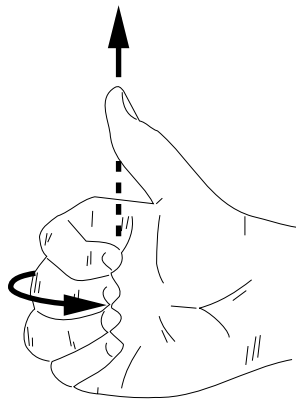


AMPERE'S LAW



AMPERE'S LAW by Kirby Morgan

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Title: **Ampere's Law**

Author: Kirby Morgan, HandiComputing, 319 E. Henry, Charlotte, MI 48813

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Evaluation: Stage 0

Length: 1 hr; 32 pages

Input Skills:

1. Vocabulary: current density (MISN-0-118); tesla (MISN-0-122).
2. Using the Ampere-Laplace-Biot-Savart equation, calculate the magnetic field due to a current in a long straight wire (MISN-0-125).
3. Take the line integral of a vector function over a specified integration path (Appendix).

Output Skills (Knowledge):

- K1. Vocabulary: line integral, solenoid, toroid, Ampere's law.
- K2. State Ampere's law for currents and define each symbol.
- K3. Describe how one can determine the direction of magnetic fields produced by a current segment.
- K4. State Ampere's law for current densities and define each symbol.

Output Skills (Problem Solving):

- S1. Use Ampere's law to calculate the magnetic field due to symmetrical configurations of current such as long straight wires, infinite planes of current, solenoids and toroids.
- S2. Given a current density in a conductor of simple geometric shape, use Ampere's law to calculate the associated magnetic field.

Post-Options:

1. "The Ampere-Maxwell Equation: Displacement Current" (MISN-0-145).
2. "Maxwell's Equations" (MISN-0-146).

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AMPERE'S LAW

by
Kirby Morgan

1. Usefulness

Ampere's law is a part of Maxwell's equations: it relates magnetic fields to electric currents that produce them.¹ Using Ampere's law, you can determine the magnetic field associated with a given current or the current associated with a given magnetic field, providing there is no time-changing electric field present. Ampere's law is particularly useful in situations where there exists a high degree of geometrical symmetry, just as is the case with Gauss's law.² Fortunately, many applications have such symmetry.

2. The Law

2a. The Integral Relationship. Gauss's law and Ampere's law have some similarities, although Gauss's law involves a surface integral,

$$\text{Gauss's law: } \oint_S \vec{E} \cdot d\vec{S} = 4\pi k_e Q_S,$$

¹See "Maxwell's Equations" (MISN-0-146).

²See "Gauss's Law and Spherically Distributed Charges" (MISN-0-132).

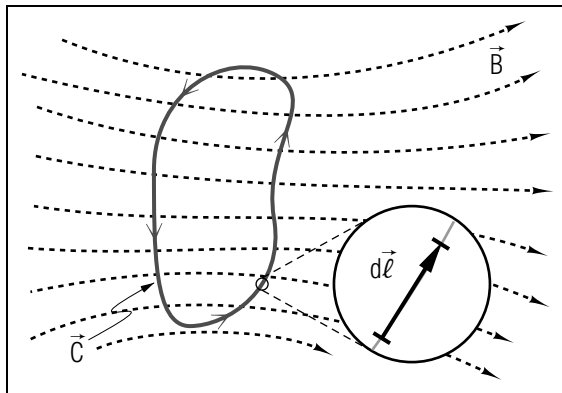


Figure 1. An integration path C in a magnetic field B . The associated currents are not shown.

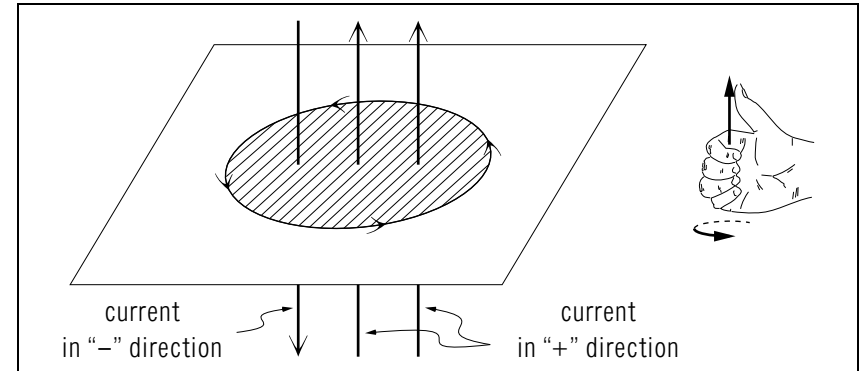


Figure 2. A right-hand rule assigns a sign to each current bounded by the loop C .

while Ampere's law involves a "line integral":³

$$\text{Ampere's law: } \oint_C \vec{B} \cdot d\vec{\ell} = 4\pi k_m I_C. \quad (1)$$

Here:

- k_m is the "magnetic force constant."
- \oint_C denotes integration along a closed imaginary line C .⁴ The closed imaginary line for any particular problem is usually called the integration "loop" or "path" for that problem. The line must pass through the point where you want to know the magnetic field.
- I_C denotes the net electric current passing through any (imaginary) surface whose boundary is the same closed line C used in \oint_C (see Fig. 1).⁵
- $d\ell$ is an infinitesimal element of length along the integration line.
- The direction of integration around the line is arbitrary, but once taken it fixes the direction of current that must be called positive. The relevant rule will be taken up later.

³See the Appendix of this module for a discussion of line integrals.

⁴The word "closed" means that the line has no end so it must be a closed loop.

⁵All surfaces bounded by the same line give the same value for I_C .

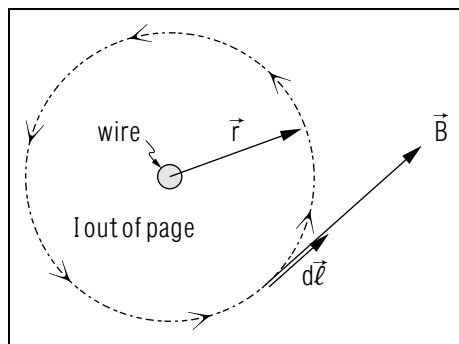


Figure 3. \vec{B} and $d\vec{\ell}$ are both tangent to C , the circular integration path.

2b. Determining Signs (\pm). The algebraic sign (\pm) of any current enclosed by the integration loop in Ampere's law is determined by a right-hand rule:

A current is taken to be positive if it points in the direction of the thumb on the right hand when the fingers of that hand encircle the loop in the direction that the line integral is taken (see Fig. 2). If it is in the opposite direction, the current must be taken as negative.

3. Simple Applications

3a. Magnetic Field Near a Long Thin Wire. The magnetic field \vec{B} at some point in space, associated with a current I in a long straight wire, can be calculated using Ampere's law. The integration path we choose is a circle, centered on the wire (see Fig. 3) and going through the point where we wish to know \vec{B} . By symmetry, we expect the magnetic field to have the same magnitude at all points on the circle and we expect the magnetic field to be tangent to the circle at each point on that circle.⁶ Since $d\vec{\ell}$ is also tangent to the circle, $\vec{B} \cdot d\vec{\ell} = B d\ell$ and the loop integral is simply $B \oint d\ell$ and $\oint d\ell$ is just the circumference of the circle. Calling the radius of the circle r , which is also the distance from the wire to the point where we wish to know \vec{B} , Ampere's law gives:

$$(B)(2\pi r) = 4\pi k_m I, \quad (2)$$

⁶See "The Magnetic Field of a Current: The Ampere-Laplace Equation" (MISN-0-125) for a proof that this is so.

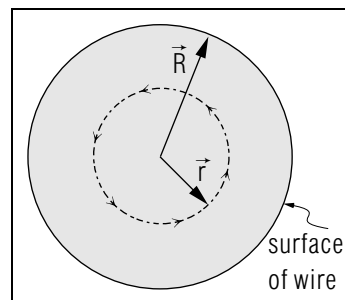


Figure 4. Circular path of integration for a current uniformly distributed throughout the cross section.

so:

$$B = 2k_m \left(\frac{I}{r} \right). \quad (\text{long straight wire}). \quad (3)$$

If the wire is not infinitely long, and no wire is, this value of B is accurate to the extent that r is much less than the distance from the field-point to either end of the wire.⁷

3b. B Outside a Long Cylindrical Conductor. Ampere's law can be used to show that the magnetic field at points outside a long circular cylinder carrying a current uniformly distributed over its cross section is the same as if all the current were concentrated in a line along the axis. For points outside the cylinder, a circular path of integration will enclose all of the current and, again by symmetry, \vec{B} and $d\vec{\ell}$ are parallel. By the same analysis that was used for the long wire, we find:

$$B = 2k_m \left(\frac{I}{r} \right); \quad r > R, \quad (4)$$

where r is the distance from the center of the wire and R is the radius of the cylinder.

3c. B Inside a Long Cylindrical Conductor. The magnetic field at a point inside a cylindrical conductor carrying a current depends on how the current is distributed. If it is uniformly distributed over its cross section and a circular path of integration is again chosen (see Fig. 4), the fraction of the current enclosed by the path will be $\pi r^2 / \pi R^2$, so that Ampere's law gives:

$$(B)(2\pi r) = 4\pi k_m I \left(\frac{\pi r^2}{\pi R^2} \right), \quad (5)$$

⁷See "Gauss's Law Applied to Cylindrical and Planar Charge Distributions" (MISN-0-133) for the electrostatic equivalent, a line of fixed charge, where the "much less than" condition is also discussed.

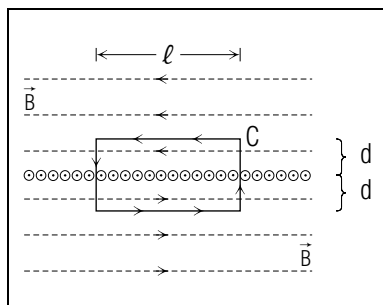


Figure 5. Rectangular integration path for the infinite plane of wires.

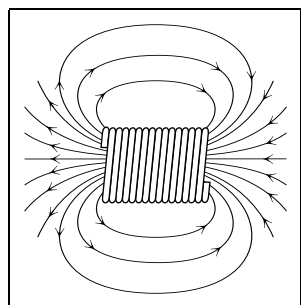


Figure 6. The magnetic field outside a solenoid of finite length.

or

$$B = 2k_m \left(\frac{I r}{R^2} \right). \quad (6)$$

Note that the magnetic field is linearly proportional to r , the distance of the field point from the axis.

Note: For the case where the current resides only on the surface of the cylinder, no current would be enclosed by the integration path and the magnetic field would be zero at all points inside such a “surface conductor.”

3d. Infinite Plane of Adjacent Wires. Ampere’s law can be used to find the magnetic field due to a conductor consisting of an infinite plane of adjacent wires. The wires are infinitely long (or are long enough to be regarded as such) and each carries a current I . By symmetry, you would expect \vec{B} to be parallel to the plane: then a rectangular integration path of length ℓ which extends a distance d on each side of the plane would be a good choice (see Fig. 5). Along the sides of the path, normal to the plane, \vec{B} is perpendicular to $d\vec{\ell}$ so $\int \vec{B} \cdot d\vec{\ell}$ is zero there. Then Ampere’s law yields:

$$\oint \vec{B} \cdot d\vec{\ell} = 2B\ell = 4\pi k_m n\ell I, \quad (7)$$

where n is the number of wires per unit length and $n\ell$ is the total number enclosed. Solving for B gives:

$$B = 2\pi k_m nI. \quad (8)$$

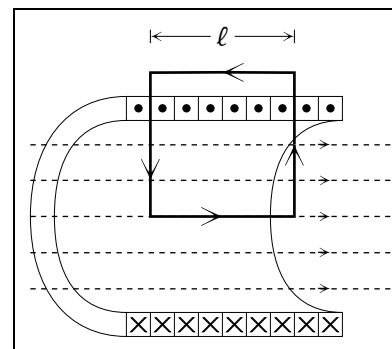


Figure 7. The integration path for a very long solenoid having $B = 0$ outside.

This equation indicates that the field associated with an infinite plane of current is independent of the distance from the plane.

4. Example Devices

4a. The Solenoid. A solenoid is a tightly wound cylindrical helix of current-carrying wire, used to make an electrical signal cause a one-directional mechanical force (for example, operating a plunger). Solenoids are frequently encountered in science and technology; there are at least several in every car. The magnetic field inside a solenoid can be easily found using Ampere’s law. The external magnetic field due to a solenoid of finite length is quite similar to that of a bar magnet (see Fig. 6). However, if the solenoid is very long, (i.e., if its length is much greater than its radius), the field outside is essentially zero, and inside the solenoid it is uniform and parallel to the solenoid’s axis (see Fig. 7).⁸

4b. Calculating the Field of a Solenoid. The magnitude of \vec{B} inside a solenoid can be found by applying Ampere’s law to the rectangular integration path shown in Fig. 7. Outside the solenoid \vec{B} is zero. Inside, \vec{B} is at right angles to the ends of the rectangle so the only non-zero contribution to the integral is along the length ℓ that is inside the solenoid. Therefore:

$$\oint_C \vec{B} \cdot d\vec{\ell} = B\ell. \quad (9)$$

⁸Look at the solenoid in Fig. 6 and notice that the magnetic field lines are much more dense inside the solenoid than outside it. Imagine making the solenoid longer and longer, during which the density inside remains constant but the density outside becomes more and more sparse.

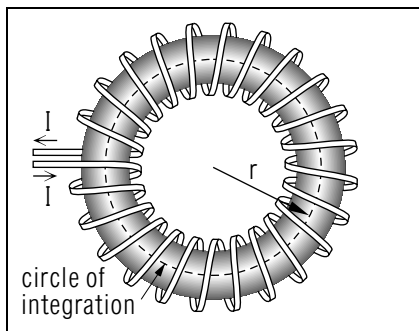


Figure 8. A toroid.

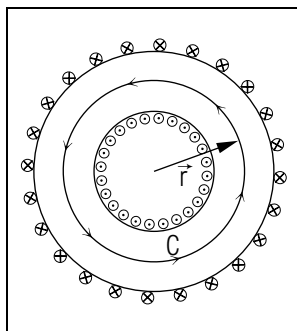


Figure 9. The circular path of integration inside a toroid.

The net current through the rectangle is $n\ell I$, where n is the number of turns per unit length over the entire length ℓ . Ampere's law then gives for the magnetic field:

$$B = 4\pi k_m n I, \quad (\text{solenoid}) \quad (10)$$

indicating a uniform field.

4c. The Toroid. A toroid is a solenoid that has been bent into a circle, assuming the space-saving shape of a doughnut (see Fig. 8). The magnetic field inside a toroid carrying a current I can be found using Ampere's law. By symmetry, the magnetic field is tangent to the circular integration path shown in Fig. 9. Therefore:

$$\oint_C \vec{B} \cdot d\vec{\ell} = (B)(2\pi r), \quad (11)$$

and the enclosed current is NI , where N is the total number of turns on the solenoid. Then:

$$B = 2k_m \left(\frac{NI}{r} \right). \quad (12)$$

Notice that, unlike the solenoid, the magnetic field inside the toroid is not constant over the cross section of the coil but varies inversely as the distance r . For points outside a toroid, it can be shown that the field is essentially zero if the turns of wire are very close together. *Help: [S-1]*

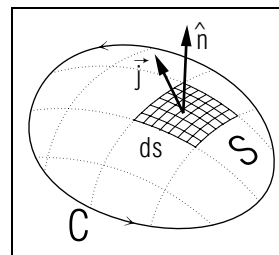


Figure 10. The current density \vec{j} is not always along the normal \hat{n} to an arbitrary surface element dS .

5. Using Current Density

5a. Introduction. Just as it is often useful to use the concept of charge density in electrostatics, in magnetics we often use the concept of *current density*. Charge density is a scalar and has three varieties: linear, surface and volume. Current density is a vector and has one variety. Current density has a non-zero value only at those space-points where there are charges flowing so there is an electric current: it is the net amount of charge going through the space-point per unit time, per unit area perpendicular to the direction of the current. The direction of the current density vector is the direction of the electric current at the space-point in question. The universal symbol for the current density is $\vec{j}(\vec{r})$, where the argument indicates that the current density may change as one moves from one space-point to another.⁹

5b. The Current Through a Surface. We now assume we know the current density \vec{j} at various space-points of interest and we want to find the I_C used in Ampere's law, Eq. (1). We start with \vec{j} at the point of a surface element dS having a normal unit vector \hat{n} . We want to know how much current dI is passing through this element of surface. Since \vec{j} is the current per unit area normal to the current, we must multiply by an element of area dA normal to the current (see Fig. 10). If we know dS , \hat{n}_S , and \hat{j} , we can get dA by (see Appendix B):

$$dA = \hat{j} \cdot \hat{n} dS.$$

Substituting $dI = j dA$ we get:

$$dI = \vec{j} \cdot \hat{n} dS. \quad (13)$$

For the special case of a uniform current flowing perpendicular to a plane surface of area A , the equation simplifies to $I = jA$; a simple statement

⁹Of course \vec{j} may also be a function of time but we are not dealing with that case here.

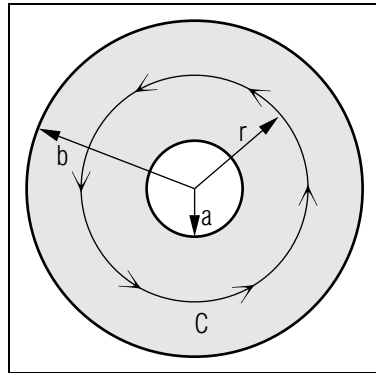


Figure 11. A hollow conducting cylinder with a non-uniform current density $j = k/r$.

that the current is the current density times area.

Finally, we integrate both sides of Eq. (13) to get:

$$I_C = \int_S \vec{j} \cdot \hat{n} dS. \quad (14)$$

5c. Ampere's Law in Terms of the Current Density. Ampere's law may be rewritten in terms of the current density, using Eqs. (1) and (14), giving:

$$\oint_C \vec{B} \cdot d\vec{\ell} = 4\pi k_m \int_S \vec{j} \cdot \hat{n} dS. \quad (15)$$

Here C is the closed path around the perimeter of the surface S .

5d. Example: Hollow Conducting Cylinder. What is the magnetic field at points inside a hollow conducting cylinder which is made such that its current density varies inversely as the distance from the center of the cylinder? The conductor is shown in Fig.11 and the current density in this problem is:

$$j = \frac{k}{r}; \quad a < r < b, \quad (16)$$

where k is a constant. If a circular integration path is chosen, the current enclosed by it, the right side of Eqs. (1) and (15), is:

$$I_C = \int_a^r \frac{k}{r'} 2\pi r' dr' = 2\pi k(r - a). \quad a < r < b \quad \text{Help: [S-2]} \quad (17)$$

Ampere's law then gives:

$$B = 4\pi k_m k \left(\frac{r - a}{r} \right), \quad a < r < b \quad (18)$$

for the magnetic field within the conducting material. *Help: [S-3]*

Acknowledgments

This module is based on an earlier version by J. Kovacs and O. McHarris. The Model Exam is taken from that version. Ray G. Van Ausdal provided editorial assistance. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.

Glossary

- **Ampere's law:** the integral form of one of Maxwell's equations:

$$\oint_C \vec{B} \cdot d\vec{\ell} = 4\pi k_m I_C.$$

It relates the integral of the magnetic field around a closed loop to the net current flowing through any surface bounded by the integration loop. Ampere's law is universally true, but is useful only when there is a high degree of symmetry.

- **current density:** a vector whose magnitude at a space point is the current per unit area normal to the direction of the current at that point and whose direction is the direction of the current at that point.
- **line integral:** the integral of a function along a specified path in space. In Ampere's law one evaluates the line integral of the tangential component of the magnetic field around a closed path that: (i) goes through the point at which one wishes to know the magnetic field; and (ii) is such that it has a constant value for the integrand so the integral can be performed trivially.
- **solenoid:** a tightly wound cylindrical helix of current-carrying wire.
- **toroid:** a solenoid bent into the shape of a doughnut.

A. Line Integrals

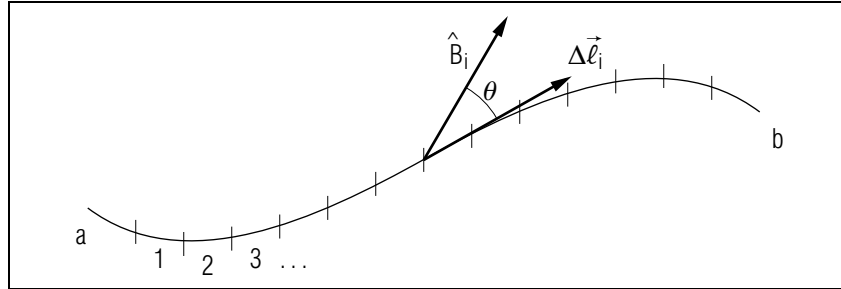


Figure 12.

The line integral

$$\int_a^b \vec{B} \cdot d\vec{\ell},$$

for the path shown above, can be approximated by dividing the path into many small segments $\Delta\vec{\ell}_i$ and for each segment the product

$$B_i \cos \theta_i \Delta\ell_i$$

can be found. Here $B_i \cos \theta_i$ is the component of \vec{B} tangent to the curve. The integral can be calculated approximately by summing these segments' terms, for example, on a computer. However, the exact value of the line integral is given by the limit:

$$\int_a^b \vec{B} \cdot d\vec{\ell} = \lim_{n \rightarrow \infty} \sum_{i=1}^n B_i \cos \theta_i \Delta\ell_i.$$

If a is joined to b , the path becomes closed and the resultant integral

$$\oint_C \vec{B} \cdot d\vec{\ell}$$

is around the “closed path” C . Often, the calculation of this integral is highly simplified by utilizing a path that takes advantage of symmetries in the problem. Two examples of such simplifications are:

(i) \vec{B} is constant and always tangent to the path:

$$\int_a^b \vec{B} \cdot d\vec{\ell} = \int_a^b B d\ell = B \int_a^b d\ell = B\ell_{ab},$$

where ℓ_{ab} is the length of the path from a to b .

(ii) \vec{B} is always perpendicular to the path:

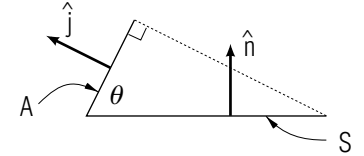
$$\int_a^b \vec{B} \cdot d\vec{\ell} = 0.$$

B. Projection of an Area

If a planar (flat) area S is projected onto another plane, the area A on the projected-onto plane is given by:

$$A = \hat{j} \cdot \hat{n} S. \quad (19)$$

Here \hat{n} is a unit vector normal to the plane of the original area and \hat{j} is a unit vector normal to the projected-onto plane (see the sketch). This is entirely equivalent to the statement that the areas are related by the cosine of the angle between the planes (again see the sketch):



$$A = S \cos \theta. \quad (20)$$

By “projection” we mean that from every point on the periphery of the original area S we drop a perpendicular to the projected-onto plane. The locus of those points on the projected-onto plane define the periphery of the projected area A .

Equations (19)-(20) are easily proved by considering infinitesimally-wide straight line elements of the area A that are normal to the line of intersection of the two planes. For each such element there is a projection of it onto the projected-onto plane, and the areas of the two elements are obviously related by the cosine of the angle between the planes. Since the areas themselves are simply the integrals of the infinitesimal areas, and since the angle between the planes is independent of where one is in one of the areas, the cosine can be pulled outside the integral and Eqs. (19)-(20) are proved.

If the area S is curved (non-planar) then Eqs. (19)-(20) apply only to infinitesimal areas (which can be considered to be planar for these

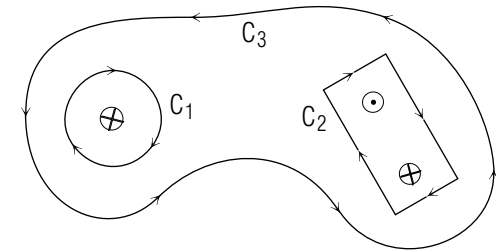
purposes):

$$dA = \hat{j} \cdot \hat{n} dS.$$

PROBLEM SUPPLEMENT

Note: Problems 8, 9, and 10 also occur in this module's *Model Exam*.

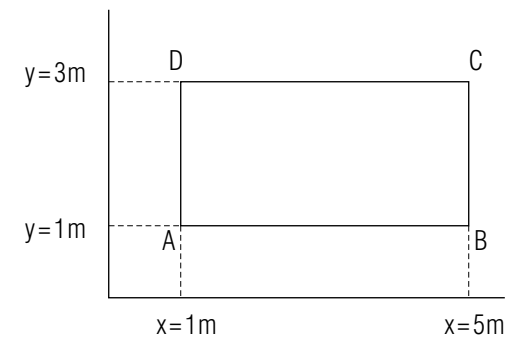
- Three infinitely long parallel wires each carry a current I in the direction shown below. What is $\int \vec{B} \cdot d\vec{\ell}$ for each of the three paths C_1 , C_2 , and C_3 ?



- The magnetic field in a certain region of space is given by

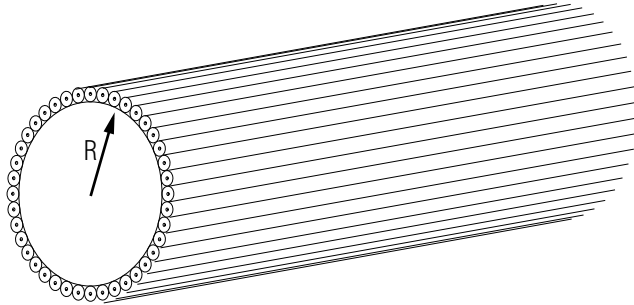
$$\vec{B} = A_0 x \hat{x}$$

where $A_0 = 3 \text{ T/m}$, x is the x -coordinate of the point, and \hat{x} is a unit vector in the x -direction. In this region, consider a rectangular path in the x - y plane whose sides are parallel to the x and y axes respectively as shown below.



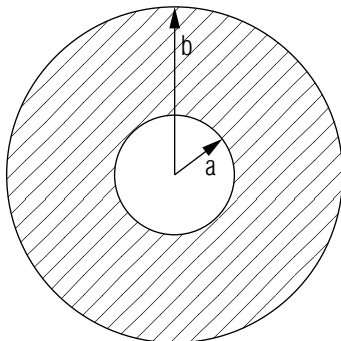
- Evaluate the line integral of \vec{B} from A to B .
- Do the same along the line from B to C .

- c. For C to D .
 - d. For D to A .
 - e. Evaluate the $\int \vec{B} \cdot d\vec{l}$ around this closed path.
 - f. Determine the net current that must be crossing the x - y plane through the rectangle $ABCD$.
3. A long cylindrical conductor of radius R has a uniform current density \vec{j} spread over its cross section. Determine the magnetic field produced at points $r < R$ and $r > R$ and sketch the magnitude of \vec{B} as a function of r .
 4. A very long non-conducting cylinder has N conducting wires placed tightly together around its circumference and running parallel to its axis as shown below:



If each wire carries a current I , find the magnetic field at points inside and outside the cylinder.

5.



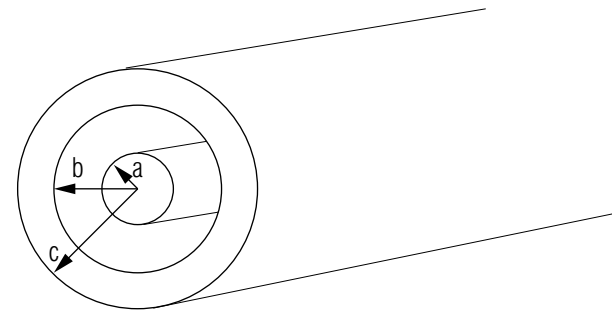
A hollow cylindrical conductor of radii a and b has a current I uniformly spread over its cross section.

- a. Show that the magnetic field inside the conductor ($a < r < b$) is:

$$B = \frac{2k_m I (r^2 - a^2)}{(b^2 - a^2) r}$$

- b. Express B in terms of the current density j .
- c. Show that when $a \rightarrow 0$ you get the same answer as in problem 3.

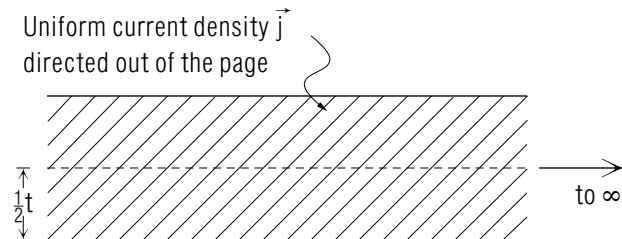
6. A long coaxial cable consists of two concentric conductors. The outside conductor carries a current I equal to that in the inside conductor, but in the opposite direction.



Find the magnetic field at these points:

- a. inside the inner conductor ($r < a$),
- b. between the conductors ($a < r < b$),
- c. inside the outer conductor ($b < r < c$), and
- d. outside the cable ($r > c$).

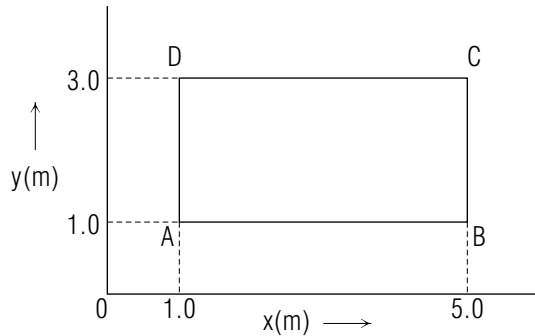
7.



An infinite, plane, conducting slab of thickness t carries a uniform current density of j amperes per square meter directed out of the page in the above diagram.

- Apply Ampere's law to determine the magnetic field at a height h above the center line of the slab for $h > t/2$. Explain carefully how you make use of symmetry in setting up your integration path.
- Suppose your integration path had been a rectangular loop with two sides parallel to the slab surface (as you must have used), but with one parallel path a distance h above the center line and the other a distance h' below the center line (both h and h' are greater than $t/2$). Explain in this case, and without prior knowledge of your final answer, why Ampere's law cannot tell you \vec{B} at points h above the slab. Then show how the use of symmetry arguments solves the problem.
- Use the answer to part (a) and Ampere's law to determine the magnetic field at points a distance y below the surface of the slab, inside the material. What is the field at the center line? Sketch the direction of the field at various points inside the slab.

8.



$$B = 1.0 \times 10^1 \hat{x} \text{ teslas everywhere.}$$

In a certain region of space, the magnetic field intensity is uniform and has the value of 10 teslas directed in the positive x -direction at every point in the region. In this region consider a rectangular path in the x - y plane from point A to point B parallel to the x -axis, B to C parallel to the y -axis and D back to A parallel to the y -axis (see the sketch above).

- Evaluate the line integral of \vec{B} from A to B . [N]
- Do the same for the line from B to C . [B]
- Do the same for the line from C to D . [I]

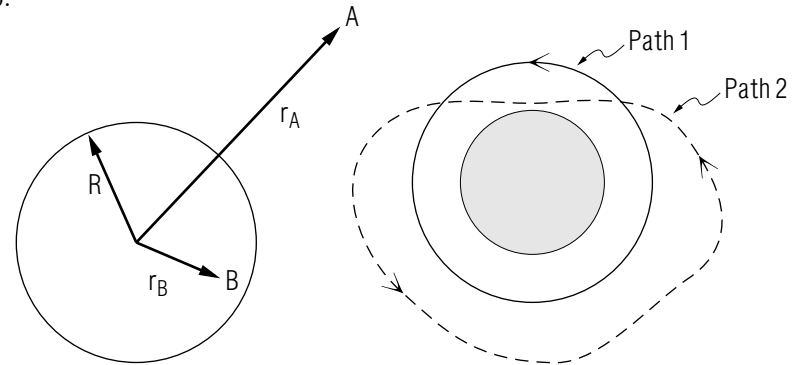
- Do the same for the line from D to A . [P]
 - Evaluate the loop integral $\int \vec{B} \cdot d\ell$ for this closed path. Use the results of parts (a)-(d) to find your answer. [J]
 - From your answer to part (e), determine the net current that must be crossing the x - y plane through this rectangle $ABCD$. [A]
9. Repeat Problem 8, parts (a) through (f) for the case where the magnetic field in this region is now given by

$$\vec{B}(x, y) = (A_0 + A_1 y) \hat{x}$$

where $A_0 = 2.0 \text{ T}$, $A_1 = 0.50 \text{ T/m}$ and y is the y -coordinate of the point.

- [C]
- [K]
- [O]
- [M]
- [H]
- [L]

10.



R = radius of the cylindrical conducting wire

j = the current per unit area (distributed uniformly) directed into the page

r_A = the distance from the center to point A outside the conductor

r_B = the distance from the center to point B inside the conductor.

Path 1 (solid line) is a circular path surrounding the cylinder concentric with the cylindrical conductor passing through point A .

Path 2 (dashed line) is an arbitrary path surrounding the conductor, also passing through point A .

- What is $\int \vec{B} \cdot d\vec{\ell}$ for each of the paths 1 and 2? [F]
- Explain how symmetry enables you to evaluate \vec{B} at point A only if you use path 1. [G]

Brief Answers:

- Circular path: Net current = I

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = 4\pi k_m I$$

Rectangular Path: Net Current = $I + (-I) = 0$

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = 0$$

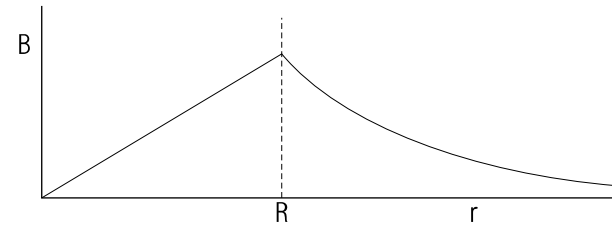
Irregular Path: Net Current = $I - I - I = -I$

$$\oint_{C_1} \vec{B} \cdot d\vec{\ell} = -4\pi k_m I$$

- 36 m T
 - zero
 - 36 m T
 - zero
 - zero
 - $\oint \vec{B} \cdot d\vec{\ell} = 0$ so $I = 0$ through rectangle $ABCD$.

- $r < R$: $B = 2\pi k_m j r$

$$r > R: B = 2\pi k_m \frac{jR^2}{r}$$



- $r < R$: $B = 0$

$$r > R: B = 2\pi k_m \frac{NI}{r}$$

- $B = 2\pi k_m j \left(\frac{r^2 - a^2}{r} \right)$

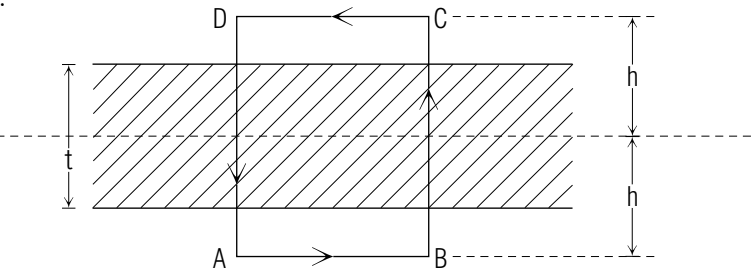
- $r < a$: $B = 2\pi k_m \frac{Ir}{a^2}$

$$\text{b. } a < r < b: B = 2\pi k_m \frac{I}{r}$$

$$\text{c. } b < r < c: B = 2\pi k_m \frac{I}{r} \left(\frac{c^2 - r^2}{c^2 - b^2} \right)$$

- $r > c$: $B = 0$

-

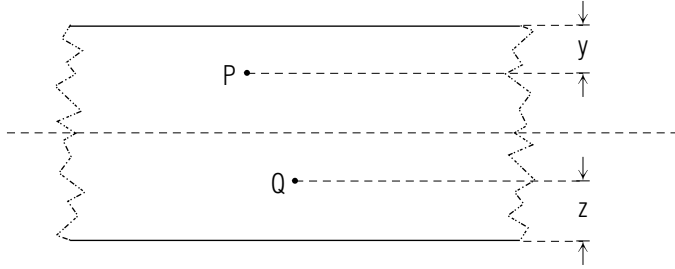


Symmetry tells you that the field, at all points on the line CD , has the same value directed to the left and this is also the same as the field at all points on line AB (but there, directed to the right).

$$B = 2\pi k_m j t,$$

independent of h if $h > t/2$ (the slab is infinitely long).

- b. If the distance below the center line had been h' in the sketch [see part (a)], then Ampere's law would give you $Bx + B'h' = 4\pi k_m jtx$, where B' is the field value at points h' below the center line. Only if $h = h'$ can you argue that $B = B'$ [as in part (a)] and then determine B .
- c.



$$B(\text{at } P) = 4\pi k_m j \left(\frac{t}{2} - y \right) \text{ directed to the left.}$$

$$B(\text{at the center line}) = 0.$$

$$B(\text{at } Q) = 4\pi k_m j \left(\frac{t}{2} - z \right) \text{ directed to the right.}$$

Both y and z are less than $t/2$.

- A. A zero net current.
- B. Zero
- C. $+1.0 \times 10^1 \text{ T m}$
- F. Because both paths completely encircle the current, $\int \vec{B} \cdot d\vec{\ell}$ is $(4\pi k_m j \pi R^2)$ for both path 1 and path 2.
- G. For path 1, symmetry tells you that B is the same (and tangent to the path) at every point on the path, so

$$\oint \vec{B} \cdot d\vec{\ell} = B \oint d\ell = 2\pi r B = 4\pi k_m j \pi R^2,$$

so at point A:

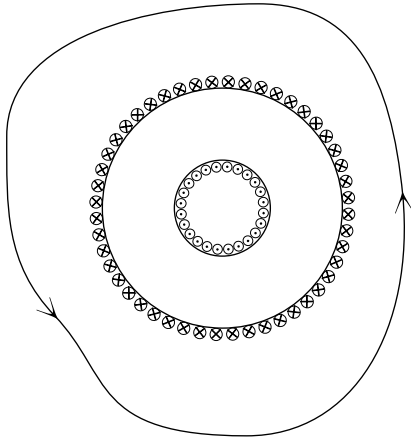
$$B = 2\pi k_m \frac{jR^2}{r}$$

- H. -4.0 T m

- I. $-4.0 \times 10^1 \text{ T m}$ *Help: [S-4]*
- J. $\oint \vec{B} \cdot d\vec{\ell} = 0$ around the closed path.
- K. Zero
- L. $3.2 \times 10^6 \text{ A}$, directed into the page.
- M. Zero
- N. $4.0 \times 10^1 \text{ T m}$
- O. -14 T m
- P. Zero

SPECIAL ASSISTANCE SUPPLEMENT

S-1 (from TX-4e)



For any shaped path enclosing the entire toroid, the net current is zero. By Ampere's law,

$$\oint_C \vec{B} \cdot d\vec{\ell} = 0,$$

which implies $B = 0$ since the path is arbitrary.

S-2 (from TX-4e)

$I = \int_S \vec{j} \cdot \hat{n} dS$ with $\vec{j} = \frac{k}{r} \hat{r}$, giving:

$$\begin{aligned} I &= \int_0^{2\pi} \int_a^r \left(\frac{k}{r'} \right) \hat{r} \cdot (\hat{r}) r' dr' d\theta \\ &= 2\pi k \int_a^r \left(\frac{1}{r'} \right) r' dr' \\ &= 2\pi k \int_a^r dr' = 2\pi k r' \Big|_a^r = 2\pi k(r - a). \end{aligned}$$

S-3 (from TX-4e)

$$\begin{aligned} \oint \vec{B} \cdot d\vec{\ell} &= 4\pi k_m I \\ (B)(2\pi r) &= 8\pi^2 k_m k(r - a) \\ B &= 4\pi k_m k \left(\frac{r - a}{r} \right) \end{aligned}$$

S-4 (from PS-problem 8)

An integral is just the limit of a sum:

$$\oint \vec{B} \cdot d\vec{\ell} = \lim_{\Delta\vec{\ell} \rightarrow 0} \sum \vec{B} \cdot \Delta\vec{\ell}.$$

Note that $\vec{B} \cdot \Delta\vec{\ell}$ is negative along the part of the path labeled $C \rightarrow D$. Therefore the sum is negative for that part of the path and hence so is the path integral for that segment of the path.

To do it formally, note that along that part of the path we have:

$$d\vec{\ell} = -\hat{x} dl$$

and

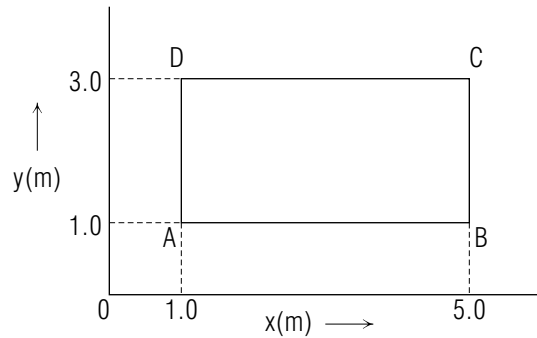
$$\vec{B} = \hat{x} B$$

so:

$$\int_C^D B \hat{x} \cdot (-\hat{x} dl) = -B \int_C^D dl = +B \int_C^D dx = B \int_{5.0 \text{ m}}^{1.0 \text{ m}} dx = -4.0B \text{ m}.$$

MODEL EXAM

1.



$$\vec{B} = 10 \hat{x} \text{ teslas everywhere.}$$

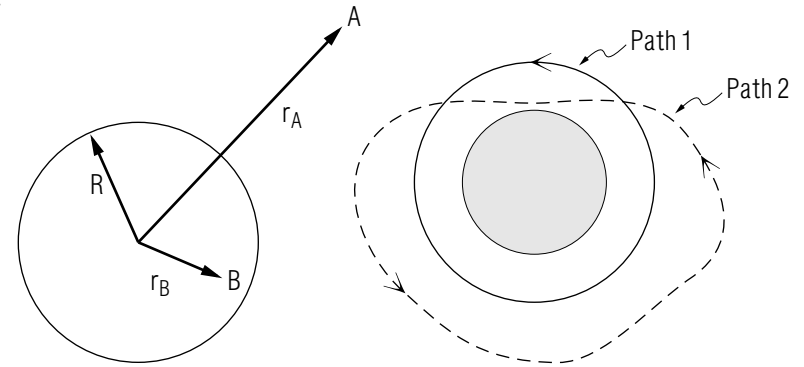
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- Evaluate the line integral of \vec{B} from A to B .
 - Do the same for the line from B to C .
 - Do the same for the line from C to D .
 - Do the same for the line from D to A .
 - Evaluate $\int \vec{B} \cdot d\ell$ around this closed path. Use the results of parts (a)-(d) to find your answer.
 - From your answer to part (e), determine the net current that must be crossing the x - y plane through this rectangle $ABCD$.
2. Repeat Problem 1, parts (a) through (f) for the case where the magnetic field in this region is given by

$$\vec{B} = (A_0 + A_1 y) \hat{x}$$

where $A_0 = 2 \text{ T}$, $A_1 = 0.5 \text{ T/m}$ and y is the y -coordinate of the point.

3.



R = radius of the cylindrical conducting wire

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r = the distance from the center to point A outside the conductor

r = the distance from the center to point B inside inside the conductor.

Path 1 (solid line) is a circular path surrounding the cylinder concentric with the cylindrical conductor passing through point A .

Path 2 (dashed line) is an arbitrary path surrounding the conductor, also passing through point A .

- What is $\int \vec{B} \cdot d\ell$ for each of the paths 1 and 2?
- Explain how symmetry enables you to evaluate \vec{B} at point A only if you use path 1.

Brief Answers:

- See Problem 8 in this module's *Problem Supplement*
- See Problem 9 in this module's *Problem Supplement*
- See Problem 10 in this module's *Problem Supplement*

