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Looking for a Book...and More, if Affinity

Julien Barral and Stéphane Jaffard

Our first meetings with Benoît Mandelbrot took place in bookshops. As young students, we used to browse in the Quartier Latin in Paris, and dream over scientific books. They were so technical... many years of study would be necessary to become familiar with the vocabulary! And, like many others, one day, we met the exception: An accessible book, which revealed large pieces of science that our masters had barely scratched; yes, we did remember the Cantor set, deeply buried among thousands of tasteless exercises... but nobody had mentioned that it was the first island announcing a rich and magical continent.

This book opened entirely new perspectives; its illustrations were arresting and would haunt our dreams; furthermore, it was understandable by students who had just a nonspecialized scientific culture. We discovered hot research topics described in simple terms that were appealing to young brains. Like many students, like many young researchers, this book and its cousins made us dream. We would not forget that dream and "young professionals" of research, we grasped eagerly the occasion to confront ourselves with these problems when we met it. Our reward was beyond our expectations. We were confronted with exciting problems, in a swirling subject where scientists from everywhere would confront their views, collaborate... and even fight! But above all, we had the great privilege to know personally the author of these books. We realized how this man loved to discuss with young scientists, infuse his enthusiasm, very generously share his ideas and his knowledge, make them feel confident and guide them towards the problems that his huge intuition made him feel were important.

Today, how could we express how much we owe him? Just, simply... thank you Benoît!

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Benefiting from Fractals

Michael Berry

Philip Morrison's review of the first English edition [22] of Benoit Mandelbrot's book hit a precise resonance with me. For the previous few years I had been studying waves reflected from irregular surfaces, motivated by an application to geophysics. All existing theories assumed random surfaces where asperities with a single length scale perturbed a plane; this disturbed me, because it created an artificial distinction between 'roughness' and 'geography' (in this case the flat earth). Before fractals, I had no idea how to convert this unease into physics.

After fractals, the way was clear: assume a rough surface with fractal dimension D, and see how waves reflected from the surface carry an imprint of D. I called such waves 'Diffractals' [1]. An exact analytical solution of the wave equation with such a boundary condition was (and remains) unavailable, even for the statistical quantities I was interested in. Existing approximation methods failed too: ray optics (short-wave asymptotics), because the transverse length scales include the wavelength; perturbation theory, because the asperities are high compared to the wavelength; and variational methods, because there is no 'nearby' exactly solvable model. Nevertheless, I made a little progress with a Kirchhoff integral approximation, that at least showed how D gets imprinted on the second moment of the wave as it propagates away from the surface. Later, this monochromatic analysis was extended to pulses (echoes) [8].

Those were the early days of quantum chaology, where a useful class of models for studying high energy levels is 'quantum billiards': waves confined within boundaries of different shapes. I wondered how a fractal boundary (or even a fractal domain) might affect the asymptotic distribution of eigenvalues, and dared to publish a speculative answer [2,7]. In a misguided attempt to be more precise than my mathematical knowledge warranted, I guessed that it would be the Hausdorff dimension that influences the asymptotics. I should have referred to D simply as the fractal dimension, because it was soon shown that the Minkowski dimension is more appropriate [13]. However, the essence of the conjecture has survived and has spawned a small literature [17,20,19,18], extending to number theory [21]. Nevertheless, an important problem has hardly been addressed: for a billiard with fractal boundary, what is the geometrical origin of the fluctuations of eigenvalue density? For smooth boundaries, the fluctuations depend on the periodic geodesics [14,6] bouncing inside the billiard, but reflection and hence geodesics are not defined for fractal boundaries. This question goes beyond the averages described by the Weyl formula and its extensions.

In the mid 1980s, talk of 'The Evil Empire' revived fears of nuclear war, and raised the possibility that smoke from the resulting fires would absorb incident sunlight but transmit radiated heat in a 'nuclear winter'. Ian Percival pointed out that the estimates of this cooling were based on models of the smoke particles as spheres, whereas it was already known that they aggregate into fractal clusters as the smoke ages. In another application of diffractals, we gave a mean-field theory [11] of the absorption of electromagnetic waves by fractal clusters; again D was implicated in a nontrivial way, which survives in more accurate computations [24]. As intuition might suggest, the absorption is greater for fractals than for spheres of the same mass, so fractality makes the nuclear winter worse [23] (see also [25]). The effect is made even worse by the fact that fractal clusters fall to earth more slowly than spheres, implying a modification of the hydrodynamic Stokes law, that I was able to estimate [3].

In diffractals, waves get imprinted with the D of objects they encounter, but they are not themselves fractal, because the wavelength provides a natural scale. It was therefore a surprise to discover that there are circumstances in which waves themselves can be fractal, in the sense of possessing self-similar structures on scales between the wavelength and the size of scattering objects. Moreover, this occurs in one of the most familiar waves, namely that diffracted by a grating with sharp-edged slits [9]. This 'Talbot effect' [26] fractal is richly anisotropic, with different D lengthwise, crosswise, and diagonally. Transferring the analysis from the paraxial wave equation to the time-dependent Schrödinger equation shows that very simple nonstationary quantum waves can be fractals too [5,10]. Another surprise was finding [16,15] and understanding [12,4,27] fractal waves in the modes of unstable lasers: simply reversing one of the mirrors in the familiar stable arrangement changes the mode from a narrow Gaussian beam to a fractal filling the laser cavity.

I would never have recognized and explored these hidden territories in my intellectual habitat of wave physics without Benoit Mandelbrot's great discovery that self-similarity is commonplace rather than pathological.

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Benoit Mandelbrot, Wizard of Science

Marc-Olivier Coppens

I will never forget how I met Professor Mandelbrot. Although this is a personal recollection, I feel the circumstances say a lot about how great a man Benoit Mandelbrot really is, and not just on how he influenced me. I was working on my undergraduate thesis in chemical engineering at the University of Ghent, Belgium, on the fractal modeling of porous catalysts. We were in April 1993. My father, Claude Coppens, a concert pianist, was about to travel to New York for work, and since these were the Easter holidays, I could accompany him and my mother. I thought this would make for an excellent opportunity to perhaps visit the IBM Thomas J. Watson Research Center where the famous Dr. Mandelbrot was working.

I had just read the *Fractal Geometry of Nature*, and was in awe of this Renaissance man, and of fractals in general. As a senior high school student, I received Peitgen and Richter's *The Beauty of Fractals* when winning the Mathematics Olympiad in Belgium, and this had started a passion. For several years, I had studied and drawn Mandelbrot sets and other fractals on my XT computer (it took an entire night to draw a picture in those days). Later on, as an undergraduate in chemical engineering, my broad interests in chemistry, engineering, mathematics, physics, and the arts, could not be better served than by accepting the challenge my thesis advisor, Prof. Froment, offered me to work on studying how the fractal morphology of porous catalysts would affect transport and chemical reactions – a topic on which I would continue to work toward a Ph.D. in Ghent. I proposed a "fractal catalyst pore model," and the first results seemed promising.

Then, the occasion to travel to New York came. E-mail was not yet common, so I sent an enthusiastic fax through a friend's fax machine, with little hope for an answer from anybody as important as Professor Mandelbrot. Sure he would have better things to do: I was this beginning researcher, unknown, without papers, and who had never been to a conference. But, you never can tell. Great was my surprise when only a day or two later, I received a reply, not an automatic reply but handwritten by the Great Man too! He wrote he had no time to see me at IBM, because he was always too busy there, but why would I not visit him at his home, on a Sunday? I could not believe my eyes, and was filled with joy. The day

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of the visit came, and Benoit Mandelbrot himself was waiting at the train station, driving me to his home on Overhill Road – a very appropriate address for a Wizard, and reminding me of that Hobbit ... The discussion was typical of the many that would follow, including topics that ranged from art to mathematics, with brackets within brackets and excursions away from measure theory and probability theory to literature, the history of science and Ligeti. The discussion would last for hours, so his wife Aliette called from the "real world" and decided that we should have lunch.

From our unusual first meeting on, Mandelbrot would be a mentor to me, in many ways. He was the first person who told me what a (scientific) "poster" was. He gave me a flyer and proposed that I should go to a meeting Prof. Vicsek organized in Budapest that summer, indeed the first conference I went to, and where I met him again. Not too long afterwards, he invited me at his 70th birthday conference in Curaçao, undoubtedly the nicest conference I ever went to, both content-wise and location-wise. Even more significantly, he offered me the opportunity to work with him on multifractals at Yale, in early 1996, where I also finished writing my Ph.D. thesis. I vividly recall my three-month visit to Yale and the discussions I had there. On certain days, I would have discussions with him that lasted six hours or more, and went well into the evening. At lunch, we would eat at a simple eatery or in an English-style Hall, where he invariably took a simple soup, possibly with a sandwich. But the diet was often followed by an ice cream around the corner, a treat he cannot resist. Discussions during meals were always interesting and would rarely involve fractals. Actually, one could pick almost any topic except perhaps sports, and start an engaging conversation with him.

Like in a discussion, organization with him has always been out of the question, and administration does not belong to his vocabulary. Fixing an appointment some time ahead, even from overseas, or arranging other practical matters is nearly impossible: without Aliette, Benoit and everybody around him is at a loss. Having grown up in an artist's family, this was not unknown to me. This is all part of Benoit Mandelbrot's artistic personality, and I quickly understood that. Many have misinterpreted this behavior, I found out later, but I always saw the chaos as part of the game. Fixing oneself to one thing is too much of a limitation, when you are Mandelbrot. Had Benoit Mandelbrot worked in streamlined administrative surroundings, with forms to be filled out and grant proposals to be written, one can only wonder whether fractals would be there today.

However, this behavior should not be confused with disinterest or arrogance, quite to the contrary. Rarely have I met a person who is so truly interested in people, not simply professionally but also personally. Small and simple things are worthy of his time. A typical phone call to chat will take at least one hour. He does more than he promises he will do. He shows genuine interest in beginning researchers, school children, family and friends, and is incredibly correct and modest in many of his endeavors. Preparing a class takes time and concentration, and here again he is the true scientist: wondering and questioning himself, his own theories and assumptions. He will defend fractals everywhere and display strong emotions towards people who hide ignorance with fashionable displays, fancy-looking formulas, or trend-followers eager to show their priority in a discovery. Rather than a display of arrogance, the fact is that hypocrisy is simply not part of his vocabulary.

Right or wrong, he is always true to his feelings, and is a true scientist and philosopher. I remember finding a possible error in one of his calculations in a paper; he would immediately start from the assumption *not* that he was right, but most likely that he was wrong. Often, he lets his intuition go wild, and his intuition is remarkable. It is this intuition that led him to combine so many fields and give birth to fractals. He will also quickly correct himself, and realize that certain assumptions or conclusions may not be right. However, it has amazed me to see results from complex computations so close to his intuition. It also keeps surprising me how often his premature visions are correct, some of the most interesting ones presented in extremely complex papers or in seldom consulted journals. Like a discussion with Benoit Mandelbrot, each paper is a developing vision, a work-in-progress, perhaps a glimpse of universality or of statistical models behind phenomena otherwise obscure. As all ideas of genius, fractals appear so "obvious" after the fact, but are of course far from obvious. Developed from unifying scaling laws in many areas of science, nature, finance and the humanities, Mandelbrot's unusual skill was and is to combine his love for art and philology with a very profound knowledge of the statistics of extreme events: What is the structure, the pattern in random phenomena with strong correlations and where the standard deviation does not necessarily converge? This combination of abstract and concrete geometric, intuitive and very precise ideas is hard to find in any man, and makes him a Picasso of science. Many had observed isolated scaling laws or even proven their occurrence, but uncovering a universal symmetry and linking it to seemingly abstract statistics and measure theory, are entirely different things.

Mandelbrot is a simple man, and therefore he is a great man. He works alone or in a very small team, interacts with people around the world, never stagnates but further develops interests in everything and anything, and lives in a healthy most loving relationship with his supporting wife Aliette, without whom fractals would not be what they are. He is a philosopher. Some thoughts are simple, and can be explained in the language of a child; other thoughts require mathematics that has not yet even been fully developed. He is like the geometry he has defined: multidisciplinary, versatile, simple in his complexity, and complex in his simplicity. Like abstract art, yet with many layers, combining the classics with the contemporary world. A man of all worlds, Benoit Mandelbrot *is* fractals.

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Mandelbrot's Vision for Mathematics

Robert L. Devaney

I remember very well the first time I met Benoit Mandelbrot. I believe it was in 1979. Benoit was at Harvard that year, and he gave a talk on discrete dynamical systems, specifically iteration of complex quadratic polynomials. The field of complex dynamics had not yet been reborn at that time, but since I was then working on things quadratic such as the logistic map in 1D and the Hénon map in 2D, I decided to attend. I remember thinking to myself both before and after his talk: Why on earth would anyone ever iterate a complex holomorphic map? Everyone knows that a small change here will make dramatic changes everywhere else. Clearly this area of research will go nowhere...

Well, I guess that paragraph says a lot more about me than it does about Benoit. Clearly, I'm not the one with vision about where mathematics should go. To me, that is Benoit's single most important contribution in mathematics: his vision for what is important and what should be studied.

Take fractals. It now seems to be painfully obvious that just about everything around me (in nature as well as dynamical systems) is a subset of fractal geometry, and that fractal geometry is a major subject in its own right. But, like they always say, it takes a person with extraordinary vision to see what later becomes so obvious. Great mathematicians of the past have, of course, investigated all sorts of topics involving fractals, from Cantor's set, to Sierpinski's triangle, and on to Hausdorff's dimension. But, before Mandelbrot, to most mathematicians, these ideas seemed to describe very special counterexamples in analysis or topology rather than the principal ingredient of a major field of mathematics and science.

Complex dynamics is another of Benoit's visionary projects. Without his input, I wonder how long it would have taken dynamicists to realize that the only way to understand things like the 1D logistic map was to pass to the complex plane. Even though such eminent mathematicians as Fatou and Julia from nearly a century ago knew about the fundamental dichotomy in quadratic dynamics, to the best of my knowledge, they never attempted to draw this picture in the parameter plane, the picture now justly called the Mandelbrot set. Of course, one can argue that Fatou and Julia did not have a computer at their disposal, but that did not deter them from studying and drawing pictures of the related Julia sets. No, it takes someone with vision to see the obvious in mathematics.

It is a credit to Benoit how far discrete dynamical systems has come since his discovery of the Mandelbrot set. We now finally understand completely the quadratic family, thanks primarily to the complex analysis associated with the Mandelbrot set. It also is a testament to his vision for what is important in mathematics that no fewer than four Fields medalists have worked on topics directly associated to the Mandelbrot set: Curt McMullen, John Milnor, William Thurston and Jean-Christophe Yoccoz. Knowing that such eminent mathematicians did not see the importance of complex dynamics until Benoit pointed it out gives me some solace about my own lack of vision in this area.

Thank you very much, Benoit, for seeing the right way to go in dynamics and for inspiring much of my own work in the field over the past quarter century.

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Benoit Mandelbrot and York

M. Maurice Dodson

Benoit Mandelbrot visited the University of York on Friday 21 May 1982 and gave a seminar entitled "Fractals" to the Mathematics Department. York was thus privileged to be one of the first institutions in Britain to hear him speak on this topic and to see the astonishingly complex and beautiful figures associated with the set that now bears his name. However, his fathering of fractals was but one element in his dazzling and comprehensive wealth of ideas and insights. These have played an important part in the emergence of the new 'chaotic' paradigm which is appropriate for the analysis of the complex irregular natural phenomena which pervade Nature. I have a long-standing interest in Hausdorff dimension through the metric theory of Diophantine approximation and I was delighted to learn that a topic that seemed somewhat recondite was in fact relevant to understanding the world about us. Not long afterwards, I had the good fortune to become involved with three biologists at York who were trying to understand the apparently anomalous size distribution of insects in trees and bushes using Mandelbrot's ideas. It turned out that the disproportionate number of small insects could be explained by fractal geometry, as the surface area of the foliage which formed the insects' range was in effect disproportionately large because of irregularities occurring at finer and finer scales [**3**].

Mandelbrot came to York on his way to Paisley University in order to participate in the celebrations of the centenary of the birth of Lewis Fry Richardson (1881–1953), a former principal (from 1929 to 1940) and another radical and highly original thinker much ahead of his time. Mandelbrot is generous in his acknowledgment of Richardson's pioneering work on self-similarity in turbulence (wittily described in the rhyme "Big whorls have little whorls that feed on their velocity, and little whorls have smaller whorls and so on to viscosity"). Richardson, who was educated in York and a Quaker, also studied conflict. His analysis revealed that the size of a war was related to its frequency by a power law and that Britain headed the list of belligerent nations! This led him to the apparently naive question "How long is the coast line of Britain?" which Mandelbrot adopted as the title of Chapter II in his two seminal books [**1**] and [**2**] for the starting point of his treatment of fractals.

Mandelbrot's knowledge and understanding extends beyond the mathematical and social worlds. The University of York is in the village of Heslington, on the outskirts of the city. The village still has some working farms and Mandelbrot was very keen to see one. He and I spent a pleasant couple of hours with a local farmer looking at livestock and particularly horses. His knowledge of and ease with the animals gained from his experience as a young man impressed a dour and canny East Riding farmer who decided that British academics were hopeless 'townies' by comparison. Mandelbrot is truly a man of many parts.

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Amérique, Ô ma Norvège !
Dominique Fourcade
Le sujet monotype
P.O.L. (1997)

Nul n'entre ici s'il n'est géomètre

Bertrand Duplantier

C'était en 1975. Avec Thierry Heidmann, mon meilleur ami à l'École normale, nous flânions chez "Offilib", librairie scientifique bien connue du quartier et au-delà, aujourd'hui disparue. Nous étions déçus par l'enseignement indigent de physique dispensé par notre chère École. Il nous fallait choisir un laboratoire d'accueil pour une thèse de troisième cycle. Nous cherchions notre voie, nous étions anxieux. Je me souviens très bien d'avoir montré à Thierry le livre *"Les objets fractals"*, premier livre de Benoît Mandelbrot. "Tiens, tu as vu, c'est étrange, ce livre... Cela a l'air intéressant..." Nous le feuilletâmes, intrigués, il n'était à nul autre pareil. J'y revois encore aujourd'hui le tracé d'une côte fractale. Nous ne l'achetâmes pas, trop incertains de la vie que nous étions.

Thierry partit à Pasteur comme "matière à penser" chez J.-P. Changeux, je m'en fus à Saclay. Cependant, l'empreinte était prise, et je me souviens qu'en thèse d'état avec Jacques des Cloizeaux, un gentihomme de la science s'il en fut, je m'entretenais avec lui de la relation $D = 1/\nu$, entre dimension fractale D et exposant de gonflement ν d'une longue chaîne polymère. C'était l'époque post-wilsonienne, et régnait alors sans partage dans l'École de Saclay le calcul des exposants critiques, même s'il fallait pour cela couper les epsilons en quatre. Personne n'y songeait alors aux fractales. Je conservai précieusement cette notion mystérieuse de *dimension fractale*, elle donnait à penser géométriquement.

Un peu plus tard, dans les années quatre-vingt, ce fut l'explosion phénoménale des notions fractales en physique statistique et de leur relation avec l'invariance d'échelle ou conforme et les phénomènes critiques.

Aujourd'hui, que de chemin parcouru ! Nous célébrons le Jubilee de Benoît. Comme dans les tableaux de Pollock, dont on mesure aujourd'hui la dimension fractale, les entrecroisements, les fourmillements de la Nature nous rappellent sans cesse que

Nul n'entre ici s'il n'est géomètre
fractal

Félicitations, Benoît, ce fut l'intuition juste.

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Amérique, Ô ma Norvège !
Dominique Fourcade
Le sujet monotype
P.O.L. (1997)

Let no one ignorant of Geometry enter here

Bertrand Duplantier

It was in 1975. Together with Thierry Heidmann, my best friend at l'École normale, I was browsing at "Offilib", the well-known scientific bookstore, recently closed. We were at that time disappointed by the poor physics teaching offered by our "chère École". Soon we were to choose a research laboratory for a "thèse de troisième cycle"; seeking our way forward, we were anxious. I have a vivid memory of my showing Thierry *"Les objets fractals"*, Benoît Mandelbrot's first book: "Hey, look at this strange book...it looks interesting..." Intrigued, we browsed through it– this book was like none other. I can still see in my mind the fractal trace of a coastline. We did not buy it; we were too uncertain of ourselves and of our future.

Thierry became "mind and matter" at J.-P. Changeux's Pasteur Institute lab; I went to Saclay. But an impression lingered on, and I remember that during my Ph.D. thesis with Jacques des Cloizeaux, a gentleman of science above all, we discussed the relation $D = 1/\nu$, which holds between the fractal dimension D and the swelling exponent ν of a long polymer chain. Those were post-Wilsonian times at Saclay, under the iron rule of the calculus of critical exponents, even if that sometimes meant splitting epsilons like hairs. No one there then thought of fractals. I privately treasured the mysterious notion of a fractal dimension, which allowed me to think geometrically. Only later, in the eighties, came the phenomenal explosion of fractal concepts in statistical physics, and of their relation to scale or conformal invariance and critical phenomena.

Today, as we celebrate Benoît's Jubilee, we have come such a long way! As in Pollock's paintings, the fractal dimensions of which are measured nowadays, the interlaced and winding patterns of Nature keep reminding us to

Let no one ignorant of
Fractal
Geometry enter here

Congratulations, Benoît, your intuition was the right one.

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A Decade of Working with a Maverick

Michael L. Frame

I met Benoit Mandelbrot in 1988 when he received an honorary doctorate at SUNY Albany. About that time, David Peak and I had started teaching a fractal geometry course for non-science students. The late James Corbitt, Benoit's host at SUNY, had planned some morning lectures on fractals as a prelude to Benoit's commencement address. Jim arranged several fairly technical talks on applications and asked me to start the morning session with a general introduction. When I arrived, I found a large lecture room already filled. After opening comments by a SUNY administrator, Jim called me forward to give the first talk. Holding a folder filled with transparencies (computer projection systems were not so common then), I came to the front of the room, turned to face the audience, and saw ... Benoit sitting in the middle of the first row! Somehow I had expected he would attend only the afternoon session. Being such a busy person he certainly wouldn't learn anything from these introductory talks. I had an uneasy feeling as if I were giving a general introduction to the Ten Commandments and found Moses sitting in the middle of the first row.

What could I do? The rumors of Benoit's arrogance just increased my worry, but there was nothing to be done but present what I had prepared. Since my graduate student days, my approach to teaching has always been to give the simplest explanation I can find and to use many examples. That appeared to work in this setting, though mine was by far the most elementary of the talks. In the break between the morning and afternoon sessions, Benoit talked with the other speakers. He seemed interested in what each had to say, but surely there was nothing he learned from my talk. Still, he made a point of finding me, complimented my skill of exposition, and said we'd have to work together sometime. Well, OK, he's much more friendly and polite than I'd expected. But he couldn't be serious about working together. We live in completely different worlds. Indeed, I didn't hear from him for several years.

Then in the spring of 1992, Benoit called to invite me to spend the next year working with him at Yale. I was completely surprised that he remembered me.

When I arrived at Yale that summer, I talked with the dean and with several people in the Mathematics Department about teaching my fractal geometry course for non-science students. Getting approval to teach the course was remarkably easy, though I received many well-meaning comments that, because it was a new course, I should not expect too many students. "If you get 20, the course will be a great success," I heard. About 180 students completed the course. Even now, a decade later, annual enrollments still average over 100.

After the success of the course became clear, Aliette [Mandelbrot] told me that Benoit had always thought his work would be taught in schools but did not expect to live to see it. Nevertheless, for some time the idea of this course had been part of Benoit's agenda. Yale produces many people who come to have considerable influence on science, education, and policy in general. How much better for science and education if part of their science experience is exciting and surprising, showing a genuinely new way to look at nature and culture? More than most other subjects, fractal geometry shows students new patterns in their own worlds.

In 1997 Benoit and I ran a small meeting, inviting about a dozen people who had developed similar courses. Even though we had worked independently, our experiences revealed many common features. Everyone found these courses excited students' interest in mathematics and science, often to a degree surprising the students. In addition, we found many had developed much common material. Sharing some of this material was the motivation for *Fractals, Graphics, and Mathematics Education* [FM] and our website

http://classes.yale.edu/fractals/index.html

Encouraged by this success, with the help of Nial Neger, in 1999 we began running summer workshops for high school and college teachers. To date, about 120 have participated in this program. Follow-up curriculum development workshops have produced a collection of lesson plans, all field tested. Now enjoying support from the National Science Foundation, this project is vigorous and growing.

Benoit participates in all the workshops — not by showing up, giving a lecture, and leaving, but rather by spending hours talking with the teachers and listening to their opinions and ideas. For example, much of the cover design of *Fractals, Graphics, and Mathematics Education* [FM] came from teachers' reactions to earlier versions. Certainly, some other brilliant scientists are interested in teaching; few devote as much time and energy to as many levels of pedagogy as I have seen Benoit do.

Another experience, early in our work together, gave me a very clear picture of Benoit's approach to science. I was investigating an aspect of lacunarity, the distribution of a fractal's gaps. This particular calculation was straightforward, but rather tedious, requiring several days of careful algebra. Benoit had an idea of how the calculation should turn out, but my work seemed to give the opposite result. I checked each step thoroughly but found no error. If Benoit were as arrogant as I had heard, he would not like to see that his prediction is wrong. Although I certainly had no indication he would act that way, I was a little nervous when presenting the calculations. What was Benoit's reply? "Marvelous. The problem is more interesting than I had expected." More than any other event, this showed me that Benoit is a real scientist: it's what is right, not what we think is right, that is important.

His smile also showed me something I did not expect but have seen often in our subsequent collaborations: intellectual play and curiosity remain an important part of Benoit's work. In this regard he is almost child-like, peering in delight at the fantastic landscape seen first by him, and now by so many others. This view, some of which can be shared with very young children, is a marvelous gift.

That the eye is important in science, that mathematics has a useful experimental aspect, that roughness can be quantified as a dimension, that symmetry under magnification is important in nature and culture—any of these would be a wonderful legacy. But my strongest impression of Benoit is that he shows what can be done by looking at the world differently and then thinking very carefully about what is seen. True intellectual mavericks are rare, in part because not belonging to any established field is a lonely calling. All the more reason we should treasure those choosing this path. Their importance to science cannot be overstated.

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Breakfast with Mandelbrot

Marc Frantz

During the Fractal Jubilee which this volume commemorates, my wife Paula and I had the pleasure of having breakfast with Benoit Mandelbrot and his charming wife Aliette. Having finally met this man whose work I had long admired, I could not help but reflect on the influence his work has had on my mathematical interests and even my choice of career.

Around 1980 I began looking for a way out of a career in the arts, which had of necessity become mostly picture framing and too little painting. Just short of my thirtieth birthday, I began teaching myself mathematics and physics, with the wild dream of becoming a scientist. Popular books by physicists, and textbooks such as Taylor and Wheeler's *Spacetime Physics* [**4**], inspired me with their subject matter, and equally with their enthusiastic style. In my narrow experience, it seemed that physicists could write with passion about their subject, while for mathematicians precision alone was sufficient.

I was therefore surprised and delighted to discover Mandelbrot's *Fractals: Form, Chance and Dimension* [**2**] on the shelf of my local public library. It was strange, beautiful, and mystifying to me. At this early stage of my mathematical education much of it seemed opaque, but it was written with passion, and it was undeniably connected to the beauty of the real world. I was hooked. Eventually I decided to pursue mathematics rather than physics. Later, when I bought my own copy of the successor book, *The Fractal Geometry of Nature* [**3**], I felt a twinge of justification for my change of plans; the first words of praise inside the dust jacket were from John Archibald Wheeler.

My choice of courses in graduate school, and my later choices in research, have always been skewed towards fractal geometry and the kinds of mathematics that can be applied to it: real analysis, complex analysis, measure theory. Of course, these subjects are beautiful in their own right, with no need for external justification. Nevertheless I doubt that I would have been led to experience that beauty without the influence of Benoit Mandelbrot. Moreover, his influence has helped bring me full circle back to art again. For several years I have taught a course in mathematics and art at Indiana University, and helped run a series of summer math and art workshops for teachers, called VIEWPOINTS. Fractal geometry is invariably a part of the course and the workshops, and it has been extremely popular with both students and teachers. Indeed, the pedagogical success of fractal geometry has been well attested to by the many contributors to Mandelbrot's recent book with Michael Frame, *Fractals, Graphics, and Mathematics Education* [**1**].

Breakfast was a good time to reflect on all this, and more. We were amazed at the Mandelbrot's busy schedule, and touched by how kind and gracious they were to us. It was clear that the "father of fractal geometry" has more than earned the title, by attending to all aspects of this ever-growing field: doing research, suggesting new directions, inspiring others, lending encouragement, and writing for and speaking to the general public. As for myself, I am deeply grateful for his lasting influence, for the encouragement he gave me that morning, and for this opportunity to express my gratitude.

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Old Memories

Jean-Pierre Kahane

Benoit was born in Poland but he spent most of his life as a French citizen. He was in France during the war, the Vichy régime and the German occupation. He was trained in French high schools. His mathematical inspiration came from Paul Lévy, his teacher at the Ecole Polytechnique, and from his uncle Szolem Mandelbrojt, professor at the College de France, my own teacher. His dissertation and all his first articles were in French. His first book on fractals was published in French. He is, by the way, a beautiful French writer. As a tribute to this aspect of Benoit, I feel that it is appropriate to write my recollections in French.

C'est donc par Szolem que j'ai connu Benoit. Szolem était un mathématicien de grande envergure, très original mais nourri de tradition classique, et de plus un homme chaleureux et un brillant causeur. Il était un mathématicien pur dans tous les sens du terme. Benoit l'aimait beaucoup, en dépit ou à cause de leurs différences de caractère et d'orientation. Il a enregistré de précieux souvenirs de Szolem, et il a été l'initiateur de l'hommage qui lui a été rendu à l'Institut de France pour le centième anniversaire de sa naissance, au tout début de 1999, sous la présidence d'Henri Cartan, avec la participation de son fils Jacques, d'Yitchak Katznelson, de Paul Malliavin, la mienne et naturellement la sienne.

Nous avions dû nous rencontrer à l'Institut Henri Poincaré lorsqu'au même moment nous passions des examens de licence. Mais j'ai connu Benoit de réputation avant de le connaître personnellement. Outre Szolem, j'en avais entendu parler par Marcel Schutzenberger, avec lequel j'avais à l'époque, à la fin des années quarante et au début des années cinquante, de fortes affinités politiques. Plus tard, au cours des années 70, c'est encore Marcel Schutzenberger qui m'a initié à ses travaux de linguistique par un exposé à la Société philomatique de Paris. Schutzenberger, qui fut le père de l'informatique théorique en France, était un esprit subtil et profond, et, médecin de formation, il était un outsider en mathématiques. Il fut l'un des premiers, le premier en France à coup sûr, à saisir l'importance des travaux de Benoit.

Ma première et seule collaboration directe avec Benoit fait suite à une visite qu'il m'avait rendue après la parution du livre "Ensembles parfaits et séries trigonométriques", que j'avais écrit avec Raphaël Salem, et que Salem, mort soudainement, n'a pas pu voir imprimé. C'est Salem qui m'avait entrainé vers les ensembles de Cantor, leur description, leurs propriétés métriques, l'outil probabiliste, et les liens merveilleux entre leurs propriétés harmoniques et arithmétiques. Les ensembles homogènes (selfsimilaires dans la terminologie de Benoit, qui est meilleure), les mesures et dimensions de Hausdorff et leurs liens à la théorie du potentiel via la caractérisation de Frostman, le rôle de la fonction de Lebesgue construite sur l'ensemble de Cantor (que Benoit désigne éloquemment comme l'escalier du diable), et surtout les ensembles aléatoires qui, grâce au brouillage des fréquences, garantissent une certaine régularité au spectre de Fourier, tout cela était nouveau dans la littérature en France, mais familier à Benoit sous des approches différentes. Il venait donc me dire que, partant de points de vue différents, nous nous occupions des mêmes choses. C'était en 1963 ou 1964, et il pensait à la turbulence, aux "random cutouts", et aux propriétés des ensembles parcourus par ce qui s'est appelé plus tard les "vols de Lévy". J'étais loin de tout comprendre, et, dans l'article que nous avons fait en commun, j'ai seulement utilisé les approches les mieux connues, celles de Lévy et de Salem. Mais le souvenir de cette conversation a joué un grand rôle dans mon évolution ultérieure.

Depuis les années 1960 Benoit s'est fait connaître et reconnaître, et mes souvenirs se fondent dans ceux, bien plus variés et considérables, des scientifiques qu'il a influencés directement ou indirectement. Nous nous sommes rencontrés dans des occasions multiples. Il m'a initié aux processus multiplicatifs. Je continue à bénéficier, de temps à autre, d'interminables conversations téléphoniques sur les sujets les plus divers de l'actualité scientifique. Mon rôle dans ces conversations est d'écouter. Je le fais volontiers, parce que j'aime jusque dans ses outrances la passion de Benoit pour la science, pour son oeuvre et pour son audience. Benoit a dû se battre, et le combat n'est pas fini, pour être parfaitement accepté dans toutes les communautés scientifiques. Il n'est pas mauvais, pour terminer, de revenir à la France, parce que la France est volontiers ingrate à l'égard de ses savants. Ce n'est que tout récemment que Fourier est pleinement reconnu dans son pays. Benoit a aujourd'hui plus de lecteurs dans le monde que n'en a jamais eu Fourier. Dans moins de deux siècles, on peut l'espérer, les Français lui rendront pleinement justice.

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My Encounters with Benoit Mandelbrot

David B. Mumford

I first met Benoit when he came to Harvard for the academic year 1979/1980 (my reconstruction of the exact timeline may be inaccurate!). I had just become dimly aware of the fact that computers were getting more powerful and more useful to non-specialists when Benoit showed me some of his first experiments with the fractals arising in complex analysis. The ones which grabbed me were not figures of the Mandelbrot set whose glories were just beginning to open up, but were the "limit sets" of Kleinian groups generated by reflections. These are a geometric aspect of one of the principal tools for attacking the classification of Riemann surfaces (so-called "moduli spaces"), the leitmotiv of my research for many years. Any new tool for exploring the moduli spaces got my attention. His crude printouts, generated by a Monte Carlo algorithm, gave a tantalizing glimpse of the amazing intricacy of these fractal sets.

This started a nearly 10-year-long interaction between Benoit and his group, David Wright and Curt McMullen (both of whom were graduate students at the time) and myself, in which we pursued the computer exploration of these striking sets. I visited Benoit at the Watson lab several times and worked with his group and with the powerful machines and graphics software IBM generously made available. I recall one memorable time when I came to IBM with one of our latest printouts of these limit sets. We had also worked out some asymptotics of its growth, so we had an estimate of its Hausdorff dimension. As a little challenge, I asked Benoit to estimate this: after a pause, he said 'about 1.8': our prediction was 1.83! And for all I know, he may have been closer than us, as we had no proof for our method. The fruits of this work, which Dave Wright, Caroline Series and I pursued off and on for another decade, have recently been published in the book *Indra's Pearls*.

But Benoit's real impact on me was not just that he had opened a new window into an area that related to moduli spaces. He reawakened my interest in computers, with which I had played as a student in high school and college (with stone age computers using relays and rheostats). His work made so clear the tremendous potential, in both pure and applied mathematics, of doing computer *experiments*, something that I have embraced ever since in my own work. It was a wholly new way of thinking about and doing mathematics. My professional career up to that point had been that of a typical pure mathematician whose only goal was to prove theorems. After finding that limit sets were computable, I began to feel one ought to use computers to get your hands on every mathematical construction, to explore and seek patterns which were inaccessible to pencil and paper.

And Benoit seemed to have looked at every field! One faculty lunch when I had started to learn a bit about neurobiology, I mentioned Ramon y Cajal and Benoit immediately began telling what a unique genius he had been and explaining to me what he had done. He attracted brilliant unusual friends like Carleton Gajducek, whose inspired detective work uncovered the cause of kuru, and he regaled us with stories of Gajducek. I think Benoit overwhelmed everyone with the scope of his inquiries. When he gave a seminar, you never knew where he would find the latest set of data exhibiting fractal structure or heavy tails—I remember a lecture where he had data on the micro-spatiotemporal distribution of rainfall during the passage of one storm. I had never noticed that rainfall was so irregular.

When I became involved in computer vision, some of his lessons came back to me. Up until 1995, with a few exceptions, researchers tended to think they could intuit the nature of natural images without looking at their empirical statistics. Teaching an introductory course one year, I remembered heavy tails and, as a lark, plotted the distribution of adjacent pixel differences in a small set of 12 images. Of course, as he would have predicted, it was clearly non-Gaussian with high kurtosis! His approach to data mining, seeking its natural patterns with a broad set of tools, has been a technique that has revolutionized the nature of computer vision in the last decade. Look at data, keep an open mind about what it wants to tell you and seek the mathematics that fits it: others may have said it before, but no one as consistently and broadly and effectively as Benoit Mandelbrot.

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Fractal Geometry and the Foundations of Physics

Laurent Nottale

In his first book in French published in 1975, "Les Objets Fractals", Benoit Mandelbrot explicitly announced in his introduction the extraordinary perspective which his already almost 25 year-long hard work had led him to unveil. He wrote: "le but essentiel de cet essai est de *fonder une nouvelle discipline scientifique*" [**A**].[1]

More than twenty five additional years later, we can affirm that he has succeeded in this foundation act. Fractals are not only a new revolutionary geometry from the viewpoint of mathematics, but also both a scientific discipline in its own right and a new and highly efficient descriptive and investigative tool in several domains of science.

In 1993, one could write: "The extraordinarily huge number of occurrences of fractals in natural (physical and biological) systems is now an unavoidable observational fact. This was established by Mandelbrot (fractal behaviour has been suggested for coastlines, the distribution of galaxies, turbulence, the structure of the lungs) and in subsequent studies (asteroids, moon craters, sun spectrum, the brain, the blood and digestive systems, hadron jets, dielectric pulling, growth phenomena...). The list is now so large that it becomes very difficult to be exhaustive." ([**B**].) Ten years later, it is now almost impossible.

The specific question I want to briefly address, in the present tribute to Benoit Mandelbrot, is the importance of fractals as regards to our present and future understanding of the foundations of physics. Let me first recall some elements of the early developments of still another application of fractals, not quoted above; namely, the use of fractal geometry, not only for describing 'objects' (that remain embedded in a Euclidean space) but also for attempting to describe the geometry of space-time itself (i.e., in an intrinsic way).

A first step toward such a goal consists of introducing fractal geometry in the general relativistic description at the level of the source terms. Indeed, the highly inhomogeneous (fractal, for several decades) distribution of galaxies in the Universe was one of Mandelbrot's main examples of applications of fractal geometry [**C**], and its discussion still plays a fundamental role in our understanding of cosmology. This proposal led him to be invited in 1974 by Jean-Claude Pecker, a Professor in Astrophysics at the Collège de France, to give conferences on his new ideas. As recalled

[1] "The central goal of this essay is *to lay the foundations for a new scientific discipline.*" (Managing editor's translation.)

by Mandelbrot [**D**], the text of his 1975 book finds its origin in these Collège de France lessons. Now, Pecker was in fact my Ph.D. (Doctorat d'Etat) thesis advisor during the years 1975–1980: I was led to consider theoretically the consequences of the inhomogeneous distribution of matter through the effects of gravitational lensing, due in particular to clusters of galaxies and large scale structures of the Universe (these effects have been observationally discovered ten years later).

But a more direct connection between fractals and fundamental physics may come from their use in describing not only the distribution of matter in space, but also the geometry of space-time itself. The great geometric discovery of the XIXth century, namely, the Gauss–Riemann curved geometry, has been incorporated into physics to explain the very nature of gravitation. What about Mandelbrot's fractal geometry? Is it not called to play as important a role in modern physics? In this context, one can be struck by the essential difference between Einstein's general relativity and quantum mechanics: the first is by essence a geometric theory and is constrained from first principles, while the second is mainly of an algebraic nature and was constructed from unexplained axioms and rules lacking a more profound basis. In 1979-1980, I have suggested that what was lacking in the quantum theory could well be a geometric description of space-time at the microscopic level. There is indeed a contradiction between the ancient statement, going back to Leibniz, that a space cannot be defined independently of the 'objects' it contains, and the status of the quantum scales, where all 'objects' are of a quantum nature while they are embedded in a space-time that is assumed to remain Minkowskian (i.e., classical and absolute). But the failure of a large number of attempts to understand the quantum behavior in terms of standard geometry indicated that such a 'quantum geometry' should be of a completely new nature. Moreover, following the lessons of Einstein's construction of a geometric theory of gravitation, it was clear that any geometric property to be attributed to space-time itself, and not only to particular objects or systems, was necessarily universal. Fortunately, the founders of quantum theory had already brought to light a universal and fundamental behavior of the quantum realm, in opposition to the classical word; namely, the explicit dependence of the measurement results on the apparatus resolution, as described by the Heisenberg uncertainty relations. This motivated me to ask the following two questions: Was it possible to describe intrinsically a space-time whose geometry would be explicitly dependent on the scale of observation? Could such a geometry be able to give rise to the quantum behavior of the objects embedded into it and participating to it?

At that time, I had begun to work at the Ecole Nationale Supérieure des Techniques Avancées (ENSTA) with Thiébaut Moulin, who was developing models based on cellular automata that he called 'arithmetic relators'. These automata were providing structures that were characterized by imbricated, hierarchical levels of organization and often showed fractal behavior. He gave me Mandelbrot's book, in which I 'entered' with delight: it appeared that Mandelbrot's intuitive definition of fractal geometry closely coincided with the founding property of the new quantum geometry to be constructed! It was only twelve years later that I discovered that, almost at the same time, Garnet Ord had reached a similar conclusion and devised a similar program [**E**].

Several aspects of the subtle points on which Mandelbrot insisted from the very beginning, and upon which he was forced to regularly come back, turned out to play a key role in such an attempt. The first consists of not reducing fractals to scaling fractals. He writes in his 1982 book [**F**]: "Here as in the standard geometry of nature, no one believes that the world is strictly homogeneous or scaling." Then Mandelbrot explicitly states the parallelism between the various levels of the laws of motion and of scale laws, which remains a clue to the construction of fractal laws going beyond self-similarity: "Standard geometry investigates straight lines as a preliminary [step]. And mechanics also views uniform rectilinear motion as merely a first step. The same is true of the study of scaling fractals." His insistence for many years on the necessity of taking into account the fluctuations in the scale space due to discrete scale invariance (for example when calculating a fractal dimension), and of the importance of the transitions between different fractal (or non-fractal) regimes also turned out to be essential: what could have been seen at first sight as secondary details actually had a deep and fundamental physical meaning (in these examples, respectively, the log-periodic behavior of crisis evolution and the suggestion of an identification of the Einstein–de Broglie length-scale, i.e., the quantum to classical transition, as a transition from fractality to standard geometry).

Every contact with Benoit Mandelbrot has been extraordinarily enriching. My first one was in 1982, just at the date of the publication of the new version of "The Fractal Geometry of Nature" [**F**]. He knew that I had written a paper (with Jean Schneider) in which I had proposed to use Robinson's Nonstandard Analysis as a tool for the intrinsic description of fractal spaces. On that occasion, in addition to a large quantity of other essential information about fractals, I learned that he had himself already discussed such a link with Robinson. . . .

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Is Randomness Partially Tamed by Fractals?

Bernard Sapoval

I first heard of Benoit Mandelbrot and fractals in 1975 after the French science magazine La Recherche had published his paper. It was at the after lunch coffee, in the "Laboratoire de physique de la matière condensée" of the Ecole Polytechnique, a solid state physics experimental group. There was an animated discussion about these strange curves, but at that time, no link was made by any of us with a possible connection with the real world.

At the beginning of the eighties, I was working on solid electrolytes (a field now known as "solid state ionics") and had close contacts with electrochemists. One of them, Alain Le Méhauté, came to me, probably in 1981, with some strange claim about relating anomalous properties of electrodes with their "fractal geometry". The idea was intriguing and since I was in charge of the last part of the studies of the students at the "Ecole", I sent two students to work with Le Mehauté for their mini-thesis work. Nothing really came out of this collaboration but the fractal virus had already contaminated me.

This is how I attended the first workshop on fractals held with the support of IBM in Courchevel in France in 1982 and this is how I first met Benoit. It was a great experience for me because I met there a large group of people from various fields with whom I had very stimulating discussions. This was the time when Richard Voss had produced his famous artificial landscapes. Benoit gave himself probably half of the Courchevel lectures. I must say that it was my first experience with a totally new "boiling" field. It was also very pleasant to encounter many open-minded people amongst the fractal beginners.

At that time, I was as an experimentalist measuring contact properties between metals and solid electrolytes and the question came to me of determining the real geometry of these contacts which were the consequence of a soldering process of some sort. This is how, with Michel Rosso and Jean-François Gouyet, we started the numerical simulation of diffused contacts and diffusion fronts. We then discovered that soldering geometry is fractal and closely related to that of percolation. We first formulated the conjecture about the dimension of the external perimeter of the incipient percolation cluster: 7/4. This was recently proved mathematically by Smirnov and Werner and it is still an active field since we have shown last year (this volume) that this exponent applies even in describing the width of non fractal diffusion fronts. There followed a number of studies about 2D and 3D gradient percolation.

As a physicist, I would like to insist, especially in this mathematics volume, on the importance of Benoit Mandelbrot's achievements in the natural sciences. The fact that there exits a general mathematical concept able to describe irregular geometry from dendrites to fractures, from soldering to lungs, from galaxy distributions to river basins, from coastlines to blood vessels, from mountains to plant roots, from corrosion figures to bacterial colonies, from dielectric breakdown geometry to plant roots (the examples are innumerable), is probably one of the most important scientific successes of the second part of the 20th century. There has been consequently a considerable broadening of the concept of universality, first introduced by Kadanoff, Fischer and Wilson in the years 1960-1970 in the study of phase transitions.

In my mind, the most significant consequence of the unveiling of fractal geometry is the possibility of understanding the properties of random objects in terms of their geometry. It is a fact that many random processes, like Brownian motion, Lévy flights, percolation, diffusion, aggregation, roughening, corrosion, etching, naturally give rise to fractal (that is, hierarchical) geometries. If the properties of these objects are due more to the hierarchy of their geometry than to the random character of this hierarchy, then understanding the properties of deterministic fractal objects with the same fractal dimension permits to understand the properties of these random objects. This personal paradigm has been part of my involvement with fractals and we have shown that it applies to fractal electrodes, fractal catalysts and fractal membranes. For instance, the response of a random electrode of dimension D_f is very close to that of a deterministic electrode with the same fractal dimension. In that sense, randomness has been tamed by fractals. The question remains open and tantalizing for fractal resonators, fractal trees and many others systems.

As I am well placed to know, physics rests upon two main pillars: experiment and courageous unhindered vision where the mathematician can lead the mind. And the originality of mathematical vision, mathematics being the universal science "par excellence", always lies on one man's shoulders; here, it lies on Benoit Mandelbrot's individuality. It is of common knowledge that Mozart died at the early age of 37, approximately Benoit's age at the time he formulated his fractal concepts. But fractals are younger than Benoit, and this is not a figure of speech as shown by the content of this volume.

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On Knowing Benoit Mandelbrot

Jean E. Taylor

It has been my pleasure to know and apreciate Benoit Mandelbrot for about 30 years. I met Benoit through Fred Almgren, my husband from 1973 until he died in 1997; I am sure Fred would also have been delighted to contribute to this volume. Fred became a fan of Benoit when Benoit told him something that seemed unlikely, but Fred then determined that it could be proved through geometric measure theory (with considerable difficulty). Benoit then spoke in Fred's Geometric Measure Theory seminar, which I attended, and subsequently came fairly often to our house for dinner.

My interactions with Benoit grew when we were both members of the "permanent faculty" of the Geometry Center. Bill Thurston made this crazy diagram showing how the research of various of us faculty were related, and somehow there was often some connection to Benoit. And I still remember Benoit giving a talk to students under the aegis of the Geometry Center, at which he was revered like a rock star.

Several times I have run a First Year Student seminar for Douglass Scholars at Rutgers University, entitled "Some New Mathematics." Each time, I have included fractals as well as soap bubbles, and the students have been astonished. One of them wrote to me how she found the awareness of fractals had entered every bit of her life; she seemed to think I had done some serious messing with her mind.

I especially appreciated Benoit's coming and giving a talk at my retirement symposium in January 2003. Once again, he packed the room to overflowing, even without much advertisement of his talk. Benoit has brought an appreciation of mathematics to a huge audience, which is a magnificent achievement in its own right.

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