DETERMINATION OF THE MASSES OF MINOR PLANETS

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ABSTRACT

The observations of minor planets are of particular interest for determining a possible rotation between the Hipparcos and the dynamical reference frames. In order to evaluate it, Hipparcos data are compared to the values calculated from the equations of motion. The associated equations of condition involve in addition orbital elements and masses of minor planets. In this paper, we present the favourable conditions of the masses determinations (close approaches) and the selection of particular cases that is being done. We show with examples which precision it is possible to expect from the data of Hipparcos minor planets. The determination of other parameters involved in the condition equations is also discussed.

Key words: minor planets; reference frame; celestial mechanics; space astrometry.

1. INTRODUCTION

Mass determinations of large minor planets have been made for several years. They often utilize repeated, moderately close encounters of objects with the largest asteroids in order to accumulate their weak perturbations in the orbits of the test minor planets (Hoffman 1989). Durations taken into account are generally of the order of a century or more. Hipparcos data of minor planets allowed us to examine the possibility of 'instantaneous' mass determination, this is to say to study a close encounter between minor planets as an actual collision.

Note that Hipparcos data can also be combined with previous data in order to improve orbital elements of asteroids. Effects of mass perturbations can be taken into account, leading to mass estimation (Viateau & Rapaport 1997).

One of the parameters to be taken into account in this problem is the rotationnal link between the Hipparcos reference frame (optical extension of the ICRS) and the dynamical one in which ephemerides of minor planets are computed. Thus, we first present a solution for these rotation parameters.

For doing that, we used two given ephemerides of the Jet Propulsion Laboratory. Study of the robustness of the estimation was also conducted (Bougeard et al. 1997).

According to these values, we study the close encounter Ceres-Ausonia in order to obtain a mass determination for Ceres.

However, the case when only the perturbated body was observed by Hipparcos and the perturbing one was not, is also to be examined. The case of minor planet 46-Hestia, non-observed, is examined in the last section, leading to an estimation of the mass of this asteroid.

2. THE O-C FORMULATION

Astrometric positions of an observed minor planet are compared with that calculated from the equations of motion. The difference 'O-C' (Observed minus Calculated) is linked to the unknowns of the problem which are the correction to the initial elements of the minor planet ($\Delta\overrightarrow{u^0}$). In addition we have to take into account a rotation $\overrightarrow{\theta}$ between the Hipparcos reference frame and the dynamical one. This vector can be written as the sum of two terms, the first one ($\overrightarrow{\theta_0}$) concerning an initial rotation between these two frames, and the second one ($\overrightarrow{w}$) concerning a drift with respect to time. This rotation induces a supplementary correction on the O-C written as:

$$\Delta\overrightarrow{z}_{rot} = R_z\overrightarrow{\theta}, \quad \text{with} \quad \overrightarrow{\theta} = \overrightarrow{\theta_0} + \overrightarrow{w}(t - t_0)$$

where R_z is a matrix which depends on the coordinates of the minor planet and t_0 the initial epoch.

We do not want to give here the whole calculation that leeds to the system of conditional equations we use for Hipparcos minor planets. The reader can refer to our paper (Bec-Borsenberger et al. 1995). We showed there we can write for a given minor planet observed by the satellite Hipparcos:

$$\mathbf{O} - \mathbf{C} = \mathbf{A}\Delta\overrightarrow{u^0} + \mathbf{B}\overrightarrow{\theta} + \mathbf{C}\Delta\vec{m} \qquad (1)$$

where we have:

A: the matrix depending on the minor planet's coordinates partial derivatives with respect to its initial elements;

Table 1. DE200: FAST solution for the rotation and spin parameters. Values of θ are given in mas, and values of ω in mas/year.

θ_1	θ_2	θ_3	w_1	w_2	w_3
3.71 $\pm$2.00	$-$11.90 $\pm$2.62	$-$12.64 $\pm$4.12	4.32 $\pm$1.18	$-$9.51 $\pm$1.47	14.91 $\pm$3.68

B: the matrix linking the Hipparcos reference frame and the dynamical frame;

C: the matrix of the minor planet's coordinates partial derivatives with respect to the perturbating masses.

The total number of observations for the global system reaches about 2700.

3. LINK BETWEEN THE HIPPARCOS AND THE DYNAMICAL REFERENCE FRAME

Astrometric positions in the Hipparcos catalogue are given within the International Celestial Reference System (ICRS). The Hipparcos reference frame has been constructed in such a way that it coincides with the ICRS.

The whole system (1) is solved in order to evaluate the unknown $\overrightarrow{\theta}$. We give in Table 1 the solution for the dynamical system materialized by the DE200 ephemeris. A solution was also computed for the DE403 ephemeris (Standish et al. 1995), but in this case the solution was undetermined, showing alignment with ICRS.

The accuracy reaches about 2.5 mas in the ecliptic plan and 4 mas around a direction perpendicular to it for the rotation parameters θ_i and about 1.5 mas in the ecliptic plan and 4 mas around a direction perpendicular to it for the ω_i. Note that the spin parameters ω_i appear to be greater than expected. This may be due to a phase effect, not taken into account for this computation.

In order to study close encounter between two minor planets, system (1) is reduced to:

$$(\mathbf{O}-\mathbf{C}) - \overrightarrow{\theta} = \mathbf{A}\Delta\overrightarrow{u^0} + \mathbf{C}\Delta m_p \tag{2}$$

The mass of the pertubing asteroid is here m_p. In a first step, we compute only the unknowns $\Delta\overrightarrow{u^0}$ without having taken into account any mass perturbation Δm_p. When possible, the orbit of the minor planet is improved. The effect of mass perturbation during close encounters will be studied over this new orbit.

The problem is now to detect which close encounters will lead to measurable effects. A systematical search is required.

4. CLOSE ENCOUNTERS

The initial epoch for all corrections is t_0 = JD 2 446 000.5. The calculated position of a minor planet was obtained by numerical integration of the motion equations (Bulirsh & Stoer 1966). The perturbations from major planets Mercury to Pluto were taken into account when integrating the equations of motion. In addition, perturbations by the minor planets 2-Pallas, 4-Vesta and 10-Hygiea were systematically taken into account: indeed these three planets are the biggest ones among minor planets and their masses have already been determined. For the computation of perturbations by major planets, we used the DE403 ephemeris of the Jet Propulsion Laboratory (Standish 1996) whereas ephemerides of minor planets were computed at Bureau des Longitudes.

In order to estimate the efficacy of a close encounter between minor planets, we compute the deviation that occurs due to this close approach. We therefore use a two-body formulation (Öpik 1976). The deviation φ is given by:

$$\cot\frac{\varphi}{2} = -\frac{bv_0^2}{Gm_p} \tag{3}$$

where v_0 is the relative velocity between the two bodies, b is the impact parameter, and m_p is the mass of the perturbing asteroid. The minimal distance at close encounter can be compute as:

$$r_m = \frac{GM}{v_0^2}\left[-1+\sqrt{1+\left(\frac{bv_0^2}{Gm_p}\right)^2}\right] \tag{4}$$

By eliminating the impact parameter b, we obtain:

$$r_m = \frac{Gm_p}{v_0^2}\left[-1+\frac{1}{\sin\frac{\varphi}{2}}\right] \tag{5}$$

$$\sin\frac{\varphi}{2} = \frac{1}{\left[1+\frac{r_m v_0^2}{Gm_p}\right]} \tag{6}$$

We have then a criterion for evaluating the 'quality' of a given close encounter by computing the values of $\sin\frac{\varphi}{2}$. For that, we neglect the differences of internal structure of asteroids and assume the same density as Ceres in order to have an estimation of the mass M. We used diameters given by Tedesco (1989). The selection we have done using this criterion and retaining a minimal distance at close approach lower than 0.15 AU (0.40 AU for the largest asteroids) is given in Table 2. It confirms the importance of the value of the perturbing mass: the criterion is larger for the four largest asteroids (Ceres, Pallas, Vesta and 10-Hygiea). We develop below the study of the perturbations due to this asteroid over the orbit of 19-Fortuna.

5. AN ESTIMATION OF THE MASS OF CERES

The study of the close encounter Ceres 63-Ausonia enables us to obtain an estimation of the mass of

Table 2. Close encounters of asteroids during (or just before) the Hipparcos mission. We retain $d_{\min} < 0.25$ AU for the larger asteroids (Ceres, Pallas, Vesta and Hygiea) and $d_{\min} < 0.10$ AU for the others. N_B is the number of Hipparcos observations for the perturbated planet.

Perturbing planet	Perturbated planet	minimal distance (AU)	date (JJ)	selection criterion	N_B (NDAC)
1-Ceres	11-Parthenope	0.206	2449522	121.732	83
1-Ceres	63-Ausonia	0.194	2447922	64.983	15
4-Vesta	28-Bellona	0.195	2446884	23.689	35
9-Metis	14-Irene	0.109	2447622	0.611	51
9-Metis	216-Kleopatra	0.080	2448056	0.399	22
10-Hygiea	88-Thisbe	0.065	2449096	92.580	36
11-Parthenope	12-Victoria	0.077	2448100	1.514	24
14-Irene	42-Isis	0.080	2448426	0.112	53
20-Massalia	44-Nysa	0.037	2447262	5.162	62
20-Massalia	129-Antigone	0.097	2446720	0.337	46
23-Thalia	42-Isis	0.073	2444962	0.063	53
23-Thalia	192-Nausicaa	0.098	2449274	0.072	33
37-Fides	63-Ausonia	0.059	2449540	0.322	15
40-Harmonia	42-Isis	0.077	2447808	0.316	53
46-Hestia	8-Flora	0,074	2449252	5.922	58
46-Hestia	19-Fortuna	0,035	2446592	15.209	30
88-Thisbe	7-Iris	0.047	2447836	3.116	36
230-Athamantis	37-Fides	0.064	2449188	0.363	40
532-Herculina	192-Nausicaa	0.037	2446666	1.177	33

Table 3. Rms of the residuals for the orbit of Fortuna for various values of the mass of 46-Hestia ranging from $m = 0$ to $m = 4.45 \times 10^{-11}\ M_\odot$.

Mass of Hestia ($\times 10^{-11}\ M_\odot$)	0.0	0.63	1.27	1.91	2.54	3.18	3.82	4.45
rms residuals (mas)	44.57	43.44	43.24	43.95	45.55	47.94	51.01	54.65

Ceres (Bange & Bec-Borsenberger 1997). It occurs at JD 2 447 922 and according to Table 2, the selection criterion is one of the largest (64.983). Unfortunately, the minor planet 63-Ausonia is one of the less observed minor planets with 30 observations in each consortium. The variation of the distance between Ceres and 63-Ausonia and the perturbations of Ceres upon the orbit of 63-Ausonia are given in Figure 1. We have assume here a value of $5.0 \times 10^{-10}\ M_\odot$ for the mass of Ceres.

A number of different orbits were calculated for Ausonia, using a different mass of Ceres in each orbit. The same set of observations was used in each calculation. The mass of Ceres and its mean error were determined by the method of Herget (1972). We obtain $m = 4.4 \pm 0.6 \times 10^{-10}\ M_\odot$.

This value is lower than previous determination of the mass of Ceres (Schubart 1974, Goffin 1991, Standish & Hellings 1989). In addition, the precision is low. Nevertheless, it is the first determination during a short period of time and secular effects or long period terms cannot affect it.

As a test to see whether the orbit of Ausonia was affected by any other minor planet, the influence of 37-Fides and 89-Julia were examined (we have found a close approach between Fides and Ausonia just after the Hipparcos mission with minimal distance equal to 0.059 AU, and also between Julia and Ausonia with minimal distance equal to 0.046 AU. 89-Julia was not observed by Hipparcos). The orbit of 63-Ausonia was recalculated including perturbations by these two minor planets but no significant difference was found.

6. THE ORBIT OF 19-FORTUNA AND THE MASS OF 46-HESTIA

The minor planet 46-Hestia was actually not observed by Hipparcos. However, minor planet 19-Fortuna, which supported a close encounter with 46-Hestia, was observed. Therefore, we applied the same method as for the Ceres mass.

The values of the rms for various values of the mass of the pertubing planet are given in Table 3. They lead to a value of $m = 1.09 \pm 0.68 \times 10^{-11}\ M_\odot$ for the mass of 46-Hestia. This value is high according the estimation of diameter given by Tedesco (164 km). Assuming an uncertainty of ± 15 per cent about this value, the density for this asteroid deduced from the value of the mass varies in the interval [5.8–14.5] g cm^{-3}. Densities of asteroids are generally supposed to vary from 1.5–4.5 g cm^{-3}.

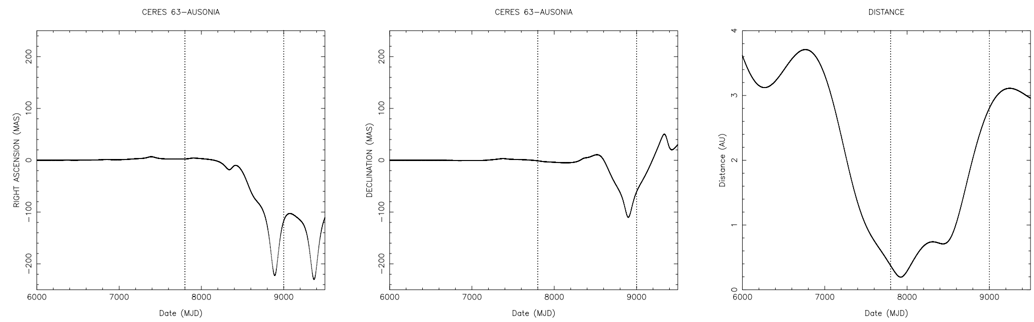

Figure 1. Perturbations of Ceres upon the orbit of 63-Ausonia assuming a value of 5.0 × 10^{-10} $M_\odot$ for the mass of Ceres, and distance between these two minor planets.

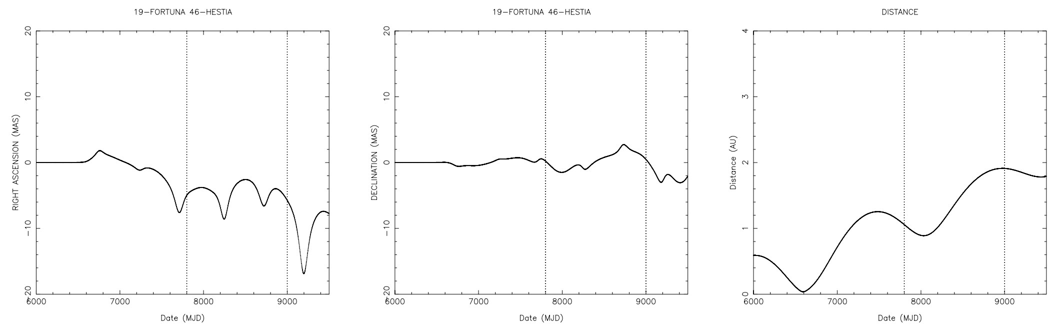

Figure 2. Perturbations of 46-Hestia upon the orbit of 19-Fortuna and distance between these two minor planets.

The method must be refined by taking into account all the effects perturbating the orbit of 19-Fortuna. Moreover, the orbit of 8-Flora can also be studied (close encounter with 46-Hestia at JD2 449 252), and the problem remains open to combine the residuals of these two orbits.

7. CONCLUSION

Our conclusions involve:

(a) the link with the dynamical reference frame: the accuracy of the link is given in Section 3. The problem now is to take into account the phase effects and to combine the data of both consortia;

(b) a search for close encounters: further attempts can be made for searching close encounters with non-Hipparcos asteroids of diameters lower than 150 km (the limit retained here). The maximal deviation criterion seems to be valuable in order to estimate the efficacy of a given close encounter;

(c) the determination of masses using short time encounter: the method applied to Hipparcos data of minor planets gives mass determination with lower accuracy than the traditionnal one. Our model is to be refined. Nevertheless, it is going to develop with the increase of the accuracy of astrometric measurements (GAIA).

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