Resonance, Tacoma Narrows bridge failure, and undergraduate physics textbooks

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The dramatic Tacoma Narrows bridge disaster of 1940 is still very much in the public eye today. Notably, in many undergraduate physics texts the disaster is presented as an example of elementary *forced resonance* of a mechanical oscillator, with the wind providing an external periodic frequency that matched the natural structural frequency. This oversimplified explanation has existed in numerous texts for a long time and continues to this day, with even more detailed presentation in some new and updated texts. Engineers, on the other hand, have studied the phenomenon over the past half-century, and their current understanding differs fundamentally from the viewpoint expressed in most physics texts. In the present article the engineers' viewpoint is presented to the physics community to make it clear where substantial disagreement exists. First it is pointed out that one misleading identification of forced resonance arises from the notion that the periodic natural vortex shedding of the wind over the structure was the source of the damaging external excitation. It is then demonstrated that the ultimate failure of the bridge was in fact related to an aerodynamically induced condition of *self-excitation* or "negative damping" in a torsional degree of freedom. The aeroelastic phenomenon involved was an *interactive* one in which developed wind forces were strongly linked to structural motion. This paper emphasizes the fact that, physically as well as mathematically, *forced resonance* and *self-excitation* are fundamentally different phenomena. The paper closes with a quantitative assessment of the Tacoma Narrows phenomenon that is in full agreement with the documented action of both the bridge itself in its final moments and a full, dynamically scaled model of it studied in the 1950s.

I. INTRODUCTION

The original Tacoma Narrows bridge, at all stages of its short life, was indeed very active in the wind (Fig. 1). Its failure on 7 November 1940 attracted wide attention at the time and has elicited recurring references ever since, notably in undergraduate physics textbooks. The occasion for this article is that the writers, who, in the course of aeroelastic research, have studied the matter closely, believe that many of the references are misleading to the reader in regard to the phenomena that were manifested at Tacoma Narrows. While the early engineering record itself was unclear and indecisive about the causative factors (see, for example, several quotations in Ref. 1), evidence available even early on[2,3] but subsequently reexamined more closely[4–9] has allowed the record to be set quite straight. What is currently offered, however, in explanation by certain textbooks to entering science and engineering students is, we believe, much too casual and often incorrect, perhaps traceable to misleading sources. The main issues in this instance are: What was the exact nature of the wind-driven occurrences at Tacoma Narrows, and, can they be considered correctly to be cases of resonance?

The occasion for this article occurred to one of us (KYB), while browsing in the bookstore and examining three currently used and popular textbooks.[10–12] These invoke inferences about the Tacoma Narrows episode that differ from present engineering understanding of the failure. However, we also point out, below, areas of at least partial agreement. Our aim is to set the record a bit straighter than it now seems to be—at least as popularly understood.

In several books, for example,[10] where the elementary concept of *resonance* is introduced and explained, the bridge disaster, complete with the sensational photographs of its failure, is cited as a pertinent example. We shall discuss a bit later to what extent it should—or should not—be considered an appropriate example of resonance. An interesting search through other introductory physics texts revealed how widespread the use of this example is. For this, five college libraries, two high-school libraries, and three public libraries, as well as three campus bookstores and two of the largest textbook-carrying stores in New York City were searched. We thus also noted the ubiquitous presence of the Tacoma Narrows bridge failure in numerous other texts.[13–42] In fact—in retrospect—it would have been of interest to note particularly those few texts which did *not* cite the Tacoma Narrows incident as a case of resonance.

The list presented above consists of introductory college physics texts used in the USA. While compiling this list, we discovered that many related works have also made similar identification of the failure. These include high-school texts,[43–45] books that are often classified as "physical sciences,"[46] more advanced texts in physics,[47] books for popular circulation,[48–50] and the short article cited earlier that accompanies a well-known film clip, now available as a video tape.[1]

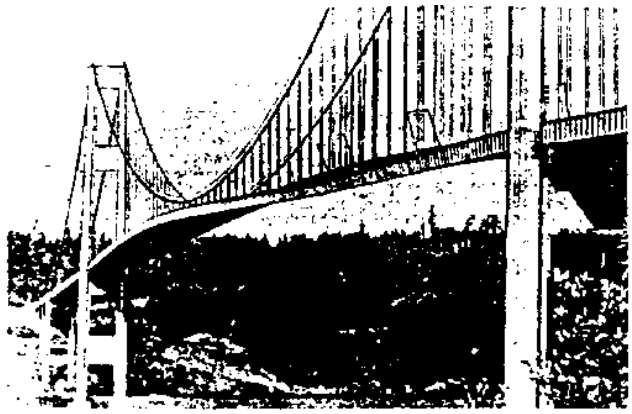

Fig. 1. The original Tacoma Narrows bridge under the action of wind.[1]

II. TEXTBOOK ACCOUNT

Typically, *resonance* is first presented qualitatively along these lines:

> In general, whenever a system capable of oscillation is acted on by a periodic series of impulses having a frequency equal to or nearly equal to one of the natural frequencies of oscillation of the system, the system is set into oscillation with a relatively large amplitude.[11]

The Tacoma Narrows bridge disaster is then suggested as an example of *resonance*:

(1) "...the central span [of the bridge] resonated until the *resonance* became so great that it eventually caused the bridge to collapse."[29]

(2) "...was destroyed by wind-generated *resonance*."[28]

(3) "...Because of *resonance*, wind blowing over the surface and support cables of the Tacoma Narrows bridge generated a very large wave disturbance that destroyed the bridge."[26]

(4) "The most famous incidence of *resonance* destroying a large structure was the collapse of the Tacoma Narrows Bridge in 1940 under the driving force of the wind."[23]

The final, catastrophic event at Tacoma Narrows did, in fact, fit part of the above qualitative definition of resonance—as we shall discuss—*if* the more penetrating question of where the "periodic series of impulses" came from is temporarily set aside, for it was indeed a single torsional mode of the bridge that was driven to destructive amplitudes by the wind, as will be discussed at a later point.

However, if we seek a more quantitative description of resonance in the common textbooks, the approach that is taken is a discussion of the classic linear single-degree-of-freedom oscillator defined by the well-known differential equation:

$$m\ddot{x} + b\dot{x} + kx = F\cos\omega_e t, \tag{1}$$

where, m, b, k are the mass, damping coefficient, and stiffness, respectively, of a linear mechanical system of displacement x, and ω_e and F are the radian frequency and the amplitude of an external exciting force, as a function of time. For this well-known system, resonance (highest response amplitude) occurs when the external forcing frequency ω_e approaches the mechanical natural frequency $2\pi f$ in the near vicinity of the value $\sqrt{k/m}$.

After the above presentation, a representative comment like the following is usually made:

> "The wind produced a fluctuating resultant force in resonance with a natural frequency of the structure. This caused a steady increase in amplitude until the bridge was destroyed."[10]

We believe that interesting facts are lost, glossed over, or misrepresented when texts are vague about just what the exciting force was and just how it (being due basically to the wind) acquired the necessary periodicity. Some texts suggest that this force was supplied "by gale winds,"[36] or "gusts of wind,"[35] etc. But "gusts" and "gale" do not connote any well-defined periodicity. Seeking such periodicity must lead to a closer investigation of the aerodynamics of bluff bodies, which falls within the writers' area of technical expertise. The so-called *periodic vortex shedding*[51–53] effect is a first, very tempting, candidate to which to attribute the necessary periodicity.

Bluff bodies (such as bridge decks) in fluid streams do in fact naturally shed periodic vortex wakes, tripped off by body shape and viscosity, that are accompanied by alternating pressures on the bodies, which oscillate in consequence. Thus some authors,[10–11] seeking a likely cause, assumed that this observed effect must have provided the necessary conditions that destroyed the bridge. Unfortunately, this explanation is incorrect. We now know that this is *not* what occurred at Tacoma Narrows.

III. VORTEX-INDUCED VIBRATION

When fixed in a fluid stream, bluff (nonstreamlined) bodies generate detached or separated flow over substantial parts of their surfaces; that is, the flow lines do not follow the contours of the body, but break away at some points. At low Reynolds number, when separation first occurs, the flow around the body remains steady. At some critical Reynolds number two thin layers—often termed the free shear layers—form to the lee of the body. These unstable layers interact nonlinearly with each other in the body wake to produce a regular periodic array of *vortices* (concentrations of rotating fluid particles) termed the *Strouhal vortices*. Such wakes were systematically investigated for circular cylinders by Bénard.[53]

These vortex arrays arrange themselves in two rows, with opposite directions of circulation. Each vortex is located opposite the midpoint of the interval between the two closest vortices in the opposite row (Fig. 2). The beauty of this "vortex street"—often termed the *Karman vortex street* after the noted aerodynamist von Karman[54]—has long attracted attention, and popular articles[55] often carry pictures of it, emphasizing the "mystery" that exists in the formation process.

The frequency of the shedding vortices over a fixed (restrained) body is often termed the Strouhal frequency (f_S) and follows the relation:

$$f_S D/U = S. \tag{2}$$

Here, U is the cross-flow velocity, D is the frontal dimen-

Fig. 2. Typical streak line pattern of vortex trail behind a bluff body.

sion, and S is the (nearly constant) Strouhal number appropriate to the body in question. In the case of the original Tacoma Narrows bridge the values of D and S are, respectively, 8 ft and about 0.11.

When the periodic vortex shedding at frequency f_S is taken into account in the bridge context, an external periodic agent is identified, and this periodic force is typically *mis*identified as the source of the "resonant" frequency that caused the bridge to fail.

(1) "The collapse was not due to the brute force of the wind but due to a resonance between the natural frequency of oscillation of the bridge and the frequency of wind-generated vortices that pushed and pulled alternately on the bridge structure."[11]

(2) "...vortices were pouring off the top and bottom of the bridge, driving the bridge at its resonant frequency, which eventually led to its collapse."[12]

(3) "Thus, vortex shedding allows us to understand the origin of the fluctuating vertical forces on the Tacoma Narrows Bridge... ."[13]

The assumption that the Strouhal frequency (f_S) matched a body natural mechanical frequency of the bridge (i.e., $f_S = f$) is frequently made. If this had been what happened during the destructive oscillation, Tacoma Narrows would have been closer to an example of "resonance"; but even this requires discussion, as we point out later. Mechanical vibration in the presence of the vortices that are shed rhythmically under the resonant condition $f_S = f$ is a well-observed phenomenon termed *vortex-induced vibration*. In the Tacoma Narrows circumstance some of the textbooks in question[11-13] are in effect concluding that the bridge failed due to this sort of phenomenon. It did not.

However, during its brief lifetime late in 1940 the bridge did experience this sort of vibration, but *safely*, as it occurred in purely vertical modes under relatively low-speed winds. In fact, the slender bridge deck gained the sobriquet "Galloping Gertie" from such oscillations, which took place repeatedly, almost from opening day, 1 July 1940. Motorists crossing the bridge sometimes experienced "roller-coaster like" travel as they watched cars ahead almost disappear vertically from sight, then reappear. Professor Burt Farquharson of the University of Washington witnessed all this, and it was reported that he did not believe, early on, that the bridge was in danger of collapse. He had begun wind-tunnel model experiments[3] that exhibited similar relatively benign undulations. Figure 3 (after Farquharson[3]) depicts the oscillation amplitudes of a full wind-tunnel model of the original Tacoma Narrows bridge. This set of graphs "says it all" in regard to what happened with that bridge. We will return to these results a little later on.

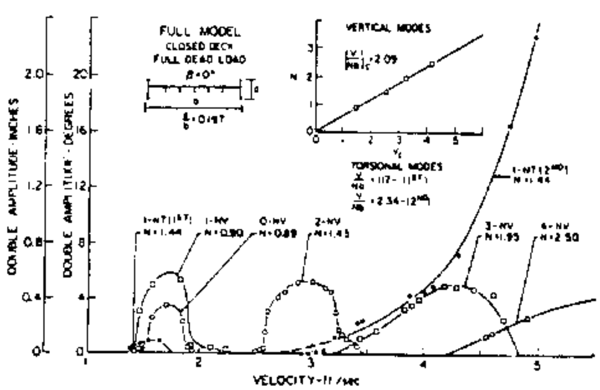

Fig. 3. Aeroelastic performance of original Tacoma Narrows full-bridge model.[3]

It has by now long since been demonstrated that from the standpoint of phenomenology, even such vortex-induced oscillations do not constitute a case of simple resonance. The wind–structure interaction phenomenon associated with natural vortex shedding has been found to be very complex, involving both externally wind-initiated forces and self-excitation forces that "lock on" to the motion of the structure. Far from a case of simple resonance, this striking phenomenon of bluff-body fluid dynamics has been—and continues to be—one of the more recondite areas of the modern field of aeroelasticity. The various extant mathematical models for it (see Ref. 52 which lists many of them) are *not* those of simple externally forced resonance. One of the present writers (KYB) argues[52] that the phenomenon is a nonlinear mode-coupling effect between body and flow periodicities, leading to a form of *parametric excitation* and subsequent amplitude saturation due to nonlinearities coming into play at higher amplitudes. While this line of mathematical modeling is peripheral to our main argument, it does illustrate just how far we are from a simple example of "resonance"—and how misleading it can be for young students of science and engineering to be given this example at a formative stage of their scientific development.

During "lock-on" the wind forces excite the structure at or near one of its resonant frequencies, but as its amplitude increases this has the effect of modifying the local fluid boundary conditions in a manner that instigates compensating, self-limiting forces. These ultimately restrict the structural motion to relatively benign amplitudes[56] (see also Fig. 3). The overall effect then resembles more a self-limiting oscillation of Van der Pol type. Vortex-induced vibration is clearly not a linear resonance even if the structure itself has linear properties, since the exciting force amplitude F is a nonlinear function of the system response.

IV. THE DESTRUCTIVE MECHANISM AT THE TACOMA NARROWS

The ultimate bridge failure at Tacoma Narrows, however, took place under a wholly different—and catastrophic—set of circumstances (cf. Fig. 1). The wind speed at that time, according to Farquharson,[3] was 42 mph, and the frequency he observed for the final destructive oscillation was 12 c/m, or 0.2 Hz. At 42 mph, the frequency of natural vortex shedding according to the Strouhal relation would be close to 1 Hz, wholly *out of synch* with the actual catastrophic oscillation then going on. It can be concluded that natural vortex shedding was *not* the cause of the collapse. This rules out one type of periodic exciting force implied by a few of our references.

Engineering interest in the problems of bridge stability under wind, a very important matter for new designs, has led to further exploration.[5-8] Some of the earliest of these explorations were already carried out by Farquharson,[3] at the University of Washington, and by Karman and Dunn,[3] at the California Institute of Technology, in the 1940s and 1950s. The final destructive, catastrophic instability was also duplicated with a scaled bridge model by Scruton[57] in

England and independently by Farquharson[3] at the University of Washington.

Referring to Fig. 3, one may observe that, in a full-bridge 1/50th scale Tacoma Narrows model several modes are identified as responding with self-limiting amplitude—except for one particular mode. This was a low torsional mode identified as "1-NT 2nd" (with $N = 1.44$ Hz) which, when the model frequency scaling factor of $\sqrt{50}$ is divided out, defines a prototype frequency of 0.2 Hz, precisely the frequency of the destructive mode identified at the site by Farquharson.[3] Under increasing wind velocity this mode proceeds to ever-increasing amplitude.

In the early 1970s Scanlan and Tomko,[4] repeating some of this work on a Tacoma Narrows section model, and carrying the work farther, demonstrated conclusively that the catastrophic mode of the old Tacoma Narrows bridge was a case of what they termed *single-degree-of-freedom torsional flutter* due to complex, separated flow. To prevent the discussion from degenerating here into mere semantics, the research will be described in some detail.

Instead of characterizing the force that excited the single-degree oscillator as a purely external function of time, it was characterized as an aerodynamic *self-excitation* effect that was able to impart a net negative damping characteristic to the system. The important (and eventually destructive) torsional motion (quite distinct from the vertical "roller coaster" motion) will be selected here for focus. The torsional oscillator (bridge deck section) may be described by

$$I\,[\ddot{\alpha} + 2\zeta_\alpha \omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] = F(\alpha,\dot{\alpha}), \qquad (3)$$

where I, ζ, ω_α are, respectively, associated inertia, damping ratio ($2\pi\zeta_\alpha$ = log dec), and natural frequency, and α is the angle of twist. The aerodynamic force $F(\alpha, \dot{\alpha})$ was postulated in the linearly self-excited form

$$F(\alpha,\dot{\alpha}) = A_2 \dot{\alpha} + A_3 \alpha, \qquad (4)$$

which, nondimensionally, became[4]

$$F(\alpha,\dot{\alpha}) = \tfrac{1}{2}\rho U^2 (2B^2)\,[KA_2^*(B\dot{\alpha}/U) + K^2 A_3^* \alpha], \qquad (5)$$

where ρ is air density, U is wind velocity, B is deck width, ω is the circular frequency of oscillation, $K = B\omega/U$, and A_2^*, A_3^* are dimensionless aerodynamic ("flutter") coefficients, functions of K. ($A_1^* \equiv 0$, a coefficient associated elsewhere with vertical motion, is not pertinent to the present discussion.) It will be especially noted that no external independent function of time is present in this formulation, i.e., Eq. (3) is a homogeneous differential equation.

Experimental determination[4] of the coefficient A_2^* revealed it to have the form plotted in Fig. 4, wherein the evolution of this damping coefficient with reduced velocity ($U/nB = 2\pi/K$) exhibits a dramatic reversal in sign. It was later found that Karman and Dunn[2,3] had determined and plotted a related Tacoma Narrows parameter, obtaining the result shown in Fig. 5, which, however, included both aerodynamic and mechanical damping effects. In this figure $\delta = 2\pi\zeta_\alpha$ is the logarithmic decrement and μ is the mass ratio $\rho B^2 g/w$, w being bridge weight per foot. The complexity of flow activity over the deck section as it oscillates under the conditions described is suggested in Fig. 6.

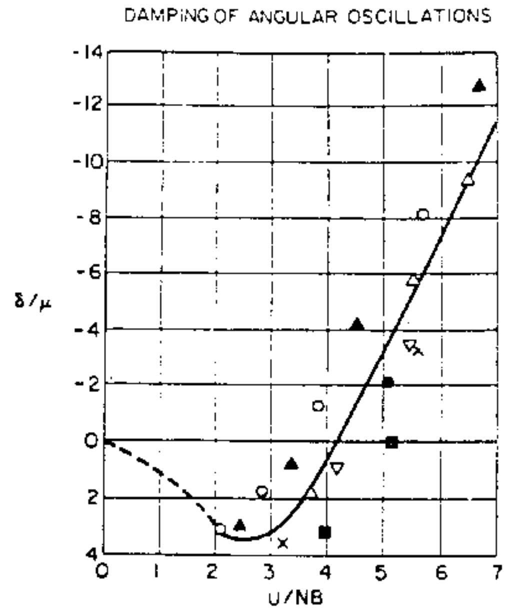

Fig. 5. Damping of angular oscillations, original Tacoma Narrows section model.[2,3]

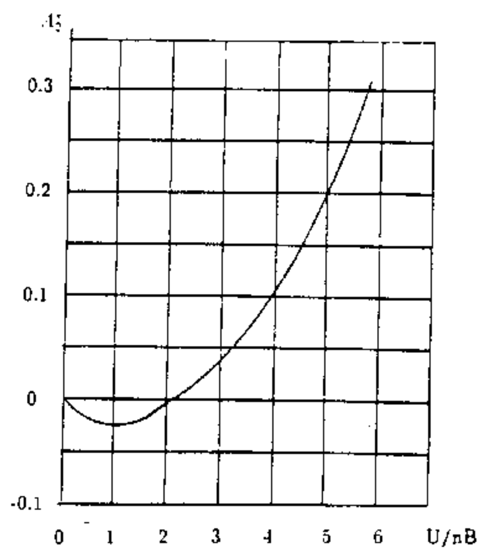

Fig. 4. Torsional damping flutter derivative,[4] original Tacoma Narrows bridge deck cross section.

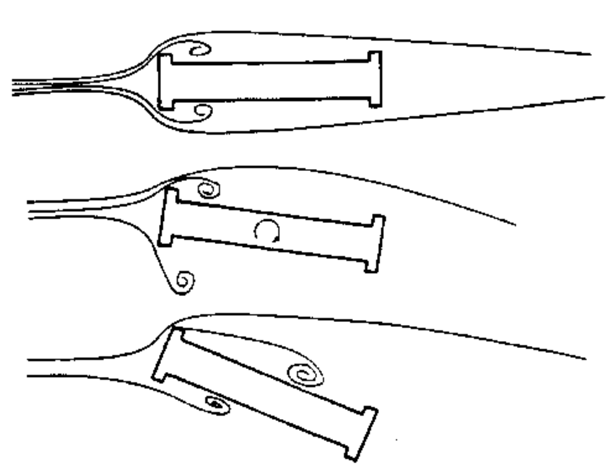

Fig. 6. Vortex pattern over rotating deck section.

When, for any reason, a body changes angle of attack in a fluid stream, it sheds new vorticity into its wake. Thus its motion is indeed associated with a vortex wake. But such a motion-induced wake will have little or nothing to do with a naturally developed Karman vortex trail. In fact, bluff bodies in oscillatory motion shed wakes containing components at *both* the oscillation and the Strouhal frequencies. Under high amplitudes of oscillation it is the former that predominate. The final, destructive oscillation of the old Tacoma Narrows bridge produced a *flutter* wake, not a Karman vortex street. This goes to the heart of a "chicken–egg" dilemma: Did the vortices cause the motion or the motion cause the vortices? In this case (flutter) it was the latter.

This action, which finally brought the bridge down, occurred in a fundamental antisymmetric torsion mode (see Figs. 1 and 3), one which had not manifested itself in the bridge response until some 45 min prior to the collapse—basically a single-degree-driven unstable oscillation with effectively negative damping representing an inflow of wind energy caused by the synchrony of motion-induced pressures with the motion itself. At the time, the wind velocity (42 mph) was far in excess of that required for incipient flutter. This is what happened on 7 November 1940 at Tacoma Narrows. Details will be presented in Sec. VI.

Could this be called a resonant phenomenon? It would appear not to contradict the qualitative definition of resonance quoted earlier, if we now identify the source of the periodic impulses as *self-induced*, the wind supplying the power, and the motion supplying the power-tapping mechanism. If one wishes to argue, however, that it was a case of *externally forced linear resonance*, the mathematical distinction between Eqs. (1) and (3) is quite clear, self-excited systems differing strongly enough from ordinary linear resonant ones. The texts we have consulted have not gone this far in explanation.

V. FURTHER CONFUSIONS

Lee Edson added additional confusion to the old Tacoma Narrows story by some informal remarks he attributed, in the first person, to von Karman in his biography. The exact quotation (Ref. 58, p. 213) is

> "...the culprit in the Tacoma disaster was the Karman Vortex Street."

We believe this attribution fails to confront the "chicken–egg" dilemma that we cited earlier. We see the *flutter* vortex trail as a consequence, not a primary cause.

The von Karman biography (as well as Ref. 12) further suggests that, von Karman's explanation of the underlying mechanism having gone to the heart of the matter, engineers subsequently were able to solve the bridge instability problem. This also is not quite accurate. We now know that von Karman's explanation—at least as reported by Edson—was off the mark. But having found experimentally that the squat H section of the original Tacoma Narrows bridge was highly unstable in *flutter* as we have described, engineers (notably Farquharson[59]) avoided it and studied many other bridge section model forms empirically in the wind tunnel, fixing finally upon a deep, open-truss section, conductive both to higher torsional structural stiffness and to aerodynamic stability, with which the Tacoma Narrows bridge was then rebuilt. The fascinating details of design[5] through which modern bridge decks, all over the world, are rendered stable will not be entered into here.

The confusion about Tacoma Narrows—an extension of much that surrounded it in earlier times—has apparently continued up to the present. The primary reason for all this, we believe, is that many *post facto* accounts or investigations were speculative or reviews of still other accounts. Reference 1 cites some six of these, while itself neglecting the underlying *fluid-dynamic* mechanism and treating the bridge *dynamics* in an oversimplified manner, though these authors cite Ref. 3 from which our Fig. 3 is taken) which properly accounts for all effects. Wind-tunnel model studies, together with theoretical studies over many years,[5,8,60,61] have led, we believe, to more penetrating insights.

Another error accompanying many accounts[62–64] has been the confusion of the phenomenon of bridge flutter with that of airplane wing flutter as though they were identical.[65–67] Unless a bridge deck is highly streamlined as is the case in some very modern decks, the eventual flutter phenomenon that it will undergo is not akin to airfoil flutter, but to a form of separated-flow flutter, which tends to excite mainly the torsional degree of freedom. The intrinsic underlying fact is that flow around highly streamlined bodies (such as airfoils) satisfies the smooth-flow trailing edge *Kutta condition* whereas flow around bluff bodies (e.g., bridges) does not. There are several sources[4–6,66] that document the fact that bridge (bluff-body) flutter is practically not comparable to airfoil flutter.

The flutter aerodynamic forces on the airfoils of modern aircraft reach magnitudes comparable to their resisting inertia and stiffness forces. As a result, flutter, when it occurs for these structures, tends to be very precipitate. Further, it represents an unstable coupling of *two* degrees of freedom (bending and torsion) into a new (binary flutter) mode, whereas each is otherwise found, individually, to be positively damped. In the quite different case of the heavy structure of bridges, the aerodynamic forces, under wind flows that are low in speed compared to those of aeronautics, are relatively weaker and do not greatly influence the responding modes—nor their frequencies.

They can and do influence the overall damping, however, reversing it in sign at the higher wind speeds. When this occurs, even if two or more modes sometimes couple, the principal driving mechanism is found to lodge in a single unstable mode—usually torsion. With bridges, unstable oscillation thus tends to grow more gradually in amplitude. The old Tacoma Narrows bridge underwent some 45 min of travail before its demise.

We note also that numerous instructional texts in mathematics[68–76] allude to the Tacoma Narrows incident, and most of these, too, could be made more precise and insightful in the light of current analyses of the problem.

VI. QUANTITATIVE ASSESSMENT OF THE OLD TACOMA NARROWS WIND-EXCITED PHENOMENA

In 1971 Scanlan and Tomko,[4] employing a spring-supported wind-tunnel section model of the original Tacoma Narrows bridge, developed the curve shown in Fig. 4 for the A_2^* torsional flutter derivative associated with the aerodynamic damping of that deck. The condition for flutter, i.e., zero damping in a torsional mode (in this case, the fundamental mode at 0.2 Hz) from Eqs. (3) and (5) is

$$(A_2^*)_{\text{crit}} = 2I\zeta_\alpha/\rho B^4, \qquad (6)$$

where I = section mass moment of inertia about the center of rotation; ρ = air density; B = deck width; and $2\pi\zeta_\alpha$ = logarithmic decrement in the torsional mode. Data for the original Tacoma Narrows case (represented first in the engineering units of pounds, slugs, and feet used in the original design study) are as follows: $\omega = 2850$ lb/ft $= 4249.1$ kg/m; $r = 15$ ft $= 4.573$ m; $g = 32.2$ ft/s^2 $= 9.81$ m/s; $I = 2(\omega/g)r^2 = 39829$ slugs ft^2/ft $= 177\,730$ (kg)m^2/m. Using

$\rho = 0.002\,378$ slugs/ft^3 $= 1.23$ kg/m^3;

$B = 39$ ft $= 11.89$ m;

it is found that

$$(A_2^*)_{\text{crit}} = \frac{2(39\,829)\zeta_\alpha}{0.002\,378(39)^4} = 14.48\zeta_\alpha .$$

From Fig. 4 the following table of values can be calculated:

Table I. OTN flutter conditions as a function of mechanical damping.

ζ_α	$(A_2^*{}_{\text{crit}})$	$(U/nB)_{\text{crit}}$	Prototype velocity U_{crit} (mph)
0.003	0.043	3.20	17.0
0.005	0.072	3.50	18.6
0.010	0.145	4.30	22.9
0.015	0.217	5.15	27.4
0.020	0.290	5.75	30.6

In comparison with this, Farquharson reported the following facts (cf. Fig. 3) from study of a full-bridge, 1:50 scale dynamic wind-tunnel model of the original Tacoma Narrows bridge:

(a) The logarithmic decrement of the mode in question was not known, but probably near to $\delta = 0.03$ (i.e., $\zeta_\alpha \cong 0.005$).

(b) Flutter in this mode, designated 1-*NT*, was incipient at 3.3 ft/s in the model. This corresponds to a prototype flutter speed of

$$U_{\text{crit}} = 3.3(\tfrac{60}{88})\sqrt{50} = 15.91 \text{ mph}.$$

This value compares quite reasonably to the (incipient) critical speeds predicted above from the section model study,[4] Table I.

In further corroboration of the incipient flutter speed, Fig. 5 (from section model studies of Karman and Dunn[2]) indicates that total damping is zero somewhere in the range

$$3.4 < U/nB < 5.1,$$

i.e., for the prototype bridge between 18 and 27 mph, which again compares reasonably with the results of Table I.

The increase of response with wind speed, beyond the incipient stage, toward higher amplitude flutter, which is influenced by structure-induced turbulence and hence progressively changing values of A_2^*, was witnessed in the full model and is documented in Fig. 3. Therein the response curve designated as 1-*NT* (2nd) is the divergent flutter response. The model frequency 1.44 Hz corresponds quite closely to a prototype frequency of

$$N = n = 1.44/\sqrt{50} = 0.20\text{Hz} = 12.2 \text{ cpm},$$

which is clearly the destructive frequency observed and reported by Farquharson[3] at the bridge site just prior to the bridge collapse. The ever-increasing response of mode 1-*NT* is seen in Fig. 3 to approach "divergent" amplitudes at a model velocity of around 5 ft/s which corresponds to a full-scale speed of about 35 mph. The actual prototype steady wind at the time of collapse was of course in excess of this value (42 mph).

In summary, Fig. 3, together with the separate examinations culminating in Figs. 4 and 5, quite accurately characterize the critical torsional oscillations of the Old Tacoma Narrows bridge. In contrast, the relatively benign and self-limiting amplitudes of vertical oscillation are all also clearly indicated in Fig. 3. This figure and Fig. 5 have been available since 1952, and Fig. 4 since 1971. No "puzzle" or "mystery" is involved here.

VII. CLOSING REMARKS

It appears to us that the accounts given in many physics as well as elementary mathematics texts are not likely to have been based on penetrating investigations of the Tacoma Narrows phenomena discussed in this paper. Many noninvestigative approaches to the Tacoma Narrows events have developed a wide range of rather loose descriptions, explanations, and speculations over the last half-century. As we have pointed out, however, good physical evidence is available in the literature corroborating the underlying mechanisms of the Tacoma Narrows events as presently understood.

The Tacoma Narrows incident will remain a celebrated example because of its spectacular nature and the freak recording of the disaster by witnessing photographers. The sensational photographs have made it an irresistible pedagogical example—and indeed, much is to be learned from it. Because it lodges itself so easily in the memory, it is doubly important for educators to draw the correct lessons from this classic and sensational event. While it is understandable how so many textbooks have, over the years, oversimplified the physics involved, it is probably time—given the more advanced state of present knowledge—to offer the next generation of students subtler, more complex, and *correct* explanations. In the engineering world proper interpretation of the aeroelastic events at Tacoma Narrows has influenced the designs of all the world's great long-span bridges built since that time.

[1] R. G. Fuller, D. A. Zollman, and T. C. Campbell, *The Puzzle of the Tacoma Narrows Bridge Collapse* (Wiley, New York, 1982) (article); American Association of Physics Teachers, 5112 Berwyn Rd., College Park, MD 20740 (video tape).

[2] O. H. Ammann, T. Von Karman, and G. B. Woodruff, "The failure of the Tacoma Narrows bridge," Report to the Federal Works Agency, 28 March 1941.

[3] *Aerodynamic Stability of Suspension Bridges*, edited by F. B. Farquharson, University of Washington Engineering Experimental Stations Bulletin, No. 116, Pts. 1–5, June 1949–June 1954.

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