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YUKTIBHĀṢĀ OF JYEṢṬHADEVA
A Book of Rationales in Indian Mathematics and Astronomy
An Analytical Appraisal

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During medieval times there had occurred in Kerala a spurt of activity in the mathematical and astronomical studies. What is particularly significant herein is that some of the works produced related to *Yukti-s* and *Upapatti-s*, i.e. rationales and proofs. For this reason, these texts flash a torch, as it were, in the dark areas of Indian mathematics and astronomy and enable us to understand how these scientists designed methodologies and derived formulae, the application of which produced reliable results.

One of the important texts of this type is the *Yuktibhāṣā of* Jyeṣṭhadeva (1500-1610). Since the work has been composed in the local language of Kerala, *viz.*, Malayalam, it has, by and large, remained beyond the ken of scholars not knowing that language, with the result that there have been but sporadic studies and that too only on certain topics dealt with therein. An attempt is made in this paper to give an idea of the contents of the work, drawing attention to certain peculiarities therein, towards making a detailed study thereof. The question of its authorship and date have also been tackled.

The work, which when printed, would cover about 300 pages, is divided into two parts, with 17 chapters in all. The First Part is related to the depiction of the rationales of Arithmetic, Algebra, Geometry and Trignometry, and the Second Part to Astronomy. The seven chapters of Part I deal with: I. Logistics, II. Ten Algebraic problems, III. Fractions, IV. Rule of Three, V. *Kuṭṭākāra*, VI. Circle, and VII. Disquisition on R-Sine. Part II deals with: Ch. VIII. Planetary motion, IX. Celestial sphere, X. Declination and Right Ascension, XI. Fifteen Problems in Right Ascension, Declination etc., XII. Directions and Shadow, XIII. Ten Problems on Spherical Triangles, XIV. *Lagna* and *Kālalagna*, XV. Eclipses and Parallax Corrections, XVI. *vyatīpāta*, and XVII. *Dṛkkarma* or Reduction to observation, and the phases of the Moon.

INTRODUCTION

During medieval times, there had been in Kerala, in the southern-most part of India, an effusion of mathematical and astronomical investigations, at least a substantial part of which has been documented in original works, commentaries and minor tracts devoted solely or primarily to the elucidation of theories and rationales of computation.[1] An important work among these is the *Yuktibhāṣā* (*YB*), 'Rationale in the Malayalam language', whose sole aim is to rationalise the theories involved in the constants and computations occurring in the *Tantrasaṅgraha*, an important astronomical work of Nīlakaṇṭha Somayāji (A.D. 1443-1560).[2] Thus, after the benedictory verses, the work commences with the statement:

> "aviṭe naṭe trantrasaṅgrahatte anusariccu grahagatiyiṅkal upayogamuḷḷa gaṇitaṅṅale muzhuvanāyi colluvān tuṭaṅṅunnēṭattu..."

"Here, commencing an elucidation in full of the rationales of planetary computations according to the *Tantrasaṅgraha*..."

The work finds its first reference in modern writings in an article by C.M. Whish in 1835,[3] where it is referred to towards verifying the date of the author of *Tantrasaṅgraha*.[4] Whish had contemplated "a farther (*sic*) account of the Yucti-Bhasha...will be given in a separate paper,"[5] which, however, does not appear to have been written or published.

Yuktibhāṣā, known also as *Gaṇitanyāyasaṅgraha* 'Compendium of astronomical rationale', has been a popular text in Kerala for more than four hundred years, since its composition towards A.D. 1530. Several manuscripts of the work are known.[6] However, since the work is couched in the Malayalam language which is spoken only in Kerala it has remained, practically, beyond the purview of scholars who did not know the language, in spite of its having been published.[7] And the few articles on this important work relate to only certain individual topics treated herein.[8] It is therefore necessary that a critical appraisal of the nature and contents of this work as a whole is made, so as to enable scholars take up the same for further study.

AUTHORSHIP OF YUKTIBHĀṢĀ

The introductory verses of the *YB* do not mention the name of its author, not do its manuscripts indicate his name at their closing colophons. However, one of its manuscripts preserved in the Sanskrit College Library, Tripunithura which had been used by Rama Varma Maru Thampuran for his edition of the work, had at its close the verse:

> alekhi yuktibhāṣā vipreṇa Brahmadattasaṃjñena|
> 'ye golapathasthās syuḥ' kalirahitāḥ śodhayantas te||

Taking the word *alekhi* in the verse to mean 'composed' instead of its natural meaning 'written, copied', the Introduction to the said edition took Brahmadatta mentioned in the verse as the author, and the date given by the Kali chronogram *ye golapathasthās syuḥ* corresponding to A.D. 1750, as the date of its composition.[9]

There are, however, evidences which point to the correct name of the author of *yuktibhāṣā* as Jyeṣṭhadeva and his date to be A.D. 1500-1610. Thus, an old palmleaf manuscript, No.755, of the Kerala University Manuscripts Library, entitled *Gaṇitayuktayaḥ* contains many astronomical tracts, in one of which, dealing with the precession of the equinoxes (*ayanacalana*), occurs the statement:

atha vikṣiptacalanasyānītau pūrvasūribhiḥ।
proktā ye matabhedās tān yakṣye tattvabubhutsayā॥

* * * * * * *

jūkakriyādike pāte svarṇam tatsādhane vidhau।
ity uktā kṣepacalanasyānītis *tantrasaṅgrahe*।
jyeṣṭhadevo 'pi *bhāṣāyam* nādhikam kiñcid uktavān॥

The *Tantrasaṅgraha* referred to here is,obviously, the work of Nilakaṇṭha Somayāji, and the *Bhāṣā* specified as the work of Jyeṣṭhadeva is the *Yuktibhāṣā*, which, as indicated above, seeks to set out and elucidate the theories and practices involved in the *Tantrasaṅgraha*.

There are more clear evidences which point to the correct name of the author of *Yuktibhāṣā* as Jyeṣṭhadeva and his date to be A.D.1500-1610. Thus, an astronomical chronology (*granthavari*) in the Malayalam language found as a post-colophonic statement in an old palmleaf manuscript of a Malayalam commentary on *Sūryasiddhānta* preserved in the Oriental Institute, Baroda, Ms. No. 9886, contains in it the statements:

"parameśvaran vaṭaśśeri nampūri, nilāyāḥ saumyatīrasthaḥ parameśvaraḥ...asya tanayo dāmodaraḥ, āsya śiṣyo nīlakaṇṭha-somayāji, iddeham *tantrasaṅgraham, āryabhaṭīyabhāṣyam* mutalāya granthaṅṅaḷku karttāvākunnu. 'lakṣmīśanihitadhyānaiḥ' iti asya kalinā kālanirṇayaḥ.

"purvokta-dāmodarasya śiṣyaḥ jyeṣṭhadevaḥ, iddeham paraṅṅoṭṭu nampūriyākunnu *yuktibhāṣa*-granthatte uṇṭākkiyatum iddeham canne

"jyeṣṭhadevante śiṣyan tṛkkaṇṭiyūru acyuta-piṣāraṭi, iddeham *sphuṭanirṇayam, goladīpikā* mutalāya grantha (ṅṅaḷkku) karttāvākunnu.

"acyuta-piṣāraṭiyuṭe śiṣyan nelputtūru nārāyaṇa bhaṭṭatiri, iddeham *nārāyaṇāyam, (prakriyā) sarvasvam* mutalāya granthaṅṅaḷkku kartā. 'āyur-ārogyasaukhyam' ityādi-kalinā kālanirṇayaḥ. ...."

'Parameśvara was a Nampūri from vaṭaśśeri (family). He resided on the northern bank of the nilā(river)...His son was Dāmodara. Nīlakaṇṭha Somayāji was his pupil. He, (the latter), is the author of the *Tantrasaṅgraha, Āryabhaṭīyabhāṣya* and other works. His date is determined by the Kali days 16, 80, 553 (A.D. 1500)

'Jyeṣṭhadeva was the pupil of the above Dāmodara. He was a Nampūri from Paraññoṭṭu (family).[10] He is also the author of the work *Yuktibhāṣā*. 'Acyuta Piṣāraṭi of Tṛkkaṇṭiyūr was the pupil of Jyeṣṭhadeva. He is the author of the *Sphuṭanirṇaya, Goladīpikā* and other works.

'Melputtūr Nārāyaṇa Bhaṭṭatiri was the pupil of Acyuta Piṣāraṭi. He is the author of the *Nārāyaṇīya, (Prakriyā) sarvasva* and other works. His date is determined by the Kali days 17, 12, 210 (A.D.1587).'

Here, it is specifically stated that Jyeṣṭhadeva is the author of *Yuktibhāṣā* and the teacher-pupil succession is: Parameśvara (A.D.1360-1455) — son, Damodara — pupils, Nīlakaṇṭha Somayāji (1443-1560) and Jyeṣṭhadeva (1500-1610 *c*.) — pupil, Acyuta Piṣāraṭi (1550-1621) — pupil, Nārāyaṇa Bhaṭṭatiri (A.D.1587).

That Jyeṣṭhadeva was the teacher of Acyuta Piṣāraṭi is stated by Piṣāraṭi himself in the concluding verse of his work on the computation of eclipses, entitled *Uparāgakriyākrama*:

'proktaḥ pravayaso dhyānāt' jyeṣṭhadevasya sadguroḥ|
vicyutāśeṣadoṣenetyacyutena kriyākramaḥ||

'Thus has been stated the (*Uparāga*)-*Kriyākramaḥ* by Ācyuta of clear thought through his contemplation of (the teachings of) his aged benigh teacher Jyeṣṭhadeva.' The Malayalam commentary to *Uparāgakriyākrama* explains that the expression *proktaḥ pravayaso dhyānāt* serves also a chronogram to give the date of completion of the work. This chronogram works out to A.D. 1592, when Acyuta Piṣāraṭi, pupil of jyeṣṭhadeva, composed the work.[11]

Whish, in his article referred to above, records a tradition that the author of the *Yuktibhāṣā* wrote also a work called *Dṛkaraṇa*.[12] The *Dṛkkaraṇa* in question, an astronomical manual in Malayalam verse, is available in manuscript from (no.C.7-C of the Kerala Univ. Mss. Library) but does not give anywhere the name of its author. However, it gives its date of composition in its final verse through the Kali chronogram *kolambe barhisūnau* (Kali days 7, 83, 391) which would be A.D.1608. In view of the tradition recorded by Whish and this date being not far from 1592 mentioned by Acyuta Piṣāraṭi, we might take the *Dṛkkaraṇa* to be a work of Jyeṣṭhadeva and that he lived up to about 1610.

In view of the facts that Dāmodara (c. 1400-1500) was a teacher both of Nīlakaṇṭha Somāyāji and Jyeṣṭhadeva, and that Jyeṣṭhadeva wrote his *Yuktibhāṣā* in the wake of

Nīlakaṇṭha's *Tantrasaṅgraha*, he must be a younger contemporary of Nīlakaṇṭha. He is remembered in 1592 by his pupil Acyuta Piṣāraṭi as *pravayas* ('very old'). His *Dṛkkaraṇa* is dated in 1608. Jyeṣṭhadeva should, therefore, have been long-lived, his date being *c.* 1500-1610. His family house Parañṅoṭṭu (Skt. Parakroḍa) still exists in the vicinity of Ālattūr and Trakkaṇṭiyūr where well known astronomers like Parameśvara, Nīlakaṇṭha and Acyuta Piṣāraṭi flourished about those times.

Scope And Extent Of Yuktibhāṣā

The entire text of *Yuktibhāṣā* occurs as one continuum, without any internal or closing colophons to mark off the subjects treated in the work. However, towards the middle of the work, where the treatment of mathematics ends and that of astronomy commences, occurs a general benedictory statement which reads: *Śrīr astu, hariḥ śrī-gaṇapataye namaḥ, avighnam astu*. This would naturally mean that the author had conceived his work as consisting of two parts, devoted respectively to mathematics and astronomy. Since the work deals with several main subjects and a number of topics under each, the needed subject and topic divisions shall have to be made editorially with suitable indication. Demarkating the work thus, the main subjects treated in Part I: Mathematics are: *Parikarma* (Logistics), *Daśapraśna* (Ten problems involving logistics), *Bhinnagaṇita* (Fractions), *Trairāśika* (Rule of three), *Kuṭṭākāra* (pulverisation), *Paridhi-vyāsa* (Relation between circumference and diameter) and *Jyānayana* (Derivation of Rsines). The subjects treated in Part II. Astronomy are: *Grahagati* (Planetary motion), *Bhagola* (Sphere of the zodiac), *Madhyagraha* (Mean Planets), *Sūryasphuṭa* (True Sun), *Grahasphuṭa* (True Planets), *Bhū-Vāyu-Bha-gola* (Spheres of the Earth, Atmosphere and Asterisms), *Ayanacalana* (Precession of the Equinoxes), *Pañcadasapraśna* (Fifteen problems relating to spherical triangles), *Digjñāna* (Orientation), *Chāyāgaṇita* (Shadow computations), *Lagna* (Rising point of the Ecliptic), *Nati-Lambana* (Parallaxes of Latitude and Longitude), *Grahaṇa* (Eclipse), *Vyatīpāta*, and *Śṛngonnati* (Elevation of the Moon's horns).

Kriyākramakarī, Yuktidīpikā And Yuktibhāṣā

There are two extensive commentaries, both by Śaṅkara Vāriyar of Tṛkkuṭaveli family (A.D. 1500-1660), being *Kriyākramakarī* and *Yuktidīpikā*, the former on the *Līlāvatī* of Bhāskara II,[13] and the latter on the *Tantrasaṅgraha* of Nīlakaṇṭha Somayāji.[14] Interestingly, there is a close affinity between the *Yuktibhāṣa* and the above-said two commentaries. Even more, there is some sequence of arguments and verbal correspondences amongst them in the treatment of identical topics. From this similitude it has been suggested that the *Yuktibhāṣā* is just a rendering into Malayalam of certain passage from that Sanskrit. It is further suggested that for this reason, there is not much that is original in the *Yuktibhāṣā*.[15] But the fact is just the other way round, namely that the Sanskrit versions are adaptations and paraphrases of the relevant portions from the *Yuktibhāṣā*. This is confirmed by Śaṅkara, the author of both the commentaries, when he states specifically in the colophonic verses of his commentary

Yuktidīpikā on the *Tantrasaṅgraha*, that what he had done in the commentary was only 'the setting out of the material elucidated in the work of the Brāhmaṇa of Parakroḍa (viz. Jyeṣṭhadeva) (author of the *Yuktibhāṣā*). *Cf., for instance one such colophonic verse:*

ity eṣa parakroḍāvāsa-dvijavara-samīrito yo 'rthaḥ|
sa tu tantrasaṅgrahasya prathame 'dhyāye mayā kathitaḥ|| (Edn., p. 77)

Yuktibhāṣā And Gaṇita-Yuktibhāṣā

There is a work entitled *Gaṇita-Yuktibhāṣā* (Ms. No. R.4382 of the Govt. or. Mss. Library, Madras) in Sanskrit and it has been suggested that it might be the source of the Malayalam *Yuktibhāṣā*. However, a detailed comparison of the two shows that the *Gaṇita-Yuktibhāṣā* is but a rough and ready translation into Sanskrit of the Malayalam original by one who lacked not only the ability of writing idiomatic Sanskrit but also an adequate knowledge of the subject. Moreover, at places there occur haplographical omissions in the Sanskrit version of passages available in the Malayalam work, which fact too confirms that the Sanskrit version is the derived form.

Presentation Of The Rationale

The mathematical and astronomical rationale presented in the *Yuktibhāṣā* relate to several aspects, to wit, concepts, theories, constants, computations, demonstration by diagrammatic representation and the like. The procedure of depiction is logical, going step by step, first presenting the fundamentals and gradually building up the argument. It is, if one might say so, 'intimate' in that it inculcates the rationales and elucidates the steps even as a teacher does to a student. The work aims at understanding and conviction by the reader. A passage might be cited to illustrate the point. The selected passage relates to the concept of the motion of the planets:

Grahaṅṅaḷuṭe gatiprakāram

Iviṭe grahaṅṅaḷellām oru vṛttamārgēṇa gamikkum, Divasattil vṛttattinte itra aṃśam gamikkum ennu niyatam tānum, Aviṭe divasattil itra yojana gamikkum ennuḷḷa yojanāgati ellā grahattinum samam, Aviṭe ceriya vṛttattiṅkal gamikkunnavattinnu kuraññoru kālam koṇṭu baṭṭam kūṭum, Valiya vṛttattiṅkal gamikkunnavattinnu perikekkālam Kūṭiyē vaṭṭam tikayū, Enniṭṭu cantrannu irupattēzhu divasam koṇṭu pantraṇṭu rāśiyiṅkalum gamiccu kūṭum, Muppattiraṇṭāṇṭu kūṭiyē śani naṭannu kūṭū, Vṛttattinte valippattinnu takkavaṇṇam kālattinte perukkam, Atāṭu graham tante vṛttattiṅkal oru vaṭṭam gamiccu kūṭunnatinu bhagaṇam ennu pēr, Caturyyugattiṅkal etra āvṛtti gamikkum tante tante vṛttattiṅkal atu tante tante yugabhagaṇam ākunnatu.

Iviṭe candrane oru nāḷ oru nakṣatrattōṭu kūṭe kaṇṭāl pittennāḷ atinte kizhakke nakṣatrattōṭu kūṭe kāṇām, Itinekkoṇṭu candranu gatiyuṇṭennum kizhakkōṭṭu gatiyennum kalpikkām, Kizhakkōṭṭu rāśikramam ennum kalpikkām.

Ī vṛttaṅṅaḷkku ellāttinum kūṭi oru pradeśatte ādiyennu kalpikkumāruṇṭu, Aviṭattinnu meṣarāśiyuṭe ādiyennu pēr, Ī goḷattiṅkal kalpikkunna vṛttaṅṅaḷe ellāttēyum irupattōrāyirattarunūru khaṇḍamāyi vibhajikkumāruṇṭu, Itil ōrō khaṇḍam iliyākunnatu| Iva valiya vṛttattiṅkal valutu, ceriya vṛttattiṅkal cerutu, Saṅkhya ellāttinkalum okkum, Atatu graham tante tante vṛttattiṅkal itra īli gamikkum ōrō divasam ennu niyatam, Graham gamikkunna vṛttattinte kendṛttiṅkal irunnu nōkkum draṣṭāvu eṅkil nityavum gati okkum ī grahattinnu ennu tōnnum, Bhūmadhyattiṅkēnnu oṭṭu cellū grahavṛttakendrzm, Bhūmīṅkalu draṣṭāvu, Ī draṣṭāviṅkal kendram āyiṭṭu oru vṛttam kalpippū grahattōṭu sparśikkumātu, Ā vṛttattiṅkal etra cenniṭṭirikkuṁ graham atra cennū meṣādiyiṅkal ninnu graham ennu tōnnum ī draṣṭāvinu, Itariyum prakāram sphuṭakriyayākunnatu."

PLANETARY MOTION

'Now, all planets move along circular (orbits). The number of minutes which each planet moves in its orbit in the course of a day is fixed.

There, the number of *yojana-s* moved per day is the same for all planets. For planets which move along smaller orbits the circle would be completed in a shorter time. For those which move along bigger orbits the circle would be completed only in a longer period. For instance, the Moon would have completely moved through the twelve signs in 27 days, while Saturn will complete it only in 30 years. The length of time taken is proportional to the size of the orbit. The completion of one motion of a planet in its orbit is called a *bhagaṇa* (of that planet). The number of times that a planet moves in its orbit during a *catur-yuga* is called its *yuga-bhagaṇa*. (revolutions per aeon).

'Now, if the Moon is seen with an asterism on a particular day, it will be seen the next day with the asterism to the east of (the first one). From this it might be understood that the Moon has eastward motion. The sequence of the signs can also be understood as eastward. For all these orbits, a particular point is taken as the commencing point. This point is termed as the First point of Aries. All the circles in a sphere are divided into 21,600 equal parts. Each part is a minute. They are larger (in length) in bigger circles and smaller in smaller circles, though the number of parts is the same in all. The number of minutes that a planet will move along its orbit during the course of a day is fixed. If one perceives the said motion placing himself at the centre of the orbit of a planet, the motion of the planet would appear equal every day. The centre of the planetary orbit is the centre of the earth. The perceiver is, however, situated on the Earth's surface. Conceive a circle with the perceiver as centre and touching the planet. The perceiver would see the planet as much advanced from the First point of Aries as the planet has advanced in the said circle. The method by which the perceiver ascertains this (advanced) point is called 'Computation of the True planet'.

ANALYTIC CONTENTS OF THE YUKTIBHĀṢĀ

As mentioned earlier, Pt. I of the *Yuktibhāṣā*, dealing with Mathematics, can be divided into seven chapters.

Ch. I on Logistics (*Parikarma*) deals with the Nature of numbers (pp. 1-2).[17] Multiplication as explained from several standpoints including the use of diagrams (pp. 2-9), Multiplication by easy methods (pp. 9-13), Divison (p. 15), Squaring in its several methods (pp. 15-29), and Roots of sums of squares and difference of squares (pp. 29-31).

Ch. II. *Daśapraśnottara* (Ten algebraic problems and their solutions) relates to the finding of two numbers when two from among the five, viz., their sum, difference, product, sum of squares and difference of squares, are given. (pp. 32-35).

Ch. III deals with Fractions (pp. 36-44). Highly analytical in presentation, the topics dealt with are: Nature of fractions, their addition, subtraction, multiplication and divison.

Ch. IV is entirely devoted to the Rule of three, where both its general and inverse rules are set out. (pp. 45-49).

In Ch. V (pp. 50-71), the importance of the Rule of three in mathematical and astronomical computations is indicated with the computing of the current Kali day.

The Mean Planets are divided therefrom. *Apavartana* (Reduction) and *Kuṭṭākāra* (Pulverisation) are introduced here and their purpose specified. An elaborate rationalisation of the principles involved in it and the practices followed in both Reduction and Pulverisation is made and explained with examples. It is also indicated how the process helps in arriving at the *Bhājya-s* (Divisers) and *Bhājaka-s* (Multiplicands) in Pulverisation by means of the *Vallī* (Series of divisons), towards computing the planets more and more accurately.

In Ch. VI (pp. 72-142), the *Yuktibhāṣā* gives several formulae for determining the circumference of a circle of a given diameter. Some of the methods involve the properties of right angled triangles, towards which the properties of right angled triangles are demonstrated graphically. For the method which involve different summations of series, the derivation of those series is also demonstrated. Among the latter are the summations of consecutive numbers, the summation of squares, the summation of cubes and higher powers, and the summation of summations. In the case of certain formulae, *Yuktibhāṣā* enunciates further rules towards making the derived results more and more accurate (pp. 120ff.). Attention of scholars might be drawn here to a series of papers wherein Prof. C.T. Rajagopal, lately Director, Ramanujan Institute of mathematics, University of Madras, and his associates have worked out, in terms of modern mathematics, the above-said series and the different formulae enunciated in the *Yuktibhāṣā*. They have also shown that these are independent of the discoveries made more than a century later by the Western scientists, James Gregory (1671), G.W. Leibniz (1673) and Isaac Newton (1670).[17a]

Ch. VII (pp. 143-290) forms a long disquision of rationales relating to Rsines, their modifications and allied subjects. The chapter commences with an elaborate exposition of the geometrical derivation of the 24 Rsines for 3° 45′ each (pp. 143-50), and explanations of allied terms like Rcosine, Rversed sine, 'arc, *bhujākhaṇḍa, koṭikhaṇḍa, jīvākhaṇḍa* and *khaṇḍajyā*, and their mutual relationship (pp. 150-60). Some of the other topics elucidated herein are: Accurate determination of the Rsines (pp. 160-65), Computation of the Rsines of any given arc (pp. 165-73), Computation of the arc of any given Rsine (pp. 173-75), Summation of Rsine differences and Rversed sine differences (pp. 176-98). Accurate determination of the circumference of a circle making use of the said summations (pp.198-206), Addition of Rsines of two angles (pp. 206-24, 237-44), Cyclic quadrilaterals and their properties (pp. 224-37, 247-64), Square of the area of a circle (pp. 264-72), Derivation of Rversed sines (pp. 272-75), Derivation of Rsine shadow (pp. 275-78), Surface area of a sphere (pp. 278-82), and Volume of a sphere (pp. 282-90). The author of the *Yuktibhāṣa* elucidates the rationales of the several items either geometrically or algebraically or using both means as the occasion demands.

Part II of *Yuktibhāṣā* contains many interesting formulae which are scarcely found in other works. For the sake of convenience, this Part is amenable to be divided into ten chapters, VIII too XVII, in consideration of the subject dealt with in them.

Ch. VIII deals with the Planetary Theory. At the outset, the concepts of the Sun, Moon and other planets, their linear velocities and angular motions are described. As in other Hindu astronomical treatises, the assumption is made that the linear velocity of all planets is the same but the angular motion varies depending on the dimension of their circular orbits. The Epicyclic Theory for the sun and the moon and for the other planets and the interaction of the *manda* and *śīghra* epicycles are explained. There is a new concept in the treatment of the epicycles as compared to the *Āryabhaṭīya* and other works.

In a nut-shell the planetary theory broadly is like this. The earth is the centre and the sun and the moon go round the earth. As for other planets, with earth as centre the *śīghra* goes round the earth with the mean motion of the sun. The mean planet moves on a circle with *śīghra* as centre. The true planet is on the *mandocca* circle with mean planet as its centre. Alternatively, the last two circles can be interchanged. This theory is advocated byNilakaṇṭhain his commentary on *Āryabhaṭīya* and practically all later Kerala authors have followed suit. In factNilakaṇṭhatries to say that it was the view of Āryabhaṭa also. If *śīghra* is identified with the sun itself then this agrees broadly with the modern theory with the positions of earth and sun reversed. In fact the western astronomer Tycho Brahe (1546-1601) appears to have adopted a similar theory.

The rationale for adopting three or four stages of operations to find the geocentric longitude of a planet is also explained. This particular aspect has been baffling the scholars as the rationale for this is not to be found in any work. It is generally believed that the Hindu astronomers were not aware that the true geocentric longitude is to be obtained in two steps by first applying the *manda* correction, and using the corrected planet the *śīghra* equation is to obtained and applied to the once corrected planet; therefore they adopted different methods involving three or four stages in different ways.[18] However later Kerala works like *Sphuṭanirṇaya* have clearly explained the procedure in two stages correctly. This is not surprising if the longitude is arrived at on the basis of the planetary theory described above. Nilakaṇṭha also in his commentary on Āryabhaṭīya has given a similar method. Then how and why these three or four stages method got in. Some explanation has been given in some papers.[19] But in the *Yuktibhāṣā* the rationale has been expounded beautifully. In short it is like this. In arriving at the *śīghra* correction the mean *mandakarṇa* is taken instead of true *mandakarṇa*. This is because the tables of *śīghra-jyās* can be constructed only on the basis of mean *mandakarṇa*. Hence a correction becomes necessary for the *śīghra-phala*. To achieve this a correction is effected in the true *manda* planet in such a way that this correction together with its effect on the tabular *śīghra-phala* will compensate for the error in *śīghra-phala* due to the difference in *mandakarṇa*. The longer commentary on *Tantrasaṅgraha* also gives this rationale as also the planetary theory. There are many small tracts[20] in Kerala wherein various alternate methods are advocated to give effect to this correction. It is a moot point whether Āryabhaṭa and other astronomers were aware of this rationale or they just hit at these methods by trial and error.

The treatment of latitude with regard to the planets in the *YB* is satisfactory. The theory is that the *śīghra* circle is always on the ecliptic plane and only the plane of the planet's path gets deflected. This accords with facts and therefore the resulting helio and geocentric latitudes also represent the correct position as also the distance between the earth and the planet making allowance for the latitude.

Ch. IX deals with Celestial Sphere. This chapter deals with the celestial sphere and the related great circles such as meridian, horizon, equator, ecliptic and their secondaries, and the small circles such as day parallels etc. The poles of great circles and their relation with mutually perpendicular great circles, the equinoctial and solstitial points are explained. First the celestial sphere for an observer on the terrestrial equator is described and then changes are explained as the observer moves to a northern latitude. Then the effects of *Ayana-calana* or the backward motion of the first point of Aries on the equator, ecliptic etc., are considered. Lastly the procedure for construction of an armillary sphere is described.

Ch. X is on Declination, Right Ascension and related problems. In the first place the declination (*krānti*) of the sun (an object of the ecliptic) is considered. The formula for this sin (declination) = sin (longitude) x sin (obliquity), is got by using the properties of similar triangles and by proportion. In addition, two more concepts which are not found in other works are introduced. These are: Draw a secondary to the equator through the first point of aries and a great circle through the solstitial points and the sun. The arcs between their point of intersection and the sun and the first point of aries are called *Krāntikoṭi* and *Nata*, respectively. (*Krāntikoṭi* is not cos *krānti* which is known as *dyujyā*). We may call them as 'inverse declination' and 'inverse R.A.'. These concepts are used to determine various other formulae which are described in the next chapter. Then the method of arriving at the R.A. or *Kālajyā* is described.

But the most important derivations in this chapter are the exact formulae for declination and R.A. of a star which is not on the ecliptic and therefore has a latitude, not necessarily of a small magnitude. This problem has not been satisfactorily solved by Indian astronomers till then and only approximate solutions have been given which are valid only if the latitude is small.[21] The formulae derived here accord with the modern formulae.

Ch. XI has tackled 15 problems. Taking the sun and the *krānti* triangle and the *krāntikoṭi* triangle there are six elements viz., longitude, R.A., declination, obliquity of the ecliptic, *nata* and *krāntikoṭi*. The problem is, given two of these six the other four are to be found out. There will be in all 15 cases which will arise. All these cases are examined and the methods for finding the other 4 elements are discussed exhaustively from fundamentals. This is a very interesting and instructived exercise to understand how the contemporary astronomer's mind worked in solving problems on spherical trigonometry.

Ch. XII deals with Direction and Shadow. This chapter starts with the method for finding accurately the east-west and north-south lines or directions. The method adopted is the familiar one, marking off the points where the shadow of a gnomon touches the circumference of a circle with centre at the foot of the gnomon, forenoon and afternoon, and joining them to get the east-west line. Since there will be a change in the declination of the sun between the forenoon and afternoon this line will not accurately depict the east-west line and a correction is provided to rectify the error. Then *Kujyā* and *Carajyā* are defined and the formulae derived. The method for arriving at the shadow for given time is next considered and the standard method is adopted. But in this two corrections are made. The sun is not a point but a sphere and the umbra is to be taken as the shadow and a correction is needed for this. Secondly the correction due to the parallax of the sun is also explained. The converse problem of finding time from shadow is tackled by the standard method after giving effect to the above two corrections. Then the problems connected with noon shadow, *samaśaṅku*, corner shadow etc., are dealt with in the usual manner.

Ch. XIII is again an interesting section concerned with shadow problems. We have five elements of the spherical triangle joining the sun, zenith and the north pole. The five elements are the three sides and two angles of the triangle. That is, zenith distance, co-declination, co-latitude which are the three sides and, the azimuth and the hour angle the two angles.(Actually the text would use either these elements or their complements). Out of these five, if any three are known, the problem is to find the other two. There will be ten cases to be solved. All these cases are taken up and the solutions are derived methodically on the basis of the properties of spherical triangles.

This subject is dealt with in *Tantrasangraha* also as it should be and based on this R.C. Gupta has presented a detailed paper.[22] He has however remarked that the rationales of the rules are not given in the work (*Tantrasangraha*) and his paper verifies the rules by the modern formulae. Well! the rationales are fully documented here in *Yuktibhāṣā*. In fact the rationales are also given in the longer commentary on *Tantrasangraha*, that is, *Yuktidīpikā*.

Ch. XIV is on Lagna and Kālalagna. The equator cuts the horizon of any place at two fixed points, east and west. The ecliptic also cuts the horizon at two points but since the position of the ecliptic varies every moment these points also very moment to moment, oscillating on either side of the east and west points. These two points at east and west are called *Udayalagna* and *Aṣṭalagna* or the rising and setting points of the ecliptic. The distance from the first point of the aries to the east point and *Udayalagna* are called the *Kālagna* and *Lagna*, respectively. This chapter deals with the method of calculating these two longitudes. Arriving at the lagna is a standard problem in all texts. The procedure followed generally is to find the rising times of the twelve signs for the local place and from the longitude of the sun obtain by interpolation the longitude of the rising point or *lagna* for the desired time. This is bound to be approximate. However, in *Yuktibhāṣā* a direct method is adopted. First the zenith distance of nonagesimal (*dṛkkṣepa*) is obtained and from that the *lagna* is arrived at. This method perhaps is again one of the Kerala methods.

Ch. XV deals with Eclipses and Parallax Corrections. In the treatment of the circumstances of an eclipse there is not much to comment. However we deal with two matters, one the second correction for the moon and the other the effect of parallax in longitude and latitude. The second correction for the moon which takes the place of the modern Evection plus the deficit in the equation of centre of the moon was known in India at least from the 10th century. There is a detailed paper by K.S. Shukla on this subject[23] wherein he has dealt with this subject as contained *Tantrasangraha* also. As can be expected *Yuktibhāṣā* follows *Tantrasangraha*, more or less, and in addition gives the basis for this correction. This basis or theory is the same as explained in Shukla's paper. However a refinement has been made in this work and that is when the moon has a latitude its distance from earth should be calculated taking the latitude also into account.

Now coming to parallax the idea is to claculate the effect of parallax in the longitude and the latitude known as *lambana* and *nati* respectively. The usual formula for these as is in vogue in Kerala texts are derived on the basis that the object is on ecliptic, that is, there is no latitude. While other texts deal with this problem differently giving in some cases only approximate results, the method followed here is exact. Further other texts do not take into account the latitude on the plea that during an eclipse the latitude of the moon is negligible. But here necessary formulae are derived to take into account the latitude also.

Then to derermine the angle of the sun's or the moon's disc at which the eclipse starts or ends and to graphically depict an eclipse the *valanas* or deflections are required to be calculated. Generally the texts deal with two deflections *ayanavalana* and *aksavalana* being the deflections due to obliquity of the ecliptic and the latitude of the place respectively. In *YB* in addition, a deflection due to latitude of the moon is also derived. This deflection is generally found in other Kerala works also.

Ch. XVI is on Vyatīpāta. *Vyatīpāta* occurs when the sun and the moon have equal declination and they are in the same *ayanabhāga* but in different quadrants or in different *ayanas* but in the same type of quardant, odd or even. This is considered very inauspicious, particularly in Kerala, and the days on which *vyatīpāta* falls are discarded for the performance of good *karmas* All Kerala works on astronomy devote a chapter for determining *vyatīpāta*. Not only this, there are full fledged works which deal with exclusively *vyatīpāta* and as in the case of eclipses the beginning, middle and ending of *vyatīpāta* are defined and the methods to arrive at these moments are also explained. From the definition it will be seen that the declinations of the sun and the moon are required to be calculated for this purpose. The declination of the sun presents no difficulty. In the case of the moon allowance has to be made for the latitude. The method of arriving at the exact declination of the moon has already been explained in Chapter IX.

In this Chapter an alternate method is given with its rationale. The method adopted is like this. The angle between the moon's path and equator is not constant but varies between 19.5 to 28.5 degs. depending upon the position of its node. The exact figure for the moment is first obtained. Then the distance between the point of intersection of the moon's path and the equator, and the moon is got by marking a correction to the node's longitude.

This correction is called *Vikṣepa-calana*. The method of arriving at these two elements, with rationale, is explained. With these two elements the declination of the moon is obtained in the same way as for the sun. This is a very interesting procedure and appears to be peculiar to Kerala.

In chapter XVII, *Dṛkkarma* or Reduction to observation is explained with rationale as also *Candraśṛṅgonnati* relating to the phases of the moon.

As is his wont, throughout the work, Jyeṣṭhadeva, the author of *Yuktibhāṣā*, elucidates the rationales of the several items, either geometrically or algebraically or using both means as the occasion demands. Towards a full evaluation of the work, it is necessary to deal in detail, in terms of modern mathematics, with the numerous rationales and theories, some of them new, anumbrated in the work. It may be added that this work has been taken up by the present writers who intend to translate with elucidation the entire text of the *Yuktibhāṣā*, which is in the Malayalam language, and also edit the *Gaṇita-Yuktibhāṣā* which is a Sanskrit rendering of the Malayalam original.

Model Demonstrations From Yuktibhāṣā

Here we demonstrate some of the derivations as explained in the *Yuktibhāṣā*. We generally follow the logic as found therein but with some modification here and there for the sake of convenience and ease of understanding.

The *Yuktibhāṣā* uses the radian measure as the radius for the sines of all angles. As a matter of fact, the angles as such are not at all used, and all angles are measured in arcs only. Thus, the full circle of 360 degrees or 21600 minutes (minutes are the usual unit of measure with sub-divisions of seconds etc., wherever needed) is taken as the circumference and, on this basis, the radius is measured as 3438 minutes which is sin 90°, and, accordingly, the sines of all angles are measured and used for various calculations. However, in out modern conventions as this will not affect the arguments used in the derivations or rationales.

Only the sine function is used though the cosine function is also well known. Whenever the cosine function is to be used the sine of the co-angle (90° minus the angle) is used or some times $\sqrt{1-\sin^2}$ is used. Sometimes the angle and its sine are used as synonyms and from the context one has to decide whether it refers to the angle or sine.

No general formulae have been derived for the solution of spherical triangles. But the properties of small and great circles and the relations between mutually perpendicular great circles and their poles, and the properties of similar triangles are used with imagination. The presentations are quite long and detailed and verbose. Anybody who reads this will get an impression that, while writing this, the author had before him a model of the celestial sphere and expects the readers also to have one before them. It may be mentioned that the procedure for constructing such an armillary sphere has also been explained in the text.

We shall discuss the solutions with their rationale to the following four problems.

i. Declination and R.A. of the Sun.
ii. One of the fifteen problems.
iii. Declination and R.A. of a star with a latitude.
iv. One of the ten problems.

I. *Declination and R.A. of the Sun*

The Hindu method of solving this problem is well known, but still for the sake of completeness we start with this.

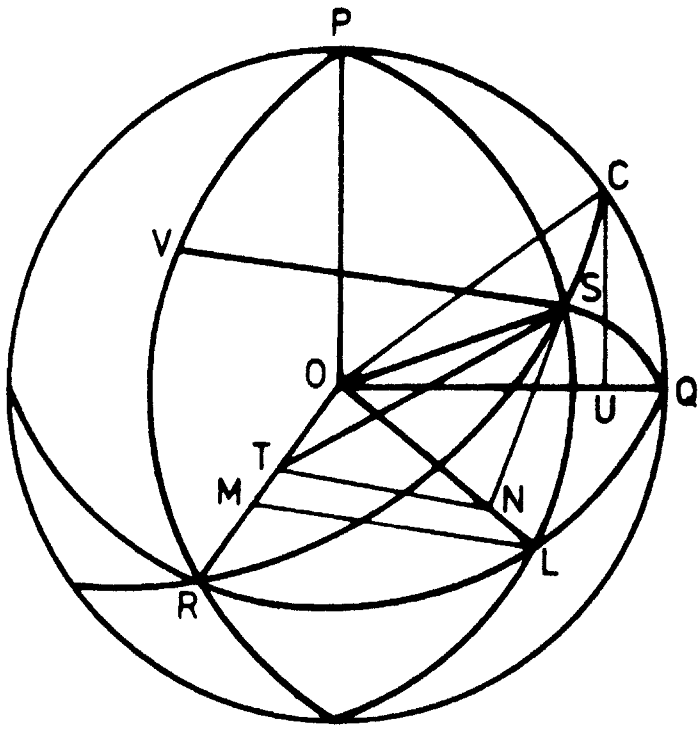

Fig. 1

In the above figure:

P Pole of Equator RLQ.
R First Point of Aries.
S Sun of Ecliptic RSC

Draw CU and SN ⊥ to the plane of Equator ROQ.
Draw NT and LM ⊥ OR on the plane of Equator.

It will be seen that

CQ or ∠ COQ Obliquity of the Ecliptic (w) and CU its sine.
RS or ∠ ROS Longitude of the Sun (l) and ST its sine.
SL or ∠ SOL Declination of the Sun (d) and SN its sine.
RL or ∠ ROL R.A. of the Sun (a) and ML its sine.

Since the two triangles COU and STN are similar we have, SN/ST = CU/CO or SN = ST × CU as CO = 1 being the Radius. So we get the declination formula

$$\sin d = \sin l \times \sin w$$

The radius of the diurnal circle of the Sun is ON and TN is the sine R.A. on this circle.

$$TN = \cos d \times \sin a \qquad \text{or } \sin a = TN/\cos d$$

Also from the two similar triangles considered above we have

$$TN/TS = OU/OC \text{ or } TN = TS \times OU \text{ or } TN = \text{Sin } l \times \cos w$$

Thus we have,

$$\sin a = \sin l \times \cos w/\cos d$$

which is the formula for R.A.

In the subsequent demonstrations we shall take into account the results of these derivations without again going into the details. This will also apply to spherical triangles in similiar situations of the type SRL considered here.

We had referred to two more elements in the main paper in this connection. They are shown in Fig.1 as SV and VR.

SV *Krāntikoṭi* or inverse declination (d_1)
VR *Nata* or inverse R.A. (a_1)

These can be calculated from triangle RSV in a similar way keeping in mind that $\angle$ SRV is $90° - w$. Thus

$$d_1 = \sin l \times \cos w \qquad \text{and } a_1 = \sin l \times \sin w/\cos d_1$$

It will be seen that

$$\sin^2 d + \sin^2 d_1 = \sin^2 l$$

II. *One of the fifteen problems*

As stated in the main paper these 15 problems are, given two out of the six elements viz., w, d, d_1, l, a, a_1 to find the other four. We shall take the case where d and a are given.

In Fig. 1 considering the triangles PSC and PLQ we have

$$\sin SC = \sin PS \times \sin LQ$$

or

$$\sin (90° - l) = \sin (90° - d) \times \sin (90° - a)$$

(This is the same as the present cosine formula)

From this l can be calculated. Then by the declination formula, w can be calculated and the rest as in I above.

Alternatively, considering triangles PVS and PRL we have

$$\sin SV = \sin PS \times \sin RL$$

Or

$$\sin d_1 = \sin (90° - d) \times \sin a$$

So d_1 is known. Obtain l from the relation

$\sin^2 l = \sin^2 d + \sin^2 d_1$. Others as before.

III. *Declination and R.A. of a star with a Latitude*

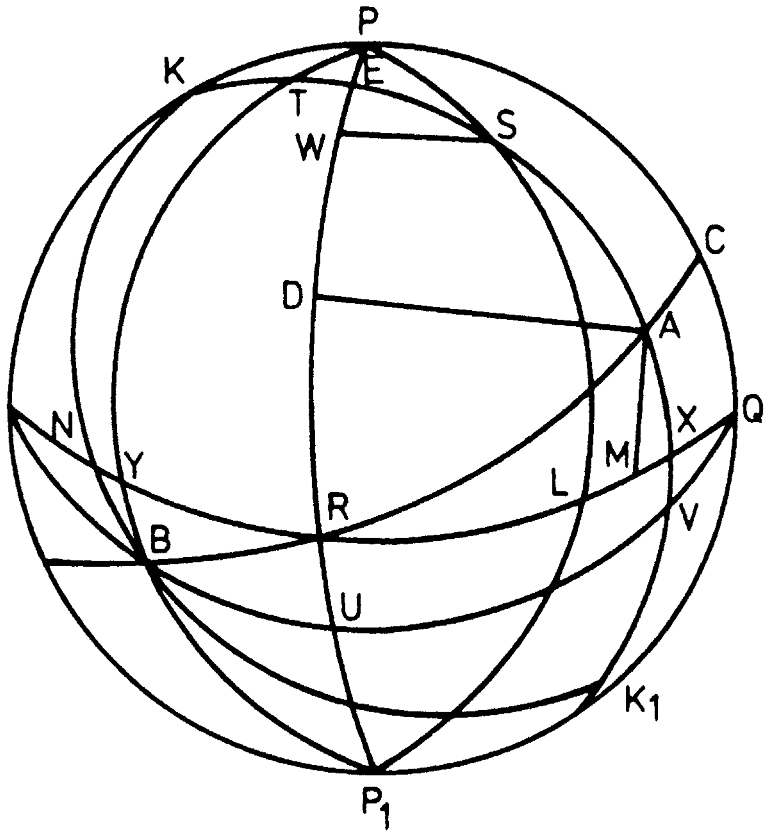

Fig.2

In the Fig. 2

P, P_1	Poles of the Equator.
K, K_1	Poles of the Ecliptic.
RQ and RC	Equator and Ecliptic.
R	First point of Aries.
S	A star away from ecliptic.
PSL and KSAX	Secondaries to Equator and Ecliptic through S.

Take a point B on the ecliptic 90° behind A. Draw PYB and KNB secondaries through B to equator and ecliptic and AM to the Equator. Draw AD and SW secondaries to PRP_1.

Given RA the longitude (l) and SA the latitude (v) find SL, the declination (d) and RL the R.A. (a).

AM is the declination of the point A on the Ecliptic, say d′
and so $\sin d' = \sin l \times \sin w$

$BK = BA = 90°$. So B is the pole of AK. Therefore BT and XT are mutually perpendicular as also XY and YT. Hence X is the pole of BP. Now consider the triangles XSL and XTY. XT and XY are equal to 90°. SL is perpendicular to LQ. Therefore

$$\sin SL = \sin SX \times \sin TY \text{ or } \sin d = \sin (v + AX) \times \sin TY \text{ or}$$
$$\sin d = \sin v \times \cos AX \times \sin TY + \cos v \times \sin AX \times \sin TY$$

From XAM we get $\sin AX \times \sin TY = \sin AM$. But $AM = d'$.

So the second term for sin d is $\cos v \times \sin d'$ where both v and d′ are known. As for the first term

$$\cos^2 AX \times \sin^2 TY = \sin^2 TY - \sin^2 AX \times \sin^2 TY$$

$$\begin{aligned} \text{L.H.S.} &= (1 - \sin^2 BY) - \sin^2 AM \\ &= 1 - \sin^2 w \times \cos^2 l - \sin^2 w \times \sin^2 l \\ &= 1 - \sin^2 w = \cos^2 w \end{aligned}$$

Hence $\cos AX \times \sin TY = \cos w$. Hence the expression for d is

$$\sin d = \sin v \times \cos w + \cos v \times \sin d'$$

which is the final form given in the book.

Putting $\sin d' = \sin l \times \sin w$ we get the modern form:

$$\sin d = \sin v \times \cos w + \cos v \times \sin l \times \sin w$$

For the R.A. a similar method is adopted. It can be seen that

$$\sin WS = \sin ES \times \sin UV = \sin (AE - v) \times \sin UV$$

Then proceeding on the same lines as for declination we get

$$\sin WS = \sin l \times \cos w \times \cos v - \sin v \times \sin w$$

But from PWS and PRL we get $\sin WS = \sin PS \times \sin RL$
and since $PS = 90° - d$ and $RL = a$, we get the expression for a,

$$\sin a = (\sin l \times \cos w \times \cos v - \sin v \times \sin w)/\cos d$$

This expression can be obtained from the modernformulae

cos w × sin l = sin w × tan v + cos l × tan a, and
cos a/cos v = cos l/cos d

IV. *One of the ten problems*

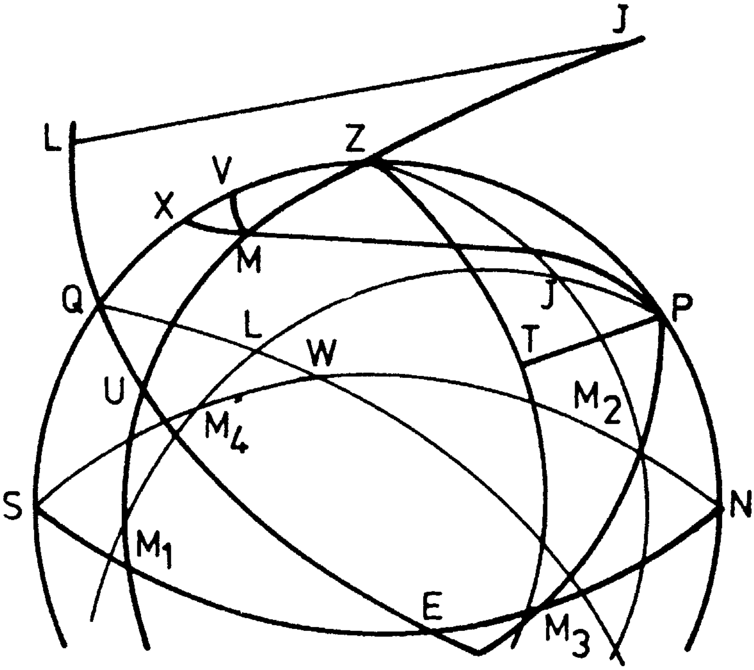

Fig. 3

In Fig. 3 SENW is the horizon, S, E, N and W being the south, east, north and west points. P is the pole, Z zenith, and M the object, and EQW equator.

Draw ZMM_1 vertical through M. Take M_3 on the horizon 90° from M_1 and LPM_3 secondary to the equator through M_3. Draw MV a small circle paralles to equator and MX perpendicular to ZQ. VQ is therefore equal to the declination of M.

In the triangle ZMP, given

Declination or its supplement	:	PM (90° − d)
Latitude of the place	:	PN φ
Azimuth (A) or A − 90°	:	M_3N_2 B

To get Altitude M_1M = a, and Hour angle $\angle$ MPZ = H,

We shall get ZM the zenith distance z, and hence a = 90° − z

Now z = ZM = ZU − MU. We shall get ZU and MU.

M_3 is the pole of M_1MZ. Therefore UM_3 = 90° and, U being on the equator, UP = 90°. Hence U is the pole of M_3PJL. Therefore UJ = UL = 90°. Now, from the triangles UJL and UZQ we get,

$$\sin QZ = \sin ZU \times \sin LJ, \text{ and so } \sin ZU = \sin QZ / \sin LJ$$
$$QZ = \varphi, \quad LJ \text{ is to be got. Now, } M_3J = PL = 90°. \text{ So, } LJ = PM_3.$$

From M_3ZN and TZP we have $\sin^2 PT + \sin^2 PN = \sin^2 PM_3$,

But from M_3TP and M_3PN we get $\sin PT = \sin ZP * \sin M_3N$
or $\sin PT = \cos \varphi \times \sin B$.

Hence the previous equation becomes $\cos^2 \varphi \times \sin^2 B + \sin^2 \varphi = \sin^2 PM_3$

$$\text{or } \sin M_3P = \surd \sin^2 \varphi + \sin^2 B \times \cos^2 \varphi) = D, \text{ say,}$$

The expression in R.H.S. is called the "Divisor" in *Tantrasaṅgraha* and denoted by D in Gupta's paper.

Substituting this value we get

$\sin UZ = \sin \varphi / D$

Now, to get UM,consider ZQU $\sin UM / \sin UZ = \sin QV / \sin QZ$

or $\sin UM = (\sin \varphi / D) \times \sin d / \sin \varphi$

or $\sin UM = \sin d / D$

we get $ZM = ZU - MU = \sin^{-1}(\sin \varphi/D) - \sin^{-1}((\sin d)/D)$

From this the altitude a is got by a = 90° − ZM.

To get the Hour Angle H,
From ZMX, $\sin MX = \sin z \times \cos B$ and
From PMX, $\sin MX = \sin H \times \cos d$ Hence we get
$\sin H = \sin z \times \cos B / \cos d$

Alternate expression for (a):

$$\sin a = \cos z = \cos (\sin^{-1} ((\sin \varphi)/D) - \sin^{-1} ((\sin d)/D))$$

Expanding R.H.S. and simplifying we get

$$[\sqrt{(D^2 - \sin^2 \varphi)} \times \sqrt{(D^2 - \sin^2 d)} - \sin \varphi \times \sin d]/D^2,$$

This is the expression given in *Tantrasaṅgraha* and quoted by Gupta in his paper.

As noted by Gupta, this expression can be got by the modern method by solving the following equation for sin a got by the modern cosine rule by transforming it into a quadratic equation.

$$\sin d = \sin \varphi \times \sin a + \cos \varphi \times \cos a \times \cos A.$$

It will be seen the method of *Yuktibhāṣā* is entirely different and does not involve any quadratic equation.

NOTES AND REFERENCES

[1]From amongst texts of this type, might be mentioned: *Ganitayuktayaḥ* (Vishveshvaranand Inst., Hoshiarpur, 1979), *Rāśigolasphuṭānīti* Hoshiarpur, 1977), *Grahaṇamaṇḍana* of Parameśvara (Hoshiarpur, 1965), *Grahaṇa-nyāya-dīpikā* of Parameśvara (Hoshiarpur, 1965), *Kriyākramakarī* of Śaṅkara Com. on the *Līlāvatī* of Bhāskara II (Hoshiarpur, 1975), *Sphuṭacandrāpti* of Mādhava (Hoshiarpur, 1973), *Jyotirmīmāṁsā* of Nīlakaṇṭha (Hoshiarpur, 1977), and *Yuktidīpikā* of Śaṅkara, Com. on the *Tantrasaṅgraha* (Hoshiarpur, 1977), all edited critically by K.V. Sarma. A large number of tracts, big and small, containing astronomical rationale, have also been identified by him.

[2]*Tantrasaṅgraha* of Nīlakaṇṭha Somayāji, ed. with two commentaries, *Yuktidīpikā* and *Laghuvivṛti*, both by Śaṅkara, ed. by K.V. Sarma, Hoshiarpur, 1977.

[3]C.M. Whish, 'On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four *Sastras*, the *Tantra Saṅgraham, Yucti Bhasha, Carana Padhati* (sic) and Sadratnamala, 'Transactions of the Royal Asiatic Society of Great Britain and Ireland, III.iii (1835), 509-23.

[4]Cf, the statement: "The testimonies as to the author (of Tantrasaṅgraha) and the period in which he lived, are the following...the mention made of him...by his commentator, the author of the *Yucti-Bhasha*, Cellalura Nambutiri", p.522. However, it has to be noted, incidentally, that Whish has made wrong identifications, when he states here: (i) that the author of Tantrasangraha is "Talaculattura Nambutiri" (p. 522), for it is correctly Gārgya Kerala Nīlakaṇṭha Somayāji; (ii) that *YB* is a commentary on the *Tantrasaṅgraha*, which it is not; and (iii) that the author of *YB* is Cellalura (i.e. Kelallur) Nambutiri (p.522), for it is actually Paraṅṅoṭṭu Nampūtiri, as would be shown below.

[5]See *op. cit.*,p.523.

[6]Several manuscripts of the work are preserved under the two titles in the Kerala University Mss. Library, Trivandrum, the Sanskrit College Library, Tripunithura, and in private possession.

[7]There is a fine edition of Pt. I of the work, now out of print, by Rama varma Maru Thampuran and A.R. Akhileswara Iyer, Mangalodayam Press, Trichur,1948 and an extremely unsastisfactory and error-ridden edition of the whole work issued by the Govt. Oriental Mass. Library, Madras, 1954.

[8]Cf., S.N. Sen, *A Bibliography of Sanskrit Works on Astronomy and Mathematics*, INSA, New Delhi, 1966, p.74; C.T. Rajagopal and M.S. Rangachari, 'On an untapped source of medieval Keralese

mathematics', *Archive for History of Exact Science*, 18 (1978)' 89-101; C.T. Rajagopal and M.S. Rangachari, 'On mediaeval Kerala Mathematics', *ib*. 35 (1986) 91-99.

[9]*Op. Cit.*, Introduction, p.5, where the A.D. date is wrongly given as 1639; Ulloor, *History of Kerala literature*, Trivendrum, vol.III, 1955 p. 439.

[10]For an independent tradition that the author of the *Yuktibhāṣā* belonged to the Parannoṭṭu faimly (Sanskritised as Parakroḍa), situated in the Ālattūr village in Malabar, see *Nampūtirimār* (Mal.), by parayil Raman Namputiri, Trichur, Kollam era 1093, (A.D. 1918), p.55.

[11]See Ms. C. 628-B, end of the Kerala Uni. Mss. Library.

[12]Whish, *loc. cit.*, p. 523.

[13]Cr. edn. with Introduction by K.V. Sarma, Vishveshvaranand Institute, Hoshiarpur, 1975.

[14]Cr. edn. with detailed Introduction by K.V. Sarma, Vishveshvaranand Institute, Hoshiarpur, 1977. The identity of the authorship of the two commentaries has been dealt with in detail herein, Intro,. pp. li-vi.

[15]See, P. Sridhara Menon, Introduction to Maru Thampuran's edn. of the work, p. 6.

[16]*Loc. cit.*, pp. 5-6

[17]The page reference are to the edition of *YB* by Rama Varma Maru Thampuran.

[17a] (i) K. Mukunda Marar and C.T. Rajagopal, 'On the Hindu quadrature of the circle', *J Bombay Branch of Royal Asiatic Soc.*, NS 20 65-82 (1944).

(ii) C.T. Rajagopas and A. Venkataraman, 'The Sine and Cosine power series in Hindu mathematics' *J Royal Asiatic Soc. of Bengal*, Sc., 15 1-13 (1949).

(iii) C.T. Rajagopal and M.S. Rangachari, 'On an untapped source of medieval Keralese mathematics', *Archive for Hist. of Exact sciences*, 18 (1978)

(iv) C.T. Rajagopal and M.S. Rangachari, 'On medieval Keralese mathematics' *Archeve for Hist. of Exact Sciences*, 35, 91-99 (1986).

[18]See, for instance, E. Burgess, *Translation of the Sūryasiddhānta*, p.86, lines 3ff.; P.C. Sengupta, *Khaṇḍakhādyaka*, p.58, last para.

[19]O. Neugebauer, 'The transmission of planetary theories in ancient and medieval astronomy', *Scripta Mathematica*, New York, 1956, App. 'Hindu planetary theory', p. 12ff.

S.N. Sen, 'Epicyclic eccentric planetary theories in ancient and medieval Indian astronomy', *Indian Jl. Hist. Science*, 9 pp. 107-21 (1974).

[20]See for instance, K.V. Sarma, *Rationales of Hindu astronomy*, Pt.I, Hoshiarpur, 1979, Tracts 24, 25 and 26.

[21]See P.C. Sengupta, *Khaṇḍakhādyaka*,pp. 189-91, Problem viii.

[22]R.C. Gupta, 'Solution of a astronomical triangle as found in the *Tantrasangraha* (1500)', *Indian Jl. of Hist. of Science*, 9, 86-99 (1974).

[23]K.S. Shukla, 'The evection and the deficit of the equation of the centre of the Moon in Hindu astronomy', *Proceedings of the Banaras Mathematical Society*, NS, 7.ii (Dec. 1945) 9-28.