Physics

Mathematics Review Sheet

Scientific Notation

5.0×10^{8} positive powers used for big numbers. 10^{8} ➔ move decimal +8 spots 500000000

4.0×10^{-6} negative powers used for small numbers 10^{-6} ➔ move decimal -6 spots .000004

E notation

When writing a number with E notation, the basic rule is that the E replaces the "x 10" in the normal scientific notation form and the exponent that is normally written as a superscript (smaller and raised slightly $^{\text{Like This}}$) is written as a normal number down on the same level and not raised up.

So basically all you need to do to put a number in E notation is take away the "x 10" and replace it with an E and then write the exponent number next to the E.

Lets see some examples.

6×10^{-4} = 6e-4
3.405×10^{6} = 3.405e6
$- 6 \times 10^{9}$ = -6e9
5.04×10^{-75} = 5.04e-75

Scientific Notation and the Calculator

When you work with numbers that are in scientific notation in a calculator, use the 'E' notation. Do not try to put the numbers in with "x 10" factors, something always goes wrong with it. It's much easier to work with the E notation and it works well with a calculator.

The TI programming calculators have an EE button located on them. When putting scientific E notation numbers in a calculator, press the EE button when it is time to put the "E" in.

Other types of calculators will either have the same EE button or have a button labeled "EXP" that you can use to put an "E" in for the "E" notation. Practice with your calculator to get used to the E notation.

A number in E notation is the same as a normal number you are used to. You can do the same arithmetic operations on E notation numbers that you can on normal form numbers. For example

60000 + 5000 = 65000
(6×10^{4}) (5×10^{3})

This can also be done with E notation on your calculator, try it.

6e4 + 5e3 = 65000

Dimension Conversions – "Factor label method"

English System Factors

1 ft = 12 in
1 yd = 3 ft
1 mi = 5280 ft
1 in = 2.54 cm
1kg = 2.2 lbs
1 ton = 2000 lbs

* SI Prefix Conversions

Tera	1 T__ = 10^{12} __	centi	1 c__ = 10^{-2} __
Giga	1 G__ = 10^{9} __	milli	1 m__ = 10^{-3} __
Mega	1 M__ = 10^{6} __	micro	1 µ__ = 10^{-6} __
kilo	1 k__ = 10^{3} __	nano	1 n__ = 10^{-9} __
deci	1 d__ = 10^{-1} __	pico	1 p__ = 10^{-12} __

* the blank line in each factor is filled with whatever base unit being used
Ex: If we were working with meters (m). Then 1 micrometer = 10^{-6} meters (1µm=10^{-6}m)

When you are converting a number from one unit to another be sure to include **units for all numbers** in your calculation and cancel each unit as you go. Conversions are best expressed by examples. Let's review some.

<u>EXAMPLE 1</u> - Convert 5 km to ? m.

Look at your factors to see what you can do. We see that 1000 m = 1 km, and we want go to 'm' so that will work right away. We need to get rid of the 'km'. To get rid of the 'km' we will put it on the bottom and divide by it so it will cancel out. So we put the m on top and the km on the bottom so that the km will cancel out.

$$5\text{ km}\left(\frac{10^3\text{ m}}{1\text{ km}}\right) =$$

Now you cancel your units.

$$5\cancel{\text{km}}\left(\frac{10^3\text{ m}}{1\cancel{\text{km}}}\right) =$$

The km cancel so all you are left with is meters. Multiply the numbers through and your Final answer is in meters = 5000 m

<u>EXAMPLE 2</u> –Convert – 1 m to ? ft

We need to go from m (meters) to ft (feet), but there is no factor that directly relates meters to feet, so we have to use more than one step. Look at the factors and see that you can go from "m to cm", so that we then can then go "cm to in", and finally "in to ft". It may be hard to see this at first, but some practice and it's easy

$$1\text{ m}\left(\frac{1\text{ cm}}{10^{-2}\text{ m}}\right) = \qquad 1\cancel{\text{m}}\left(\frac{1\text{ cm}}{10^{-2}\cancel{\text{m}}}\right) = \quad 100\text{ cm}$$

now we have 'cm' so we move from 'cm' to 'in'

$$100\text{ cm}\left(\frac{1\text{ in}}{2.54\text{ cm}}\right) = \qquad 100\cancel{\text{cm}}\left(\frac{1\text{ in}}{2.54\cancel{\text{cm}}}\right) = \quad 39.37\text{ in}$$

now we have 'in' so we move from 'in' to 'ft'

$$39.37\text{ in}\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \qquad 39.37\cancel{\text{in}}\left(\frac{1\text{ ft}}{12\cancel{\text{in}}}\right) = \quad 3.28\text{ ft}$$

… We are now in feet units which was our goal.

We can save time by doing this all in one step rather than piece by piece.

$$1\text{ m}\left(\frac{1\text{ cm}}{10^{-2}\text{ m}}\right)\left(\frac{1\text{ in}}{2.54\text{ cm}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \qquad 1\cancel{\text{m}}\left(\frac{1\cancel{\text{cm}}}{10^{-2}\cancel{\text{m}}}\right)\left(\frac{1\cancel{\text{in}}}{2.54\cancel{\text{cm}}}\right)\left(\frac{1\text{ ft}}{12\cancel{\text{in}}}\right) = \quad 3.28\text{ ft}$$

You can see that all the units cancel through as you work left to right, and all you are left with is feet. Multiply and divide all the numbers through and you find that: 1m = 3.28 ft.

EXAMPLE 3 - Units on the bottom.

Convert $5\ \frac{m}{s}$ to $??\frac{m}{hr}$

To get rid of something when its on the bottom, you have to multiply by it rather than divide (when you have something on the bottom, you want to put it on top when you are converting so it will cancel out) … lets see how it works. We will use the fact that there are 3600 seconds in 1 hr to make this conversion.

$$5\frac{m}{\cancel{s}}\left(\frac{3600\,\cancel{s}}{1\,hr}\right) = 18000\,\frac{m}{hr}$$

the seconds cancel out leaving us with 'hr' left over

EXAMPLE 4 – Convert 15 m/s (meters per second) to ? mph (miles per hour)

For this problem you have to do two conversions. First, you have to convert the seconds to hours (which is on the bottom since its m/s to mi/hr). Then you have to convert the 'm' to 'mi'. Lets do it in two steps so it will be easier to see. First we will convert seconds into hours

$$15\frac{m}{\cancel{s}}\left(\frac{3600\,\cancel{s}}{1\,hr}\right) =$$ 54000 m / hr

Now we convert the **'m'** part into **'mi'**

$$54000\ \frac{\cancel{m}}{hr}\ \left(\frac{1\ \cancel{cm}}{10^{-2}\ \cancel{m}}\right)\left(\frac{1\ \cancel{in}}{2.54\ \cancel{cm}}\right)\left(\frac{1\ \cancel{ft}}{12\ \cancel{in}}\right)\left(\frac{1\ mi}{5280\ \cancel{ft}}\right) =$$ 33.6 mi / hr (mph)

<u>Conversions when units are raised to a power</u> – There is a trick to doing conversion problems with units that are squared or cubed (ft^2 or m^3). Very few people know how to do this, and it is difficult to understand. Examples are presented here for your reference, though it will be discussed in class to clarify.

EX 1 – convert 10 square miles to ? square ft

$$10\,mi^2\left[\left(\frac{5280\,ft}{1\,mi}\right)\right]^2 = 2.78 \times 10^8\ ft^2$$

The trick is that you have to square or cube all of the conversion factors when doing this.

EX 2 - Convert 100 cm^2 to ? ft^2

$$100\,cm^2\left[\left(\frac{1\,in}{2.54\,cm}\right)\left(\frac{1\,ft}{12\,in}\right)\right]^2 = 100\,cm^2\left[\left(\frac{1\,\cancel{in}}{2.54\,cm}\right)\left(\frac{1\,ft}{12\,\cancel{in}}\right)\right]^2 = 100\,cm^2\left[.0328\,\frac{ft}{cm}\right]^2 =$$

$$100\,\cancel{cm^2}\,(.0328^2)\,\frac{ft^2}{\cancel{cm^2}} =$$ .108 ft^2

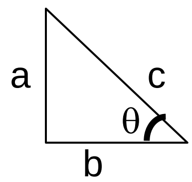
EX 3 – convert 3 yd^3 to ? cm^3

$$3\,yd^3\left[\left(\frac{3\,ft}{1\,yd}\right)\left(\frac{12\,in}{1\,ft}\right)\left(\frac{2.54cm}{1\,in}\right)\right]^3 =$$

$$3\,yd^3\left[\left(\frac{3\,\cancel{ft}}{1\,\cancel{yd}}\right)\left(\frac{12\,\cancel{in}}{1\,\cancel{ft}}\right)\left(\frac{2.54cm}{1\,\cancel{in}}\right)\right]^3 = 3\,yd^3\left[91.44\left(\frac{cm}{yd}\right)\right]^3 = 3\,\cancel{yd^3}\,(91.44)^3\left(\frac{cm^3}{\cancel{yd^3}}\right) =$$ 2293664.6 cm^3

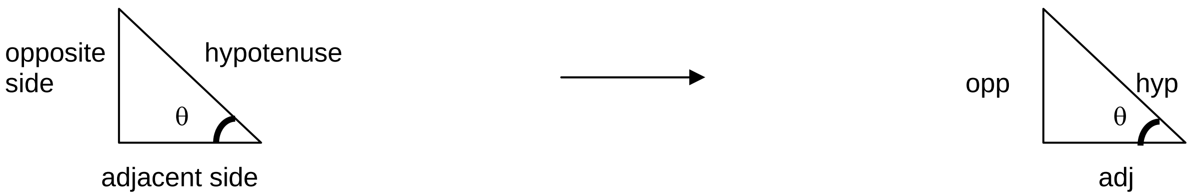
Geometry Review

Triangle review



Side "c" is a special side in this triangle because it is the only one that is diagonal. This side is called the "hypotenuse"

Sides a and b are further defined as to how they relate to the unknown angle. Meaning that in the picture above side 'a' is said to be 'opposite' from the angle θ and side 'b' is said to be 'adjacent' (next to) the angle θ.



Trig functions – sin, cos, tan. It is not real important that you know what these are, just that you know how to use them. These functions are used to find the sides and angles and triangles, that's what you will use them for.

Sin θ = opp / hyp (side a/ side c)

Cos θ = adj / hyp (side b / side c)

Tan θ = opp / adj (side a / side b)

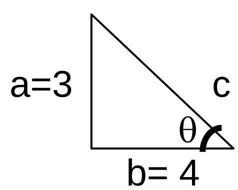
You can remember these by knowing SOHCAHTOA

(Sin Opp Hyp, Cos Adj Hyp, Tan Opp Adj)

<u>Finding an Unknown side to a triangle</u>

Method 1 – Use The Pythagorean Theorem = $c^2 = a^2 + b^2$

EXAMPLE 1 – finding unknown side c



Using $c^2 = a^2 + b^2$ ➔

$c^2 = 3^2 + 4^2$

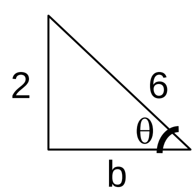
$c^2 = 9 + 16 = 25$

$c^2 = \sqrt{25} = 5$

so side c = 5

EXAMPLE 2 – finding unknown side other than c

This example often gives students a problem if they are not paying attention and not showing all their work, note the slight difference here:



Using $c^2 = a^2 + b^2$ ➔

$b^2 = c^2 - a^2$

$b^2 = 6^2 - 2^2 =$

$b^2 = 36 – 4 = 32$

$b = \sqrt{32} = 5.66$

Method 2 - Using the Trig Functions to find a side

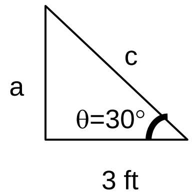
NOTE: When using any trig function, your calculator should be set to "degree" mode.

A regular non graphing calculator is usually set to degree mode, but **if you use a TI type graphing calculator, you need to switch the mode from 'radian' to 'degree'**… Any time you replace the batteries in your graphing calculator you need to make this change as well. See Mr. Wells or your math teacher if you do not know how to set your calculator to degree mode. You will get incorrect answers if you are in the wrong mode.

To test it, Try entering (sin 30) … if this comes out to 0.5, you are ok.

Using trig to find an unknown side.

Suppose you had the following problem: Find the length of the diagonal of the triangle shown below.



Suppose we are looking to find side 'a' in the triangle above. We can’t use pyhtagorean theorem because we are missing two sides, so we know we have to use trig since we know an angle and 1 side.
We are looking for Side 'a' = the opposite (opp) side.
We know the angle and the adjacent (adj) side. Lets list what we have

adj = 3 ft
θ = 30°
opp = ??? (lets call it 'x')

Look for a trig formula that has all those terms in it and we see the 'tan' formula contains all those items.

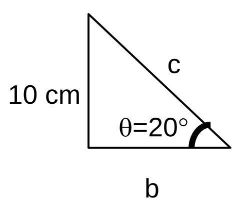
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan(30^\circ) = \frac{x}{3\,\text{ft}}$$

$$x = 3 * (\tan(30))$$

$$x = 1.73\,\text{ft}$$

Here is another example - find side (c) in the triangle below.



Listing what we have:
Opp = 10 cm
θ = 20°
hyp = ??? call it 'x'

Looking at the trig function, we see that 'sin' has both 'opp' and 'hyp' in it so we can use that.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\sin(20^\circ) = \frac{10\,\text{cm}}{x}$$

$$x \sin(20^\circ) = 10\,\text{cm} \qquad x = \frac{10\,\text{cm}}{\sin(20^\circ)} \qquad x = 29.24\,\text{cm}$$

Finding an unknown angle in a triangle.

You can determine the unknown angle θ using your calculator and the inverse sine, cosine or tangent functions. These functions are part of all scientific calculators and are written as , $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$. These functions are usually accessible as a secondary (2^{nd}) feature of the sin, cos, and tan functions on a calculator.

Since

Sin θ = opp / hyp

Cos θ = adj / hyp

Tan θ = opp / adj

It follows that if we know the sides of the triangle we could find the angle as well. Algebraic manipulations reveals that finding unknown angles can be performed using one of the formulas below derived from the relationships above.

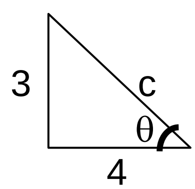
$\theta = \sin^{-1}$ (opp / hyp)

$\theta = \cos^{-1}$ (adj / hyp)

$\theta = \tan^{-1}$ (opp / adj)

You can use any one of these to find an unknown angle; they will all give you the same answer. If you know all the sides then you can use any of the formula to find the angle.

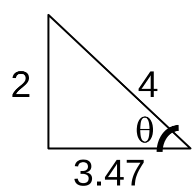
Example 1 – find the unknown angle given the following



Since we already know the 'adj' and 'opp' sides we do not need to solve for 'c' first. We can just use the inverse tan function with the two known sides to find the angle.

$\theta = \tan^{-1}$ (opp / adj) $= \tan^{-1}$ (3 / 4) = 36.87°

Example 2 – find the unknown angle given the following



Since we know all the sides we can use any of the inverse trig functions. Since the opp and hyp sides are whole numbers, lets use the inverse sin function to find the angle.

$\theta = \sin^{-1}$ (opp / hyp) $= \sin^{-1}$ (2 / 4) = 30°

You should do these problems above on your calculator to make sure you can get the same answer.