Models of the Solar interior

(1) Basic Solar Parameters

(2) Equations of stellar structure and evolution

(3) Inputs to evolution codes

(4) Solar models

(5) Using the Sun as a laboratory

The Sun like all stars is a self gravitating ball of gas.

There are two basic forces at play:

(1) **GRAVITY**, that causes stars to collapse

(2) **PRESSURE**, that causes stars to expand

The interplay between pressure and gravity plays out throughout a star's life.

There are stages in a star's life where gravity wins and the star begins to collapse.

There are stages where pressure wins and the star begins to expand.

For the most part though, gravity and pressure balance each other, a state known as **hydrostatic equilibrium**. The Sun is currently in that state of life.

While modelling the Sun we have the advantage that we know a lot more about the Sun than for other stars.

But this also makes modeling more challenging!

Any model of the Sun must satisfy the following requirements:

Mass = 1.989 x 10^{33} g
Radius = 6.959 x 10^{10} cm
Luminosity (energy output) = 3.8418 x 10^{33} ergs/s
Age = 4.57 x 10^{9} years

The effective temperature of the Sun is determined assuming that the Sun emits like a black body, i.e.,

$$L = 4\pi R^2 \sigma T_{eff}^4$$

Mass is an input to solar models.

Whether we have succeeded in making a solar model is determined by whether or not the model has 1Ro, 1Lo at 4.57 Gyr.

Equations governing stellar structure and Evolution:

For most parts, stars are **spherically symmetric**, i.e., their internal structure is only a function of radius and not of latitude or longitude.

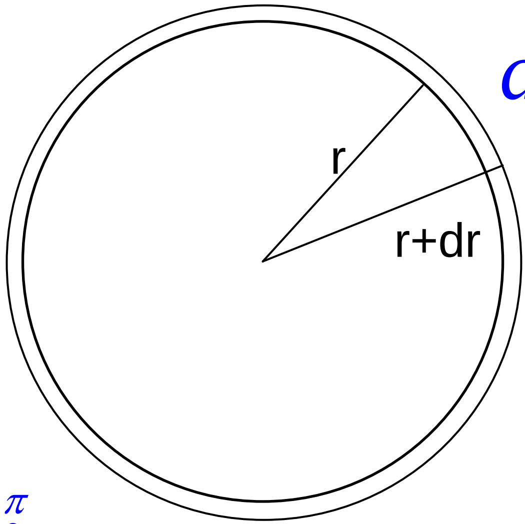
This means that we can express the properties of stars using a set of 1D equations, rather than a full set of 3D equations. The main equations concern the following physical principles:

(1) Conservation of Mass
(2) Conservation of momentum
(3) Conservation of energy
(4) Transport of radiation
(5) Nuclear reaction rates
(6) Change of abundances by various processes

Equation 1: Conservation of Mass – I

The mass in a spherical shell must be accounted for by its density in the absence of flows:

density $\rho = {dm}/{dV}$



$$dV = r^2 \sin\theta d\theta d\phi dr$$

In the spherically symmetric case:

$$dV = r^2 dr \int_{-\pi}^{\pi} \sin\theta d\theta \int_{0}^{2\pi} d\varphi$$

$$= 4\pi r^2 dr$$

$$dm = 4\pi r^2 \rho \, dr$$

Equation 1: Conservation of Mass – II

The radius or a star can change in the course of its life.

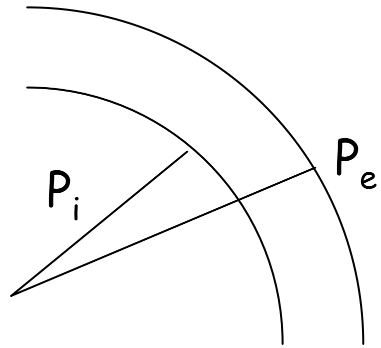
Mass of stars (particularly low-mass stars like the Sun) do not change much.

Therefore, customary to use **mass** rather than the radius as the independent variable.

Thus we write:

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \qquad (1)$$

Equation 2: Conservation of Momentum– I



Pressure (force/area) acts outwards:

Difference in pressure = P_i-P_e

= -(dP/dr)dr = f_P

Force due to gravity acts inwards:

=acceleration * mass

= -g $4\pi r^2 \rho dr$

Force per area due to gravity

= -g $4\pi r^2 \rho dr / 4\pi r^2$= -g $\rho dr = f_g$

At equilibrium, sum of forces is 0

Therefore,

fP+fg = 0 $\Rightarrow$ -(dP/dr)dr -g ρdr=0,

or,

$(dP/dr) = -g\,\rho$

$(dP/dr) = -(Gm/r^2)\rho$

Converting dr to dm we get

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \qquad (2)$$

Equation 2: Conservation of Momentum– II

What happens when the forces do not balance? The shells accelerate:

$$\left(\frac{dm}{4\pi r^2}\right)\frac{d^2r}{dt^2} = f_P + f_g$$
$$= -\left(\frac{dP}{dr}\right)dr - g\rho$$
$$= -\left(\frac{dP}{dm}\right)dm - \frac{Gm}{4\pi r^4}$$

$$\frac{d^2r}{dt^2} = 0 \Rightarrow \text{hydrostatic equilibrium}$$

$$\text{and } \frac{dr}{dt} = 0, \text{ or } \frac{dr}{dt} = \text{const.}$$

What happens when pressure vanishes? Free fall! Assume time taken to collapse is τ_{ff}

$$\frac{d^2r}{dt^2} \approx \frac{R}{\tau_{ff}^2} = g = \frac{Gm}{r^2},$$

$$\Rightarrow \tau_{ff} = \left(\frac{R}{g}\right)^{1/2}$$

For the Sun, τ is 27 min

Equation 2: Conservation of Momentum– III

What happens when gravity vanishes? The star explodes!

$$\frac{dm}{4\pi r^2}\frac{d^2 r}{dt^2} = -\left(\frac{dP}{dm}\right)dm$$

$$\Rightarrow \frac{d^2 r}{dt^2} = -4\pi r^2\left(\frac{dP}{dm}\right) = -\frac{1}{\rho}\left(\frac{dP}{dr}\right)$$

$$\therefore \frac{R}{\tau_{\exp}^2} = \frac{P}{R}\rho \Rightarrow \frac{R^2}{P}\rho = \tau_{\exp}^2 \Rightarrow \tau_{\exp} = R\left(\frac{\rho}{P}\right)^{1/2}$$

But $\frac{P}{\rho} \approx c_s^2$,

$\therefore \tau_{\exp} \approx \frac{R}{c_s}$, the sound travel time, again about 27 min for the Sun.

Digression:

$$c_s^2 = \Gamma_1 \frac{P}{\rho},$$

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{\text{ad}}$$

Digression: Ideal gases

Ideal gases have a simple relation between pressure, density, temperature and composition, i.e., a simple Equation of State.

For an ideal gas:

$$P = nkT \Rightarrow P = \frac{\rho}{\bar{m}} kT, \quad \bar{m} \text{ is the mass of particle}$$

$$\bar{m} = \frac{\text{total mass of particles}}{\text{total no. of particles}}$$

$$\mu = \text{mean molecular wt.} = \frac{\bar{m}}{m_H},$$

$$\therefore P = \frac{\rho}{\mu m_H} kT = \frac{R}{\mu} \rho T$$

Thus for ideal gases:

$$\alpha \equiv \left(\frac{\partial \ln \rho}{\partial \ln P} \right)_{T,\mu} = 1; \quad \delta \equiv -\left(\frac{\partial \ln \rho}{\partial \ln T} \right)_{P,\mu} = 1; \quad \phi \equiv \left(\frac{\partial \ln \rho}{\partial \ln \mu} \right)_{T,P} = 1$$

Digression: The mean molecular weight

How do we calculate the mean molecular weight?

We assume that the star is made predominantly of Hydogen, and Helium, with a very small fraction of other elements.

Let mass fraction of Hydrogem = X

Mass fraction of Helium = Y

Mass fraction of everything else = Z

Then of course $X+Y+Z=1$

Assume mass of gas provided by nuclei, but electrons contribute to total number of particles. Thus, e.g., each hydrogen atom contributes two particles, but 1 mass unit, therefore mean mol. wt. of ionized hydrogen is (1/2). We can show that

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{Z}{2}$$

Digression: The Virial Theorem – I

The virial theorem states that for a system in equilibrium,

Potential energy+2 Kinetic Energy =0

For a star, we can show that this leads to

$$E_g + \int_0^M \frac{3P}{\rho} dm = 0, \quad E_g = -\int_0^M \frac{Gm}{r} dm$$

If the star were made of ideal gas, then we can relate the second term to the internal energy and get $E_g+2U=0$, *or* $U=-E_g/2$

The total energy of a star is $E_{tot}=E_g+U$

When a star contracts, E_g becomes more negative, i.e. $dEg/dt < 0$,

Energy is therefore released during gravitational contraction.

From the virial theorem we can show that half this energy goes into increasing the internal energy (i.e. makes the star hotter), the other half is radiated away (i.e., the star becomes more luminous).

Digression: The Virial Theorem – II

How long would a star live if its energy were provided by release of gravitational energy through contraction of a star?

The KELVIN-HELMHOLTZ time scale

$$\frac{dE_g}{dt} = 2L, \text{ or}$$

$$\frac{E_g}{\tau} = 2L, \Rightarrow \tau = \frac{E_g}{2L} \Rightarrow \tau \approx \frac{GM^2}{2RL}$$

For the Sun, the KH time scale is about 10^7 years.

Equation 3: Conservation of Energy

Define dl = luminosity of a mass shell, i.e., $\int_0^M \frac{dl}{dm} dm = L$

Let ε be nuclear energy released per unit time per unit mass, then

$$\frac{dl}{dm} = \varepsilon$$

When a star expands or contracts, need to account of energy absorbed or released, then we can show that

$$\frac{dl}{dm} = \varepsilon + \varepsilon_g = \varepsilon - C_P \frac{dT}{dt} + \frac{\delta}{\rho} \frac{dP}{dt} \qquad (3)$$

Equation 4: The temperature distribution

In principle, a very simple equation

$$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4} \Rightarrow \frac{dT}{dP}\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}\frac{dT}{dP} \Rightarrow$$

$$\frac{dT}{dm} = -\frac{Gm}{4\pi r^4}\frac{dT}{dP}$$

$$\text{Defining, } \nabla = \frac{d\ln T}{d\ln P}$$

$$\boxed{\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P}\nabla} \qquad \text{———} \quad (4)$$

Question is, what is ∇?

Digression: How is heat transported?

The expression for the non-dimensional temperature gradient ∇ depends on how heat is transported within the star.

There are two main forms of heat transport in normal stars

(1) Radiation (i.e., heat is transported by photons)

(2) Convection (i.e., heat is transported by the bulk movement of matter.

When do we have radiative transport and when convective? Depends on ∇.

For any mode of heat transportation, we can write

$$F = -k\nabla T$$

This can be interpreted as we need a certain temperature gradient to transport a given amount of flux for a given "conductivity."

Digression: Radiative heat transport

Stellar interiors are very opaque to radiation, and the mean free path of photons is of a order of 1-2 cm. Thus when photons transport energy, they do so as a diffusive process. We know how to write the flux for diffusion. We get,

$$F = -\frac{4}{3}\frac{c}{\kappa\rho}aT^3\frac{dT}{dr}$$

We need (dlnT/dlnP) for Eq. (4), since $l=4\pi r^2 F$, we get

$$\frac{dT/dm}{dP/dm} = \frac{-3\kappa l}{64\pi^2 acr^4T^3} \times \frac{-4\pi r^4}{Gm} = \frac{3\kappa l}{16\pi acGmT^4}$$

Or,

$$\nabla_{rad} = \frac{d\ln T}{d\ln P} = \frac{3}{16\pi acG}\frac{\kappa l P}{mT^3}$$

Digression: Radiative heat transport

To calculate ∇ for radiative heat transport, we need to know the opacity κ.

κ is a measure of how opaque material is to radiation, and the mean free path of a photon in a dense medium is given by $d = \frac{1}{\kappa\rho}$

Usually the opacity is a function of the frequency of the radiation. The opacity that occurs in the expression for ∇ is the so-called "Rosseland" mean opacity, given by

$$\frac{1}{\kappa} = \frac{\pi}{acT^3} \int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T} d\nu$$

B is the Planck function, κ_ν are the monochromatic opacities.

Digression: What about convection?

To transport a given amount of flux by radiation, we need to have a certain ∇.

But each material has a maximum ∇ beyond which the material becomes unstable and convection sets in.

This maximum value of ∇ is $\left(\frac{d\ln T}{d\ln P}\right)_{\rm ad}$ and is determined by the equation of state (called $\nabla_{\rm ad}$).

Thus convection sets in when $\nabla_{\rm rad} > \nabla_{\rm ad}$, the so-called "Schwarzchild Criterion"

$$\nabla_{\rm ad} = \frac{P\delta}{T\rho C_P} = \frac{2}{5} \text{ for an ideal gas}$$

Digression: A simple case of instability

Assume that a star is made of ideal gases, then:

$$\frac{P}{\rho} = \frac{\mathcal{R}}{\mu} T \;\Rightarrow\; \frac{dP}{P} - \frac{d\rho}{\rho} = \frac{dT}{T} \;\Rightarrow\; \frac{1}{P}\frac{dP}{dr} - \frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{T}\frac{dT}{dr},$$

But, $\frac{dP}{dr} = -g\rho$ (hydrostatic equilibrium)

$$\Rightarrow -g\rho \frac{\mu}{\rho T \mathcal{R}} - \frac{1}{\rho}\frac{d\rho}{dr} = \frac{1}{T}\frac{dT}{dr}, \text{ or } \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{T}\frac{dT}{dr} - \frac{g\mu}{\mathcal{R}T}$$

$\frac{dT}{dr}$ is negative (T increases as r decreases), $\therefore -\frac{1}{T}\frac{dT}{dr}$ is positive

$\frac{g\mu}{\mathcal{R}T}$ is positive.

$\therefore \frac{d\rho}{dr} > 0$, i.e, heavier material on lighter material (unstable situation) if

$$-\frac{1}{T}\frac{dT}{dr} > \frac{g\mu}{\mathcal{R}}, \text{ i.e., } -\frac{dT}{dr} > \frac{g\mu}{\mathcal{R}}$$

Digression: ∇ for convection

Cannot be calculated from first principles.

Usual to use the so-called "Mixing Length" formalism.

The formalism states that all convective eddies come in one size:

$l_m = \alpha H_P$, where,

α = mixing length parameter

$$H_P = \text{pressure scale height} = -\frac{dr}{d\ln P}$$

∇ is actual gradient

$\nabla_{\rm rad}$ is gradient assuming all flux is carried by radiation

Define $U = \dfrac{3acT^3}{C_P \rho^2 \kappa l_m^2}\sqrt{\dfrac{8H_P}{g}}$, $W = \nabla - \nabla_{rad}$, and $\xi^2 = \nabla - \nabla_{ad} + U^2$

For a given U, we get ξ by solving

$$(\xi - U)^3 + \frac{8}{9}U(\xi^2 - U^2 - W) = 0$$

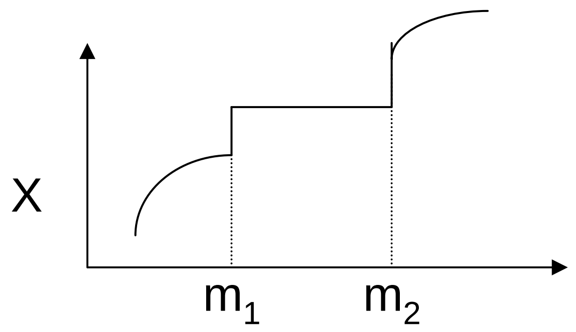
Since $\nabla_{\rm ad}$ is known from the EOS, we know ∇

Equation 5: Composition

Composition changes can occur because of three reasons:

(1) Nuclear reaction :

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left[\sum_j r_{ji} - \sum_k r_{ik}\right], \quad i = 1....N$$



(2) In a convection zone

$$\overline{X_i} = \frac{1}{m_2 - m_1}\int_{m_1}^{m_2} X_i \, dm$$

$$\frac{\partial \overline{X_i}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{m_2 - m_1}\int_{m_1}^{m_2} X_i \, dm\right)$$

$$= \frac{1}{m_2 - m_1}\left[\int_{m_1}^{m_2}\frac{\partial X_i}{\partial t}dm + \frac{\partial m_2}{\partial t}(X_{i,2} - \overline{X_i}) - \frac{\partial m_1}{\partial t}(X_{i,1} - \overline{X_i})\right]$$

(3) The gravitational settling of elements

$$\frac{\partial X_i}{\partial t} = D\nabla^2 X_i$$

An overview of the equations

(1) $$dm = 4\pi r^2 \rho \, dr$$

(2) $$\frac{dP}{dm} = -\frac{Gm}{4\pi r^4}$$

(3) $$\frac{dl}{dm} = \varepsilon + \varepsilon_g = \varepsilon - C_P \frac{dT}{dt} + \frac{\delta}{\rho}\frac{dP}{dt}$$

(4) $$\frac{dT}{dm} = -\frac{GmT}{4\pi r^4 P}\nabla$$

(5) $$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left[\sum_j r_{ji} - \sum_k r_{ik}\right], \; i = 1....N$$

There are 5 equations in 6 unknowns *(r, P, T, ρ , l, X_i)*
Need a relation to connect ρ to P,T,X_i – the Equation of State.

Some salient points:

Equations (1) and (2) are mechanical and are connected to the other equations through ρ. Thus, if we have an independent prescription for r, we can solve these equations without reference to others.

Equation (5) is the chemical part, and because of the time scales involved, it can generally be decoupled from the other 4. $\therefore$ we can solve Eq. (1) to (4) if $X_i(m)$ is given at a given time. These are called "static models".

Solving the equations

Normally Eq. (1)-(4) are solved for a given time. The time is advanced, Eq. (5) is solved, and then Eq. (1)-(4) solved again.

Thus we consider two independent variables: *m and t.*

So we look for solutions in the interval

$$0 \le m \le M \quad \text{(stellar structure)}$$

$$t \ \ge \ t_o \quad \text{(stellar evolution)}$$

To solve these equations we need 4 boundary conditions, either at m=0 or m=M, and we need initial conditions at $t=t_0$

The boundary conditions: centre

Unfortunately for us, the boundary conditions are not all at one boundary. That makes solving tricky.

There are two boundary conditions at the centre: We can show that

$r(m=0)=0$ and $l(m=0)=0$.

We do not get central boundary conditions on T and P. For T and P we can only show that

$$P - P_c = -\frac{3G}{8\pi}\left(\frac{4\pi}{3}\rho_c\right)^{4/3} m^{2/3}$$

$$T - T_c = -\frac{T_c}{P_c}\nabla_c\left(P - P_c\right)$$

The boundary conditions: surface

We need boundary conditions for T and P at the surface. A simplistic boundary condition would be

$T(m=M)=0$, and $P(m=M)=0$

In reality, pressure is non-zero, though small, and T can be a few thousand Kelvin.

We therefore, take refuge in models of stellar atmospheres. The simplest is the so called Eddington approximation, which says that in the atmosphere

$$T^4 = \frac{3}{4}\left(\tau + \frac{2}{3}\right)T_{\text{eff}}^4,$$

$$T_{\text{eff}}^4 = \frac{L}{\sigma 4\pi R^2}$$

Assuming that acceleration due to gravity, g, is const. in the stellar atmosphere, the condition of hydrostatic equilibrium gives

$$\frac{dP}{dr} = -g\rho = -\frac{GM}{R^2}\rho, \text{ but}$$

$$d\tau = \kappa\rho dr, \text{ assuming } \kappa = \text{const., we get}$$

$$\frac{dP}{d\tau} = -\frac{GM}{R^2}\frac{1}{\kappa}$$

We know at surface, T is T_{eff}, we can find P corresponding to that and use these values of T and P as the boundary conditions. May have to iterate since the P and Teff from surface may not be that obtained from centre.

Initial Conditions:

Initial conditions depend on where one starts the evolution.

If one begins the evolution in the so-called pre-MS phase (i.e., before core hydrogen fusion), the initial structure is chemically homogeneous (because the models are fully convective), and stratified accordingly. Deuterium fusion occurs before core hydrogen burning and causes the star to become somewhat chemically inhomogeneous before the star enters the hydrogen fusion stage.

If evolution is begun on the zero-age main sequence (i.e., at the onset of hydrogen fusion), then a proper ZAMS model must be used. The models are not completely chemically homogeneous because they need not be fully convective.

INPUTS – I

(I) **The equation of state.**

This gives ρ as a function of T, P, X_i. Also gives C_p, δ, ∇_{ad}, ϕ.

Probably the simplest EOS is the ideal gas EOS. But that does not apply everywhere. Does not include ionization, radiation pressure, degeneracy etc.

Modern equations of state are given in tabular forms and we need to interpolate to get what we need. The effects include ionization, radiation pressure, degeneracy, pressure ionization, etc.

EOS most commonly used for the Sun: OPAL. Works well for stars of most masses, but not for very low mass stars.

INPUTS – II

(II) <u>Opacities:</u>

Usually Rosseland mean opacities are provided as a funtion of T, ρ and Z. Opacity increases with increase in Z.

Opacity tables generally used for solar models: OPAL, or OP.

All good opacity tables include the four basic causes of opacity:

(1) Scattering of photons by electrons – important when material is fully ionized.

(2) Free-Free transitions (i.e., Bremsstrahlung), free electrons and nuclei form momentary dipoles that can emit or absorb radiation.

(3) Bound-free transitions (i.e., ionization of material): the process of ionization requires absorption of photons to ionize.

(4) Bound-bound transitions, i.e., formation of spectral lines. Most difficult part since there are millions and millions of spectral lines that need to be accounted for.

In addition, opacity tables needed for very low mass stars need to include molecular opacities, caused by molecules such as water, methane, oxides of titanium and vanadium, etc.,

Rough dependences of opacities

(1) Electron scattering: Frequency independent

$$\kappa_e = \frac{\sigma_T n_e}{\rho} = 0.2(1 + X)$$

(2) Free-Free:

$$\kappa_{ff} = \kappa_0 \rho T^{-3.5}, \quad \kappa_0 = 3.8 \times 10^{22} (1 + X)(1 - Z)\overline{g}_{ff}$$

(3) Bound-free:

$$\kappa_{bf} = \kappa_0 \rho T^{-3.5}, \kappa_0 = 4.34 \times 10^{25} Z(1 + Z) \frac{\overline{g_{bf}}}{t}$$

INPUTS – III

(III) **Nuclear Reaction Rates:**

The rate at which nuclear reactions occur, and the amount of energy released per reaction is one of the required inputs. Different reactions occur at different rates that depend on the cross section of the reactions, the temperature, the density and the abundance of the different nuclei.

Binding energy per nucleon :

$$\text{Mass Defect} = \Delta m = (m_{\text{nuclus}} - (A - Z)m_n - Zm_p)$$

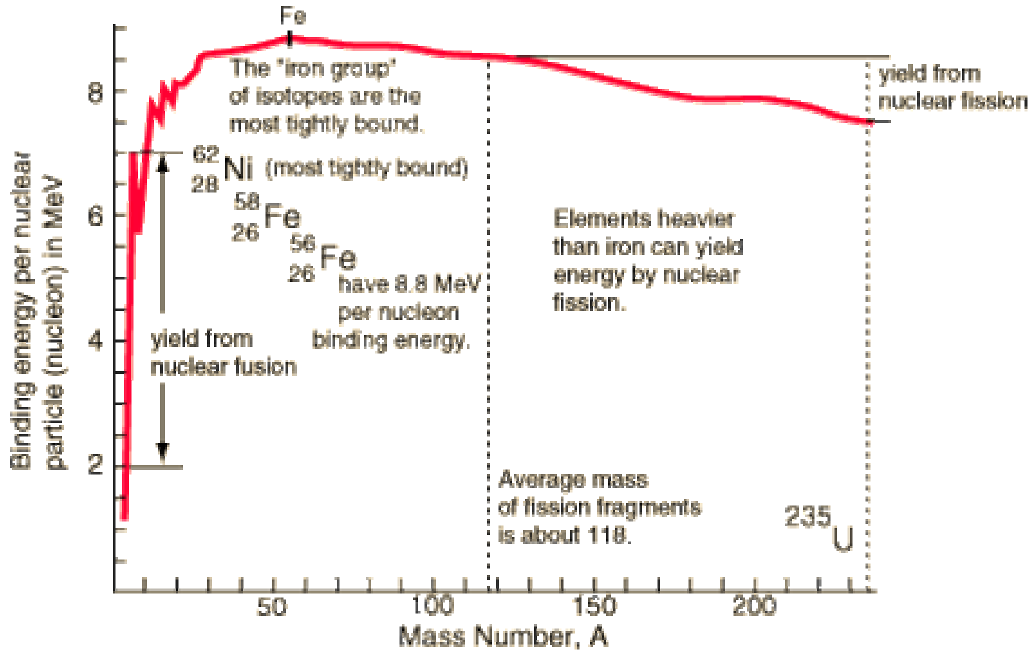
$$\text{Binding Energy} = \Delta m\, c^2$$

$$\text{Binding Energy per nucleon} = \Delta m\, c^2 / A$$

$$\text{Binding energy per nucleon of He} = 6.6\text{MeV}$$

$$\text{Binding energy per nucleon of Fe} = 8.5\text{MeV}$$

$$\varepsilon = \varepsilon_0 \rho T^n$$



The p-p Chain

$$H^1 + H^1 \rightarrow H^2 + e^+ + \nu$$
$$H^2 + H^1 \rightarrow He^3 + \gamma$$

$T > 10^7 K$

$$He^3 + He^3 \rightarrow He^4 + 2\,H^1$$

26.2MeV

$$He^3 + He^4 \rightarrow Be^7 + \gamma$$

$$Be^7 + e^- \rightarrow Li^7 + \nu$$
$$Li^7 + H^1 \rightarrow He^4 + He^4$$

25.67MeV

$$Be^7 + H^1 \rightarrow B^8 + \gamma$$
$$B^8 \rightarrow Be^8 + e^+ + \nu$$
$$Be^8 \rightarrow 2\,He^4$$

19.2MeV

The CNO Cycle

$$C^{12} + H^{1} \rightarrow N^{13} + \gamma$$

$$N^{13} \rightarrow C^{13} + e^{+} + \nu$$

$$C^{13} + H^{1} \rightarrow N^{14} + \gamma$$

$$N^{14} + H^{1} \rightarrow O^{15} + \gamma$$

24.97 MeV

$$O^{15} \rightarrow N^{15} + e^{+} + \nu$$

$$N^{15} + H^{1} \rightarrow C^{12} + He^{4}$$

$$N^{15} + H^{1} \rightarrow O^{16} + \gamma$$

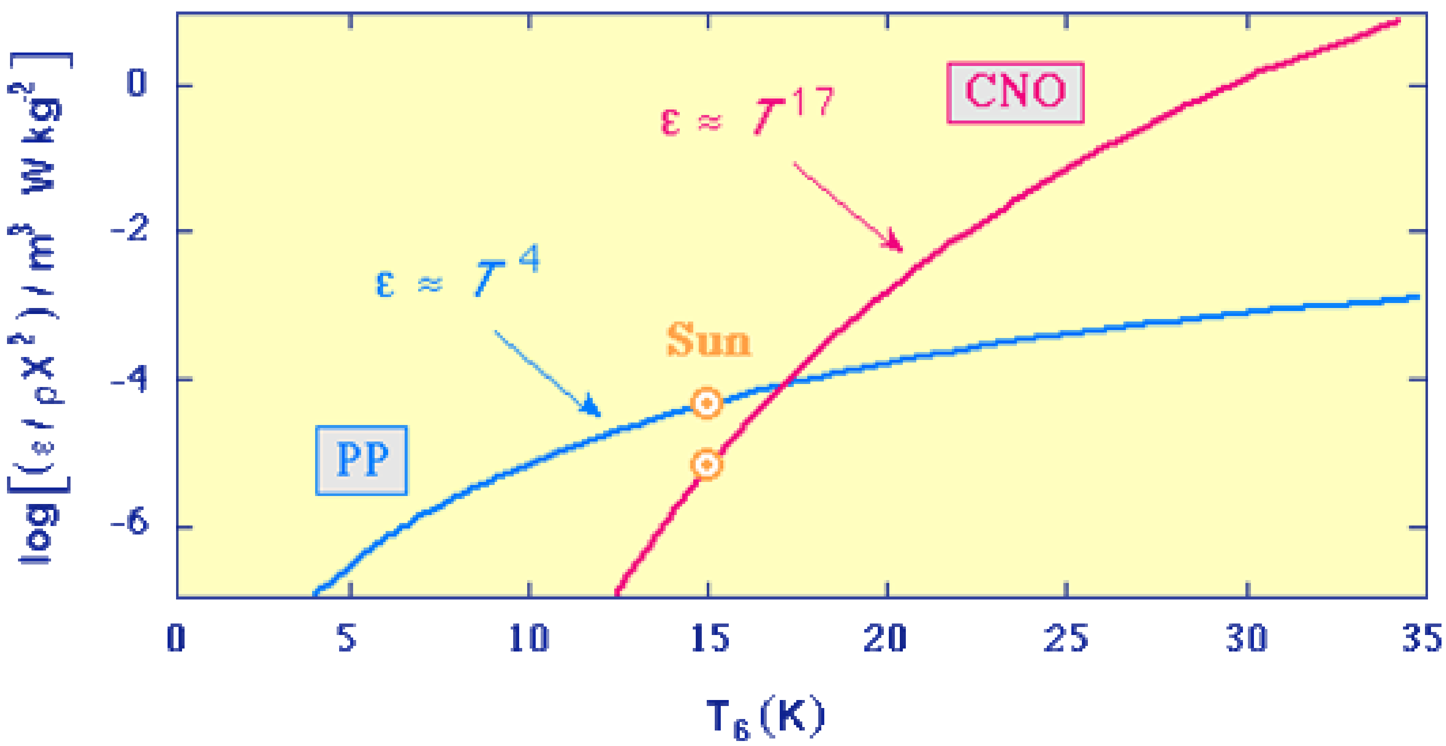
$$O^{16} + H^{1} \rightarrow F^{17} + \gamma$$

$$F^{17} \rightarrow O^{17} + e+ + \nu$$

$$O^{17} + H^{1} \rightarrow N^{14} + He^{4}$$

10^{4} times less probable

Relative contributions of pp and CNO



After the core-hydrogen burning stage

The 3α reaction:

$He^4 + He^4 \rightarrow Be^8$

$Be^8 + He^4 \rightarrow C^{12} + \gamma$

Other reactions involving He:

$C^{12} + He^4 \rightarrow O^{16} + \gamma$

$O^{16} + He^4 \rightarrow Ne^{20} + \gamma$

$Ne^{16} + He^4 \rightarrow Mg^{24} + \gamma$

Carbon Burning:

$C^{12} + C^{12} \rightarrow Mg^{24} + \gamma$

$C^{12} + C^{12} \rightarrow Na^{23} + p$

$C^{12} + C^{12} \rightarrow Ne^{20} + He^4$

$C^{12} + C^{12} \rightarrow Mg^{23} + n$

$C^{12} + C^{12} \rightarrow O^{16} + 2\ He^4$

Oxygen Burning:

$O^{16} + O^{16} \rightarrow S^{32} + \gamma$

$O^{16} + O^{16} \rightarrow P^{31} + p$

$O^{16} + O^{16} \rightarrow S^{31} + n$

$O^{16} + O^{16} \rightarrow Si^{28} + He^4$

$O^{16} + O^{16} \rightarrow Mg^{24} + 2\ He^4$

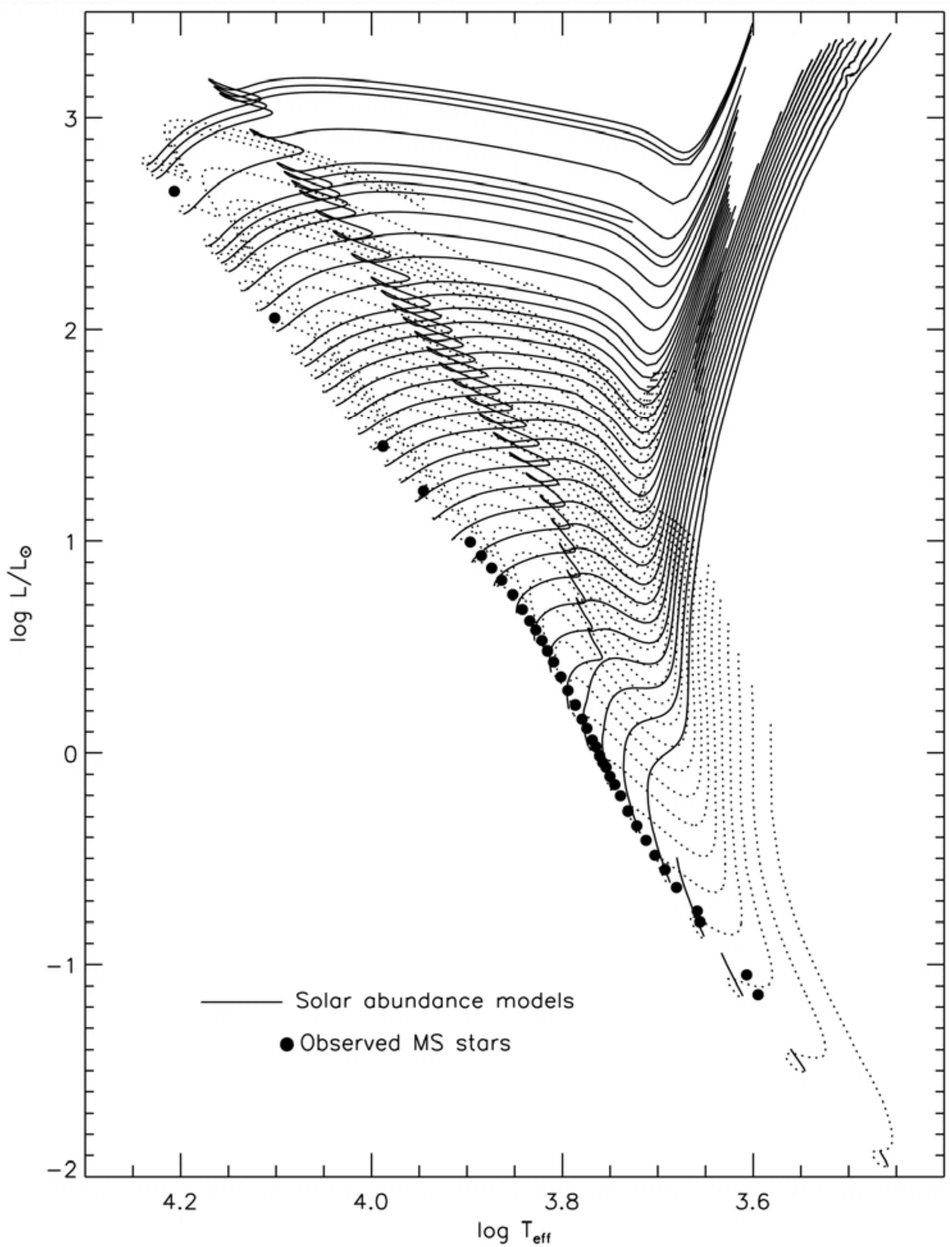
What else do we need to know?

(1) The mass
(2) The heavy-element and helium abundances.

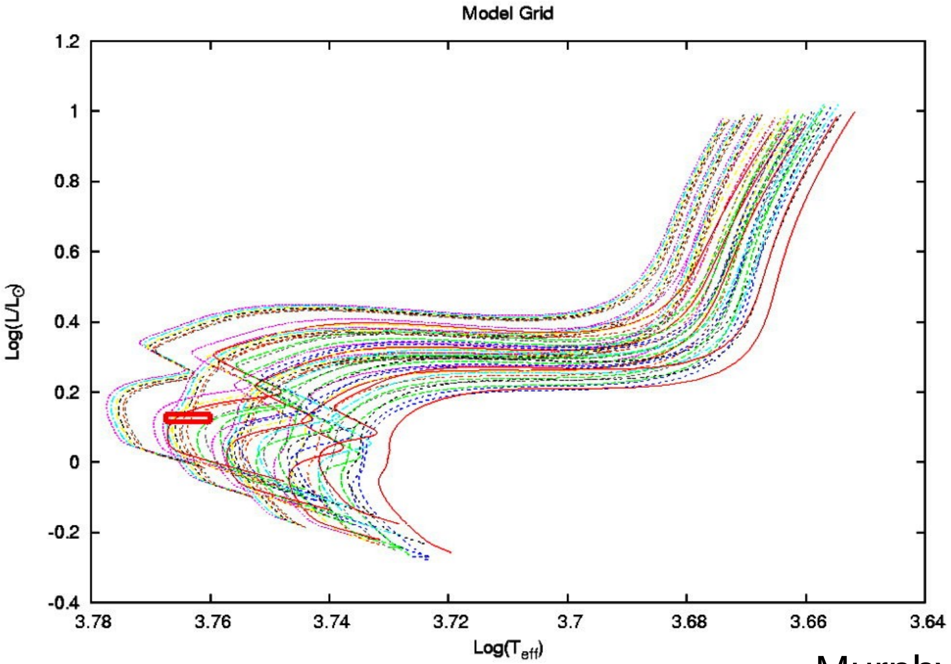
Once we have these quantities, we can start to model a star and calculate how it would evolve.

We also need to decide what we will use as the mixing length parameter.

Evolutionary tracks on the HR diagram



How does one model stars? Example: 51 Peg



Murphy & Demarque 2004

$L_{\rm bol} = 1.343^{+0.025}_{-0.037}\ L_\odot$ $T_{\rm eff} = 5805 \pm 50$ K, [Fe/H]=0.21± 0.06

How do we model the Sun?

We have two constraints at $t=4.57$Gyr:

(1) The luminosity of the Sun

(2) The radius of the Sun

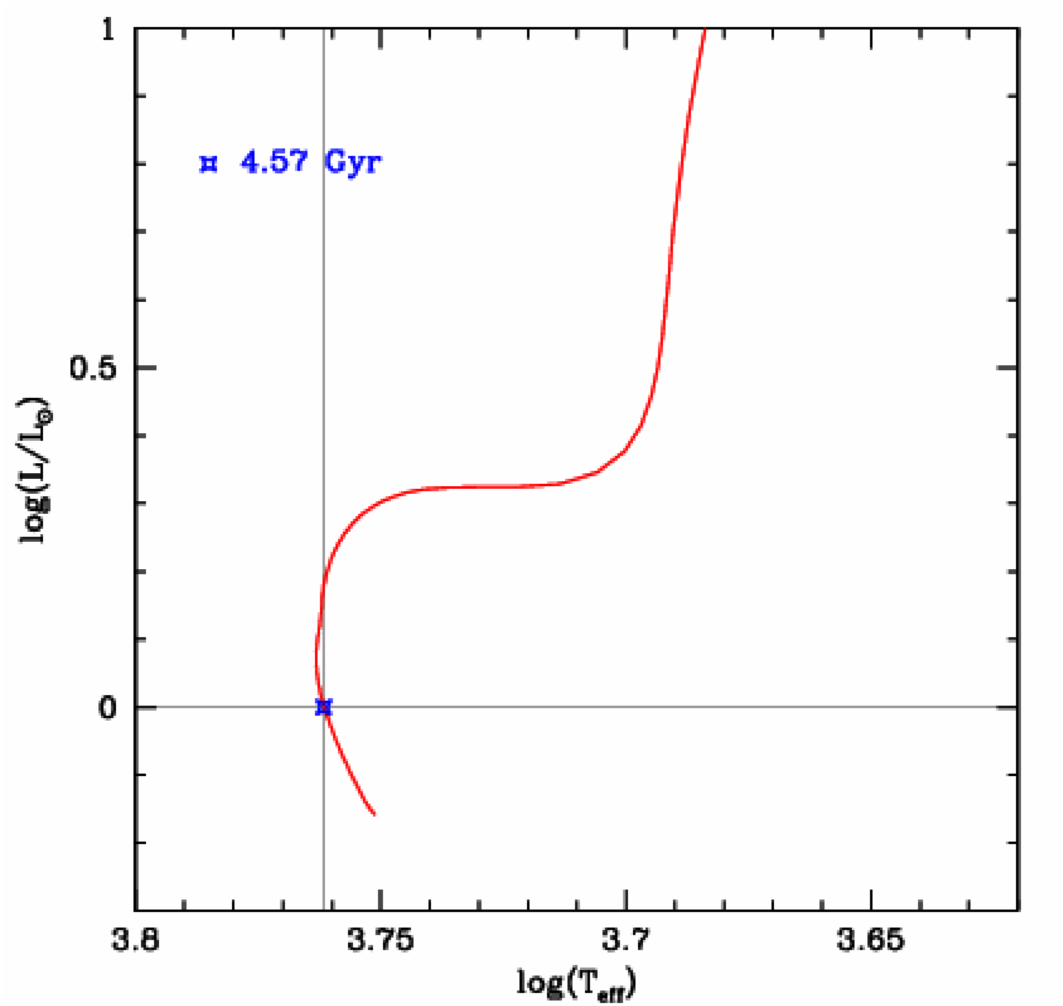
We have two "free" parameters to play with

(1) The initial helium abundance, Y_0, i.e., the helium abundance of the Sun at $t=0$

(2) The mixing length parameter α

We can, therefore, iterate. Start with a given Y_0 and a, evolve till 4.57 Gyr. Test how close the luminosity and radius is to $1L_O$ and $1R_O$. Find corrections to Y_0 and α, evolve again, repeat till convergence is reached. Also need to change initial Z to get observed Z/X today.

The Solar Track



1Msun,

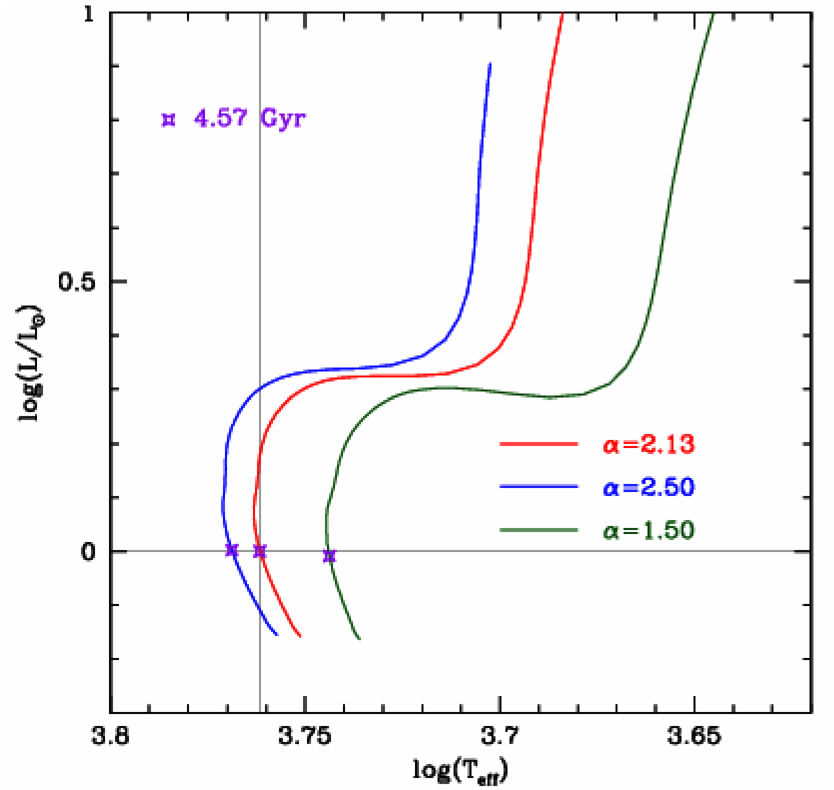
1Lsun, 1Rsun at 4.57 Gyr

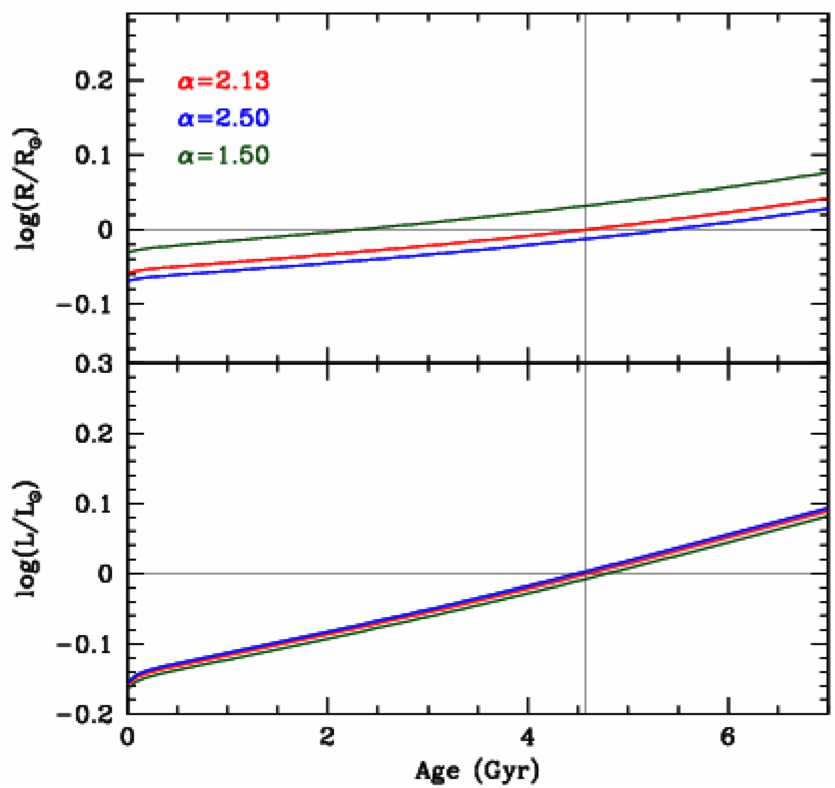
Z/X=0.023

α=2.13

Y_o=0.278

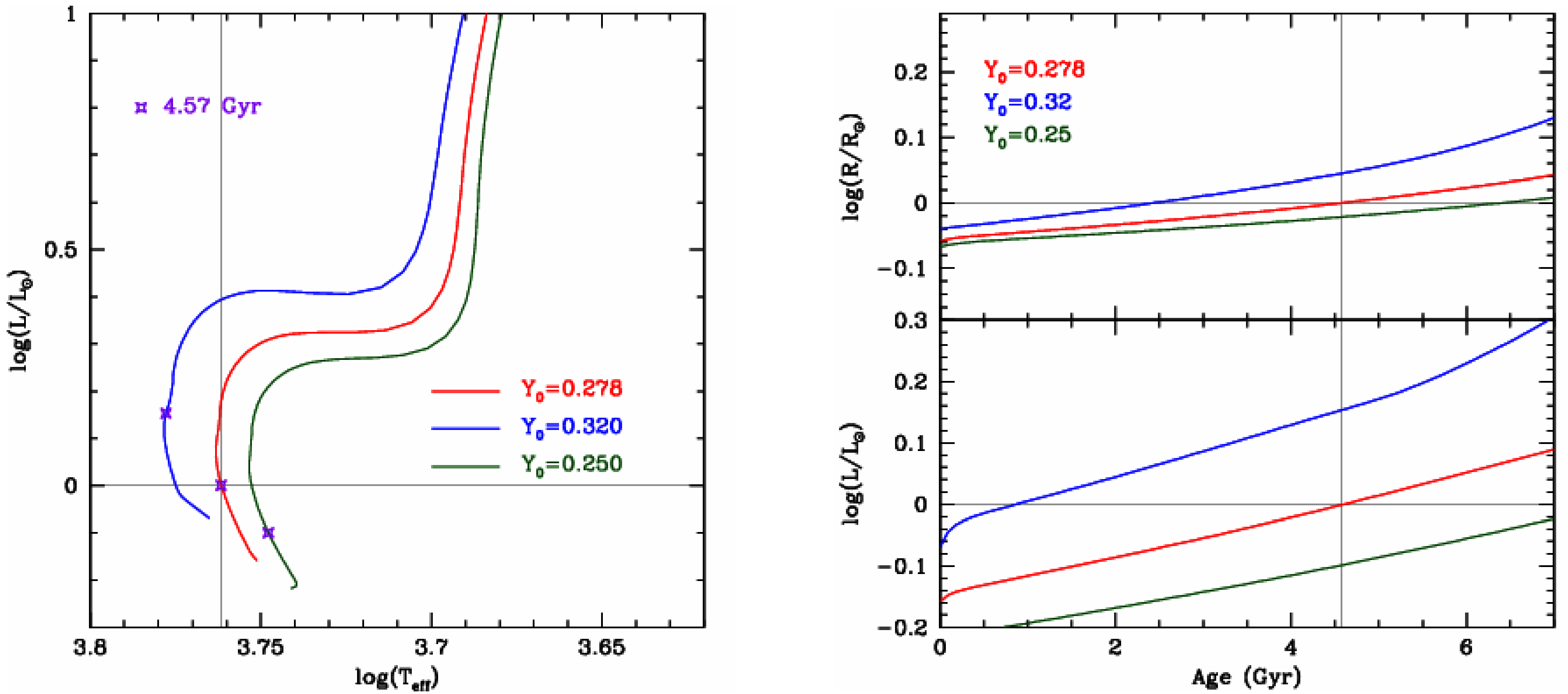
Effect of α





Increasing α makes stars bluer. Main effect is on radius — radius is smaller for larger α. Luminosity is not affected much.

Effect of initial Helium abundance

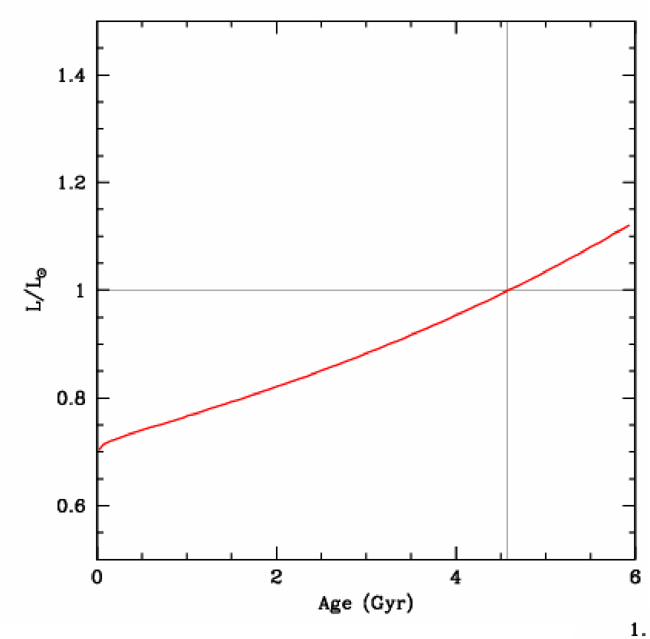


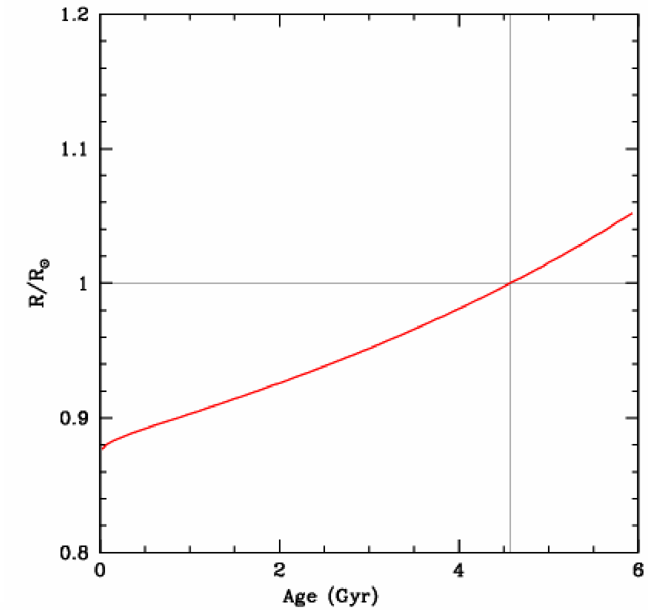
Increasing Y makes stars bluer and more luminous. Main effect is on luminosity. Effect on radius is much smaller.

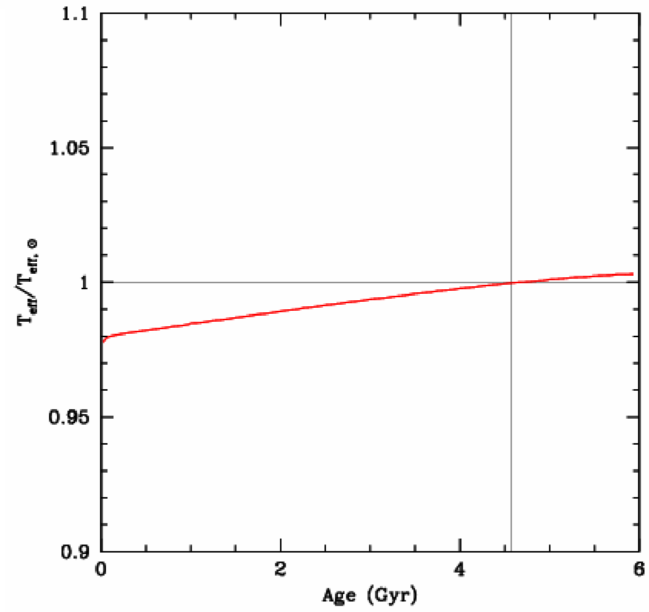
Standard Solar Models

A standard solar model is one where the physical inputs are not changed to bring the model in better agreement with the Sun. The input physics (nuclear reaction rates, diffusion coefficients, opacity tables, equation of state) are input as they are. The agreement or otherwise between the Sun and the model is a indication of how good the input parameters.

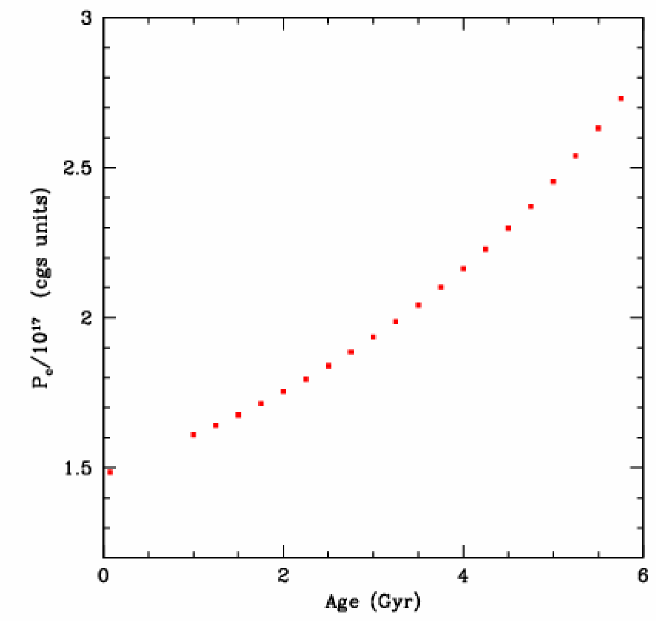
The evolving Sun – I

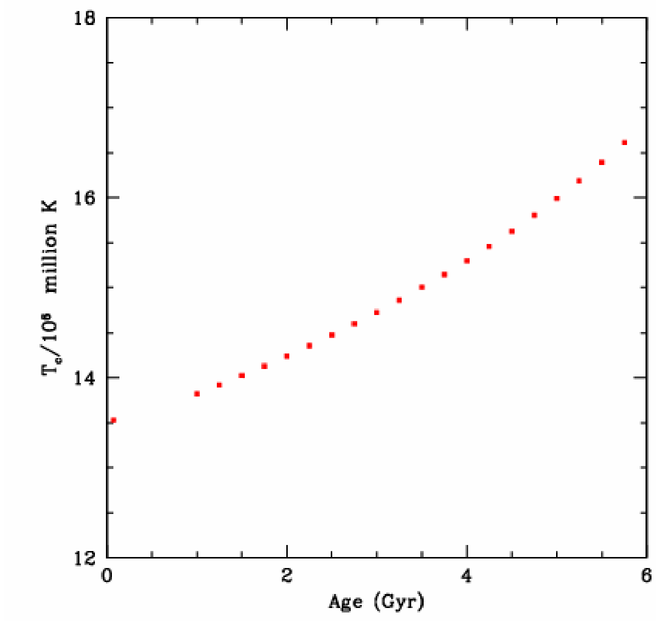


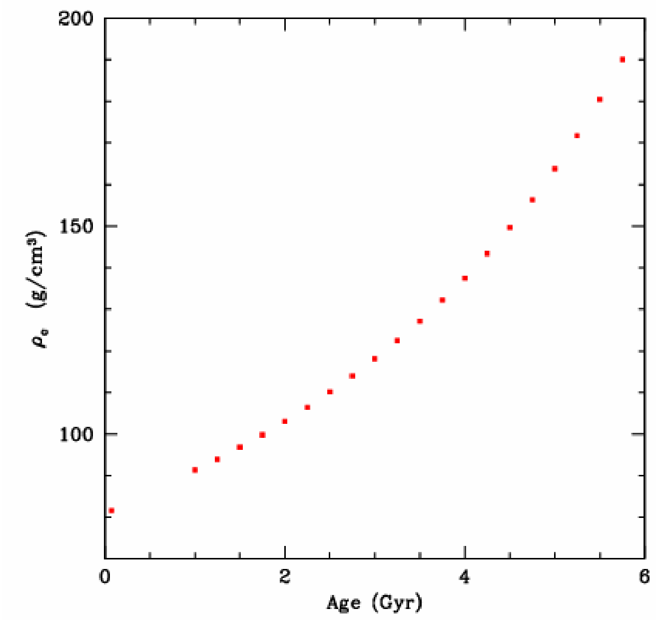




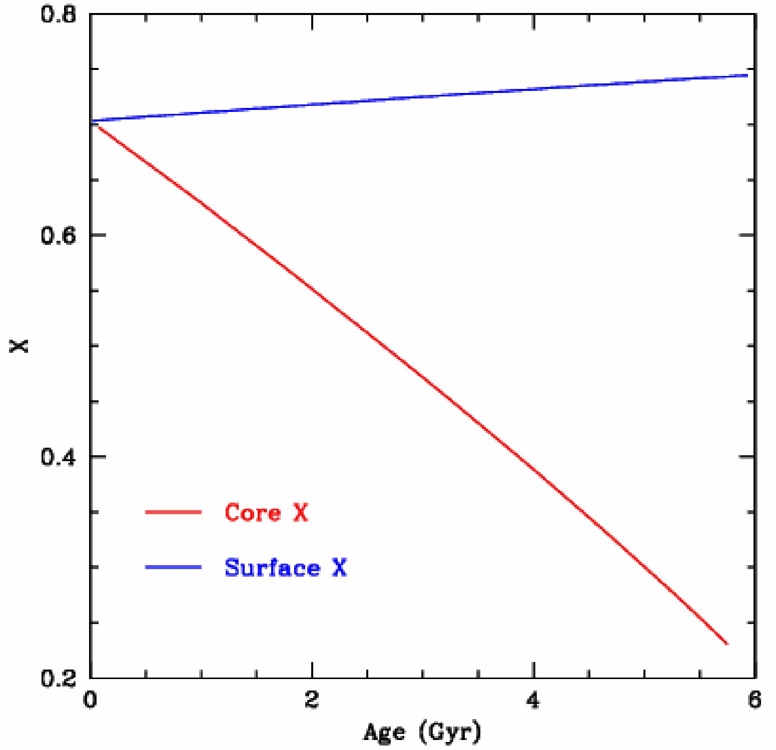
The evolving Sun – II

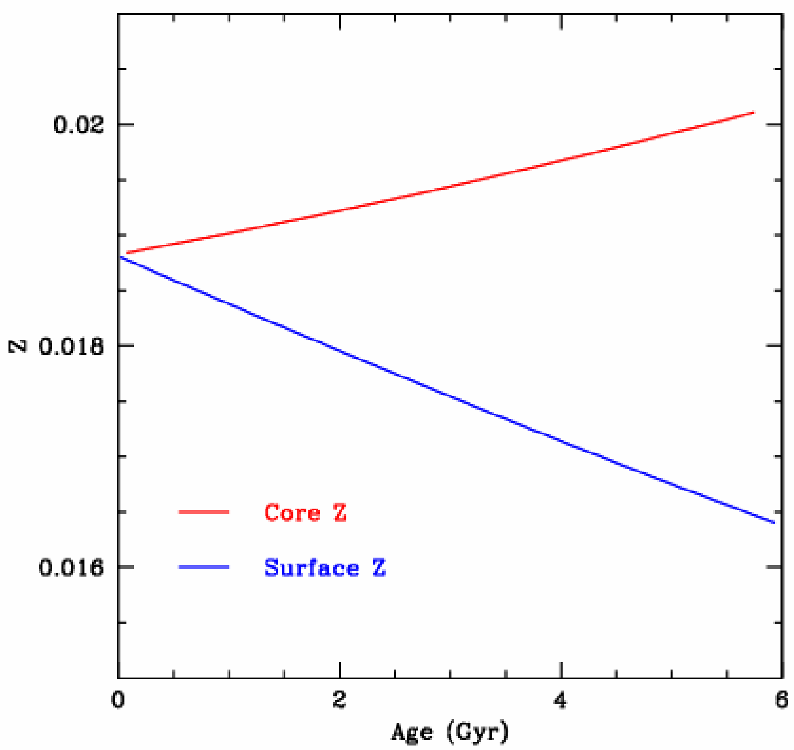




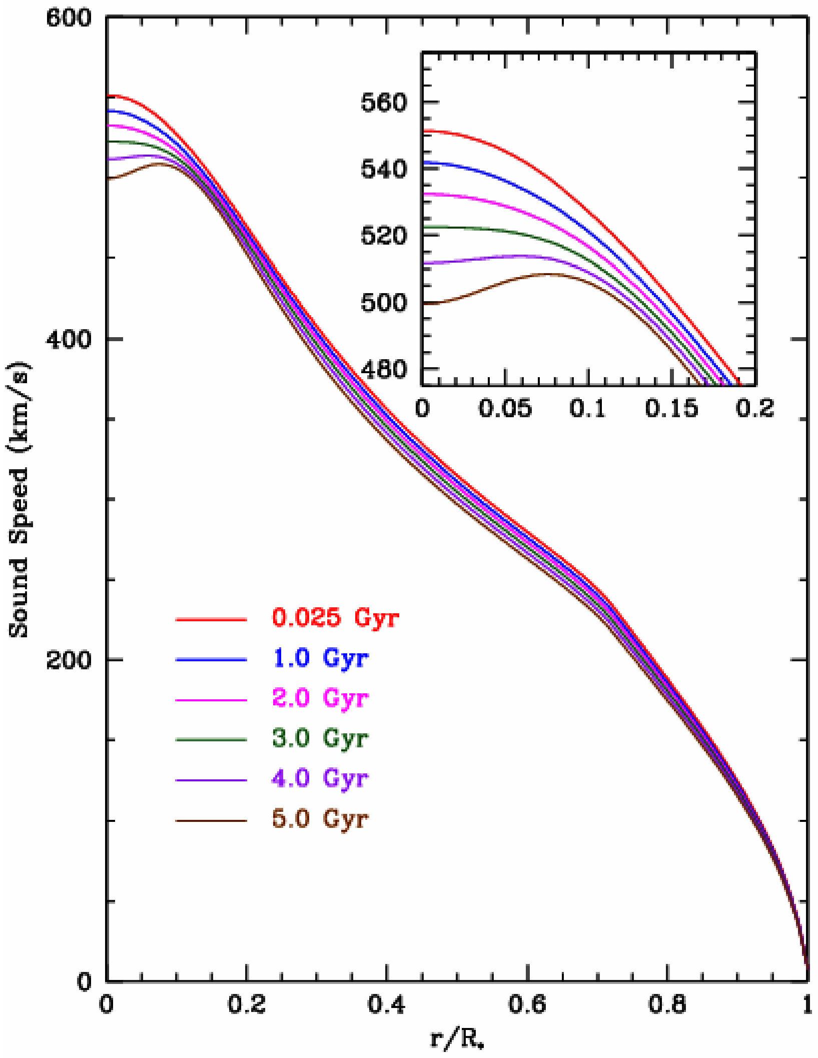


The evolving Sun – III



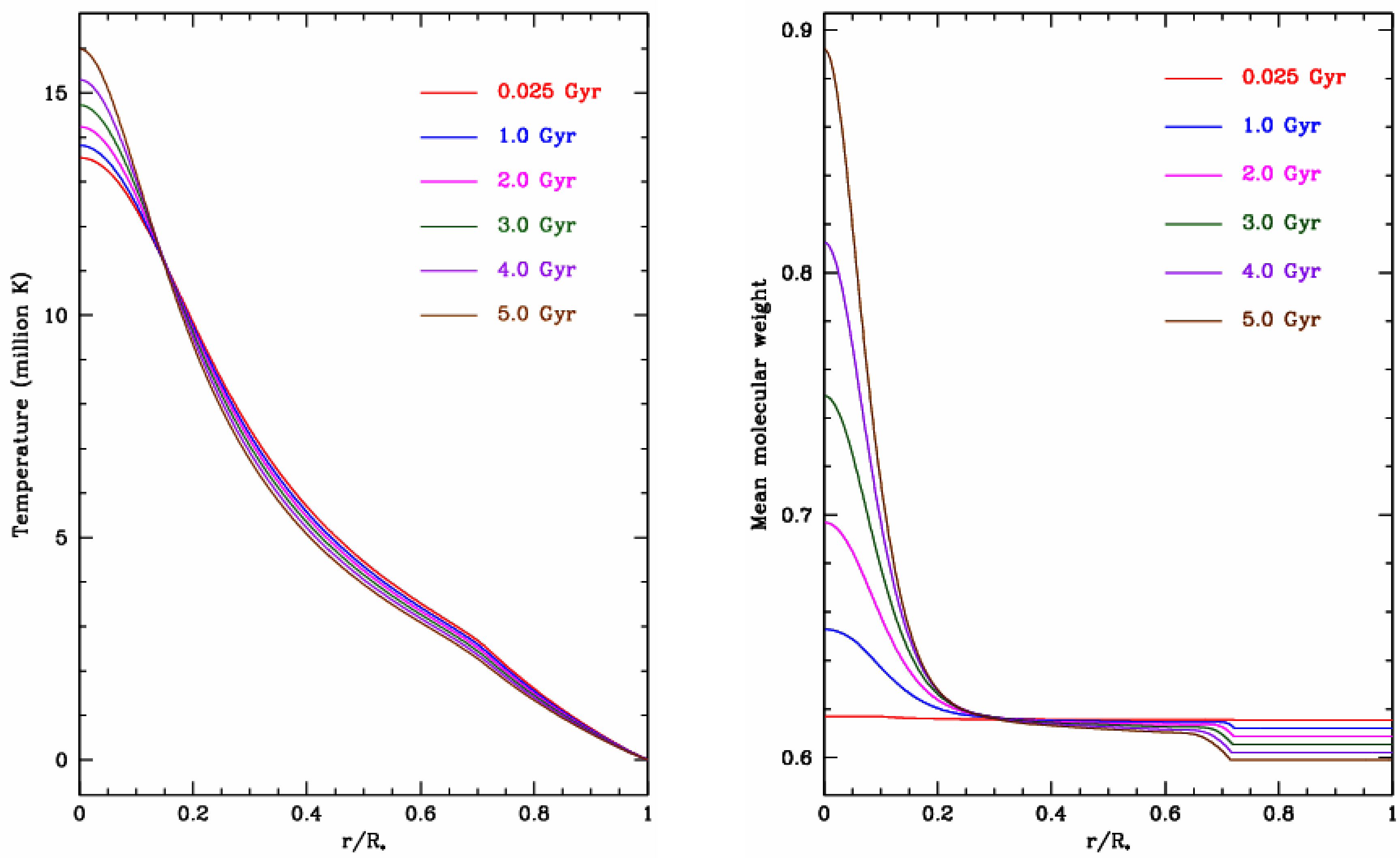


The Sun across the ages – I

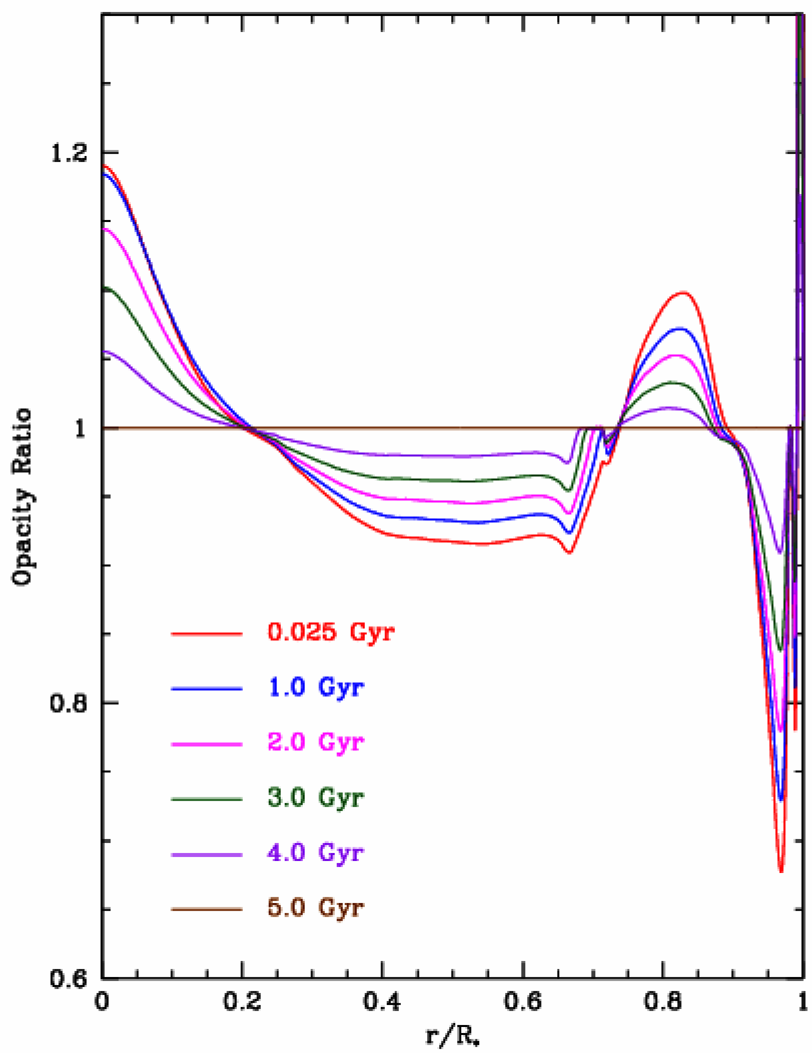


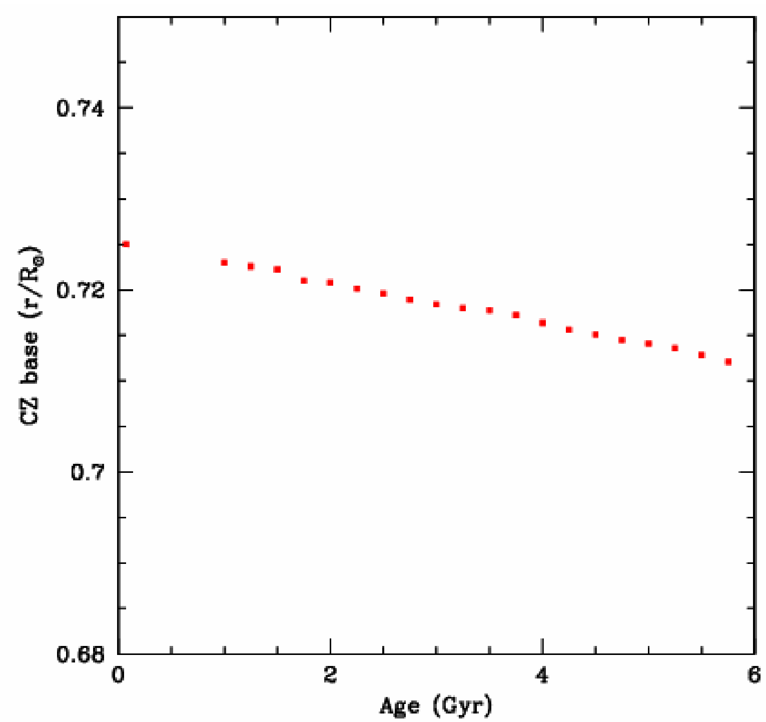
The Sun across the ages – II

$$c^2 \propto \frac{T}{\mu}$$

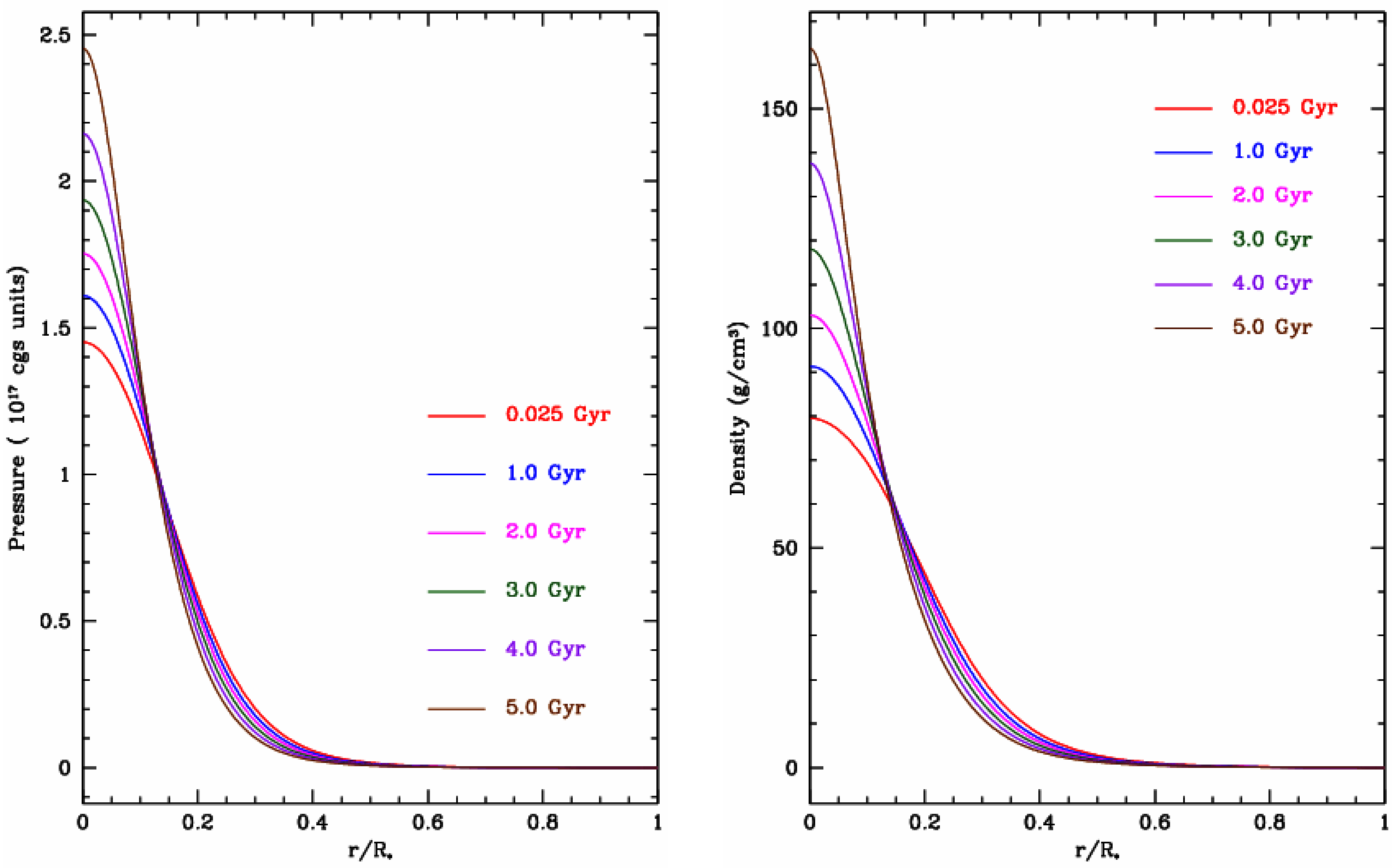


The Sun across the ages – III

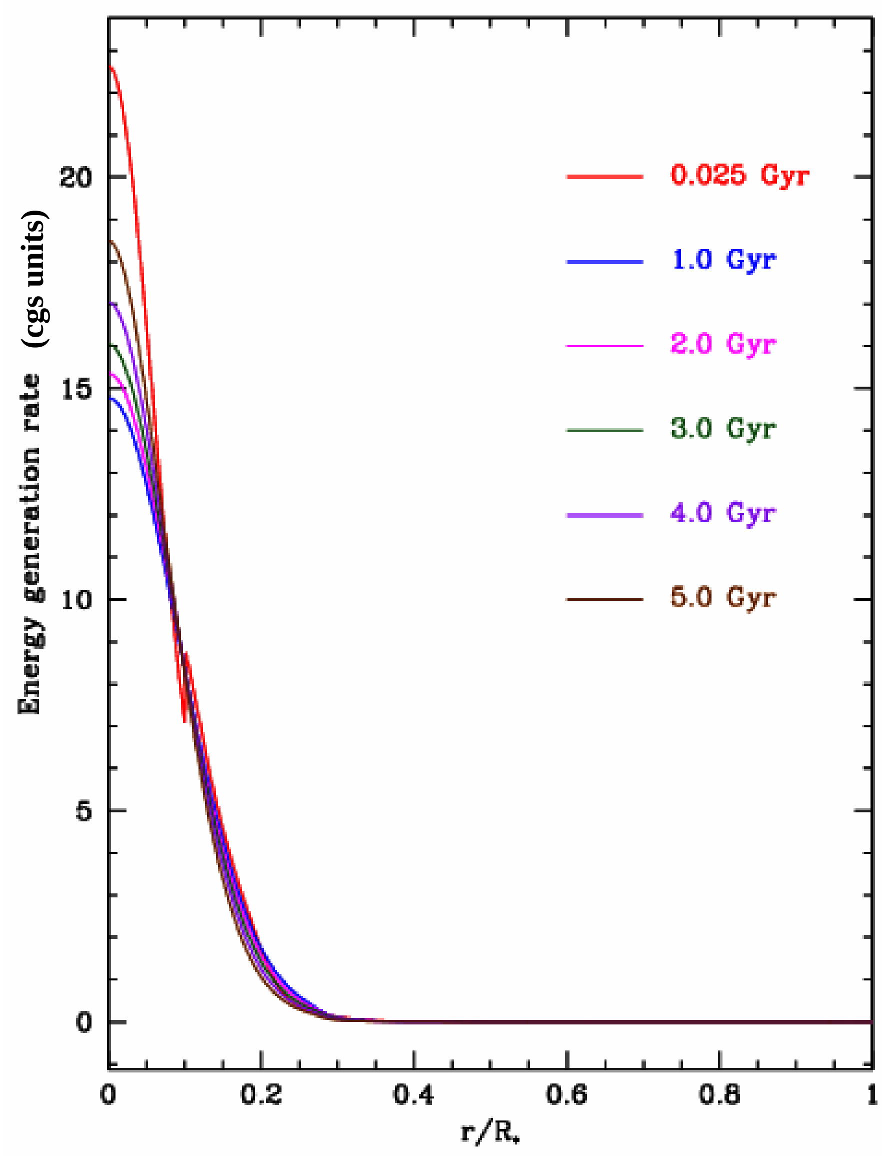




The Sun across the ages – IV



The Sun across the ages – V



Why is the Sun getting more centrally condensed?

For hydrogen burning, $\varepsilon \propto \rho X^2 T^n$

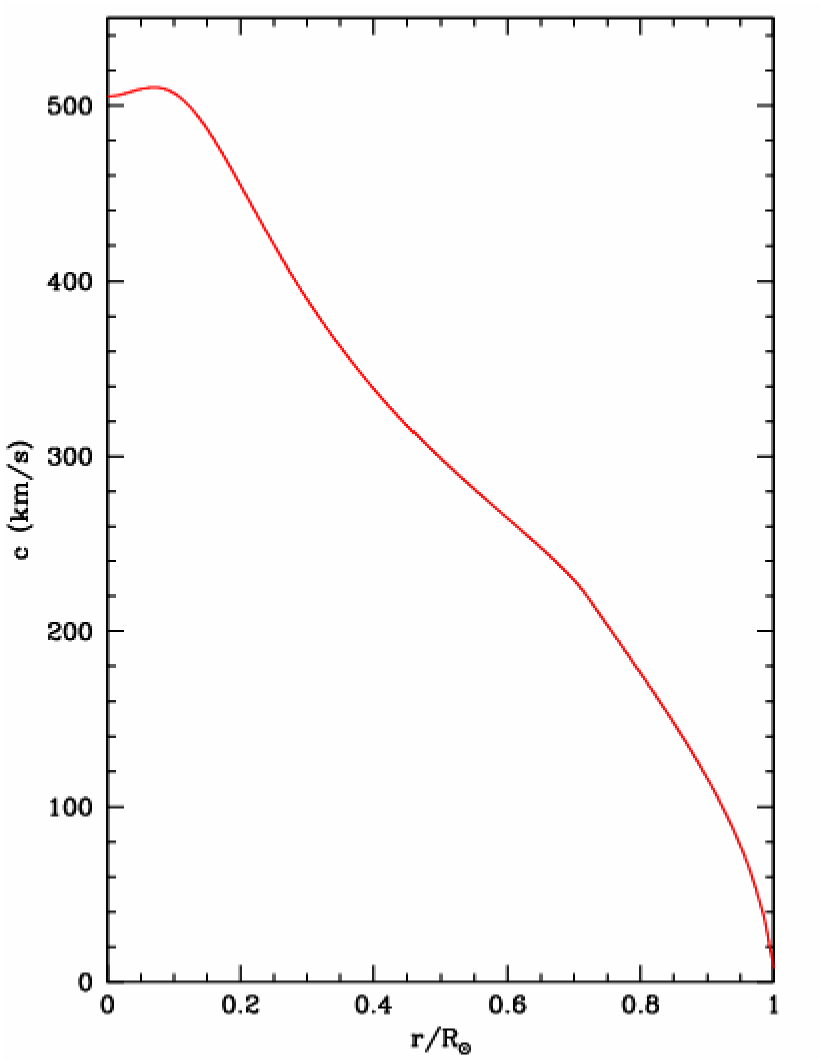
As H is used up, ε will decrease unless ρ or T (or both) increases. P also decreases because H-fusion reduces the number of particles.

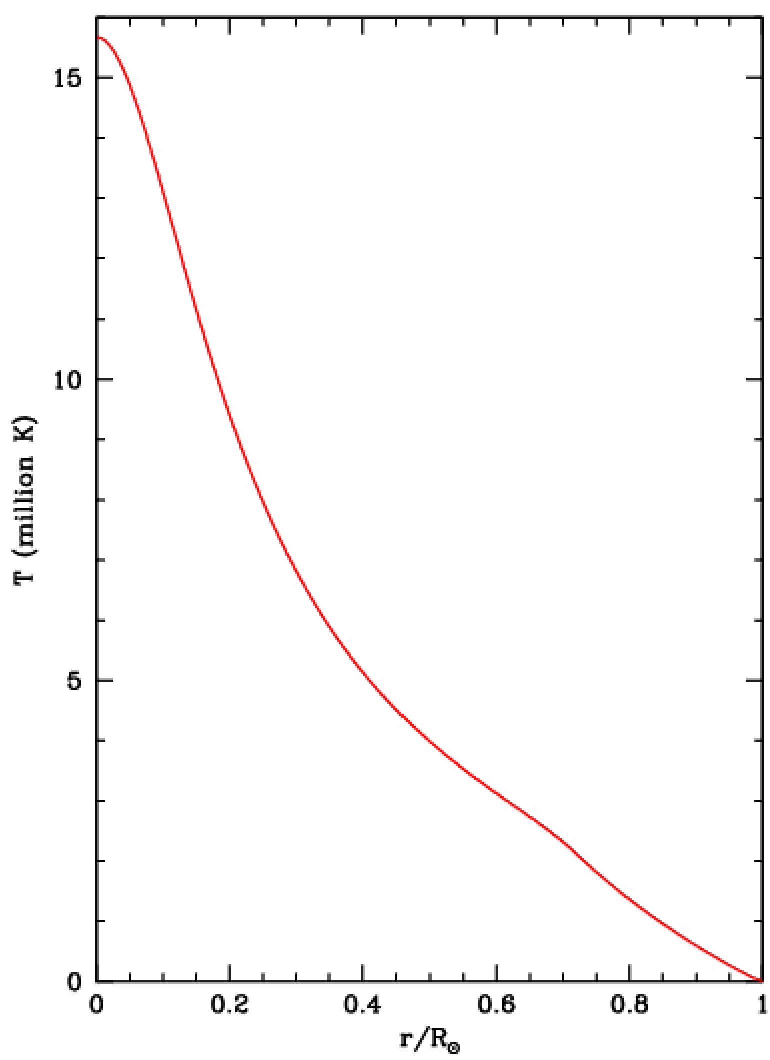
The decrease in P causes the core to contract, increasing P, ρ, T and ε

When the core contracts, the outer layers of the star expands. Thus one gets an increased density and pressure in the core compared to the outer regions.

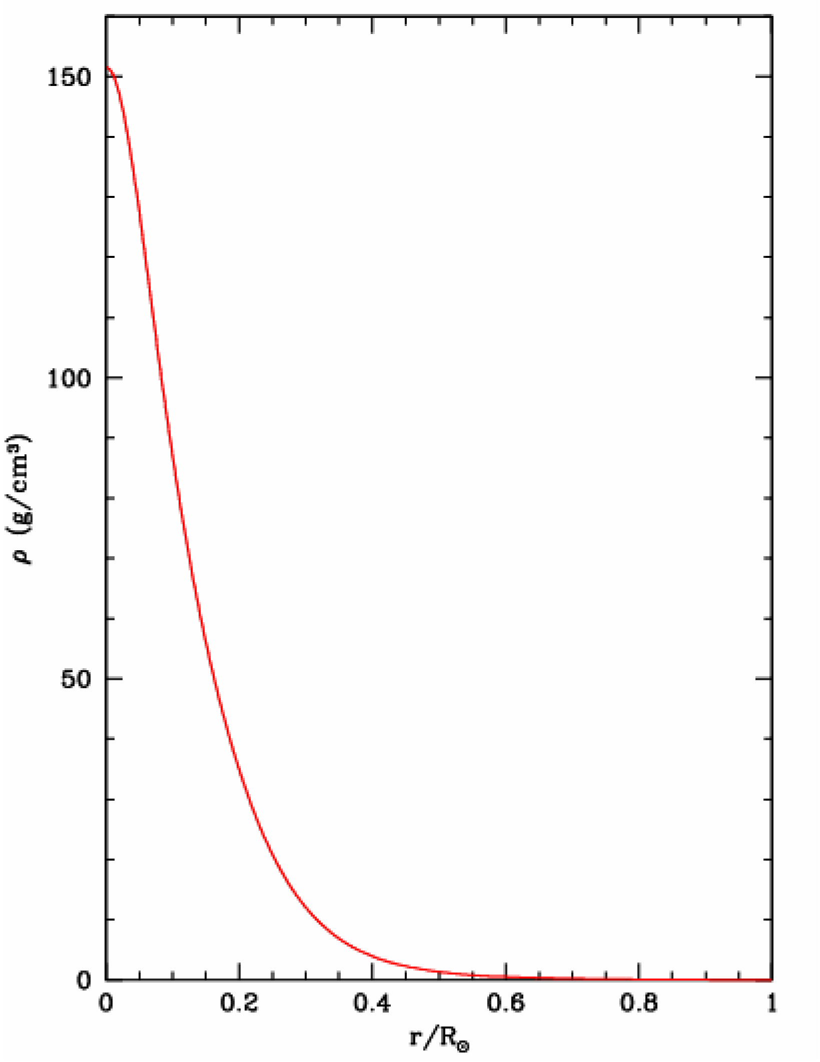
Rule of thumb in stellar evolution: *If core contracts, envelope expands, if core expands envelope contracts.* Why? When core contracts, T increases, therefore T gradient increases. However, T gradient must be compatible with flux that needs to be transported, therefore envelope (the outer parts of the star) expands to decrease the T gradient to original value. The opposite happens if core expands — T gradient decreases, there envelope has to contract.

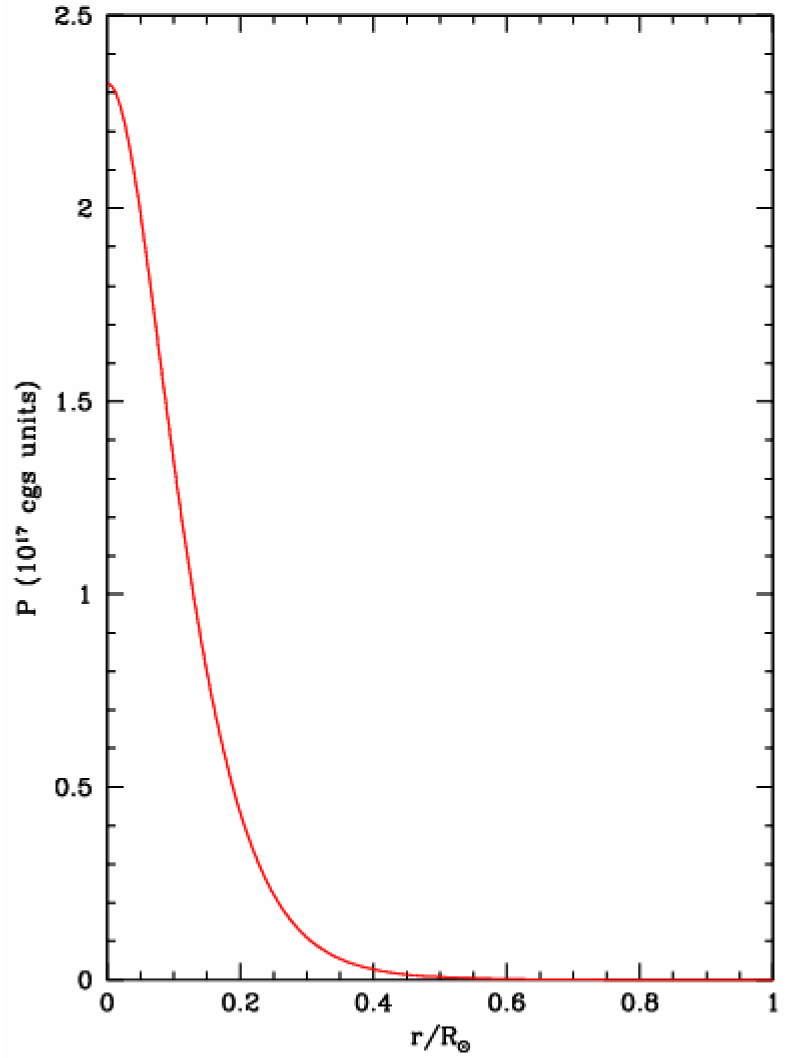
Model of the present-day Sun – I



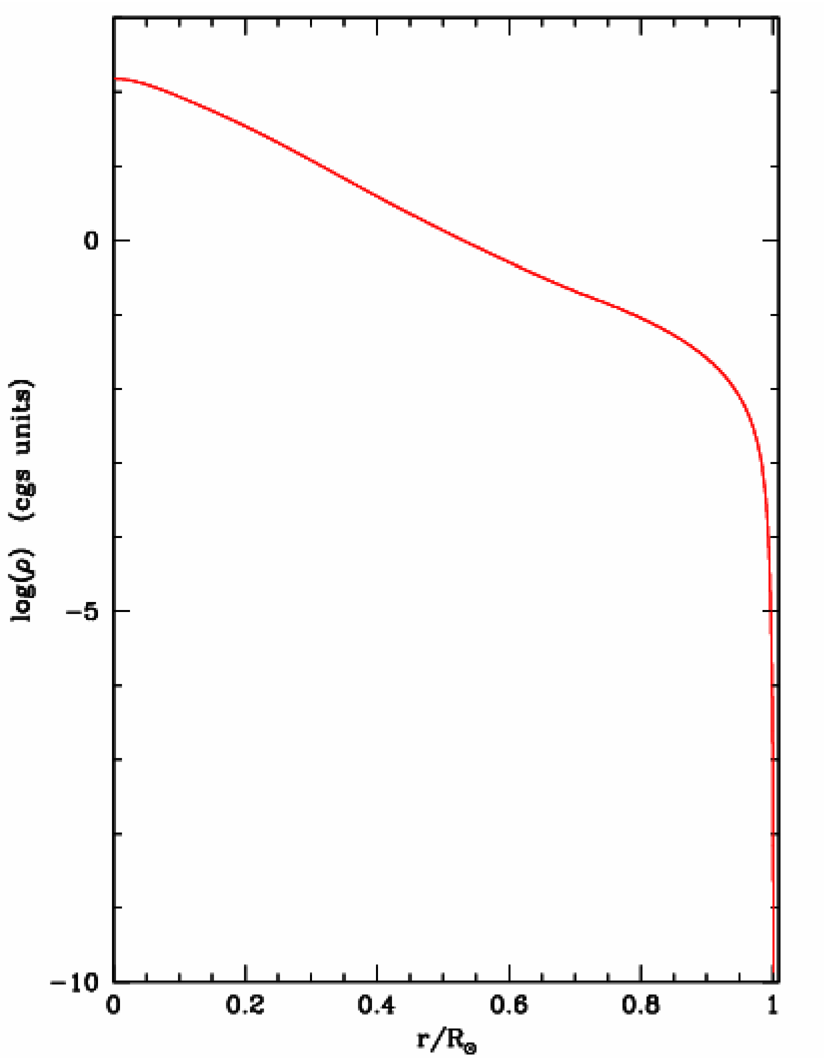


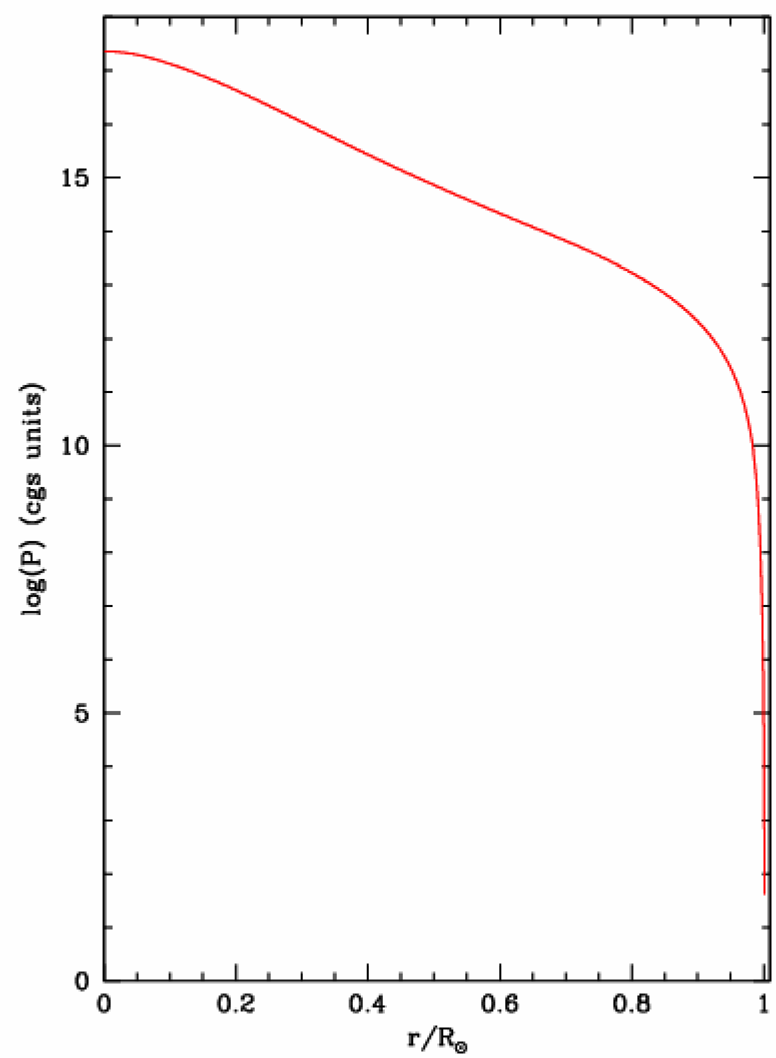
Model of the present-day Sun– II



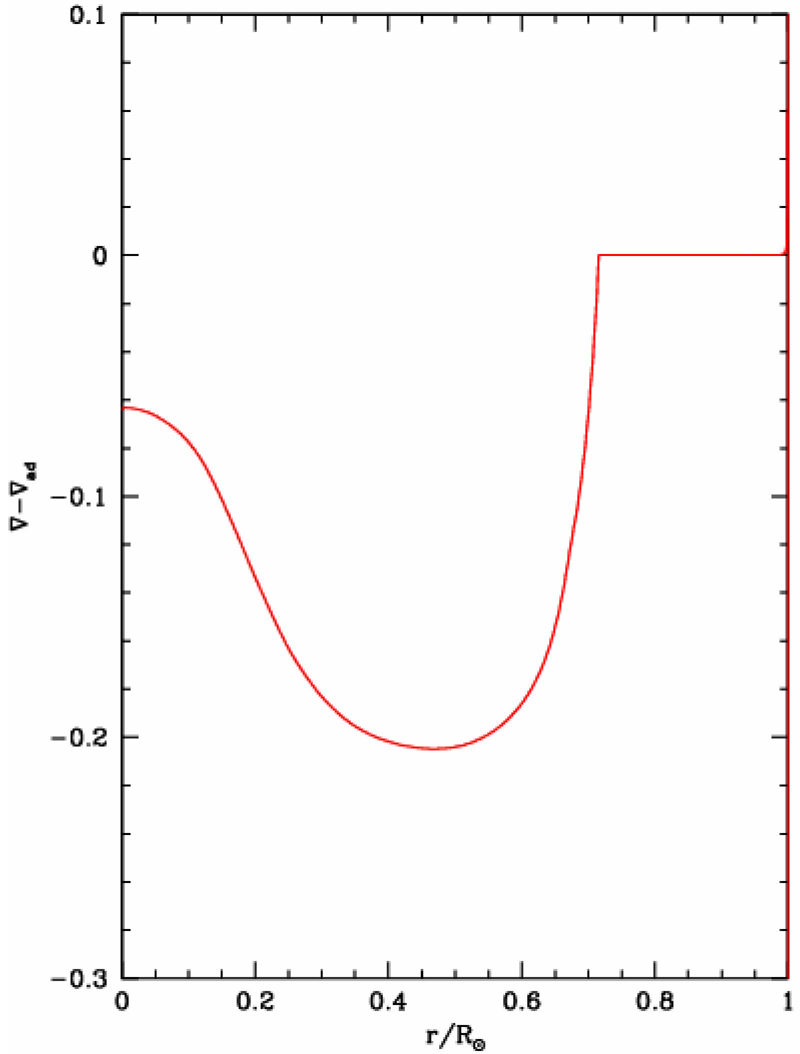


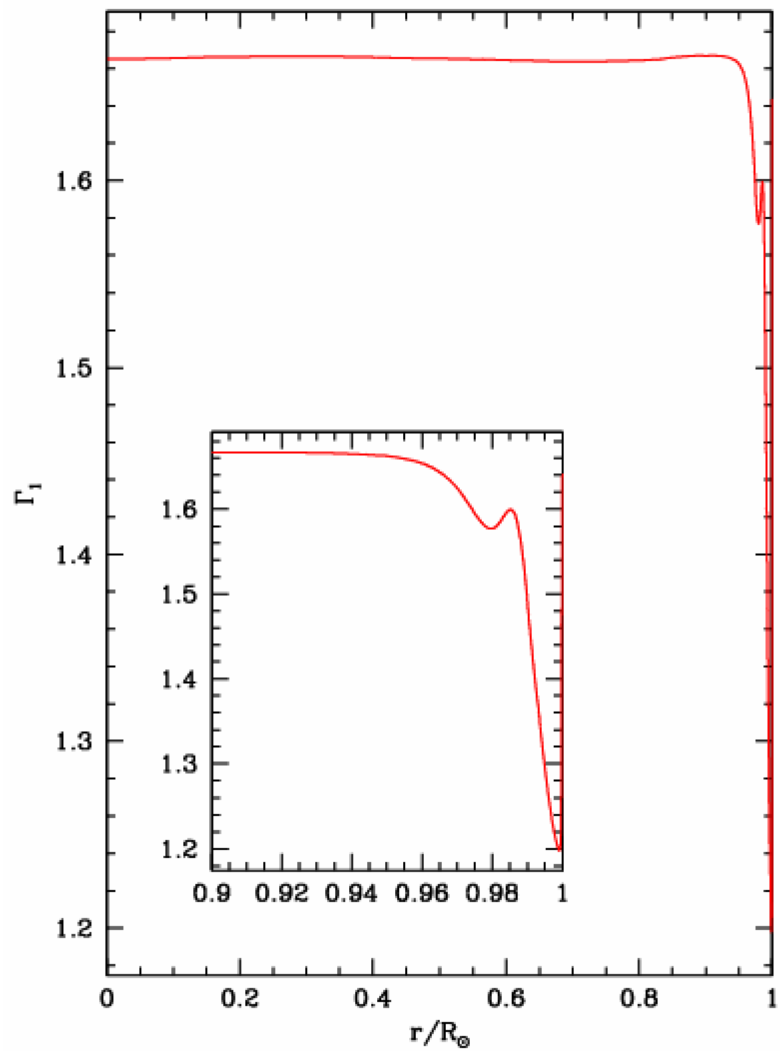
The large variation of ρ and P



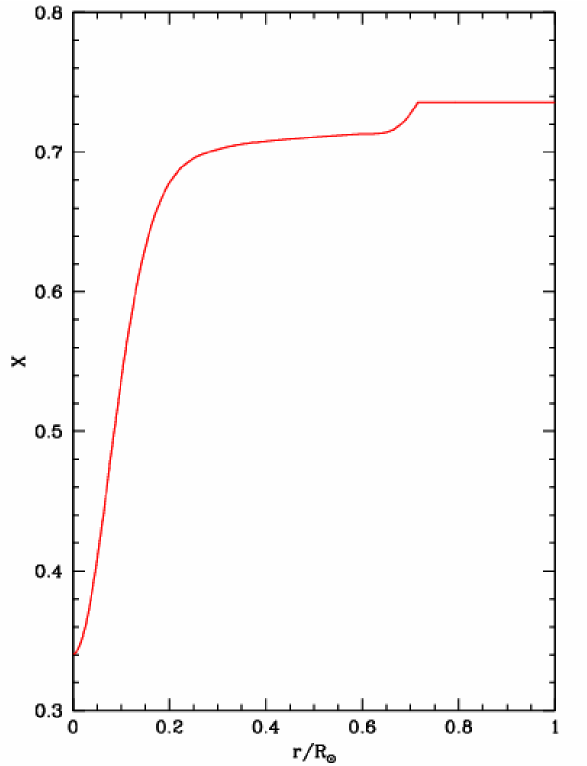


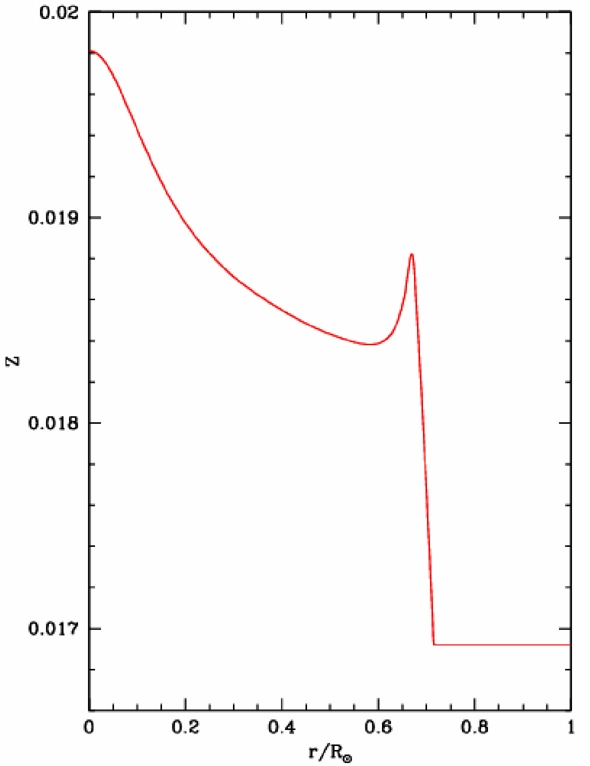
Model of the present-day Sun – III

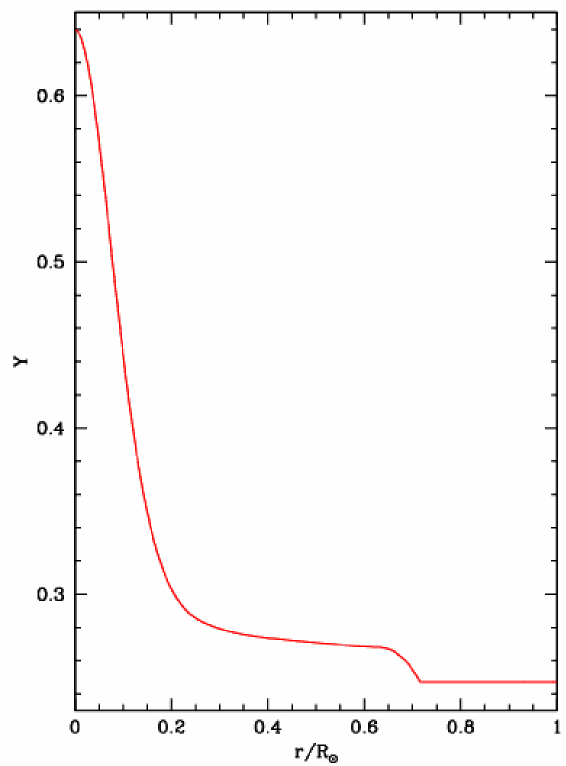




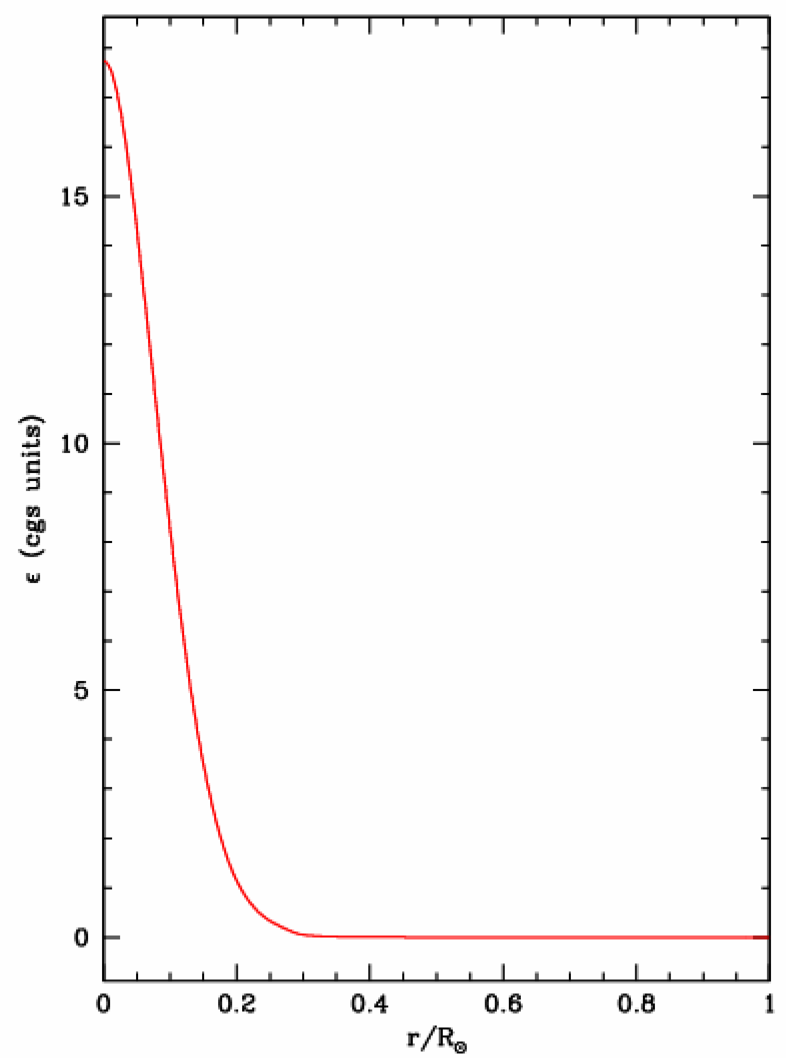
Model of the present-day Sun – IV







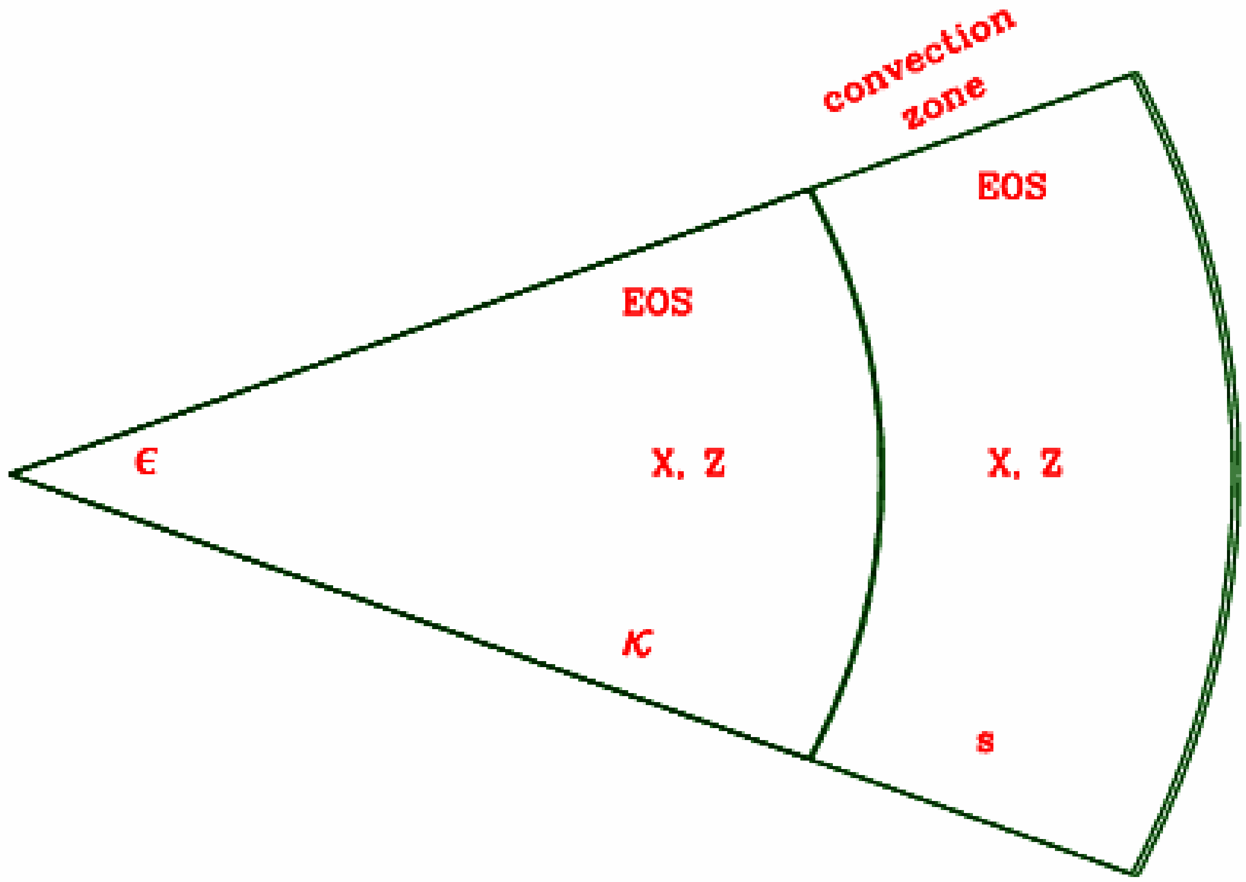
Model of the present-day Sun – V



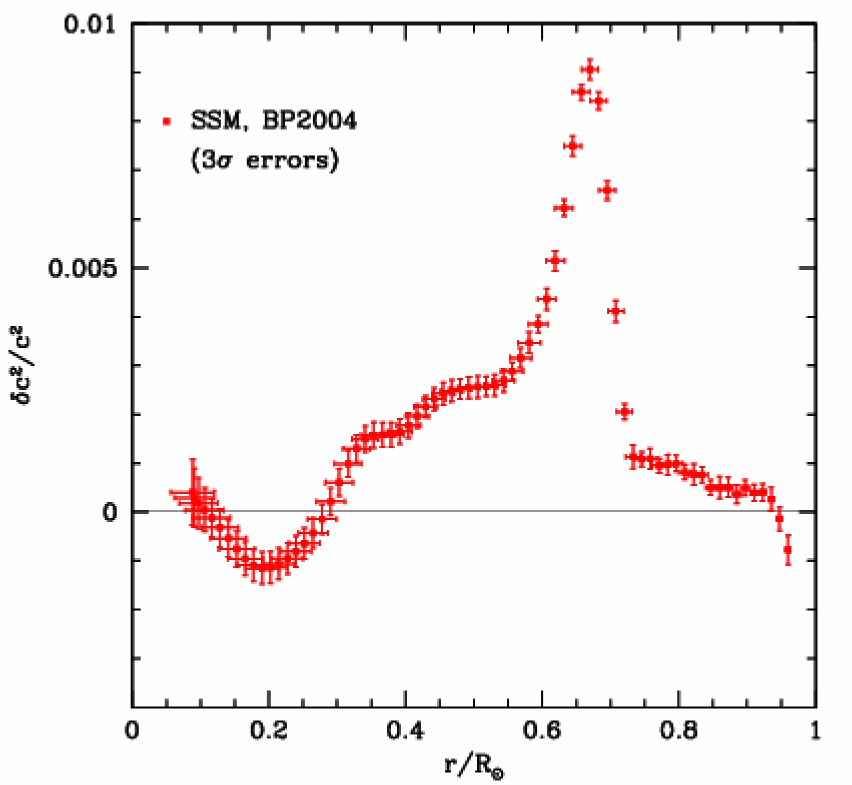
How do we know that a solar model is good?
Helioseismology!

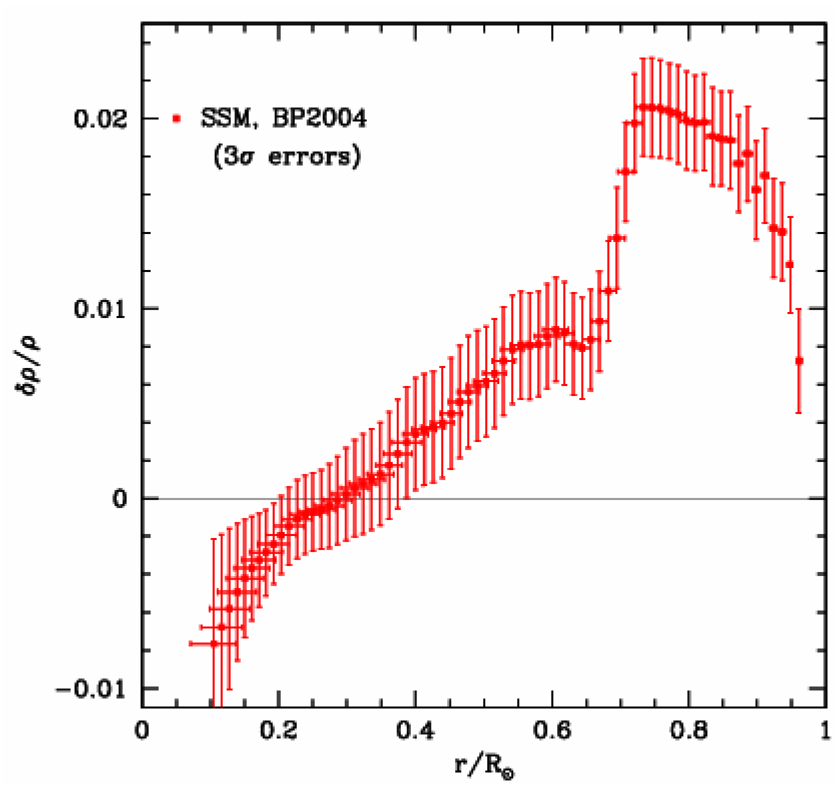
We know
(1) The position of the CZ base (0.713R)
(2) The CZ helium abundance (0.249)
(3) The sound speed, density and Γ_1 profiles.

The Schematic Sun: What input affects where?



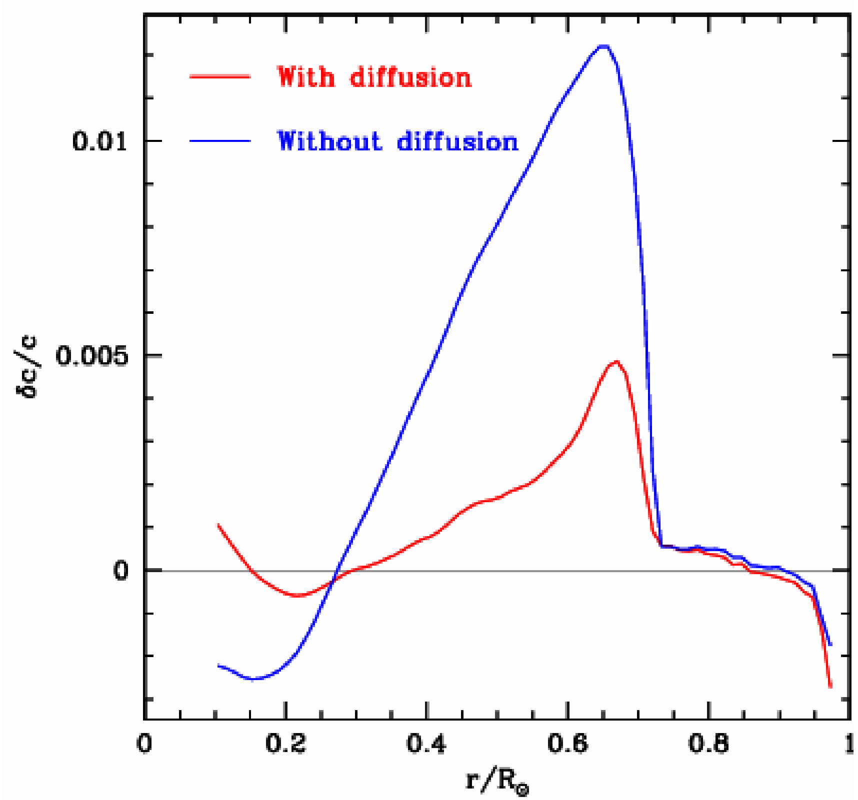
A standard solar model

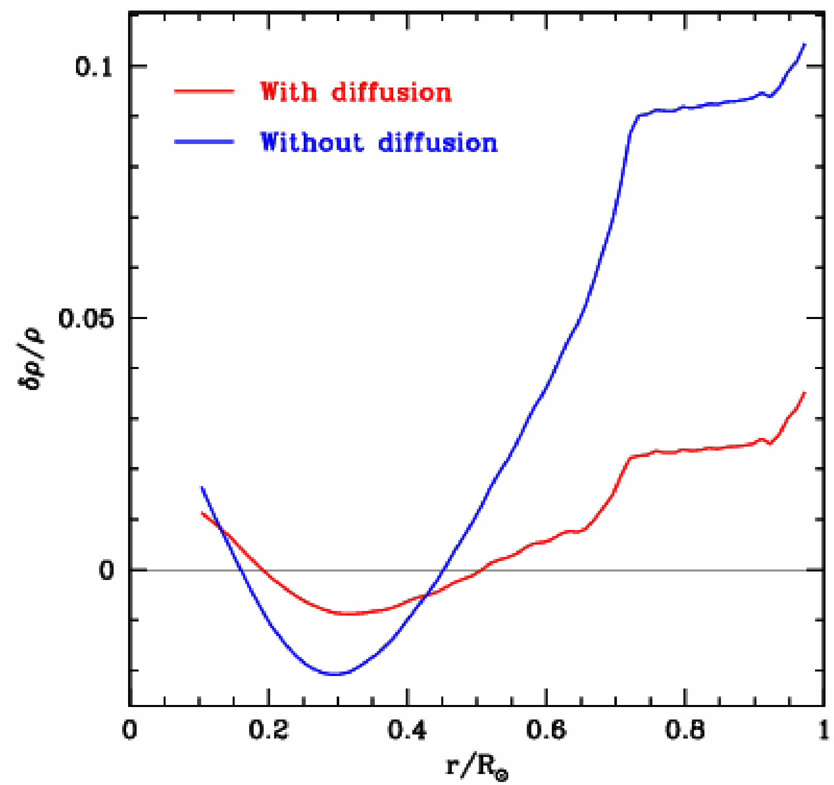




Model from Bahcall, Basu & Serenelli (2005)

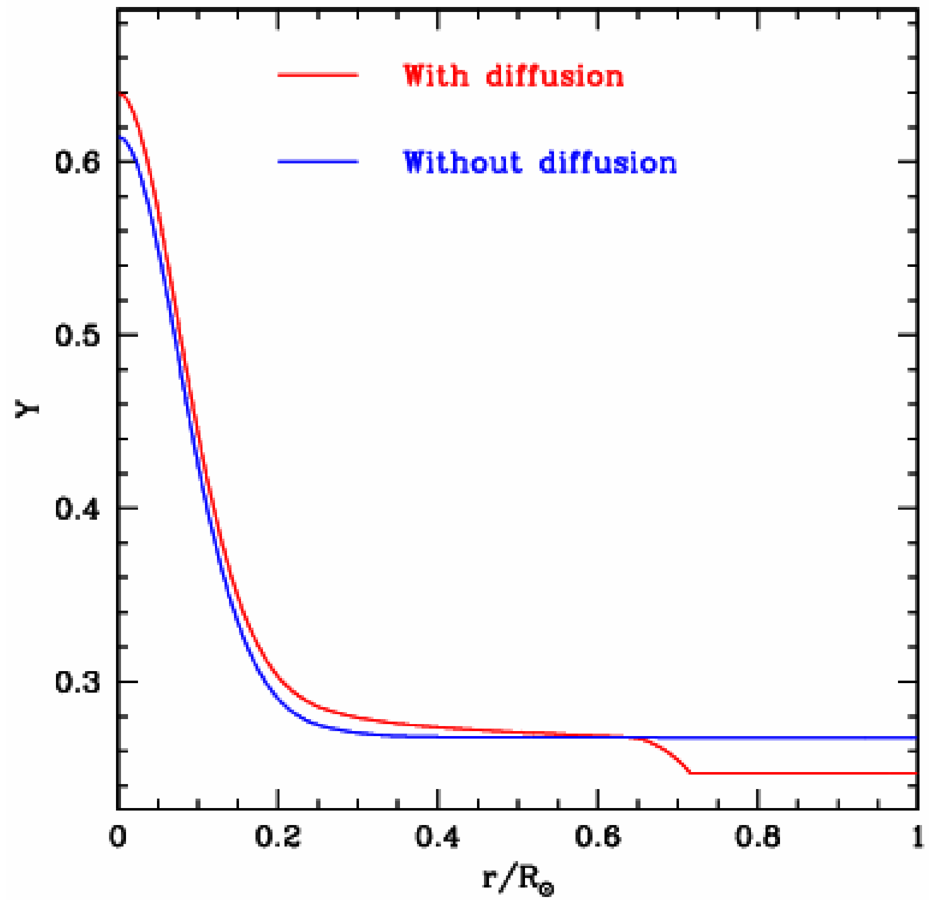
The effect of diffusion



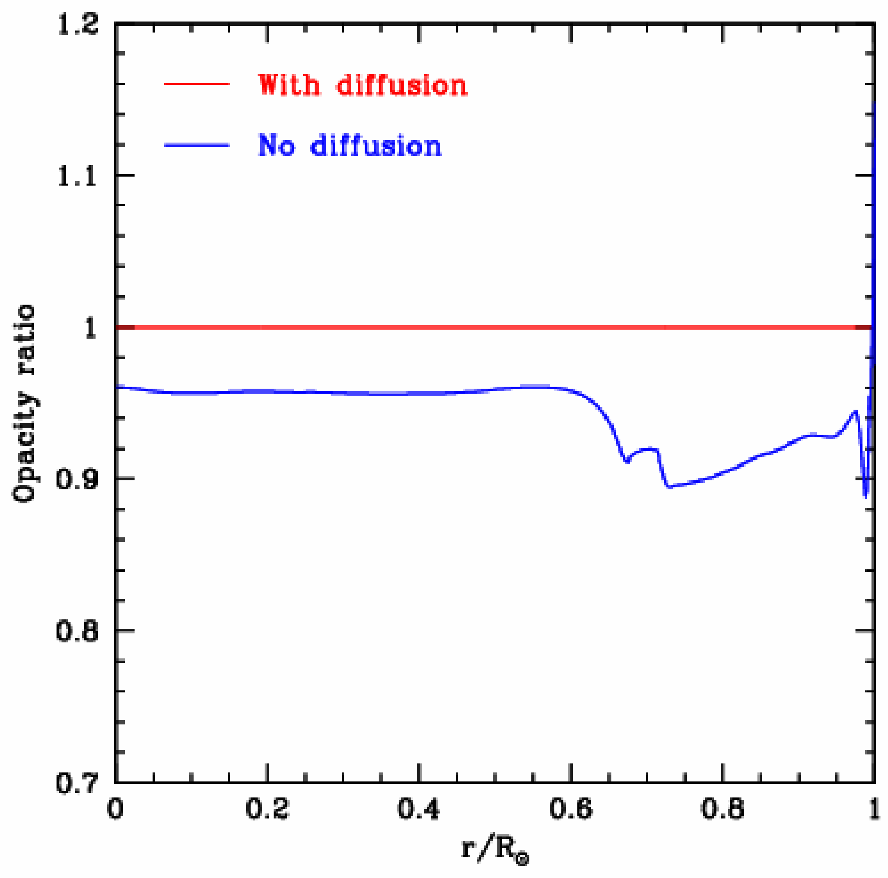


Main reason for disagreement: mismatch on position of CZ base

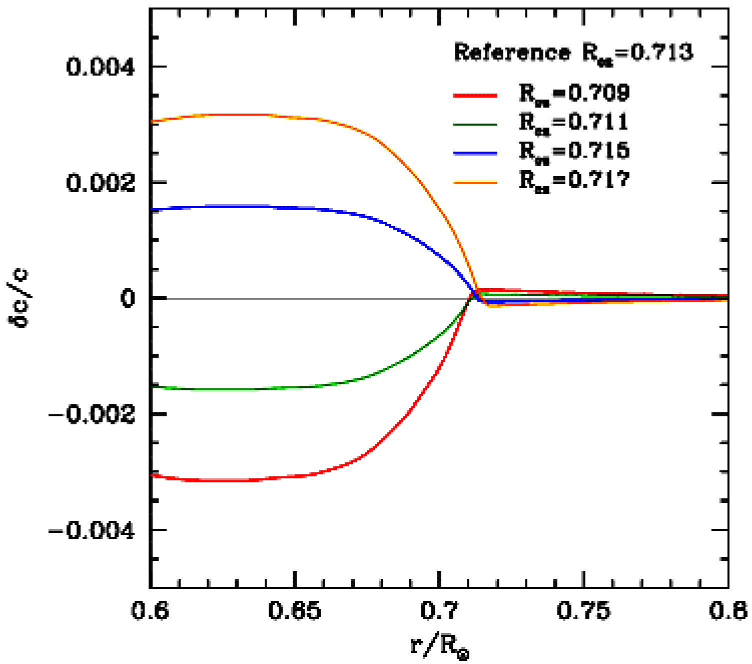
Diffusion and the abundance profile

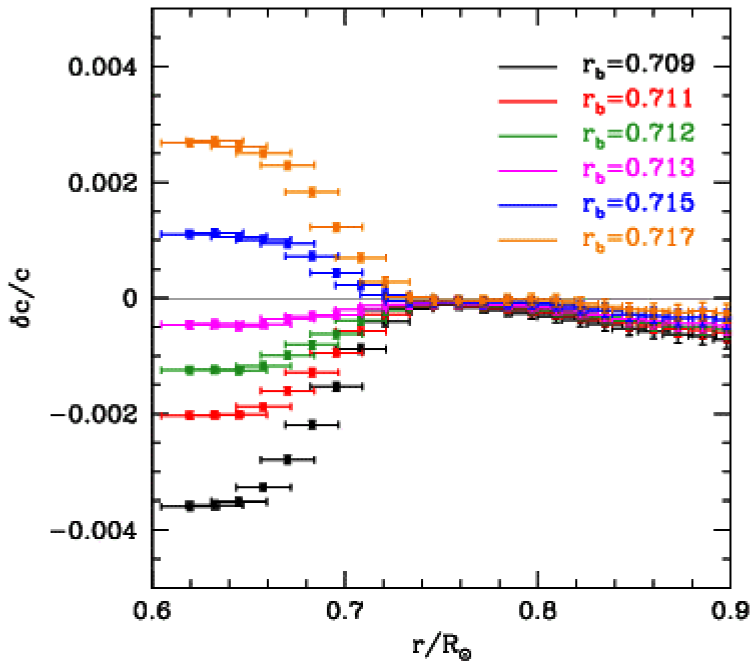


Diffusion and opacity



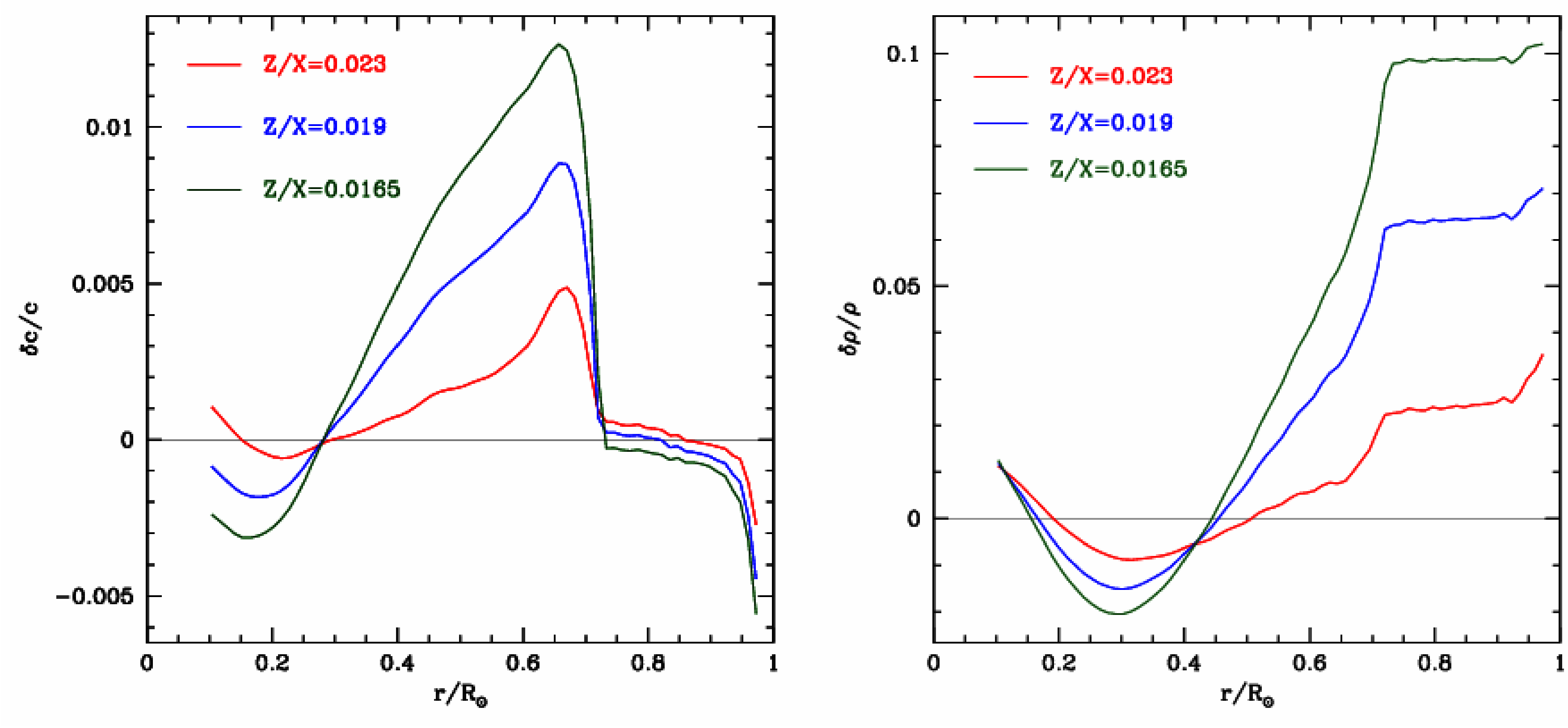
How do we know where the solar CZ base is?



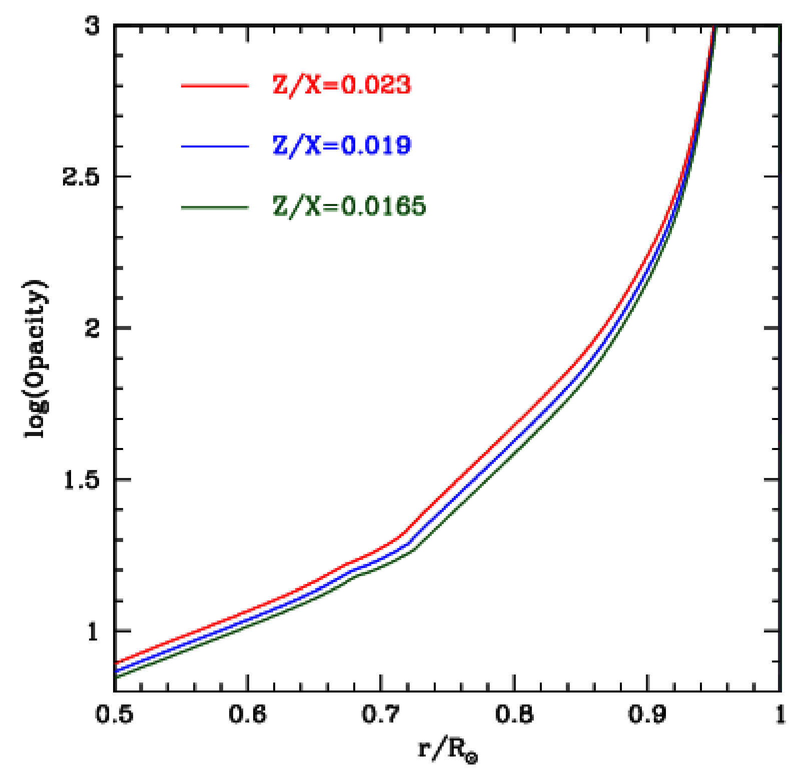


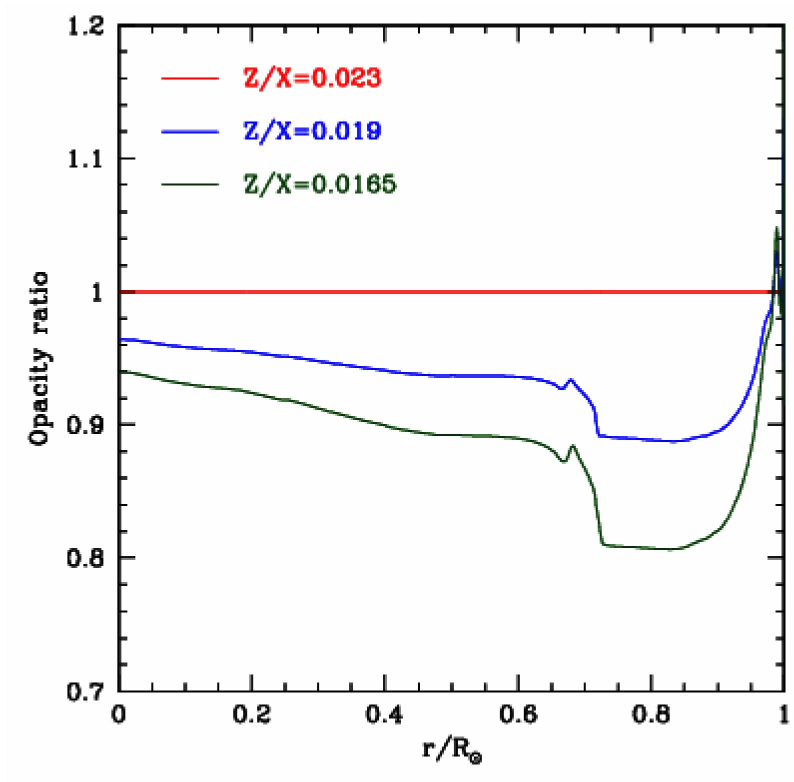
Solar CZ base is at 0.713 ± 0.001 Rsun

Effect of Heavy Element Abundance (Z) – I

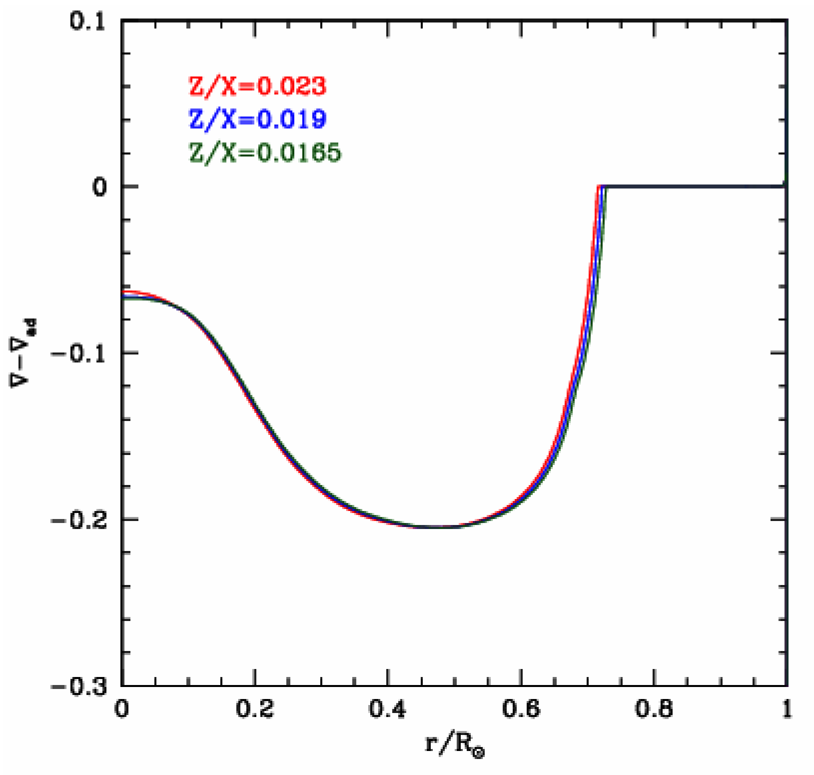


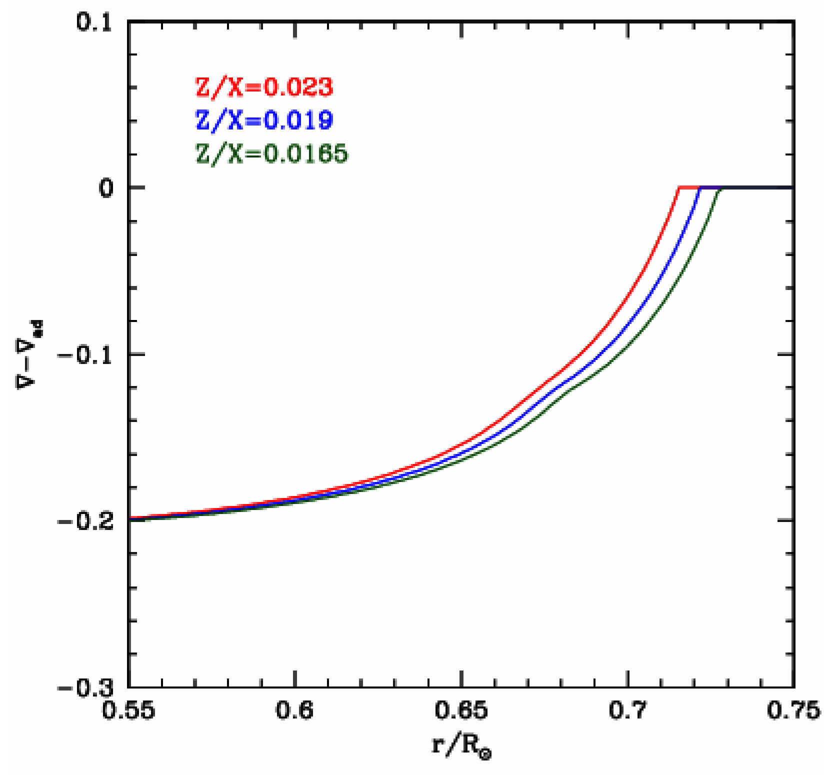
Effect of Heavy Element Abundance (Z) – II





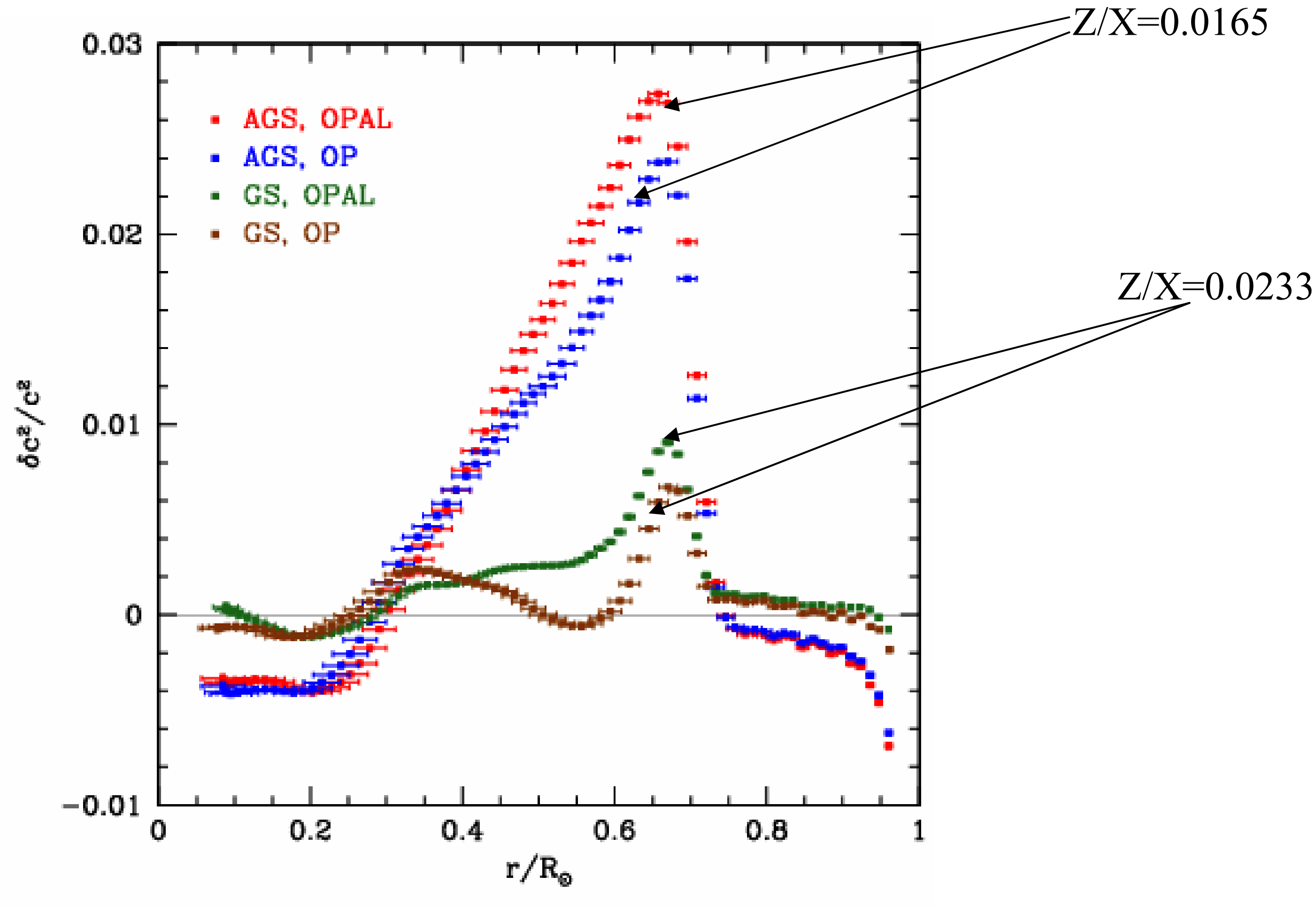
Effect of Heavy Element Abundance (Z) – III



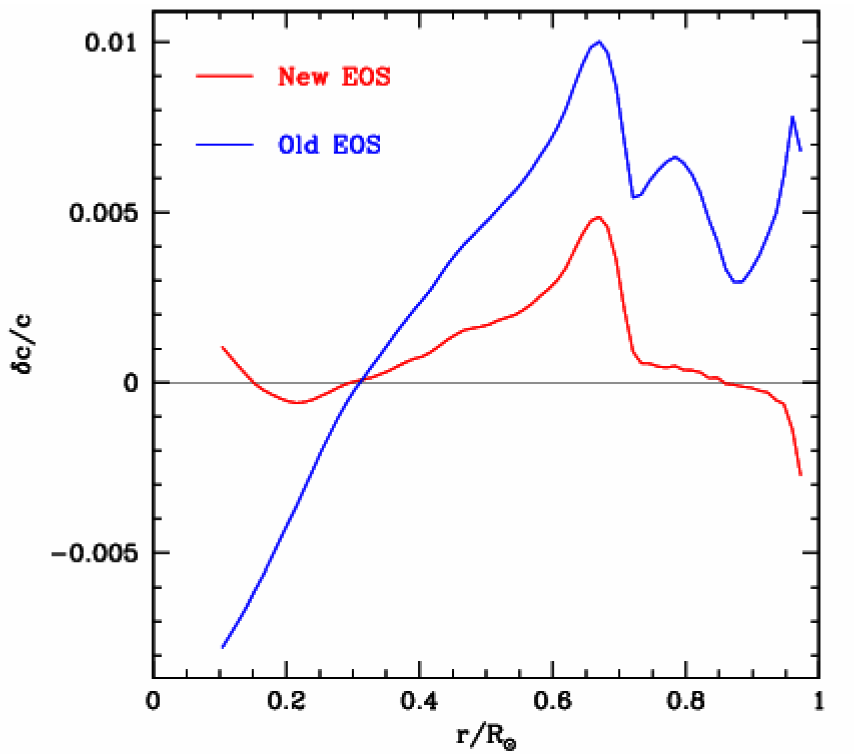


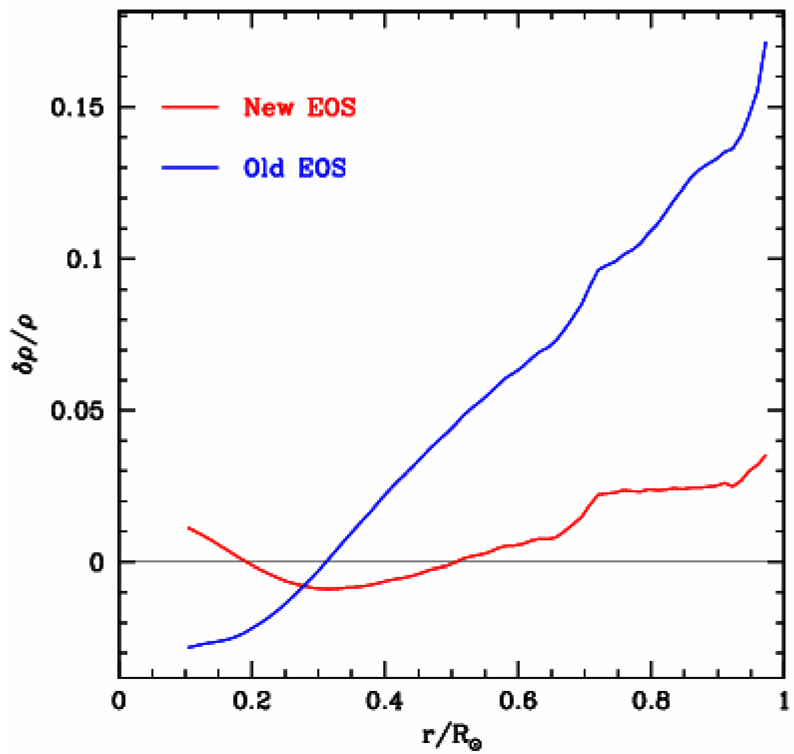
∇-∇_{ad} < 0 in radiative zone, ∇-∇_{ad} > 0, but very small in CZ

The effect of opacity



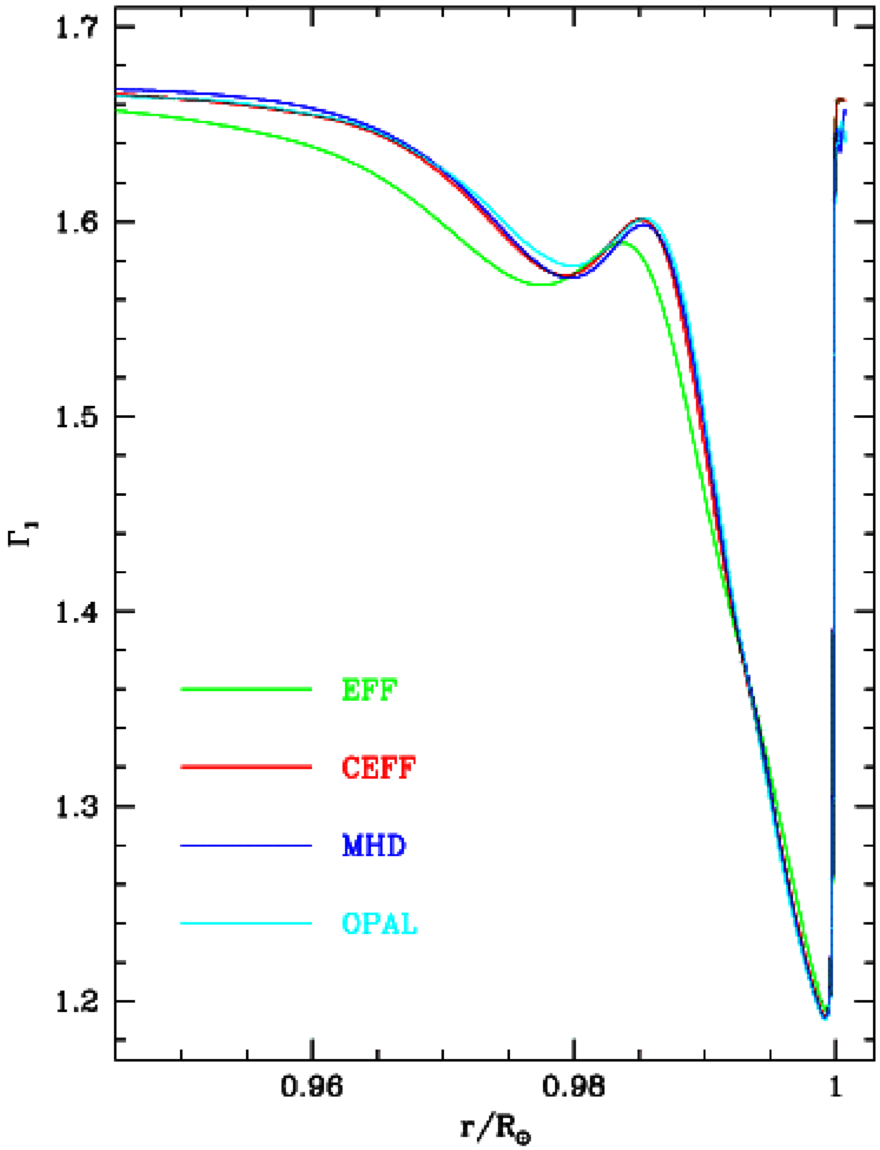
Effect of the Equation of State – I

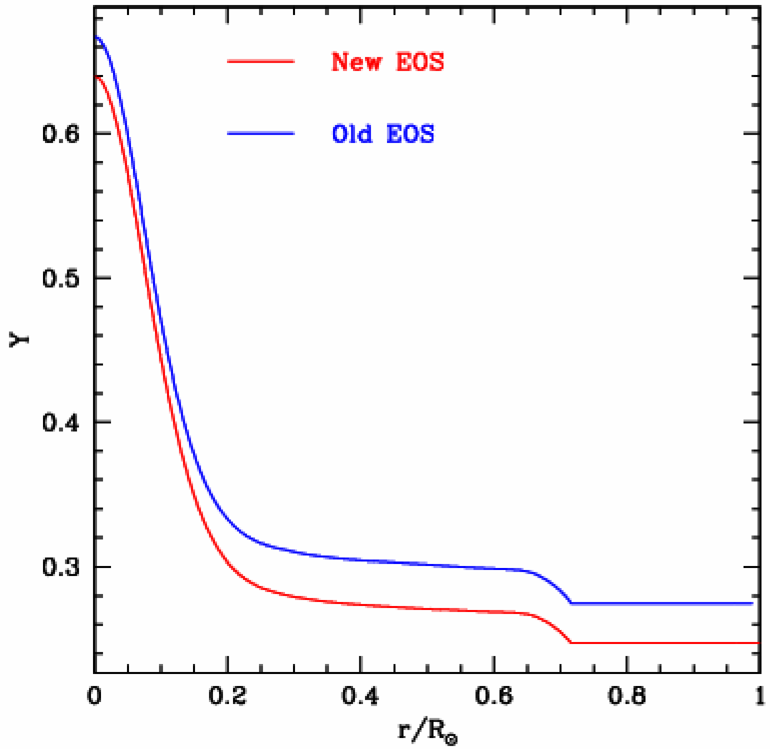




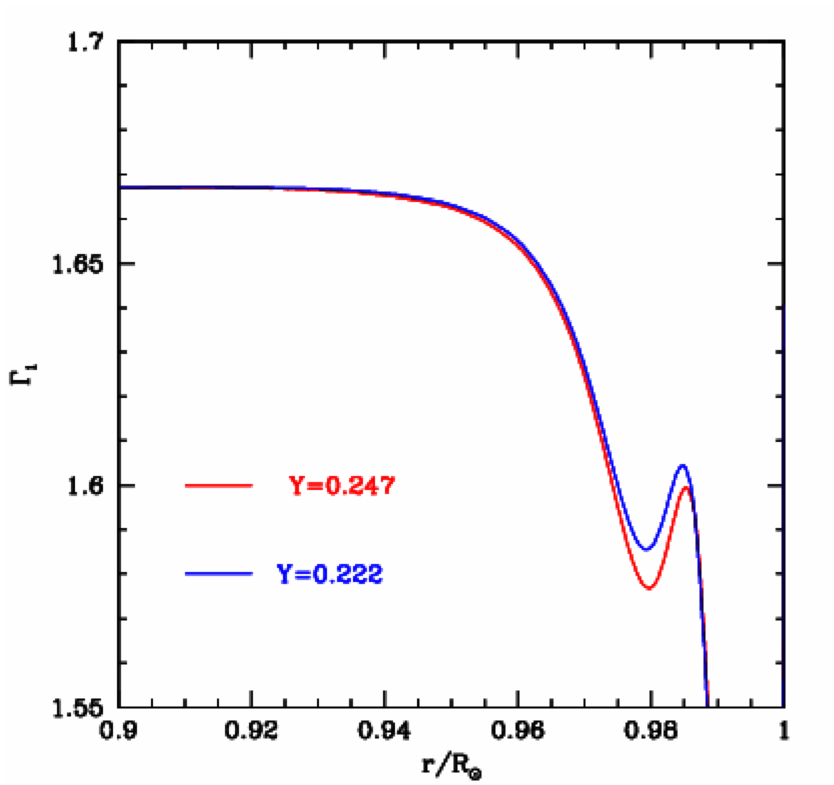
The way you know that the EOS is deficient is that the disagreement in the CZ becomes very bad!

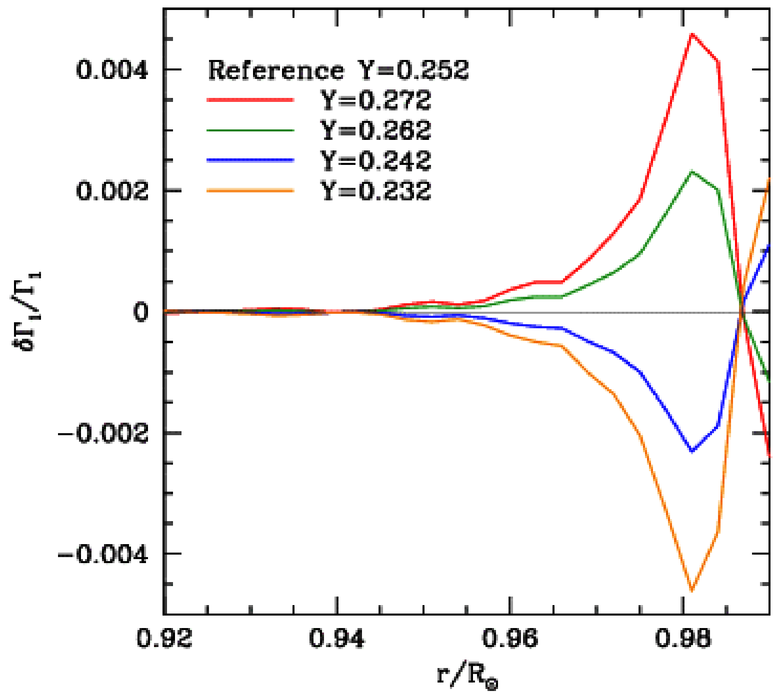
Effect of the Equation of State – II





Why are we bothered about Γ_1?
Determining Helium Abundance!





The helium abundance (Y) of the solar envelope is 0.249 ± 0.003

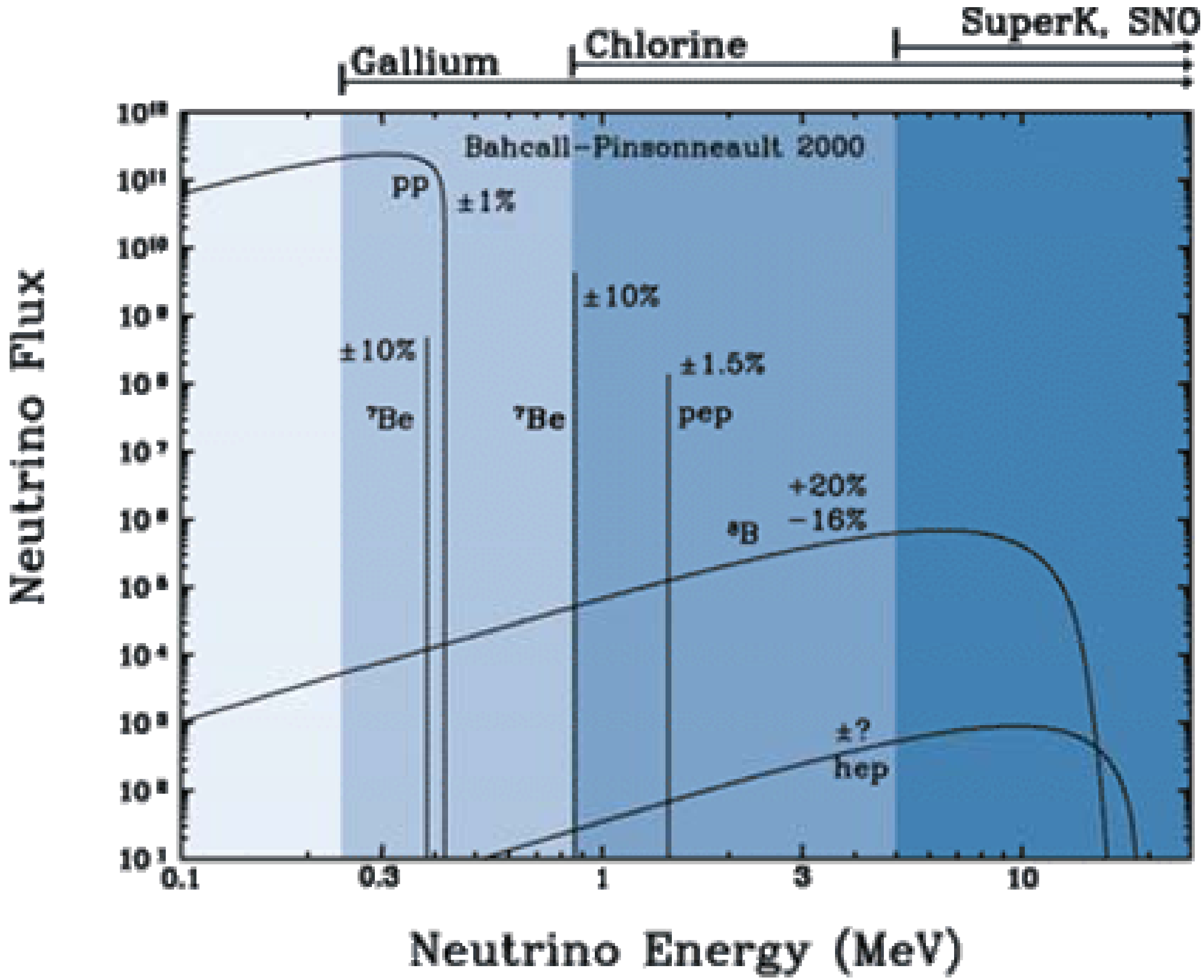
Using the Sun as a Laboratory

The Solar Neutrino Problem
The p-p chain

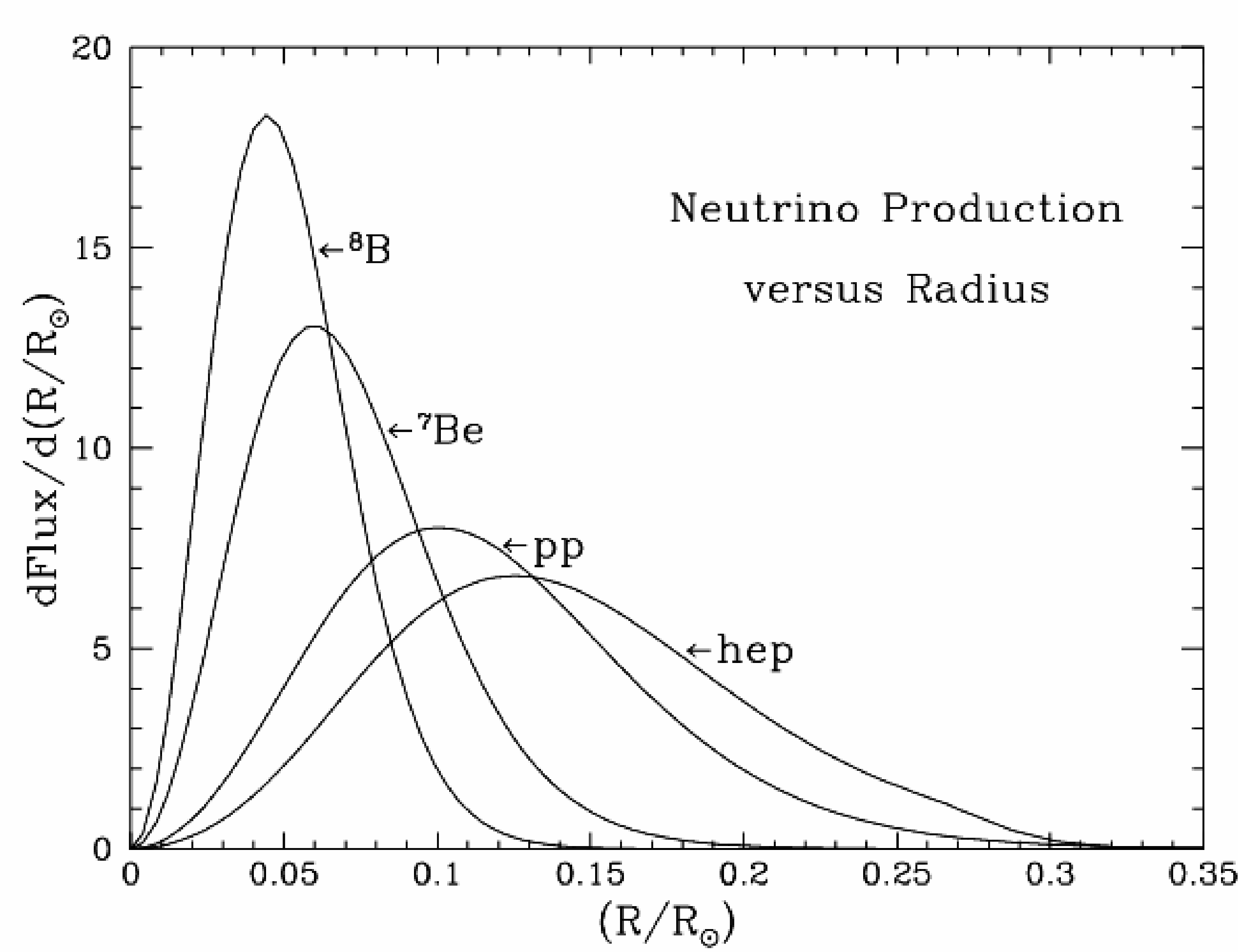
REACTION	TERM (%)	ν ENERGY (MeV)
$p + p \rightarrow {}^2H + e^+ + \nu_e$	(99.96)	≤ 0.423
or		
$p + e^- + p \rightarrow {}^2H + \nu_e$	(0.44)	1.445
${}^2H + p \rightarrow {}^3He + \gamma$	(100)	
${}^3He + {}^3He \rightarrow \alpha + 2p$	(85)	
or		
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$	(15)	
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$	(15)	$\begin{cases} 0.863 & 90\% \\ 0.385 & 10\% \end{cases}$
${}^7Li + p \rightarrow 2\alpha$		
or		
${}^7Be + p \rightarrow {}^8B + \gamma$	(0.02)	
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$		< 15
${}^8Be^* \rightarrow 2\alpha$		
or		
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$	(0.00003)	< 18.8

Neutrino terminations from BP2000 solar model. Neutrino energies include solar corrections: J. Bahcall, Phys. Rev. C, 56, 3391 (1997).

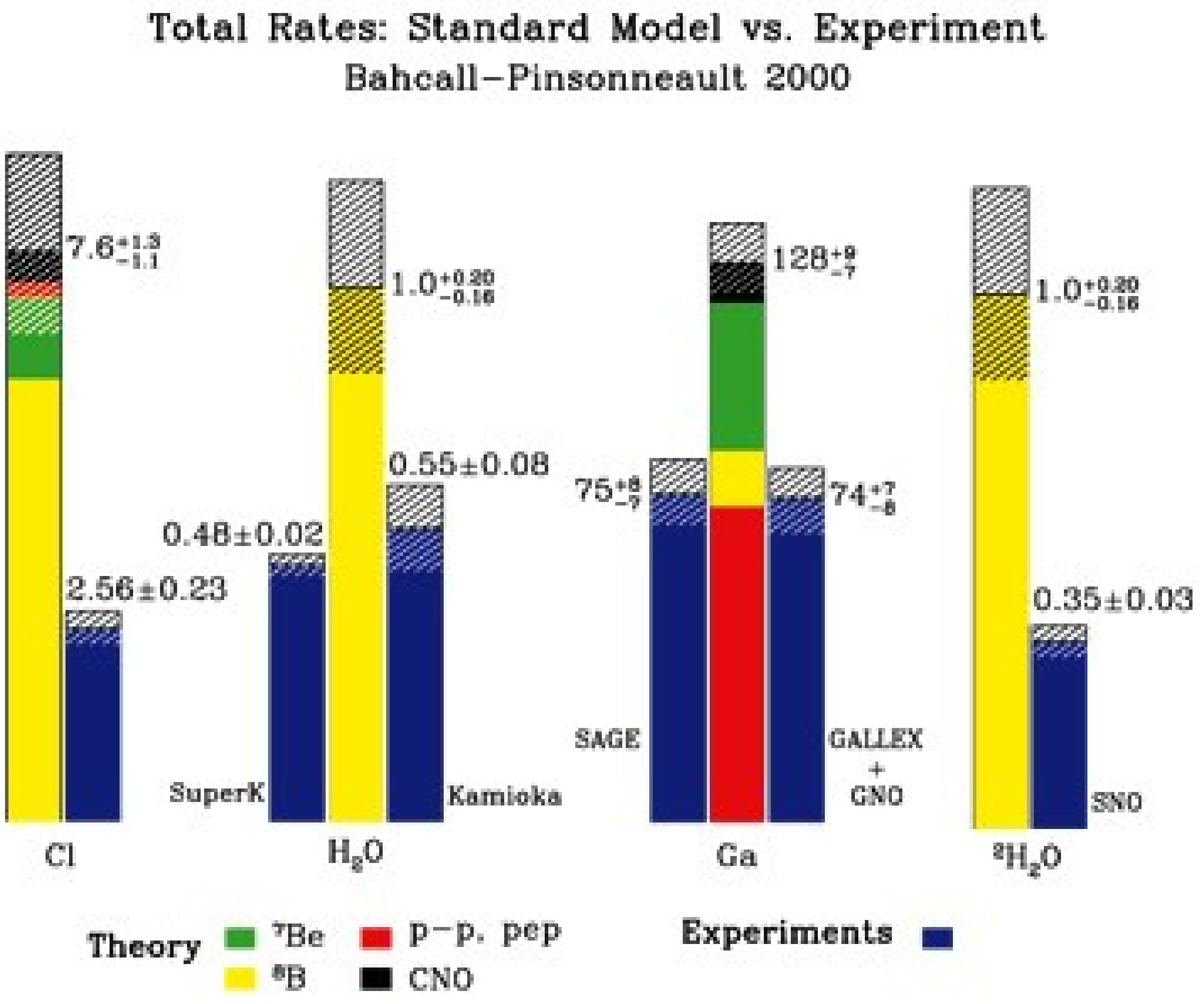
The neutrino spectrum



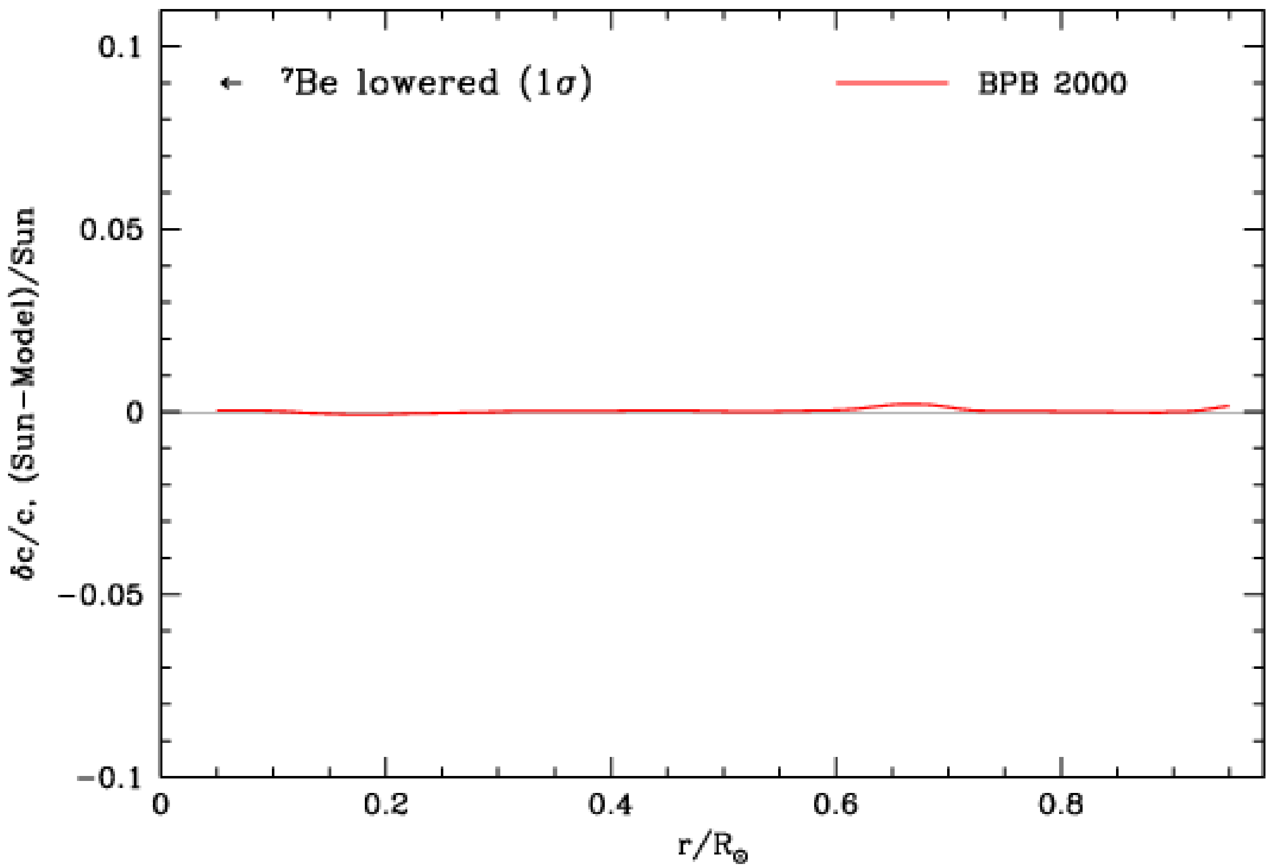
Where are the neutrinos produced?

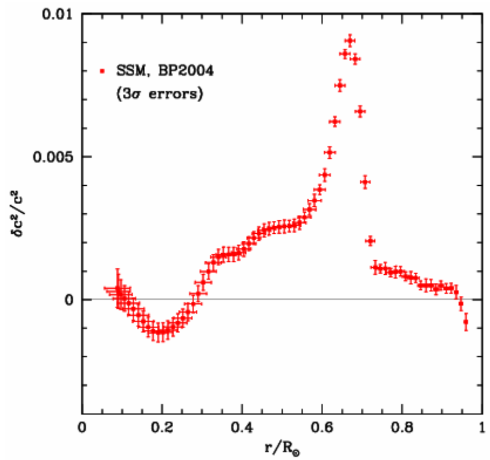


The Solar Neutrino Problem – Early Observations

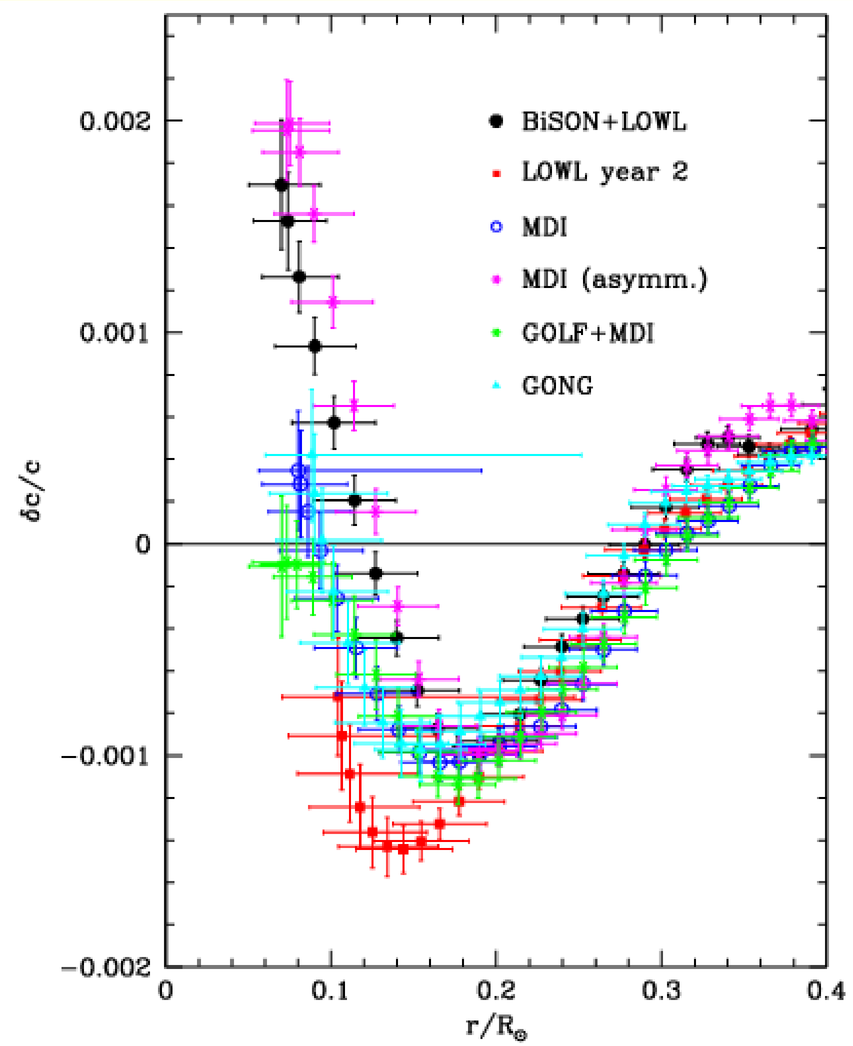


What of solar structure?



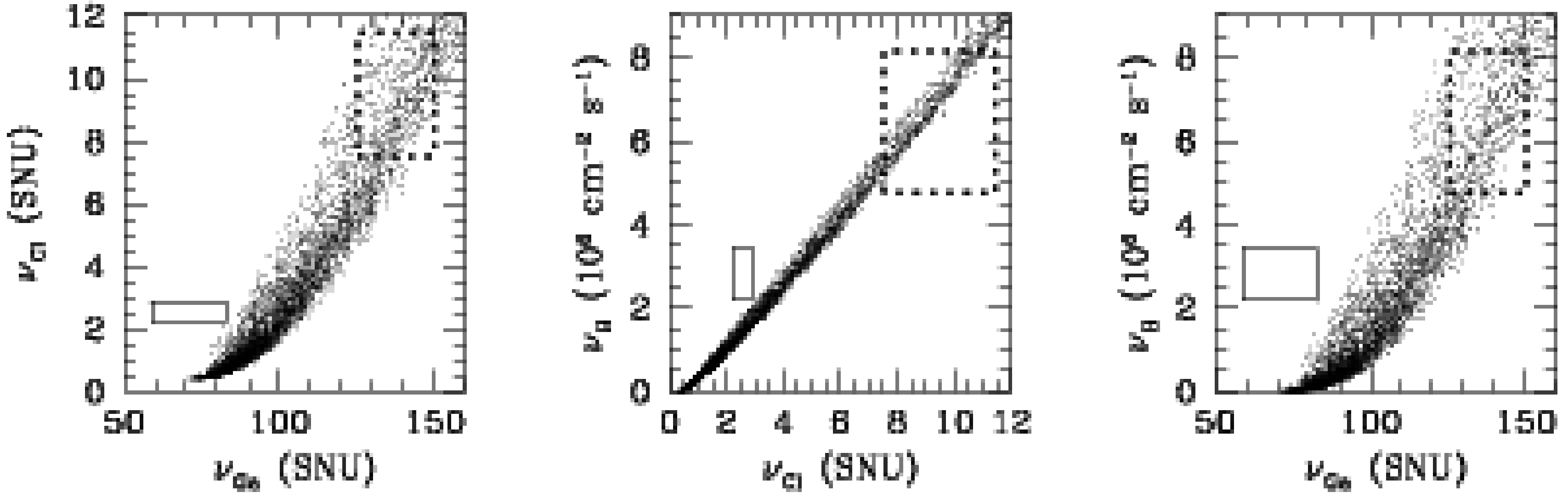


Effect of different data sets



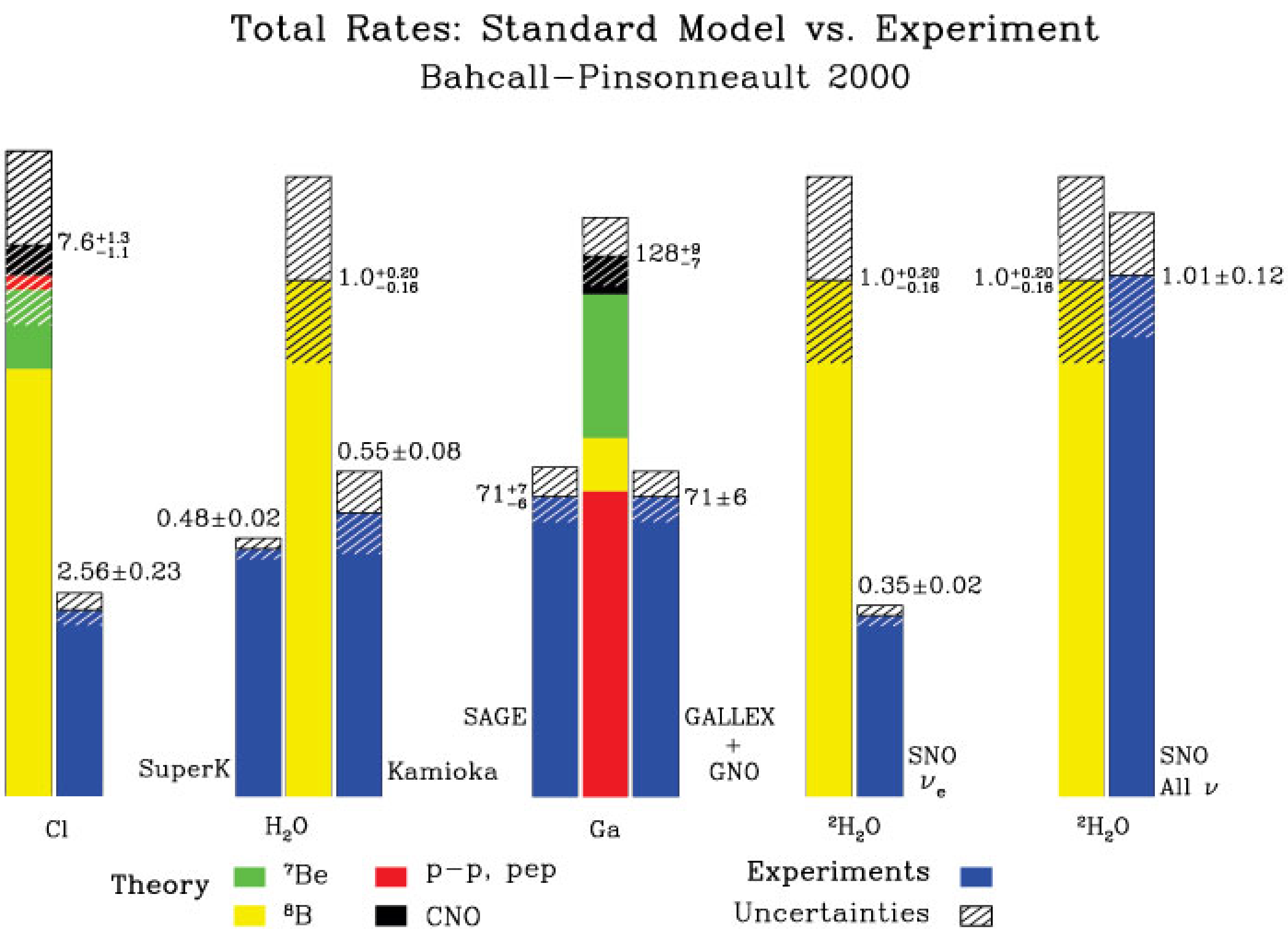
Changing data sets do not change the core structure.

Other evidence

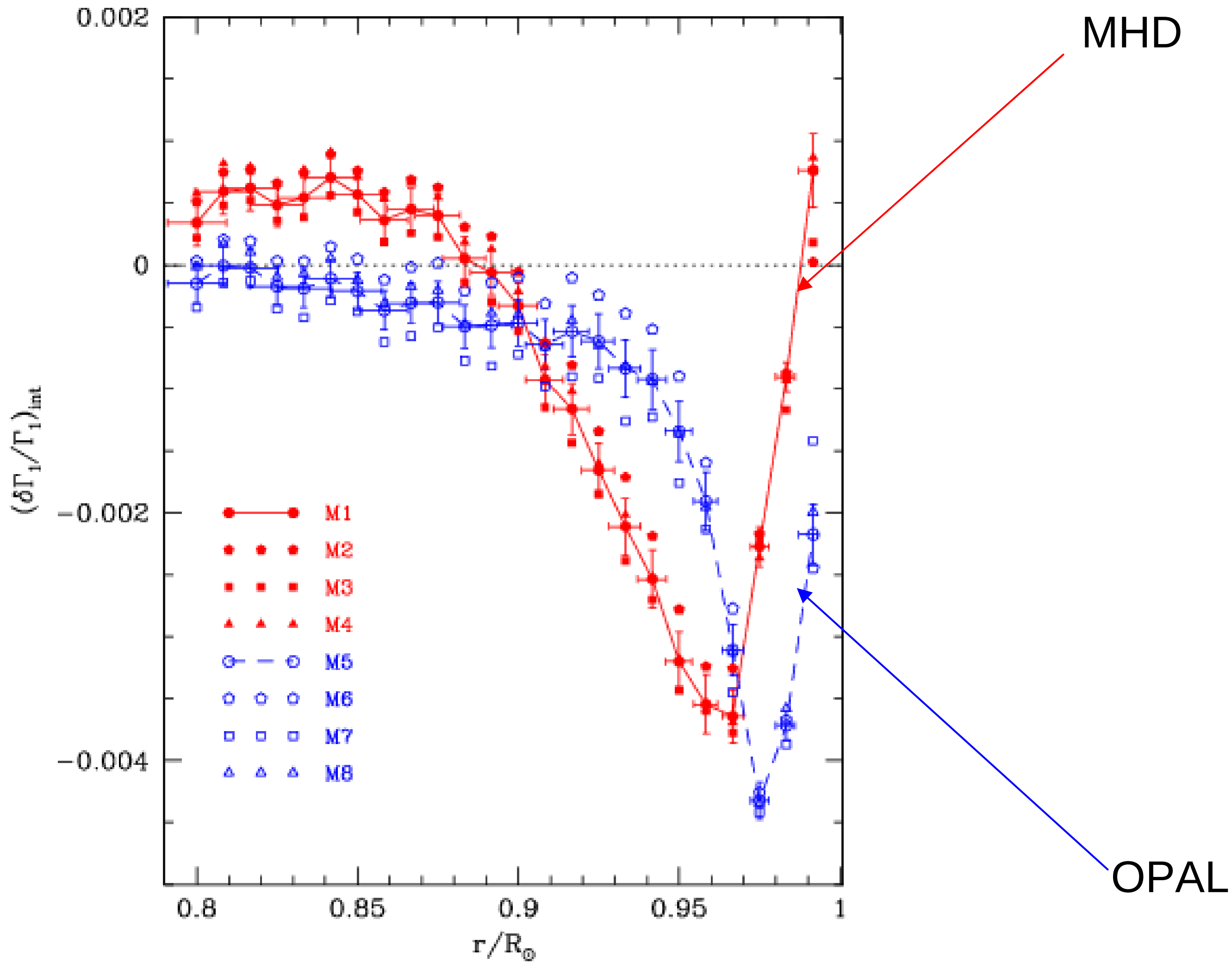


From Antia & Chitre (1997)

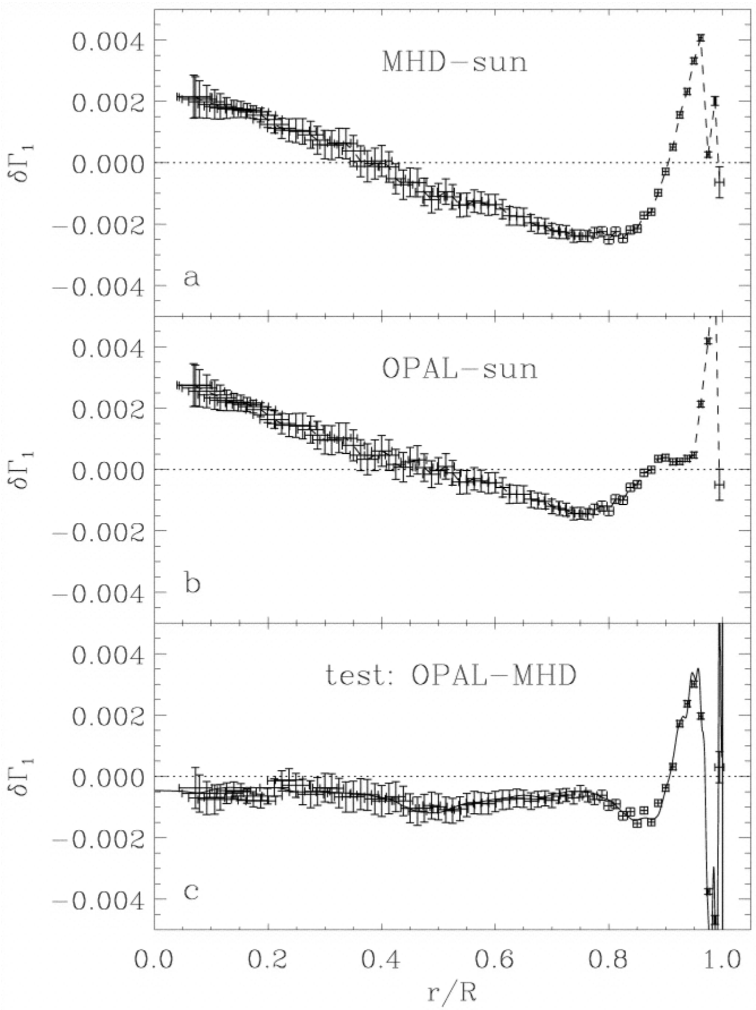
Solar Neutrinos Today



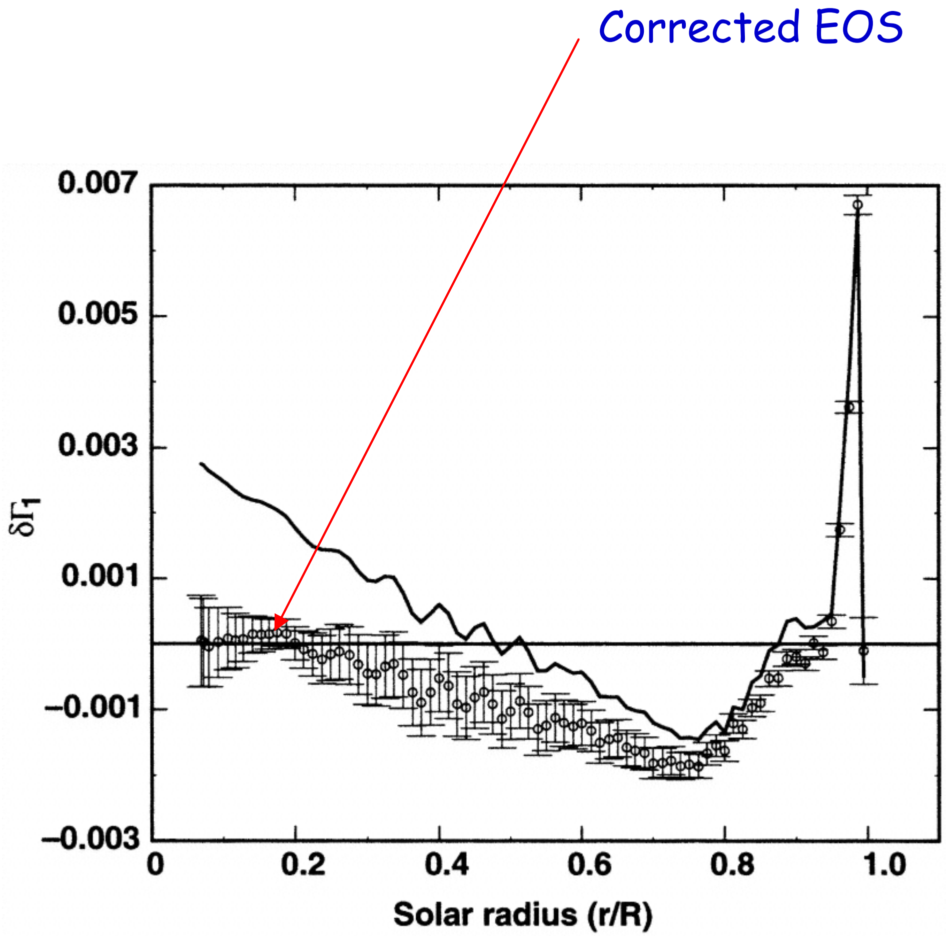
The equation of state



The Equation of state in the solar core



From Elloitt & Kosovichev 1998



From Nayfonov & Rogers 2002

Summary:

(1) How stars such as the Sun evolve can be determined using a few basic equations.

(2) The models rely critically on physics inputs such as opacities, nuclear reaction rates, opacities, etc.

(3) The structure of the present day Sun is known very well from helioseismology, and this can be used to test solar models.

(4) One of the triumphs of the field was to show that the solar neutrino problem has a particle physics solution and that it was not a problem with the models.