables, since most of them are related to geometric feature variables of complicated suspension configuration. All of these are considered to be disadvantages for application of ordinary optimization techniques based on gradient information. Thus, this paper applies the genetic algorithm based optimization method for complicated mechanical systems design

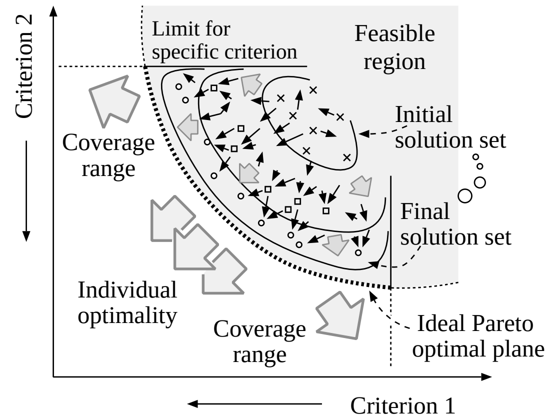

Figure 3 GA based multi-objective optimization

problems (Fujita *et al.*, 1998) to such a suspension design problem.

Besides, we assume that the optimization problem can be represented as the following form with a design variable vector $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T$ for the latter discussion:

$$\left.\begin{array}{lllll} \text{minimize} & f_i(\boldsymbol{x}) & & & (i = 1,2,\cdots,r) \\ \text{subject to} & h_j(\boldsymbol{x}) & = & 0 & (j = 1,2,\cdots,p) \\ & g_k(\boldsymbol{x}) & \le & 0 & (k = 1,2,\cdots,q) \\ & x_l^L \le x_l \le x_l^U & & & (l = 1,2,\cdots,n) \end{array}\right\} \quad (1)$$

This form considers the side constraints with lower bounds x_l^L and upper bounds x_l^U for design variables specifically. It also includes equality constraints for generality.

5 GENETIC ALGORITHM BASED MULTI-OBJECTIVE OPTIMIZATION

The genetic algorithms (Goldberg, 1989) are the probabilistic group-based optimization method that originated in the analogy to national selection. While their abilities against various difficulties in optimization problems have been proven through their past applications, a different optimization problem requires another configuration of a genetic algorithm that are suitable for its characteristics in some cases.

The genetic algorithm that is applied to the optimization problem fundamentally follows Simple GA described by Goldberg (1989), and we introduce several extensions to it for the constrained multi-objective optimization problem of the complicated real design problem (Fujita *et al.*, 1998). Figure 3 shows the concept of the method. The method aims to converge a set of solutions to Pareto optima by means of individuals in genetic algorithms. For this purpose, the algorithm

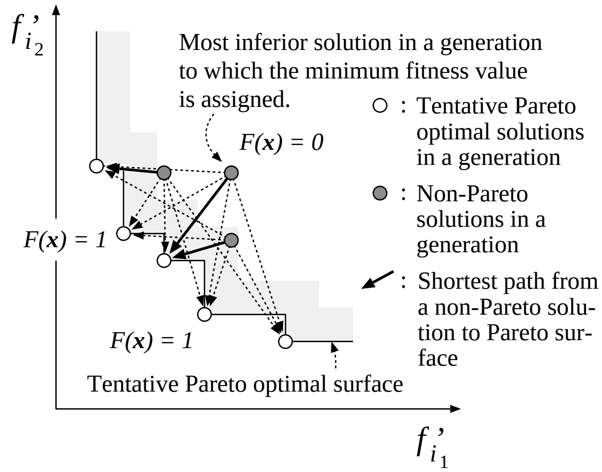

Figure 4 Fitness function for multiple objectives

should have the abilities for both optimizing each solution and evenly distributing whole solutions within the feasible and acceptable region.

The following subsections describe the extensions to Simple GA introduced in the optimization method for multi-objectives, diversification of solutions and real design variables.

5.1 Fitness function

While an objective function is used as a fitness function for single-objective optimization problems with several scaling techniques, this optimization method uses the distance from the tentative Pareto surface in each generation as the fitness function. Before calculating the fitness function, constraints are included into all objective functions as penalty terms with the following equation:

$$\begin{aligned} f'_i(\boldsymbol{x}) &= f_i(\boldsymbol{x}) + p_E(t)\sum_{j=1}^{p}\left|h_j(\boldsymbol{x})\right|^2 \\ &\quad + p_I(t)\sum_{k=1}^{q}\{\max(g_k(\boldsymbol{x}),0)\}^2 \end{aligned} \quad (2)$$

Where, $p_E(t)$ and $p_I(t)$ are the penalty coefficient functions for equality and inequality constraints, respectively, that are grown as the generation t gains.

Figure 4 shows the concept on how to calculate the fitness function $F(\boldsymbol{x})$ from $\boldsymbol{f}'(\boldsymbol{x})$. In the figure, all Pareto solutions take the best value $F(\boldsymbol{x}) = 1$, the worst non-Pareto solution takes the worst value $F(\boldsymbol{x}) = 0$, and the other take intermediate values corresponding to how far it is from the tentative Pareto surface, respectively (Osyczka and Kundu, 1995).

Further, the fitness function is adjusted as $F'(\boldsymbol{x})$ through σ truncation and linear scaling to prevent premature convergence, and the resulted values are arranged to $F''(\boldsymbol{x})$ based on the crowdedness of solutions to obtain distributed Pareto solutions with the following equations:

$$F''(\boldsymbol{x}_i) = \frac{F'(\boldsymbol{x}_i)}{nc_i} \quad (3)$$

$$\begin{cases} nc_i = \sum_j sh\left(d\left(\boldsymbol{f}'(\boldsymbol{x}_i), \boldsymbol{f}'(\boldsymbol{x}_j)\right)\right) \\ sh(d) = \max\left(0,\ 1 - \dfrac{d}{\sigma_{share}}\right) \end{cases} \quad (4)$$

Where, nc_i is the niche count and σ_{share} is the niche size. The distance d is Euclidean norm in the objective function space, $\boldsymbol{f}'(\boldsymbol{x})$. $\sum_j$ means the sum across the solutions in a generation. The niche size σ_{share} means the region of Pareto surface that the individual solution should stand for, and it is determined with a revised method from Fonseca and Fleming (1993)'s method with the following equation:

$$N = \frac{\prod_{i=1}^{r}\left(f'_{i_{max}} - f'_{i_{min}} + 2\dfrac{\sigma_{share}}{\alpha_{share}}\right) - \prod_{i=1}^{r}\left(f'_{i_{max}} - f'_{i_{min}}\right)}{\left(2\dfrac{\sigma_{share}}{\alpha_{share}}\right)^r} \quad (5)$$

Where, $f'_{i_{max}}$ and $f'_{i_{min}}$ are the maximal and minimal values of $f'_i(\boldsymbol{x})$ in each generation, and N is the number of solutions in a generation. α_{share} is a revising coefficient against the excessive arrangement for sharing distribution $(0 \le \alpha_{share} \le 1)$.

5.2 Similarity based selection

Since the optimization problem has a lot of design variables and they are inter-related in a complicated way, the random mating of individuals may generate the solutions that violate constraints or that are inferior in objectives. To avoid this, pairs for crossover operation are selected based on similarity between each pair in addition to the fitness function.

Figure 5 shows how to calculate the selection probability for a pair of solutions, not for a solution, where C_1 and C_2 are the coefficients for controlling the probability. In the figure, the weighting factor w_{ij} for selection probability of a pair of solutions is introduced based on the distance d_{ij} between the solutions, and the resulting selection probability for the pair is determined as $w_{ij}F''_iF''_j$ with the fitness values, F''_i and F''_j of respective solutions. Finally, selection

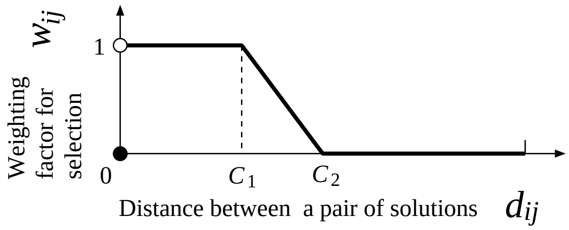

Figure 5 Mating probability for diversification

is performed for all possible pairs of solutions with the remainder-stochastic-sampling-without-replacement strategy (Goldberg, 1989).

5.3 Life span limit for Pareto solutions

The 'elitist plan' is introduced for tentative Pareto optimal solutions, but the generation number where each solution is preserved is limited within a constant number T for preventing that a generation is almost occupied by preserved solutions (such situation can easily occur for the cases with large number of objectives).

5.4 Real number direct coding and crossover

The design variable vector is used as a coding method for the genetic algorithm in order to insure preciseness in optimized design variables. The crossover for real numbers is performed by interpolating parent solutions with interpolation ratio that is randomly generated under a normal distribution $N(0,\sigma^2)$ (Furukawa and Yagawa, 1995). Further, since the design variables in mechanical systems design problems are often restricted within the side constraints, a monotonously increasing threshold mapping is used between design variables and intermediate variables for such restriction.

The crossover operation from two design variable vectors $\boldsymbol{x}^{\alpha}$ and $\boldsymbol{x}^{\beta}$ in a parent generation to two ones $\boldsymbol{x}^{\gamma}$ and $\boldsymbol{x}^{\delta}$ in a child generation is performed with the following linear interpolation equation over the intermediate variables translated with the sigmoid function $Sig_i(\widehat{x}_i)$:

$$\left\{ \begin{array}{lll} x_i^{\gamma} & = & Sig_i\big(\quad \mu_i \quad \cdot Sig_i^{-1}(x_i^{\alpha}) \\ & & \quad + (1-\mu_i)\cdot Sig_i^{-1}\left(x_i^{\beta}\right)\big) \\ x_i^{\delta} & = & Sig_i\big((1-\mu_i)\cdot Sig_i^{-1}(x_i^{\alpha}) \\ & & \quad + \quad \mu_i \quad \cdot Sig_i^{-1}\left(x_i^{\beta}\right)\big) \end{array} \right. \tag{6}$$

$$Sig_i(\widehat{x}_i) = \frac{x_i^U + x_i^L \exp(-\widehat{x}_i)}{1 + \exp(-\widehat{x}_i)} \tag{7}$$

Where, $x_i^{(\bullet)}$ is the i-th design variable of a solution $\boldsymbol{x}^{(\bullet)}$ in the generation, and μ_i is an interpolation coefficient that is randomly generated for each design variable under a normal distribution $N(0,\sigma^2)$. This crossover operation can be arranged by standard deviation σ and crossover probability P_c.

Besides, since this crossover includes the characteristic of mutation, the algorithm does not include any mutation.

6 COMPUTATIONAL EXAMPLE OF DESIGN OPTIMIZATION

This section shows a computational example of the design optimization of multi-link suspension system toward totally superior performance of drivability such as handling ability, stability and comfortability.

The setting for optimization calculation is as follows: The number of individuals is 100. The penalty coefficient function for inequality constraints is $p_I(t) = 100 \times 2^{\frac{t}{10}}$, while the formulation includes no equality constraint. The revising coefficient for sharing α_{share} is 0.3. The parameters for similarity-based selection are that C_1 is equal to the average distance across all possible pairs of solutions in design variable space minus its standard deviation, and C_2 is the average distance, in respective generations. The life span limit T is 10 generations. The crossover probability is $P_c = 1.0$. The standard deviation for crossover operation is $\sigma = 0.5$.

6.1 Convergence History

Figure 6 shows the optimization history where a set of individuals is converged into the Pareto solutions as the generation of the genetic algorithm is proceeded. While the optimization problem has eleven objective function as aforementioned, the figure shows with the weighted sums of three categories of items; the items related to analytic operations [1] and [2], the items related to analytic operation [4], and the items related to analytic operations [5] and [6] ([3] relates only to constraints). The categories are called as '*Straight running stability*,' '*Turnablity*' and '*Comfortability*,' respectively in the following. In the figure all objectives are translated to be minimized, and non-Pareto solutions are eliminated. In the optimization history, it can be confirmed that the individuals are going to close to ideal Pareto solutions generation by generation in the early generations, while they slightly tend to gather to a central spot. It is also seen that after these situations the individuals are improved to spread to wider range of Pareto solutions in the late generations.

Besides, we examine the effects of similarity based selection, life span limit and so forth by the comparison with the optimization calculation without such effects. The comparison ascertains their effects

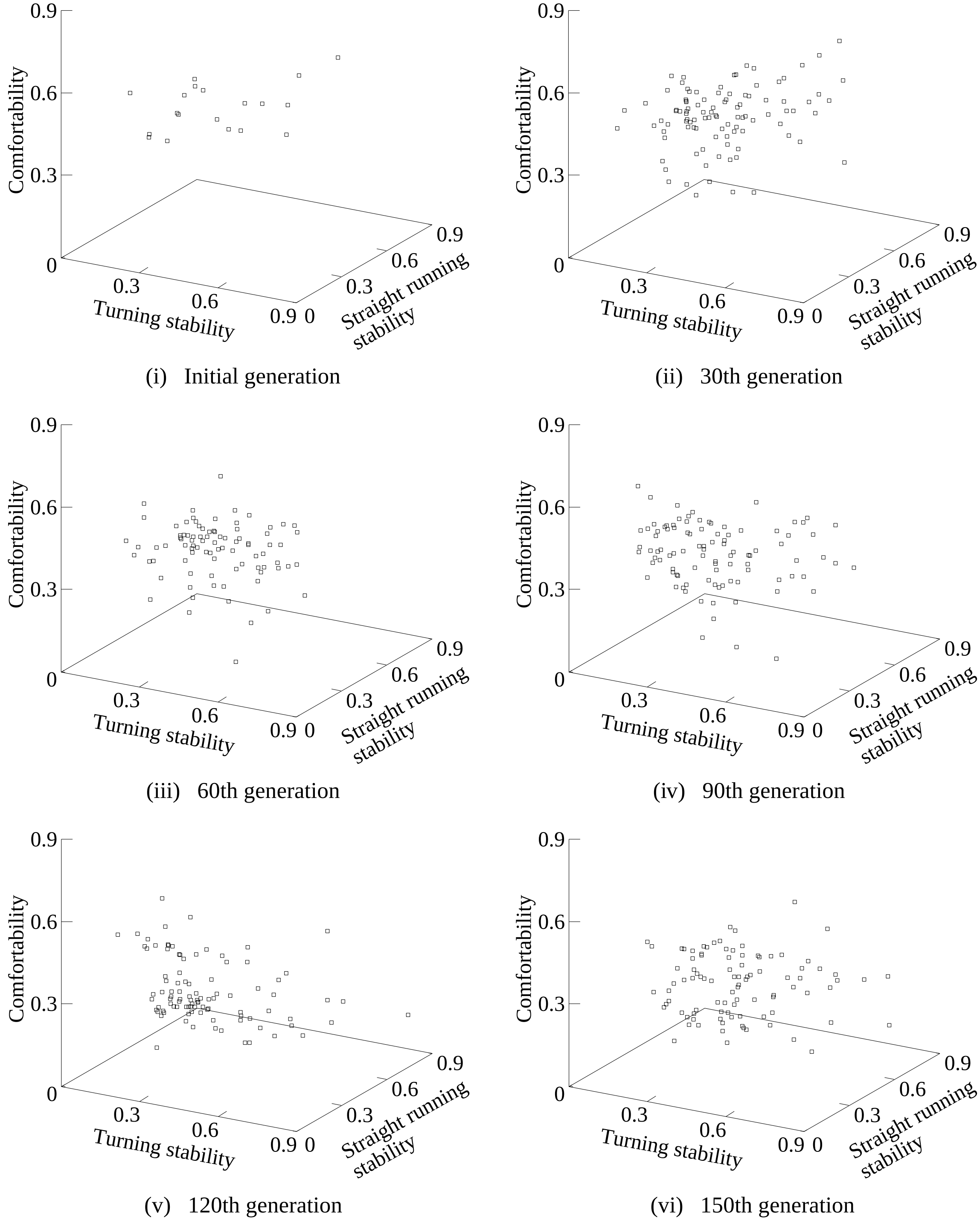

Figure 6 Optimization history

for superior optimization performance on complicated multi-objective optimization problems.

6.2 Pareto Solutions

The above result shows that the genetic algorithm based optimization method (Fujita *et al.*, 1998) is effective for the mathematically operational part of the suspension design problem, and that a relevant set of Pareto

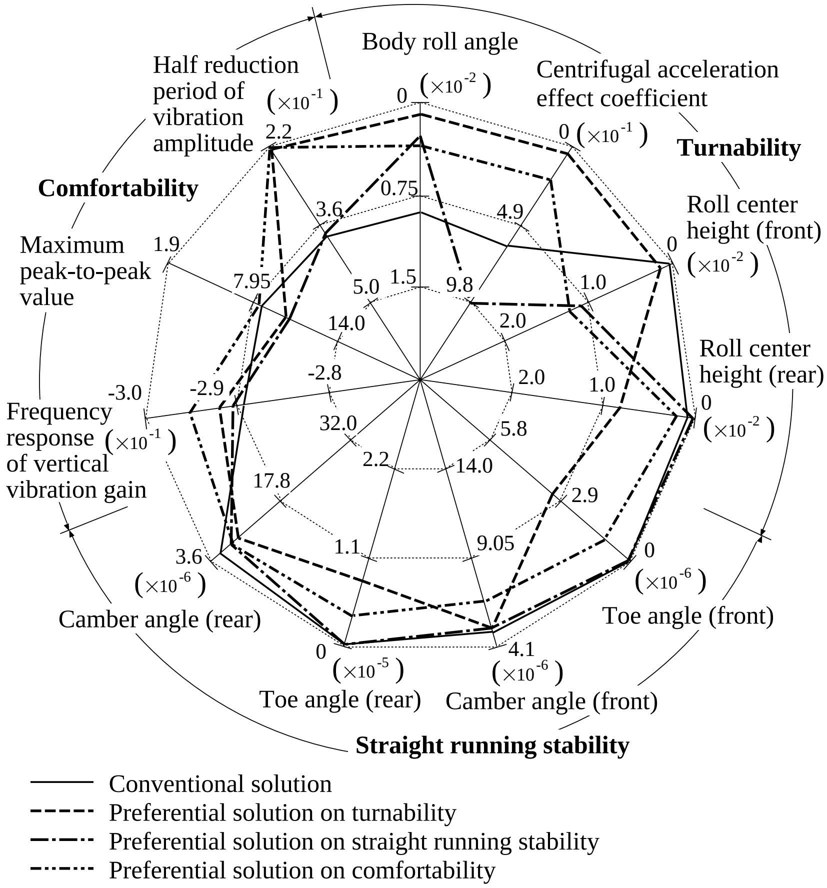

Figure 7 Several Pareto optimal solutions

optima can be gotten with it. After such solutions are obtained, it is necessary to select a preferable solution from a set of Pareto optima for the realization of a design. Figure 7 shows a relative comparison of several representative Pareto optima that are selected from the result shown in Fig. 6 (vi), where the outside of the chart indicates to be superior on respective objectives. The tendency of the gotten Pareto solutions, in addition to ones shown in Fig. 7, shows that there is a tradeoff between turnability and straight running stability and that there is no significant tradeoff between comfortability and the others. It also shows that the conventional design is relatively close to the preferential solution on straight running stability.

While the final solution must be selected by a designer from this kind of understanding of design tendency such as tradeoff among objectives, it must be valid and effective to provide designers essential information on design optimality of a complicated engineering system through a sequence of the ISM based systematic formulation of the optimization problem and the genetic algorithm based optimization calculation for seeking a relevant set of Pareto optima.

7 SUMMARY

This paper discussed the optimal design problem of a vehicle using multi-link suspension system, and proposed an ISM based systematic structurization procedure and a genetic algorithm based optimization method for a class of complicated engineering system where the problem belongs. The hierarchical arrangement of the problem was useful for formulating the optimization problem in a mathematically appropriate form, and the computational example showed that the genetic algorithms are robust enough for complicated optimization problems and that some extensions introduced in this paper are novel toward superior optimization performance. The overall procedure for design optimization that was applied to multi-link suspension systems must be useful as a good reference for the introduction of systematically rational design approach to other complicated design problems.

ACKNOWLEDGMENTS

This research is partially supported by “Research for the Future” Program of the Japan Society for the Promotion of Science (JSPS) under "Methodology for Emergent Synthesis" Project (project number 96P00702).

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