

# CHAPTER 2

## PRINCIPLE OF OPERATION OF AN ACCELEROMETER

In the following work the term ‘accelerometer’ is used to designate the entire transducer, normally comprising a mechanical sensing element and conversion of the signal from the mechanical to the electrical domain.

### 2.1 Derivation of the Motion Equation

The measurement of acceleration always relies on classical Newton’s mechanics. Normally the acceleration to which a body is subjected is of interest; the accelerometer being rigidly attached to that body. The transducers which are the subject of this research programme make use of a sensing element consisting of a proof mass (also referred to as seismic mass) which is suspended by a spring; acceleration causes a force to act on the mass which is consequently deflected by a

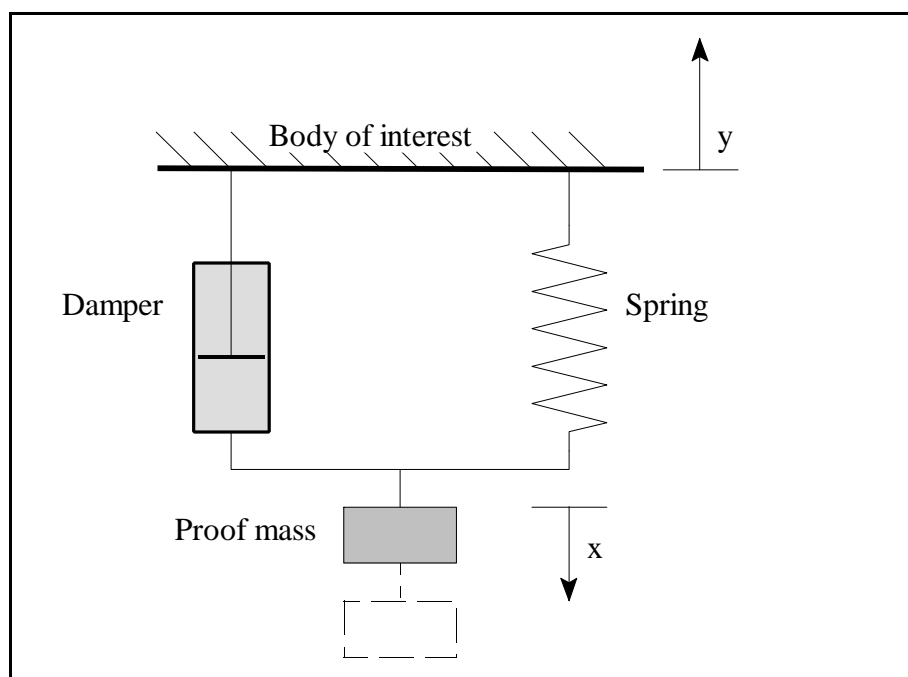


Fig. 2.1: Principle of operation of a sensing element of an accelerometer.

distance 'x' as shown in fig. 2.1. Some form of damping is required, otherwise the system would oscillate at its natural frequency 'T<sub>n</sub>' for any input signal.

To derive the motion equation of the system, D'Alembert's principle is applied, where all real forces acting on the proof mass are equal to the inertia force on the proof mass, Timoshenko and Young [100]. From the stationary observer's point of view, the sum of all forces in the y direction is:

$$0 = m \frac{d^2(y+x)}{dt^2} + b \frac{dx}{dt} + kx$$

$$m \frac{d^2y}{dt^2} = m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx \quad \dots(2.1)$$

where:

m: mass of the proof mass,

y: movement of the body of interest,

x: movement of the proof mass,

b: damping factor (assumed to be constant, for this consideration),

k: spring constant.

The second derivative of 'y' with respect to time is the acceleration 'a' of the body of interest.

## 2.2 Steady State Performance

In the steady state condition, that is with constant acceleration 'a' and constant deflection 'x', eq. (2.1) yields:

$$ma = kx$$

$$a = \frac{kx}{m} \quad \dots(2.2)$$

Here the sensitivity 'S' of an accelerometer is defined by:

$$S = \frac{x}{a} = \frac{m}{k} \quad \dots(2.3)$$

### 2.3 Dynamic Performance

For the dynamic performance it is easier to consider the Laplace transform of eq. (2.1):

$$\frac{x(s)}{a(s)} = \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} \quad \dots(2.4)$$

From this equation, the natural frequency 'T<sub>n</sub>' is

$$T_n = \sqrt{\frac{k}{m}} \quad \dots(2.5)$$

An upper boundary of the bandwidth of an open loop accelerometer is its natural frequency. It can be seen by comparing eq. (2.3) and eq. (2.5) that the bandwidth of an accelerometer sensing element has to be traded off with its sensitivity since  $S \propto 1/T_n^2$  (This trade-off can be partly overcome by applying feedback, which is discussed later).

For the dynamic performance of an accelerometer the damping factor 'b' is crucial. For maximum bandwidth the proof mass should be critically damped; it can be shown that for  $b = 2mT_n$  this is the case. It should be noted here that in micromachined accelerometers the damping originates from the movement of the proof mass in a viscous medium. However, depending on the mechanical design, the damping coefficient 'b' cannot be assumed to be constant; this phenomena will be discussed in sec. 5.3.

## 2.4 Closed Loop Accelerometer

The accelerometer discussed above, in which the deflection of the proof mass provides a measure for the acceleration, is commonly referred to as an open loop accelerometer. High performance transducers have traditionally made use of some form of feedback where an external force is used to compensate the inertia force on the proof mass due to the acceleration, thus keeping the proof mass at zero deflection ( $x = 0$ ). This can be approximately achieved by a closed loop structure depicted in fig. 2.2

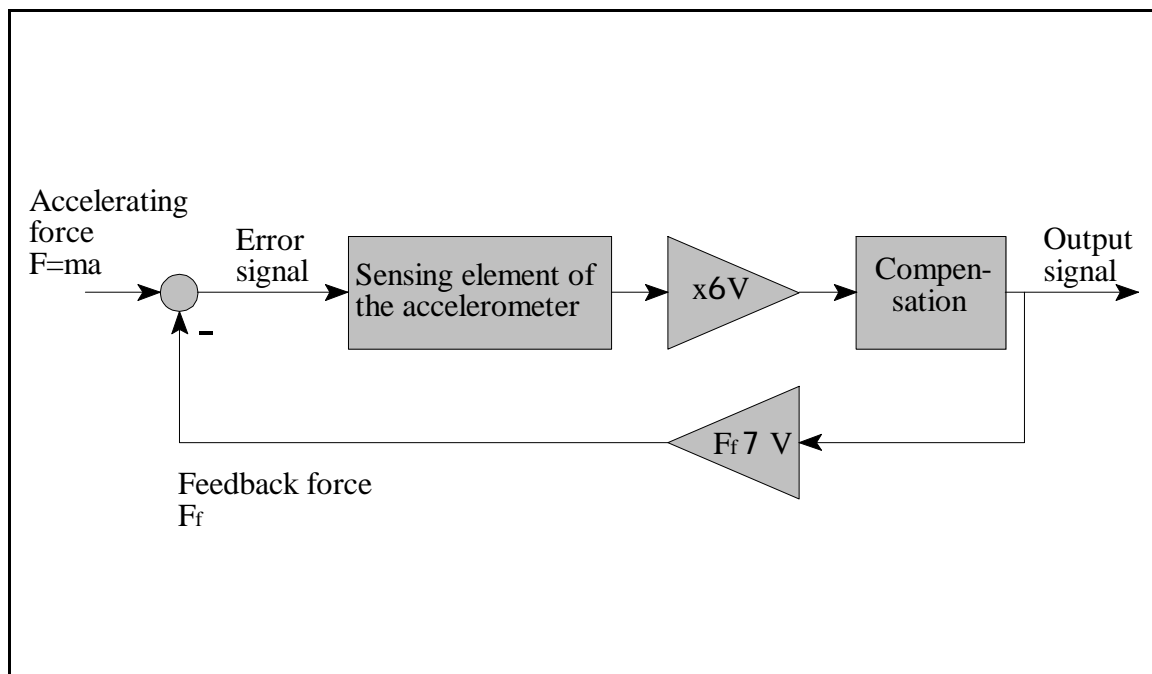


Fig. 2.2: Closed loop accelerometer.

The external force has to be derived by a residual deflection of the proof mass. The position of the proof mass is sensed and converted into a voltage, providing the output signal. The same voltage is used to generate a feedback force on the proof mass opposing the accelerating input force. Consequently, the proof mass is only being deflected by the difference between these two forces which can be considered as the error signal. Some sort of compensation may be required to ensure the stability of the loop.

In a closed loop accelerometer the sensitivity 'S' is not inversely proportional to the square of the

natural frequency ' $T_n$ ' of the sensing element but depends on the loop gain and the form of compensation. Another advantage is that possible nonlinearities of spring and the damping are reduced considerably if the proof mass stays at or close to its rest position. Furthermore the bandwidth of the accelerometer can be increased past the natural frequency of the sensing element.

In conventional, closed loop accelerometers the feedback force is commonly generated by using electro-magnetic forces, Sundstrand data sheet [97]. However, in micromachined devices the dimensions are so small that electrostatic forces can be used to provide the feedback. This has the advantage of lower power consumption and overcomes the problems of manufacturing micromachined inductors, Ward and King [109]. At the present time no micromachined accelerometer is described in the literature that uses electro-magnetic feedback or reset. The drawback of electrostatic feedback with the proof at a defined potential is that the forces are proportional to the square of the voltage, hence they are always positive and consequently negative feedback is difficult to generate. For this reason the feedback relationship is nonlinear. These problems are discussed in detail in chapter 6.