Chapter IV

# CLASSICAL ELECTRODYNAMICS

What is light? The study of relativity left us completely in the dark, even though we had embarked in it precisely to find an answer to that question. True, we have learned how the motion of light compares with that of objects. We also learned that light is a moving entity that cannot be stopped; but we haven't learned anything about its nature. The answer to this long-standing question emerges only from the study of those types of motion that are *not* related to gravitation, such as the ways magicians levitate objects.

## 13. Liquid electricity, invisible fields and maximum speed

Page 150 Revisiting the list of motors found in this world, we remark that gravitation hardly describes any of them. Neither the motion of sea waves, fire and earthquakes, nor that of a gentle breeze are due to gravity. The same applies to the motion of muscles. Have you Challenge 917 e ever listened to your own heart beat with a stethoscope? Without having done so, you cannot claim to have experienced the mystery of motion. Your heart has about 3000 million beats in your lifetime. Then it stops.

It was one of the most astonishing discoveries of science that heart beats, sea waves and most other cases of everyday motion, as well as the nature of light itself, are connected to observations made thousands of years ago using two strange stones. These stones show that all examples of motion, which are called *mechanical* in everyday life, are, without exception, of *electrical* origin.

In particular, the solidity, the softness and the impenetrability of matter are due to Ref. 484 internal electricity; also the emission of light is an electrical process. As these aspects are part of everyday life, we will leave aside all complications due to gravity and curved space-time. The most productive way to study electrical motion is to start, as in the case of gravity, with those types of motion which are generated without any contact between the bodies involved.

### Amber, lodestone and mobile phones

The story of electricity starts with trees. Trees have a special relation to electricity. When a tree is cut, a viscous resin appears. With time it solidifies and, after millions of years, it forms *amber*. When amber is rubbed with a cat fur, it acquires the ability to attract small objects, such as saw dust or pieces of paper. This was already known to Thales of


FIGURE 221 Objects surrounded by fields: amber, lodestone and mobile phone

Miletus, one of the original seven sages, in the sixth century BCE. The same observation can be made with many other polymer combinations, for example with combs and hair, with soles of the shoe on carpets, and with a TV screen and dust. Children are always surprised by the effect, shown in Figure 222, that a comb rubbed on wool has on running tap water. Another interesting effect can be observed when a rubbed comb is put near a burning candle. (Can you imagine what happens?)

Challenge 918 ny

Another part of the story of electricity involves an iron mineral found in certain caves around the world, e.g. in a region (still) called Magnesia in the Greek province of Thessalia, and in some regions in central Asia. When two stones of this mineral are put near each other, they attract or repel each other, depending on their relative orientation. In addition, these stones attract objects made of cobalt, nickel or iron.

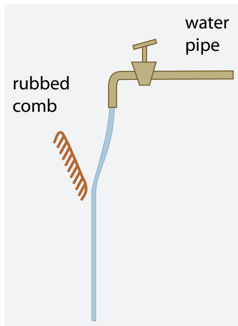


FIGURE 222 How to amaze kids

Today we also find various small objects in nature with more sophisticated properties, as shown in Figure 221. Some objects enable you to switch on a television, others unlock car doors, still others allow you to talk with far away friends.

All these observations show that in nature there are situations where bodies exert influence on others *at a distance*. The space surrounding a body exerting such an influence is said to contain a field. A *(physical) field* is thus an entity that manifests itself by accelerating other bodies in its region of space. A field is some 'stuff' taking up space. Experiments show that fields have no mass. The field surrounding the mineral found in Magnesia is called a *magnetic field* and the stones are called *magnets*.* The field around amber – called ἤλεκτρον in Greek, from a root meaning 'brilliant, shining' – is called an *electric field*. The name is due to a proposal by the famous English part-time physicist William Gilbert (1544–1603) who was physician to Queen Elizabeth I. Objects surrounded by a permanent electric field are called *electrets*. They are much less common than magnets; among others, they are used in certain loudspeaker systems.**

* A pretty book about the history of magnetism and the excitement it generates is JAMES D. LIVINGSTON, *Driving Force – the Natural Magic of Magnets*, Harvard University Press, 1996.

** The Kirlian effect, which allows one to make such intriguingly beautiful photographs, is due to a time-

TABLE 41 Searches for magnetic monopoles, i.e., for magnetic charges

| Search | Magnetic charge |
|---|---|
| Smallest magnetic charge suggested by quantum theory | $g = \frac{h}{e} = \frac{eZ_0}{2\alpha} = 4.1\,\text{pWb}$ |
| Search in minerals | none Ref. 485 |
| Search in meteorites | none Ref. 485 |
| Search in cosmic rays | none Ref. 485 |
| Search with particle accelerators | none Ref. 485 |

FIGURE 223 Lightning: a picture taken with a moving camera, showing its multiple strokes (© Steven Horsburgh)

The field around a mobile phone is called a *radio* field or, as we will see later, an *electromagnetic* field. In contrast to the previous fields, it oscillates over time. We will find out later that many other objects are surrounded by such fields, though these are often very weak. Objects that emit oscillating fields, such as mobile phones, are called radio transmitters or radio emitters.

Fields influence bodies over a distance, without any material support. For a long time, this was rarely found in everyday life, as most countries have laws to restrict machines that use and produce such fields. The laws require that for any device that moves, produces sound, or creates moving pictures, the fields need to remain inside them. For this reason a magician moving an object on a table via a hidden magnet still surprises and entertains his audience. To feel the fascination of fields more strongly, a deeper look into a few experimental results is worthwhile.

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varying electric field.

TABLE 42 Some observed magnetic fields

| Observation | Magnetic field |
|---|---|
| Lowest measured magnetic field (e.g., fields of the Schumann resonances) | 1 fT |
| Magnetic field produced by brain currents | 0.1 pT to 3 pT |
| Intergalactic magnetic fields | 1 pT to 10 pT |
| Magnetic field in the human chest, due to heart currents | 100 pT |
| Magnetic field of our galaxy | 0.5 nT |
| Magnetic field due to solar wind | 0.2 to 80 nT |
| Magnetic field directly below high voltage power line | 0.1 to 1 µT |
| Magnetic field of Earth | 20 to 70 µT |
| Magnetic field inside home with electricity | 0.1 to 100 µT |
| Magnetic field near mobile phone | 100 µT |
| Magnetic field that influences visual image quality in the dark | 100 µT |
| Magnetic field near iron magnet | 100 mT |
| Solar spots | 1 T |
| Magnetic fields near high technology permanent magnet | max 1.3 T |
| Magnetic fields that produces sense of coldness in humans | 5 T or more |
| Magnetic fields in particle accelerator | 10 T |
| Maximum static magnetic field produced with superconducting coils | 22 T |
| Highest static magnetic fields produced in laboratory, using hybrid magnets | 45 T |
| Highest *pulsed* magnetic fields produced without coil destruction | 76 T |
| Pulsed magnetic fields produced, lasting about 1 µs, using imploding coils | 1000 T |
| Field of white dwarf | $10^4$ T |
| Fields in petawatt laser pulses | 30 kT |
| Field of neutron star | from $10^6$ T to $10^{11}$ T |
| Quantum critical magnetic field | 4.4 GT |
| Highest field ever measured, on magnetar and soft gamma repeater SGR-1806-20 | 0.8 to $1 \cdot 10^{11}$ T |
| Field near nucleus | 1 TT |
| Maximum (Planck) magnetic field | $2.2 \cdot 10^{53}$ T |

## How can one make lightning?

Everybody has seen a lightning flash or has observed the effect it can have on striking a tree. Obviously lightning is a moving phenomenon. Photographs such as that of Figure 223 show that the tip of a lightning flash advance with an average speed of around 600 km/s. But *what* is moving? To find out, we have to find a way of making lightning for ourselves.

In 1995, the car company General Motors accidentally rediscovered an old and simple

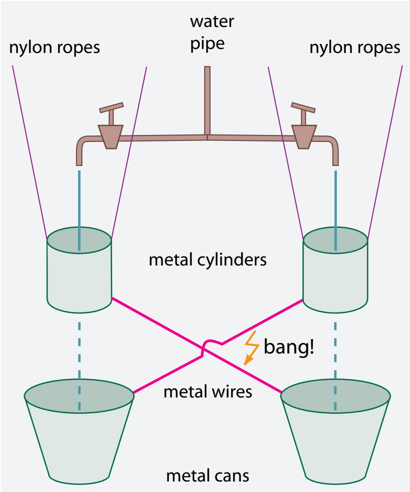


FIGURE 224 A simple Kelvin generator

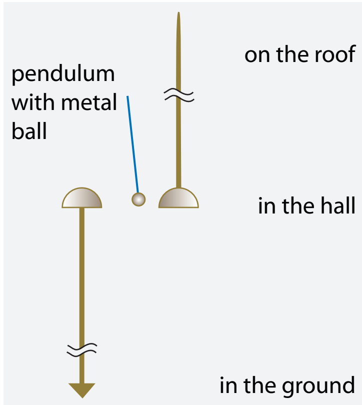


FIGURE 225 Franklin's personal lightning rod

method of achieving this. Their engineers had inadvertently built a spark generating mechanism into their cars; when filling the petrol tank, sparks were generated, which sometimes lead to the explosion of the fuel. They had to recall 2 million vehicles of its Opel brand. Ref. 486

What had the engineers done wrong? They had unwittingly copied the conditions for a electrical device which anyone can build at home and which was originally invented by William Thomson.* Repeating his experiment today, we would take two water taps, four empty bean or coffee cans, of which two have been opened at both sides, some nylon rope and some metal wire. Ref. 487

Putting this all together as shown in Figure 224, and letting the water flow, we find a strange effect: large sparks periodically jump between the two copper wires at the point where they are nearest to each other, giving out loud bangs. Can you guess what condition for the flow has to be realized for this to work? And what did Opel do to repair the cars they recalled? Challenge 919 n

If we stop the water flowing just before the next spark is due, we find that both buckets are able to attract sawdust and pieces of paper. The generator thus does the same that rubbing amber does, just with more bang for the buck(et). Both buckets are surrounded by electric fields. The fields increase with time, until the spark jumps. Just after the spark,

* William Thomson (1824–1907), important Irish Unionist physicist and professor at Glasgow University. He worked on the determination of the age of the Earth, showing that it was much older than 6000 years, as several sects believed. He strongly influenced the development of the theory of magnetism and electricity, the description of the aether and thermodynamics. He propagated the use of the term 'energy' as it is used today, instead of the confusing older terms. He was one of the last scientists to propagate mechanical analogies for the explanation of phenomena, and thus strongly opposed Maxwell's description of electromagnetism. It was mainly for this reason that he failed to receive a Nobel Prize. He was also one of the minds behind the laying of the first transatlantic telegraphic cable. Victorian to his bones, when he was knighted, he chose the name of a small brook near his home as his new name; thus he became Lord Kelvin of Largs. Therefore the unit of temperature obtained its name from a small Scottish river.

the buckets are (almost) without electric field. Obviously, the flow of water somehow builds up an entity on each bucket; today we call this *electric charge*. Charge can flow in metals and, when the fields are high enough, through air. We also find that the two buckets are surrounded by two different types of electric fields: bodies that are attracted by one bucket are repelled by the other. All other experiments confirm that there are *two* types of charges. The US politician and part-time physicist Benjamin Franklin (1706–1790) called the electricity created on a glass rod rubbed with a dry cloth *positive*, and that on a piece of amber *negative*. (Previously, the two types of charges were called 'vitreous' and 'resinous'.) Bodies with charges of the same sign repel each other, bodies with opposite charges attract each other; charges of opposite sign flowing together cancel each other out.*

In summary, electric fields start at bodies, provided they are charged. Charging can be achieved by rubbing and similar processes. Charge can flow: it is then called an electric *current*. The worst conductors of current are polymers; they are called *insulators* or *dielectrics*. A charge put on an insulator remains at the place where it was put. In contrast, metals are good conductors; a charge placed on a conductor spreads all over its surface. The best conductors are silver and copper. This is the reason that at present, after a hundred years of use of electricity, the highest concentration of copper in the world is below the surface of Manhattan.

Of course, one has to check whether natural lightning is actually electrical in origin. In 1752, experiments performed in France, following a suggestion by Benjamin Franklin, published in London in 1751, showed that one can indeed draw electricity from a thunderstorm via a long rod.** These French experiments made Franklin famous worldwide; they were also the start of the use of lightning rods all over the world. Later, Franklin had
Ref. 488 a lightning rod built through his own house, but of a somewhat unusual type, as shown in Figure 225. Can you guess what it did in his hall during bad weather, all parts being
Challenge 920 n made of metal? (Do not repeat this experiment; the device can kill.)

## Electric charge and electric fields

If all experiments with charge can be explained by calling the two charges positive and negative, the implication is that some bodies have more, and some less charge than an uncharged, *neutral* body. Electricity thus only flows when two differently charged bodies are brought into contact. Now, if charge can flow and accumulate, we must be able to somehow measure its amount. Obviously, the *amount* of charge on a body, usually abbreviated $q$, is defined via the influence the body, say a piece of sawdust, feels when subjected to a field. Charge is thus defined by comparing it to a standard reference charge. For a charged body of mass $m$ accelerated in a field, its charge $q$ is determined by the relation

$$\frac{q}{q_{\text{ref}}} = \frac{ma}{m_{\text{ref}}a_{\text{ref}}} , \qquad (390)$$

* In fact, there are many other ways to produces sparks or even *arcs*, i.e. sustained sparks; there is even a complete subculture of people who do this as a hobby at home. Those who have a larger budget do it professionally, in particle accelerators. See the http://www.kronjaeger.com/hv/ website.

** The details of how lightning is generated and how it propagates are still a topic of research. An introduction is given on page 597.

TABLE 43 Properties of *classical* electric charge

| Electric charges | Physical property | Mathematical name | Definition |
|---|---|---|---|
| Can be distinguished | distinguishability | element of set | Page 646 |
| Can be ordered | sequence | order | Page 1195 |
| Can be compared | measurability | metricity | Page 1205 |
| Can change gradually | continuity | completeness | Page 1214 |
| Can be added | accumulability | additivity | Page 69 |
| Do not change | conservation | invariance | $q$ = const |
| Can be separated | separability | positive or negative | |

i.e., by comparing it with the acceleration and mass of the reference charge. This definition reflects the observation that mass alone is not sufficient for a complete characterization of a body. For a full description of motion we need to know its electric charge; charge is therefore the second intrinsic property of bodies that we discover in our walk.

Nowadays the unit of charge, the *coulomb*, is defined through a standard flow through metal wires, as explained in Appendix B. This is possible because all experiments show that charge is conserved, that it flows, that it flows continuously and that it can accumulate. Charge thus behaves like a fluid substance. Therefore we are forced to use for its description a scalar quantity $q$, which can take positive, vanishing, or negative values.

In everyday life these properties of electric charge, listed also in Table 43, describe observations with sufficient accuracy. However, as in the case of all previously encountered classical concepts, these experimental results for electrical charge will turn out to be only approximate. More precise experiments will require a revision of several properties. However, no counter-example to charge conservation has as yet been observed.

A charged object brought near a neutral one polarizes it. *Electrical polarization* is the separation of the positive and negative charges in a body. For this reason, even neutral objects, such as hair, can be attracted to a charged body, such as a comb. Generally, both insulators and conductors can be polarized; this occurs for whole stars down to single molecules.

Attraction is a form of acceleration. Experiments show that the entity that accelerates charged bodies, the *electric field*, behaves like a small arrow fixed at each point $\mathbf{x}$ in space; its length and direction do not depend on the observer. In short, the electric field $\mathbf{E}(\mathbf{x})$ is a *vector* field. Experiments show that it is best defined by the relation

$$q\mathbf{E}(\mathbf{x}) = m\mathbf{a}(\mathbf{x}) \quad (391)$$

taken at every point in space $\mathbf{x}$. The definition of the electric field is thus based on how it *moves* charges.* The field is measured in multiples of the unit N/C or V/m.

Challenge 922 e

To describe the motion due to electricity completely, we need a relation explaining how charges *produce* electric fields. This relation was established with precision (but not for

Challenge 921 ny

* Does the definition of electric field given here assume a charge speed that is much less than that of light?

TABLE 44 Values of electrical charge observed in nature

| Observation | Charge |
|---|---|
| Smallest measured non-vanishing charge | $1.6 \cdot 10^{-19}$ C |
| Charge per bit in computer memory | $10^{-13}$ C |
| Charge in small capacitor | $10^{-7}$ C |
| Charge flow in average lightning stroke | 1 C to 100 C |
| Charge stored in a fully-charge car battery | 0.2 MC |
| Charge of planet Earth | 1 MC |
| Charge separated by modern power station in one year | $3 \cdot 10^{11}$ C |
| Total charge of positive (or negative) sign observed in universe | $10^{62\pm2}$ C |
| Total charge observed in universe | 0 C |

TABLE 45 Some observed electric fields

| Observation | Electric field |
|---|---|
| Field 1 m away from an electron in vacuum | Challenge 923 n |
| Field values sensed by sharks | down to 0.1 μV/m |
| Cosmic noise | 10 μV/m |
| Field of a 100 W FM radio transmitter at 100 km distance | 0.5 mV/m |
| Field inside conductors, such as copper wire | 0.1 V/m |
| Field just beneath a high power line | 0.1 to 1 V/m |
| Field of a GSM antenna at 90 m | 0.5 V/m |
| Field inside a typical home | 1 to 10 V/m |
| Field of a 100 W bulb at 1 m distance | 50 V/m |
| Ground field in Earth's atmosphere | 100 to 300 V/m |
| Field inside thunder clouds | up to over 100 kV/m |
| Maximum electric field in air before sparks appear | 1 to 3 MV/m |
| Electric fields in biological membranes | 10 MV/m |
| Electric fields inside capacitors | up to 1 GV/m |
| Electric fields in petawatt laser pulses | 10 TV/m |
| Electric fields in $U^{91+}$ ions, at nucleus | 1 EV/m |
| Maximum practical electric field in vacuum, limited by electron pair production | 1.3 EV/m |
| Maximum possible electric field in nature (corrected Planck electric field) | $2.4 \cdot 10^{61}$ V/m |

the first time) by Charles-Augustin de Coulomb on his private estate, during the French Revolution.* He found that around any small-sized or any spherical charge $Q$ *at rest* there is an electric field. At a position **r**, the electric field **E** is given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\frac{\mathbf{r}}{r} \quad \text{where} \quad \frac{1}{4\pi\varepsilon_0} = 9.0\,\mathrm{GV\,m/C}\,. \tag{392}$$

Later we will extend the relation for a charge in motion. The bizarre proportionality constant, built around the so-called *permittivity of free space* $\varepsilon_0$, is due to the historical way the unit of charge was defined first.** The essential point of the formula is the decrease of

Challenge 924 n the field with the square of the distance; can you imagine the origin of this dependence?

The two previous equations allow one to write the interaction between two charged bodies as

$$\frac{\mathrm{d}\mathbf{p}_1}{\mathrm{d}t} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{\mathbf{r}}{r} = -\frac{\mathrm{d}\mathbf{p}_2}{\mathrm{d}t}\,, \tag{393}$$

where $\mathrm{d}\mathbf{p}$ is the momentum change, and **r** is the vector connecting the two centres of mass. This famous expression for electrostatic attraction and repulsion, also due to Coulomb, is valid only for charged bodies that are of small size or spherical, and most of all, that are *at rest*.

Electric fields have two main properties: they contain energy and they can polarize bodies. The energy content is due to the electrostatic interaction between charges. The strength of the interaction is considerable. For example, it is the basis for the force of our muscles. Muscular force is a macroscopic effect of equation 393. Another example is the material strength of steel or diamond. As we will discover, all atoms are held together by electrostatic attraction. To convince yourself of the strength of electrostatic attraction, answer the following: What is the force between two boxes with a gram of protons each,

Challenge 925 n located on the two poles of the Earth? Try to guess the result before you calculate the astonishing value.

Coulomb's relation for the field around a charge can be rephrased in a way that helps to generalize it to non-spherical bodies. Take a closed surface, i.e., a surface than encloses a certain volume. Then the integral of the electric field over this surface, the electric flux, is the enclosed charge $Q$ divided by $\varepsilon_0$:

$$\int_{\mathrm{closedsurface}} \boldsymbol{E}\,\mathrm{d}\boldsymbol{A} = \frac{Q}{\varepsilon_0}\,. \tag{394}$$

Challenge 926 n This mathematical relation, called *Gauss's 'law'*, follows from the result of Coulomb. (In the simplified form given here, it is valid only for static situations.) Since inside conductors the electrical field is zero, Gauss's 'law' implies, for example, that if a charge $q$ is sur-

* Charles-Augustin de Coulomb (b. 1736 Angoulême, d. 1806 Paris), French engineer and physicist. His careful experiments on electric charges provided a firm basis for the study of electricity.

** Other definitions of this and other proportionality constants to be encountered later are possible, leading to *unit systems* different from the SI system used here. The SI system is presented in detail in Appendix B. Among the older competitors, the Gaussian unit system often used in theoretical calculations, the Heaviside–Lorentz unit system, the electrostatic unit system and the electromagnetic unit system are

Ref. 489 the most important ones.

rounded by a uncharged metal sphere, the outer surface of the metal sphere shows the
Challenge 927 e same charge $q$.

Owing to the strength of electromagnetic interactions, separating charges is not an easy task. This is the reason that electrical effects have only been commonly used for about a hundred years. We had to wait for practical and efficient devices to be invented for separating charges and putting them into motion. Of course this requires energy. Batteries, as used in mobile phones, use chemical energy to do the trick.* Thermoelectric elements, as used in some watches, use the temperature difference between the wrist and the air to separate charges; solar cells use light, and dynamos or Kelvin generators use kinetic energy.

Do uncharged bodies attract one other? In first approximation they do not. But when the question is investigated more precisely, one finds that they can attract one other. Can
Challenge 929 n you find the conditions for this to happen? In fact, the conditions are quite important, as our own bodies, which are made of neutral molecules, are held together in this way.

What then is electricity? The answer is simple: *electricity is nothing in particular*. It is the name for a field of inquiry, but not the name for any specific observation or effect. Electricity is neither electric current, nor electric charge, nor electric field. Electricity is not a specific term; it applies to *all* of these phenomena. In fact the vocabulary issue hides a deeper question that remains unanswered at the beginning of the twenty-first century: what is the nature of electric charge? In order to reach this issue, we start with the following question.

## Can we detect the inertia of electricity?

If electric charge really is something *flowing* through metals, we should be able to observe the effects shown in Figure 226. Maxwell has predicted most of these effects: electric charge should fall, have inertia and be separable from matter. Indeed, each of these effects has been observed.** For example, when a long metal rod is kept vertically, we can measure an electrical potential difference, a voltage, between the top and the bottom. In other words, we can measure the *weight* of electricity in this way. Similarly, we can meas-
Ref. 490 ure the potential difference between the ends of an accelerated rod. Alternatively, we can measure the potential difference between the centre and the rim of a rotating metal disc. The last experiment was, in fact, the way in which the ratio $q/m$ for currents in metals was first measured with precision. The result is

$$q/m = 1.8 \cdot 10^{11}\ \mathrm{C/kg} \qquad (395)$$

for all metals, with small variations in the second digit. In short, electrical current has mass. Therefore, whenever we switch on an electrical current, we get a *recoil*. This simple
Ref. 491 effect can easily be measured and confirms the mass to charge ratio just given. Also, the emission of current into air or into vacuum is observed; in fact, every television tube uses this principle to generate the beam producing the picture. It works best for metal
Ref. 492 objects with sharp, pointed tips. The rays created this way – we could say that they are

Challenge 928 n * Incidentally, are batteries sources of charges?

** Maxwell also performed experiments to detect these effects (apart from the last one, which he did not predict), but his apparatuses where not sensitive enough.

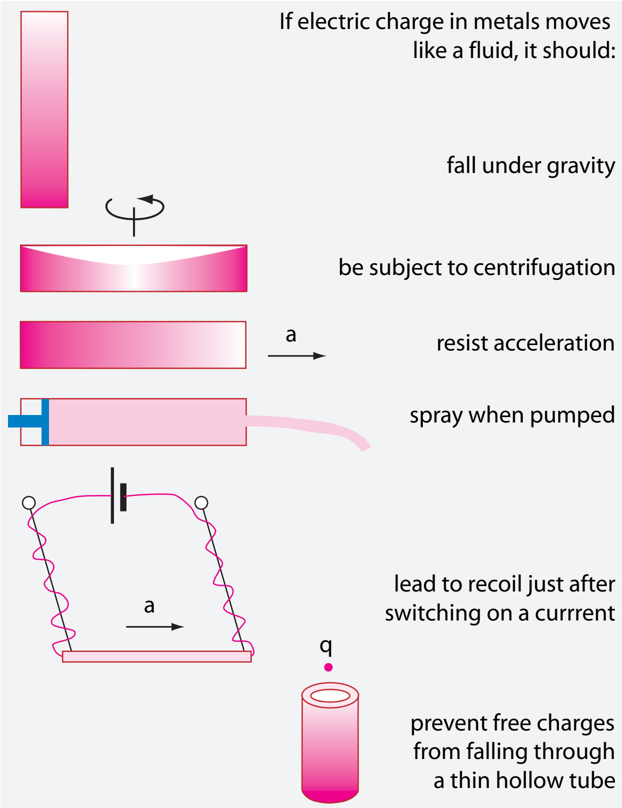


FIGURE 226 Consequences of the flow of electricity

'free' electricity – are called *cathode rays*. Within a few per cent, they show the same mass to charge ratio as expression (395). This correspondence thus shows that charges move almost as freely in metals as in air; this is the reason metals are such good conductors.

If electric charge *falls* inside vertical metal rods, we can make the astonishing deduction that cathode rays – as we will see later, they consist of free electrons* – should not be able to fall through a vertical metal tube. This is due to exact compensation of the acceleration by the electrical field generated by the displaced electricity in the tube and the acceleration of gravity. Thus electrons should not be able to fall through a long thin cylinder. This would not be the case if electricity in metals did not behave like a fluid. The experiment has indeed been performed, and a reduction of the acceleration of free fall for electrons of 90 % has been observed. Can you imagine why the ideal value of 100 % is not achieved?

Challenge 930 e

Ref. 493

Challenge 931 n

If electric current behaves like a liquid, one should be able to measure its speed. The first to do so, in 1834, was Charles Wheatstone. In a famous experiment, he wire of a

* The name 'electron' is due to George Stoney. Electrons are the smallest and lightest charges moving in metals; they are, usually – but not always – the 'atoms' of electricity – for example in metals. Their charge is small, 0.16 aC, so that flows of charge typical of everyday life consist of large numbers of electrons; as a result, electrical charge behaves like a continuous fluid. The particle itself was discovered and presented in 1897 by the Prussian physicist Johann Emil Wiechert (1861–1928) and, independently, three months later, by the British physicist Joseph John Thomson (1856–1940).

TABLE 46 Some observed electric current values

| Observation | Current |
|---|---|
| Smallest regularly measured currents | 1 fA |
| Human nerve signals | 20 μA |
| Lethal current for humans | as low as 20 mA, typically 100 mA |
| Current drawn by a train engine | 600 A |
| Current in a lightning bolt | 10 to 100 kA |
| Highest current produced by humans | 20 MA |
| Current inside the Earth | around 100 MA |

quarter of a mile length, to produce three sparks: one at the start, one at the middle, and one at the end. He then mounted a rapidly moving mirror on a mechanical watch; by noting who much the three spark images were shifted against each other on a screen, he determined the speed to be 450 Mm/s, though with a large error. Latter, more precise measurements showed that the speed is always below 300 Mm/s, and that it depends on the metal and the type of insulation of the wire. The high value of the speed convinced many people to use electricity for transmitting messages. A modern version of the experiment, for computer fans, uses the 'ping' command. The 'ping' command measures the time for a computer signal to reach another computer and return back. If the cable length between two computers is known, the signal speed can be deduced. Just try.

Ref. 494

Challenge 932 e

## Feeling electric fields

Why is electricity dangerous to humans? The main reason is that the human body is controlled by 'electric wires' itself. As a result, outside electricity interferes with the internal signals. This has been known since 1789. In that year the Italian medical doctor Luigi Galvani (1737–1798) discovered that electrical current makes the muscles of a dead animal contract. The famous first experiment used frog legs: when electricity was applied to them, they twitched violently. Subsequent investigations confirmed that all nerves make use of electrical signals. Nerves are the 'control wires' of animals. However, nerves are not made of metal: metals are not sufficiently flexible. As a result, nerves do not conduct electricity using electrons but by using *ions*. The finer details were clarified only in the twentieth century. Nerve signals propagate using the motion of sodium and potassium ions in the cell membrane of the nerve. The resulting signal speed is between 0.5 m/s and 120 m/s, depending on the type of nerve. This speed is sufficient for the survival of most species – it signals the body to run away in case of danger.

Being electrically controlled, all mammals can sense strong electric fields. Humans can sense fields down to around 10 kV/m, when hair stands on end. In contrast, several animals can sense weak electric and magnetic fields. Sharks, for example, can detect fields down to 1 μV/m using special sensors, the Ampullae of Lorenzini, which are found around their mouth. Sharks use them to detect the field created by prey moving in water; this allows them to catch their prey even in the dark. Several freshwater fish are also able to detect electric fields. The salamander and the platypus, the famous duck-billed mammal, can also sense electric fields. Like sharks, they use them to detect prey in water which is too muddy to see through. Certain fish, the so-called *weakly-electric fish*, even generate a weak field in order to achieve better prey detection.*

No land animal has special sensors for electric fields, because any electric field in air is strongly damped when it encounters a water-filled animal body. Indeed, the usual atmosphere has an electric field of around 100 V/m; inside the human body this field is damped to the μV/m range, which is much less than an animal's internal electric fields. In other words, humans do not have sensors for low electric fields because they are land animals. (Do humans have the ability to sense electric fields in water? Nobody seems to know.) However, there a few exceptions. You might know that some older people can sense approaching thunderstorms in their joints. This is due the coincidence between the electromagnetic field frequency emitted by thunderclouds – around 100 kHz – and the resonant frequency of nerve cell membranes.

Challenge 933 ny

Page 564

The water content of the human body also means that the electric fields in air that are found in nature are rarely dangerous to humans. Whenever humans do sense electric fields, such as when high voltage makes their hair stand on end, the situation is potentially dangerous.

The high impedance of air also means that, in the case of time-varying electromagnetic fields, humans are much more prone to be affected by the magnetic component than by the electric component.

## Magnets

The study of magnetism progressed across the world independently of the study of electricity. Towards the end of the 12th century, the compass came into use in Europe. At that time, there were heated debates on whether it pointed to the north or the south. In 1269, the French military engineer Pierre de Maricourt (1219–1292) published his study of magnetic materials. He found that every magnet has *two* points of highest magnetization, and he called them *poles*. He found that even after a magnet is cut, the resulting pieces always retain two poles: one points to the north and the other to the south when the stone is left free to rotate. Magnets are dipoles. Atoms are either dipoles or unmagnetic. There are no magnetic monopoles. Despite the promise of eternal fame, no magnetic monopole has ever been found, as shown in Table 41. Like poles repel, and unlike poles attract.

Ref. 495

Magnets have a second property: magnets transform unmagnetic materials into magnetic ones. There is thus also a magnetic polarization, similar to the electric polarization. Unlike the electric case, some magnetic materials retain the induced magnetic polarization: they become magnetized. This happens when the atoms in the material get aligned by the external magnet.

* It took until the year 2000 for technology to make use of the same effect. Nowadays, airbag sensors in cars often use electric fields to sense whether the person sitting in the seat is a child or an adult, thus changing the way that the bag behaves in an accident.

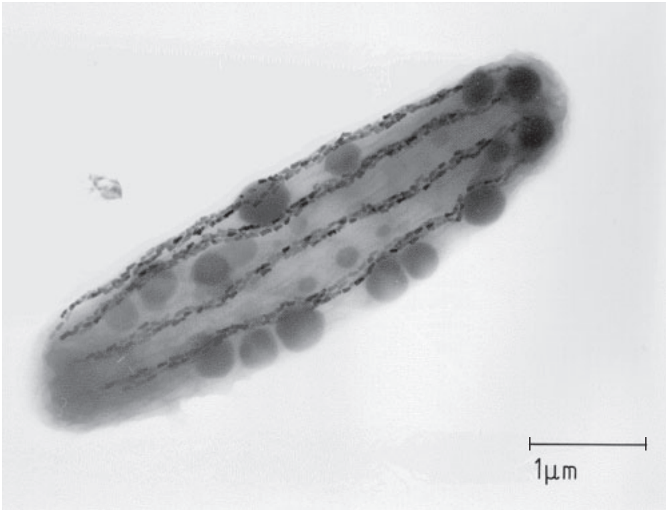


FIGURE 227 The magentotactic bacterium *Magnetobacterium bavaricum* with its magnetosomes (© Marianne Hanzlik)

## Can humans feel magnetic fields?

> Any fool can ask more questions than seven sages can answer.
>
> Antiquity

It is known that honey bees, sharks, pigeons, salmon, trout, sea turtles and certain bacteria can feel magnetic fields. One speaks of the ability for magnetoreception. All these life forms use this ability for navigation. The most common detection method is the use of small magnetic particles inside a cell; the cell then senses how these small built-in magnets move in a magnetic field. The magnets are tiny, typically around 50 nm in size. These small magnets are used to navigate along the magnetic field of the Earth. For higher animals, the variations of the magnetic field of the Earth, 20 to 70 µT, produce a landscape that is similar to the visible landscape for humans. They can remember it and use it for navigation.

Ref. 496

Can humans feel magnetic fields? Magnetic material seems to be present in the human brain, but whether humans can feel magnetic fields is still an open issue. Maybe you can devise a way to check this?

Challenge 934 ny

Are magnetism and electricity related? François Arago* found out that they were. He observed that a ship that had survived a bad thunderstorm and had been struck by lightning, needed a new compass. Thus lightning has the ability to demagnetize compasses. Arago knew, like Franklin, that lightning is an electrical phenomena. In other words, electricity and magnetism must be related. More precisely, magnetism must be related to the *motion* of electricity.

* Dominique-François Arago (1786–1853) French physicist.

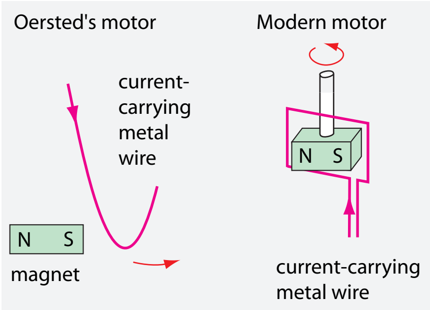


FIGURE 228 An old and a newer version of an electric motor

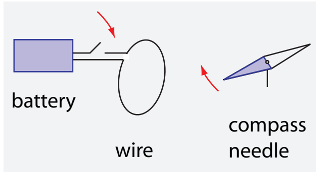


FIGURE 229 An electrical current always produces a magnetic field

## How can one make a motor?

> "Communism is the power of the local councils plus electricification of the whole country.
> Lenin.*"

The reason for Lenin's famous statement were two discoveries. One was made in 1820 by the Danish physicist Hans Christian Oersted (1777–1851) and the other in 1831 by the English physicist Michael Faraday.** The consequences of these experiments changed the world completely in less than one century.

On the 21st of July of 1821, Oersted published a leaflet, in Latin, which took Europe by storm. Oersted had found (during a lecture demonstration to his students) that when a current is sent through a wire, a nearby magnet is put into motion. In other words, he found that the flow of electricity can move bodies.

Further experiments show that *two* wires in which charges flow attract or repel each other, depending on whether the currents are parallel or antiparallel. These and other experiments show that wires in which electricity flows behave like magnets.*** In other words, Oersted had found the definite proof that electricity could be turned into magnetism.

Shortly afterwards, Ampère**** found that *coils* increase these effects dramatically.

* Lenin (b. 1870 Simbirsk, d. 1924 Gorki), founder of the Union of Soviet Socialist Republics, in 1920 stated this as the centre of his development plan for the country. In Russian, the local councils of that time were called soviets.

** Michael Faraday (b. 1791 Newington Butts, d. 1867 London) was born to a simple family, without schooling, and of deep and naive religious ideas. As a boy he became assistant to the most famous chemist of his time, Humphry Davy (1778–1829). He had no mathematical training, but late in his life he became member of the Royal Society. A modest man, he refused all other honours in his life. He worked on chemical topics, the atomic structure of matter and, most of all, developed the idea of (magnetic) fields and field lines through all his experimental discoveries, such as effect. Fields were later described mathematically by Maxwell, who at that time was the only person in Europe to take over Faraday's field concept.

*** In fact, if one imagines tiny currents moving in circles inside magnets, one gets a unique description for all magnetic fields observed in nature.

**** André-Marie Ampère (b. 1775 Lyon, d. 1836 Marseille), French physicist and mathematician. Autodidact, he read the famous *Encyclopédie* as a child; in a life full of personal tragedies, he wandered from maths to chemistry and physics, worked as a school teacher, and published nothing of importance until 1820. Then

Coils behave like small magnets. In particular, coils, like magnetic fields, always have two poles, usually called the north and the south pole. Opposite poles attract, like poles repel each other. As is well known, the Earth is itself a large magnet, with its magnetic north pole near the geographic south pole, and vice versa.

Moving electric charge produces magnetic fields. This result explains why magnetic fields always have two poles. The lack of magnetic monopoles thus becomes clear. But one topic is strange. If magnetic fields are due to the motion of charges, this must be also the case for a normal magnet. Can this be shown?

In 1915, two men in the Netherlands found a simple way to prove that even in a magnet, something is moving. They suspended a metal rod from the ceiling by a thin thread and then put a coil around the rod, as shown in Figure 230. They predicted that the tiny currents inside the rod would become aligned by the magnetic field of the coil. As a result, they expected that a current passing through the coil would make the rod turn around its axis. Indeed, when they sent a strong current through the coil, the rod rotated. (As a result of the current, the rod was magnetized.) Today, this effect is called the *Einstein–de Haas effect* after the two physicists who imagined, measured and explained it.* The effect thus shows that even in the case of a permanent magnet, the magnetic field is due to the internal motion of charges. The size of the effect also shows that the moving particles are electrons. (Twelve years later it became clear that the angular momentum of the electrons responsible for the effect is a mixture of orbital and spin angular momentum; in fact, the electron spin plays a central role in the effect.)

Ref. 497

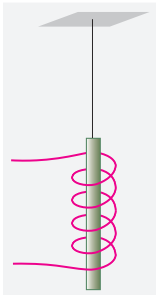

FIGURE 230 Current makes a metal rods rotate

Since magnetism is due to the alignment of microscopic rotational motions, an even more surprising effect can be predicted. Simply rotating a ferromagnetic material** should magnetize it, because the tiny rotating currents would then be aligned along the axis of rotation. This effect has indeed been observed; it is called the *Barnett effect* after its discoverer. Like the Einstein–de Haas effect, the Barnett effect can also be used to determine the gyromagnetic ratio of the electron; thus it also proves that the spins of electrons (usually) play a larger role in magnetism than their orbital angular momentum.

Ref. 498
Page 760

## Magnetic fields

Experiments show that the magnetic field always has a given direction in space, and a magnitude common to all (resting) observers, whatever their orientation. We are tempted

the discovery of Oersted reached all over Europe: electrical current can deviate magnetic needles. Ampère worked for years on the problem, and in 1826 published the summary of his findings, which lead Maxwell to call him the 'Newton of electricity'. Ampère named and developed many areas of electrodynamics. In 1832, he and his technician also built the first dynamo, or rotative current generator. Of course, the unit of electrical current is named after him.

* Wander Johannes de Haas (1878–1960), Dutch physicist. De Haas is best known for two additional magneto-electric effects named after him, the Shubnikov–de Haas effect (the strong increase of the magnetic resistance of bismuth at low temperatures and high magnetic fields) and the de Haas–Van Alphen effect (the diamagnetic susceptibility of bismuth at low temperatures is a periodic function of the magnetic field).

** A ferromagnetic material is a special kind of paramagnetic material that has a permanent magnetization.

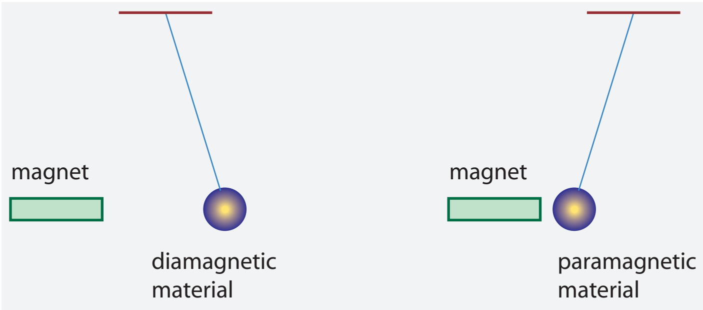


FIGURE 231 The two basic types of magnetic material behaviour (tested in an inhomogeneous field): diamagnetism and paramagnetism

to describe the magnetic field by a vector. However, this would be wrong, since a magnetic field does not behave like an arrow when placed before a mirror. Imagine that a system produces a magnetic field directed to the right. You can take any system, a coil, a machine, etc. Now build or imagine a second system that is the exact mirror version of the first: a mirror coil, a mirror machine, etc. The magnetic system produced by the mirror system does not point to the left, as maybe you expected: it still points to the right. (Check by yourself.) In simple words, magnetic fields do *not* behave like arrows. Challenge 935 e

In other words, it is not completely correct to describe a magnetic field by a vector $\mathbf{B} = (B_x, B_y, B_z)$, as vectors behave like arrows. One also speaks of a *pseudovector*; angular momentum and torque are also examples of such quantities. The precise way is to describe the magnetic field by the quantity*

$$\mathrm{B} = \begin{pmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{pmatrix} , \qquad (396)$$

called an *antisymmetric tensor*. In summary, *magnetic fields* are defined by the acceleration they impart on moving charges. This acceleration turns out to follow

$$\mathbf{a} = \frac{e}{m}\mathbf{v}\mathrm{B} = \frac{e}{m}\mathbf{v} \times \mathbf{B} \qquad (397)$$

a relation which is often called *Lorentz acceleration*, after the important Dutch physicist Hendrik A. Lorentz (b. 1853 Arnhem, d. 1928 Haarlem) who first stated it clearly.** (The relation is also called the *Laplace acceleration*.) The unit of the magnetic field is called tesla and is abbreviated T. One has $1\,\mathrm{T} = 1\,\mathrm{N\,s/C\,m} = 1\,\mathrm{V\,s/m^2} = 1\,\mathrm{V\,s^2/A\,m}$.

The Lorentz acceleration is the effect at the root of any electric motor. An electric motor is a device that uses a magnetic field as efficiently as possible to accelerate charges flowing

* The quantity B was not called the 'magnetic field' until recently. We follow here the modern, logical definition, which supersedes the traditional one, where B was called the 'magnetic flux density' or 'magnetic induction' and another quantity, **H**, was called – incorrectly, but for over a century – the magnetic field. This quantity **H** will not appear in this walk, but it is important for the description of magnetism in materials.

Challenge 936 ny ** Does the definition of magnetic field given here assume a charge speed much lower than that of light?

in a wire. Through the motion of the charges, the wire is then also moved. Electricity is thus transformed into magnetism and then into motion. The first efficient motor was built back in 1834.

As in the electric case, we need to know how the *strength* of a magnetic field is determined. Experiments such as Oersted's show that the magnetic field is due to moving charges, and that a charge moving with velocity **v** produces a field B given by

$$\mathrm{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \mathbf{r}}{r^3} \quad \text{where} \quad \frac{\mu_0}{4\pi} = 10^{-7}\,\mathrm{N/A}^2 \,. \tag{398}$$

This is called *Ampère's 'law'*. Again, the strange factor $\mu_0/4\pi$ is due to the historical way in which the electrical units were defined. The constant $\mu_0$ is called the *permeability of the vacuum* and is defined by the fraction of newtons per ampere squared given in the formula. It is easy to see that the magnetic field has an intensity given by $\mathbf{v}\mathbf{E}/c^2$, where **E**
Challenge 937 e is the electric field measured by an observer moving *with* the charge. This is the first hint that magnetism is a relativistic effect.

We note that equation (398) is valid only for small velocities and accelerations. Can
Challenge 938 n you find the general one?

In 1831, Michael Faraday discovered an additional piece of the puzzle, one that even the great Ampère had overlooked. He found that a *moving* magnet could cause a current flow in an electrical circuit. Magnetism can thus be turned into electricity. This important discovery allowed the production of electrical current flow by generators, so-called *dynamos*, using water power, wind power or steam power. In fact, the first dynamo was built in 1832 by Ampère and his technician. Dynamos started the use of electricity throughout the world. Behind every electrical plug there is a dynamo somewhere.

Additional experiments show that magnetic fields also lead to electric fields when one changes to a moving viewpoint. You might check this on any of the examples of Figures 228 to 240. *Magnetism indeed is relativistic electricity*. Electric and magnetic fields are partly transformed into each other when switching from one inertial reference frame to the other. Magnetic and electrical fields thus behave like space and time, which are also mixed up when changing from one inertial frame to the other. The theory of special relativity thus tells us that there must be a single concept, an *electromagnetic field*, describing them both. Investigating the details, one finds that the electromagnetic field F surrounding charged bodies has to be described by an antisymmetric 4-*tensor*

$$\mathrm{F}^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \text{ or } \mathrm{F}_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} . \tag{399}$$

Obviously, the electromagnetic field $F$, and thus every component of these matrices, depends on space and time. The matrices show that electricity and magnetism are two faces of the same effect.* In addition, since electric fields appear only in the topmost row and

* Actually, the expression for the field contains everywhere the expression $1/\sqrt{\mu_o\varepsilon_0}$ instead of the speed of light $c$. We will explain the reason for this substitution shortly.

leftmost column, the expressions show that in everyday life, for small speeds, electricity
Challenge 939 n and magnetism *can* be separated. (Why?)

The total influence of electric and magnetic fields on fixed or moving charges is then given by the following expression for the relativistic force-acceleration relation $\mathbf{K} = m\mathbf{b}$:

$$m\mathbf{b} = q\mathrm{F}u \quad \text{or}$$
$$m\,\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} = q\mathrm{F}^\mu{}_\nu u^\nu \quad \text{or}$$
$$m\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\gamma c\\ \gamma v_x\\ \gamma v_y\\ \gamma v_z\end{pmatrix} = q\begin{pmatrix}0 & E_x/c & E_y/c & E_z/c\\ E_x/c & 0 & B_z & -B_y\\ E_y/c & -B_z & 0 & B_x\\ E_z/c & B_y & -B_x & 0\end{pmatrix}\begin{pmatrix}\gamma c\\ \gamma v_x\\ \gamma v_y\\ \gamma v_z\end{pmatrix} \quad \text{or}$$
$$W = q\mathbf{E}\mathbf{v} \quad \text{and} \quad d\mathbf{p}/\mathrm{d}t = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})\ , \tag{400}$$

which show how the work $W$ and the three-force $dp/\mathrm{d}t$ depend on the electric and magnetic fields. All four expressions describe the same content; the simplicity of the first one is the reason for the involved matrices (399) of the electromagnetic field. In fact, the extended *Lorentz relation* (400) is the *definition* of the electromagnetic field, since the field is defined as that 'stuff' which accelerates charges. In particular, all devices that put charges into motion, such as batteries and dynamos, as well as all devices that are put into motion by flowing charges, such as electric motors and muscles, are described by this relation. That is why this relation is usually studied, in simple form, already in school. The Lorentz relation describes all cases in which the motion of objects can be seen by the naked eye or felt by our senses, such as the movement of an electrical motor in a high speed train, in a lift and in a dental drills, the motion of the picture generating electron beam in a television tube, or the travelling of an electrical signal in a cable and in the nerves of the
Ref. 499, Ref. 500 body.

In equation (400) it is understood that one sums over indices that appear twice. The electromagnetic field tensor F is an *antisymmetric* 4-tensor. (Can you write down the
Challenge 940 ny relation between $\mathrm{F}^{\mu\nu}$, $\mathrm{F}_{\mu\nu}$ and $\mathrm{F}^\mu{}_\nu$?) Like any such tensor, it has two invariants, i.e., two deduced properties that are the same for every observer: the expression $B^2 - E^2/c^2 = \frac{1}{2}\mathrm{tr}\,\mathrm{F}^2$ and the product $4\mathbf{EB} = -c\ \mathrm{tr}\,\mathrm{F}^*\mathrm{F}$. (Can you confirm this, using the definition of
Challenge 941 n trace as the sum of the diagonal elements?)

The first invariant expression turns out to be the Lagrangian of the electromagnetic field. It is a scalar and implies that if $E$ is larger, smaller, or equal to $cB$ for one observer, it also is for all other observers. The second invariant, a pseudoscalar, describes whether the angle between the electric and the magnetic field is acute or obtuse for all observers.*

* There is in fact a third Lorentz invariant, much less known. It is specific to the electromagnetic field and is a combination of the field and its vector potential:

$$\begin{aligned}\kappa_3 &= \frac{1}{2}A_\mu A^\mu \mathrm{F}_{\rho\nu}\mathrm{F}^{\nu\rho} - 2A_\rho \mathrm{F}^{\rho\nu}\mathrm{F}_{\nu\mu}A^\mu\\ &= (\mathbf{AE})^2 + (\mathbf{AB})^2 - |\mathbf{A}\times\mathbf{E}|^2 - |\mathbf{A}\times\mathbf{B}|^2 + 4\frac{\varphi}{c}(\mathbf{AE}\times\mathbf{B}) - (\frac{\varphi}{c})^2(E^2+B^2)\ .\end{aligned} \tag{401}$$

Ref. 501 This expression is Lorentz (but not gauge) invariant; knowing it can help clarify unclear issues, such as the lack of existence of waves in which the electric and magnetic fields are parallel. Indeed, for plane mono-

The application of electromagnetic effects to daily life has opened up a whole new world that did not exist before. Electrical light, electric motors, radio, telephone, X-rays, television and computers have changed human life completely in less than one century. For example, the installation of electric lighting in city streets has almost eliminated the previously so common night assaults. These and all other electrical devices exploit the fact that charges can flow in metals and, in particular, that electromagnetic energy can be transformed

— into mechanical energy – as used in loudspeakers, motors, piezo crystals;
— into light – as in lamps and lasers;
— into heat – as in ovens and tea pots;
— into chemical effects – as in hydrolysis, battery charging and electroplating;
— into coldness – as in refrigerators and Peltier elements;
— into radiation signals – as in radio and television;
— into stored information – as in magnetic records and in computers.

## How motors prove relativity to be right

> The only mathematical operation I performed in my life was to turn the handle of a calculator.
> Michael Faraday

All electric motors are based on the result that electric currents interact with magnetic fields. The simplest example is the attraction of two wires carrying parallel currents. This observation alone, made in 1820 by Ampère, is sufficient to make motion larger than a certain maximal speed impossible. Ref. 502

The argument is beautifully simple. We change the original experiment and imagine two long, electrically charged rods of mass $m$, moving in the same direction with velocity $v$ and separation $d$. An observer moving with the rods would see an electrostatic repulsion between the rods given by Challenge 943 e

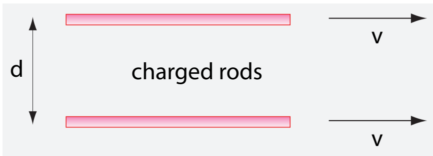


FIGURE 232 The relativistic aspect of magnetism

$$ma_e = -\frac{1}{4\pi\varepsilon_0}\frac{2\lambda^2}{d} \qquad (402)$$

where $\lambda$ is the charge per length of the rods. A second, *resting* observer sees two effects: the electrostatic repulsion and the attraction discovered by Ampère. The second observer therefore observes Challenge 944 e

$$ma_{em} = -\frac{1}{4\pi\varepsilon_0}\frac{2\lambda^2}{d} + \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d} . \qquad (403)$$

This expression must be consistent with the observation of the first observer. This is the

chromatic waves all three invariants *vanish* in the Lorentz gauge. Also the quantities $\partial_\mu J^\mu$, $J_\mu A^\mu$ and $\partial_\mu A^\mu$ are Lorentz invariants. (Why?) The latter, the frame independence of the divergence of the four-potential, reflects the invariance of gauge choice. The gauge in which the expression is set to zero is called the *Lorentz gauge*. Challenge 942 n case only if both observers find repulsions. It is easy to check that the second observer sees a repulsion, as does the first one, only if

$$v^2 < \frac{1}{\varepsilon_0 \mu_0} . \qquad (404)$$

This maximum speed, with a value of 0.3 GM/s, is thus valid for any object carrying charges. But *all* everyday objects contain charges: there is thus a maximum speed for matter.

Challenge 945 ny Are you able to extend the argument for a maximum speed to neutral particles as well? We will find out more on this limit velocity, which we know already, in a minute.

Another argument for magnetism as a relativistic effect is the following. In a wire with electrical current, the charge is zero for an observer at rest with respect to the wire. The reason is that the charges enter and exit the wire at the same time for that observer. Now imagine an observer who flies along the wire. The entrance and exit events do not occur simultaneously any more; the wire is charged for a moving observer. (The charge depends on the direction of the observer's motion.) In other words, if the observer himself were charged, he would experience a force. Moving charges experience forces from current-carrying wires. This is exactly why magnetic fields were introduced: they only produce forces on *moving* charges. In short, current carrying wires are surrounded by magnetic fields.

In summary, electric effects are due to flow of electric charges and to electric fields; magnetism is due to *moving* electric charges. It is *not* due to magnetic charges.* The strength of magnetism, used in any running electric motor, proves relativity right: there is a maximum speed in nature. Both electric and magnetic fields carry energy and momentum. They are two faces of the same coin. However, our description of electromagnetism is not complete yet: we need the final description of the way charges *produce* an electromagnetic field.

## Curiosities and fun challenges about things electric and magnetic

> Et facta mirari et intellectua assequi.
> Augustine of Hippo

Before we study the motion of an electromagnetic field in detail, let's have some fun with electricity.

* *

Nowadays, having fun with sparks is straightforward. Tesla coils, named after Nikola Tesla ** are the simplest devices that allow to produce long sparks at home. Attention: this is dangerous; that is the reason that such devices cannot be bought anywhere. The basic

Page 590
Challenge 946 ny

* 'Electrons move in metal with a speed of about 1 μm/s; thus if I walk with the same speed along a cable carrying a constant current, I should not be able to sense any magnetic field.' What is wrong with this argument?

** Никола Тесла (1856 Smiljan–1943 New York City), Serbian engineer and inventor. He invented and promoted the polyphase alternating current system, the alternating current electric motor, wireless communic-

TABLE 47 Voltage values observed in nature

| Observation | Voltage |
| --- | --- |
| Smallest measured voltage | *c.* 10 fV |
| Human nerves | 70 mV |
| Voltaic cell ('battery') | 1.5 V |
| Mains in households | 230 V or 110 V |
| Electric eel | 100 to 600 V |
| Sparks when rubbing a polymer pullover | 1 kV |
| Electric fence | 0.7 to 10 kV |
| Colour television tube | 30 kV |
| X-ray tube | 30 to 200 kV |
| Electron microscopes | 0.5 kV to 3 MV |
| Stun gun | 65 to 600 kV |
| Lightning stroke | 10 to 100 MV |
| Record accelerator voltage | 1 TV |
| Planck voltage, highest value possible in nature | $1.5 \cdot 10^{27}$ V |

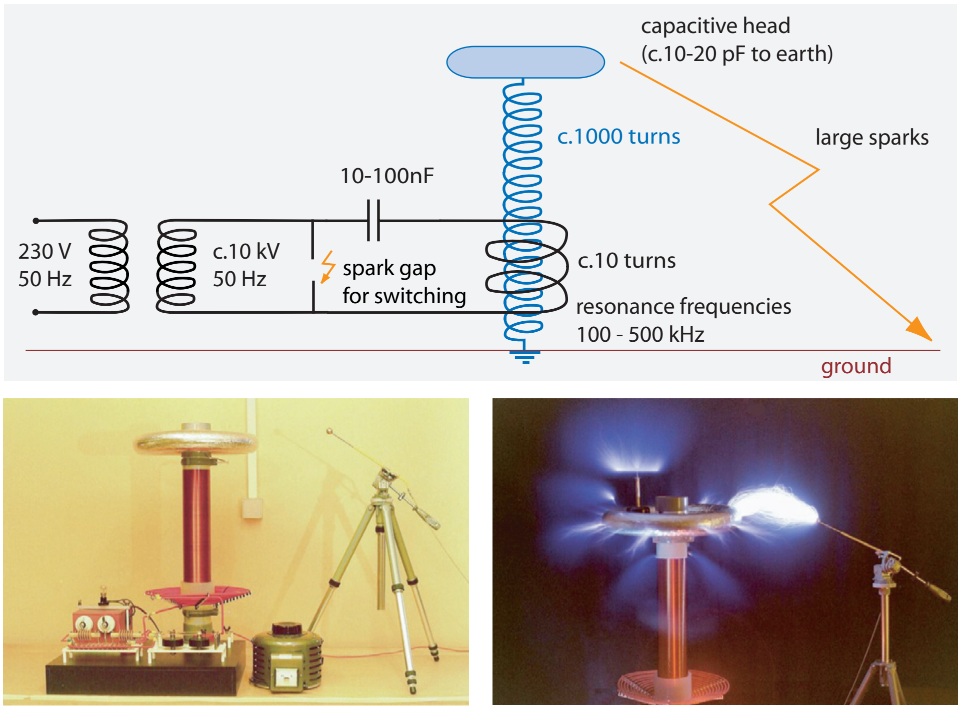


FIGURE 233 The schematics, the realization and the operation of a Tesla coil, including spark and corona discharges (© Robert Billon)

ation, fluorescent lighting and many other applications of electricity. He is also one of the inventors of radio. The SI unit of the magnetic field is named after him. A flamboyant character, his ideas were sometimes diagram and an example is shown in Figure 233. Tesla coils look like large metal mushrooms (to avoid unwanted discharges) and plans for their construction can be found on numerous websites or from numerous enthusiast's clubs, such as http://www.stefan-kluge.de.

* *

If even knocking on a wooden door is an electric effect, we should be able to detect fields when doing so. Can you devise an experiment to check this? Challenge 947 ny

* *

Birds come to no harm when they sit on unprotected electricity lines. Nevertheless, one almost never observes any birds on tall, high voltage lines of 100 kV or more, which transport power across longer distances. Why? Challenge 948 n

* *

How can you distinguish a magnet from an unmagnetized metal bar of the same size and material, using no external means? Challenge 949 n

* *

How do you wire up a light bulb to the mains and three switches so that the light can be switched on at any of the switches and off at any other switch? And for four switches? Challenge 950 n Nobody will take a physicist seriously who is able to write Maxwell's equations but cannot solve this little problem.

* *

The first appliances built to generate electric currents were large rubbing machines. Then, in 1799 the Italian scientist Alessandro Volta (1745–1827) invented a new device to generate electricity and called it a *pile*; today it is called a (voltaic) cell or, less correctly, a *battery*. Voltaic cells are based on chemical processes; they provide much more current and are smaller and easier to handle than electrostatic machines. The invention of the battery changed the investigation of electricity so profoundly that Volta became world famous. At last, a simple and reliable source of electricity was available for use in experiments; unlike rubbing machines, piles are compact, work in all weather conditions and make no noise.

An apple or a potato with a piece of copper and one of zinc inserted is one of the simplest possible voltaic cells. It provides a voltage of about 1 V and can be used to run digital clocks or to produce clicks in headphones. Volta was also the discoverer of the charge law $q = CU$ of capacitors ($C$ being the capacity, and $U$ the voltage) and the inventor of the high sensitivity capacitor electroscope. A modest man, nevertheless, the unit of electrical potential, or 'tension', as Volta used to call it, was deduced from his name. A 'battery' is a large number of voltaic cells; the term was taken from an earlier, almost purely military use.* A battery in a mobile phone is just an elaborated replacement for a

unrealistic; for example he imagined that Tesla coils could be used for wireless power transmission.

* A pile made of sets of a zinc plate, a sheet of blotting paper soaked with salt water and a copper coin is easily constructed at home. Challenge 951 ny

number of apples or potatoes.

* *

A PC or a telephone can communicate without wires, by using radio waves. Why are these and other electrical appliances not able to obtain their *power* via radio waves, thus eliminating power cables? Challenge 952 n

* *

Objects that are not right–left symmetric are called *chiral*, from the Greek word for 'hand'. Can you make a mirror that does not exchange left and right? In two different ways? Challenge 953 n

* *

A Scotch tape roll is a dangerous device. Pulling the roll quickly leads to light emission (through triboluminescence) and to small sparks. It is suspected that several explosions in mines were triggered when such a spark ignited a combustible gas mixture.

* *

Take an envelope, wet it and seal it. After letting it dry for a day or more, open it in the dark. At the place where the two sides of paper are being separated from each other, the envelope glows with a blue colour. Why? Is it possible to speed up the test using a hair dryer? Challenge 954 n

* *

Electromagnetism is full of surprises and offers many effects that can be reproduced at home. The internet is full of descriptions of how to construct Tesla coils to produce sparks, coil guns or rail guns to shoot objects, electrostatic machines to make your hair stand on end, glass spheres with touch-sensitive discharges and much more. If you like experiments, just search for these terms.

* *

A high voltage can lead to current flow through air, because air becomes conductive in high electric fields. In such discharges, air molecules are put in motion. As a result, one can make objects that are attached to a pulsed high tension source lift up in the air, if one optimizes this air motion so that it points downwards everywhere. The high tension is thus effectively used to accelerate ionized air in one direction and, as a result, an object will move in the opposite direction, using the same principle as a rocket. An example is shown in Figure 234, using the power supply of a PC monitor. (Watch out: danger!) Numerous websites explain how to build these so-called lifters at home; in Figure 234, the bottle and the candle are used as high voltage insulator to keep one of the two thin high voltage wires (not visible in the photograph) high enough in the air, in order to avoid discharges to the environment or to interfere with the lifter's motion. Unfortunately, the majority of websites – not all – give incorrect or confused explanations of the phenomenon. These websites thus provide a good challenge for one to learn to distinguish fact from speculation. Challenge 955 e

* *

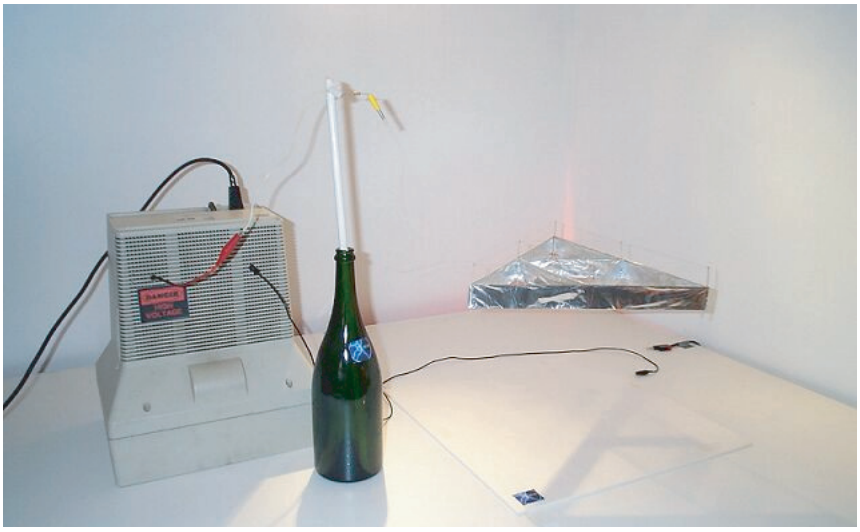

FIGURE 234 Lifting a light object – covered with aluminium foil – using high a tension discharge (© Jean-Louis Naudin at http://www.jlnlabs.org)

The electric effects produced by friction and by liquid flow are usually small. However, in the 1990s, a number oil tankers disappeared suddenly. The sailors had washed out the oil tanks by hosing sea water onto the tank walls. The spraying led to charging of the tank; a discharge then led to the oil fumes in the tank igniting. This led to an explosion and subsequently the tankers sank. Similar accidents also happen regularly when chemicals are moved from one tank to another.

* *

Rubbing a plastic spoon with a piece of wool charges it. Such a charged spoon can be used to extract pepper from a salt–pepper mixture by holding the spoon over the mixture.
Challenge 956 n Why?

* *

When charges move, they produce a magnetic field. In particular, when ions inside the Earth move due to heat convection, they produce the Earth's magnetic field. When the ions high up in the stratosphere are moved by solar wind, a geomagnetic storm appears; its field strength can be as high as that of the Earth itself. In 2003, an additional mechanism was discovered. When the tides move the water of the oceans, the ions in the salt water produce a tiny magnetic field; it can be measured by highly sensitive magnetometers in satellites orbiting the Earth. After two years of measurements from a small satellite it was
Ref. 503 possible to make a beautiful film of the oceanic flows. Figure 235 gives an impression.

* *

The names electrode, electrolyte, ion, anode and cathode were suggested by William Whewell (1794–1866) on demand of Michael Faraday; Faraday had no formal education and asked his friend Whewell to form two Greek words for him. For anode and cathode, Whewell took words that literally mean 'upward street' and 'downward street'. Faraday

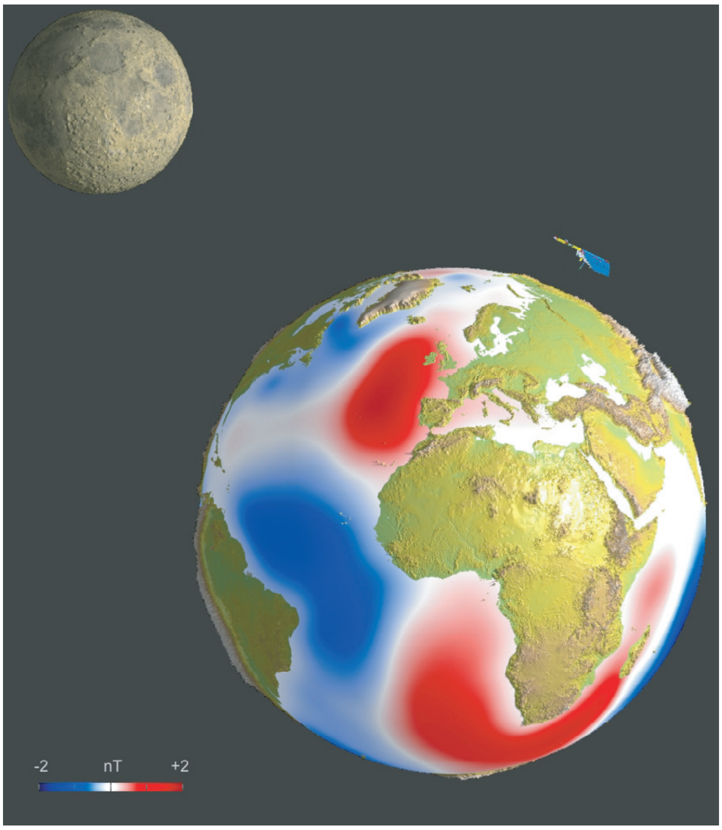


FIGURE 235 The magnetic field due to the tides (© Stefan Maus)

then popularized these terms, like the other words mentioned above.

* *

The shortest light pulse produced so far had a length of 100 as. To how many wavelengths of green light would that correspond?

Challenge 957 n

* *

Why do we often see shadows of houses and shadows of trees, but never shadows of the electrical cables hanging over streets?

Challenge 958 n

* *

How would you measure the speed of the tip of a lightning bolt? What range of values do you expect?

Challenge 959 n

* *

Ref. 504

One of the simplest possible electric motors was discovered by Faraday in 1831. A magnet suspended in mercury will start to turn around its axis if a current flows through it. (See Figure 236.) In addition, when the magnet is forced to turn, the device (often also called Barlow's wheel) also works as a current generator; people have even tried to generate

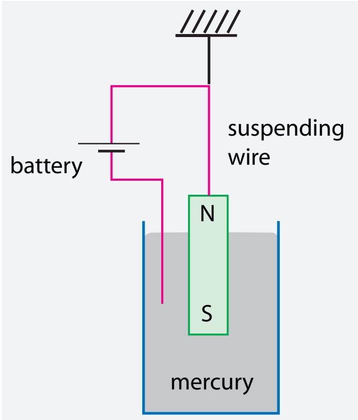

FIGURE 236 A unipolar motor

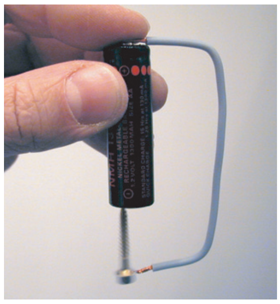
FIGURE 237 The simplest motor (© Stefan Kluge)

Challenge 960 n domestic current with such a system! Can you explain how it works?

The modern version of this motor makes use of a battery, a wire, a conductive samarium–cobalt magnet and a screw. The result is shown in Figure 237.

* *

The magnetic field of the Earth is much higher than that of other planets because of the Earth's Moon. The field has a dipole strength of $7.8 \cdot 10^{22}$ A m$^2$. It shields us from lethal solar winds and cosmic radiation particles. Today, a lack of magnetic field would lead to high radiation on sunny days; but in the past, its lack would have prevented the evolution of the human species. We owe our existence to the magnetic field. Ref. 505

* *

The ionosphere around the Earth has a resonant frequency of 7 Hz; for this reason any apparatus measuring low frequencies always gets a strong signal at this value. Can you give an explanation of the frequency? Challenge 961 n

* *

Challenge 962 ny The Sun is visible to the naked eye only up to a distance of 50 light years. Is this true?

* *

At home, electricity is mostly used as alternating current. In other words, no electrons actually flow through cables; as the drift speed of electrons in copper wires is of the order of 1 μm/s, electrons just move back and forward by 20 nm. Nothing flows in or out of the cables! Why do the electricity companies require a real flow of money in return, instead of being satisfied with a back and forth motion of money? Page 590 Challenge 963 e

* *

Comparing electricity with water is a good way of understanding electronics. Figure 238 shows a few examples that even a teenager can use. Can you fill in the correspondence for the coil, and thus for a transformer? Challenge 964 n

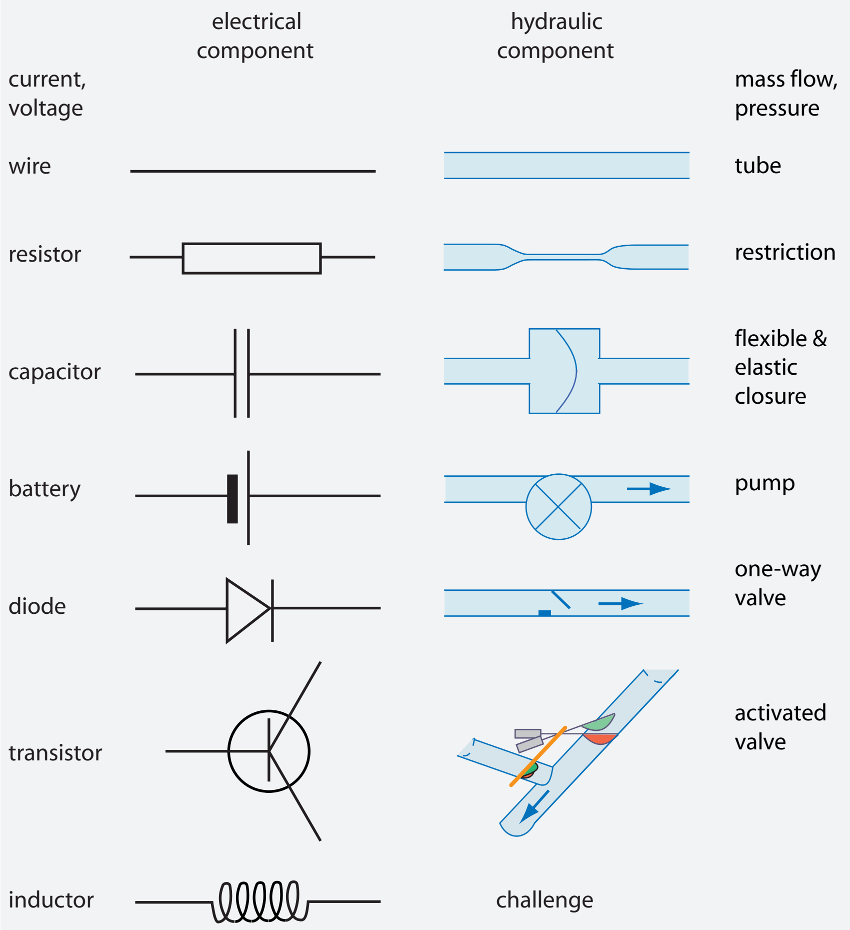


FIGURE 238 The correspondence of electronics and water flow

* *

Challenge 965 ny Do electrons and protons have the same charge? Experiments show that the values are equal to within at least twenty digits. How would you check this?

* *

Challenge 966 ny Charge is also velocity-independent. How would you check this?

* *

Magnets can be used, even by school children, to climb steel walls. Have a look at the http://www.physicslessons.com/TPNN.htm website.

* *

Extremely high magnetic fields have strange effects. At fields of $10^{10}$ T, vacuum becomes effectively birefringent, photons can split and coalesce, and atoms get squeezed. Hydrogen atoms, for example, are estimated to get two hundred times narrower in one direction. Fortunately, these conditions exist only in specific neutron stars, called magnetars.

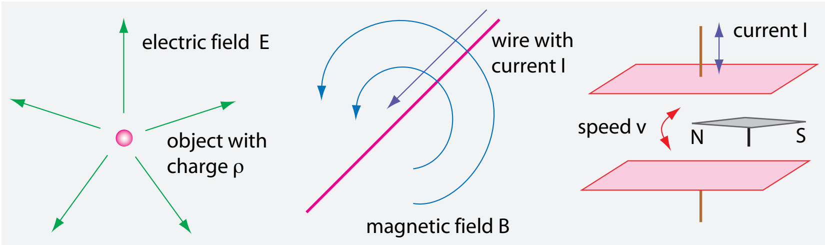


FIGURE 239 The first of Maxwell's equations

* *

A good way to make money is to produce electricity and sell it. In 1964, a completely new method was invented by Fletcher Osterle. The method was presented to a larger public in
Ref. 506 a beautiful experiment in 2003. One can take a plate of glass, add a conducting layers on each side, and then etch a few hundred thousand tiny channels through the plate, each around 15 μm in diameter. When water is made to flow through the channels, a current is generated. The contacts at the two conducting plates can be used like battery contacts.

This simple device uses the effect that glass, like most insulators, is covered with a charged layer when it is immersed in a liquid. Can you imagine why a current is gener-
Challenge 967 n ated? Unfortunately, the efficiency of electricity generation is only about 1%, making the method much less interesting than a simple blade wheel powering a dynamo.

## The description of electromagnetic field evolution

In the years between 1861 and 1865, taking in the details of all the experiments known to him, James Clerk Maxwell produced a description of electromagnetism that forms one of the pillars of physics.* Maxwell took all the experimental results and extracted their common basic principles, as shown in Figures 239 and 240. Twenty years later, Heaviside and Hertz extracted the main points of Maxwell ideas, calling their summary *Maxwell's*
Page 1201 *theory of the electromagnetic field*. It consists of two equations (four in the non-relativistic case).

The first equation is the precise statement that electromagnetic fields *originate at*

* James Clerk Maxwell (b. 1831 Edinburgh, d. 1879 Cambridge), Scottish physicist. He founded electromagnetism by theoretically unifying electricity and magnetism, as described in this chapter. His work on thermodynamics forms the second pillar of his activity. In addition, he studied the theory of colours and developed the now standard horseshoe colour diagram; he was one of the first people to make a colour photograph. He is regarded by many as the greatest physicist ever. Both 'Clerk' and 'Maxwell' were his family names.

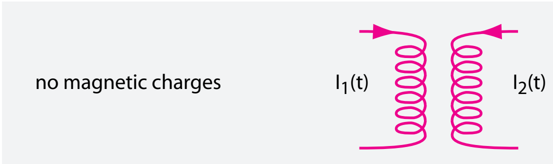


FIGURE 240 The second of Maxwell's equations

*charges*, and nowhere else. The corresponding equation is variously written*

$$dF = j\sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{or}$$

$$d^{\nu}F_{\mu\nu} = j^{\mu}\sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{or} \tag{405}$$

$$(\partial_t/c, -\partial_x, -\partial_y, -\partial_z)\begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} = \sqrt{\frac{\mu_0}{\varepsilon_0}}(\rho, j_x/c, j_y/c, j_z/c) \quad \text{or}$$

$$\nabla\mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \text{and} \quad \nabla\times\mathbf{B} - \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t} = \mu_0\mathbf{j}\ .$$

Each of these four equivalent ways to write the equation makes a simple statement: *electrical charge carries the electromagnetic field*. This statement, including its equations, are equivalent to the three basic observations of Figure 239. It describes Coulomb's relation, Ampère's relation, and the way changing electrical fields induce magnetic effects, as you may want to check for yourself.

Challenge 968 ny

The second half of equation (405) contains the right hand rule for magnetic fields around wires, through the vector product. The equation also states that changing electric fields induce magnetic fields. The effect is essential for the primary side of transformers. The factor $1/c^2$ implies that the effect is small; that is why coils with many windings or strong electric currents are needed to find it. Due to the vector product, all induced magnetic field lines are closed lines.

Challenge 969 e

The second result by Maxwell is the precise description of how changing electric fields create magnetic fields, and vice versa. In particular, an electric field can have vortices only when there is a changing magnetic field. In addition it expresses the observation that in nature there are no magnetic charges, i.e. that magnetic fields have no sources. All these

* Maxwell generalized this equation to cases where the charges are not surrounded by vacuum, but located inside matter. We will not explore these situations in our walk because, as we will see during our mountain ascent, the apparently special case of vacuum in fact describes all of nature.

results are described by the relation variously written

$$
\begin{aligned}
& d\,{}^{*}\mathrm{F} = 0 \quad \text{with} \quad {}^{*}\mathrm{F}^{\rho\sigma} = \frac{1}{2}\varepsilon^{\rho\sigma\mu\nu}\mathrm{F}_{\mu\nu} \quad \text{or} \\
& \varepsilon_{\mu\nu\rho}\partial_\mu \mathrm{F}_{\nu\rho} = \partial_\mu \mathrm{F}_{\nu\rho} + \partial_\nu \mathrm{F}_{\rho\mu} + \partial_\rho \mathrm{F}_{\mu\nu} = 0 \quad \text{or} \\
& \begin{pmatrix} \gamma\frac{1}{c}\partial_t \\ \gamma\partial_x \\ \gamma\partial_y \\ \gamma\partial_z \end{pmatrix} \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z/c & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & E_x/c & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \qquad (406) \\
& \nabla \mathbf{B} = 0 \quad \text{and} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \,.
\end{aligned}
$$

The relation expresses the *lack of sources for the dual field tensor*, usually written ${}^{*}\mathrm{F}$: there are no magnetic charges, i.e. no magnetic monopoles in nature. In practice, this equation is always needed together with the previous one. Can you see why? Challenge 970 ny

Since there are no magnetic charges, magnetic field lines are always closed; they never start or end. For example, field lines continue inside magnets. This is often expressed mathematically by stating that the *magnetic flux* through a closed surface $S$ – such as a sphere or a cube – *always vanishes*: $\int_S \mathbf{B}\mathrm{d}\mathbf{A} = 0$.

The second half of equation (406), also shown in Figure 240, expresses that changes in magnetic fields produce electric fields: this effect is used in the secondary side of transformers and in dynamos. The cross product in the expression implies that an electric field generated in this way – also called an *electromotive field* – has no start and endpoints. The electromotive field lines thus run in circles: in most practical cases they run along electric circuits.

Together with Lorentz' evolution equation (400), which describes how charges move given the motion of the fields, Maxwell's evolution equations (405) and (406) describe *all* electromagnetic phenomena occurring on everyday scales, from mobile phones, car batteries, to personal computers, lasers, lightning, holograms and rainbows. We now have a system as organized as the expression $a = GM/r$ or as Einstein's field equations for gravitation.

We will not study many applications of the field equations but will continue directly towards our aim to understand the connection to everyday motion and to the motion of light. In fact, the electromagnetic field has an important property that we mentioned right at the start: the field itself itself can move.

## Colliding charged particles

A simple experiment clarifies the properties of electromagnetic fields defined above. When two charged particles collide, their total momentum is *not* conserved.

Imagine two particles of identical mass and identical charge just after a collision, when they are moving away from one another. Imagine also that the two masses are large, so that the acceleration due to their electrical repulsion is small. For an observer at the centre of gravity of the two, each particle feels an acceleration from the electric field of the other. The electric field $E$ is given by the so-called *Heaviside formula* Challenge 971 ny

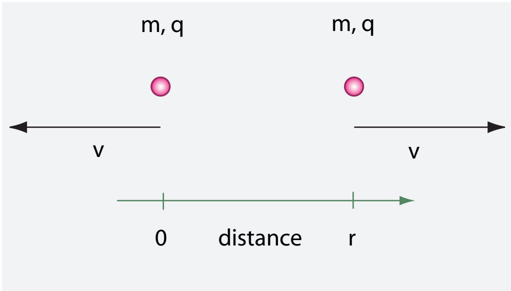


FIGURE 241 Charged particles after a collision

$$E = \frac{q(1 - v^2/c^2)}{4\pi e_0 r^2} . \qquad (407)$$

In other words, the total system has a vanishing total momentum.

Take a second observer, moving with respect to the first with velocity $v$, so that the first charge will be at rest. Expression (407) leads to two *different* values for the electric fields, one at the position of each particle. In other words, the system of the two particles is not in inertial motion, as we would expect; the total momentum is not conserved. Where did it go?

Ref. 507

Challenge 972 n

This at first surprising effect has been put in the form of a theorem by Van Dam and Wigner. They showed that for a system of particles interacting at a distance the total particle energy–momentum cannot remain constant in all inertial frames.

Ref. 508

The total momentum of the system is conserved only because the electromagnetic field itself also carries momentum. If electromagnetic fields have momentum, they are able to *strike* objects and to be struck by them. As we will show below, light is also an electromagnetic field. Thus we should be able to move objects by shining light on to them. We should even be able to suspend particles in mid air by shining light on to them from below. Both predictions are correct, and some experiments will be presented shortly.

We conclude that any sort of field leading to particle interactions must carry energy and momentum, as the argument applies to all such cases. In particular, it applies to nuclear interactions. Indeed, in the second part of our mountain ascent we will even find an additional result: all fields are themselves composed of particles. The energy and momentum of fields then become an obvious state of affairs.

## The gauge field – the electromagnetic vector potential

The study of moving fields is called *field theory* and electrodynamics is the prime example. (The other classical example is fluid dynamics; moving electromagnetic fields and moving fluids are very similar mathematically.) Field theory is a beautiful topic; field lines, equipotential lines and vortex lines are some of the concepts introduced in this domain. They fascinate many.* However, in this mountain ascent we keep the discussion focused

Challenge 973 n

* What is the relation, for static fields, between field lines and (equi-) potential surfaces? Can a field line cross a potential surface twice? For more details on topics such as these, see the *free* textbook by Bo Thidé, *Electromagnetic Field Theory*, on his http://www.plasma.uu.se/CED/Book website. And of course, in English, have a look at the texts by Schwinger and by Jackson.

Ref. 489

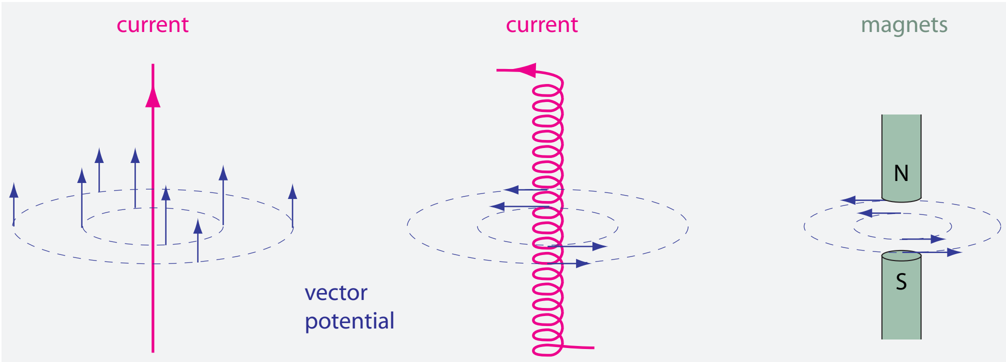


FIGURE 242 Vector potentials for selected situations

on motion.

We have seen that fields force us to extend our concept of motion. Motion is not only the change in state of objects and of space-time, but also the *change in state of fields*. We therefore need, also for fields, a complete and precise description of their state. The observations with amber and magnets have shown us that *fields possess energy and momentum*. They can impart it to particles. The experiments with motors have shown that objects can add energy and momentum to fields. We therefore have to define a *state function* which allows us to define energy and momentum for electric and magnetic fields. Since electric and magnetic fields transport energy, their motion follows the speed limit in nature.

Maxwell defined the state function in two standard steps. The first step is the definition of the *(magnetic) vector potential*, which describes the momentum per charge that the field provides: Ref. 509

$$\mathbf{A} = \frac{\mathbf{p}}{q} . \tag{408}$$

When a charged particle moves through a magnetic potential $\mathbf{A}(\mathbf{x})$, its momentum changes by $q\Delta\mathbf{A}$; it changes by the difference between the potential values at the start and end points, multiplied by its charge. Owing to this definition, the vector potential has the property that

$$\mathrm{B} = \nabla \times \mathbf{A} = \mathrm{curl}\ \mathbf{A} \tag{409}$$

i.e. that the magnetic field is the curl of the magnetic potential. The curl is called the *rotation*, abbreviated rot in most languages. The curl (or rotation) of a field describes, for each point of space, the direction of the local, imagined axis of rotation, as well as (twice) the rotation speed around that axis. For example, the curl for the velocities of a rotating solid body is everywhere $2\omega$, or twice the angular velocity. Challenge 974 ny

The vector potential for a long straight current-carrying wire is parallel to the wire; it has the magnitude Ref. 510 Challenge 975 d

$$A(r) = -\frac{\mu_0 I}{4\pi} \ln \frac{r}{r_0} , \tag{410}$$

which depends on the radial distance $r$ from the wire and an integration constant $r_0$. This expression for the vector potential, pictured in Figure 242, shows how the moving current produces a linear momentum in the (electro-) magnetic field around it. In the case of a solenoid, the vector potential 'circulates' around the solenoid. The magnitude obeys

$$A(\mathrm{r}) = -\frac{\Phi}{4\pi}\frac{1}{r} , \tag{411}$$

where $\Phi$ is the magnetic flux inside the solenoid. We see that, in general, the vector potential is *dragged along* by moving charges. The dragging effect decreases for larger distances. This fits well with the image of the vector potential as the momentum of the electromagnetic field.

This behaviour of the vector potential around charges is reminiscent of the way honey is dragged along by a spoon moving in it. In both cases, the dragging effect decreases with distance. However, the vector potential, unlike the honey, does *not* produce any friction that slows down charge motion. The vector potential thus behaves like a frictionless liquid.

Inside the solenoid, the magnetic field is constant and uniform. For such a field B we find the vector potential

Challenge 976 ny

$$\mathbf{A}(\mathrm{r}) = -\frac{1}{2}\mathbf{B}\times\mathbf{r} . \tag{412}$$

In this case, the magnetic potential thus increases with increasing distance from the origin.* In the centre of the solenoid, the potential vanishes. The analogy of the dragged honey gives exactly the same behaviour.

However, there is a catch. The magnetic potential is *not* defined uniquely. If $\mathbf{A}(\mathbf{x})$ is a vector potential, then the different vector potential

$$\mathbf{A}'(\mathbf{x}) = \mathbf{A}(\mathbf{x}) + \operatorname{grad}\Lambda , \tag{413}$$

where $\Lambda(t,\mathbf{x})$ is some scalar function, is *also* a vector potential for the same situation. (The magnetic field B stays the same, though.) Worse, can you confirm that the corresponding (absolute) momentum values also change? This unavoidable ambiguity, called *gauge invariance*, is a central property of the electromagnetic field. We will explore it in more detail below.

Challenge 977 ny

Not only the momentum, but also the energy of the electromagnetic field is defined ambiguously. Indeed, the second step in the specification of a state for the electromagnetic field is the definition of the *electric potential* as the energy $U$ per charge:

Ref. 509

$$\varphi = \frac{U}{q} \tag{414}$$

In other words, the potential $\varphi(\mathbf{x})$ at a point $\mathbf{x}$ is the energy needed to move a unit charge to the point $\mathbf{x}$ starting from a point where the potential vanishes. The potential energy is thus given by $q\varphi$. From this definition, the electric field $\mathbf{E}$ is simply the *change* of the

* This is only possible as long as the field is constant; since all fields drop again at large distances – because the energy of a field is always finite – also the vector potential drops at large distances.

potential with position corrected by the time dependence of momentum, i.e.

$$\mathbf{E} = -\nabla\varphi - \frac{\partial}{\partial t}\mathbf{A} , \qquad (415)$$

Obviously, there is a freedom in the choice of the definition of the potential. If $\varphi(\mathbf{x})$ is a possible potential, then

$$\varphi'(\mathbf{x}) = \varphi(\mathbf{x}) - \frac{\partial}{\partial t}\mathbf{\Lambda} \qquad (416)$$

is also a potential function for the same situation. This freedom is the generalization of the freedom to define energy up to a constant. Nevertheless, the electric field **E** remains the same for all potentials.

Ref. 509 To be convinced that the potentials really are the energy and momentum of the elec-
Challenge 978 ny tromagnetic field, we note that for a moving charge we have

$$\begin{aligned} \frac{d}{dt}(\frac{1}{2}mv^2 + q\varphi) &= \frac{\partial}{\partial t}q(\varphi - \mathbf{vA}) \\ \frac{d}{dt}(m\mathbf{v} + q\mathbf{A}) &= -\nabla q(\varphi - \mathbf{vA}) , \end{aligned} \qquad (417)$$

which show that the changes of generalized energy and momentum of a particle (on the left-hand side) are due to the change of the energy and momentum of the electromagnetic field (on the right-hand side).*

In relativistic 4-vector notation, the energy and the momentum of the field appear together in one quantity. The state function of the electromagnetic field becomes

$$A^\mu = (\varphi/c, \mathbf{A}) . \qquad (418)$$

It is easy to see that the description of the field is complete, since we have

$$\mathrm{F} = d\,A \quad \text{or} \quad \mathrm{F}^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \qquad (419)$$

which means that the electromagnetic field $F$ is completely specified by the 4-potential $A$. But as just said, the 4-potential itself is *not* uniquely defined. Indeed, any other gauge field $A'$ is related to $A$ by the *gauge transformation*

$$A'^{\,\mu} = A^\mu + \partial^\mu\Lambda \qquad (420)$$

where $\Lambda = \Lambda(t, x)$ is any arbitrarily chosen scalar field. The new field $A'$ leads to the *same* electromagnetic field, and to the same accelerations and evolutions. The gauge 4-field $A$ is thus an *overdescription* of the physical situation as several *different* gauge fields correspond to the *same* physical situation. Therefore we have to check that all measurement results are independent of gauge transformations, i.e. that all observables are gauge

* This connection also shows why the expression $P^\mu - qA^\mu$ appears so regularly in formulae; indeed, it plays a central role in the quantum theory of a particle in the electromagnetic field.

invariant quantities. Such gauge invariant quantities are, as we just saw, the fields F and *F, and in general all classical quantities. We add that many theoretical physicists use the term 'electromagnetic field' loosely for both the quantities $F^{\mu\nu}$ and $A_\mu$.

There is a simple image, due to Maxwell, to help overcoming the conceptual difficulties of the vector potential. It turns out that the closed line integral over $A_\mu$ is gauge invariant, because

Challenge 979 e

$$\oint A_\mu \mathrm{d}x^\mu = \oint (A_\mu + \partial_\mu \Lambda) \mathrm{d}x^\mu = \oint A'_\mu \mathrm{d}x^\mu \ . \qquad (421)$$

In other words, if we picture the vector potential as a quantity allowing one to associate a number to a tiny ring at each point in space, we get a good, gauge invariant picture of the vector potential.*

Now that we have defined a state function that describes the energy and momentum of the electromagnetic field, let us look at what happens in more detail when electromagnetic fields move.

## Energy and linear and angular momentum of the electromagnetic field

The description so far allows us to write the *total* energy $E_{\text{nergy}}$ of the electromagnetic field as

$$E_{\text{nergy}} = \frac{1}{8\pi} \int \varepsilon_0 \mathbf{E}^2 + \frac{\mathbf{B}^2}{\mu_0} \, \mathrm{d}V \ . \qquad (422)$$

Energy is thus quadratic in the fields.

For the total linear momentum one obtains

$$\mathbf{P} = \frac{\varepsilon_0}{4\pi} \int \mathbf{E} \times \mathbf{B} \, \mathrm{d}V \ . \qquad (423)$$

The expression is also called the *Poynting vector*.**

For the total angular momentum one has

$$\mathbf{L} = \frac{\varepsilon_0}{4\pi} \int \mathbf{E} \times \mathbf{A} \, \mathrm{d}V \ , \qquad (424)$$

where **A** is the magnetic vector potential.

## The Lagrangian of electromagnetism

The motion of charged particles and the motion of the electromagnetic field can also be described using a Lagrangian instead of using the three equations given above. It is not hard to see that the action $S_{\text{CED}}$ for a particle in classical electrodynamics can be symbolically defined by***

Challenge 980 ny

Ref. 511

* In the second part of the text, on quantum mechanics, we will see that the exponent of this expression, namely $\exp(iq \oint A_\mu \mathrm{d}x^\mu)/\hbar$, usually called the *phase factor*, can indeed be directly observed in experiments.

** John Henry Poynting (1852–1914) introduced the concept in 1884.

$$S_{\text{CED}} = -mc^2 \int \mathrm{d}\tau - \tfrac{1}{4\mu_0} \int \mathrm{F} \wedge * \mathrm{F} - \int j \wedge A \,, \qquad (425)$$

which in index notation becomes

$$S_{\text{CED}} = -mc \int_{-\infty}^{\infty} \sqrt{\eta_{\mu\nu} \frac{\mathrm{d}x_n^{\mu}(s)}{\mathrm{d}s} \frac{\mathrm{d}x_n^{\nu}(s)}{\mathrm{d}s}}\, \mathrm{d}s - \int_{\mathbf{M}} \left( \tfrac{1}{4\mu_0} \mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} + j_{\mu} A^{\mu} \right) \mathrm{d}^4 x \,. \qquad (426)$$

What is new is the measure of the change produced by the electromagnetic field. Its internal change is given by the term $F^* F$, and the change due to interaction with matter is given by the term $jA$.

The least action principle, as usual, states that the change in a system is always as small as possible. The action $S_{\text{CED}}$ leads to the evolution equations by requiring that the action be stationary under variations $\delta$ and $\delta'$ of the positions and of the fields which vanish at infinity. In other terms, the principle of least action requires that

$$\begin{aligned} \delta S = 0 \quad & \text{when} \quad x_{\mu} = x_{\mu} + \delta_{\mu} \quad \text{and} \quad A_{\mu} = A_{\mu} + \delta'_{\mu} \quad , \\ & \text{provided} \quad \delta x_{\mu}(\theta) \to 0 \quad \text{for} \quad |\theta| \to \infty \\ & \text{and} \quad \delta A_{\mu}(x_{\nu}) \to 0 \quad \text{for} \quad |x_{\nu}| \to \infty \,. \end{aligned} \qquad (427)$$

Page 180
Challenge 981 ny

In the same way as in the case of mechanics, using the variational method for the two variables $A$ and $x$, we recover the evolution equations for particle and fields

$$b^{\mu} = \frac{q}{m} \mathrm{F}^{\mu}_{\nu}\, u^{\nu} \quad , \quad \partial_{\mu} \mathrm{F}^{\mu\nu} = j^{\nu} \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad , \quad \text{and} \quad \varepsilon^{\mu\nu\rho\sigma}\, \partial_{\nu} \mathrm{F}_{\rho\sigma} = 0 \,, \qquad (428)$$

which we know already. Obviously, they are equivalent to the variational principle based on $S_{\text{CED}}$. Both descriptions have to be completed by specifying *initial conditions* for the particles and the fields, as well as *boundary conditions* for the latter. We need the first and zeroth derivatives of the position of the particles, and the zeroth derivative for the electromagnetic field.

Are you able to specify the Lagrangian of the pure electrodynamic field using the fields **E** and **B** instead of $F$ and $^*F$?

Challenge 982 ny

The form of the Lagrangian implies that electromagnetism is *time reversible*. This means that every example of motion due to electric or magnetic causes can also take place backwards. This is easily deduced from the properties of the Lagrangian. On the other hand, everyday life shows many electric and magnetic effects which are not time invariant, such as the breaking of bodies or the burning of electric light bulbs. Can you explain how this fits together?

Challenge 983 ny

In summary, with the Lagrangian (425) all of classical electrodynamics can be described and understood. For the rest of this chapter, we look at some specific topics from this vast field.

*** The product described by the symbol $\wedge$, 'wedge' or 'hat', has a precise mathematical meaning, defined for this case in equation (426). Its background, the concept of *(mathematical) form*, carries us too far from our walk.

Ref. 512

## Symmetries – the energy–momentum tensor

We know from classical mechanics that we get the definition of energy and momentum tensor by using Noether's theorem, if we determine the conserved quantity from the Lorentz symmetry of the Lagrangian. For example, we found that relativistic particles have an energy–momentum *vector*. At the point at which the particle is located, it describes the energy and momentum.

Since the electromagnetic field is not a localized entity, like a point particle, but an extended entity, we need to know the *flow* of energy and momentum at every point in space, separately *for each direction*. This makes a description with a *tensor* necessary. The Page 424 result is the *energy–momentum tensor* of the electromagnetic field

$$T^{\mu\nu} = \left( \begin{array}{c|c} \text{energy density} & \text{energy flow or momentum density} \\ \hline \text{energy flow or momentum density} & \text{momentum flow density} \end{array} \right)$$
$$= \left( \begin{array}{c|c} u & S/c = cp \\ \hline cp & T \end{array} \right) = \left( \begin{array}{c|c} 0 & \varepsilon_0 c \boldsymbol{E} \times \boldsymbol{B} \\ \hline \varepsilon_0 c \cdot & -\varepsilon_0 E_i E_j - B_i B_j/\mu_0 \\ \boldsymbol{E} \times \boldsymbol{B} & 1/2\delta_{ij}(\varepsilon_0 E^2 + B^2/\mu_0) \end{array} \right) \qquad (429)$$

The energy–momentum tensor shows again that electrodynamics is both Lorentz and gauge invariant.

Both the Lagrangian and the energy–momentum tensor show that electrodynamics is symmetric under motion inversion. If all charges change direction of motion – a situation often incorrectly called 'time inversion' – they move backwards along the exact paths they took when moving forward.

We also note that charges and mass destroy a symmetry of the vacuum that we mentioned in special relativity: only the vacuum is invariant under conformal symmetries. In particular, only the vacuum is invariant under the spatial inversion $r \to 1/r$.

To sum up, electrodynamic motion, like all other examples of motion that we have encountered so far, is deterministic, reversible and conserved. This is no big surprise. Nevertheless, two symmetries of electromagnetism deserve special mention.

## What is a mirror?

We will study the strange properties of mirrors several times during our walk. We start with the simplest one first. Everybody can observe, by painting each of their hands in a different colour, that a mirror does *not* exchange right and left, as little as it exchanges up and down; however, a mirror does exchange right and left *handedness*. In fact, it does so by exchanging front and back.

Electrodynamics give a second answer: a mirror is a device that switches magnetic Challenge 984 n north and south poles. Can you confirm this with a diagram?

But is it always possible to distinguish left from right? This seems easy: this text is quite different from a ɒɘɿoɿɿim version, as are many other objects in our surroundings. But take a simple landscape. Are you able to say which of the two pictures of Figure 243 is the original?

FIGURE 243 Which one is the original landscape? (NOAA)

Astonishingly, it is actually impossible to distinguish an original picture of nature from its mirror image if it does not contain any human traces. In other words, everyday nature is somehow left–right symmetric. This observation is so common that all candidate exceptions, from the jaw movement of ruminating cows to the helical growth of plants, such as hops, or the spiral direction of snail shells, have been extensively studied.* Can you name a few more?

Page 939

Challenge 985 n

The left–right symmetry of nature appears because everyday nature is described by gravitation and, as we will see, by electromagnetism. Both interactions share an important property: substituting all coordinates in their equations by the negative of their values leaves the equations unchanged. This means that for any solution of these equations, i.e. for any naturally occurring system, a mirror image is a possibility that can also occur naturally. Everyday nature thus cannot distinguish between right and left. Indeed, there are right *and* left handers, people with their heart on the left *and* others with their heart on the right side, etc.

To explore further this strange aspect of nature, try the following experiment: imagine you are exchanging radio messages with a Martian; are you able to explain to him what right and left are, so that when you meet, you are sure you are talking about the same thing?

Challenge 986 n

Actually, the mirror symmetry of everyday nature – also called its *parity invariance* – is so pervasive that most animals cannot distinguish left from right in a deeper sense. Most animals react to mirror stimuli with mirror responses. It is hard to teach them different ways to react, and it is possible almost only for mammals. The many experiments performed in this area gave the result that animals have symmetrical nervous systems,

Ref. 514

* The most famous is the position of the heart. The mechanisms leading to this disposition are still being investigated. Recent research suggests that the oriented motion of the cilia on embryos, probably in the region called the *node*, determines the right–left asymmetry. The deep origin of this asymmetry is not yet elucidated, however.

Ref. 513

Most human bodies have more muscles on the right side for right-handers, such as Albert Einstein and Pablo Picasso, and correspondingly on the left side for left-handers, such as Charlie Chaplin and Peter Ustinov. This asymmetry reflects an asymmetry of the human brain, called lateralization, which is essential to human nature.

Another asymmetry of the human body is the hair whirl on the back of the head; the majority of humans have only one, and in 80 % of the cases it is left turning. But many people have more than one.

and possibly only humans show *lateralization*, i.e. a preferred hand and different uses for the left and the right parts of the brain.

To sum up this digression, classical electrodynamics is left–right symmetric, or parity invariant. Can you show this using its Lagrangian?

Challenge 987 n

A concave mirror shows an inverted image; so does a plane mirror if it is partly folded along the horizontal. What happens if this mirror is rotated around the line of sight?

Challenge 988 n

Why do metals provide good mirrors? Metals are strong absorbers of light. Any strong absorber has a metallic shine. This is true for metals, if they are thick enough, but also for dye or ink crystals. Any material that strongly absorbs a light wavelength also reflects it efficiently. The cause of the strong absorption of a metal is the electrons inside it; they can move almost freely and thus absorb most visible light frequencies.

## What is the difference between electric and magnetic fields?

Obviously, the standard answer is that electric fields have sources, and magnetic fields do not; moreover, magnetic fields are small relativistic effects of importance only when charge velocities are high or when electrical fields cancel out.

For situations involving matter, this clear distinction is correct. Up to the present day, no particle with a magnetic charge, called a *magnetic monopole*, has ever been found, even though its existence is possible in several unified models of nature. If found, the action (425) would have to be modified by the addition of a fourth term, namely the magnetic current density. However, no such particle has yet been detected, despite intensive search efforts.

Page 942

In empty space, when matter is not around, it is possible to take a completely different view. In empty space the electric and the magnetic fields can be seen as two faces of the same quantity, since a transformation such as

$$\begin{aligned} \mathbf{E} &\rightarrow c\,\mathbf{B} \\ \mathbf{B} &\rightarrow -\mathbf{E}/c \end{aligned} \qquad (430)$$

called (electromagnetic) *duality* transformation, transforms each vacuum Maxwell equation into the other. The minus sign is necessary for this. (In fact, there are even more such transformations; can you spot them?) Alternatively, the duality transformation transforms F into $^*$F. In other words, in empty space we *cannot* distinguish electric from magnetic fields.

Challenge 989 n

Matter would be symmetric under duality only if magnetic charges, also called magnetic monopoles, could exist. In that case the transformation (430) could be extended to

$$c\rho_e \rightarrow \rho_m \quad , \quad \rho_m \rightarrow -c\rho_e \; . \qquad (431)$$

It was one of the great discoveries of theoretical physics that even though classical electrodynamics with matter is not symmetric under duality, nature is. In 1977, Claus Montonen and David Olive showed that quantum theory allows duality transformations even *with* the inclusion of matter. It has been known since the 1930s that quantum theory allows magnetic monopoles. We will discover the important ramifications of this result in the

third part of the text. This duality turns out to be one of the essential stepping stones that leads to a unified description of motion. (A somewhat difficult question: extending this duality to quantum theory, can you deduce what transformation is found for the fine structure constant, and why it is so interesting?)
Challenge 990 ny

Duality, by the way, is a symmetry that works *only* in Minkowski space-time, i.e. in space-times of 3 + 1 dimensions. Mathematically, duality is closely related to the existence of quaternions, to the possibility of interpreting Lorentz boosts as rotations in 3 + 1 dimensions, and last, but not least, to the possibility of defining other smooth mathematical structures than the standard one on the space $R^4$. These mathematical connections are mysterious for the time being; they somehow point to the special role that *four* space-time dimensions play in nature. More details will become apparent in the third part of our mountain ascent.

## Electrodynamic challenges and curiosities

### Could electrodynamics be different?

Any interaction such as Coulomb's rule (392), which acts, for one given observer, between two particles independently of 3-velocity, must depend on 3-velocity for other inertial observers.* It turns out that such an interaction cannot be independent of the 4-velocity either. Such an interaction, even though it would indeed be 3-velocity dependent, would change the rest mass, since the 4-acceleration would not be 4-orthogonal to the 4-velocity.

The next simplest case is the one in which the acceleration is proportional to the 4-velocity. Together with the request that the interaction leaves the rest mass constant, we then recover electrodynamics. Ref. 515

In fact, the requirements of gauge symmetry and of relativity symmetry also make it impossible to modify electrodynamics. In short, it does not seem possible to have a behaviour different from $1/r^2$ for a classical interaction.

An inverse square dependence implies a vanishing mass of light and light particles, the photons. Is the mass really zero? The issue has been extensively studied. A massive photon would lead to a wavelength dependence of the speed of light in vacuum, to deviations from the inverse square 'law', to deviations from Ampère's 'law', to the existence of longitudinal electromagnetic waves and more. No evidence for these effects has ever been found. A summary of these studies shows that the photon mass is below $10^{-53}$ kg, or maybe $10^{-63}$ kg. Some arguments are not universally accepted, thus the limit varies somewhat from researcher to researcher. Ref. 516

A small non-vanishing mass for the photon would change electrodynamics somewhat. The inclusion of a tiny mass poses no special problems, and the corresponding Lagrangian, the so-called *Proca Lagrangian*, has already been studied, just in case. Ref. 516

Strictly speaking, the photon mass cannot be said to vanish. In particular, a photon with a Compton wavelength of the radius of the visible universe cannot be distinguished from one with zero mass through any experiment. This gives a mass of $10^{-69}$ kg for the photon. One notes that the experimental limits are still much larger. Photons with such a small mass value would not invalidate electrodynamics as we know it.

* This can be deduced from special relativity from the reasoning of page 536 or from the formula in the footnote of page 324.

Interestingly, a non-zero mass of the photon implies the lack of magnetic monopoles, as the symmetry between electric and magnetic fields is broken. It is therefore important on the one hand to try to improve the experimental mass limit, and on the other hand to explore whether the limit due to the universe's size has any implications for this issue. This question is still open.

### The toughest challenge for electrodynamics

Electrodynamics faces an experimental and theoretical issue that physicist often avoid. The process of thought is electric in nature. Physics faces two challenges in this domain. First, physicists must find ways of modelling the thought process. Second, measurement technology must be extended to allow one to measure the currents in the brain.

Even though important research has been carried out in these domains, researchers are still far from a full understanding. Research using computer tomography has shown, for example, that the distinction between the conscious and the unconscious can be measured and that it has a biological basis. Psychological concepts such as repression can be observed in actual brain scans. Modellers of the brain mechanisms must thus learn to have the courage to take some of the concepts of psychology as descriptions for actual physical processes. This approach requires one to translate psychology into physical models, an approach that is still in its infancy.

Similarly, research into magnetoencephalography devices is making steady progress. The magnetic fields produced by brain currents are as low as 10 fT, which require sensors at liquid helium temperature and a good shielding of background noise. Also the spatial resolution of these systems needs to be improved.

The whole programme would be considered complete as soon as, in a distant future, it was possible to use sensitive measuring apparatus to detect what is going on inside the brain and to deduce or 'read' the thoughts of a person from these measurements. In fact, this challenge might be the most complex of all challenges that science is facing. Clearly, the experiment will require involved and expensive machinery, so that there is no danger for a misuse of the technique. It could also be that the spatial resolution required is beyond the abilities of technology. However, the understanding and modelling of the brain will be a useful technology in other aspects of daily life as well.*

## 14. What is light?

The nature of light has fascinated explorers of nature since at least the time of the ancient Greeks. In 1865, Maxwell summarized all data collected in the 2500 years before him by deducing a basic consequence of the equations of electrodynamics. He found that in the case of empty space, the equations of the electrodynamic field could be written as
Ref. 517

$$\Box\, \mathbf{A} = 0 \quad \text{or, equivalently} \quad \varepsilon_0 \mu_0 \frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = 0\ . \qquad (432)$$

* This vision, formulated here in 2005, is so far from realization that it is unclear whether it will come true in the twenty-first or in any subsequent century.

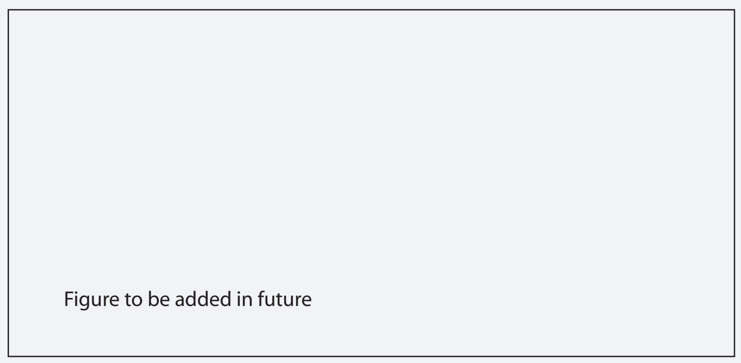


FIGURE 244 A plane, monochromatic and linearly polarized electromagnetic wave, with the fields as described by the field equations of electrodynamics

Challenge 991 e

This is called a *wave equation*, because it admits solutions of the type

$$\mathbf{A}(t,\mathbf{x}) = \mathbf{A}_0 \sin(\omega t - \mathbf{k}\mathbf{x} + \delta) = \mathbf{A}_0 \sin(2\pi f t - 2\pi \mathbf{x}/\lambda + \delta) \ , \qquad (433)$$

which are commonly called *plane waves*. Such a wave satisfies equation (432) for any value of *amplitude* $A_0$, of *phase* $\delta$, and of *angular frequency* $\omega$, provided the *wave vector* $\mathbf{k}$ satisfies the relation

$$\omega(\mathbf{k}) = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} k \quad \text{or} \quad \omega(\mathbf{k}) = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \sqrt{\mathbf{k}^2} \ . \qquad (434)$$

The relation $\omega(\mathbf{k})$ between the angular frequency and the wave vector, the so-called *dispersion relation*, is the main property of any type of wave, be it a sound wave, a water wave, an electromagnetic wave, or any other kind. Relation (434) specifically characterizes electromagnetic waves in empty space, and distinguishes them from all other types of waves.*

Equation (432) for the electromagnetic field is *linear* in the field; this means that the sum of two situations allowed by it is itself an allowed situation. Mathematically speaking, any *superposition* of two solutions is also a solution. For example, this means that two waves can cross each other without disturbing each other, and that waves can travel across static electromagnetic fields. Linearity also means that any electromagnetic wave can be described as a superposition of pure sine waves, each of which is described by expression (433). The simplest possible electromagnetic wave, a harmonic plane wave with linear polarization, is illustrated in Figure 244. Page 562

After Maxwell predicted the existence of electromagnetic waves, in the years between 1885 and 1889 Heinrich Hertz** discovered and studied them. He fabricated a very simple

Page 205

* For completeness, we remember that a *wave* in physics is any propagating imbalance.

** Heinrich Rudolf Hertz (b. 1857 Hamburg, d. 1894 Bonn), important Hamburger theoretical and experimental physicist. The unit of frequency is named after him. Despite his early death, Hertz was a central figure in the development of electromagnetism, in the explanation of Maxwell's theory and in the unfolding of radio communication technology. More about him on page 152.

FIGURE 245 The first transmitter (left) and receiver (right) of electromagnetic (micro-) waves

transmitter and receiver for 2 GHz waves. Waves around this frequency are used in the last generation of mobile phones. These waves are now called *radio waves*, since physicists tend to call all moving force fields *radiation*, recycling somewhat incorrectly a Greek term that originally meant 'light emission.'

Hertz also measured the speed of these waves. In fact, you can also measure the speed at home, with a chocolate bar and a kitchen microwave oven. A microwave oven emits radio waves at 2.5 GHz – not far from Hertz's value. Inside the oven, these waves form standing waves. Just put the chocolate bar (or a piece of cheese) in the oven and switch the power off as soon as melting begins. You will notice that the bar melts at regularly spaced spots. These spots are half a wavelength apart. From the measured wavelength value and the frequency, the speed of light and radio waves simply follows as the product of the two.

Heinrich Hertz

If you are not convinced, you can measure the speed directly, by telephoning a friend on another continent, if you can make sure of using a satellite line (choose a low cost provider). There is about half a second additional delay between the end of a sentence and the answer of the friend, compared with normal conversation. In this half second, the signal goes up to the geostationary satellite, down again and returns the same way. This half second gives a speed of $c \approx 4 \cdot 36\,000\,\mathrm{km}/0.5\,\mathrm{s} \approx 3 \cdot 10^5\,\mathrm{km/s}$, which is close to the precise value. Radio amateurs who reflect their signals from the Moon can perform the same experiment and achieve higher precision.

But Maxwell did more. He strengthened earlier predictions that *light* itself is a solution of equation (433) and therefore an electromagnetic wave, albeit with a much higher frequency. Let us see how we can check this.

It is easy to confirm the *wave* properties of light; indeed they were known already long before Maxwell. In fact, the first to suggest that light is a wave was, around the year 1678, the important physicist Christiaan Huygens. You can confirm that light is a wave with your own fingers. Simply place your hand one or two centimetres in front of your eye, look towards the sky through the gap between the middle and the index finger and

Page 78

FIGURE 246 The primary and secondary rainbow, and the supernumerary bows below the primary bow (© Antonio Martos and Wolfgang Hinz)

let the two fingers almost touch. You will see a number of dark lines crossing the gap. These lines are the interference pattern formed by the light behind the slit created by the fingers. *Interference* is the name given to the amplitude patterns that appear when several waves superpose.* The interference patterns depend on the spacing between the fingers. This experiment therefore allows you to estimate the wavelength of light, and thus, if you know its speed, its frequency. Can you do this? Challenge 993 n

Historically, another effect was central in convincing everybody that light was a wave: supernumerary rainbows, the additional bows below the main or primary rainbow. If we look carefully at a rainbow, below the main red–yellow–green–blue–violet bow, we observe weaker, additional green, blue and violet bows. Depending on the intensity of the rainbow, several of these supernumerary rainbows can be observed. They are due to an interference effect, as Thomas Young showed around 1803.** Indeed, the repetition distance of the supernumerary bows depends on the radius of the average water droplets that form them. (Details about the normal rainbows are given below.) Supernumerary rainbows were central in convincing people that light is a wave. It seems that in those times scientists either did not trust their own fingers, or did not have any. Ref. 518 Page 574

There are many other ways in which the wave character of light can be made apparent. Maybe the most beautiful is an experiment carried out by a team of Dutch physicists in 1990. They simply measured the light transmitted through a *slit* in a metal plate. It turns out that the transmitted intensity depends on the width of the slit. Their surprising result is shown in Figure 247. Can you explain the origin of the unexpected intensity steps in the curve? Ref. 519 Challenge 994 ny

* Where does the energy go in an interference pattern? Challenge 992 n

** Thomas Young (1773 Milverton–1829), read the bible at two, spoke Latin at four; a doctor of medicine, he became a professor of physics. He introduced the concept of *interference* into optics, explaining Newtonian rings and supernumerary rainbows; he was the first person to determine light's *wavelength*, a concept that he also introduced, and its dependence on colour. He was the first to deduce the three-colour vision explanation of the eye and, after reading of the discovery of polarization, explained light as a transverse wave. In short, Young discovered most of what people learn at secondary school about light. He was a universal talent: he also worked on the deciphering of hieroglyphs, on ship building and on engineering problems. Young collaborated with Fraunhofer and Fresnel. In Britain his ideas on light were not accepted, since Newton and his followers crushed all opposing views. Towards the end of his life, his results were finally made known to the physics community by Fresnel and Helmholtz.

Numerous other experiments on the creation, detection and measurement of electromagnetic waves were performed in the nineteenth and twentieth centuries. For example, in 1800, William Herschel discovered *infrared light* using a prism and a thermometer. (Can you guess how?) In 1801, Johann Wilhelm Ritter (1776–1810) a colourful figure of natural Romanticism, discovered *ultraviolet light* using silver chloride, AgCl, and again a prism. The result of all these experiments is that electromagnetic waves can be primarily distinguished by their wavelength or frequency. The main categories are listed in Table 48. For visible light, the wavelength lies between 0.4 μm (pure violet) and 0.8 μm (pure red).

Challenge 995 n

Page 564

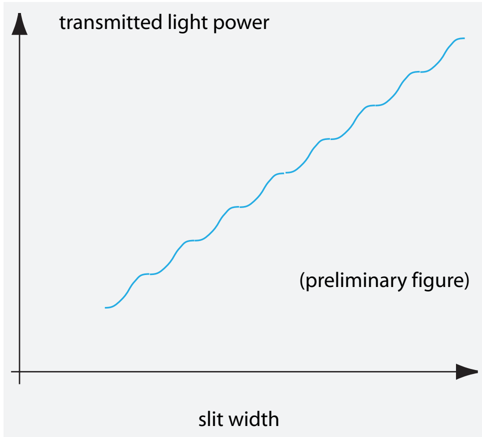


FIGURE 247 The light power transmitted through a slit as function of its width

At the end of the twentieth century the final confirmation of the wave character of light became possible. Using quite sophisticated experiments researchers, measured the oscillation frequency of light *directly*. The value, between 375 and 750 THz, is so high that its detection was impossible for a long time. But with these modern experiments the dispersion relation (434) of light has finally been confirmed in all its details.

Ref. 520

Ref. 521

We are left with one additional question about light. If light oscillates, in which direction does this occur? The answer is hidden in the parameter $\mathbf{A}_0$ in expression (433). Electromagnetic waves oscillate in directions *perpendicular* to their motion. Therefore, even for identical frequency and phase, waves can still differ: they can have different *polarization* directions. For example, the polarization of radio transmitters determines whether radio antennas of receivers have to be kept horizontal or vertical. Also for light, polarization is easily achieved, e.g. by shining it through a stretched plastic film. When the polarization of light was discovered in 1808 by the French physicist Louis Malus (1775–1812), it definitively established the wave nature of light. Malus discovered it when he looked at the strange double images produced by feldspar, a transparent crystal found in many minerals. Feldspar ($KAlSi_3O_8$) splits light beams into two – it is *birefringent* – and polarizes them differently. That is the reason that feldspar is part of every crystal collection. Calcite ($CaCO_3$) shows the same effect. If you ever get hold of a piece of feldspar or transparent calcite, do look through it at some written text.

By the way, the human eye is unable to detect polarization, in contrast to the eyes of many insects, spiders and certain birds. Honey bees use polarization to deduce the position of the Sun, even when it is hidden behind clouds. Some beetles of the genus *Scarabeus* use the polarization of the Moon light for navigation, and many insects use polarization to distinguish water surfaces from mirages. Can you find out how? Despite the human inability to detect polarization, both the cornea and the lens of the human eye are birefringent.

Ref. 522

Challenge 996 n

Ref. 523

Note that all possible polarizations of light form a continuous set. However, a general plane wave can be seen as the superposition of two orthogonal, linearly polarized waves with different amplitudes and different phases. Most books show pictures of plane, linear-

ized electrodynamic waves. Essentially, electric fields look like water waves generalized to three dimensions, the same for magnetic fields, and the two are perpendicular to each
Challenge 997 ny other. Can you confirm this?

Interestingly, a generally polarized plane wave can also be seen as the superposition of right and left *circularly polarized waves*. However, no illustrations of circularly polarized
Challenge 998 n waves are found in any textbook. Can you explain why?

So far it is clear that light is a wave. To confirm that light waves are indeed *electromagnetic* is more difficult. The first argument was given by Bernhard Riemann in 1858;* he deduced that any electromagnetic wave must propagate with a speed $c$ given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} . \tag{435}$$

Already ten years before him, in 1848, Kirchoff had noted that the measured values on both sides agreed within measurement errors. A few years later, Maxwell gave a beautiful confirmation by deducing the expression from equation (434). You should be able to re-
Challenge 999 ny peat the feat. Note that the right-hand side contains electric and magnetic quantities, and the left-hand side is an optical entity. Riemann's expression thus unifies electromagnetism and optics.

Of course, people were not yet completely convinced. They looked for more ways to show that light is electromagnetic in nature. Now, since the evolution equations of the electrodynamic field are linear, additional electric or magnetic fields alone do not influence
Challenge 1000 e the motion of light. On the other hand, we know that electromagnetic waves are emitted only by accelerated charges, and that all light is emitted from matter. It thus follows that matter is full of electromagnetic fields and accelerated electric charges. This in turn implies that the influence of matter on light can be understood from its internal electromagnetic fields and, in particular, that subjecting matter to an *external* electromagnetic field should change the light it emits, the way matter interacts with light, or generally, the material properties as a whole.

Searching for effects of electricity and magnetism on matter has been a main effort of physicists for over a hundred years. For example, electric fields influence the light transmission of oil, an effect discovered by John Kerr in 1875.** The discovery that certain gases change colour when subject to a field yielded several Nobel Prizes for physics. With time, many more influences on light-related properties by matter subjected to fields were found. An extensive list is given in the table on page 605. It turns out that apart from a few exceptions the effects can *all* be described by the electromagnetic Lagrangian (425),
Page 553 or equivalently, by Maxwell's equations (428). In summary, classical electrodynamics indeed unifies the description of electricity, magnetism and optics; all phenomena in these fields, from the rainbow to radio and from lightning to electric motors, are found to be different aspects of the evolution of the electromagnetic field.

* Bernhard Riemann (b. 1826 Breselenz, d. 1866 Selasca), important German mathematician. He studied curved space, providing several of the mathematical and conceptual foundations of general relativity, but then died at an early age.
** John Kerr (1824–1907), Scottish physicist, friend and collaborator of William Thomson.

TABLE 48 The electromagnetic spectrum

| Frequency | Wavelength | Name | Main properties | Appearance | Use |
|---|---|---|---|---|---|
| $3\cdot 10^{-18}$ Hz | $10^{26}$ m | **lower frequency limit** | | see the section on cosmology | |
| < 10 Hz | > 30 Mm | **quasistatic fields** | | intergalactic, galactic, stellar and planetary fields, brain, electrical fish | power transmission, accelerating and deflecting cosmic radiation |
| | | **radio waves** | | electronic devices | |
| 10 Hz–50 kHz | 30 Mm–6 km | ELW | go round the globe, penetrate into water, penetrate metal | nerve cells, electromechanical devices | power transmission, communication through metal walls, communication with submarines http://www.vlf.it |
| 50 – 500 kHz | 6 km–0.6 km | LW | follow Earth's curvature, felt by nerves ('bad weather nerves') | emitted by thunderstorms | radio communications, telegraphy, inductive heating |
| 500 – 1500 kHz | 600 m–200 m | MW | reflected by night sky | | radio |
| 1.5 – 30 MHz | 200 m–10 m | SW | circle world if reflected by the ionosphere, destroy hot air balloons | emitted by stars | radio transmissions, radio amateurs, spying |
| 15 – 150 MHz | 20 m–2 m | VHF | allow battery operated transmitters | emitted by Jupiter | remote controls, closed networks, tv, radio amateurs, radio navigation, military, police, taxi |
| 150 – 1500 MHz | 2 m–0.2 m | UHF | *idem*, line of sight propagation | | radio, walkie-talkies, tv, mobile phones, internet via cable, satellite communication, bicycle speedometers |
| | | **microwaves** | | | |
| 1.5 – 15 GHz | 20 cm–2 cm | SHF | *idem*, absorbed by water | night sky, emitted by hydrogen atoms | radio astronomy, used for cooking (2.45 GHz), telecommunications, radar |
| 15 – 150 GHz | 20 mm–2 mm | EHF | *idem*, absorbed by water | | |

| Frequency | Wavelength | Name | Main properties | Appearance | Use |
|---|---|---|---|---|---|
| | | **infrared** | go through clouds | emitted by every warm object | satellite photography of Earth, astronomy |
| 0.3 – 100 THz | 1000 –3 μm | IRC or far infrared | | sunlight, living beings | seeing through clothes, envelopes and teeth |
| 100 – 210 THz | 3 μm– 1.4 μm | IRB or medium infrared | | sunlight | used for optical fibre communications for telephone and cable TV |
| 210 – 385 THz | 1400– 780 nm | IRA or near infrared | penetrates for several cm into human skin | sunlight, radiation from hot bodies | healing of wounds, rheumatism, sport physiotherapy, hidden illumination |
| 375 – 750 THz | 800– 400 nm | **light** | not absorbed by air, detected by the eye (up to 850 nm at sufficient power) | heat ('hot light'), lasers & chemical reactions e.g. phosphor oxidation, fireflies ('cold light') | definition of straightness, enhancing photosynthesis in agriculture, photodynamic therapy, hyperbilirubinaemia treatment |
| 375 – 478 THz | 780–627 nm | red | penetrate flesh | blood | alarm signal, used for breast imaging |
| | 700 nm | pure red | | rainbow | colour reference for printing, painting, illumination and displays |
| 478 – 509 THz | 627–589 nm | orange | | various fruit | attracts birds and insects |
| | 600 nm | standard orange | | | |
| 509 – 530 THz | 589–566 nm | yellow | | majority of flowers | *idem*; best background for reading black text |
| | 580 nm | standard yellow | | | |
| 530 – 606 THz | 566– 495 nm | green | maximum eye sensitivity | algae and plants | highest brightness per light energy for the human eye |
| | 546.1 nm | pure green | | rainbow | colour reference |
| 606 – 688 THz | 495–436 nm | blue | | sky, gems, water | |
| | 488 nm | standard cyan | | | |

| Frequency | Wavelength | Name | Main properties | Appearance | Use |
|---|---|---|---|---|---|
| | 435.8 nm | pure blue | | rainbow | colour reference |
| 688 – 789 THz | 436–380 nm | indigo, violet | | flowers, gems | |
| | | **ultraviolet** | | | |
| 789 – 952 THz | 380–315 nm | UVA | penetrate 1 mm into skin, darken it, produce vitamin D, suppress immune system, cause skin cancer, destroy eye lens | emitted by Sun and stars | seen by certain birds, integrated circuit fabrication |
| 0.95 – 1.07 PHz | 315–280 nm | UVB | *idem*, destroy DNA, cause skin cancer | *idem* | *idem* |
| 1.07 – 3.0 PHz | 280–100 nm | UVC | form oxygen radicals from air, kill bacteria, penetrate 10 μm into skin | *idem* | disinfection, water purification, waste disposal, integrated circuit fabrication |
| 3 –24 PHz | 100–13 nm | EUV | | | sky maps, silicon lithography |
| | | **X-rays** | penetrate materials | emitted by stars, plasmas and black holes | imaging human tissue |
| 24 – 240 PHz | 13–1.3 nm | soft X-rays | *idem* | synchrotron radiation | *idem* |
| > 240 PHz or > 1 keV | < 1.2 nm | hard X-rays | *idem* | emitted when fast electrons hit matter | crystallography, structure determination |
| > 12 EHz or > 50 keV | < 24 pm | **$\gamma$-rays** | *idem* | radioactivity, cosmic rays | chemical analysis, disinfection, astronomy |
| $2 \cdot 10^{43}$ Hz | $\approx 10^{-35}$ m | **Planck limit** | | see part three of this text | |

## The slowness of progress in physics

The well-known expression

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (436)$$

for the speed of light is so strange that one should be astonished when one sees it. Something essential is missing.

Indeed, the speed is *independent* of the proper motion of the observer measuring the electromagnetic field. In other words, the speed of light is independent of the speed of the lamp and independent of the speed of the observer. All this is contained in expression (436). Incredibly, for five decades, *nobody* explored this strange result. In this way, the theory of relativity remained undiscovered from 1848 to 1905. As in so many other cases, the progress of physics was much slower than necessary.

The constancy of the speed of light is the essential point that distinguishes special relativity from Galilean physics. In this sense, any electromagnetic device, making use of expression (436), is a working proof of special relativity.

## How does the world look when riding on a light beam?

At the end of the nineteenth century, the teenager Albert Einstein read a book discussing the speed of light. The book asked what would happen if an observer moved at the same speed as light.* Einstein thought much about the issue, and in particular, asked himself what kind of electromagnetic field he would observe in that case. Einstein later explained that this Gedanken experiment convinced him already at that young age that *nothing* could travel at the speed of light, since the field observed would have a property not found
Challenge 1001 n in nature. Can you find out which one he meant?

Riding on a light beam situation would have strange consequences.

— You would have no mirror image, like a vampire.
— Light would not be oscillating, but would be a static field.
— Nothing would move, like in the tale of sleeping beauty.

But also at speeds *near* the velocity of light observations would be interesting. You would:

— see a lot of light coming towards you and almost no light from the sides or from behind; the sky would be blue/white in the front and red/black behind;
— observe that everything around happens very very slowly;
— experience the smallest dust particle as a deadly bullet.

Challenge 1002 n Can you think of more strange consequences? It is rather reassuring that our planet moves rather slowly through its environment.

## Does light travel in a straight line?

Usually light moves in straight lines. Indeed, we even use light to *define* 'straightness.'
Ref. 524 However, there are a number of exceptions that every expert on motion should know.

In sugar syrup, light beams curve, as shown in Figure 248. In fact, light beams bend at any material interface. This effect, called *refraction*, also changes the appearance of the shape of our feet when we are in the bath tub and makes aquaria seem less deep than they actually are. Refraction is a consequence of the change of light speed from material

* This was the book series in twenty volumes by Aaron Bernstein, *Naturwissenschaftliche Volksbücher*, Duncker, 1873-1874. The young Einstein read them, between 1892 and 1894, with 'breathless attention', as he wrote later on.

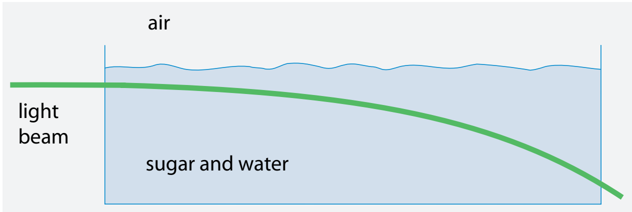


FIGURE 248 Sugar water bends light

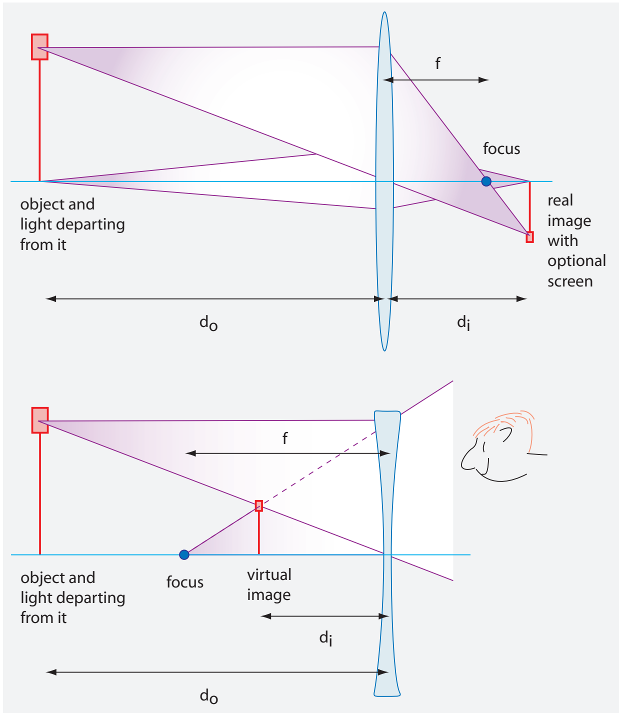


FIGURE 249 The real image produced by a converging lens and the virtual image produced by a diverging lens

Challenge 1003 n to material. Are you able to explain refraction, and thus explain the syrup effect?

Refraction is chiefly used in the design of lenses. Using glass instead of water, one can produce curved surfaces, so that light can be *focused*. Focusing devices can be used to produce images. The two main types of lenses, with their focal points and the images they produce, are shown in Figure 249. When an object is put between a converging lens and its focus, works as a *magnifying glass*. It also produces a *real* image, i.e., an image that can be projected onto a screen. In all other cases lenses produce so-called *virtual images*: such images can be seen with the eye but not be projected onto a screen. Figure 249 also allows one to deduce the *thin lens formula* that connects the lengths $d_o$, $d_o$ and $f$. What

Challenge 1004 n is it?

Even though glasses and lenses have been known since antiquity, the Middle Ages had to pass by before two lenses were combined to make more elaborate optical instruments. The *telescope* was invented in, or just before, 1608 by the German–Dutch lens grinder

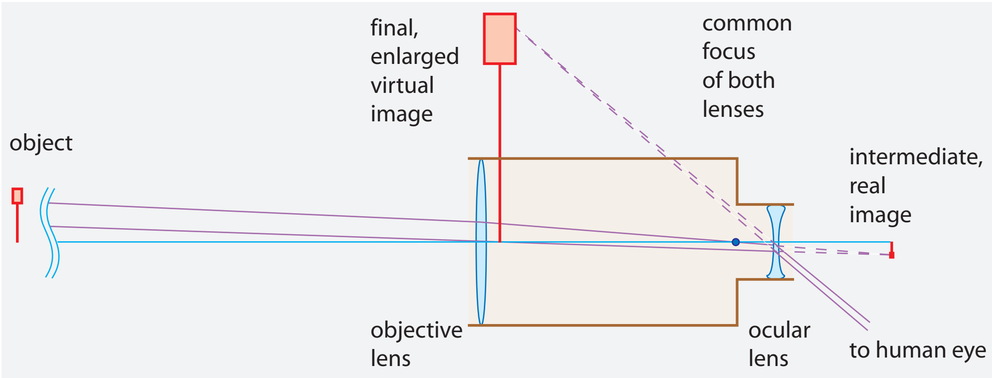


FIGURE 250 Refraction as the basis of the telescope – shown here in the original Dutch design

Johannes Lipperhey (*c.* 1570–1619), who made a fortune by selling it to the Dutch military. When Galileo heard about the discovery, he quickly took it over and improved it. In 1609, Galileo performed the first astronomical observations; they made him world-famous. The Dutch telescope design has a short tube yielding a bright and upright image. It is still used today in opera glasses. Many other ways of building telescopes have been developed over the years. *

Another way to combine two lenses leads to the *microscope*. Can you explain to a non-physicist how a microscope works?** Werner Heisenberg almost failed his Ph.D. exam because he could not. The problem is not difficult, though. Indeed, the inventor of the microscope was an autodidact of the seventeenth century: the Dutch technician Antoni van Leeuwenhoek (1632–1723) made a living by selling over five hundred of his microscopes to his contemporaries.

Challenge 1005 n

No ray tracing diagram, be it that of a simple lens, of a telescope or a microscope, is really complete if the eye, with its lens and retina, is missing. Can you add it and convince yourself that these devices really work?

Challenge 1006 ny

Refraction is often colour-dependent. For that reason, microscopes or photographic cameras have several lenses, made of different materials. They compensate the colour effects, which otherwise yield coloured image borders. The colour dependence of refraction in water droplets is also the basis of the rainbow, as shown below, and refraction in ice crystals in the atmosphere is at the basis of the halos and the many other light patterns often seen around the Sun and the Moon.

Page 574
Ref. 525

Also the lens in the human eye has colour-dependent diffraction. This is effect is not corrected in the eye, but in the brain. The dispersion of the eye lens can be noticed if this correction is made impossible, for example when red or blue letters are printed on a black

* A fascinating overview about what people have achieved in this domain up to now is given by Peter Manly, *Unusual Telescopes*, Cambridge University Press, 1991. Images can also be made with mirrors. Since mirrors are cheaper and more easy to fabricate with high precision, most large telescopes have a mirror instead of the first lens.

By the way, telescopes also exist in nature. Many spiders have two types of eyes. The large ones, made to see far away, have two lenses arranged in the same way as in the telescope.

** If not, read the beautiful text by Elizabeth M. Slater & Henry S. Slater, *Light and Electron Microscopy*, Cambridge University Press, 1993.

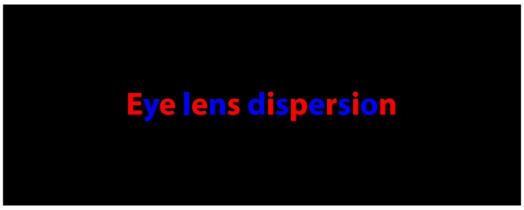


FIGURE 251 Watching this graphic at higher magnification shows the dispersion of the lens in the human eye: the letters float at different depths


FIGURE 252 In certain materials, light beams can spiral around each other

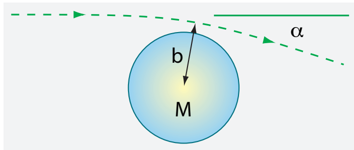


FIGURE 253 Masses bend light

background, as shown in Figure 251. One gets the impression that the red letters float in front of the blue letters. Can you show how dispersion leads to the floating effect? Challenge 1007 n

A second important observation is that light goes around corners, and the more so the sharper they are. This effect is called *diffraction*. In fact, light goes around corners in the same way that sound does. Diffraction is due to the wave nature of light (and sound). You probably remember the topic from secondary school.

Because of diffraction, it is impossible to produce strictly parallel light beams. For example, every laser beam diverges by a certain minimum amount, called the *diffraction limit*. Maybe you know that the world's most expensive Cat's-eyes are on the Moon, where they have been deposited by the Lunakhod and the Apollo missions. Ref. 526 Can you determine how wide a laser beam with minimum divergence has become when it arrives at the Moon and returns back to Earth, assuming that it was 1 m wide when it left Earth? How wide would it be when it came back if it had been 1 mm wide at the start? Challenge 1008 n

Diffraction implies that there are no perfectly sharp images: there exists a *limit on resolution*. This is true for every optical instrument, including the eye. The resolution of the eye is between one and two minutes of arc, i.e. between 0.3 and 0.6 mrad. The limit is due to the finite size of the pupil. Therefore, for example, there is a maximum distance at which humans can distinguish the two headlights of a car. Can you estimate it? Challenge 1009 n

Resolution limits also make it impossible to see the Great Wall in northern China from the Moon, contrary to what is often claimed. In the few parts that are not yet in ruins, the wall is about 6 metres wide, and even if it casts a wide shadow during the morning or the evening, the angle it subtends is way below a second of arc, so that it is completely invisible to the human eye. In fact, three different cosmonauts who travelled to the Moon performed careful searches and confirmed that the claim is absurd. Ref. 527 The story is one of the most tenacious urban legends. (Is it possible to see the Wall from the space shuttle?) Challenge 1010 ny The largest human-made objects are the polders of reclaimed land in the Netherlands; they *are* visible from outer space. So are most large cities as well as the highways in Belgium at night; their bright illumination makes them stand out clearly from the dark side of the

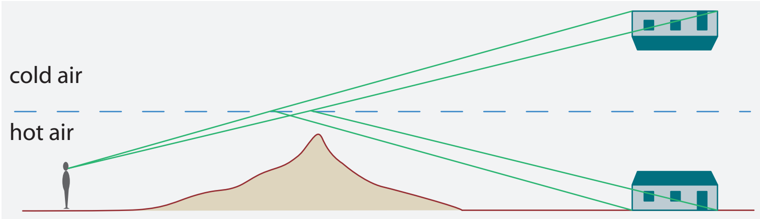


FIGURE 254 Reflection at air interfaces is the basis of the Fata Morgana

Earth.

Diffraction also means that behind a small disc illuminated along its axis, the centre of the shadow shows, against all expectations, a bright spot. This 'hole' in the shadow was predicted in 1819 by Denis Poisson (1781–1840) in order to show to what absurd consequences the wave theory of light would lead. He had just read the mathematical description of diffraction developed by Augustin Fresnel* on the basis of the wave description of light. But shortly afterwards, François Arago (1786–1853) actually observed Poisson's point, converting Poisson, making Fresnel famous and starting the general acceptance of the wave properties of light.

Additional electromagnetic fields usually do not influence light directly, since light has no charge and since Maxwell's equations are linear. But in some materials the equations are non-linear, and the story changes. For example, in certain photorefractive materials, two nearby light beams can even *twist* around each other, as was shown by Segev and Ref. 528 coworkers in 1997.

A final way to bend light is gravity, as discussed already in the chapters on universal gravity and on general relativity. The effect of gravity between two light beams was also Page 475 discussed there.

In summary, light travels straight only if it travels *far from other matter*. In everyday life, 'far' simply means more than a few millimetres, because all gravitational and electromagnetic effects are negligible at these distances, mainly due to lights' truly supersonic speed.

## The concentration of light

If one builds a large lens or a curved mirror, one can collect the light of the Sun and focus it on a single spot. Everybody has used a converging lens as a child to burn black spots on newspapers in this way. In Spain, wealthier researchers have even built a curved mirror as large as a house, in order to study solar energy use and material behaviour at high temperature. Essentially, the mirror provides a cheap way to fire an oven. Indeed, 'focus' is the Latin word for 'oven'.

Kids find out quite rapidly that large lenses allow them to burn things more easily than small ones. It is obvious that the Spanish site is the record holder in this game. However, building a larger mirror does not make sense. Whatever its size may be, such a set-up

* Augustin Jean Fresnel (1788–1827), engineer and part time physicist; he published in 1818 his great paper on wave theory for which he got the prize of the French Academy of Sciences in 1819. To improve his finances, he worked in the commission responsible for lighthouses, for which he developed the well-known Fresnel lens. He died prematurely, partly of exhaustion due to overwork.

cannot reach a higher temperature than that of the original light source. The surface temperature of the Sun is about 5800 K; indeed, the highest temperature reached so far is about 4000 K. Are you able to show that this limitation follows from the second law of thermodynamics?

Challenge 1011 ny

In short, nature provides a *limit* to the concentration of light energy. In fact, we will encounter additional limits in the course of our exploration.

FIGURE 255 The last mirror of the solar furnace at Odeillo, in the French Pyrenees (© Gerhard Weinrebe)

## Can one touch light?

If a little glass bead is put on top of a powerful laser, the bead remains suspended in mid-air, as shown in Figure 256.* This means that light has momentum. Therefore, contrary to what we said in the beginning of our mountain ascent, images *can* be touched! In fact, the ease with which objects can be pushed even has a special name. For stars, it is called the *albedo*, and for general objects it is called the *reflectivity*, abbreviated as $r$.

Like each type of electromagnetic field, and like every kind of wave, light carries energy; the energy flow $T$ per surface and time is

Challenge 1012 e

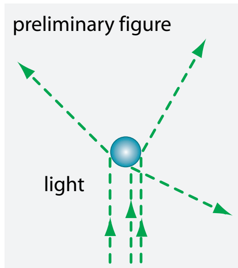


FIGURE 256 Levitating a small glass bead with a laser

$$\mathbf{T} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \quad \text{giving an average} \quad \langle T\rangle = \frac{1}{2\mu_0}E_{\max}B_{\max}\ . \qquad (437)$$

Obviously, light also has a momentum $P$. It is related to the energy $E$ by

$$P = \frac{E}{c}\ . \qquad (438)$$

Challenge 1013 e

As a result, the pressure $p$ exerted by light on a body is given by

$$p = \frac{T}{c}(1+r) \qquad (439)$$

where for black bodies we have that a reflectivity $r = 0$ and for mirrors $r = 1$; other bodies have values in between. What is your guess for the amount of pressure due to sunlight on a black surface of one square metre? Is this the reason that we feel more pressure during the day than during the night?

Challenge 1014 n

In fact, rather delicate equipment is needed to detect the momentum of light, in other words, its radiation pressure. Already around 1610, Johannes Kepler had suggested in *De cometis* that the tails of comets exist only because the light of the Sun hits the small dust

* The heaviest object that has been levitated with a laser had a mass of 20 g; the laser used was enormous, and the method also made use of a few additional effects, such as shock waves, to keep the object in the air.

particles that detach from it. For this reason, the tail always points *away* from the Sun, as
Challenge 1015 e you might want to check at the next opportunity. Today, we know that Kepler was right; but proving the hypothesis is not easy.

In 1873, William Crookes * invented the *light mill radiometer*. He had the intention of demonstrating the radiation pressure of light. The light mill consists of four thin plates, black on one side and shiny on the other, that are mounted on a vertical axis, as shown in Figure 257. However, when Crookes finished building it – it was similar to those sold in shops today – he found, like everybody else, that it turned in the wrong direction, namely with the shiny side towards
Challenge 1016 n the light! (Why is it wrong?) You can check it by yourself by shining a laser pointer on to it. The behaviour has been a puzzle for quite some time. Explaining it involves the tiny amount of gas left over in the glass bulb and takes us too far from
Ref. 529 the topic of our mountain ascent. It was only in 1901, with the advent of much better pumps, that the Russian physicist Peter/Pyotr Lebedev managed to create a sufficiently good vacuum to al-
Ref. 530 low him to measure the light pressure with such an improved, true radiometer. Lebedev also confirmed the predicted value of the light pressure and proved the correctness of Kepler's hypothesis. Today it is even possible to build tiny propellers that start to turn
Ref. 531 when light shines on to them, in exactly the same way that the wind turns windmills.

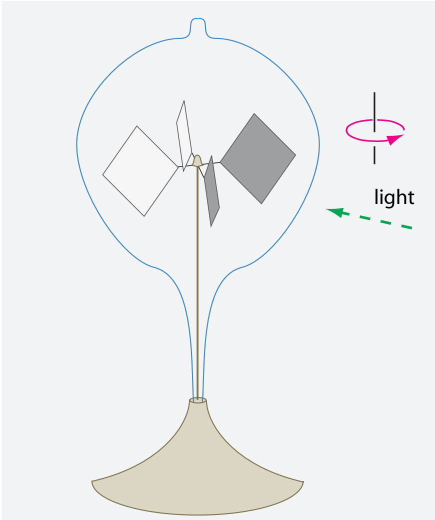

FIGURE 257 A commercial light mill turns *against* the light

But light cannot only touch and be touched, it can also *grab*. In the 1980s, Arthur Ashkin and his research group developed actual *optical tweezers* that allow one to grab,
Ref. 532 suspend and move small transparent spheres of 1 to 20 μm diameter using laser beams. It is possible to do this through a microscope, so that one can also observe at the same time what is happening. This technique is now routinely used in biological research around the world, and has been used, for example, to measure the force of single muscle fibres, by chemically attaching their ends to glass or Teflon spheres and then pulling them apart with such optical tweezers.

But that is not all. In the last decade of the twentieth century, several groups even
Ref. 532 managed to *rotate* objects, thus realizing actual *optical spanners*. They are able to rotate particles at will in one direction or the other, by changing the optical properties of the laser beam used to trap the particle.

In fact, it does not take much to deduce that if light has linear momentum, circularly polarized light also has *angular* momentum. In fact, for such a wave the angular momentum L is given by

$$L = \frac{E_{\text{nergy}}}{\omega} \;. \qquad (440)$$

* William Crookes (b. 1832 London, d. 1919 London), English chemist and physicist, president of the Royal Society, discoverer of thallium.

Challenge 1017 e
Ref. 533
Challenge 1018 ny

Equivalently, the angular momentum of a wave is $\lambda/2\pi$ times its linear momentum. For light, this result was already confirmed in the early twentieth century: a light beam can put certain materials (which ones?) into rotation, as shown in Figure 258. Of course, the whole thing works even better with a laser beam. In the 1960s, a beautiful demonstration was performed with microwaves. A circularly polarized microwave beam from a maser – the microwave equivalent of a laser – can put a metal piece absorbing it into rotation. Indeed, for a beam with cylindrical symmetry, depending on the sense of rotation, the angular momentum is either parallel or antiparallel to the direction of propagation. All these experiments confirm that light also carries angular momentum, an effect which will play an important role in the second part of our mountain ascent.

We note that not for all waves angular momentum is energy per angular frequency. This is only the case for waves made of what in quantum theory will be called spin 1 particles. For example, for gravity waves the angular momentum is *twice* this value, and they are therefore expected to be made of spin 2 particles.

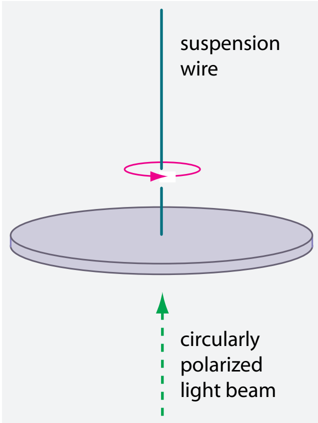

FIGURE 258 Light can rotate objects

In summary, light can touch and be touched. Obviously, if light can rotate bodies, it can also *be* rotated. Could you imagine how this can be achieved?

Challenge 1019 n

## War, light and lies

From the tiny effects of equation (439) for light pressure we deduce that light is not an efficient tool for hitting objects. On the other hand, light is able to heat up objects, as we can feel when the skin is touched by a laser beam of about 100 mW or more. For the same reason even cheap laser pointers are dangerous to the eye.

In the 1980s, and again in 2001, a group of people who had read too many science fiction novels managed to persuade the military – who also indulge in this habit – that lasers could be used to shoot down missiles, and that a lot of tax money should be spent on developing such lasers. Using the definition of the Poynting vector and a hitting time of about 0.1 s, are you able to estimate the weight and size of the battery necessary for such a device to work? What would happen in cloudy or rainy weather?

Challenge 1020 ny

Other people tried to persuade NASA to study the possibility of propelling a rocket using emitted light instead of ejected gas. Are you able to estimate whether this is feasible?

Challenge 1021 ny

## What is colour?

We saw that radio waves of certain frequencies are visible. Within that range, different frequencies correspond to different colours. (Are you able to convince a friend about this?) But the story does not finish here. Numerous colours can be produced either by a single wavelength, i.e. by *monochromatic* light, or by a *mixture* of several different colours. For example, standard yellow can be, if it is pure, an electromagnetic beam of 600 nm wavelength or it can be a mixture of standard green of 546.1 nm and standard red of 700 nm. The eye cannot distinguish between the two cases; only spectrometers can. In everyday life, all colours turn out to be mixed, with the exceptions of those of yellow

Challenge 1022 n

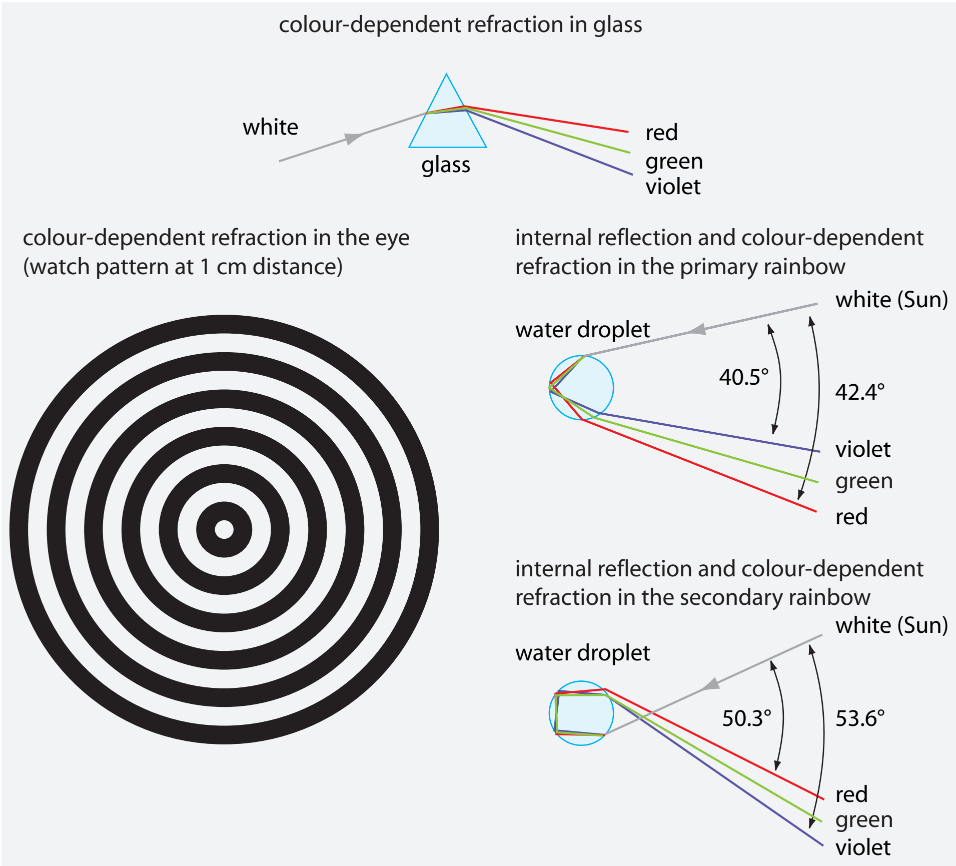


FIGURE 260 Proving that white light is a mixture of colours

street lamps, of laser beams and of the rainbow. You can check this for yourself, using an umbrella or a compact disc: they decompose light mixtures, but not pure colours.

Challenge 1023 e

In particular, white light is a mixture of a continuous range of colours with a given intensity per wavelength. To check that white light is a mixture of colours, simply hold the left-hand side of Figure 260 so near to your eye that you cannot focus the stripes any more. The unsharp borders of the white stripes have a pink or a green shade. These colours are due to the imperfections of the lens in the human eye, its so-called *chromatic aberrations*. Aberrations have the consequence that not all light frequencies follow the same path through the lens of the eye, and therefore they hit the retina at different spots. This is the same effect that occurs in prisms or in water drops showing a rainbow. By the way, the shape of the rainbow tells something about the shape of the water droplets. Can you deduce the connection?

Challenge 1024 n

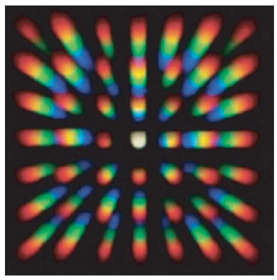

FIGURE 259 Umbrellas decompose white light

The right side of Figure 260 explains how rainbows form. The main idea is that internal reflection inside the water droplets in the sky is responsible for throwing back the light coming from the Sun, whereas the wavelength-dependent refraction at the air–water sur-

face is responsible for the different paths of each colour. The first two persons to verify this explanation were Theodoricus Teutonicus de Vriberg (*c.* 1240 to *c.* 1318), in the years from
Ref. 534 1304 to 1310 and, at the same time, the Persian mathematician Kamal al-Din al-Farisi. To check the explanation, they did something smart and simple; anybody can repeat this at
Challenge 1025 e home. They built an enlarged water droplet by filling a thin spherical (or cylindrical) glass container with water; then they shone a beam of white light through it. Theodoricus and al-Farisi found exactly what is shown in Figure 260. With this experiment, each of them was able to reproduce the angle of the main or primary rainbow, its colour sequence, as well as the existence of a secondary rainbow, its observed angle and its inverted colour
Page 561 sequence.* All these bows are visible in Figure 246. Theodoricus's beautiful experiment is sometimes called the most important contribution of natural science in the Middle Ages.

Even pure air splits white light. This is the reason that the sky and far away mountains look blue or that the Sun looks red at sunset and at sunrise. (The sky looks black even during the day from the Moon.) You can repeat this effect
Challenge 1027 e by looking through water at a black surface or at a lamp. Adding a few drops of milk to the water makes the lamp yellow and then red, and makes the black surface blue (like the sky seen from the Earth as compared to the sky seen from the Moon). More milk increases the effect. For the same reason, sunsets are especially red after volcanic eruptions.

FIGURE 261 Milk and water simulate the evening sky (© Antonio Martos)

Incidentally, at sunset the atmosphere itself also acts as a prism; that means that the Sun is split into different images, one for each colour, which are slightly shifted with respect to each other, a bit like a giant rainbow in which not only the rim, but the whole disc is coloured. The total shift is about 1/60th of the diameter. If the weather is favourable and if the air is clear and quiet up to and beyond the horizon, for a few seconds it is possible to see, after the red, orange and yellow images of the setting Sun, the rim of the green–blue image.
Ref. 536 This is the famous 'rayon vert' described by Jules Verne in his novel of the same title. It is often seen on islands, for example in Hawaii.**

To clarify the difference between colours in physics and colour in human perception and language, a famous linguistic discovery deserves to be mentioned: colours in human language have a natural *order*. Colours are ordered by all peoples of the world, whether they come from the sea, the desert or the mountains, in the following order: 1st black and white, 2nd red, 3rd green and yellow, 4th blue, 5th brown; 6th come mauve, pink, orange, grey and sometimes a twelfth term that differs from language to language. (Colours that refer to objects, such as aubergine or sepia, or colours that are not generally applicable,

Challenge 1026 ny * Can you guess where the tertiary and quaternary rainbows are to be seen? There are rare reported sightings of them. The hunt to observe the fifth-order rainbow is still open. (In the laboratory, bows around droplets up
Ref. 535 to the 13th order have been observed.) For more details, see the beautiful website at http://www.sundog.clara.co.uk/atoptics/phenom.htm. There are several formulae for the angles of the various orders of rainbows; they follow from straightforward geometric considerations, but are too involved to be given here.

** For this and many other topics on colours in nature, such as, for example, the halos around the Moon and the Sun or the colour of shadows,, see the beautiful book by Marcel Minnaert mentioned on page 74.

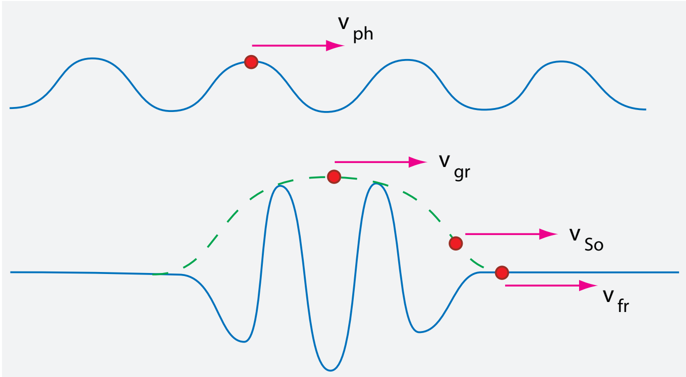


FIGURE 262 The definition of important velocities in wave phenomena

such as blond, are excluded in this discussion.) The precise discovery is the following: if a particular language has a word for any of these colours, then it also has a word for all the preceding ones. The result also implies that people use these basic colour classes even if their language does *not* have a word for each of them. These strong statements have been confirmed for over 100 languages. Ref. 537

## What is the speed of light? – Again

Physics talks about motion. Talking is the exchange of sound; and sound is an example of a signal. A *(physical) signal* is the transport of information using the transport of energy. There are no signals without a motion of energy. Indeed, there is no way to store information without storing energy. To any signal we can thus ascribe a propagation speed. The highest possible signal speed is also the maximal velocity of the general influences, or, to use sloppy language, the maximal velocity with which effects spread causes.

If the signal is carried by matter, such as by the written text in a letter, the signal velocity is then the velocity of the material carrier, and experiments show that it is limited by the speed of light.

For a wave carrier, such as water waves, sound, light or radio waves, the situation is less evident. What is the speed of a wave? The first answer that comes to mind is the speed with which wave crests of a sine wave move. This already introduced *phase velocity* is given by the ratio between the frequency and the wavelength of a monochromatic wave, i.e. by

$$v_{ph} = \frac{\omega}{k} . \quad (441)$$

For example, the phase velocity determines interference phenomena. Light in a vacuum has the same phase velocity $v_{ph} = c$ for all frequencies. Are you able to imagine an experiment to test this to high precision? Challenge 1028 n

On the other hand, there are cases where the phase velocity is greater than $c$, most notably when light travels through an absorbing substance, and when at the same time the frequency is near to an absorption maximum. In these cases, experiments show that the phase velocity is *not* the signal velocity. For such situations, a better approximation Ref. 538

to the signal speed is the *group velocity*, i.e. the velocity at which a group maximum will travel. This velocity is given by

$$v_{gr} = \left.\frac{d\omega}{dk}\right|_{k_0} , \qquad (442)$$

where $k_0$ is the central wavelength of the wave packet. We observe that $\omega = c(k)k = 2\pi v_{ph}/\lambda$ implies the relation

$$v_{gr} = \left.\frac{d\omega}{dk}\right|_{k_0} = v_{ph} - \lambda \frac{dv_{ph}}{d\lambda} . \qquad (443)$$

This means that the sign of the last term determines whether the group velocity is larger or smaller than the phase velocity. For a travelling group, as shown by the dashed line in Figure 262, this means that new maxima appear either at the end or at the front of the group. Experiments show that this is only the case for light passing *through matter*; for light *in vacuum*, the group velocity has the same value $v_{gr} = c$ for all values of the wave vector $k$.

You should be warned that many publications are still propagating the incorrect statement that the group velocity *in a material* is never greater than $c$, the speed of light in vacuum. Actually, the group velocity in a material can be zero, infinite or even negative; this happens when the light pulse is very narrow, i.e. when it includes a wide range of frequencies, or again when the frequency is near an absorption transition. In many (but not all) cases the group is found to widen substantially or even to split, making it difficult to define precisely the group maximum and thus its velocity. Many experiments have confirmed these predictions. For example, the group velocity in certain materials has been measured to be *ten times* that of light. The refractive index then is smaller than 1. However, in all these cases the group velocity is *not* the same as the signal speed.*

Challenge 1029 ny

Ref. 539

What then is the best velocity describing signal propagation? The German physicist Arnold Sommerfeld** almost solved the main problem in the beginning of the twentieth century. He defined the signal velocity as the velocity $v_{So}$ of the front slope of the pulse, as shown in Figure 262. The definition cannot be summarized in a formula, but it does have the property that it describes signal propagation for almost all experiments, in particular those in which the group and phase velocity are larger than the speed of light. When studying its properties, it was found that for no material is Sommerfeld's signal velocity greater than the speed of light in vacuum.

Ref. 538

Sometimes it is conceptually easier to describe signal propagation with the help of the energy velocity. As previously mentioned, every signal transports energy. The *energy velocity* $v_{en}$ is defined as the ratio between the power flow density **P**, i.e. the Poynting

* In quantum mechanics, Schrödinger proved that the velocity of an electron is given by the group velocity of its wave function. Therefore the same discussion reappeared in quantum theory, as we will find out in the second part of the mountain ascent.

** Arnold Sommerfeld (b. 1868 Königsberg, d. 1951 München) was a central figure in the spread of special and general relativity, of quantum theory, and of their applications. A professor in Munich, an excellent teacher and text book writer, he worked on atomic theory, on the theory of metals and on electrodynamics, and was the first to understand the importance and the mystery around 'Sommerfeld's famous fine structure constant.'

vector, and the energy density $W$, both taken in the direction of propagation. For electromagnetic fields – the only ones fast enough to be interesting for eventual superluminal signals – this ratio is

$$\mathbf{v}_{\text{en}} = \frac{\text{Re}(\mathbf{P})}{W} = \frac{2c^2\,\mathbf{E}\times\mathbf{B}}{\mathbf{E}^2 + c^2\mathbf{B}^2} \ . \tag{444}$$

However, as in the case of the front velocity, in the case of the energy velocity we have to specify if we mean the energy transported by the main pulse or by the front of it. In vacuum, neither is ever greater than the speed of light.* (In general, the velocity of energy in matter has a value slightly different from Sommerfeld's signal velocity.) Ref. 538

In recent years, the progress in light detector technology, allowing one to detect even the tiniest energies, has forced scientists to take the fastest of all these energy velocities to describe signal velocity. Using detectors with the highest possible sensitivity we can use as signal the first point of the wave train whose amplitude is different from zero, i.e. the first tiny amount of energy arriving. This point's velocity, conceptually similar to Sommerfeld's signal velocity, is commonly called the *front velocity* or, to distinguish it even more clearly from Sommerfeld's case, the *forerunner velocity*. It is simply given by Challenge 1030 n

$$v_{\text{fr}} = \lim_{\omega\to\infty} \frac{\omega}{k} \ . \tag{445}$$

The forerunner velocity is *never* greater than the speed of light in a vacuum, even in materials. In fact it is precisely $c$ because, for extremely high frequencies, the ratio $\omega/k$ is independent of the material, and vacuum properties take over. The forerunner velocity is the true signal velocity or the *true velocity of light*. Using it, all discussions on light speed become clear and unambiguous.

Challenge 1031 n To end this section, here are two challenges for you. Which of all the velocities of light is measured in experiments determining the velocity of light, e.g. when light is sent to the Moon and reflected back? And now a more difficult one: why is the signal speed of light less inside matter, as all experiments show? Challenge 1032 n

## 200 years too late – negative refraction indices

In 1968 the Soviet physicist Victor Veselago made a strange prediction: the index of refraction could have *negative* values without invalidating any known law of physics. A negative index means that a beam is refracted to the same side of the vertical, as shown in Figure 263.

In 1996, John Pendry and his group proposed ways of realizing such materials. In 2000, a first experimental confirmation for microwave refraction was published, but it met with strong disbelief. In 2002 the debate was in full swing. It was argued that negative refraction indices imply speeds greater than that of light and are only possible for either phase velocity or group velocity, but not for the energy or true signal velocity. The Ref. 542

* Signals not only carry energy, they also carry negative entropy ('information'). The entropy of a transmitter increases during transmission. The receiver decreases in entropy (but less than the increase at the transmitter, of course). Ref. 540

Note that the negative group velocity implies energy transport against the propagation velocity of light. This is possible only in *energy loaded* materials. Ref. 541

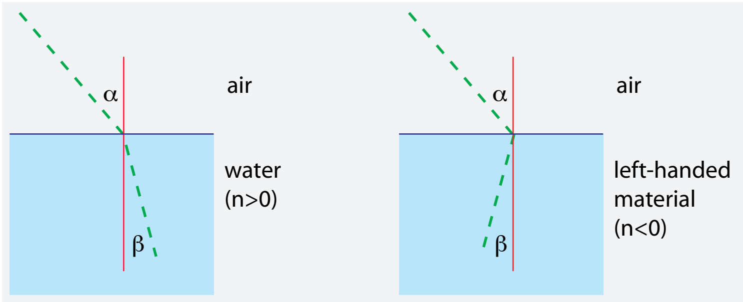


FIGURE 263 Positive and negative indices of refraction

conceptual problems would arise only because in some physical systems the refraction angle for phase motion and for energy motion differ.

Ref. 543 Today, the consensus is the following: a positive index of refraction less than one is impossible, as it implies an energy speed of greater than one. A negative index of refraction, however, is possible if it is smaller than −1. Negative values have indeed been frequently observed; the corresponding systems are being extensively explored all over the world. The materials showing this property are called *left-handed*. The reason is that the vectors of the electric field, the magnetic field and the wave vector form a left-handed triplet, in contrast to vacuum and most usual materials, where the triplet is right-handed. Such materials consistently have negative magnetic permeability and negative dielectric coefficient (permittivity). Ref. 544

Left-handed materials have negative phase velocities, i.e., a phase velocity opposed to the energy velocity, they show reversed Doppler effects and yield obtuse angles in the Çerenkov effect (emitting Çerenkov radiation in the backward instead of the forward direction).

But, most intriguing, negative refraction materials are predicted to allow the construction of lenses that are completely flat. In addition, in the year 2000, John Pendry gained the attention of the whole physics community world-wide by predicting that lenses made with such materials, in particular for $n = -1$, would be *perfect*, thus beating the usual diffraction limit. This would happen because such a lens also images the evanescent parts of the waves, by amplifying them accordingly. First experiments seem to confirm the prediction. Discussion on the topic is still in full swing. (Ref. 545; Ref. 544)

Challenge 1033 ny Can you explain how negative refraction differs from diffraction?

## Signals and predictions

When one person reads a text over the phone to a neighbour who listens to it and maybe repeats it, we speak of communication. For any third person, the speed of communication is always less than the speed of light. But if the neighbour already knows the text, he can recite it without having heard the readers' voice. To the third observer such a situation appears to imply motion that is faster than light. Prediction can thus *mimic* communication and, in particular, it can mimic faster-than-light (superluminal) communication. Ref. 546 Such a situation was demonstrated most spectacularly in 1994 by Günter Nimtz, who seemingly transported music – all music is predictable for short time scales – through a

TABLE 49 Experimental properties of (flat) vacuum and of the 'aether'

| Physical property | Experimental value |
|---|---|
| permeability | $\mu_0$ =1.3 μH/m |
| permittivity | $\varepsilon_0$ =8.9 pF/m |
| wave impedance/resistance | $Z_0$ = 376.7 Ω |
| conformal invariance | applies |
| spatial dimensionality | 3 |
| topology | $R^3$ |
| mass and energy content | not detectable |
| friction on moving bodies | not detectable |
| motion relative to space-time | not detectable |

'faster-than-light' system. To distinguish between the two situations, we note that in the case of prediction, no transport of energy takes place, in contrast to the case of communication. In other words, the definition of a signal as a transporter of information is not as useful and clear-cut as the definition of a signal as a *transporter of energy*. In the above-mentioned experiment, no energy was transported faster than light. The same distinction between prediction on the one hand and signal or energy propagation on the other will be used later to clarify some famous experiments in quantum mechanics.

> “If the rate at which physics papers are being published continues to increase, physics journals will soon be filling library shelves faster than the speed of light. This does not violate relativity since no useful information is being transmitted.
>
> David Mermin”

## Does the aether exist?

Gamma rays, light and radio waves are moving electromagnetic waves. All exist in empty space. What is oscillating when light travels? Maxwell himself called the 'medium' in which this happens the *aether*. The properties of the aether measured in experiments are listed in Table 49.

Of course, the values of the permeability and the permittivity of the vacuum are related to
Page 1155 the definition of the units henry and farad. The last item of the table is the most important:
Ref. 547 despite intensive efforts, nobody has been able to detect any *motion* of the aether. In other words, even though the aether supposedly oscillates, it does not move. Together with the other data, all these results can be summed up in one sentence: there is no way to
Challenge 1034 n distinguish the aether from the vacuum: they are one and the same.

Sometimes one hears that certain experiments or even the theory of relativity show that the aether does not exist. This is not strictly correct. In fact, experiments show something more important. All the data show that the aether is indistinguishable from the vacuum. Of course, if we use the change of curvature as the definition for motion of the vacuum, the vacuum *can* move, as we found out in the section on general relativity; but

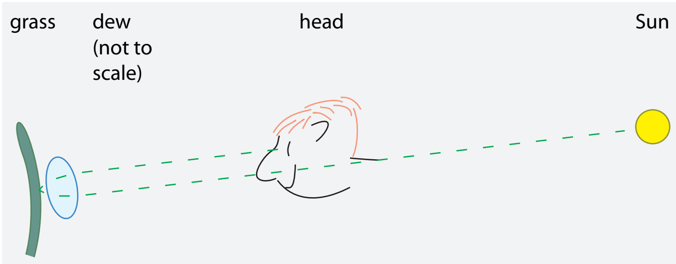


FIGURE 264 The path of light for the dew on grass that is responsible for the aureole

the aether still remains indistinguishable from it.*

Later we will even find out that the ability of the vacuum to allow the propagation of light and its ability to allow the propagation of particles are equivalent: both require the same properties. Therefore the aether remains *indistinguishable* from a vacuum in the rest of our walk. In other words, the aether is a superfluous concept; we will drop it from our walk from now on. Despite this result, we have not yet finished the study of the vacuum; vacuum will keep us busy for a long time, starting with the intermezzo following this chapter. Moreover, quite a few of the aspects in Table 49 will require some amendments later.

Ref. 548

## Curiosities and fun challenges about light

### How to prove you're holy

Light reflection and refraction are responsible for many effects. The originally Indian symbol of holiness, now used throughout most of the world, is the *aureole*, also called *halo* or *Heiligenschein*, a ring of light surrounding the head. You can easily observe it around your own head. You need only to get up early in the morning and look into the wet grass while turning your back to the Sun. You will see an aureole around your shadow. The effect is due to the morning dew on the grass, which reflects the light back predominantly in the direction of the light source, as shown in Figure 264. The fun part is that if you do this in a group, you will see the aureole around only *your own* head.

Ref. 549

Retroreflective paint works in the same way: it contains tiny glass spheres that play the role of the dew. A large surface of retroreflective paint, a traffic sign for example, can also show your halo if the light source is sufficiently far away. Also the so-called 'glow' of the eyes of a cat at night is due to the same effect; it is visible only if you look at the cat with a light source behind you. By the way, do Cat's-eyes work like a cat's eyes?

Ref. 550

Challenge 1035 n

Ref. 548

* In fact, the term 'aether' has been used as an expression for several different ideas, depending on the author. First of all it was used for the idea that a vacuum is not empty, but *full*; secondly, that this fullness can be described by *mechanical models*, such as gears, little spheres, vortices, etc.; thirdly, it was imagined that a vacuum is *similar to matter*, being made of the same substrate. Interestingly, some of these issues will reappear in the third part of our mountain ascent.

FIGURE 265 A limitation of the eye

## Do we see what exists?

Sometimes we see *less* than there is. Close your left eye, look at the white spot in Figure 265, bring the page slowly towards your eye, and pay attention to the middle lines. At a distance
Challenge 1036 n of about 15 to 20 cm the middle line will seem uninterrupted. Why?

On the other hand, sometimes we see *more* than there is, as Figures 266 and 267 show. The first shows that parallel lines can look skewed, and the second show a so-called *Hermann lattice*, named after its discoverer.* The Hermann lattice of Figure 267, discovered by Elke Lingelbach in 1995, is especially striking. Variations of these lattices are now used
Ref. 551 to understand the mechanisms at the basis of human vision. For example, they can be used to determine how many light sensitive cells in the retina are united to one signal pathway towards the brain. The illusions are angle dependent because this number is also angle dependent.

Our eyes also 'see' things *differently:* the retina sees an *inverted* image of the world. There is a simple method to show this, due to Helmholtz.** You need only a needle and a piece of paper, e.g. this page of text. Use the needle to make two holes inside the two letters 'oo'. Then keep the page as close to your eye as possible, look through the two holes towards the wall, keeping the needle vertical, a few centimetres behind the paper. You will see two images of the needle. If you now cover the *left* hole with your finger, the *right* needle will disappear, and vice versa. This shows that the image inside the eye, on the
Challenge 1037 ny retina, is inverted. Are you able to complete the proof?

---

* Ludimar Herrmann (1838–1914), Swiss physiologist. The lattices are often falsely called 'Hering lattices' after the man who made Hermann's discovery famous.

** See Hermann von Helmholtz, *Handbuch der physiologischen Optik*, 1867. This famous classic is available in English as *Handbook of Physiological Optics*, Dover, 1962. The Prussian physician, physicist and science politician born as Hermann Helmholtz (b. 1821 Potsdam, d. 1894 Charlottenburg) was famous for his works on optics, acoustics, electrodynamics, thermodynamics, epistemology and geometry. He founded several physics institutions across Germany. He was one of the first to propagate the idea of conservation of energy. His other important book, *Die Lehre von den Tonempfindungen*, published in 1863, describes the basis of acoustics and, like the handbook, is still worth reading.

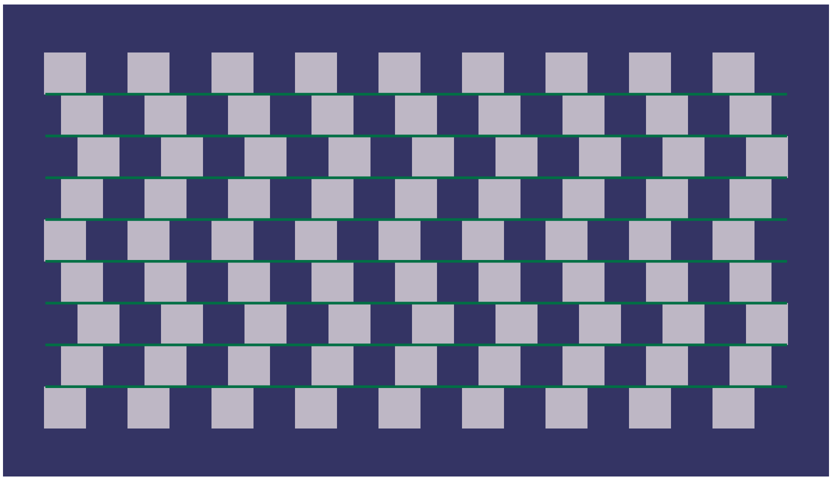

FIGURE 266 What is the angle between adjacent horizontal lines?

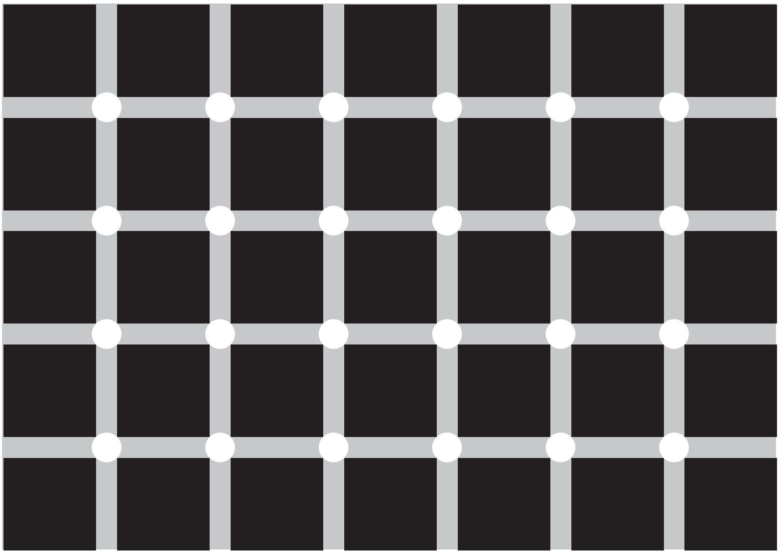

FIGURE 267 The Lingelbach lattice: do you see white, grey, or black dots?

Two other experiments can show the same result. If you push very lightly on the *inside* of your eye (careful!), you will see a dark spot appear on the *outside* of your vision field. And if you stand in a dark room and ask a friend to look at a burning candle, explore his eye: you will see three reflections: two upright ones, reflected from the cornea and from the lens, and a dim third one, *upside-down*, reflected form the retina.

Another reason that we do not see the complete image of nature is that our eyes have a limited sensitivity. This sensitivity peaks around 560 nm; outside the red and the violet, the eye does not detect radiation. We thus see only part of nature. For example, infrared photographs of nature, such as the one shown in Figure 268, are interesting because they show us something different from what we see usually.

FIGURE 268 An example of an infrared photograph, slightly mixed with a colour image (© Serge Augustin)

Every expert of motion should also know that the sensitivity of the eye does *not* correspond to the brightest part of sunlight. This myth has been spread around the world by the numerous textbooks that have copied from each other. Depending on whether frequency or wavelength or wavelength logarithm is used, the solar spectrum peaks at 500 nm, 880 nm or 720 nm. They human eye's spectral sensitivity, like the completely different sensitivity of birds or frogs, is due to the chemicals used for detection. In short, the human eye can only be understood by a careful analysis of its particular evolutionary history.

Ref. 552

An urban legend says that newborn babies see everything upside down. Can you explain why this idea is wrong?

Challenge 1038 n

In summary, we have to be careful when maintaining that seeing means observing. Examples such as these lead to ask whether there are other limitations of our senses which are less evident. And our walk will indeed uncover several of them.

## How does one make pictures of the inside of the eye?

The most beautiful pictures so far of a *living* human retina, such as that of Figure 269, were made by the group of David Williams and Austin Roorda at the University at Rochester in New York. They used adaptive optics, a technique that changes the shape of the imaging lens in order to compensate for the shape variations of the lens in the human eye.*

Ref. 553

* Nature uses another trick to get maximum resolution: the eye continuously performs small movements, called *micronystagmus*. The eye continuously oscillates around the direction of vision with around 40 to 50 Hz. In addition, this motion is also used to allow the cells in the retina to recharge.

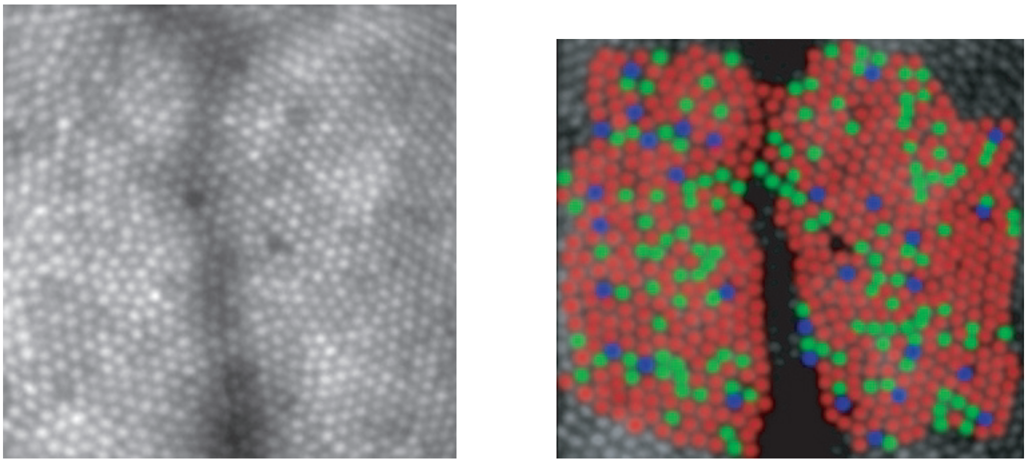

FIGURE 269 A high quality photograph of a live human retina, including a measured (false colour) indication of the sensitivity of each cone cell (© Austin Roorda)

The eyes see colour by averaging the intensity arriving at the red, blue and green sensitive cones. This explains the possibility, mentioned above, of getting the same impression of colour, e.g. yellow, either by a pure yellow laser beam, or by a suitable mixture of red and green light.

Page 574

But if the light is focused on to one cone only, the eye makes mistakes. If, using this adaptive optics, a red laser beam is focused such that it hits a green cone only, a strange thing happens: even though the light is *red*, the eye sees a *green* colour!

Incidentally, Figure 269 is quite puzzling. In the human eye, the blood vessels are located *in front* of the cones. Why don't they appear in the picture? And why don't they disturb us in everyday life? (The picture does not show the other type of sensitive light cells, the *rods*, because the subject was in ambient light; rods come to the front of the retina only in the dark, and then produce black and white pictures.

Challenge 1039 n

Of all the mammals, only primates can see *colours*. Bulls for example, don't; they cannot distinguish red from blue. On the other hand, the best colour seers overall are the birds. They have cone receptors for red, blue, green, UV and, depending on the bird, for up to three more sets of colours. A number of birds (but not many) also have a better eye resolution than humans. Several birds also have a faster temporal resolution: humans see continuous motion when the images follow with 30 to 70 Hz (depending on the image content); some insects can distinguish images up to 300 Hz.

## How does one make holograms and other three-dimensional images?

Our sense of sight gives us the impression of depth mainly due to three effects. First, the two eyes see different images. Second, the images formed in the eyes are position dependent. Third, our eye needs to focus differently for different distances.

A simple photograph does not capture any of the three effects. A photograph corresponds to the picture taken by one eye, from one particular spot and at one particular focus. In fact, all photographic cameras are essentially copies of a single static eye.

Challenge 1040 e

Any system wanting to produce the perception of depth must include at least one of

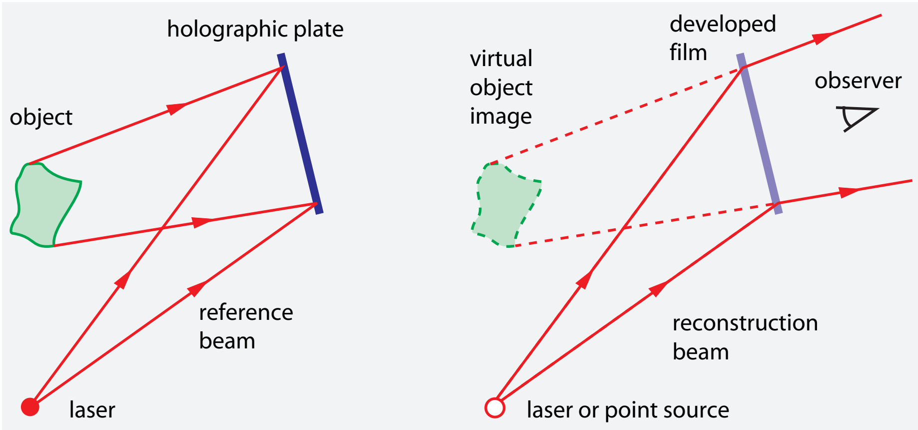


FIGURE 270 The recording and the observation of a hologram

the three effects just mentioned. In all systems so far, the third and weakest effect, varying focus with distance, is never used, as it is too weak. Stereo photography and virtual reality systems extensively use the first effect by sending two different images to the eyes. Also certain post cards and computer screens are covered by thin cylindrical lenses that allow one to send two different images to the two eyes, thus generating the same impression of depth.

But obviously the most spectacular effect is obtained whenever position dependent images can be created. Some virtual reality systems mimic this effect using a sensor attached to the head, and creating computer–generated images that depend on this position. However, such systems are not able to reproduce actual situations and thus pale when compared with the impression produced by holograms.

Holograms reproduce all that is seen from any point of a region of space. A *hologram* is thus a stored set of position dependent pictures of an object. It is produced by storing amplitude *and phase* of the light emitted by an object. To achieve this, the object is illuminated by a *coherent* light source, such as a laser, and the interference pattern is stored. Illuminating the developed photographic film by a coherent light source then allows one to see a full three-dimensional image. In particular, due to the reproduction of the situation, the image appears to float in free space. Holograms were developed in 1947 by the Hungarian physicist Dennis Gabor (1900–1979), who received the 1971 Nobel Prize for physics for this work.

Holograms can be made to work in reflection or transmission. The simplest holograms use only one wavelength. Most coloured holograms are rainbow holograms, showing false colours that are unrelated to the original objects. Real colour holograms, made with three different lasers, are rare but possible.

Is it possible to make *moving* holograms? Yes; however, the technical set-ups are still extremely expensive. So far, they exist only in a few laboratories and cost millions of euro. By the way, can you describe how you would distinguish a moving hologram from a real body, if you ever came across one, without touching it?

Challenge 1041 n

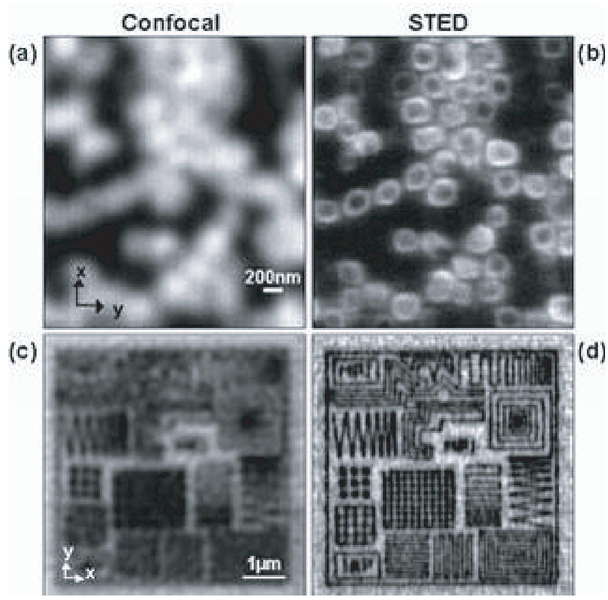


FIGURE 271 Sub-wavelength optical microscopy using stimulated emission depletion (© MPI für biophysikalische Chemie/Stefan Hell)

## Imaging

Producing images is an important part of modern society. The quality images depends on the smart use of optics, electronics, computers and materials science. Despite long experience in this domain, there are still new results in the field. Images, i.e. two or three-dimensional reproductions, can be taken by at least four methods:

— *Photography* uses a light source, lenses and film or another large area detector. Photography can be used in reflection, in transmission, with phase-dependence and in many other ways.
— *Holography* uses lasers and large area detectors, as explained above. Holography allows to take three-dimensional images of objects. It is usually used in reflection, but can also be used in transmission.
— *Scanning techniques* construct images point by point through the motion of the detector, the light source or both.
— *Tomography*, usually in transmission, uses a source and a line detector that are both rotated around an object. This allows to image cross sections of materials.

In all methods, the race is for images with the highest resolution possible. The techniques of producing images with resolutions *less* than the wavelength of light have made great progress in recent years.

A recent technique, called *stimulated emission depletion microscopy*, allows spot sizes of molecular size. The conventional diffraction limit for microscopes is

$$d \geqslant \frac{\lambda}{2n \ \sin\alpha} , \tag{446}$$

where $\lambda$ is the wavelength, $n$ the index of refraction and $\alpha$ is the angle of observation. The new technique, a special type of fluorescence microscopy developed by Stefan Hell, modifies this expression to

$$d \geqslant \frac{\lambda}{2n\ \sin\alpha; \sqrt{I/I_{\mathrm{sat}}}} , \qquad (447)$$

so that a properly chosen saturation intensity allows one to reduce the diffraction limit to arbitrary low values. So far, light microscopy with a resolution of 16 nm has been performed, as shown in Figure 271. This and similar techniques should become commonplace in the near future.

Ref. 554

## LIGHT AS WEAPON?

In many countries, there is more money to study assault weapons than to increase the education and wealth of their citizen. Several types of assault weapons using electromagnetic radiation are being researched. Two are particularly advanced.

The first weapon is a truck with a movable parabolic antenna on its roof, about 1 m in size, that emits a high power (a few kW) microwave (95 GHz) beam. The beam, like all microwave beams, is invisible; depending on power and beam shape, it is painful or lethal, up to a distance of a 100 m. This terrible device, with which the operator can make many many victims without even noticing, was ready in 2006. (Who expects that a parabolic antenna is dangerous?) Efforts to ban it across the world are slowly gathering momentum.

The second weapon under development is the so-called *pulsed impulse kill laser*. The idea is to take a laser that emits radiation that is not absorbed by air, steam or similar obstacles. An example is a pulsed deuterium fluoride laser that emits at 3.5 μm. This laser burns every material it hits; in addition, the evaporation of the plasma produced by the burn produces a strong hit, so that people hit by such a laser are hurt and hit at the same time. Fortunately, it is still difficult to make such a device rugged enough for practical use. But experts expect battle lasers to appear soon.

In short, it is probable that radiation weapons will appear in the coming years.*

# 15. CHARGES ARE DISCRETE – THE LIMITS OF CLASSICAL ELECTRODYNAMICS

One of the most important results of physics: *electric charge is discrete* has already been mentioned a number of times. Charge does not vary continuously, but changes in fixed steps. Not only does nature show a smallest value of entropy and smallest amounts of matter; nature also shows a smallest charge. Electric charge is quantized.

In metals, the quantization of charge is noticeable in the flow of electrons. In electrolytes, i.e. electrically conducting liquids, the quantization of charge appears in the flow of charged atoms, usually called *ions*. All batteries have electrolytes inside; also water is an electrolyte, though a poorly conducting one. In plasmas, like fire or fluorescent lamps,

* What a man working on such developments tells his children when he comes home in the evening is not clear.

both ions and electrons move and show the discreteness of charge. Also in radiation – from the electron beams inside TVs, channel rays formed in special glass tubes, and cosmic radiation, up to radioactivity – charges are quantized.

From all known experiments, the same smallest value for charge change has been found. The result is

$$\Delta q \geqslant e = 1.6 \times 10^{-19}\,\mathrm{C}\,. \qquad (448)$$

In short, like all flows in nature, the flow of electricity is due to a flow of discrete particles.

A smallest charge change has a simple implication: classical electrodynamics is *wrong*. A smallest charge implies that no infinitely small test charges exist. But such infinitely small test charges are necessary to define electric and magnetic fields. The limit on charge size also implies that there is no correct way of defining an instantaneous electric current and, as a consequence, that the values of electric and magnetic field are always somewhat fuzzy. Maxwell's evolution equations are thus only approximate.

We will study the main effects of the discreteness of charge in the part on quantum Page 753 theory. Only a few effects of the quantization of charge can be treated in classical physics. An instructive example follows.

## How fast do charges move?

In vacuum, such as inside a colour television, charged particles accelerated by a tension Challenge 1042 n of 30 kV move with a third of the speed of light. In modern particle accelerators charges move so rapidly that their speed is indistinguishable from that of light for all practical purposes.

Inside a metal, electric signals move with speeds of the order of the speed of light. The precise value depends on the capacity and impedance of the cable and is usually in the range $0.3c$ to $0.5c$. This high speed is due to the ability of metals to easily take in arriving charges and to let others depart. The ability for rapid reaction is due to the high mobility of the charges inside metals, which in turn is due to the small mass and size of these charges, the electrons.

The high signal speed in metals appears to contradict another determination. The drift speed of the electrons in a metal wire obviously obeys

$$v = \frac{I}{Ane}\,, \qquad (449)$$

where $I$ is the current, $A$ the cross-section of the wire, $e$ the charge of a single electron and $n$ the number density of electrons. The electron density in copper is $8.5 \cdot 10^{28}\,\mathrm{m}^{-3}$. Using a typical current of 0.5 A and a typical cross-section of a square millimetre, we get a drift speed of 0.37 µm/s. In other words, electrons move a thousand times slower than ketchup inside its bottle. Worse, if a room lamp used direct current instead of alternate current, the electrons would take several days to get from the switch to the bulb! Nevertheless, the lamp goes on or off almost immediately after the switch is activated. Similarly, the electrons from an email transported with direct current would arrive much later than a paper letter sent at the same time; nevertheless, the email arrives quickly. Are you able to Challenge 1043 n explain the apparent contradiction between drift velocity and signal velocity?

Inside liquids, charges move with a different speed from that inside metals, and their charge to mass ratio is also different. We all know this from direct experience. Our *nerves* work by using electric signals and take (only) a few milliseconds to respond to a stimulus, even though they are metres long. A similar speed is observed inside semiconductors and inside batteries. In all these systems, moving charge is transported by *ions*; they are charged atoms. Ions, like atoms, are large and composed entities, in contrast to the tiny electrons.

In other systems, charges move both as electrons and as ions. Examples are neon lamps, fire, plasmas and the Sun. Inside atoms, electrons behave even more strangely. One tends to think that they orbit the nucleus (as we will see later) at a rather high speed, as the orbital radius is so small. However, it turns out that in most atoms many electrons do not orbit the nucleus at all. The strange story behind atoms and their structure will be told in
Page 754 the second part of our mountain ascent.

## Challenges and curiosities about charge discreteness

Charge discreteness is one of the central results of physics.

* *

Challenge 1044 n How would you show experimentally that electrical charge comes in smallest chunks?

* *

The discreteness of charge implies that one can estimate the size of atoms by observing
Challenge 1045 ny galvanic deposition. How?

* *

Cosmic radiation consists of charged particles hitting the Earth. (We will discuss this
Page 897 in more detail later.) Astrophysicists explain that these particles are accelerated by the
Ref. 555 magnetic fields around the Galaxy. However, the expression of the Lorentz acceleration shows that magnetic fields can only change the direction of the velocity of a charge, not
Challenge 1046 ny its magnitude. How can nature get acceleration nevertheless?

* *

What would be the potential of the Earth in volt if we could take away all the electrons of
Challenge 1047 n a drop of water?

* *

When a voltage is applied to a resistor, how long does it take until the end value of the current, given by Ohm's 'law', is reached? The first to answer this question was Paul Drude.* in the years around 1900. He reasoned that when the current is switched on, the speed $v$ of an electron increases as $v = (eE/m)t$, where $E$ is the electrical field, $e$ the charge and $m$ the mass of the electron. Drude's model assumes that the increase of electron speed stops

* Paul Karl Ludwig Drude (1863–1906), German physicist. A result of his electron gas model of metals was the prediction, roughly correct, that the ratio between the thermal conductivity and the electronic conductivity at a given temperature should be the same for all metals. Drude also introduced $c$ as the symbol for the speed of light.

when the electron hits an atom, loses its energy and begins to be accelerated again. Drude
Challenge 1048 ny deduced that the average time $\tau$ up to the collision is related to the specific resistance by

$$\rho = \frac{2m}{\tau e^2 n} , \qquad (450)$$

with $n$ being the electron number density. Inserting numbers for copper ($n = 10.3 \cdot 10^{28}\,/\mathrm{m}^{-3}$ and $\rho = 0.16 \cdot 10^{-7}\,\Omega\mathrm{m}$), one gets a time $\tau = 42\,\mathrm{ps}$. This time is so short that the switch-on process can usually be neglected.

## 16. ELECTROMAGNETIC EFFECTS AND CHALLENGES

Classical electromagnetism and light are almost endless topics. Some aspects are too beautiful to be missed.

* *

Since light is a wave, something must happen if it is directed to a hole less than its
Challenge 1049 n wavelength in diameter. What exactly happens?

* *

Electrodynamics shows that light beams always push; they never pull. Can you confirm
Challenge 1050 e that 'tractor beams' are impossible in nature?

* *

It is well known that the glowing material in light bulbs is tungsten wire in an inert gas. This was the result of a series of experiments that began with the grandmother of all lamps, namely the cucumber. The older generation knows that a pickled cucumber, when attached to the 230 V of the mains, glows with a bright green light. (Be careful; the experiment is dirty and somewhat dangerous.)

* *

If you calculate the Poynting vector for a charged magnet – or simpler, a point charge near a magnet – you get a surprising result: the electromagnetic energy flows in circles around
Challenge 1051 n the magnet. How is this possible? Where does this angular momentum come from?

Worse, any atom is an example of such a system – actually of two such systems. Why
Ref. 556 is this effect not taken into account in calculations in quantum theory?

* *

Ohm's law, the observation that for almost all materials the current is proportional to the voltage, is due to a school teacher. Georg Simon Ohm* explored the question in great depth; in those days, such measurements were difficult to perform. This has changed now.

* Georg Simon Ohm (b. 1789 Erlangen, d. 1854 München), Bavarian school teacher and physicist. His efforts were recognized only late in his life, and he eventually was promoted to professor at the University in München. Later the unit of *electrical resistance*, the proportionality factor between voltage and current, was named after him.

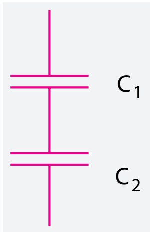

FIGURE 272 Capacitors in series

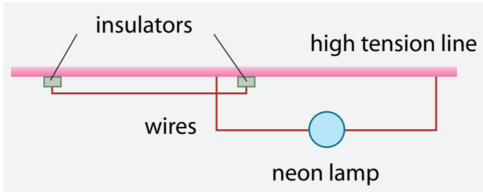

FIGURE 273 Small neon lamps on a high voltage cable

Recently, even the electrical resistance of single atoms has been measured: in the case of
Ref. 557 xenon it turned out to be about $10^5\,\Omega$. It was also found that lead atoms are ten times
Challenge 1052 ny more conductive than gold atoms. Can you imagine why?

* *

Ref. 558 The charges on two capacitors in series are not generally equal, as naive theory states. For perfect, leak-free capacitors the voltage ratio is given by the inverse capacity ratio $V_1/V_2 = C_2/C_1$, due to the equality of the electric charges stored. This is easily deduced from Figure 272. However, in practice this is only correct for times between a few and a
Challenge 1053 n few dozen minutes. Why?

* *

Does it make sense to write Maxwell's equations in vacuum? Both electrical and magnetic fields require charges in order to be measured. But in vacuum there are no charges at all. In fact, only quantum theory solves this apparent contradiction. Are you able to imagine
Challenge 1054 d how?

* *

Ref. 559 Grass is usually greener on the other side of the fence. Can you give an explanation based
Challenge 1055 n on observations for this statement?

* *

The maximum force in nature limits the maximum charge that a black hole can carry.
Challenge 1056 ny Can you find the relation?

* *

On certain high voltage cables leading across the landscape, small neon lamps shine when the current flows, as shown in Figure 273. (You can see them from the train when riding
Challenge 1057 ny from Paris to the Roissy airport.) How is this possible?

* *

'Inside a conductor there is no electric field.' This statement is often found. In fact the truth is not that simple. First, a *static* field or a *static* charge on the metal surface of a body does not influence fields and charges inside it. A closed metal surface thus forms a
Challenge 1058 n shield against an electric field. Can you give an explanation? In fact, a tight metal layer is

not required to get the effect; a cage is sufficient. One speaks of a *Faraday cage*.

The detailed mechanism allows you to answer the following question: do Faraday cages for gravity exist? Why?

For *moving* external fields or charges, the issue is more complex. Fields due to accelerated charges – radiation fields – decay exponentially through a shield. Fields due to charges moving at constant speed are strongly reduced, but do not disappear. The reduction depends on the thickness and the resistivity of the metal enclosure used. For sheet metal, the field suppression is very high; it is not necessarily high for metal sprayed plastic. Such a device will not necessarily survive a close lightning stroke. Ref. 560

In practice, there is no danger if lightning hits an aeroplane or a car, as long they are made of metal. (There is one film on the internet of a car hit by lightning; the driver does not even notice.) However, if your car is hit by lightning in dry weather, you should wait a few minutes before getting out of it. Can you imagine why?

Faraday cages also work the other way round. (Slowly) changing electric fields changing that are inside a Faraday cage are not felt outside. For this reason, radios, mobile phones and computers are surrounded by boxes made of metal or metal-sprayed plastics. The metal keeps the so-called *electromagnetic smog* to a minimum.

There are thus three reasons to surround electric appliances by a grounded shield: to protect the appliance from outside fields, to protect people and other machines from electromagnetic smog, and to protect people against the mains voltage accidentally being fed into the box (for example, when the insulation fails). In high precision experiments, these three functions can be realized by three separate cages.

For purely magnetic fields, the situation is more complex. It is quite difficult to shield the inside of a machine from outside magnetic fields. How would you do it? In practice one often uses layers of so-called *mu-metal*; can you guess what this material does? Challenge 1059 ny

* *

*Electric polarizability* is the property of matter responsible for the deviation of water flowing from a tap caused by a charged comb. It is defined as the strength of electric dipole induced by an applied electric field. The definition simply translates the observation that many objects acquire a charge when an electric field is applied. Incidentally, how precisely combs get charged when rubbed, a phenomenon called *electrification*, is still one of the mysteries of modern science. Page 518

* *

A pure magnetic field cannot be transformed into a pure electric field by change of observation frame. The best that can be achieved is a state similar to an equal mixture of magnetic and electric fields. Can you provide an argument elucidating this relation? Challenge 1060 ny

* *

Researchers are trying to detect tooth decay with the help of electric currents, using the observation that healthy teeth are bad conductors, in contrast to teeth with decay. How would you make use of this effect in this case? (By the way, it might be that the totally unrelated technique of imaging with terahertz waves could yield similar results.) Ref. 561 Challenge 1061 ny

* *

A team of camera men in the middle of the Sahara were using battery-driven electrical equipment to make sound recordings. Whenever the microphone cable was a few tens of metres long, they also heard a 50 Hz power supply noise, even though the next power supply was thousands of kilometres away. An investigation revealed that the high voltage lines in Europe lose a considerable amount of power by irradiation; these 50 Hz waves are reflected by the ionosphere around the Earth and thus can disturb recording in the middle of the desert. Can you estimate whether this observation implies that living directly near a high voltage line is dangerous? Challenge 1062 ny

* *

Ref. 562 When two laser beams cross at a small angle, they can form light pulses that seem to move faster than light. Does this contradict special relativity? Challenge 1063 n

* *

It is said that astronomers have telescopes so powerful that they can see whether somebody is lighting a match on the Moon. Can this be true? Challenge 1064 ny

* *

When solar plasma storms are seen on the Sun, astronomers first phone the electricity company. They know that about 24 to 48 hours later, the charged particles ejected by the storms will arrive on Earth, making the magnetic field on the surface fluctuate. Since power grids often have closed loops of several thousands of kilometres, additional electric currents are induced, which can make transformers in the grid overheat and then switch off. Other transformers then have to take over the additional power, which can lead to their overheating, etc. On several occasions in the past, millions of people have been left without electrical power due to solar storms. Today, the electricity companies avoid the problems by disconnecting the various grid sections, by avoiding large loops, by reducing the supply voltage to avoid saturation of the transformers and by disallowing load transfer from failed circuits to others.

* *

Is it really possible to see stars from the bottom of a deep pit or of a well, even during the day, as is often stated? Challenge 1065 n

* *

If the electric field is described as a sum of components of different frequencies, its so-called *Fourier components*, the amplitudes are given by Ref. 563

$$\hat{\mathbf{E}}(k,t) = \frac{1}{(2\pi)^3/2} \int \mathbf{E}(x,t) \mathrm{e}^{-i\mathbf{kx}} \, \mathrm{d}^3 x \tag{451}$$

and similarly for the magnetic field. It then turns out that a Lorentz invariant quantity $N$, describing the energy per circular frequency $\omega$, can be defined:

$$N = \frac{1}{8\pi} \int \frac{|\mathbf{E}(k,t)|^2 + |\mathbf{B}(k,t)|^2}{c|\mathbf{k}|} \, \mathrm{d}^3 k \; . \tag{452}$$

FIGURE 274 How natural colours (top) change for three types of colour blind: deutan, protan and tritan (© Michael Douma)

Challenge 1066 n Can you guess what $N$ is physically? (Hint: think about quantum theory.)

* *

Faraday discovered how to change magnetism into electricity, knowing that electricity could be transformed into magnetism. (The issue is subtle. Faraday's law is not the dual of Ampère's, as that would imply the use of magnetic monopoles; neither is it the reciprocal, as that would imply the displacement current. But he was looking for a link and he found a way to relate the two observations – in a novel way, as it turned out.) Faraday also discovered how to transform electricity into light and into chemistry. He then tried to
Challenge 1067 ny change gravitation into electricity. But he was not successful. Why not?

* *

At high altitudes above the Earth, gases are completely ionized; no atom is neutral. One speaks of the *ionosphere*, as space is full of positive ions and free electrons. Even though both charges appear in exactly the same number, a satellite moving through the iono-
Challenge 1068 n sphere acquires a negative charge. Why? How does the charging stop?

* *

A capacitor of capacity $C$ is charged with a voltage $U$. The stored electrostatic energy is $E = CU^2/2$. The capacitor is then detached from the power supply and branched on to an empty capacitor of the same capacity. After a while, the voltage obviously drops to $U/2$. However, the stored energy now is $C(U/2)^2$, which is half the original value. Where did
Challenge 1069 n the energy go?

* *

Colour blindness was discovered by the great English scientist John Dalton (1766–1844)
Challenge 1070 ny – on himself. Can you imagine how he found out? It affects, in all its forms, one in 20 men.

FIGURE 275 Cumulonimbus clouds from ground and from space (NASA)

In many languages, a man who is colour blind is called *daltonic*. Women are almost never
Ref. 564 daltonic, as the property is linked to defects on the X chromosome. If you are colour blind, you can check to which type you belong with the help of Figure 274.

* *

Perfectly spherical electromagnetic waves are impossible in nature. Can you show this
Challenge 1071 n using Maxwell's equation of electromagnetism, or even without them?

* *

Light beams, such as those emitted from lasers, are usually thought of as lines. However, light beams can also be *tubes*. Tubular laser beams, or Bessel beams of high order, are used in modern research to guide plasma channels.

## Is lightning a discharge? – Electricity in the atmosphere

Looking carefully, the atmosphere is full of electrical effects. The most impressive electrical phenomenon we observe, lightning, is now reasonably well understood. Inside a thunderstorm cloud, especially inside tall *cumulonimbus* clouds,* charges are separated by collision between the large 'graupel' ice crystals falling due to their weight and the small
Ref. 566 'hail' ice crystallites rising due to thermal upwinds. Since the collision takes part in an elec-
Page 521 tric field, charges are separated in a way similar to the mechanism in the Kelvin generator. Discharge takes place when the electric field becomes too high, taking a strange path influenced by ions created in the air by cosmic rays. It seems that cosmic rays are at least partly

* Clouds have Latin names. They were introduced in 1802 by the English explorer Luke Howard (1772–1864), who found that all clouds could be seen as variations of three types, which he called *cirrus*, *cumulus* and
Ref. 565 *stratus*. He called the combination of all three, the rain cloud, *nimbus* (from the Latin 'big cloud'). Today's internationally agreed system has been slightly adjusted and distinguishes clouds by the height of their lower edge. The clouds starting above a height of 6 km are the cirrus, the cirrocumulus and the cirrostratus; those starting at heights of between 2 and 4 km are the altocumulus, the altostratus and the nimbostratus; clouds starting below a height of 2 km are the stratocumulus, the stratus and the cumulus. The rain or thunder cloud, which crosses all heights, is today called cumulonimbus.

responsible for the zigzag shape of lightning.* Lightning flashes have strange properties. First, they appear at fields around 200 kV/m (at low altitude) instead of the 2 MV/m of normal sparks. Second, lightning emits radio pulses. Third, they emit gamma rays. Russian researchers, from 1992 onwards explained all three effects by a newly discovered discharge mechanism. At length scales of 50 m and more, cosmic rays can trigger the appearance of lightning; the relativistic energy of these rays allows for a discharge mechanism that does not exist for low energy electrons. At relativistic energy, so-called runaway breakdown leads to discharges at much lower fields than usual laboratory sparks. The multiplication of these relativistic electrons also leads to the observed radio and gamma ray emissions. Ref. 568

Incidentally, you have a 75 % chance of survival after being hit by lightning, especially if you are completely wet, as in that case the current will flow outside the skin. Usually, wet people who are hit loose all their clothes, as the evaporating water tears them off. Rapid resuscitation is essential to help somebody to recover after a hit.**

As a note, you might know how to measure the distance of a lightning by counting the seconds between the lightning and the thunder and multiplying this by the speed of sound, 330 m/s; it is less well known that one can estimate the *length* of the lightning bolt by measuring the *duration* of the thunder, and multiplying it by the same factor.

In the 1990s more electrical details about thunderstorms became known. Airline pilots and passengers sometime see weak and coloured light emissions spreading from the top of thunderclouds. There are various types of such emissions: blue *jets* and mostly red *sprites* and *elves*, which are somehow due to electric fields between the cloud top and the ionosphere. The details are still under investigation, and the mechanisms are not yet clear.***

All these details are part of the electrical circuit around the Earth. This fascinating part of geophysics would lead us too far from the aim of our mountain ascent. But every physicist should know that there is a vertical electric field of between 100 and 300 V/m on a clear day, as discovered already in 1752. (Can you guess why it is not noticeable in everyday life? And why despite its value it cannot be used to extract large amounts of energy?) The field is directed from the ionosphere down towards the ground; in fact the Earth is permanently negatively charged, and in clear weather current flows downwards through the clear atmosphere, trying to *discharge* our planet. The current of about 1 kA is spread over the whole planet; it is possible due to the ions formed by cosmic radiation. (The resistance between the ground and the ionosphere is about 200 Ω, so the total voltage drop is about 200 kV.) At the same time, the Earth is constantly being *charged* by several effects: there is a dynamo effect due to the tides of the atmosphere and there are currents induced by the magnetosphere. But the most important effect is lightning. In other words, contrary Challenge 1072 n

Ref. 567 * There is no ball lightning even though there is a Physics Report about it. Ball lightning is one of the favourite myths of modern pseudo-science. Actually, they would exist if we lived in a giant microwave oven. To show this, just stick a toothpick into a candle, light the toothpick, and put it into (somebody else's) microwave at maximum power.

** If you are ever hit by lightning and survive, go to the hospital! Many people died three days later having failed to do so. A lightning strike often leads to coagulation effects in the blood. These substances block the kidneys, and one can die three days later because of kidney failure. The remedy is to have dialysis treatment.

*** For images, have a look at the interesting http://sprite.gi.alaska.edu/html/sprites.htm, http://www.fma-research.com/spriteres.htm and http://paesko.ee.psu.edu/Nature websites.

Ref. 569 to what one may think, lightning does not discharge the ground, it actually charges it up!* Of course, lightning does discharge the cloud to ground potential difference; but by doing so, it actually sends a negative charge down to the Earth as a whole. Thunderclouds are batteries; the energy from the batteries comes from the the thermal uplifts mentioned above, which transport charge *against* the global ambient electrical field.

Using a few electrical measurement stations that measure the variations of the electrical field of the Earth it is possible to locate the position of all the lightning that comes Ref. 570 down towards the Earth at a given moment. Present research also aims at measuring the activity of the related electrical sprites and elves in this way.

The ions in air play a role in the charging of thunderclouds via the charging of ice crystals and rain drops. In general, all small particles in the air are electrically charged. When aeroplanes and helicopters fly, they usually hit more particles of one charge than of the other. As a result, aeroplanes and helicopters are charged up during flight. When a helicopter is used to rescue people from a raft in high seas, the rope pulling the people upwards must first be earthed by hanging it in the water; if this is not done, the people on the raft could die from an electrical shock when they touch the rope, as has happened a few times in the past.

The charges in the atmosphere have many other effects. Recent experiments have con-Ref. 571 firmed what was predicted back in the early twentieth century: lightning emits X-rays. The confirmation is not easy though; it is necessary to put a detector near the lightning flash. To achieve this, the lightning has to be directed into a given region. This is possible using a missile pulling a metal wire, the other end of which is attached to the ground. These experimental results are now being collated into a new description of lightning which also explains the red-blue sprites above thunderclouds. In particular, the processes Ref. 572 also imply that inside clouds, electrons can be accelerated up to energies of a few MeV.

Why are sparks and lightning blue? This turns out to be a material property: the colour comes from the material that happens to be excited by the energy of the discharge, usually air. This excitation is due to the temperature of 30 kK inside the channel of a typical lightning flash. For everyday sparks, the temperature is much lower. Depending on the situation, the colour may arise from the gas between the two electrodes, such as oxygen or nitrogen, or it may due to the material evaporated from the electrodes by the discharge. For an explanation of such colours, as for the explanation of all colours due to materials, we need to wait for the next part of our walk.

But not only electric fields are dangerous. Also time-varying electromagnetic fields can be. In 1997, in beautiful calm weather, a Dutch hot air balloon approached the powerful radio transmitter in Hilversum. After travelling for a few minutes near to the antenna, the gondola suddenly detached from the balloon, killing all the passengers inside.

An investigation team reconstructed the facts a few weeks later. In modern gas balloons the gondola is suspended by high quality nylon ropes. To avoid damage by lightning and in order to avoid electrostatic charging problems all these nylon ropes contain thin metal wires which form a large equipotential surface around the whole balloon. Unfortunately, in the face of the radio transmitter, these thin metal wires absorbed radio energy from the transmitter, became red hot, and melted the nylon wires. It was the first time that this had ever been observed.

Challenge 1073 ny * The Earth is thus charged to about −1 MC. Can you confirm this?

## Does gravity make charges radiate?

We learned in the section on general relativity that gravitation has the same effects as acceleration. This means that a charge kept fixed at a certain height is equivalent to a charge accelerated by $9.8\,\mathrm{m/s^2}$, which would imply that it radiates electromagnetically, since all accelerated charges radiate. However, the world around us is full of charges at fixed heights, and there is no such radiation. How is this possible?

Ref. 573 The question has been a pet topic for many years. Generally speaking, the concept of radiation is not observer invariant: If one observer detects radiation, a second one does not necessarily do so as well. The exact way a radiation field changes from one observer to the other depends on the type of relative motion and on the field itself.

A precise solution of the problem shows that for a uniformly accelerated charge, an observer undergoing the same acceleration only detects an electrostatic field. In contrast, an Ref. 574 inertial observer detects a radiation field. Since gravity is (to a high precision) equivalent to uniform acceleration, we get a simple result: gravity does not make electrical charges radiate for an observer at rest with respect to the charge, as is observed. The results holds true also in the quantum theoretical description.

## Research questions

The classical description of electrodynamics is coherent and complete; nevertheless there are still many subjects of research. Here are a few of them.

The origin of magnetic field of the Earth, the other planets, the Sun and even of the galaxy is a fascinating topic. The way that the convection of fluids inside the planets generates magnetic fields, an intrinsically three-dimensional problem, the influence of turbulence, of nonlinearities and of chaos makes it a surprisingly complex question.

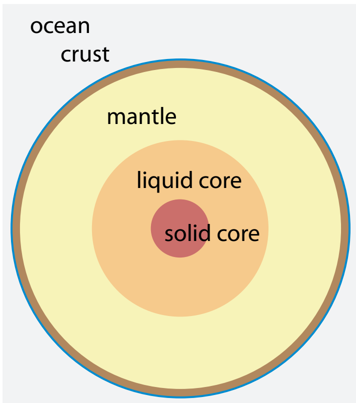


FIGURE 276 The structure of our planet

The details of the generation of the magnetic field of the Earth, usually called the *geodynamo*, began to appear only in the second half of the twentieth century, when the knowledge of the Earth's interior reached a Ref. 505 sufficient level. The Earth's interior starts below the Earth's crust. The *crust* is typically 30 to 40 km thick (under the continents), though it is thicker under high mountains and thinner near volcanoes or under the oceans. As already mentioned, the crust consists of large segments, the *plates*, that move with respect to one other. The Earth's interior is divided into the *mantle* – the first 2900 km from the surface – and the *core*. The core is made up of a liquid *outer* core, 2210 km thick, and a solid *inner* core of 1280 km radius. (The temperature of the core is not well known; it is believed to be 6 to 7 kK. Can you Challenge 1074 ny find a way to determine it? The temperature might have decreased a few hundred kelvin during the last 3000 million years.)

The Earth's core consists mainly of iron that has been collected from the asteroids that collided with the Earth during its youth. It seems that the liquid and electrically conducting outer core acts as a dynamo that keeps the magnetic field going. The magnetic energy comes from the kinetic energy of the outer core, which rotates with respect to the Earth's surface; the fluid can act as a dynamo because, apart from rotating, it also *convects* from deep inside the Earth to more shallow depths, driven by the temperature gradients between the hot inner core and the cooler mantle. Huge electric currents flow in complex ways through these liquid layers, maintained by friction, and create the magnetic field. Why this field switches orientation at irregular intervals of between a few tens of thousands and a few million years, is one of the central questions. The answers are difficult; experiments are not yet possible, 150 years of measurements is a short time when compared with the last transition – about 730 000 years ago – and computer simulations are extremely involved. Since the field measurements started, the dipole moment of the magnetic field has steadily diminished, presently by 5% a year, and the quadrupole moment has steadily increased. Maybe we are heading towards a surprise.* (By the way, the study of *galactic* magnetic fields is even more complex, and still in its infancy.)

Another important puzzle about electricity results from the equivalence of mass and energy. It is known from experiments that the size $d$ of electrons is surely smaller than
Ref. 576 $10^{-22}$ m. This means that the electric field surrounding it has an energy content $E$ given
Challenge 1075 ny by at least

$$E_{\text{nergy}} = \frac{1}{2}\varepsilon_0 \int E^2_{\text{lectric field}}\, \mathrm{d}V = \frac{1}{2}\varepsilon_0 \int_d^\infty \Big(\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\Big)^2 4\pi r^2 \mathrm{d}r$$
$$= \frac{q^2}{8\pi\varepsilon_o}\frac{1}{d} > 1.2\,\mu\mathrm{J}\,. \qquad (453)$$

On the other hand, the *mass* of an electron, usually given as 511 keV/c$^2$, corresponds to an energy of only 82 fJ, ten million times *less* than the value just calculated. In other words, classical electrodynamics has considerable difficulty describing electrons. In fact, a consistent description of charged point particles within classical electrodynamics is
Ref. 577 impossible. This pretty topic receives only a rare – but then often passionate – interest nowadays, because the puzzle is solved in a different way in the upcoming, second part of our mountain ascent.

Even though the golden days of materials science are over, the various electromagnetic properties of matter and their applications in devices do not seem to be completely explored yet. About once a year a new effect is discovered that merits inclusion in the list
Page 605 of electromagnetic matter properties of Table 50. Among others, some newer semiconductor technologies will still have an impact on electronics, such as the recent introduction of low cost light detecting integrated circuits built in CMOS (complementary metal oxide silicon) technology.

The building of light sources of high quality has been a challenge for many centuries and remains one for the future. Light sources that are intense, tunable and with large coherence length or sources that emit extreme wavelengths are central to many research pursuits. As one example of many, the first X-ray lasers have recently been built; however, they are several hundred metres in size and use modified particle accelerators. The construction of compact X-ray lasers is still many years off – if it is possible at all.

* In 2005, it has been reported that the inner core of the Earth seems to rotate faster than the Earth's crust
Ref. 575 by up to half a degree per year.

Electrodynamics and general relativity interact in many ways. Only a few cases have been studied up to now. They are important for black holes and for empty space. For example, it seems that magnetic fields increase the stiffness of empty space. Many such topics will appear in the future. Ref. 578

But maybe the biggest challenge imaginable in classical electrodynamics is to decode the currents inside the brain. Will it be possible to read our thoughts with an apparatus placed outside the head? One could start with a simpler challenge: Would it be possible to distinguish the thought 'yes' from the thought 'no' by measuring electrical or magnetic fields around the head? In other words, is mind-reading possible? Maybe the twenty-first century will come up with a positive answer. If so, the team performing the feat will be instantly famous. Challenge 1076 r

## Levitation

We have seen that it is possible to move certain objects without touching them, using a magnetic or electric field or, of course, using gravity. Is it also possible, without touching an object, to keep it fixed, floating in mid-air? Does this type of rest exist?

It turns out that there are several methods of levitating objects. These are commonly divided into two groups: those that consume energy and those who do not. Among the methods that consume energy is the floating of objects on a jet of air or of water, the floating of objects through sound waves, e.g. on top of a siren, or through a laser beam coming from below, and the floating of conducting material, even of liquids, in strong radio-frequency fields. Levitation of liquids or solids by strong ultrasound waves is presently becoming popular in laboratories. All these methods give *stationary* levitation. Another group of energy consuming methods sense the way a body is falling and kick it up again in the right way via a feedback loop; these methods are *non-stationary* and usually use magnetic fields to keep the objects from falling. The magnetic train being built in Shanghai by a German consortium is levitated this way. The whole train, including the passengers, is levitated and then moved forward using electromagnets. It is thus possible, using magnets, to levitate many tens of tonnes of material. Ref. 579 Ref. 580 Ref. 581

For levitation methods that do *not* consume energy – all such methods are necessarily stationary – a well-known limitation can be found by studying Coulomb's 'law' of electrostatics: no static arrangement of electric fields can levitate a *charged* object in free space or in air. The same result is valid for gravitational fields and *massive* objects;* in other words, we cannot produce a local minimum of potential energy in the middle of a box using electric or gravitational fields. This impossibility is called *Earnshaw's theorem*. Speaking mathematically, the solutions of the Laplace equation $\Delta\varphi = 0$, the so-called *harmonic functions*, have minima or maxima only at the border, and never inside the domain of definition. (You proved this yourself on page 121.) The theorem can also be proved by noting that given a potential minimum in free space, Gauss' theorem for a sphere around that minimum requires that a source of the field be present inside, which is in contradiction with the original assumption. Ref. 582

* To the disappointment of many science-fiction addicts, this would even be true if a negative mass existed. And even though gravity is not really due to a field, but to space-time curvature, the result still holds in general relativity. Page 80

We can deduce that it is also impossible to use electric fields to levitate an electrically *neutral* body in air: the potential energy $U$ of such a body, with volume $V$ and dielectric constant $\varepsilon$, in an environment of dielectric constant $\varepsilon_0$, is given by

$$\frac{U}{V} = -\frac{1}{2}(\varepsilon - \varepsilon_0)E^2 \ . \quad (454)$$

Challenge 1077 ny Since the electric field $E$ never has a maximum in the absence of space charge, and since for all materials $\varepsilon > \varepsilon_0$, there cannot be a minimum of potential energy in free space for a neutral body.*

To sum up, using static electric or static gravitational fields it is impossible to keep an object from falling; neither quantum mechanics, which incorporates phenomena such as antimatter, nor general relativity, including phenomena such as black holes, change this basic result.

For static *magnetic* fields, the argument is analogous to electrical fields: the potential energy $U$ of a magnetizable body of volume $V$ and permeability $\mu$ in a medium with permeability $\mu_0$ containing no current is given by (Challenge 1079 ny)

$$\frac{U}{V} = -\frac{1}{2}\left(\frac{1}{\mu} - \frac{1}{\mu_0}\right)B^2 \quad (455)$$

and due to the inequality $\Delta B^2 \geqslant 0$, isolated maxima of a static magnetic field are not possible, only isolated minima. Therefore, it is impossible to levitate paramagnetic ($\mu > \mu_o$) or ferromagnetic ($\mu \gg \mu_0$) materials such as steel, including bar magnets, which are all attracted, and not repelled to magnetic field maxima. (Challenge 1080 ny)

There are thus two ways to get magnetic levitation: levitating a diamagnet or using a time dependent field. Diamagnetic materials ($\mu < \mu_o$) can be levitated by static magnetic fields because they are attracted to magnetic field minima; the best-known example is the levitation of superconductors, which are, at least those of type I, perfects diamagnets ($\mu = 0$). Strong forces can be generated, and this method is also being tested for the levitation of passenger trains in Japan. In some cases, superconductors can even be *suspended* in mid-air, below a magnet. Single atoms with a magnetic moment are also diamagnets; they are routinely levitated this way and have also been photographed in this state. (Ref. 584, Ref. 581, Ref. 585)

Also single neutrons, which have a magnetic dipole moment, have been kept in magnetic bottles in this way, until they decay. Recently, scientists have levitated pieces of wood, plastic, strawberries, water droplets, liquid helium droplets as large as 2 cm, grasshoppers, fish and frogs (all alive and without any harm) in this way. They are, like humans, all made of diamagnetic material. Humans themselves have not yet been levitated, but the feat is being planned and worked on. (Ref. 586)

Diamagnets levitate if $\nabla B^2 > 2\mu_0 \rho g/\chi$, where $\rho$ is the mass density of the object and $\chi = 1 - \mu/\mu_0$ its magnetic susceptibility. Since $\chi$ is typically about $10^{-5}$ and $\rho$ of order $1000\,\mathrm{kg/m^3}$, field gradients of about $1000\,\mathrm{T^2/m}$ are needed. In other words, levitation re- (Challenge 1081 ny)

Ref. 583 * It is possible, however, to 'levitate' gas bubbles in liquids – 'trap' them to prevent them from rising would be a better expression – because in such a case the dielectric constant of the environment is higher than that of the gas. Can you find a liquid–gas combination where bubbles fall instead of rise? (Challenge 1078 ny)

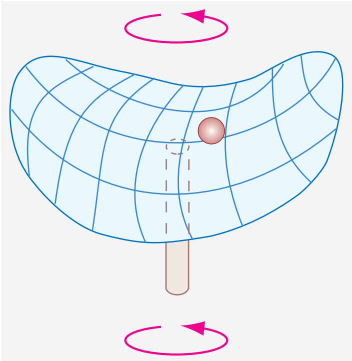

FIGURE 277 Trapping a metal sphere using a variable speed drill and a plastic saddle

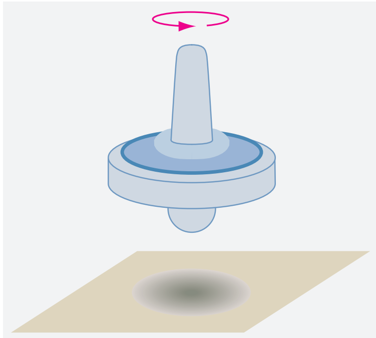

FIGURE 278 Floating 'magic' nowadays available in toy shops

quires fields changes of 10 T over 10 cm, which is nowadays common for high field laboratory magnets.

Finally, *time dependent* electrical or magnetic fields, e.g. periodic fields, can lead to levitation in many different ways without any consumption of energy. This is one of the methods used in the magnetic bearings of turbomolecular vacuum pumps. Also single charged particles, such as ions and electrons, are now regularly levitated with Paul traps and Penning traps. The mechanical analogy is shown in Figure 277.

Ref. 579

Ref. 579

Figure 278 shows a toy that allows you to personally levitate a spinning top in mid-air above a ring magnet, a quite impressive demonstration of levitation for anybody looking at it. It is not hard to build such a device yourself.

Ref. 587

Ref. 588

Even free electrons can be levitated, letting them float above the surface of fluid helium. In the most recent twist of the science of levitation, in 1995 Stephen Haley predicted that the suspension height of small magnetic particles above a superconducting ring should be quantized. However, the prediction has not been verified by experiment yet.

Ref. 589

For the sake of completeness we mention that nuclear forces cannot be used for levitation in everyday life, as their range is limited to a few femtometres. However, we will see later that the surface matter of the Sun is prevented from falling into the centre by these interactions; we could thus say that it is indeed levitated by nuclear interactions.

Page 922

## Matter, levitation and electromagnetic effects

The levitation used by magicians mostly falls into another class. When David Copperfield, a magician performing for the MTV generation at the end of the twentieth century, 'flies' during his performances, he does so by being suspended on thin fishing lines that are rendered invisible by clever lighting arrangements. In fact, if we want to be precise, we should count fishing lines, plastic bags, as well as every table and chair as levitation devices. (Journalists would even call them 'anti-gravity' devices.) Contrary to our impression, a hanging or lying object is not really in contact with the suspension, if we look at the critical points with a microscope.* More about this in the second part of our walk.

Ref. 590

Challenge 1082 ny

* The issue is far from simple: which one of the levitation methods described above is used by tables or chairs?

But if this is the case, why don't we fall through a table or through the floor? We started the study of mechanics by stating that a key property of matter its *solidity*, i.e. the impossibility of having more than one body at the same place at the same time. But what is the origin of solidity? Again, we will be able to answer the question only in the second part of our adventure, but we can already collect the first clues at this point. Page 840

Solidity is due to electricity. Many experiments show that matter is constituted of charged particles; indeed, matter can be moved and influenced by electromagnetic fields in many ways. Over the years, material scientists have produced a long list of such effects, all of which are based on the existence of charged constituents. Can you find or imagine a new one? For example, can electric charge change the colour of objects? Ref. 591 Challenge 1083 r

TABLE 50 Selected matter properties related to electromagnetism, showing among other things the role it plays in the constitution of matter; at the same time a short overview of atomic, solid state, fluid and business physics

| Property | Example | Definition |
|---|---|---|
| thermal radiation or heat radiation or incandescence | every object | temperature-dependent radiation emitted by any macroscopic amount of matter |
| **Interactions with charges and currents** | | |
| electrification | separating metals from insulators | spontaneous charging |
| triboelectricity | glass rubbed on cat fur | charging through rubbing |
| barometer light | mercury slipping along glass | gas discharge due to triboelectricity Ref. 592 |
| insulation | air | no current flow below critical voltage drop |
| semiconductivity | diamond, silicon or gallium arsenide | current flows only when material is impure ('doped') |
| conductivity | copper, metals | current flows easily |
| superconductivity | niobium | current flows indefinitely |
| ionization | fire flames | current flows easily |
| localization (weak, Anderson) | disordered solids | resistance of disordered solids |
| resistivity, Joule effect | graphite | heating due to current flow |
| thermoelectric effects: Peltier effect, Seebeck effect, Thomson effect | ZnSb, PbTe, PbSe, BiSeTe, $Bi_2Te_3$, etc. | cooling due to current flow, current flow due to temperature difference, or due to temperature gradients |
| acoustoelectric effect | CdS | sound generation by currents, and vice versa |
| magnetoresistance | iron, metal multilayers | resistance changes with applied magnetic field Ref. 593 |
| recombination | fire alarms | charge carriers combine to make neutral atoms or molecules |
| annihilation | positron tomography | particle and antiparticle, e.g. electron and positron, disappear into photons |

| Property | Example | Definition |
|---|---|---|
| Penning effect | Ne, Ar | ionization through collision with metastable atoms |
| Richardson effect, thermal emission | $BaO_2$, W, Mo, used in tv and electron microscopes | emission of electrons from hot metals |
| skin effect | Cu | high current density on exterior of wire |
| pinch effect | InSb, plasmas | high current density on interior of wire |
| Josephson effect | Nb-Oxide-Nb | tunnel current flows through insulator between two superconductors |
| Sasaki–Shibuya effect | n-Ge, n-Si | anisotropy of conductivity due to applied electric field |
| switchable magnetism | InAs:Mn | voltage switchable magnetization Ref. 594 |
| **Interactions with magnetic fields** | | |
| Hall effect | silicon; used for magnetic field measurements | voltage perpendicular to current flow in applied magnetic field |
| Zeeman effect | Cd | change of emission frequency with magnetic field |
| Paschen–Back effect | atomic gases | change of emission frequency in strong magnetic fields |
| ferromagnetism | Fe, Ni, Co, Gd | spontaneous magnetization; material strongly attracted by magnetic fields |
| paramagnetism | Fe, Al, Mg, Mn, Cr | induced magnetization parallel to applied field; attracted by magnetic fields |
| diamagnetism | water, Au, graphite, NaCl | induced magnetization opposed to applied field; repelled by magnetic fields |
| magnetostriction | $CeB_6$, $CePd_2Al_3$ | change of shape or volume by applied magnetic field |
| magnetoelastic effect | Fe, Ni | change of magnetization by tension or pressure |
| acoustomagnetic effect | metal alloys, anti-theft stickers | excitation of mechanical oscillations through magnetic field |
| spin valve effect | metal multilayers | electrical resistance depends on spin direction of electrons with respect to applied magnetic field |
| magneto-optical activity or Faraday effect or Faraday rotation | flint glass | polarization angle is rotated with magnetic field; different refraction index for right and left circularly polarized light, as in magneto-optic (MO) recording |
| magnetic circular dichroism | gases | different absorption for right- and left-circularly polarized light; essentially the same as the previous one |
| Majorana effect | colloids | specific magneto-optic effect |

| Property | Example | Definition |
|---|---|---|
| photoelectromagnetic effect | InSb | current flow due to light irradiation of semiconductor in a magnetic field |
| Voigt effect | vapours | birefringence induced by applied magnetic field |
| Cotton–Mouton effect | liquids | birefringence induced by applied magnetic field |
| Hanle effect | Hg | change of polarization of fluorescence with magnetic field |
| Shubnikov–de Haas effect | Bi | periodic change of resistance with applied magnetic field |
| thermomagnetic effects: Ettinghausen effect, Righi–Leduc effect, Nernst effect, magneto–Seebeck effect | BiSb alloys | relation between temperature, applied fields and electric current |
| Ettinghausen–Nernst effect | Bi | appearance of electric field in materials with temperature gradients in magnetic fields |
| photonic Hall effect | $CeF_3$ | transverse light intensity depends on the applied magnetic field Ref. 595 |
| magnetocaloric effect | gadolinium, GdSiGe alloys | material cools when magnetic field is switched off Ref. 596 |
| cyclotron resonance | semiconductors, metals | selective absorption of radio waves in magnetic fields |
| magnetoacoustic effect | semiconductors, metals | selective absorption of sound waves in magnetic fields |
| magnetic resonance | most materials, used for imaging in medicine for structure determination of molecules | selective absorption of radio waves in magnetic fields |
| magnetorheologic effect | liquids, used in advanced car suspensions | change of viscosity with applied magnetic fields |
| Meissner effect | type 1 superconductors, used for levitation | expulsion of magnetic field from superconductors |
| **Interactions with electric fields** | | |
| polarizability | all matter | polarization changes with applied electric field |
| ionization, field emission, Schottky effect | all matter, tv | charges are extracted at high fields |
| paraelectricity | $BaTiO_3$ | applied field leads to polarization in same direction |
| dielectricity | water | in opposite direction |

| Property | Example | Definition |
|---|---|---|
| ferroelectricity | $BaTiO_3$ | spontaneous polarization below critical temperature |
| piezoelectricity | the quartz lighter used in the kitchen | polarization appears with tension, stress, or pressure |
| electrostriction | platinum sponges in acids | shape change with applied voltage Ref. 597 |
| pyroelectricity | $CsNO_3$, tourmaline, crystals with polar axes; used for infrared detection | change of temperature produces charge separation |
| electro-osmosis or electrokinetic effect | many ionic liquids | liquid moves under applied electric field Ref. 598 |
| electrowetting | salt solutions on gold | wetting of surface depends on applied voltage |
| electrolytic activity | sulphuric acid | charge transport through liquid |
| liquid crystal effect | watch displays | molecules turn with applied electric field |
| electro-optical activity: Kerr effect, Pockels effect | liquids (e.g. oil), crystalline solids | material in electric field rotates light polarization, i.e. produces birefringence |
| Freederichsz effect, Schadt–Helfrichs effect | nematic liquid crystals | electrically induced birefringence |
| Stark effect | hydrogen, mercury | colour change of emitted light in electric field |
| field ionization | helium near tungsten tips in field ion microscope | ionization of gas atoms in strong electric fields |
| Zener effect | Si | energy-free transfer of electrons into conduction band at high fields |
| field evaporation | W | evaporation under strong applied electric fields |
| **Interactions with light** | | |
| absorption | coal, graphite | transformation of light into heat or other energy forms (which ones?) Challenge 1084 n |
| blackness | coal, graphite | complete absorption in visible range |
| colour, metallic shine | ruby | absorption depending on light frequency |
| photostriction | PbLaZrTi | light induced piezoelectricity |
| photography | AgBr, AgI | light precipitates metallic silver |
| photoelectricity, photoeffect | Cs | current flows into vacuum due to light irradiation |
| internal photoelectric effect | Si p–n junctions, solar cells | voltage generation and current flow due to light irradiation |
| photon drag effect | p-Ge | current induced by photon momentum |
| emissivity | all bodies | ability to emit light |

| Property | Example | Definition |
|---|---|---|
| transparency | glass, quartz, diamond | low reflection, low absorption, low scattering |
| reflectivity | metals | light bounces on surface |
| polarization | pulled polymer sheets | light transmission depending on polarization angle |
| optical activity | sugar dissolved in water, quartz | rotation of polarization |
| birefringence | feldspar,cornea | refraction index depends on polarization direction, light beams are split into two beams |
| dichroism | feldspar, andalusite | absorption depends on polarization |
| optically induced anisotropy, Weigert effect | AgCl | optically induced birefringence and dichroism |
| second harmonic generation | $LiNbO_3$, $KPO_4$ | light partially transformed to double frequency |
| luminescence: general term for opposite of incandescence | GaAs, tv | cold light emission |
| fluorescence | $CaF_2$, X-ray production, light tubes, cathode ray tubes | light emission during and after light absorption or other energy input |
| phosphorescence | $TbCl_3$ | light emission due to light, electrical or chemical energy input, continuing *long after* stimulation |
| electroluminescence | ZnS | emission of light due to alternating electrical field |
| photoluminescence | ZnS : Cu, $SrAlO_4$ : Eu, Dy, hyamine | light emission triggered by UV light, used in safety signs |
| chemoluminescence | $H_2O_2$, phenyl oxalate ester, dye | cold light emission used in light sticks for divers and fun |
| bioluminescence | glow-worm, deep sea fish | cold light emission in animals |
| triboluminescence | sugar | light emission during friction or crushing |
| thermoluminescence | quartz, feldspar | light emission during heating, used e.g. for archaeological dating of pottery Ref. 599 |
| bremsstrahlung | X-ray generation | radiation emission through fast deceleration of electrons |
| Compton effect | momentum measurements | change of wavelength of light, esp. X-rays and gamma radiation, colliding with matter |

| Property | Example | Definition |
|---|---|---|
| Čerenkov effect | water, polymer particle detectors | light emission in a medium due to particles, e.g. emitted by radioactive processes, moving faster than the speed of light in that medium |
| transition radiation | any material | light emission due to fast particles moving from one medium to a second with different refractive index |
| electrochromicity | wolframates | colour change with applied electric field |
| scattering | gases, liquids | light changes direction |
| Mie scattering | dust in gases | light changes direction |
| Raleigh scattering | sky | light changes direction, sky is blue |
| Raman effect or Smekal–Raman effect | molecular gases | scattered light changes frequency |
| laser activity, superradiation | beer, ruby, He–Ne | emission of stimulated radiation |
| sonoluminescence | air in water | light emission during cavitation |
| gravitoluminescence | does not exist; Challenge 1085 n why? | |
| switchable mirror | LaH | voltage controlled change from reflection to transparency Ref. 600 |
| radiometer effect | bi-coloured windmills | irradiation turns mill (see page 573) |
| luminous pressure | *idem* | irradiation turns mill directly |
| solar sail effect | future satellites | motion due to solar wind |
| acoustooptic effect | $LiNbO_3$ | diffraction of light by sound in transparent materials |
| photorefractive materials | $LiNbO_3$, GaAs, InP | light irradiation changes refractive index |
| Auger effect | Auger electron spectroscopy | electron emission due to atomic reorganization after ionization by X-rays |
| Bragg reflection | crystal structure determination | X-ray diffraction by atomic planes |
| Mößbauer effect | Fe, used for spectroscopy | recoil-free resonant absorption of gamma radiation |
| pair creation | Pb | transformation of a photon in a charged particle–antiparticle pair |
| photoconductivity | Se, CdS | change of resistivity with light irradiation |
| optoacoustic affect, photoacoustic effect | gases, solids | creation of sound due to absorption of pulsed light |
| optogalvanic effect | plasmas | change of discharge current due to light irradiation |
| optical nonlinear effects: parametric amplification, frequency mixing, saturable absorption, $n$-th harmonic generation, optical Kerr effect, etc. | | |
| phase conjugated mirror activity | gases | reflection of light with opposite phase |

| Property | Example | Definition |
|---|---|---|
| **Material properties** | | |
| solidity, impenetrability | floors, columns, ropes, buckets | at most one object per place at a given time |
| **Interactions with vacuum** | | |
| Casimir effect | metals | attraction of uncharged, conducting bodies |

All matter properties in the list can be influenced by electric or magnetic fields or directly depend on them. This shows that the nature of all these material properties is electromagnetic. In other words, charges and their interactions are an essential and fundamental part of the structure of objects. The table shows so many different electromagnetic properties that the motion of charges inside each material must be complex indeed. Most effects are the topic of solid state physics,* fluid and plasma physics.

Solid state physics is by far the most important part of physics, when measured by the impact it has on society. Almost all effects have applications in technical products, and give employment to many people. Can you name a product or business application for Challenge 1086 e any randomly chosen effect from the table?

In our mountain ascent however, we look at only one example from the above list: thermal radiation, the emission of light by hot bodies.

Earnshaw's theorem about the impossibility of a stable equilibrium for charged particles at rest implies that the charges inside matter must be *moving*. For any charged particle in motion, Maxwell's equations for the electromagnetic field show that it radiates energy by emitting electromagnetic waves. In short, classical mechanics thus predicts that matter must radiate electromagnetic energy.

Interestingly, everybody knows from experience that this is indeed the case. Hot bodies light up depending on their temperature; the working of light bulbs thus proves that metals are made of charged particles. *Incandescence*, as it is called, requires charges. Actually, *every* body emits radiation, even at room temperature. This radiation is called *thermal radiation*; at room temperature it lies in the infrared. Its intensity is rather weak Ref. 601 in everyday life; it is given by the general expression

$$I(T) = f\,T^4\,\frac{2\pi^5 k^4}{15c^2h^3} \quad \text{or} \quad I(T) = f\sigma T^4 \quad \text{with} \quad \sigma = 56.7\,\text{nW/K}^4\text{m}^2\,, \qquad (456)$$

where $f$ is a material-, shape- and temperature-dependent factor, with a value between zero and one, and is called the *emissivity*. The constant $\sigma$ is called the *Stefan–Boltzmann black body radiation constant* or *black body radiation constant*. A body whose emissivity is given by the ideal case $f = 1$ is called a *black body*, because at room temperature such a Challenge 1087 n body also has an ideal absorption coefficient and thus appears black. (Can you see why?) The heat radiation such a body emits is called *black body radiation*.

Ref. 602 By the way, which object radiates more energy: a human body or an average piece of Challenge 1088 n the Sun of the same mass? Guess first!

* Probably the best and surely the most entertaining introductory English language book on the topic is the

## Why can we see each other?

Physicists have a strange use of the term 'black'. Most bodies at temperatures at which they are red hot or even hotter are excellent approximations of black bodies. For example, the tungsten in incandescent light bulbs, at around 2000 K, emits almost pure black body radiation; however, the glass then absorbs much of the ultraviolet and infrared components. Black bodies are also used to define the colour *white*. What we commonly call pure white is the colour emitted by a black body of 6500 K, namely the Sun. This definition Ref. 603 is used throughout the world, e.g. by the Commission Internationale d'Eclairage. Hotter black bodies are bluish, colder ones are yellow, orange or red.* The stars in the sky are classified in this way, as summarized on page 186.

Let us make a quick summary of black body radiation. Black body radiation has two important properties: first, the emitted light power increases with the fourth power of the temperature. With this relation alone you can check the temperature of the Sun, mentioned above, simply by comparing the size of the Sun with the width of your thumb when Challenge 1089 d your arm is stretched out in front of you. Are you able to do this? (Hint: use the excellent Ref. 604 approximation that the Earth's average temperature of about 14.0°C is due to the Sun's irradiation.)

The precise expression for the emitted energy density $u$ per frequency $\nu$ can be deduced from the radiation law for black bodies discovered by Max Planck**

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1} . \qquad (457)$$

He made this important discovery, which we will discuss in more detail in the second part of our mountain ascent, simply by comparing this curve with experiment. The new constant $h$, *quantum of action* or *Planck's constant*, turns out to have the value $6.6 \cdot 10^{-34}$ Js, Page 705 and is central to all quantum theory, as we will see. The other constant Planck introduced, the Boltzmann constant $k$, appears as a prefactor of temperature all over thermodynamics, as it acts as a conversion unit from temperature to energy.

Challenge 1090 ny The radiation law gives for the total emitted energy density the expression

$$u(T) = T^4 \frac{8\pi^5 k^4}{15 c^3 h^3} \qquad (458)$$

Challenge 1091 ny from which equation (456) is deduced using $I = uc/4$. (Why?)

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one by Neil Ashcroft & David Mermin, *Solid State Physics*, Holt Rinehart & Winston, 1976.

* Most bodies are not black, because colour is not only determined by emission, but also by absorption of light.

** Max Planck (1858–1947), professor of physics in Berlin, was a central figure in thermostatics. He discovered and named *Boltzmann's constant* $k$ and the *quantum of action* $h$, often called Planck's constant. His introduction of the quantum hypothesis gave birth to quantum theory. He also made the works of Einstein known in the physical community, and later organized a job for him in Berlin. He received the Nobel Prize for physics in 1918. He was an important figure in the German scientific establishment; he also was one of the very few who had the courage to tell Adolf Hitler *face to face* that it was a bad idea to fire Jewish professors. (He got an outburst of anger as answer.) Famously modest, with many tragedies in his personal life, he was esteemed by everybody who knew him.

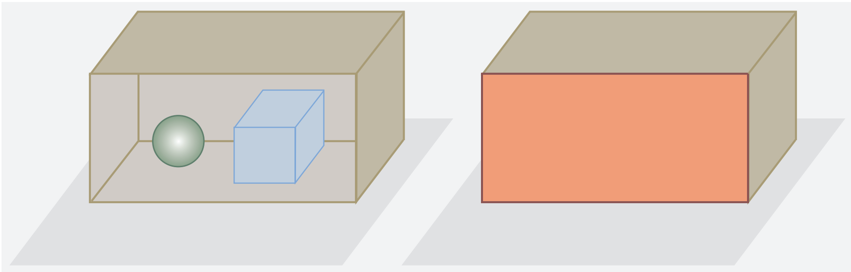

FIGURE 279 Bodies inside a oven at room temperature (left) and red hot (right)

The second property of black body radiation is the value of the peak wavelength, i.e. the wavelength emitted with the highest intensity. This wavelength determines their colour; it is deduced from equation (457) to be

Challenge 1092 ny

$$\lambda_{max} = \frac{hc}{4.956\,k}\frac{1}{T} = \frac{2.9\,\mathrm{mm\,K}}{T} \quad \text{but} \quad \hbar\nu_{max} = 2.82\,kT = (3.9\cdot10^{-23}\,\mathrm{J/K})\cdot T\,. \quad (459)$$

Either of these expressions is called *Wien's colour displacement* after its discoverer.* The colour change with temperature is used in optical thermometers; this is also the way the temperatures of stars are measured. For 37°C, human body temperature, it gives a peak wavelength of 9.3 µm or 115 THz, which is therefore the colour of the bulk of the radiation emitted by every human being. (The peak wavelength does not correspond to the peak frequency. Why?) On the other hand, following the telecommunication laws of many countries, any radiation emitter needs a licence to operate; it follows that strictly in Germany only dead people are legal, and only if their bodies are at absolute zero temperature.

Challenge 1093 ny

Note that a black body or a star can be blue, white, yellow, orange or red. It is never green. Can you explain why?

Challenge 1094 ny

Above, we predicted that any material made of charges emits radiation. Are you able to find a simple argument showing whether heat radiation is or is not this classically predicted radiation?

Challenge 1095 ny

But let us come back to the question in the section title. The existence of thermal radiation implies that any hot body will cool, even if it is left in the most insulating medium there is, namely in vacuum. More precisely, if the vacuum is surrounded by a wall, the temperature of a body in the vacuum will gradually approach that of the wall.

Interestingly, when the temperature of the wall and of the body inside have become the same, something strange happens. The effect is difficult to check at home, but impressive photographs exist in the literature.

Ref. 605

One arrangement in which walls and the objects inside them are at the same temperature is an *oven*. It turns out that it is *impossible* to see objects in an oven using the light coming from thermal radiation. For example, if an oven and all its contents are red hot, taking a picture of the inside of the oven (without a flash!) does not reveal anything; no contrast nor brightness changes exist that allow one to distinguish the objects from the walls or their surroundings. Can you explain the finding?

Challenge 1096 n

* Wilhelm Wien (b. 1864 Gaffken, d. 1824 München), East-Prussian physicist; he received the Nobel Prize for physics in 1911 for the discovery of this relation.

In short, we are able to see each other only because the light sources we use are at a *different* temperature from us. We can see each other only because we do *not* live in thermal equilibrium with our environment.

### A summary of classical electrodynamics and of its limits

In general, classical electrodynamics can be summarized in a few main ideas.

— The electromagnetic field is a physical observable, as shown e.g. by compass needles.
— The field sources are the (moving) charges and the field evolution is described by Maxwell's evolution equations, as shown, for example, by the properties of amber, lodestone, batteries and remote controls.
— The electromagnetic field changes the motion of electrically charged objects via the Lorentz expression as, for example, shown by electric motors.
— The field behaves like a continuous quantity, a distribution of little arrows, and propagates as a wave, as shown, for example, by radios and mobile phones.
— The field can exist and move in empty space, as shown, for example, by the stars.

As usual, the motion of the sources and the field is reversible, continuous, conserved and deterministic. However, there is quite some fun in the offing; even though this description is correct in everyday life, during the rest of our mountain ascent we will find that *each* of the bullet points is in fact wrong. A simple example shows this.

At a temperature of zero kelvin, when matter does not radiate thermally, we have the paradoxical situation that the charges inside matter cannot be moving, since no emitted radiation is observed, but they cannot be at rest either, due to Earnshaw's theorem. In short, the simple existence of matter – with its discrete charge values – shows that classical electrodynamics is wrong.

Page 605

In fact, the overview of material properties of Table 50 makes the same point even more strongly; classical electrodynamics can describe many of the effects listed, *but it cannot explain the origin of any of them*. Even though few of the effects will be studied in our walk – they are not essential for our adventure – the general concepts necessary for their description will be the topic of the second part of this mountain ascent, that on quantum theory.

## 17. Classical physics in a nutshell – one and a half steps out of three

The description of general relativity and classical electrodynamics concludes our walk hrough classical physics. In order to see its limitations, we summarize what we have found out. In nature, we learned to distinguish and to characterize objects, radiation and space-time. All these three can move. In all motion we distinguish the fixed, intrinsic properties from the varying state. All motion happens in such a way as to minimize change.

Looking for all the *fixed, intrinsic* aspects of objects, we find that all sufficiently small objects or particles are described completely by their mass and their electric charge. There is no magnetic charge. Mass and electric charge are thus the only localized intrinsic properties of classical, everyday objects. Both mass and electric charge are defined by the accelerations they produce around them. Both quantities are conserved; thus they can be added. Mass, in contrast to charge, is always positive. Mass describes the interaction of objects with their environment, charge the interaction with radiation.

All *varying* aspects of objects, i.e. their state, can be described using momentum and position, as well as angular momentum and orientation. All can vary continuously in amount and direction. Therefore the set of all possible states forms a space, the so-called *phase space*. The state of extended objects is given by the states of all its constituent particles. These particles make up all objects and somehow interact electromagnetically.

The state of a particle depends on the observer. The state is useful to calculate the change that occurs in motion. For a given particle, the change is independent of the observer, but the states are not. The states found by different observers are related: the relations are called the 'laws' of motion. For example, for different times they are called *evolution equations*, for different places and orientations they are called *transformation relations*, and for different gauges they are called *gauge transformations*. All can be condensed in the principle of least action.

We also observe the motion of a massless entity: *radiation*. Everyday types of radiation, such as light, radio waves and their related forms, are travelling electromagnetic waves. They are described by same equations that describe the interaction of charged or magnetic objects. The speed of massless entities is the maximum possible speed in nature and is the same for all observers. The *intrinsic properties* of radiation are its dispersion relation and its energy–angular momentum relation. The *state* of radiation is described by its electromagnetic field strength, its phase, its polarization and its coupling to matter. The motion of radiation describes the motion of images.

The space-time *environment* is described by space and time coordinates. Space-time is also able to move, by changing its curvature. The intrinsic properties of space-time are the number of dimensions, its signature and its topology. The state is given by the metric, which describes distances and thus the local warpedness. The warpedness can oscillate and propagate, so that empty space can move like a wave.

Our environment is finite in age. It has a long history, and on large scales, all matter in the universe moves away from all other matter. The large scale topology of our environment is unclear, as is unclear what happens at its spatial and temporal limits.

*Motion* follows a simple rule: change is always as small as possible. This applies to matter, radiation and space-time. All energy moves in the way space-time dictates it, and space moves the way energy dictates it. This relation describes the motion of the stars, of thrown stones, of light beams and of the tides. Rest and free fall are the same, and gravity is curved space-time. Mass breaks conformal symmetry and thus distinguishes space from time.

Energy and mass speed is bound from above by a universal constant $c$, and energy change per time is bound from above by a universal constant $c^5/4G$. The speed value $c$ is realized for the motion of massless particles. It also relates space to time. The power value $c^5/4G$ is realized by horizons. They are found around black holes and at the border of the universe. The value also relates space-time curvature to energy flow and thus describes the elasticity of space-time.

No two objects can be at the same spot at the same time. This is the first statement that humans encounter about electromagnetism. More detailed investigation shows that electric charge accelerates other charges, that charge is necessary to define length and time intervals, and that charges are the source of electromagnetic fields. Also light is such a field. Light travels at the maximum possible velocity. In contrast to objects, light can interpenetrate. In summary, we learned that of the two naive types of object motion, namely motion due to gravity – or space-time curvature – and motion due to the electromagnetic field, only the latter is genuine.

Above all, classical physics showed us that motion, be it linear or rotational, be it that of matter, radiation or space-time, is conserved. Motion is continuous. More than that, motion is similar to a continuous substance: it is never destroyed, never created, but always redistributed. Owing to conservation, all motion, that of objects, images and empty space, is predictable and reversible. Owing to conservation of motion, time and space can be defined. In addition, we found that classical motion is also right–left symmetric. Classical physics showed us that motion is predictable: there are *no* surprises in nature.

## The future of planet Earth

Maybe nature shows no surprises, but it still provides many adventures. On the 8th of March 2002, a 100 m sized body almost hit the Earth. It passed at a distance of only 450 000 km from our planet. On impact, it would have destroyed a region the size of London. A few months earlier, a 300 m sized body missed the Earth by 800 000 km; the record for closeness so far was in 1994, when the distance was only 100 000 km.* Several other adventures can be predicted by classical physics, as shown in Table 51. Many are problems facing humanity in the distant future, but some, such as volcanic eruptions or
Ref. 606 asteroid impacts, could happen at any time. All are research topics.

TABLE 51 Examples of disastrous motion of possible future importance

| Critical situation | Years from now |
|---|---|
| End of fundamental physics | *c.* 30 (around year 2030) |
| Giant tsunami from volcanic eruption at Canary islands | *c.* 10-200 |
| Major nuclear material accident or weapon use | unknown |
| Ozone shield reduction | *c.* 100 |
| Rising ocean levels due to greenhouse warming | *c.* 100-1 000 |
| End of applied physics | > 200 |
| Explosion of volcano in Greenland, leading to long darkening of sky | unknown |
| Several magnetic north and south poles appear, allowing solar storms to disturb radio and telecommunications, to interrupt electricity supplies, to increase animal mutations and to disorient migrating animals such as wales, birds and tortoises | *c.* 800 |
| Our interstellar gas cloud detaches from the solar systems, changing the size of the heliosphere, and thus expose us more to aurorae and solar magnetic fields | *c.* 3 000 |

* The web pages around http://cfa-www.harvard.edu/iau/lists/Closest.html provide more information on such events.

| Critical situation | Years from now |
|---|---|
| Reversal of Earth's magnetic field, implying a time with almost no magnetic field, with increased cosmic radiation levels and thus more skin cancers and miscarriages | unknown |
| Atmospheric oxygen depletion due to forest reduction and exaggerated fuel consumption | > 1000 |
| Upcoming ice age | *c.* 15 000 |
| Possible collision with interstellar gas cloud assumed to be crossed by the Earth every 60 million years, maybe causing mass extinctions | *c.* 50 000 |
| Explosion of Yellowstone or other giant volcano leading to year-long volcanic winter | 0 to 100 000 |
| Possible genetic degeneration of homo sapiens due to Y chromosome reduction | *c.* 200 000 |
| Africa collides with Europe, transforming the Mediterranean into a lake that starts evaporating | around $3 \cdot 10^6$ |
| Gamma ray burst from within our own galaxy, causing radiation damage to many living beings | between 0 and $5 \cdot 10^6$ |
| Asteroid hitting the Earth, generating tsunamis, storms, darkening sunlight, etc. | between 0 and $50 \cdot 10^6$ |
| Neighbouring star approaching, starting comet shower through destabilization of Oort cloud and thus risk for life on Earth | $> 10^6$ |
| American continent collides with Asia | $> 100 \cdot 10^6$ |
| Instability of solar system | $> 100 \cdot 10^6$ |
| Low atmospheric $CO_2$ content stops photosynthesis | $> 100 \cdot 10^6$ |
| Collision of Milky Way with star cluster or other galaxy | $> 150 \cdot 10^6$ |
| Sun ages and gets hotter, evaporating seas | $> 250 \cdot 10^6$ |
| Ocean level increase due to Earth rotation slowing/stopping (if not evaporated before) | $> 10^9$ |
| Temperature rise/fall (depending on location) due to Earth rotation stop | $> 10^9$ |
| Sun runs out of fuel, becomes red giant, engulfs Earth | $5.0 \cdot 10^9$ |
| Sun stops burning, becomes white dwarf | $5.2 \cdot 10^9$ |
| Earth core solidifies, removing magnetic field and thus Earth's cosmic radiation shield | $10.0 \cdot 10^9$ |
| Nearby nova (e.g. Betelgeuse) bathes Earth in annihilation radiation | unknown |
| Nearby supernova (e.g. Eta Carinae) blasts over solar system | unknown |
| Galaxy centre destabilizes rest of galaxy | unknown |
| Universe recollapses – if ever (see page 377) | $> 20 \cdot 10^9$ |
| Matter decays into radiation – if ever (see Appendix C) | $> 10^{33}$ |
| Problems with naked singularities | unknown, controversial |
| Vacuum becomes unstable | unknown, controversial |

Despite the fascination of the predictions, we leave aside these literally tremendous issues and continue on our adventure.

## The essence of classical physics – the infinitely small implies the lack of surprises

We can summarize classical physics with a simple statement: nature lacks surprises because *classical physics is the description of motion using the concept of the infinitely small*. All concepts used so far, be they for motion, space, time or observables, assume that the infinitely small exists. Special relativity, despite the speed limit, still allows infinitely small velocities; general relativity, despite its black hole limit, still allows infinitely small force and power values. Similarly, in the description of electrodynamics and gravitation, both integrals and derivatives are abbreviations of mathematical processes that use infinitely small intermediate steps.

In other words, the classical description of nature introduces the infinitely small in the description of motion. The classical description then discovers that there are no surprises in motion. The detailed study of this question lead us to a simple conclusion: the infinitely small implies determinism.* Surprises contradict the existence of the infinitely small.

On the other hand, both special and general relativity have eliminated the existence of the infinitely large. There is no infinitely large force, power, size, age or speed.

## Why have we not yet reached the top of the mountain?

> "The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote... Our future discoveries must be looked for in the sixth place of decimals.
>
> Albert Michelson.** "

We might think that we know nature now, as did Albert Michelson at the end of the nineteenth century. He claimed that electrodynamics and Galilean physics implied that the major laws of physics were well known. The statement is often quoted as an example of flawed predictions, since it reflects an incredible mental closure to the world around him. General relativity was still unknown, and so was quantum theory.

At the end of the nineteenth century, the progress in technology due to the use of electricity, chemistry and vacuum technology had allowed better and better machines and apparatuses to be built. All were built with classical physics in mind. In the years between 1890 and 1920, these classical machines completely destroyed the foundations of classical physics. Experiments with these apparatuses showed that matter is made of atoms, that electrical charge comes in the smallest amounts and that nature behaves randomly. Nature does produce surprises – through in a restricted sense, as we will see. Like

* No surprises also imply no miracles. Classical physics is thus in opposition to many religions. Indeed, many religions argue that infinity is the necessary ingredient to perform miracles. Classical physics shows that this is not the case.

** From his 1894 address at the dedication ceremony for the Ryerson Physical Laboratory at the University of Chicago.

the British Empire, the reign of classical physics collapsed. Speaking simply, classical physics does not describe nature at small scales.

But even without machines, the Victorian physicist could have predicted the situation. (In fact, many more progressive minds did so.) He had overlooked a contradiction between electrodynamics and nature, for which he had no excuse. In our walk so far we found that clocks and metre bars are necessarily made of matter and based on electromagnetism. But as we just saw, classical electrodynamics does not explain the stability of matter. Matter is made of small particles, but the relation between these particles, electricity and the smallest charges is not clear. If we do not understand matter, we do not yet understand space and time, since they are defined using measurement devices made of matter.

Worse, the Victorian physicist overlooked a simple fact: the classical description of nature does not allow one to understand *life*. The abilities of living beings – growing, seeing, hearing, feeling, thinking, being healthy or sick, reproducing and dying – are all unexplained by classical physics. In fact, all these abilities contradict classical physics. Understanding matter and its interactions, including life itself, is therefore the aim of the second part of our ascent of Motion Mountain. The understanding will take place at small scales; to understand nature, we need to study particles. Indeed, the atomic structure of matter, the existence of a smallest charge and the existence of a smallest entropy makes us question the existence of the infinitely small. There is something to explore. Doing so will lead us from surprise to surprise. To be well prepared, we first take a break.

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$$\frac{L}{M} = \frac{2m}{e} \cdot \frac{1}{g} , \qquad (460)$$

where $e$ is the electron charge and $m$ its mass. Both $L$ and $M$ are measurable. The first measurements were published with a $g$-value of 1, most probably because the authors expected the value. In later experiments, de Haas found other values. Measurements by other researchers gave values nearer to 2 than to 1, a fact that was only understood with the discovery of spin. The original publications are A. Einstein & W.J. de Haas, Proefondervinderlijk bewijs voor het bestaan der moleculaire stroomen van Ampère, *Konninklijke Akademie der Wetenschappen te Amsterdam, Verslagen* **23**, p. 1449, 1915, and A. Einstein & W.J. de Haas, Experimental proof of the existence of Ampère's molecular currents, *Konninklijke Akademie der Wetenschappen te Amsterdam, Proceedings* **18**, p. 696, 1916. Cited on page 532.

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**536** There are also other ways to see the green ray, for longer times, namely when a fata morgana appears at sunset. An explanation with colour photograph is contained in M. Vollmer, Gespiegelt in besonderen Düften ...– Oasen, Seeungeheuer und weitere Spielereien der Fata Morgana, *Physikalische Blätter* **54**, pp. 903–909, 1998. Cited on page 576.

**537** This famous discovery is by Brent Berlin & Paul Kay, *Basic Color Terms: Their Universality and Evolution*, University of California Press, 1969. Their ongoing *world colour survey* is eagerly awaited. Of course there are also ongoing studies to find possible exceptions; the basic structure is solid, as shown in the conference proceedings C.L. Hardin & Luisa Maffi, *Colour Categories in Thought and Language*, Cambridge University Press, 1997. Cited on page 577.

**538** For a thorough discussion of the various velocities connected to wave trains, see the classic text by Louis Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York, 1960. It expands in detail the theme discussed by Arnold Sommerfeld, Über die Fortpflanzung des Lichtes in dispergierenden Medien, *Annalen der Physik,* 4th series, **44**, pp. 177–202, 1914. See also Arnold Sommerfeld, *Optik*, Dietrichssche Verlagsbuchandlung, Wiesbaden 1950, section 22. An English translation Arnold Sommerfeld, *Lectures on Theoretical Physics: Optics*, 1954, is also available. Cited on pages 577, 578, and 579.

**539** Changing the group velocity in fibers is now even possible on demand, as shown by M. González-Herráez, K.-Y. Song & L. Thévenaz, Optically controlled slow and fast light in optical fibers using stimulated Brillouin scattering, *Applied Physics Letters* **87**, p. 081113, 2005. They demonstrate group velocities from $0.24c$ to plus infinity and beyond, to negative values. Cited on page 578.

Another experiment was carried out by S. Chu & S. Wong, Linear pulse propagation in an absorbing medium, *Physical Review Letters* **48**, pp. 738–741, 1982. See also S. Chu & D. Styer, Answer to question #52. Group velocity and energy propagation, *American Journal of Physics* **66**, pp. 659–661, 1998. Another example was described in 1993 by the group of Raymond Chiao for the case of certain nonlinear materials in R. Chiao, P.G. Kwait & A.M. Steinberg, Faster than light?, *Scientific American* **269**, p. 52, August 1993, and R.Y. Chiao, A.E. Kozhekin & G. Kurizki, Tachyonlike excitations in inverted two-level media, *Physical Review Letters* **77**, pp. 1254–1257, 1996. On still another experimental set-up using anomalous dispersion in caesium gas, see L.J. Wang, A. Kuzmich & A. Dogarin, Gain-assisted superluminal light propagation, *Nature* **406**, pp. 277–279, 20 July 2000.

**540** Y.P. Terletskii, *Paradoxes in the Theory of Relativity*, Plenum Press, 1968. Cited on page 579.

**541** See the excellent explanation by Kirk T. McDonald, Negative group velocity, *American Journal of Physics* **69**, pp. 607–614, 2001. Cited on page 579.

**542** The prediction of negative refraction is due to V.G. Veselago, The electrodynamics of substances with simultaneously negative values of $\varepsilon$ and $\mu$, *Soviet Physics Uspekhi* **10**, p. 509,

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**543** The first example of material system with a negative refraction index were presented by David Smith and his team. R.A. Schelby, D.R. Smith & S. Schultz, Experimental verification of a negative index of refraction, *Science* **292**, p. 77-79, 2001. More recent examples are A.A. Houck, J.B. Brock & I.L. Chuang, Experimental observations of a left-handed material that obeys Snell's law, *Physical Review Letters* **90**, p. 137401, 2003, C.G. Parazzoli, R.B. Greegor, K. Li, B.E.C. Koltenbah & M. Tanielian, Experimental verification and simulation of negative index of refraction using Snell's law, *Physical Review Letters* **90**, p. 107401, 2003. S. Foteinopoulou, E.N. Economou & C.M. Soukoulis, Refraction in media with a negative refractive index, *Physical Review Letters* **90**, p. 107402, 2003. Cited on page 580.

**544** S.A. Ramakrishna, Physics of negative refractive index materials, *Reorts on Progress of Physics* **68**, pp. 449–521, 2005. Cited on page 580.

**545** J. Pendry, Negegative refraction makes a perfect lens, *Physical Review Letters* **85**, p. 3966, 2000. Cited on page 580.

**546** G. Nimtz, A. Enders & H. Spieker, *Journal de Physique I (Paris)* **4**, p. 565, 1994. Unfortunately, Nimtz himself seems to believe that he transported energy or signals faster than light; he is aided by the often badly prepared critics of his quite sophisticated experiments. See A. Enders & G. Nimtz, *Physikalische Blätter* **49**, p. 1119, Dezember 1993, and the weak replies in *Physikalische Blätter* **50**, p. 313, April 1994. See also A.M. Steinberg, *Journal de Physique I (Paris)* **4**, p. 1813, 1994, A.M. Steinberg, P.G. Kwiat & R.Y. Chiao, *Physical Review Letters* **71**, pp. 708–711, 1993, and A. Ranfagni, P. Fabeni, G.P. Pazzi & D. Mugnai, *Physical Review E* **48**, p. 1453, 1993. Cited on page 580.

**547** A summary of all evidence about the motion of the aether is given by R.S. Shankland, S.W. McCuskey, F.C. Leone & G. Kuerti, New analysis of the interferometer observations of Dayton C. Miller, *Review of Modern Physics* **27**, pp. 167–178, 1955. An older text is H. Witte, *Annalen der Physik* **26**, p. 235, 1908. Cited on page 581.

**548** The history of the concept of vacuum can be found in the book by E. Grant, *Much Ado About Nothing*, Cambridge University Press, 1981, and in the extensive reference text by Edmund T. Whittaker, *A History of the Theories of Aether and Electricity*, Volume 1: *The Classical Theories*, Volume 2: *The Modern Theories*, Tomash Publishers, American Institute of Physics 1951, 1987. Cited on page 582.

The various *aether models* – gears, tubes, vortices – proposed in the nineteenth century were dropped for various reasons. Since many models used to explain electric and magnetic fields as motion of some entities, it was concluded that the speed of light would depend on electric or magnetic fields. One type of field was usually described by linear motion of the entities, the other by rotatory or twisting motion; both assignments are possible. As a consequence, aether must be a somewhat strange fluid that flows perfectly, but that resists rotation of volume elements, as McCullogh deduced in 1839. However, experiments show that the speed of light in vacuum does not depend on electromagnetic field intensity. Vortices were dropped because real world vortices were found out to be unstable. All models received their final blow when they failed to meet the requirements of special relativity.

**549** This happened to Giovanni Bellini (*c.* 1430–1516) the great Venetian Renaissance painter, who even put this experience into writing, thus producing one of the greatest 'gaffes' ever. If you take a photograph of the effect with a remotely controlled camera, you can prove that your camera is holy as well. Cited on page 582.

**550** S.R. Wilk, How retroreflectors really work, *Optics & Photonics News,* pp. 6–7, December 1993. Cited on page 582.

**551** W.H. Ehrenstein & B. Lingelbach, Das Hermann–Gitter, *Physik in unserer Zeit* **6**, pp. 263–268, 2002. The journal also shows a colour variation of these lattices. Cited on page 583.

**552** The eye sensitivity myth is debunked in detail by B.H. Soffer & D.K. Lynch, Some paradoxes, errors, and resolutions concerning the spectral optimization of human vision, *American Journal of Physics* **67**, pp. 946–953, 1999. Cited on page 585.

**553** David R. Williams, Supernormal Vision, *Science News* **152**, pp. 312–313, 15 November 1997. See also http://www.cvs.rochester.edu/people/d_williams/d_williams.html as well as the photographs at http://www.cvs.rochester.edu/people/~aroorda/ao_research.html of the interior of living human eyes. Their last publication is A. Roorda, A. Metha, P. Lennie & D.R. Williams, Packing arrangement of the three cone classes in the primate retina, *Vision Research* **41**, pp. 1291–1306, 2001. Cited on page 585.

**554** S.W. Hell, Strategy for far-field optical imaging and writing without diffraction limit, *Physics Letters A* **326**, pp. 140–145, 2004, see also V. Westphal & S.W. Hell, Nanoscale resolution in the focal plane of an optical microscope, *Physical Review Letters* **94**, p. 143903, 2005, and V. Westphal, J. Seeger, T. Salditt & S.W. Hell, Stimulated emission depletion microscopy on lithographic microstructures, *Journal of Physics B* **38**, pp. S695–S705, 2005. Cited on page 589.

**555** A.D. Erlykin & A.W. Wolfendale, The origin of cosmic rays, *European Journal of Physics* **20**, pp. 409–418, 1999, Cited on page 591.

**556** D. Singleton, Electromagnetic angular momentum and quantum mechanics, *American Journal of Physics* **66**, pp. 697–701, 1998, Cited on page 592.

**557** A. Yazdani, D.M. Eigler & N.D. Lang, Off-resonance conduction through atomic wires, *Science* **272**, pp. 1921–1924, 28 June 1996. For aluminium, gold, lead, niobium, as well as the influence of chemical properties, see Elke Scheer, The signature of chemical valence in the electric conduction through a single-atom contact, *Nature* **394**, pp. 154–157, 9 July 1998. Cited on page 593.

**558** See L. Kowalski, A myth about capacitors in series, *The Physics Teacher* **26**, pp. 286–287, 1988, and A.P. French, Are the textbook writers wrong about capacitors?, *The Physics Teacher* **31**, pp. 156–159, 1993. Cited on page 593.

**559** This problem was suggested by Vladimir Surdin. Cited on page 593.

**560** See for example, J.M. Aguirregabiria, A. Hernandez & M. Rivas, Velocity fields inside a conducting sphere near a slowly moving charge, *American Journal of Physics* **62**, pp. 462–466, 1994. Cited on page 594.

**561** Philip Cohen, Open wide, this won't hurt a bit, *New Scientist* p. 5, 3 February 1996. Cited on page 594.

**562** Such a claim was implicitly made by D. Mugnai, A. Ranfagni & R. Ruggieri, Observation of superluminal behaviors in wave propagation, *Physical Review Letters* **84**, p. 4830, 2000. An excellent explanation and rebuttal was given by W.A. Rodrigues, D.S. Thober & A.L. Xavier, Causal explanation for observed superluminal behavior of microwave propagation in free space, http://www.arxiv.org/abs/physics/0012032. Cited on page 595.

**563** J.E. Avron, E. Berg, D. Goldsmith & A. Gordon, Is the number of photons a classical invariant?, *European Journal of Physics* **20**, pp. 153–159, 1999. Cited on page 595.

**564** If you want to see more on how the world looks for the different types of colour blind, have a look at the http://webexhibits.org/causesofcolor/2.html or the http://www.vischeck.com/examples web pages. Cited on page 597.

**565** About the life-long passion that drove Luke Howard, see the book by Richard Hamblyn, *The Invention of Clouds*, Macmillan 2001. Cited on page 597.

**566** J. Latham, The electrification of thunderstorms, *Quartely Journal of the Royal Meteorological Society* **107**, pp. 277–289, 1981. For a more recent and wider review, see Earle R. Williams, The tripole structure of thunderstorms, *Journal of Geophysical Research* **94**, pp. 13151–13167, 1989. See also the book by the National Research Council Staff, *The Earth's Electrical Environment*, Studies in Geophysics, National Academy Press, 1986. Cited on page 597.

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**572** J.R. Dwyer, A fundamental limit on electric fields in air, *Geophysical Research Letters* **30**, p. 2055, 2003. Cited on page 599.

**573** For a recent summary, see S. Parrott, http://www.arxiv.org/abs/gr-qc/9711027. See also T.A. Abbott & D.J. Griffiths, Acceleration without radiation, *American Journal of Physics* **53**, pp. 1203–1211, 1985. See also A. Kovetz & G.E. Tauber, Radiation from an accelerated charge and the principle of equivalence, *American Journal of Physics* **37**, pp. 382–385, 1969. Cited on page 600.

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**587** The well-known toy allows levitation without the use of any energy source and is called the 'Levitron'. It was *not* invented by Bill Hones of Fascination Toys & Gifts in Seattle, as the http://www.levitron.com website explains. The toy is discussed by Ron Edge, Levitation using only permanent magnets, *Physics Teacher* **33**, p. 252, April 1995. It is also discussed in M.V. Berry, The Levitron$^{TM}$: an adiabatic trap for spins, *Proceedings of the Royal Society A* **452**, pp. 1207–1220, 1996, (of Berry's phase fame) as well as by M.D. Simon, L.O. Heflinger & S.L. Ridgeway, Spin stabilized magnetic levitation, *American Journal of Physics* **65**, pp. 286–92, 1997, and by T.B. Jones, M. Washizu & R. Gans, Simple theory for the Levitron, *Journal of Applied Physics* **82**, pp. 883–889, 1997. Cited on page 604.

**588** The drill trick and the building of a Levitron are described in the beautiful lecture script by JOSEF ZWECK, *Physik im Alltag*, Skript zur Vorlesung im WS 1999/2000 der Universität Regensburg. Cited on page 604.

**589** The prediction about quantized levitation is by STEPHEN B. HALEY, Length quantization in levitation of magnetic microparticles by a mesoscopic superconducting ring, *Physical Review Letters* 74, pp. 3261–3264, 1995. The topic is discussed in more detail in STEPHEN B. HALEY, Magnetic levitation, suspension, and superconductivity: macroscopic and mesoscopic, *Physical Review B* **53**, p. 3506, 1996, reversed in order with STEPHEN B. HALEY, Quantized levitation of superconducting multiple-ring systems, *Physical Review B* **53**, p. 3497, 1996, as well as STEPHEN B. HALEY, Quantized levitation by multiply-connected superconductors, LT-21 Proceedings, in *Czechoslovak Journal of Physics* **46**, p. 2331, 1996. In 1998, there was not yet an experimental confirmation (Stephen Haley, private communication). Cited on page 604.

**590** All the illusions of the flying act look as if the magician is hanging on lines, as observed by many, including the author. (Photographic flashes are forbidden, a shimmery background is set up to render the observation of the lines difficult, no ring is ever actually pulled over the magician, the aquarium in which he floats is kept open to let the fishing lines pass through, always the same partner is 'randomly' chosen from the public, etc.) Information from eyewitnesses who have actually seen the fishing lines used by David Copperfield explains the reasons for these set-ups. The usenet news group alt.magic.secrets, in particular Tilman Hausherr, was central in clearing up this issue in all its details, including the name of the company that made the suspension mechanism. Cited on page 604.

**591** Detailed descriptions of many of these effects can be found in the excellent overview edited by MANFRED VON ARDENNE, GERHARD MUSIOL & SIEGFRIED REBALL, *Effekte der Physik und ihre Anwendungen*, Harri Deutsch, 2004. Cited on page 605.

**592** R. BUDDAKIAN, K. WENINGER, R.A. HILLER & SETH J. PUTTERMAN, Picosecond discharges and stick–slip friction at a moving meniscus of mercury in glass, *Nature* **391**, pp. 266–268, 15 January 1998. See also *Science News* **153**, p. 53, 24 January 1998. Cited on page 605.

**593** HENK SWAGTEN & REINDER COEHOORN, Magnetische tunneljuncties, *Nederlands tijdschrift voor natuurkunde* **64**, pp. 279–283, November 1998. Cited on page 605.

**594** H. OHNO, D. CHIBA, F. MATSUKURA, T. OMIYA, E. ABE, T. DIETL, Y. OHNO & K. OHTANI, Electric-field control of ferromagnetism, *Nature* **408**, pp. 944–946, 21-28 December 2000. Cited on page 606.

**595** This effect was discovered by GEERT RIKKEN, BART VAN TIGGELEN & ANJA SPARENBERG, Lichtverstrooiing in een magneetveld, *Nederlands tijdschrift voor natuurkunde* **63**, pp. 67–70, maart 1998. Cited on page 607.

**596** VITALIJ PECHARSKY & KARL A. GSCHNEIDNER, Giant magnetocaloric effect in Gd5(Si2Ge2), *Physical Review Letters* **78**, pp. 4494–4497, 1995, and, from the same authors, Tunable magnetic regenerator alloys with a giant magnetocaloric effect for magnetic refrigeration from ~20 to ~2990 K, *Applied Physics Letters* **70**, p. 3299, 1997. Cited on page 607.

**597** J. WEISSMÜLLER, R.N. VISWANATH, D. KRAMER, P. ZIMMER, R. WÜRSCHUM & H. GLEITER, Charge-induced reversible strain in a metal, *Science* **300**, pp. 312–315, 11 April 2003. Cited on page 608.

**598** A. AJDARI, Electro-osmosis on inhomogeneously charged surfaces, *Physical Review Letters* **75**, pp. 755–758, 1995. Cited on page 608.

**599** M.J. AITKEN, *Thermoluminescence Dating*, Academic Press, 1985. The precision of the method is far worse that C14 dating, however, as shown by H. Huppertz, *Thermolumineszenzdatierung: eine methodologische Analyse aufgrund gesicherter Befunde*, Peter Lang Verlag, 2000. Cited on page 609.

**600** This effect was discovered by J.N. Huiberts, R. Griessen, J.H. Rector, R.J. Wijngarden, J.P. Dekker, D.G. de Groot & N.J. Koeman, Yttrium and lanthanum hydride films with switchable optical properties, *Nature* 380, pp. 231–234, 1996. A good introduction is R. Griessen, Schaltbare Spiegel aus Metallhydriden, *Physikalische Blätter* 53, pp. 1207–1209, 1997. Cited on page 610.

**601** See any book on thermostatics, such as Linda Reichl, *A Modern Course in Statistical Physics*, Wiley, 2nd edition, 1998. Cited on page 611.

**602** The Sun emits about $4 \cdot 10^{26}$ W from its mass of $2 \cdot 10^{30}$ kg, about 0.2 mW/kg; a person with an average mass of 75 kg emits about 100 W (you can check this in bed at night), i.e. about 500 times more. Cited on page 611.

**603** See its http://www.cie.co.at/cie website. Cited on page 612.

**604** P.D. Jones, M. New, D.E. Parker, S. Martin & I.G. Rigor, Surface air temperature and its changes over the past 150 years, *Reviews of Geophysics* 37, pp. 173–199, May 1999. Cited on page 612.

**605** Pictures of objects in a red hot oven and at room temperature are shown in C.H. Bennett, Demons, engines and the second law, *Scientific American* 255, pp. 108–117, November 1987. Cited on page 613.

**606** Bill McGuire, *A Guide to the End of the World: Everything you Never Wanted to Know*, Oxford University Press, 2002. On past disasters, see introduction by Tony Hallam, *Catastrophes and Lesser Calamities – the Causes of Mass Extinctions*, Oxford University Press, 2004. Cited on page 616.