THE MATHEMATICAL LEGACY OF ANCIENT EGYPT - A RESPONSE TO ROBERT PALTER

by
Beatrice Lumpkin
Chicago City Colleges, retired

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The Mathematical Legacy of Ancient Egypt - A Response to Robert Palter

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Martin Bernal has credited my work in multicultural mathematics with an insistence on the profound influence of racism within conventional scholarship.[1] In fact I believe that every facet of life in the United States suffers from the profound influence of racism. So in 1968, when my mathematics students in the Chicago City Colleges demanded "Black Studies," I was sympathetic to their demands. But if they (or other students) had not made demands, I probably would have continued to teach in the old way. The English literature teacher, next door to my classroom, was unwilling to add so much as a single African American author to the reading list. Students began to picket her room, charging that African and African American achievements had been omitted from the curriculum.

Mathematics and "Black Studies" in the 60's

I found nothing in the mathematics textbooks that could help me meet my students' demands to include something from their heritage in my classes. Although I also had a degree in history, my college courses never touched on African or African American contributions to mathematics, or to any other subject for that matter. I could not teach what I did not know. Since African American students were proud of the contributions of ancient Egypt, I began my search by reading about the mathematics of that ancient civilization. My efforts were rewarded with a lot of good material for my classes. I found that almost every history of mathematics had an opening chapter on ancient Egypt. However, I was saddened to find derogatory statements in some of the standard histories of mathematics. Some examples are:

A. Neugebauer: "Ancient science was the product of a very few men; and these few happened not to be Egyptians."[2]

Morris Kline: (Compared to the Greeks) "The mathematics of the Egyptians and Babylonians is the scrawling of children just learning how to write, as opposed to great literature."[3]

Florian Cajori: "Thus a Semitic race was, during the Dark Ages, the custodian of the Aryan intellectual possessions" (referring to Islamic development of mathematics).[4]

My students' complaint about bias in the curriculum was justified. The history of mathematics was only a special case of the general omission of African achievements. In archaeology, Alison Brooks, who has made significant finds in Central Africa, told *Science*, "A lot of people don't want to ascribe any independent discoveries to Africa."[5] One of the most valuable sections of *Black Athena*, in my opinion, is the documentation of the changes in European and North American historiography that accompanied attempts to justify slavery.

Although slavery ended, racism continued, as a new era of European exploitation of Africa, Asia, and Latin America began. Racism in the United States continued to have an economic base by guaranteeing a large pool of low-paid labor. What Bernal calls the "Aryan Model" of history is still in place, slightly broadened to include "Phoenicians," the State of Israel, and Babylonians (although the State of Iraq is out of favor.) The "Aryan," and "Broad Aryan" Models are described by some writers as "Eurocentrism," for example, in the work of George G. Joseph, the Syrian Indian-born historian of mathematics.[6] The African Centered Curriculum movement developed, in part, to counteract Eurocentrism. But more on that later.

Palter Joins the Detractors of Ancient Egyptian Science

Black Athena seems to have touched a nerve, especially among those who refuse to admit possible influence of racism among scholars. Jacques Berlinerblau, writing in the *Nation* has characterized criticism of the books as ranging from polite to homicidal.[7] Although so much highly critical material had been written about *Black Athena,* Robert Palter thought that something was missing which he could supply. He notes:

> "I have discovered almost nothing on the history of science. It is true that in these volumes Bernal makes only scattered comments concerning science (all in *BA*, i), but the general drift of his views is clear enough ..."[8]

For the next 60 pages of his "*Black Athena*, Afro-Centrism, and the History of Science", Palter discusses a wide range of subjects including my article on "Mathematics and Engineering in the Nile Valley,"[9] and my note on "The Egyptians and Pythagorean Triples."[10] I welcome this opportunity to respond. In general, Palter follows in the direction of the derogatory quotations given above and his discussion of ancient Egyptian mathematics contains nothing new. For the purpose of discussion, we can accept Palter's restatement of Bernal's position on Egyptian science:

> "First Bernal maintains that there were scientific elements in Egyptian medicine, mathematics, and astronomy long before there was any Greek science at all, and, second, he maintains that Egyptian medicine, mathematics, and astronomy critically influenced the corresponding Greek disciplines."[11]

Evidence for Egyptian influence on Greek mathematics has been cited by many writers, including Sir Thomas Heath, the distinguished historian of Greek and Alexandrian mathematics.[12] That will not be the focus of this paper. Instead, the rest of this discussion will examine "the scientific elements" in ancient Egyptian mathematics from which the early Greeks had much to learn.

Scientific Elements in Egyptian Mathematics

Palter does not define "scientific elements," nor does he define "mathematics." For the field of education, the National Research Council described mathematics as "a science of pattern and order."[13] This definition is certainly broad enough. As an alternative to providing a definition of mathematics, Joseph offers a helpful description from which we excerpt:

> "Mathematics has developed into a worldwide language with a particular kind of logical structure. It contains a body of knowledge relating to number and space and prescribes a set of methods for reaching conclusions about the physical world. And it is an intellectual activity which calls for both intuition and imagination in deriving proofs and reaching conclusions."[14]

Egyptian Numerals

Palter begins his put-down of Egyptian mathematics with his discussion of the Egyptian numerals. "The important point here is what the Egyptians did *not* do: they did not use a so-called place value notation."[15] I suggest that what a culture "did *not* do" is an inappropriate way to open a discussion of mathematics history. Palter seems to have missed the important point on what the Egyptians *did* do with their numerals.

In contrast to Palter, the eminent historian Carl Boyer concluded that, "The introduction by the Egyptians of the idea of cipherization constitutes a decisive contribution to the development of numeration and is in every way comparable in significance to that of the Babylonians in adopting the positional principle."[16] The introduction of cipherization replaced tally marks with abstract symbols for numerals. Tallies were cumbersome because they had to be repeated until the count reached the desired value. The hieroglyphic numerals used abstract symbols for powers of 10, but these symbols still had to be repeated and counted. For example, 1 was the symbol for 10 but had to be repeated to show 90 as 111111111. Babylonian numerals used vertical tallies for units and horizontal tallies for tens, repeating tallies as needed. For example, 49 would be shown in Babylonian numerals as:

< < 1 1 1 1 1
< < 1 1 1 1

The big advantage of positional value, as introduced by the Babylonians, was its extension to fractions, known as sexagesimal fractions because the Babylonian base was 60.

Most of the surviving papyri are written in a cursive form of the hieroglyphs that we now call "hieratic," a script that came into use at a very early date.[17] The hieratic numerals introduced a new principle for the representation of numbers. Hieroglyphic numerals used different symbols for each power of 10 up to a million. These symbols were grouped and repeated as needed to represent a value. On the other hand, hieratic numerals used single symbols or ciphers, as Boyer wrote, "…for each of the first nine integral multiples of integral powers of ten." He called the hieratic numerals system, "decimal cash-register cipherization," referring to old-style cash registers which sent up a flag for each decimal place.[18]

The following example from problem 79 of the Rhind Mathematical Papyrus, written by the scribe Ahmose, illustrates the difference between hieroglyphic and hieratic numerals. The value that we write as 19,607 requires 23 hieroglyphs, but the hieratic needs only 4 symbols. The hieroglyphic numerals were written by a modern Egyptologist as an aid in understanding the hieratic, but the scribe used only hieratic in the papyrus. To illustrate the different principles involved in hieroglyphic, hieratic, and Babylonian numerals, the Babylonian cuneiform numeral for 19, 607 is also shown.[19]

insert figure 1.

Boyer suggested, "It would be interesting to speculate on the part which the flexibility afforded by Egyptian writing materials may here have played in making available new symbols with which to establish the principle of cipherization, and to compare this with the extent to which in Mesopotamia a characteristic inflexibility, with a corresponding need for an economy of symbols, may have operated to lead to the other great principle of numeration, local value."[20] Here Boyer contrasted the flexible brush and papyrus writing materials of the Egyptians with the inflexible stylus and clay tablets of the Mesopotamians. In their time, the flexible Egyptian materials were more elegant and certainly more portable than the clay tablets of Mesopotamia. Unfortunately, papyri were subject to decay, and were even burned for fuel in relatively recent times. It appears that we shall never gain as detailed a knowledge of ancient Egypt as we will learn about Mesopotamia when the millions of clay tablets are translated.

It is interesting to note that the earlier Attic numerals, adopted in Greek cities around 500 BCE, used the same repetitive principle as the Egyptian hieroglyphic numerals.[21] Attic numerals then gave way to the Ionian numerals that used the same cipherization principle as Egyptian hieratic numerals. The thrust of Boyer's article in *Isis* was to "rehabilitate" the Ionian numerals by showing that they were not inferior to Babylonian numerals which had positional value but did not use ciphers. Boyer believed that Ionian numerals were patterned on Egyptian hieratic numerals and may have had a direct influence on the development of the Hindu-Arabic (or Indo-Arabic) numerals.[22]

Egyptian Fractions

Palter questions my statement that Egyptian unit fractions continued in use for thousands of years, up to the modern period.[23] Except for 2/3, Egyptian fractions were always stated as a sum of unit fractions with numerators of 1. For example, 3/4 was written as 1/2 + 1/4. Today we would prefer to work with decimals. But for those early times, the development of unit fractions in ancient Egypt was a big advance, a way to refine measurement, and a way to express the results of division. In practice, scribes in ancient Egypt were able to perform any needed computations with accuracy. In Alexandrian Egypt, astronomers used the Babylonian sexagesimal fractions for calculation but still expressed their results in unit fractions.

Egyptian scribes had good reason to pride themselves on their astounding ability to perform complex operations with fractions. In the proof for problem 33 of the Ahmose Papyrus, for example, the scribe took 2/3 of the value 16+ 1/56+1/679+1/776, and got 10 2/3+1/84+1/1358+1/4074+1/1164. Then the scribe added 1/2 and then 1/7 of 16+ 1/56+1/679+1/776, and showed that it all added to 37.[24] Any reader who tries this computation will gain new respect for the scribes. In practice, tables of fractions lightened the load. With the aid of their fractions, the Egyptians built their monuments with a precision that was not equaled for thousands of years.

When Gillings wrote that Egyptians had "no means of writing even the common fraction p/q,"some misinterpreted this to mean that the scribes could not compute with common fractions. I wondered about reading a length of 13/16 finger on a cubit stick. The cubit is divided into 7 palms, and each palm into 4 fingers. A cubit stick in the Louvr has divisions of the finger, starting with 1/2, 1/3, 1/4, down to 1/16 of a finger. Suppose a cubit stick was used to measure a length of 2 cubits, 3 palms, and 13/16 of a finger. How would the ancient Egyptians have read 13/16 of a finger? Would they count the sixteenth marks, going from 1 to 13? Then what would they call the reading? I sent the question to Robins and Shute (private communication). Professor Shute's reply was that an Egyptian would have noted that the endpoint on the rod was closer to the 1 finger than to the 0 finger mark. The scribe would have counted back 3 subdivisions to get to 13. He would have immediately known that 13 could be partitioned into 8 + 4 + 1. Then he would have given the reading in fingers as 1/2 + 1/4 + 1/16. Shute added: "The Egyptians did in fact use fractions with numerators greater than unity in the course of calculations. They did not have a conceptual block against this, but chose to express the end result in terms of unit fractions, possibly for aesthetic reasons."

There are many examples in the Ahmose papyrus of computations with the equivalents of common fractions. The scribal technique for addition of fractions was basically the same as ours, by use of a common denominator and finding the "new" numerators for the equivalent fractions. The scribe used red ink for the "new" numerators. See, for example, Ahmose problem 7.[25]

Why unit fractions persisted so long, I do not know. But then I don't understand why the United States is still mired in common fractions when the rest of the world has adopted metric. No doubt unit fractions were considered elegant in their time. Heath writes that 'The Greeks had a preference for expressing ordinary proper fractions as the sum of two or more submultiples; in this they followed the Egyptians who always expressed fractions in this way, ..."[26] It is true that later Alexandrian astronomers used the Babylonian sexagesimal fractions for calculations.[27] Heron of Alexander, c 100 CE, "writing for the common man, seems to have preferred unit fractions. ... The old Egyptian addiction to unit fractions continued in Europe for at least a thousand years after the time of Heron."[28] As late as Leonardo of Pisa's

Liber abaci, 1202 CE, tables are provided for conversion from common fractions to unit fractions. As Boyer said, "Fibonacci evidently was fond of unit fractions - or he thought his readers were..."[29]

Richard J. Gillings

Some of the pioneering work on Egyptian fractions was accomplished by Richard J. Gillings. He analyzed number theory that the scribes used to find the most elegant decomposition of common fractions into unit fractions. For example, a commonly used identity was $1/(2n) = 1/(3n) + 1/(6n)$. Gillings also wrote the first book-length treatment of Egyptian mathematics.[30] Gillings' conclusion, in a way a summary of his life's work, was that Egyptian mathematics "reached a relatively high level of mathematical sophistication." Palter uses a single sentence from Gillings' book to not only blunt Gillings' thrust, but to distort it and turn it into its opposite. It is Gillings' second sentence about Egyptian mathematics that Palter liked so much, when he quoted the following paragraph from *Mathematics in the Time of the Pharaohs:*

"...it is not proper or fitting that we of the twentieth century should compare too critically methods with those of the Greeks or any other nation of later emergence, who, as it were, stood on their shoulders. We tend to forget they were a people who had no plus, minus, multiplication or division signs, no equals or square-root signs, no zero and no decimal point, no coinage, no indices, and no means of writing even the common fraction p/q; in fact, nothing even approaching a mathematical notation, nothing beyond a very complete knowledge of a twice-times table, and the ability to find two-thirds of any number, whether integral or fractional. With these restrictions they reached a relatively high level of mathematical sophistication."[31]

Palter said that, "Gillings is attempting to evaluate Egyptian mathematics in its own terms."[32] I would agree, with the addition of, "in its own time." The Gillings approach I would call a "historical" approach. But Palter is troubled because I labeled the following statement by Neugebauer as "unhistoric:" "The role of Egyptian mathematics is probably best described as a retarding force upon numerical procedures."[33] That sentence is how Neugebauer closes his discussion of Egyptian mathematics, a discussion which he opened with his self-proclaimed fact: "The fact that Egyptian mathematics did not contribute positively to the development of mathematical knowledge..."[34] However, as we have seen, there are other respected historians of mathematics who disagree with this negative assessment.

Gillings' overly modest statement about Egyptian mathematics needs some modification. The jacket of the hard-cover edition of his book says that, "Gillings gives examples showing that the Egyptians were able, for example, to solve problems in direct and inverse proportion, to evaluate certain square roots, to introduce the concept of a 'harmonic mean' between two numbers, to solve linear equations of the first degree, and two simultaneous equations, one of the second degree; to find the sums of terms of arithmetic and geometric progressions, to calculate the area of a circle and of cylindrical (possibly even spherical) surfaces, to calculate the volumes of truncated pyramids and cylindrical granaries, and to make use of rudimentary trigonometric functions in describing the slopes of pyramids." This is a fair outline of Gillings' analysis of Egyptian mathematics.

An update: Symbols and The Egyptian Zero

As Gillings shows in his book, the Ahmose papyrus does contain the beginnings of the use of symbols. Egyptians used the word for heap (variously transliterated as aha and hau) for the unknown quantity, much as cosa was used in 15th-16th century Italy.[35] Some symbols were used for addition and subtraction: a pair of feet would walk one number towards another for addition. For subtraction, a pair of feet would walk the number away (RMP 29).[36]

Gillings' statement about Egyptians having no zero also needs modification. It is true that a zero *placeholder* was not used (or needed) in the Egyptian "cash register" decimal system. However, the Egyptians did use a zero symbol for at least two other applications of the zero concept. The first occurred at some Old Kingdom construction sites where the hieroglyph "nfr" was used to label a zero point on number lines that serve as guidelines. For example, a series of horizontal leveling lines were used as construction guides for the Old Kingdom Meidum pyramid. Lines above the zero level were labeled 1 cubit above zero, 2 cubits above zero, and so on. Lines below the zero level were labeled according to the number of cubits below zero.[37] The zero symbol was the triliteral hieroglyph, nfr. This very early use of the concept of directed numbers, where above and below are comparable to positive and negative, deserves more than passing notice.

Figure 2, Number line at Meidum, adapted from Arnold

The same nfr symbol was also used to express zero remainders in a monthly account sheet from the Middle Kingdom, dynasty XIII c. 1770 BCE. It looks like a double entry account sheet with separate columns for each type of goods. At the end of the month, the account was balanced. For each item, income was added; then disbursements were totaled. Finally, total disbursement was subtracted from the total income for each column. Four columns had zero remainders, shown by the nfr symbol.[38] I found it especially interesting that the Egyptians used the same symbol for two different applications of the zero concept.

Egyptologists, such as Borchardt, Petrie, and Reisner all knew of the Egyptian nfr symbol Old Kingdom constructions where it was used to label the zero level in a series of equally-spaced horizontal guide lines.[39] Scharff[40] and Gardiner[41] knew that the Egyptian nfr symbol had been used to show a zero remainder in account books. However historians of mathematics, including Gillings, were probably not aware of the Egyptian zero symbol because it did not appear in the surviving mathematics papyri. Boyer found evidence of the use of zero as a number in a much later Egyptian deed from Edfu:

> "A surviving deed from Edfu, dating from a period 1500 years after Ahmes, gives examples of triangles, trapezoids, rectangles and more general quadrilaterals: the rule for finding the area of a general quadrilateral is to take the product of the arithmetic means of the opposite sides. Inaccurate though the rule is, the author of the deed deduced from it a corollary: that the area of a triangle is half the sum of two sides multiplied by half the third side. This is a striking instance of the search for relationships among geometric figures, as well an early use of the zero concept as a replacement for a magnitude in geometry."[42]

In discussing Mesopotamian mathematics, Ifrah wrote, "...that the notion of □nothing□ was not yet conceived as a number." He adds,

> "In a mathematical text from Susa, the scribe, obviously not knowing how to express the result of subtracting 20 from 20, concluded in this way: '20 minus 20 ... you see.□\' And in another such text from Susa, at a place where we would expect to find zero as the result of a distribution of grain, the scribe simply wrote, 'The grain is exhausted.' "[43]

Perhaps the concept of a zero remainder will one day be discovered in some cuneiform record of commercial accounts in Mesopotamia. Hopefully, other examples will also be found in Egypt, once Egyptologists begin such a search. Incidentally, nfr is also the Egyptian word for good, or beauty. It is interesting to note that the Maya also made a positive association with the concept of zero, using a shell symbol which had good connotations.[44]

Proportions

Some mathematical achievements of ancient Egypt are known only from construction sites or artwork, rather than mathematical texts. Palter restricted his inquiry to “a dozen or so” texts, some of them on pottery fragments. But the use of mathematics was not restricted to texts. Recognition of mathematical thinking outside of school mathematics has opened up the new field of *ethnomathematics,* which is finding evidence of mathematical ability among all people, including societies that were not literate.[45]

The Egyptian sources all reveal highly developed sense of proportional relations. Artists followed a canon of proportions for drawings and sculpture of the human body. Artwork was proportionally enlarged by placing a net of squares over a drawing and making an enlarged copy, square by square. Proportional reasoning was also widely used to solve problems with false position. The method of false position remained in use up to the twentieth century C.E. Ahmose employed false position solutions in problems 24 - 29, 35 - 38, 40, and 76.[46]

Ahmose problem 72 could have been solved by a simple proportion, the method used by the scribe for problems 73 and 75. Possibly for instructional purposes, in problem 72 the scribe extended the proportion of a/b = c/d to get a/c = b/d, and (a-b)/b = (c-d)/d. Gillings concluded that, “...we can only be amazed at such an achievement in 1850 B.C.,□ and suggest that □here perhaps, as the scribe wrote it, we are looking at the very earliest example of rhetorical algebra to come to the attention of the historian of mathematics.”[47]

The scribe also made use of harmonic proportions and did not fall into the trap set for students in the following modern version of Ahmose problem 76:

> For a round trip of 10 miles each way, a driver averaged 20 miles per hour going from home to the destination point, but averaged 30 miles per hour for the return trip. What was the average speed for the whole trip?

The answer is not 25! The correct answer is 24 miles per hour; more time is spent at the slower speed than at the faster speed. The equivalent problem in Ahmose involved an exchange of loaves of bread of different pesu. If 1 hekat of grain made 10 loaves, that bread would be rated □pesu 10□ with each loaf having twice the value of bread ratd pesu 20 (1 hekat of grain stretched to make 20 loaves). The problem calls for an exchange of 1000 loaves of pesu 10 for an equal number of loaves of pesu 20 and pesu 30. The solution is not the arithmetic mean of 25 between 20 and 30, but the harmonic mean, which is 24. The exchange is for 12 loaves of pesu 20 and 12 loaves of pesu 30.

We see here a forerunner of the class of problems that involve adding the reciprocals of the given numbers. Another common example of this type asks for the time required to fill a cistern when two or more pipes bring in fluid at different rates.[48]

Series in Egyptian Mathematics

A papyrus from Kahun, was described by the translator F. Ll Griffith, as, " ... one I fear is beyond hope: it was beautifully written in columns, and still contains the most tantalizing phrase, multiply by 1/2 to infinity ..." This reminded Griffith of Zeno's paradox which ends up with a denial of motion, because one can always travel half the remaining distance but there will still be the other half to cover.[49] Unfortunately, the papyrus breaks off and we do not know what the Egyptian mathematician found who let n increase indefinitely in the sequence $(1/2)^n$. We know that the sequence 1 /2, 1 /4, 1/8, 1/16 ... 1/64 was of great interest to the Egyptians who used these "Horus-eye" fractions to measure grain.

The Egyptian method of multiplication gave the scribes a powerful tool for computation and reveals some of their insights into subjects such as power series, field properties of integers, and inverse operations. First the commutative property of multiplication was used to select the smaller factor as the multiplier. Starting with the multiplicand taken once, multiplier and multiplicand were then successively doubled, probably by adding.[50] That process produced the partial products whose sum made up the total product. The distributive property was used much as we do today except that the multiplier was partitioned into powers of 2 instead of powers of 10. For example, to multiply 32 x 17, multiply 17 x 32:

1	32 /
2	64
4	128
8	256
16	512 /

Now add the first and last products to get

17(32) = 1(32) + 16(32) = 32 + 512 = 544

In this method, the multiplier was expressed as a base 2 number: $17 = 2^1 + 2^4$. After studying the Egyptian method, Curtis concluded that, "The ancient Egyptians were masters of the geometric progression, and constructed their entire system of basic arithmetic operations around it."[51] It is in the work with series and geometry in the Ahmose papyrus, and the second degree equations and geometry in the Moscow and Berlin papyri, that we begin to get a glimpse of the heights reached by ancient Egyptian mathematics.

In Ahmose Problem 64, he asks for a division of 10 measures of grain among 10 men so that there is a constant difference of 1/8 between portions. [52] Ahmose solves the problem by using the equivalent of the modern formula for the sum of an arithmetic series. The Ahmose method also has the virtue of making the derivation of this formula more intuitive. He finds the average share, then adds to it the product of the common difference times half the number of differences. This gives him the last, or highest share and other shares are found by subtraction of the common difference.

Ahmose□s Problem 40 asks for a division of 100 loaves among five persons so there is a common difference. Also, the sum of the highest three must be 7 times the sum of the lowest two. A full explanation of the Ahmose method for problems 64 and 40 is given in the appendix.

The scribe also shows expert knowledge of geometric series in the famous Ahmose problem 79. Some think this problem was the forerunner of Mother Goose's "As I was walking to St. Ives, I met a man with seven wives... ." The problem was theoretical and/or recreational, rather than the purely practical type that some claim was the limit of Egyptian mathematics. The problem was given in tabular form:

houses	7
cats	49
mice	343
spelt (grain)	2401
hekats (1/8 bushel)	16807
Total	19607

The words, "The sum according to the rule," introduce a multiplication that Gilling believes shows deep knowledge of geometric series. Note that 2801 is 1 more than 7 + 49 + 343 + 2401. Gillings describes an inductive process that the ancient Egyptians could have used to get the equivalent of:

$$a + a^2 + a^3 + a^4 + a^5 + ... + a^n = a(1 + a + a^2 + a^3 + a^4 + a^5 + ... + a^{n-1}).$$

1	2801
2	5602
4	11204
Total	19607

It is of more than passing interest that Fibonacci, also known as Leonardo of Pisa, returned to Italy from North Africa with a similar problem about the year 1200:[53]

The Right-Triangle Theorem

The mathematics that developed from the use of grids for architecture and art was studied by Robins and Shute[54] It is hard to make such a study and still hold to the belief that Egyptians knew nothing about the right triangle theorem of $c^2 = a^2 + b^2$.[55] I believe the right-triangle theorem is a better name than Pythagorean Theorem because nothing in the record of his times connects Pythagoras with the theorem.[56] Moreover, other cultures -- Babylonian, Chinese, and Indian, have records that date their use of the theorem to a time earlier than Pythagoras.[57]

Incidentally, Bernal credits me with showing the connection between pyramid sekeds of 5 1/4 and 3,4,5 triangles. (Seked is the Egyptian term for the cotangent of the angle of inclination of the pyramid sides.) The connection itself is valid. Consider a right triangle with sides made up of the horizontal run of 5 1/4 palms, and the vertical rise of 7 palms. Then the hypotenuse would be 8 3/4 palms. The sides of this triangle would be in the ratio of 3:4:5. However, this material should be credited to Robin and Shute and appeared in their *Historia Mathematica* article. The article did cite my note in *Historia Mathematica*[58] for examples of the 3,4,5 triplet in the papyri.

As I remember it, the editors of *Historia Mathematica* published my note based on a recommendation by R. J. Gillings. Unfortunately, Gillings is no longer with us to throw his weight on the side of those who want to keep the question open, “Did the ancient Egyptians know the right-triangle theorem?” My note referred to two problems in the Berlin Mathematical Papyrus. The problems ask for the lengths of the sides of two squares, such that the sum of the areas of these squares is equal to the area of a third, given square. The ratio of the sides of the two small squares is also given. The solution values for the problems are 6, 8, 10 cubits and 12, 16, 20 cubits, in both cases multiples of (3, 4, 5). Unless one takes the view that the right triangle theorem sprang full-blown from the oracle’s head, we can assume that discovery of the theorem was preceded by geometric experimentation. These Berlin Papyrus problems suggest this type of geometric experimentation. Details of the problems are given in Appendix 1.

On the use of the remen to halve the area of a figure, I disagree with Palter's assessment that the evidence is "flimsy." I also disagree with his claim that the remen and double remen were not known to have been used to measure land.[59] The double remen was the length of the diagonal of a 1-cubit square, or $\sqrt{2}$ cubits. The remen equaled half that length, measuring $(1/2)\sqrt{2}$ cubits. The area measure called "remen" was a square of 100 remens x 100 remens, and was half the area of a set (called aroura in Greek.) The *set* was a square of 100 cubits x 100 cubits. Despite Palter's doubts, the remen was indeed used to measure land. F. Ll. Griffith reported that:

"The sign *remen,* at Edfu, for the half aroura is remarkable. We must connect it with *remen*, "upper arm?" of the cubit rods, this being of 5 palms, while the royal cubit is 7 palms, so that sq.(remen sign): sq. (Cubit sign) :: 25 : 49, practically 1:2"[60]

Here Griffith has documented the practical use of the remen-cubit relationship in land measurement. Area is halved by simply changing the unit of length from cubits to remens. Did that mean that ancient Egyptians knew that the sum of the squares on the legs of an isosceles right triangle is equal to the square on the hypotenuse of the triangle? I think it does, although some will want more evidence. A fair position would be to leave the issue of the right triangle theorem in Egypt as an open question. However, on the question of "incommensurability of the side and the diagonal" of the cubit square, Palter is correct. Neither the Egyptian nor the Babylonian scribes considered the length of the diagonal and the length of the side as different kinds of numbers. [61]

Rectangular Coordinates at Saqqara, 2700 BCE

From an architect's plan on a limestone ostracon found at Saqqara, we have an example of the use of rectangular coordinates c. 2700 BCE. Somers and Engelbach called it "of great importance."[62] This artifact has an architect's drawing for a curved section of a roof. For horizontal coordinates spaced 1 cubit apart, the vertical (height) is given for points which define a curve. The curve in the sketch exactly matches the curve of a nearby temple roof. This appears to be the earliest use of rectangular coordinates and is another example of sophisticated mathematical concepts found in practical applications outside of the surviving mathematics papyri.

As a mathematics teacher, I must say I'm horrified by Palter's "Ghalioungui really should know that building and orienting pyramids does not require any advanced mathematics, ..."[63] That's what students who don't want to study mathematics have been saying all along. I wonder if the architects and engineers who planned the construction of the American obelisk (the Washington monument) would agree with Palter's statement. Even for the so-called example of Roman engineers who produced great construction without producing great mathematics, I doubt that Roman construction could have been so successful without access to the accumulated Egyptian-Babylonian-Greek-Hellenistic mathematics available to them.

Volume of a Truncated Pyramid

In what has been called the greatest Egyptian pyramid of all, which Palter says, "is often considered the high point of Egyptian mathematics,"[64] the volume of a truncated pyramid is given as: $V = (h/3)(a^2 + ab + b^2)$. Here *h* is the remaining height of the truncated pyramid, *a* is the length of the square base, and *b* is the length of the square top. Mathematicians have entertained themselves for many years trying to devise constructions that would produce this formula through empirical means. Cheikh Anta Diop calls these attempts "silly" and claims that so accurate a formula could only be derived analytically. He gives the example of an incorrect Babylonian formula that he believes was derived empirically, $V = (h/2)(a^2 + b^2)$.[65] Victor Katz, in a multicultural section of his textbook, asks students to find the percentage error if the Babylonian formula is used for a truncated pyramid "of lower base 10, upper base 8, and height 2."[66] The Egyptian formula yields a volume of 162 2/3 (probably cubits^3) compared to the Babylonian 164 cubits^3, an error of only 0.8%. For most practical purposes, the Babylonian formula was probably close enough. In view of the closeness of these results, the possibility of an analytical Egyptian derivation for their absolutely correct formula should not be dismissed out of hand. The question is, did the Egyptians have any analytical tools that they could have used to derive such a formula.

Most historians assume that Egyptians knew the volume of a pyramid. Then the volume of a truncated pyramid would be the remainder after a small pyramid of height k is cut off from the top of the original pyramid of height j. Could the Egyptian scribes have developed their formula from this difference of $(j/3)a^2 - (k/3)b^2$? The scribes did have some ability to simplify and rearrange what we call algebraic terms. An example was cited above, for the case of proportions. For problem 19 of the Moscow Papyrus, Gillings shows Egyptians could, and did solve a first degree equation by transposition and division by the coefficient, rather than by false position.. Gillings gives the steps in modern form:

$(1\ 1/2)x + 4 = 10$

(1 1/2x) = 10 - 4

x = 6 / (1 1/2)

x = 4

Gillings also showed that once the formula for volume was obtained, the Egyptian scribes had the tools to prove its validity.[67] Students of the history of mathematics may want to occupy themselves with attempts to use tools available to ancient Egyptians to find analytical derivations of the volume formula. In this connection, it might be well to explore an old suggestion by Lancelot Hogben to make use of the ancient Egyptian facility with series.[68]

Curved Figures

By finding excellent formulas for the area of curved figures, Egyptian mathematicians made some big breakthroughs. A diagram for Ahmose problem 48 suggests the use of an octagon to approximate the area of a circle. Palter cannot find too many derogatory things to say about the Egyptian formulas for the area of a circle and the surface area of a hemisphere, so his thrust is to deny that any Egyptian results on volumes of solids influenced the Greek geometrical tradition. Palter writes about Eudoxus, a student of Archytas, that, “his mathematical proofs must have been of the rigorous axiomatic sort characteristic of that tradition. It is difficult to exaggerate the difference between such proofs and *anything* found in Egyptian mathematics.”[69] Perhaps it is difficult, but Palter has succeeded. To claim that Egyptian discovery of formulas for the volume of solids could not have inspired or influenced the later Greek mathematicians who wrote proofs of these formulas is ludicrous. At least Palter did not follow in Neugebauer□s footsteps on the debate about Moscow Papyrus problem 10: Was it a cylinder or a hemisphere? Bernal was right to be shocked when Neugebauer proclaimed that the more “primitive” interpretation (the cylinder) was preferable.[70]

For some reason, Palter writes as though the Egyptians and Babylonians are in some kind of competition, and that he is supposed to be on the side of the Babylonians. The question is placed as though admitting the achievements of one culture takes something away from another culture. Clearly Bernal does not think along those lines; neither do I. For example, Palter describes Egyptian calculation of the square root of 1 9/16, saying, “All of this may sound impressive enough for the first mathematicians in history, but once again, Babylonian mathematicians of about the same period were even more impressive ...” Again, □But the Egyptians also computed areas and volumes, and here they surpassed the Babylonians.”[71].

Mathematical Proof

What could the Greeks have learned about logical arguments from the Egyptians? It was said that Solon borrowed Egyptian laws and introduced them to Greece.[72] There was much in the ancient Egyptian mathematics and literature that showed development of logical, deductive methods. Debate and argument were held in high regard in the literature and culture. The story of "The Eloquent Peasant"[73] tells about a peasant□s suit for restitution which he pleaded so eloquently in court that it was reported to the king. The king ordered the man detained (while his family was taken care of) so that they could hear the eloquent peasant day after day until they finally granted his plea and gave him a rich reward. Egyptians pleaded their own cases in court and they were a litigious society, with some cases extending for 3 generations.[74] Cases were decided according to a body of law. Although no single document survives that shows the entire code, there are paintings of court sessions showing scrolls of laws, and substantial surviving manuscripts which give detailed accounts. For example, a small papyrus in the Leiden Museum lists 11 items stolen by a female servant. The penalty is triple the value of the item stolen with the exception of a very valuable piece where the penalty is double.[75]

Within the mathematical papyri themselves, Gillings found many examples of concrete solutions given to illustrate a general method. He devotes his Appendix 1 on "The Nature of Proof" to this discussion. In the following quotation, he considers the rigor of Egyptian-style proofs, and the differences between the Egyptian use of logic and the later axiomatic approach:

> " Twentieth-century students of the history and philosophy of science, in considering the contributions of the ancient Egyptians, incline to the modern attitude that an argument or logical proof must be *symbolic* if it is to be regarded as rigorous, and that one or two specific examples using selected numbers cannot claim to be scientifically sound. But this is not true! A nonsymbolic argument or proof can be quite rigorous when given for a particular value of the variable; the conditions for rigor are that the particular value of the variable should be *typical,* and that a further generalization to *any* value should be *immediate.* In any of the topics mentioned in this book where the scribes' treatment follows such lines, both these requirements are satisfied, so that the arguments adduced to the scribes are already *rigorous;* the concluding proofs are really not necessary, only confirmatory. The rigor is implicit in the method.
>
> We have to accept the circumstance that the Egyptians did not think and reason as the Greeks did. If they found some exact method (however they may have discovered it), they did not ask

themselves *why* it worked. They did not seek to establish its universal truth by an a priori symbolic argument that would show clearly and logically their thought processes. What they did was to explain and define in an ordered sequence the steps necessary in the proper procedure, and at the conclusion they added a verification of proof that the steps outlined did indeed lead to a correct solution of the problem. This was science as they knew it ... ."[76]

Alexandrian Mathematics C a Synthesis

When Alexander appointed his general, Ptolemy I, to rule Egypt, he also appointed Seleucus I to rule Mesopotamia (Babylonia). But those who are so eager to downgrade the African influence call the science of Ptolemaic Egypt "Greek," while they call science and mathematics of Seleucid Mesopotamia "Mesopotamian." That ends up with some historians comparing the mathematical work from Seleucid Mesopotamia not with the contemporary work of Euclid, but with Ahmose whose work was 1500 years earlier. In the example of astronomy, Neugebauer writes, "Early Mesopotamian astronomy appeared to be crude and merely qualitative, quite similar to contemporary Egyptian astronomy. ... only the last three centuries B.C. furnished us with texts ... . fully comparable to the corresponding Greek systems ... ."[77] Outstanding among these so-called Greek systems were those of Alexandria, especially the Ptolemaic, which Neugebauer calls Greek, instead of Egyptian. Although Mesopotamia of that time was also under Greek rule, Neugebauer (and others) call the Seleucid astronomy Mesopotamian!

On the subject of Alexandrian mathematics, I would like to pose a question. If the influence of Egyptian mathematics was so minor or non-existent as Palter claims, why does Neugebauer find

"...two widely separate types of Greek mathematics. One is represented by the strictly logical approach of Euclid, Archimedes, Appolonius, etc.; the other group is only a part of general Hellenistic mathematics, the roots of which lie in the Babylonian and Egyptian procedures. The writings of Heron and Diophantus and works known only from fragments or from papyrus documents form part of this oriental tradition which can be followed into the Middle Ages both in the Arabic and in the western world."[78]

Does this not suggest that the two currents of ancient mathematics, the axiomatic geometry and the applied algebra, existed side by side in all of the mathematical centers of North Africa, Southern Europe, Western, and Central Asia, and created a blend that was much more than just "Greek" mathematics? Some historians have moved towards this view and use the term, "Hellenistic," to describe the mathematics of this part of the world in the period from Euclid to Hypatia. For example, Boyer writes:

“Under Alexander there had been a gradual blending of Hellenic and Oriental customs and learning, so that it was more appropriate to speak of the newer civilization as Hellenistic, rather than Hellenic. Moreover, the new city of Alexandria, established by the world conqueror, now took the place of Athens as the center of the mathematical world.”[79] Joseph also adds that scholars came to Alexandria in Egypt from as far away as India. Joseph’s work also reminds us that important mathematics was being developed in that period in non-Hellenistic parts of the world also, including China, India, and Central America.[80]

“Afrocentrism” “Eurocentrism,” Race and Racism

Starting with the title of his article, Palter uses the term “Afrocentrism” as an accusation, but never says how he defines the term. A letter dated January 10, 1997, written to educators by Erich Martel, in behalf of Mary Lefkowitz, offers a ”clarification.” [81] The letter solicited names of schools and programs using “Afrocentric curricular material” to be listed in an appendix for a new edition of *Not Out of Africa - How Afrocentrism Became an Excuse to Teach Myth as History.* Martel;s letter states, “For purpose of clarity, the term *‘Afrocentrism’ or ‘Afrocentric’* as employed in this questionnaire, refers to undocumented or misinterpreted historical and scientific claims and assertions about ancient Africa, ancient Egypt and their influence on the formation of other world cultures and civilizations, especially that of ancient Greece.”

This was not a definition but an indictment and a conviction without a trial. I thought it was only fair to seek a definition by a leader of the African-centered curriculum movement. Jacob Carruthers, currently acting director of Northeastern Illinois University□s Center for Inner City Studies, supplied me with a copy of his paper in which he says, “...the major and central objective of the African Centered Curriculum ... is to restore the truth about Africa to the world.” He adds, “Such a result we feel is good for everybody.”[82] Evidently he believes that some of the programs called “multicultural” continue to omit Africa. In his paper, Carruthers “recommends a massive infusion into the curriculum of correct and relevant information and materials about Africa.”[83] Since Carruthers’ African Centered Curriculum project insists on “correct and relevant information,” it would seem not to fall under Martel’s criteria for Afrocentrism.

Were the ancient Egyptians Africans? Were they Black? Palter does not raise the question but Mary Lefkowitz does in her introduction to *Black Athena Revisited*. She complains, “Bernal would prefer to emphasize that Egypt is a part of Africa, rather than try to determine the exact proportion of darker-skinned central Africans in the population. For to speak of the ancient (or modern) Egyptians as ‘black’ is misleading in the extreme. ... ‘black’ in the modern sense of ‘Negroid.’ ”[84] Lefkowitz earlier resorted to invoking the name of W.E.B. DuBois, as one who was opposed to Garvey's concepts of"Egypt gave the world civilization," and Negro professors taught in the universities in Alexandria."[85]

Misusing the DuBois name was a big mistake. This is what DuBois actually said in *The World and Africa,* a passage in which he also replies to concerns about "darker-skinned" or, "were the Egyptians brown or black?"

:

"The word 'Negro' was used for the first time in the world's history to tie color to race and blackness to slavery and degradation. The white race was pictured as 'pure' and superior; the black race as dirty, stupid, and inevitably inferior; the yellow race as sharing, in deception and cowardice, much of this color inferiority; while mixture of races was considered the prime cause of degradation and failure in civilization. Everything great, everything fine, everything really successful in human culture was white.

"In order to prove this, even black people in India and Africa were labeled as 'white' if they showed any trace of progress; and on the other hand, any progress by colored people was attributed to some intermixture, ancient or modern, of white blood or some influence of white civilization."[86]

And further, in his chapter on Egypt, DuBois flatly takes the position, "We conclude, therefore, that the Egyptians were Negroids." [87]

The discussion of race and racism may seem out of date in 1997 when any biological base for race has been completely demolished.[88] But that doesn't mean that racism is not an issue. The African American playwright, Lorraine Hansberry, put it very well in her *play, Les Blancs*:

Tshembe: I said racism is a device that, of itself, explains nothing. It is simply a means. An invention to justify the rule of some men over others.

Charlie: But I agree with you entirely! Race hasn't a thing to do with it actually.

Tshembe: Ah, but it *has!* ... I am simply saying that a device *is* a device, but that it also has consequences: once invented it takes on a life, a reality of its own. So, in one country, men invoke the device of religion to cloak their conquests. In another, race. Now in both cases you and I may recognize the fraudulence of the device, but the fact remains that a man who has a sword run through him because he refuses to become a Moslem or a Christian, or who is shot in Zatembe or Mississippi because he is black, is suffering the utter *reality* of the device. And it is pointless to pretend that it doesn't *exist*, merely because it is a *lie!*

Les Blancs, p. 92

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The facility of the Egyptians with arithmetic series is illustrated by Ahmose's solution of problem 40:

Divide 100 loaves among five persons so there is a common difference. Also, the sum of the highest three portions must be 7 times the sum of the lowest two.

Ahmose used the method of false position in which a value is assumed (the false value). By trying the false value in the problem, a proportional correction factor is determined. Ahmose started by taking the common difference to be 5 1/2, a happy choice, and assuming a value of 1 for the lowest share. The shares would be:
1, 6 1/2, 12, 17 1/2, and 23.

The sum of the three highest terms, 23 + 17 1/2 + 12, is 7 times the sum of the two lowest, 1 + 6 1/2, as required. But the sum of the series is 60, not the required 100. As many times as 60 is multiplied to give 100, so each term of the series must be multiplied. The correction factor is 100/60 or 5/3 and the desired shares are:
1 2/3, 10 5/6, 20, 29 1/6, 38 1/3.

Was the trial value of 5 1/2 for the common difference just a lucky choice? The translator of the Ahmose papyrus, Arnold Chace, thinks not. He suggests a method that Ahmose could have used to find this value. He believes that the Egyptians experimented with various common differences, starting with 1, 2, and so on. For the first term, they usually used an assumed value of 1.

Series	Difference	Sum of smallest two - 1/7 Sum of largest three
1, 2, 3, 4, 5	(difference = 1)	3 - 1 5/7 = 1 2/7
1, 3, 5, 7, 9	(difference = 2)	4 - 3 = 1
1, 4, 7, 10, 13	(difference = 3)	5 - 4 2/7 = 5/7

Each time the common difference increases 1 unit, there is a decrease of 2/7 in the difference between the sum of the smallest two and 1/7 the sum of the largest three. How much must the common difference be increased to make the two sums equal? Divide 1 2/7 by 2/7 to get 4 1/2. Add 4 1/2 to 1 for a common difference of 5 1/2.[i]

Berlin Papyrus 6619 problem

The false position method also appears in other Egyptian papyri, for example Kahun LV,3 and the Berlin 6619 papyrus [ii]. The Berlin papyrus problems involve a system of two equations, one of second degree. One problem asks for two squares such that the sum of the two areas equals a third square of area 100 cubits, and one square has a side 3/4 the length of the side of the other square. What are the lengths of the sides?

"Always take a square of side 1," the scribe recommends for the false position solution. Then the squares have assumed lengths of 1, and 3/4 cubit, and the sum of the assumed squares is 1 9/16 square cubits. To find the side of a square of 1 9/16 square cubits, the scribe takes the square root and finds it to be 1 1/4 cubits. But the required sum is 100, with side 10. Divide 10 by 1 1/4 to get the correction factor of 8 for the sides. Notice that the correction factor was not the ratio of the areas but the ratio of the square roots. The required sides are 8 and 6 cubits.[iii] Since the side of the large square is 10 cubits, the values of the sides are a multiple of the right triangle triplet (3, 4, 5). The solution for a similar problem in the Berlin Papyrus provides two squares of sides 12 and 16 cubits. The sum of the areas is equal to a square of side 20 cubits, again a multiple of the right triangle triplet (3, 4, 5).

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