Realising the metre

Lecture 1 of 2 on length metrology

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Dimensional Metrology, NPL

New frontiers for metrology
Module I – Physical metrology
4 July 2019, Varenna Summer School

Andrew Lewis – facts and figures

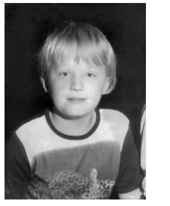
- Moderately stable 1.72 m length artefact
- Fastest speed achieved w.r.t Earth: 2,100 km/h
- Furthest from Earth mean surface: 17.7 km
- Significant average locations in history:
  - 18 years in Leicester (achieving 1.72 m height)
  - 3 years in Cambridge (improving my stored power – 'Knowledge is power')
  - 31 years in Teddington (working out how to measure things…. ongoing)

- Work:

  Science Area Leader, Dimensional Metrology, CCL WG ex-chair, EURAMET TC-L ex-chair, Hon Prof. University College London

- Non-work:

  Gardening, flying (trainee pilot), karting, juggling, walking



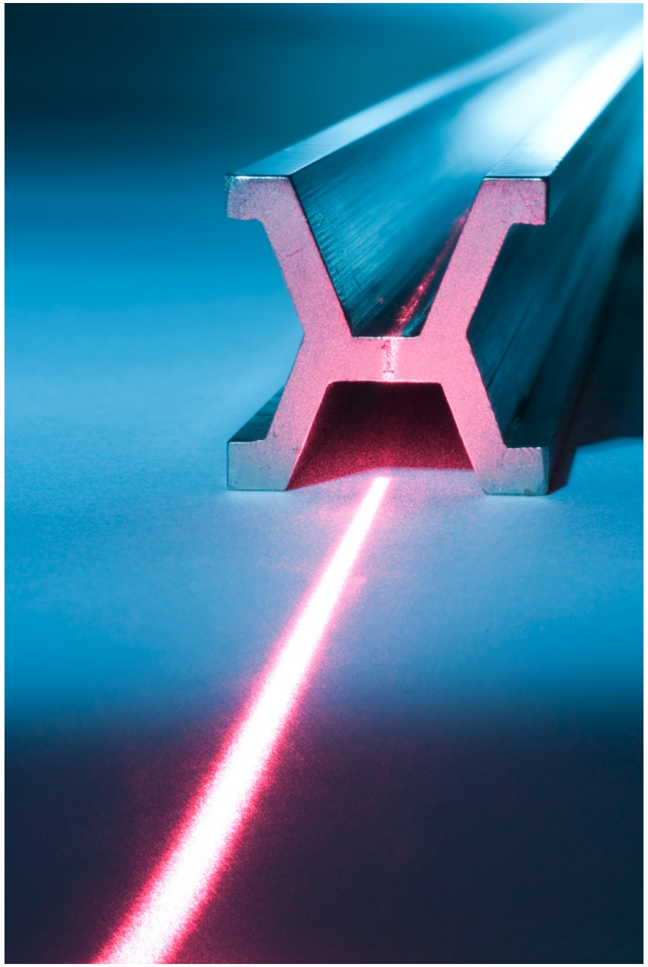
What this lecture is about

- What is a metre – what is length?
  - Why do I care?

- How have we defined the metre
  - How the metre definition has changed over time
  - What is the latest (20 May 2019) definition

- Working with the latest definition:
  - What does it mean in reality
  - How do we use the definition
  - What problems do we encounter
  - What might the future look like

- (Lecture 2 tomorrow focuses on dimensional metrology)



Why do I care what a metre is?

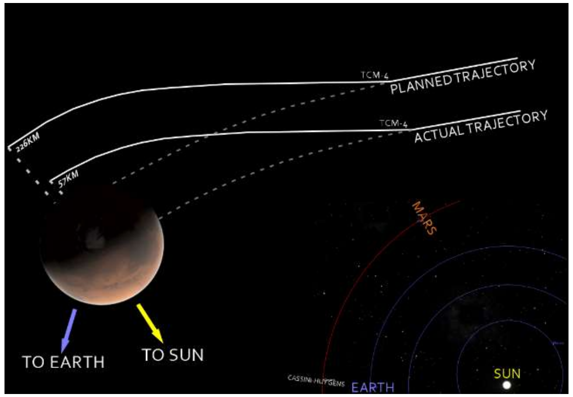
Traceability matters

Legal metrology

(Egyptian cubit)

Intercompatibility

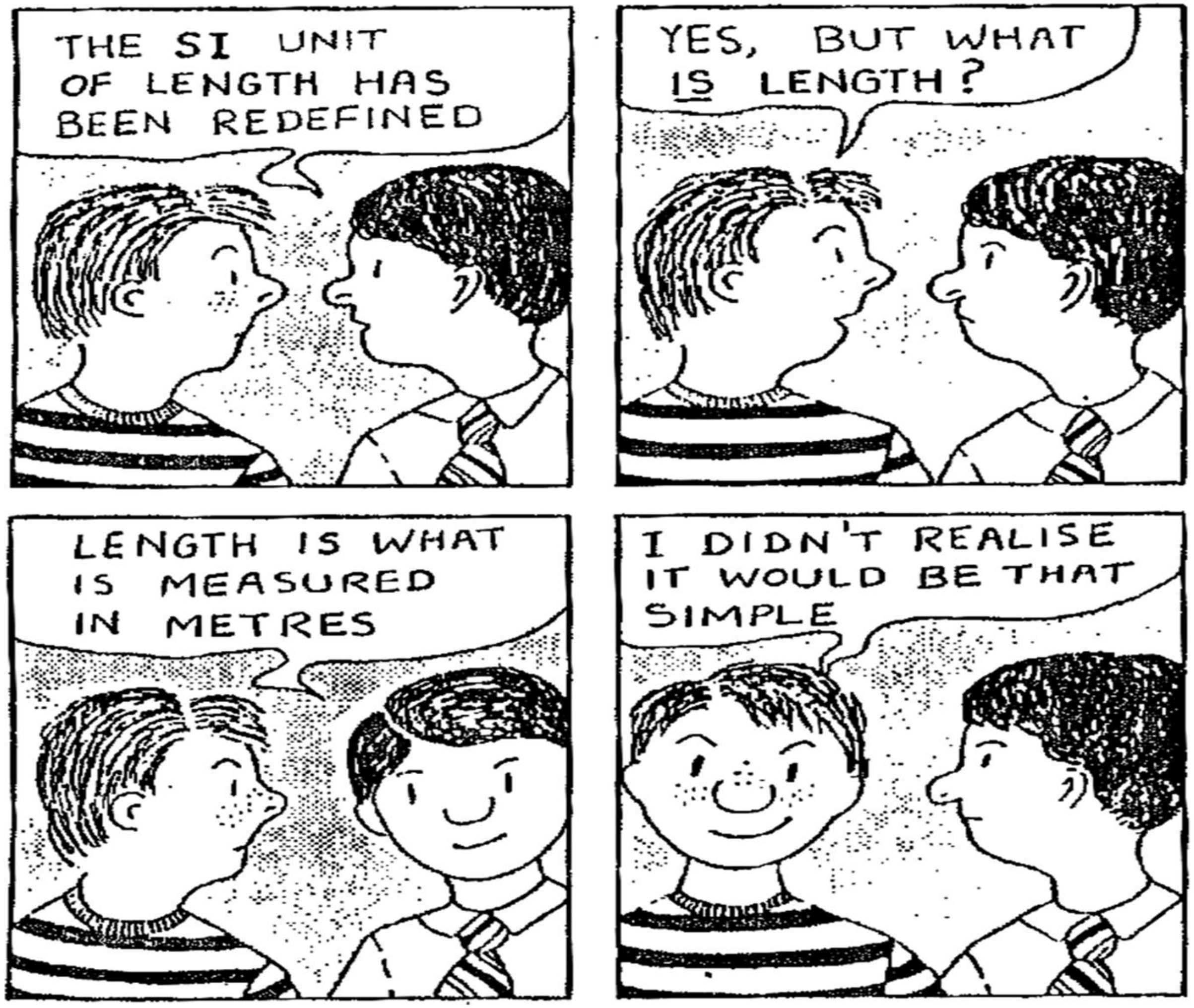
(precision phone component dimensions)



Checks and balances

(Mars climate orbiter lb-s/N-s; Software Interface Specification)

So, what is a metre?

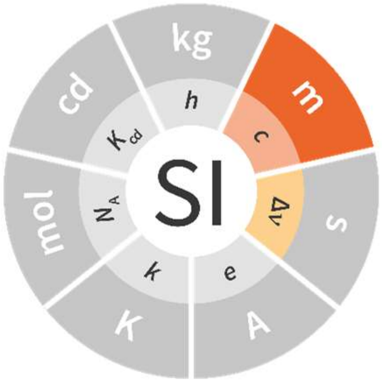


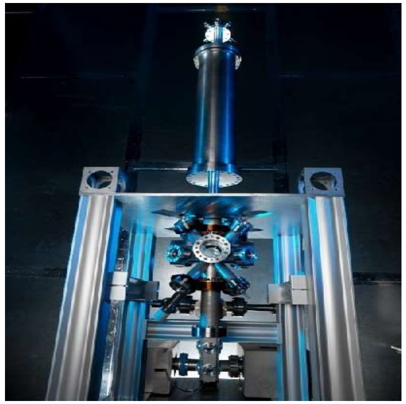
The End!

The definition of the metre

As enacted on 20 May 2019

The metre, symbol m, is the SI unit of length. It is defined by taking the numerical value of the speed of light in vacuum *c* to be 299 792 458 when expressed in the unit m s^{-1}, where the second is defined in terms of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$.

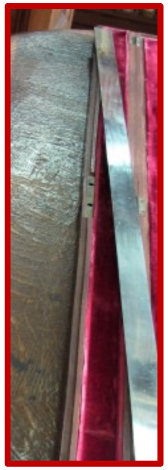


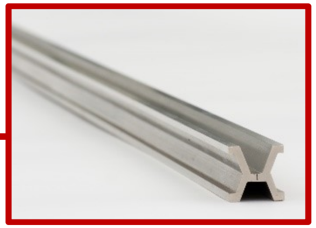


The metre throughout history

~100 years

1791 Commission chooses polar quadrant of Earth, as basis for length standard
1792 Delambre and Méchain commence measurements along the Paris meridian
1793 National Convention (France) adopts 1 m = 443.44 *lignes* of *Toise de Perou*
1799 *Metre des Archives*, (platinum), 25.3 mm × 4 mm set to 1/10 000 000 of Earth quadrant
1872 Construction started of 30 new Prototype Metres 90 % Pt, 10 % Ir, design by Tresca
1875 Convention du Metre signed
1889 Work on thirty prototype metres completed, deposited at the BIPM





~100 years

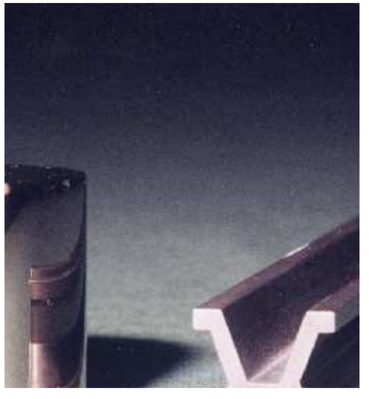
1889 Definition of metre in terms of Prototype Metre
1893 Michelson and Benoit compare cadmium red line to Prototype Metre
1927 Re-definition of the metre
1956 Engraved lines on metre bars re-ruled for use at 0 ºC and 20 ºC
1960 Re-definition of metre in terms of ^{86}Kr wavelength (4 $\times 10^{-9}$)
1975 Speed of light fixed at c = 299 792 458 m s^{-1}
1983 Definition of metre as distance travelled by light in 1/299 792 458 s
2019 Revision of the SI base unit definitions to explicit constant format

Photos: Quinn, Lewis, NIST

1889 definition of the metre

From the 1st Conférence Générales des Poids et Mesures

This prototype, at the temperature of melting ice, will henceforth represent the unit of length



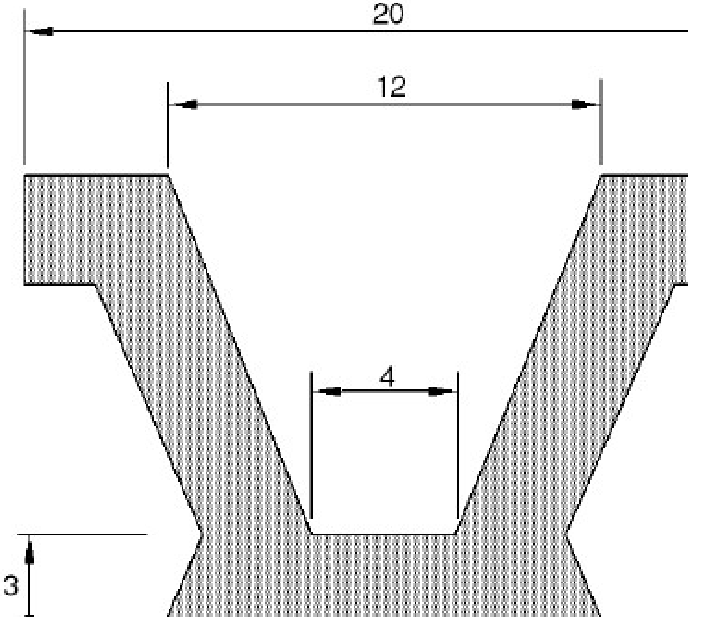
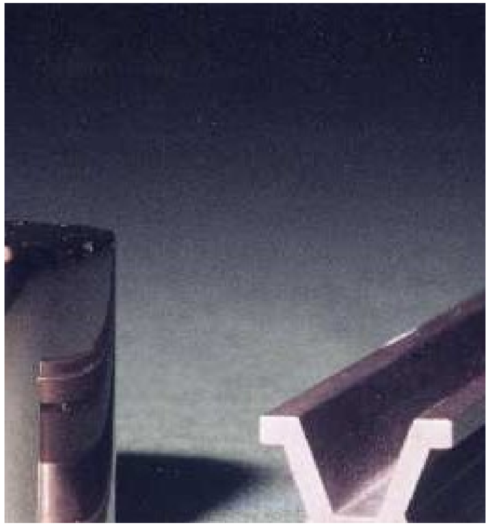


Photo: BIPM

1927 definition of the metre

Refinements from the 7th Conférence Générales des Poids et Mesures

The unit of length is the metre, defined as the distance at 0 ° between the two lines engraved in the platinum iridium bar, deposited in the BIPM, and declared as the Prototype Metre by the 1st CGPM, this standard supported at normal atmospheric pressure on two rollers, of less than 1 cm diameter, situated symmetrically in a horizontal plane, at a distance of 571 mm from each other

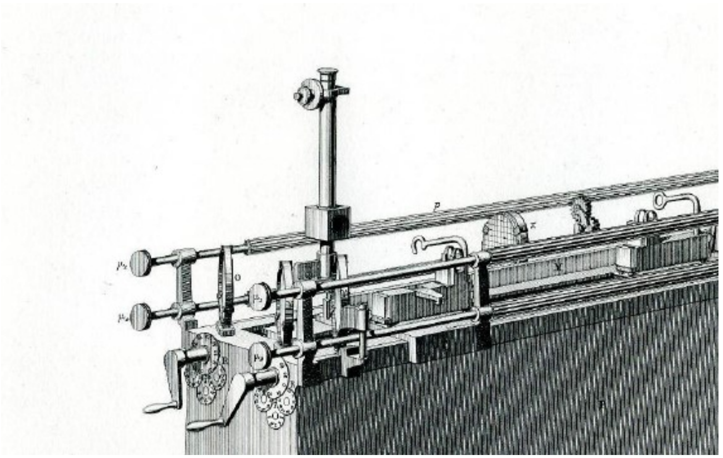


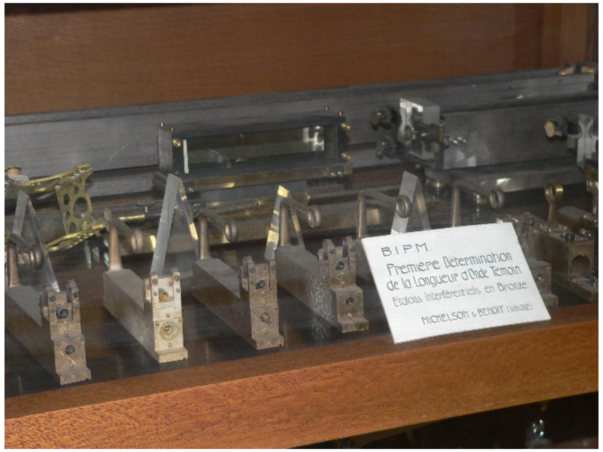
Already, dimensional metrology issues must be considered

Photo: BIPM

Michelson and wavelengths

- In 1892 Michelson measured the International Prototype of the metre in terms of the wavelength of the red line of cadmium: 1 m = 1 553 163.8 wavelengths
- In 1906 Benoît and Fabry re-measured the bar and obtained 1 m = 1 553 164.13 wavelengths
- Equivalent to λ_{cd} = 643.846 96 nm
- 7th General Conference adopted λ_{cd} = 6438.4696 Angströms to define the Angström
- Several other spectral lamps are then investigated…



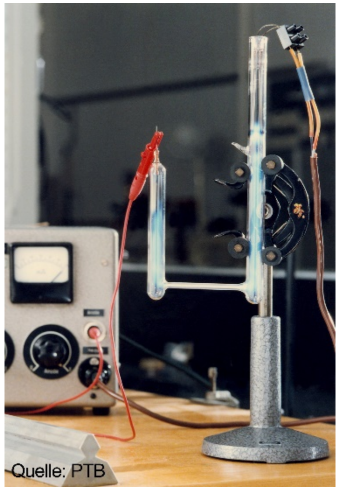


Images: Quinn, Lewis (BIPM)

1960* definition of the metre

Abandonment of the metal bar, transition to wavelengths

1. The metre is the length equal to 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the energy levels $2p_{10}$ and $5d_5$ of an atom of krypton 86.
2. The definition of the metre in use since 1889, based on the International Prototype of platinum iridium, is abrogated.
3. The International Prototype Metre sanctioned by the first CGPM of 1889 will be conserved at the BIPM in the same conditions that it was placed in 1889.



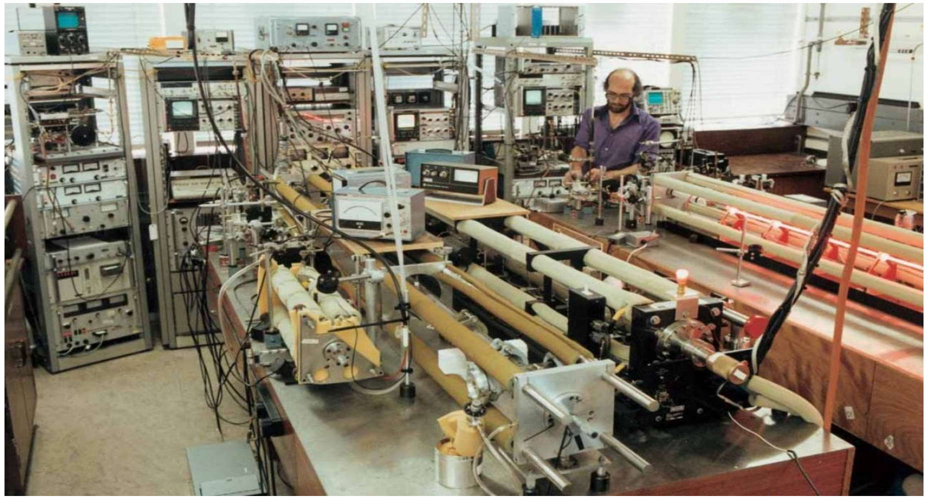
Conforming to paragraph 1 of resolution 2 of the 11th CGPM (October 1960), the CIPM recommends that the radiation of krypton 86 adopted as the fundamental standard of length should be realised by means of a hot cathode discharge tube containing krypton 86 of a purity not less than 99 per cent in a sufficient quantity to ensure the presence of solid krypton at a temperature of 64 K; the lamp having a capillary with the following characteristics: internal diameter 2 to 4 mm, wall thickness about 1 mm.

*1960 was also the year the laser was invented

Photos: PTB, BIPM

Measuring the speed of light in the 1970s

Linking the caesium second to the krypton metre



Uncertainty in speed of light similar to uncertainty in krypton wavelength

Lasers being frequency inter-compared with smaller uncertainty than their absolute values

Danger of a laser wavelength scale separate from the SI

Simultaneous measurement of frequency and wavelength of light in 1972

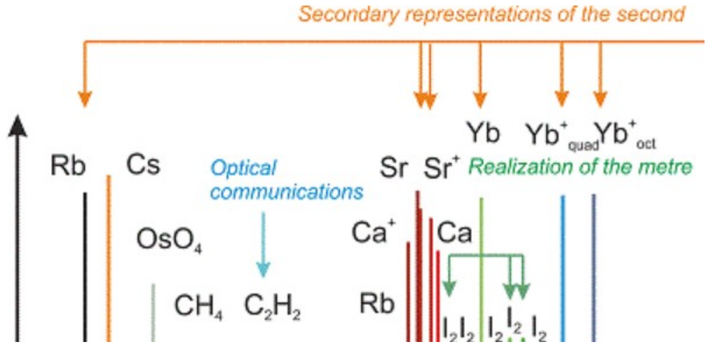
New measured value for *c* with $U(c)$ = 1.2 m s^{-1}

CGPM states new value for c = 299 792 458 m s^{-1} in 1975

Linking the caesium and krypton standards

caesium	33 mm	9 192 MHz
krypton	605.78 nm	494 886 516 MHz

Lots of stabilised laser designs evolve....



1983 definition of the metre

How to avoid the problem of choosing a single wavelength

The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

The definition of the metre in use since 1960, based on the transition between the two lines $2p_{10}$ and $5d_5$ of the krypton atom, is abrogated.

1983 realisation of the metre definition

Mise en pratique of the definition of the metre

The metre should be realized by one of the following methods:

1. by means of the **length *l* of the path travelled in vacuum by a plane electromagnetic wave in a time *t***; this length is obtained from the measured time *t*, using the relation $l = c_0 \cdot t$ and the value of the speed of light in vacuum c_0= 299 792 458 m s^{-1},
2. by means of the **wavelength in vacuum of a plane electro-magnetic wave of frequency *f***; this wavelength is obtained from the measured frequency *f* using the relation $\lambda = c_0/f$ and the value of the speed of light in vacuum c_0 = 299 792 458 m s^{-1},
3. by means of **one of the radiations from the list given here**, whose stated wavelength in vacuum or whose stated frequency can be used with the uncertainty shown, provided that the given specifications and accepted good practice are followed.

Applicability of the three methods in the MeP

- Method 1 is essentially time of flight

  Most suited for long ranges (ideally in vacuum)

  Extreme example, distance to Voyager 1, currently 2.17 × 10^{10} km (20.11 light hours)

- Method 2 is determination of a wavelength given a frequency

$$c = f \times \lambda$$

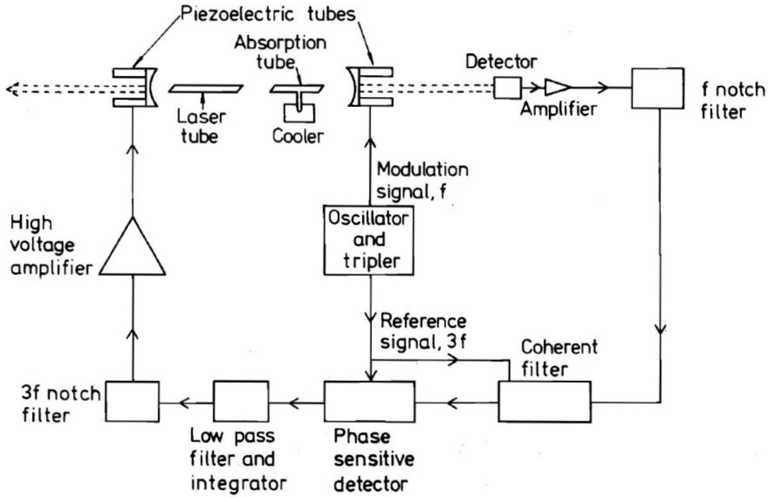
- Method 3 is using a laser or radiation source, built and operated according to 'MeP' rules

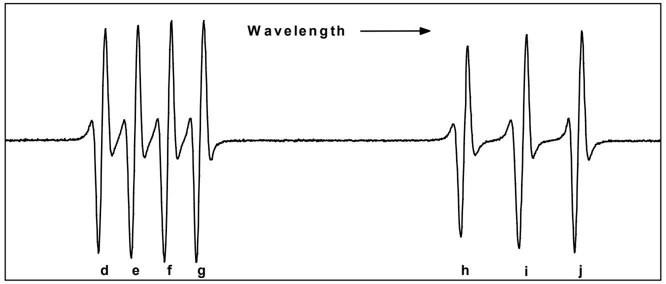
  Classical example is 633 nm iodine stabilised He-Ne laser

  Older standards (krypton lamp, cadmium lamp (Michelson), are also in the list)

Iodine-stabilised He-Ne laser at 633 nm

The most common metre realisation at NMIs





MeP specifies the values of the frequencies (and uncertainties) of the hyperfine components

e.g. when locked to component "f", the laser frequency is 473 612 353 604 kHz ($u(f)$ = 10 kHz)

MeP 633 nm laser specifics

Operation according to the 'recipe' in the MeP to get 2.1 × 10^{-11}

Third harmonic locking

Removes background variation, better SNR

Saturated absorption

Two beams to remove Doppler shifted signals

Cell wall temperature (25 ± 5) ºC

To control frequency shift due to collisions

Cold-finger temperature (15.0 ± 0.2) ºC

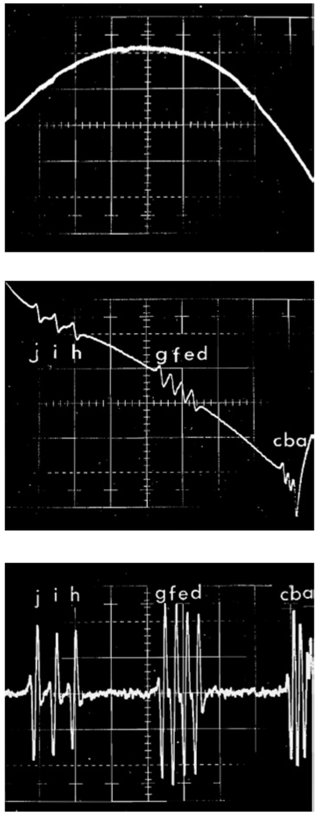
To set the iodine pressure

Frequency modulation width, peak to peak (6.0 ± 0.3) MHz

One-way intracavity power (10 ± 5) mW

Power shift coefficient below 1.0 kHz/mW

And of course, isotopically pure iodine

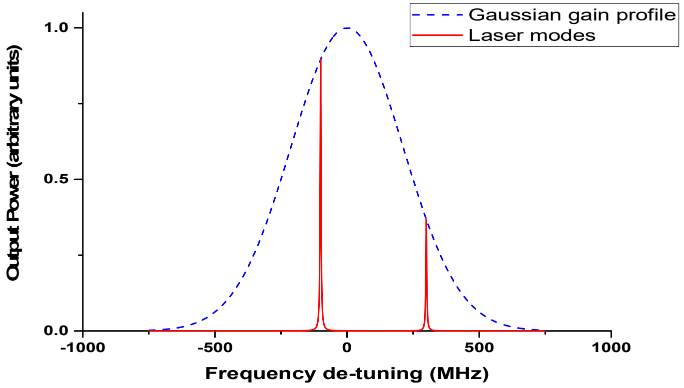


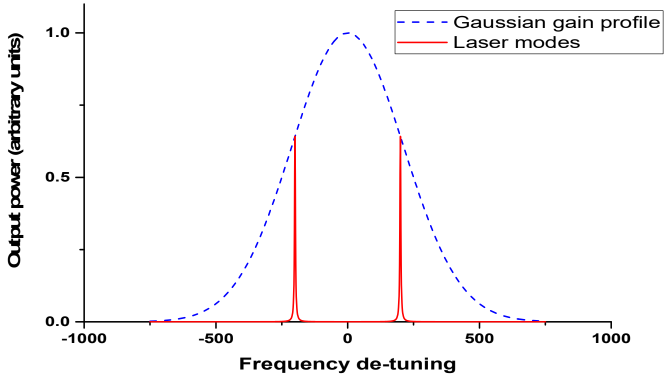
Turn-key commercially available frequency stabilised lasers

- Simpler stabilisation methods developed for 633 nm (early 1970s) and other He-Ne lines (*e.g.* 543 nm and 612 nm) in the late 1980s. Commercially developed by several companies.

Two & three-mode balanced frequency stabilisation

- Adjacent laser modes are orthogonally linearly polarised
- Frequency stabilisation by balancing mode intensities
- Typical frequency stabilities and day-to-day reproducibilities of ~10^{-8} (5 MHz) can be achieved
- Developed for displacement interferometer systems

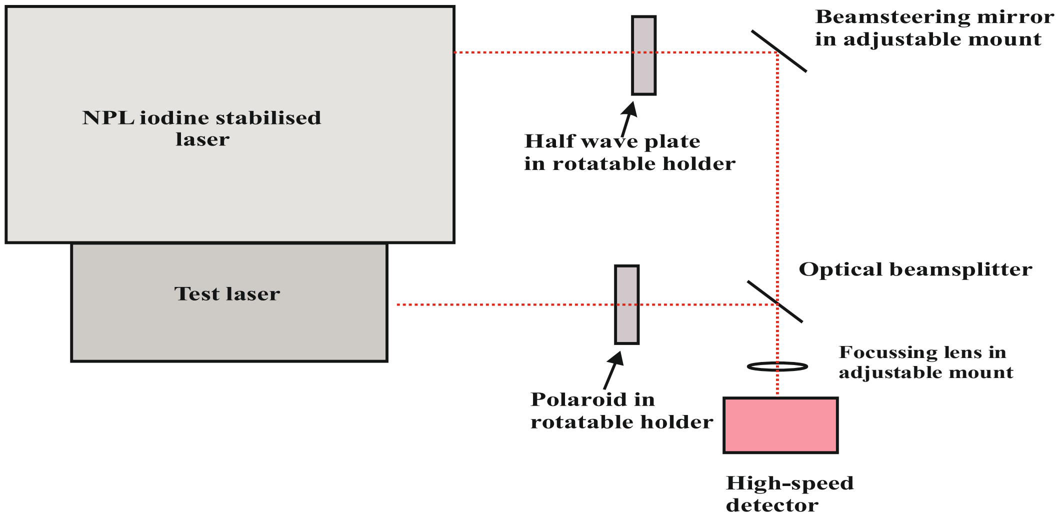


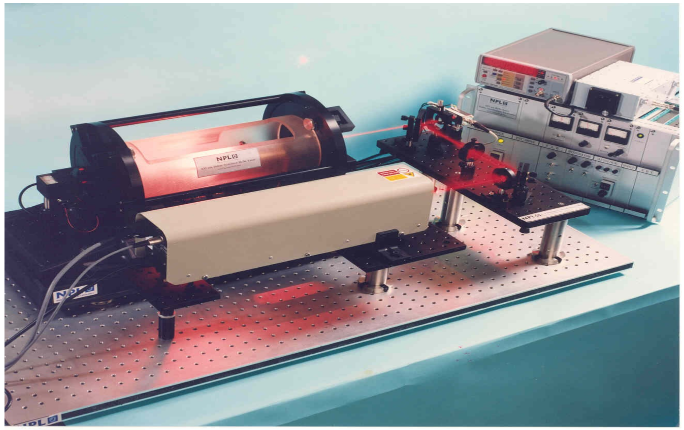


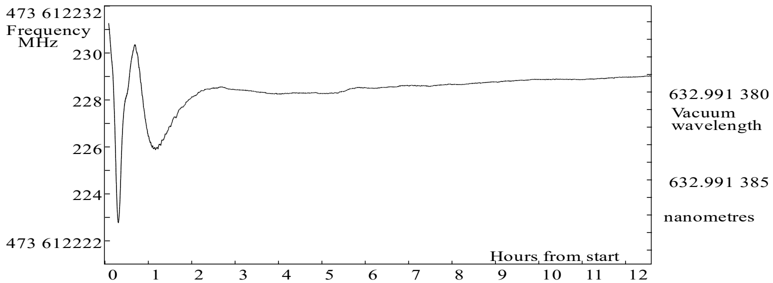
Laser modes before (left) and after (right) frequency lock

Stabilised laser (laser interferometer) calibrations

Beat frequency operation at 633 nm and 543 nm







A high-speed detector records the frequency difference between the two lasers (typically in the region 10 MHz to few GHz). Results are recorded *via* a counter interfaced to a computer

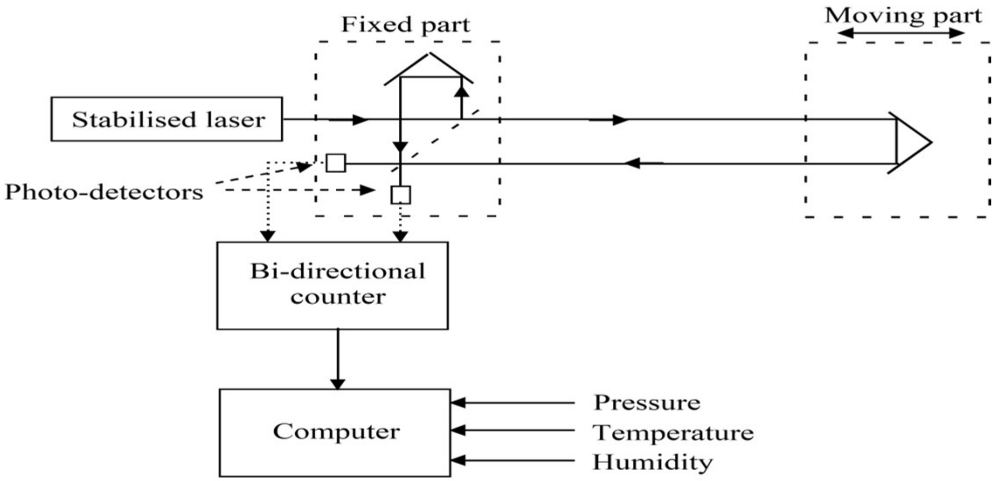
Interferometry basics

- To be covered in detail in later talk
- This is a brief introduction to understand the importance of a frequency stabilised laser in dimensional metrology
- The laser frequency is stabilised, but many measurements are performed in air and $\lambda_{air} = \lambda_{vac}/n$ (n is the refractive index)

For the most precise measurements over long distances, the laser should be single frequency. The displacement *L* is then:

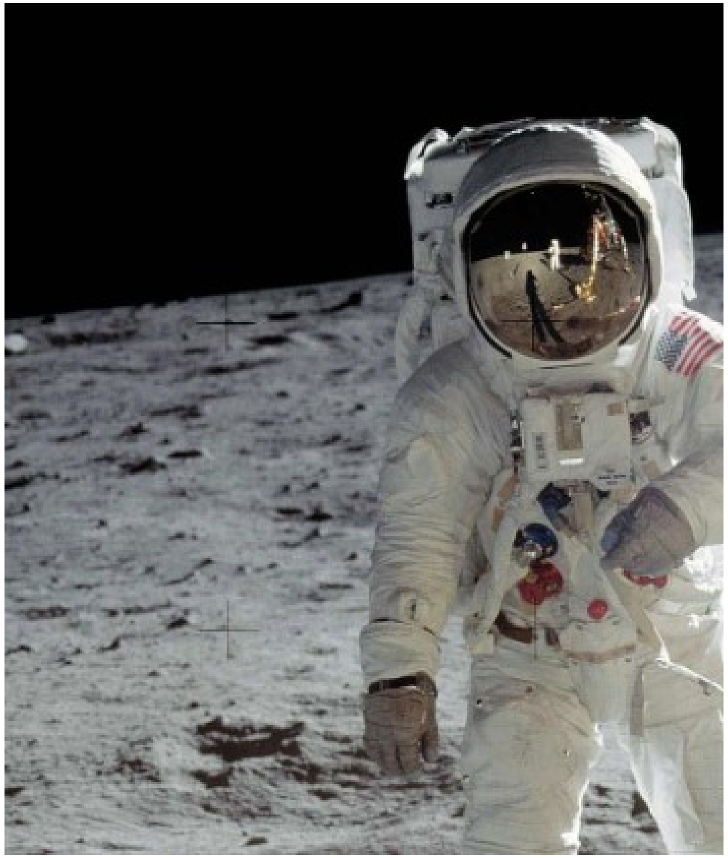
$$L = \tfrac{1}{2} N \lambda_{air}$$

(N = fringe count)



Refractive index

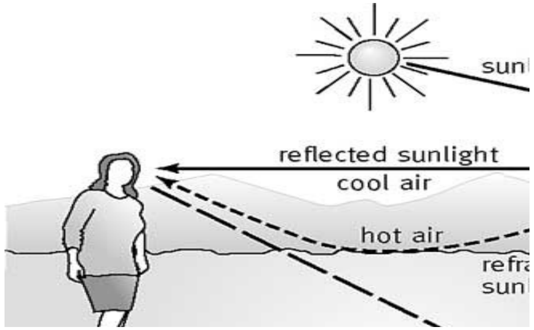
The 'elephant in the room'

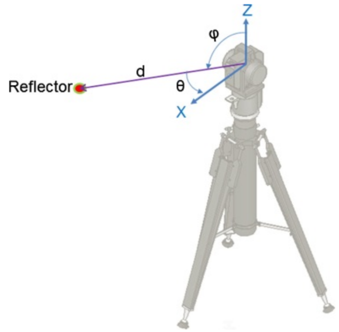


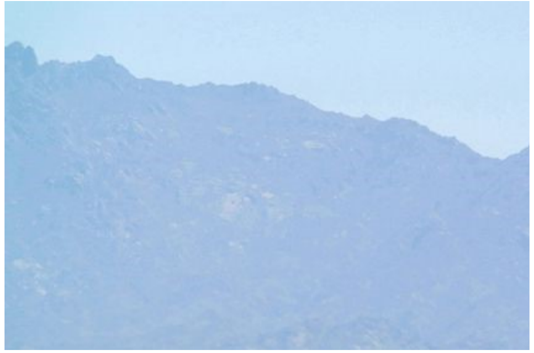
Refractive index slows light:
speed < c_0

Wavelength is reduced too

Refractive index gradient due to pressure gradient causes refraction (beam bending) – more important for Large Volume Metrology using laser trackers







Refractive index

Empirical equations

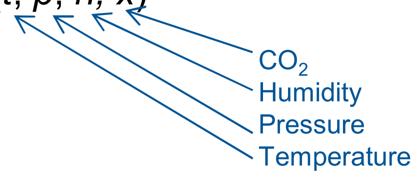
- Edlen – original (1953 & 1966)
- Birch & Downs – new T_{90}, better density term (1993-4)
- Ciddor – extended to infra-red (1996)
- Bönsch & Potulski – updated Edlen/Birch & Downs (1998)

Refractive index n

Refractivity $(n-1)_{tph} = K_{\lambda} D_{tph}$

a dispersion term $K(\lambda)$

a density term $D(t, p, h, x)$



Dispersion, σ is the wavenumber in μm^{-1} ($\sigma = 1/\lambda$)

$$(n-1)_N \cdot 10^8 = 8091.37 + \frac{2333983}{130-\sigma^2} + \frac{15518}{38.9-\sigma^2}$$

CO_2 molar fraction x

$$(n-1)_x = (n-1)_N \cdot [1 + 0.5327(x - 0.0004)]$$

Temperature, t/°C & pressure, p/Pa

$$(n-1)_{tp} = \frac{p \cdot (n-1)_x}{93214.60} \times \frac{1 + 10^{-8} \cdot (0.5953 - 0.009876 \cdot t) \cdot p}{1 + 0.0036610 \cdot t}$$

Humidity, partial pressure f/Pa

$$n_{tpf} - n_{tp} = -f \cdot (3.802 - 0.0384 \cdot \sigma^2) \cdot 10^{-10}$$

Refractive index

Sensitivity & uncertainty

- Standard uncertainty from the equations: 1×10^{-8}

- Sensitivities

Parameter	Symbol	Sensitivity	Normal lab: standard and daily variation	Refractive index *change*
Pressure	$\frac{\partial n}{\partial p}$	$+2.7 \times 10^{-7}$ hPa^{-1}	1013.25 hPa ± 25 hPa	6.75×10^{-6}
Temperature	$\frac{\partial n}{\partial t}$	-9.2×10^{-7} °C^{-1}	20 °C ± 2 °C	1.84×10^{-6}
Humidity	$\frac{\partial n}{\partial h}$	$+1{\cdot}0 \times 10^{-8}$ (RH %)$^{-1}$	50 % RH ± 15 % RH	1.5×10^{-7}
CO_2	$\frac{\partial n}{\partial x}$	$+1.4 \times 10^{-10}$ ppm^{-1}	400 ppm ± 100 ppm	1.4×10^{-8}

Refractive index

Sensitivity & uncertainty

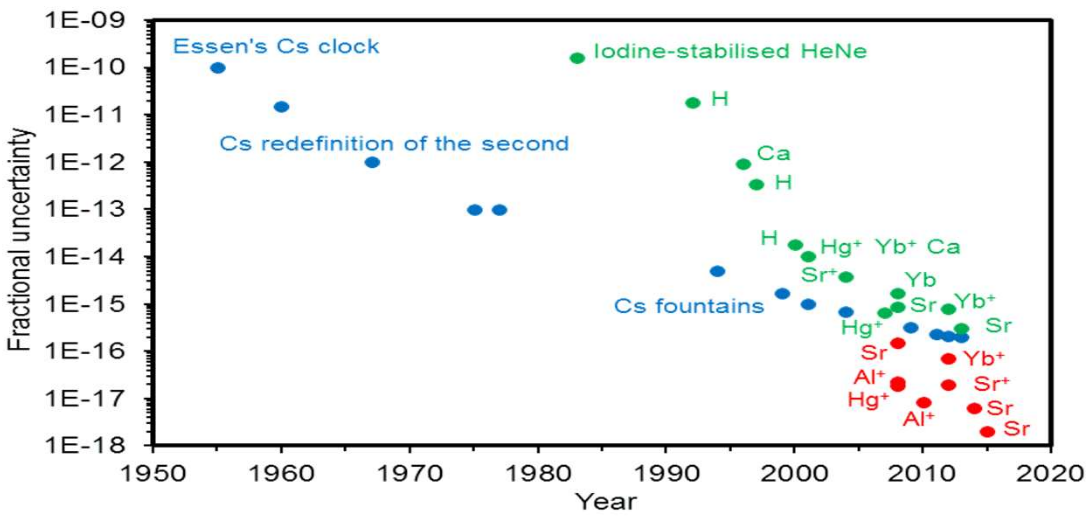
Uncertainties from high-accuracy sensors used in dimensional applications

Parameter	Sensor	Typical cost (including calibration)	Best standard uncertainty in application	Influence on refractive index
Equations	-	-	1×10^{-8}	1.0×10^{-8}
Pressure	Vibrating Si barometer	£3,000	0.05 hPa	1.4×10^{-8}
Temperature	Platinum resistance thermometer, AC bridge	£7,000	0.05 °C	-4.6×10^{-8}
Humidity	Chilled mirror dewpoint meter	£3,000	0.7 %RH	0.7×10^{-8}
CO_2	IR absorption in cell	£1,000	50 ppm	0.7×10^{-8}
TOTAL		**£14,000**	**[summed in quadrature]**	$\mathbf{5 \cdot 10^{-8}}$

The future for wavelength-based metre realisations

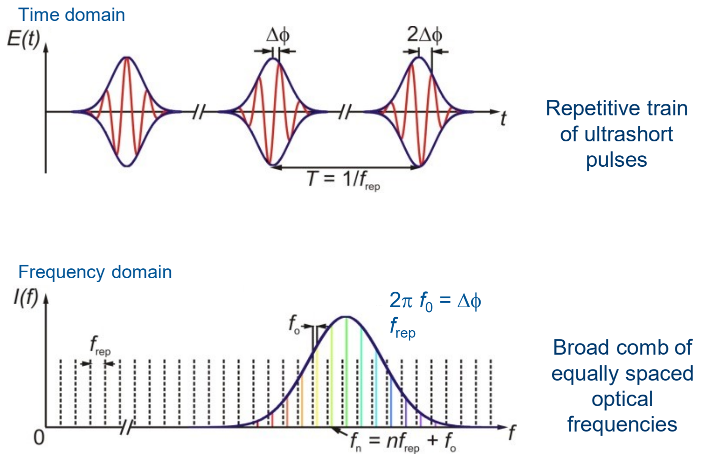
How accurately can we measure frequency?

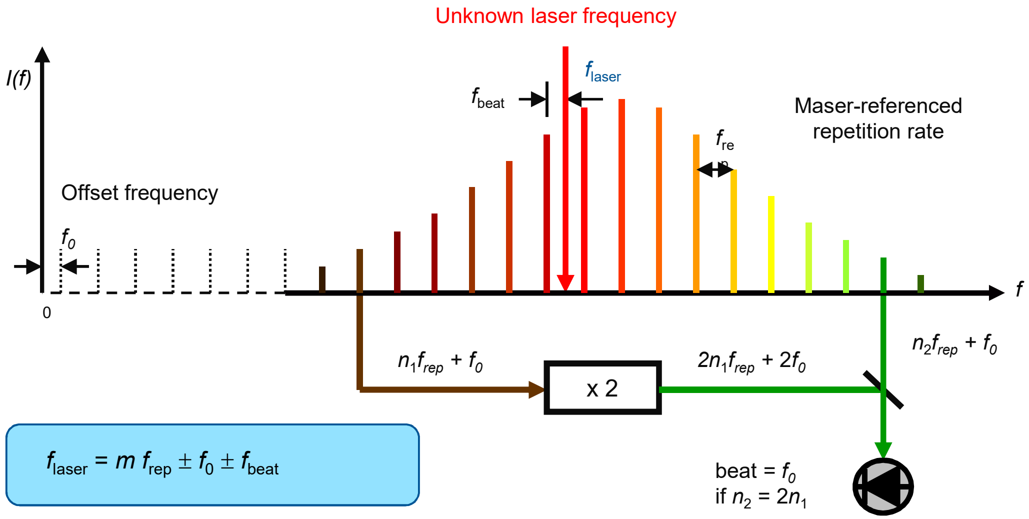
- Best Cs clocks now claim uncertainties of ~2×10^{-16}. This limits the uncertainty of absolute frequency measurements.
- Optical frequencies can be compared using different femtosecond combs to parts in 10^{21}!



Traceability to new frequency standards

Use frequency comb to link any wavelength to a high accuracy frequency standard



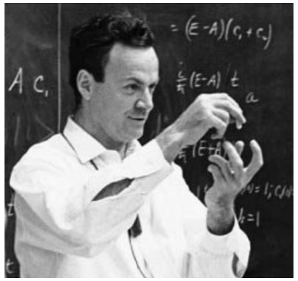


- Replacement for the room-sized 1970s experiment !
- ‘Scientific’ uses can do useful things with the accuracy (seismometry)
- But do we need such frequency accuracy in dimensional metrology?
- Many other issues to solve in practical aspects for dimensional metrology– see lecture 2

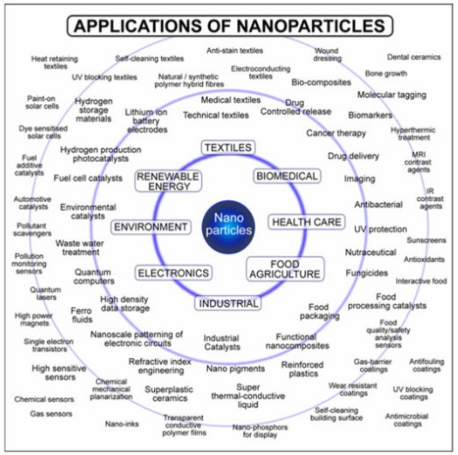
Secondary realisation of the metre

- For the first time, the 2019 Mise en Pratique for the definition of the metre included a secondary realisation of the metre
- This is only intended for dimensional nanometrology but recognises the importance of extending the SI length scale to the very small (very large is already covered by time of flight – Method 1 of the old MeP)
- So what is special about nanometrology?

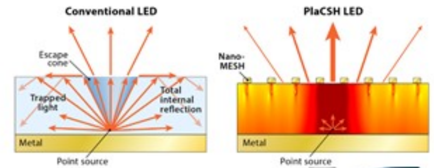
Nanotechnology



- Started with a talk "There's Plenty of Room at the Bottom" by Richard Feynman in 1959
- Feynman predicted that scientists would be able to manipulate and control individual atoms and molecules.
- Offered a $1000 prize for construction of a rotating electric motor in 1/64 inch cube
- William McLellan won the prize in 1960 (2000 rpm, 250 μg motor, 13 separate parts)
- …but used a top down approach right at the limits
- 1974 Prof. Norio Taniguchi coined the term 'nanotechnology'
- Nanotechnology is big business



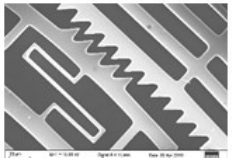
Optical coatings

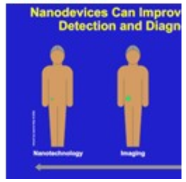


Lighting

Coatings for cutting tools

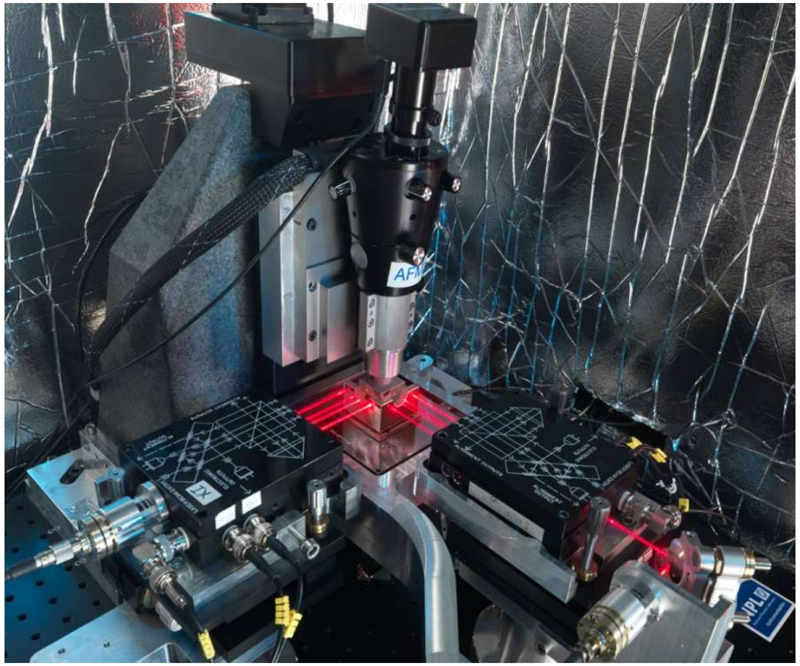
lubricants

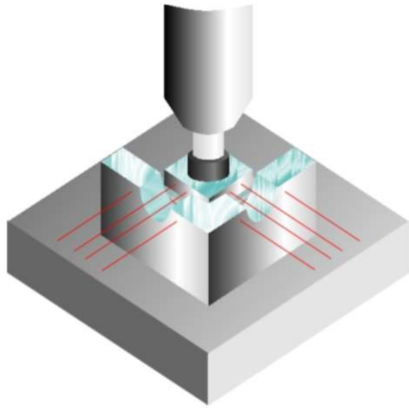




Traceable nanometrology

The metrological atomic force microscope





Mirror fixed to PZT tube

Mirror on stage that holds sample

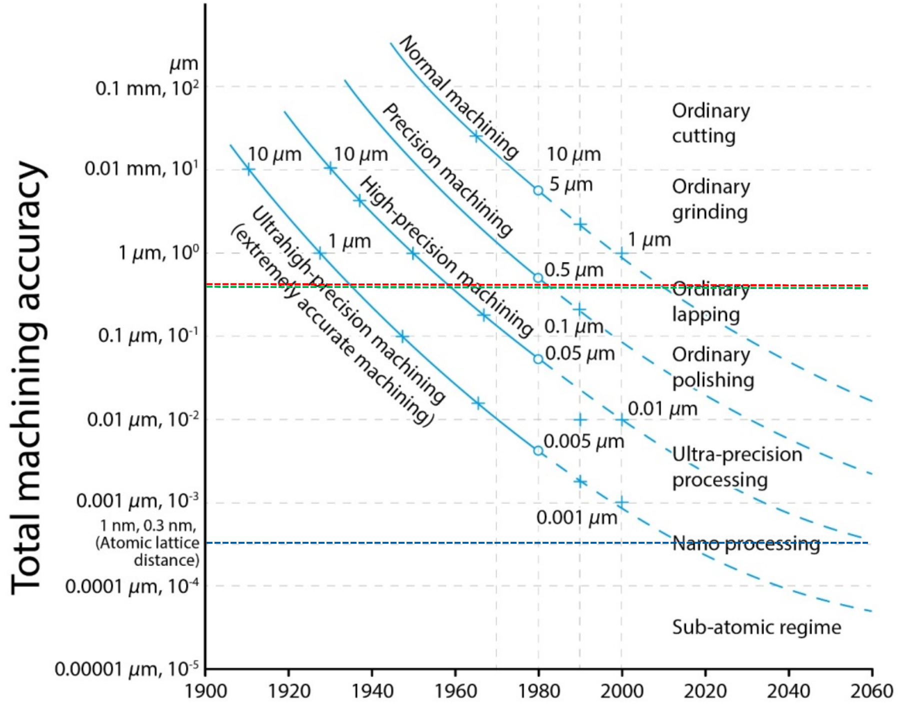
Both are 3 sided orthogonal mirrors

Optical interferometers measure the relative displacement of the AFM tip and the sample in x, y and z axes.

Turn off and not use the scanning PZT as it is highly non-linear

Nanotechnology and nanometrology

Staying ahead of the game

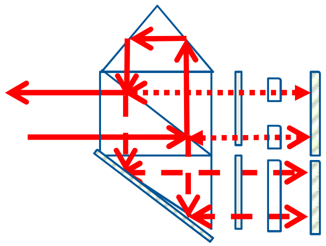


Taniguchi's 1983 prediction extrapolated

- 25 - 30 year time lag between precision levels, each ×10 improvement
- Similar to Moore's Law in computing: (× 2 transistor count every 2 years)
- But metrology must be in advance of the machining
  - Stage and sensor manufacturers claiming sub-nm positioning capability but need independent verification
  - Quantum metrology exploits quantum mechanical effects and will require precise construction of nanostructures
  - Linewidths for semiconductor industry requiring nm nanometre
  - Drive towards miniaturization; reduced measurement tolerances
  - Ability to measure at nanometre level with low uncertainty
- Wavelengths become too big to sub-divide (need better than 1/1000 wavelength)

Optical interferometry at the nm level

Limiting factors of fringe sub-division



Even with double pass, reflected beams, one fringe is 158 nm
With 10 V signals at 16 bits, should get down to 2 pm
However:

- bit noise on A2D loses resolution
- detector noise, non-linear photodiode behaviour
- electronic noise
- stray reflections, ‘optical rattle’, non-perfect 90° phase quadrature, beam overlap position changes

all cause nonlinearity which is normally cyclic with integer fractional periods of the fringe period (½ , ¼, ⅛, …)

‘Heydemann correction’ can negate much but still leaves up to 200 pm error

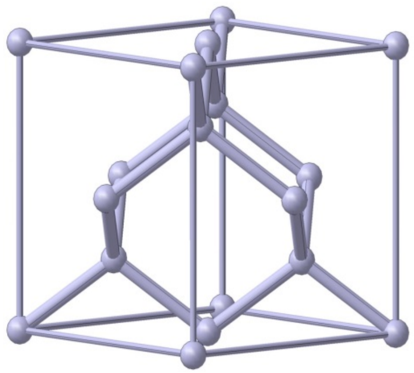
Basically the wavelengths are too big to sub-divide

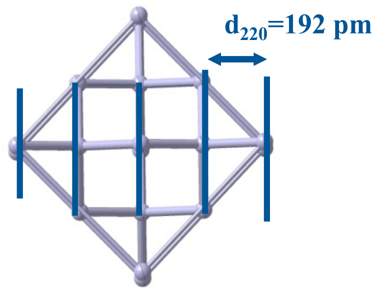
Better to use a natural standard of similar dimensions…

The silicon lattice parameter

Secondary realisation of the metre for dimensional nanometrology

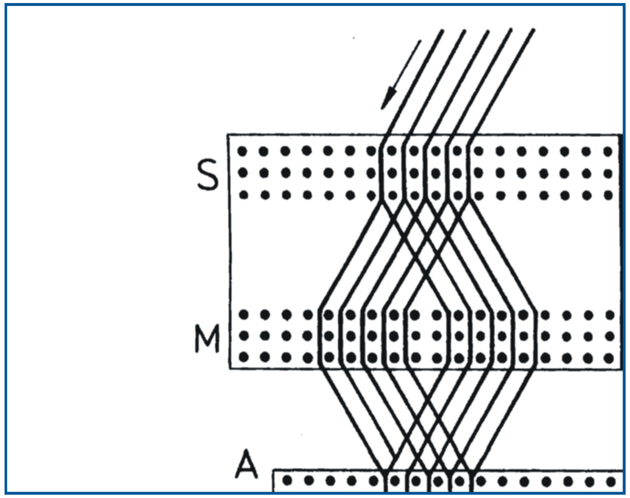
- d_{220} lattice parameter of silicon is 192 pm
- Exact value is traceably known to 1 part in 10^8 (quoted in CODATA)



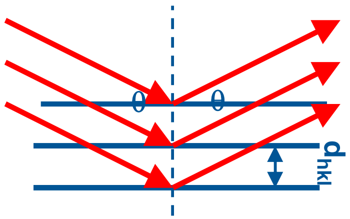


X-ray interferometry

- Developed by Bonse and Hart in 1960s
- Three parallel equally spaced lamellae (< 1 mm thick)
- Oriented so that x-rays diffract from a set of crystallographic planes
- At first lamella (S) two diffracted beams produced
- At second lamella (M) two more pairs of beams produced.
- Inner beams from each pair recombine at third lamella to form an interference pattern.
- Fringe spacing equal to lattice spacing of planes (NOT wavelength)
- Too small to see, but use of third lamella gives a Moiré fringe pattern
- As 3rd lamella translated, x-ray intensity varies to produce a fringe pattern

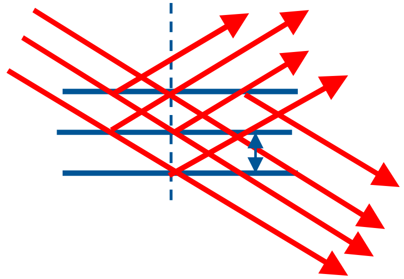


Bragg diffraction



$$2d_{hkl}\sin\theta = n\lambda$$

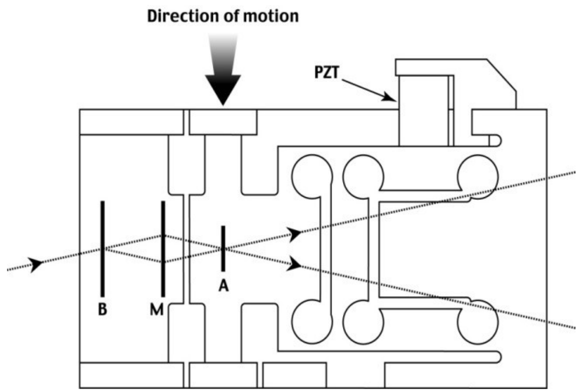
d_{hkl} = spacing between planes
θ = Bragg angle
(note θ is angle x-ray makes with planes, NOT the normal to the planes)
λ = wavelength of x-rays

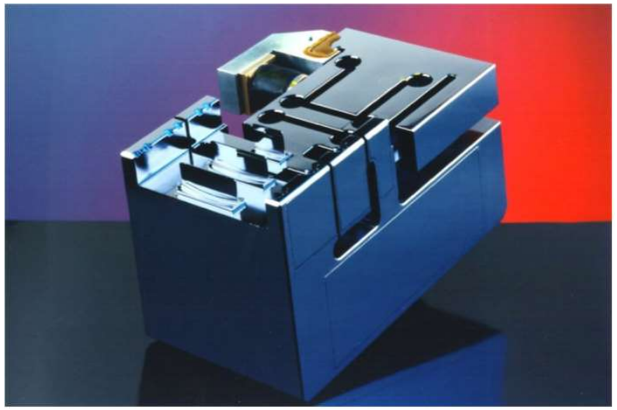


X-ray interferometer

- Atomic scale ruler/translation stage
- Moiré fringe pattern period given by lattice spacing NOT wavelength of x-rays
- d(220)= 0.192 nm
- Range of 10 μm
- Displacement is traceable to metre definition
- Flexure stage driven by PZT actuator
- Three optical mirrors on side of XRI

- Used for:
  Metrological translation stage for STM
  Calibration of linear encoders
  Verifying Fabry-Perot interferometers (~8 pm)

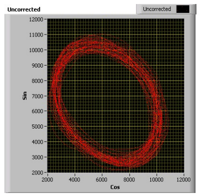


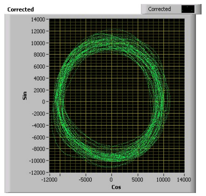


Ultimate limits for XRI?

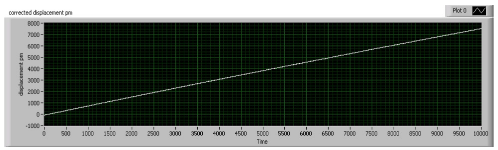
How low can you go?

Quadrature x-ray fringe detection
Sub-fringe ‘phase’ measurement
Heydemann correction of non-linearity

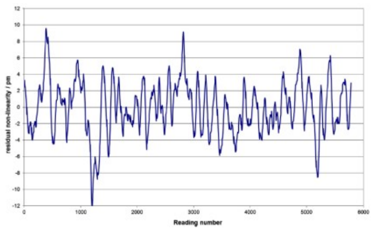




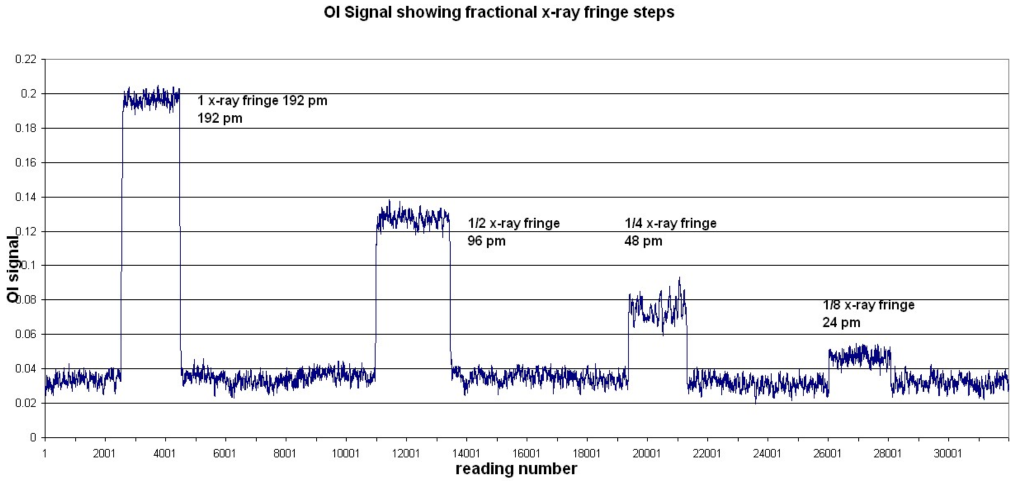
1 revolution
= 192 pm



8000 pm
scan

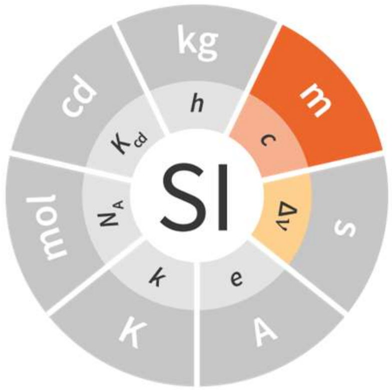


residuals 6
to 12 pm



- Potentially can lock XRI onto sub-fringe linear regions to provide quantized steps of 24 pm

Recap of lecture 1



- **The metre is defined based on the speed of light, *c*, and a time interval**

- For long ranges, this implies simple timing of light transit time

- At moderate length scales, interferometry is used to access *c* via $c = f \times \lambda$
  - Stabilised lasers offer well-defined λ (2.1×10^{-11} in vacuum)
  - Refractive index compensation in air

- At very short scales (nano) interferometry is not accurate
  - Secondary realisation using silicon lattice
  - X-Ray Interferometer links physical length or transducers to Si lattice
  - Extends SI-traceable length scale to few picometres

Tomorrow

- Dimensional metrology in practice

THANK YOU

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