End-to-End Differentiable Proving

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Whiteson Research Lab

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Machine Reading Lab

[3]Bloomsbury AI

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6th of December 2017

Combining Deep and Symbolic Reasoning

Neural Networks

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Neural Networks

- Trained end-to-end
- Strong generalization

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- **Induce interpretable rules** end-to-end from proof success

Machine Learning & Logic

- Fuzzy Logic (Zadeh, 1965)

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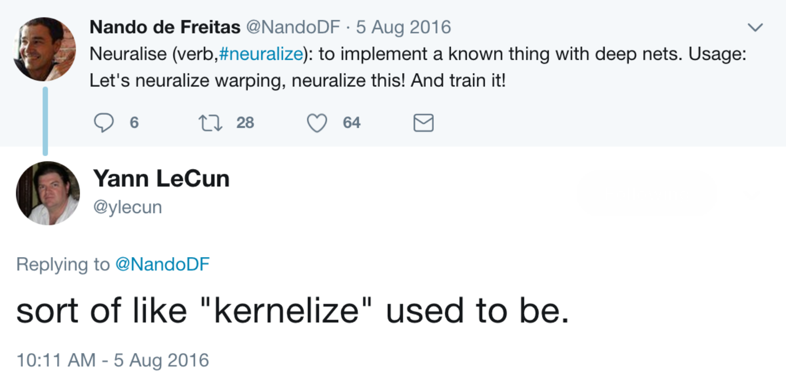
  - Statistical Predicate Invention (Kok and Domingos, 2007)
- Neural-symbolic Connectionism
  - Propositional rules: EBL-ANN (Shavlik and Towell, 1989), KBANN (Towell and Shavlik, 1994), C-LIP (Garcez and Zaverucha, 1999)
  - First-order inference (no training of symbol representations): Unification Neural Networks (Holldöbler, 1990; Komendantskaya 2011), SHRUTI (Shastri, 1992), Neural Prolog (Ding, 1995), CLIP++ (Franca et al. 2014), Lifted Relational Networks (Sourek et al. 2015)

Approach


Nando de Freitas @NandoDF · 5 Aug 2016
Neuralise (verb,#neuralize): to implement a known thing with deep nets. Usage: Let's neuralize warping, neuralize this! And train it!
6
28
64

Approach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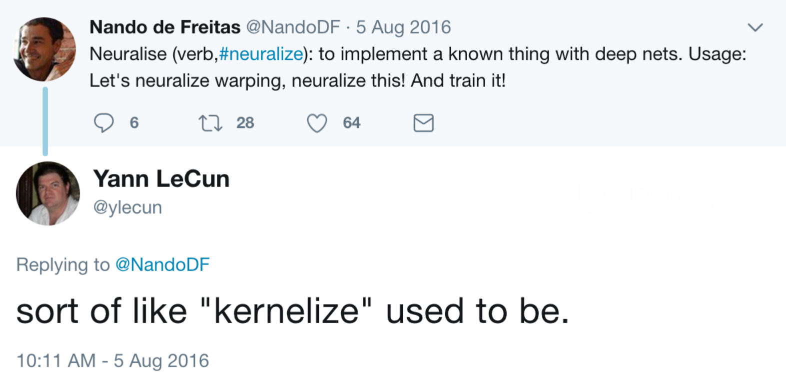
Yann LeCun
@ylecun
Replying to @NandoDF
sort of like "kernelize" used to be.
10:11 AM - 5 Aug 2016

Approach



Let's **neuralize** Prolog's Backward Chaining using a Radial Basis Function **kernel** for unifying vector representations of symbols!

Prolog's Backward Chaining

Example Knowledge Base:

1. `fatherOf`(ABE, HOMER).
2. `parentOf`(HOMER, BART).
3. `grandfatherOf`(X, Y) :–
   `fatherOf`(X, Z),
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Intuition:

- Backward chaining translates a query into subqueries via rules, *e.g.*,
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  $\texttt{grandfatherOf}(\text{ABE}, \text{BART}) \xrightarrow{3.} \texttt{fatherOf}(\text{ABE}, Z), \texttt{parentOf}(Z, \text{BART})$
- It attempts this for all rules in the knowledge base and thus specifies a depth-first search

Unification

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Query

`grandfatherOf` ABE BART

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grandfatherOf	ABE	BART

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FAIL SUCCESS FAIL

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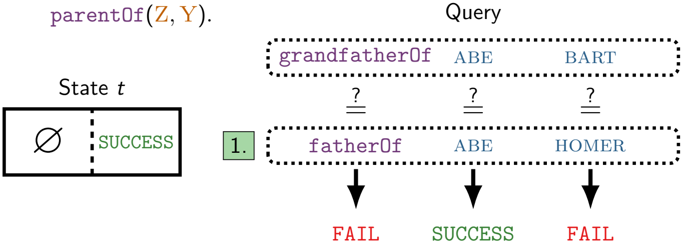

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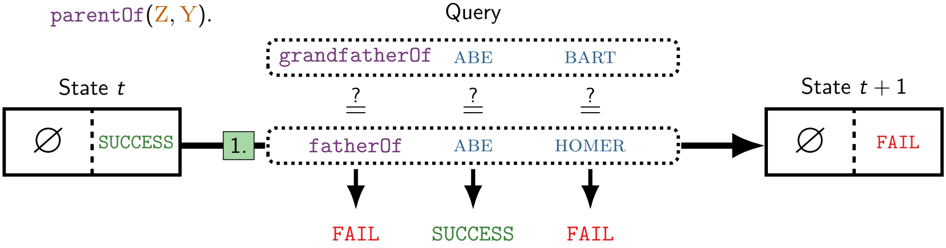

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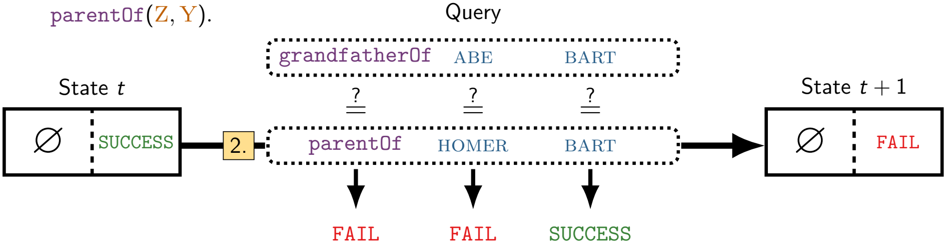

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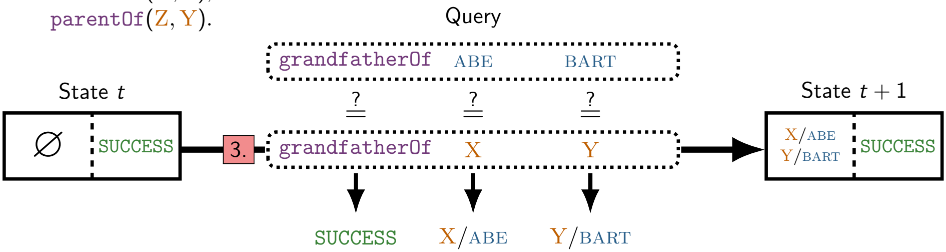

Unification Failure

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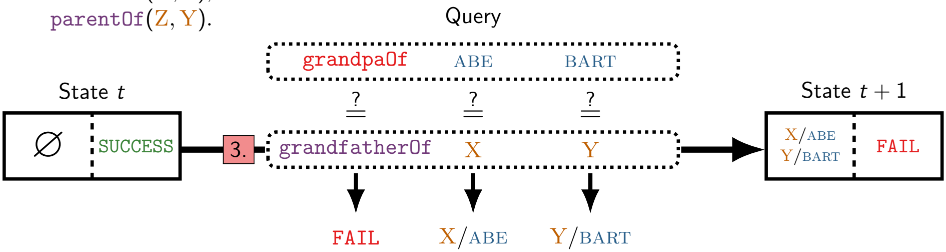

Neural Unification

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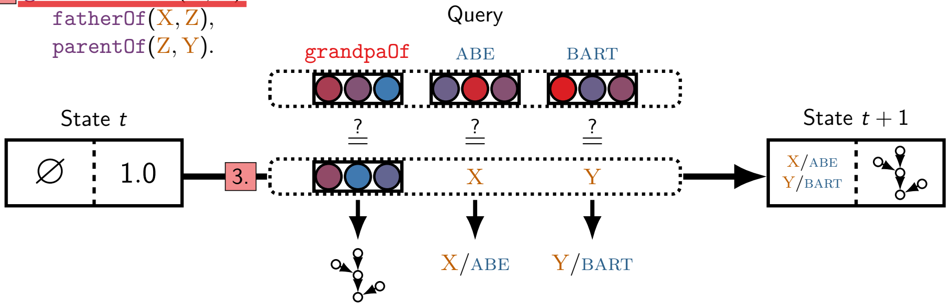

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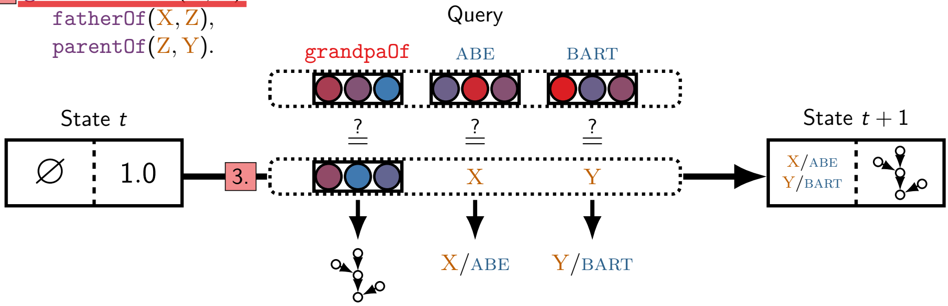

$$\min\left(1.0, \exp\left(\frac{-\|\mathbf{v}_{\texttt{grandpaOf}} - \mathbf{v}_{\texttt{grandfatherOf}}\|_2}{2\mu^2}\right)\right)$$

Differentiable Prover

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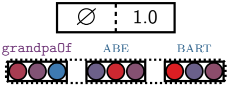

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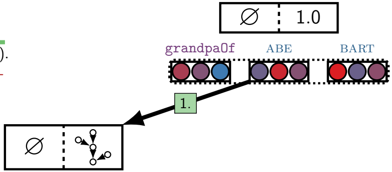

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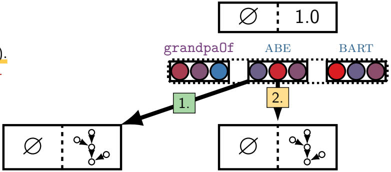

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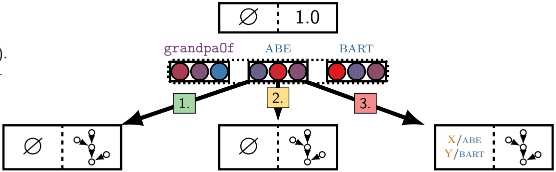

3.1 fatherOf(X, Z)
3.2 parentOf(Z, Y)

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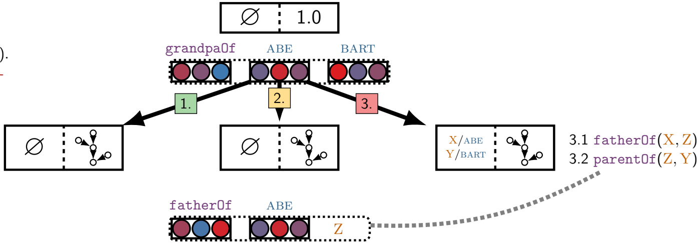

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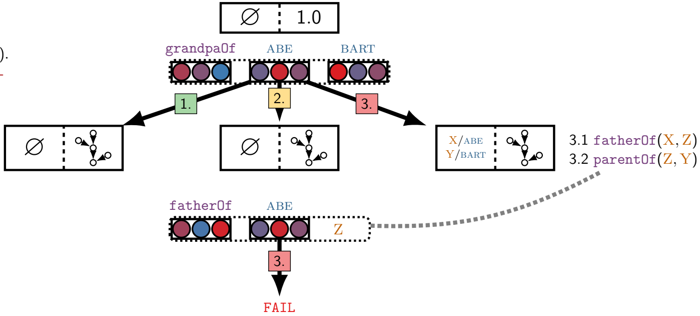

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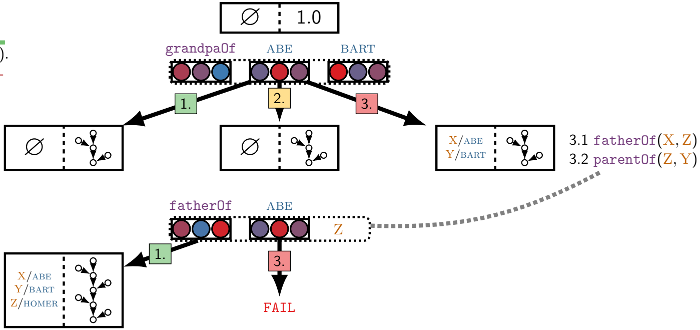

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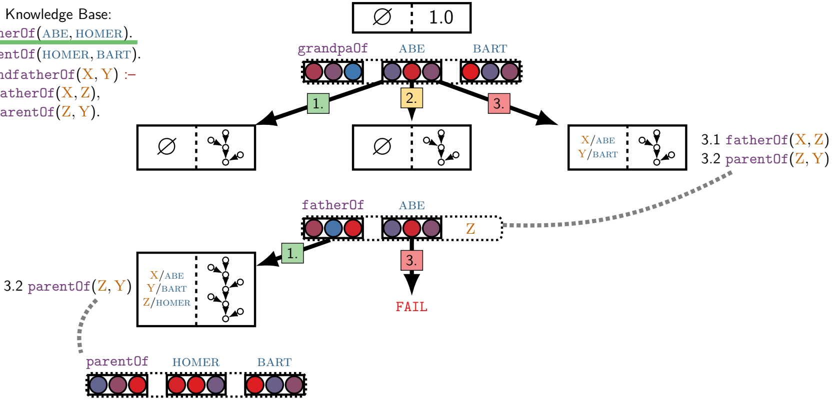

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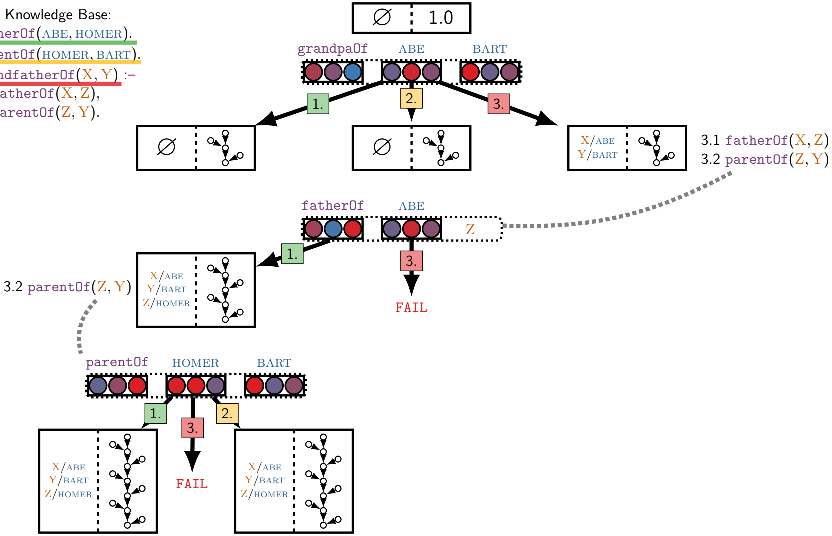

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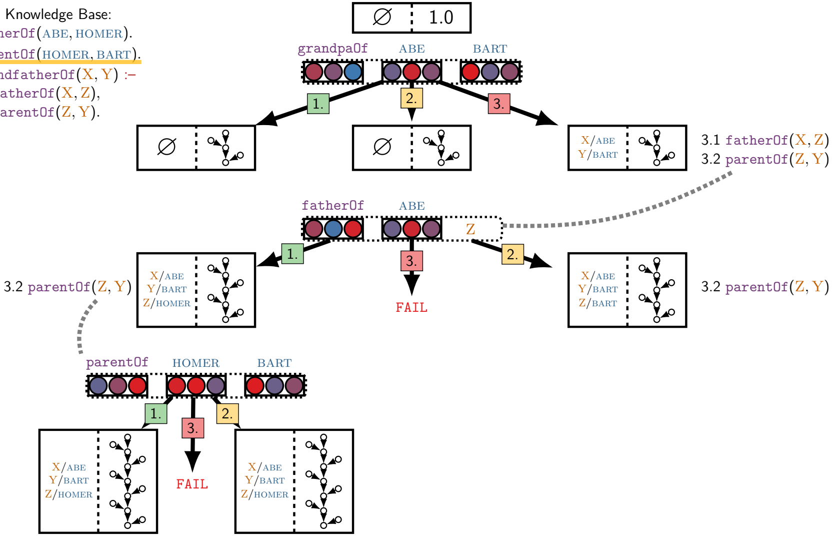

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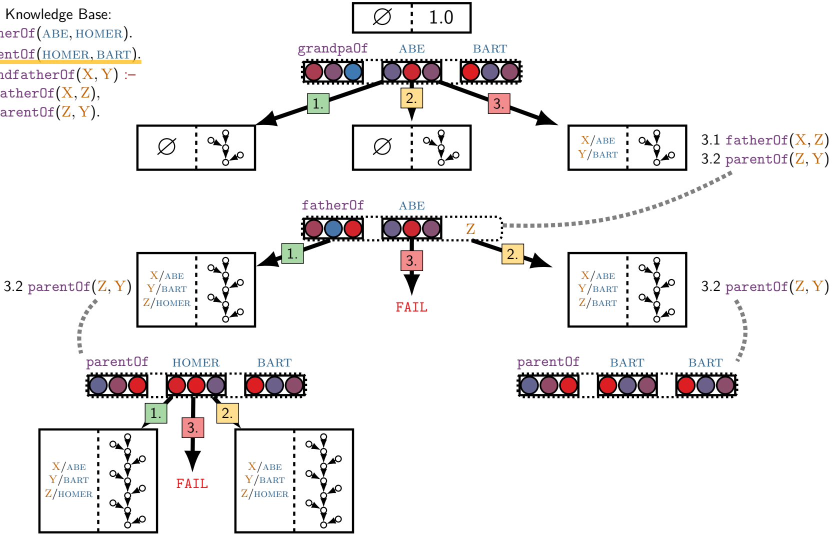

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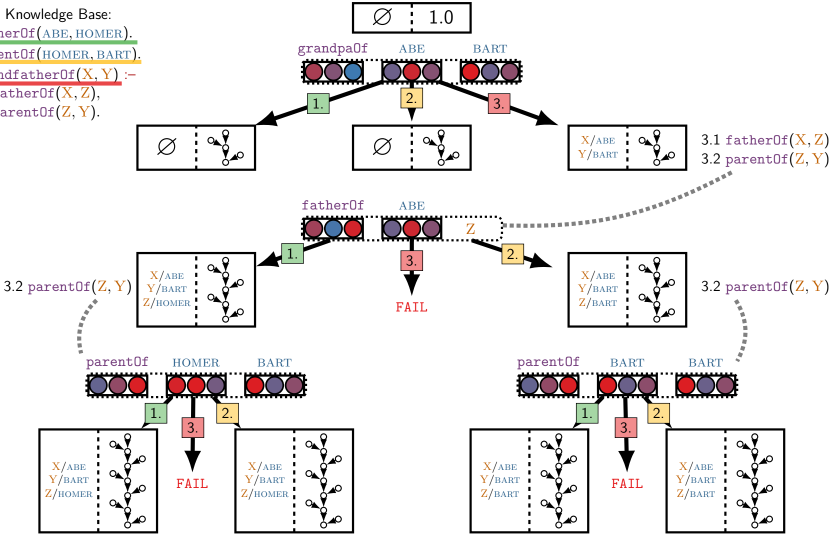

Neural Program Induction

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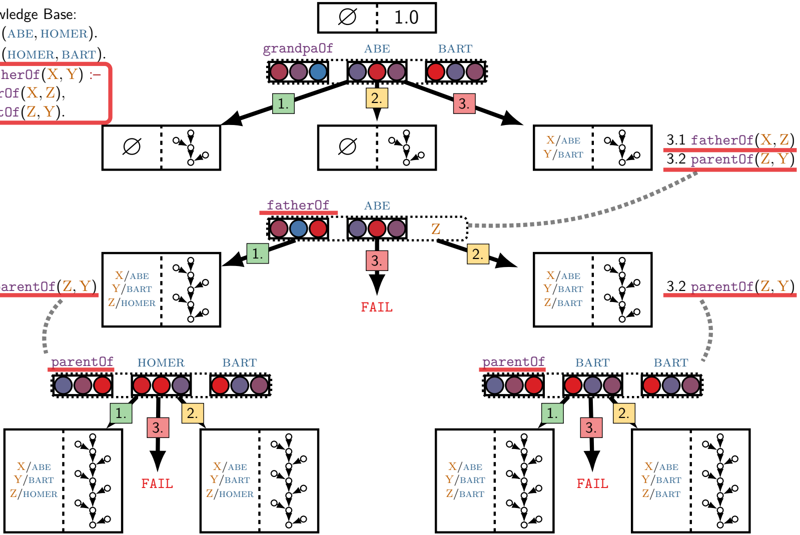

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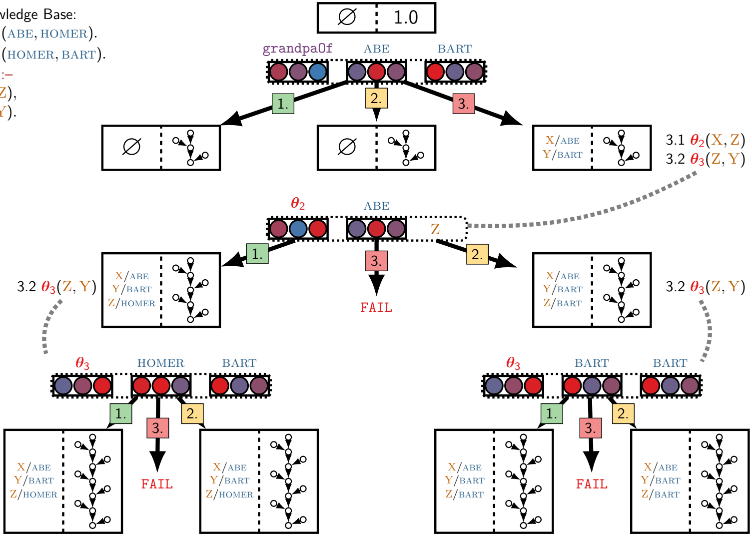
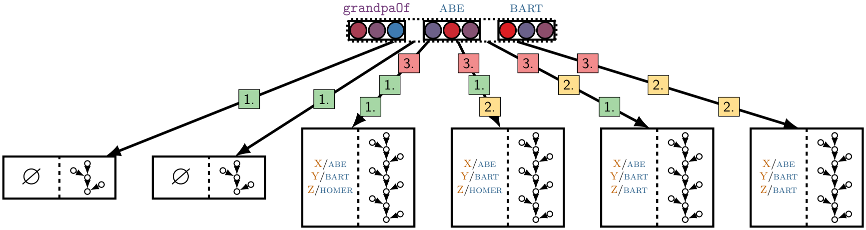


Training Objective



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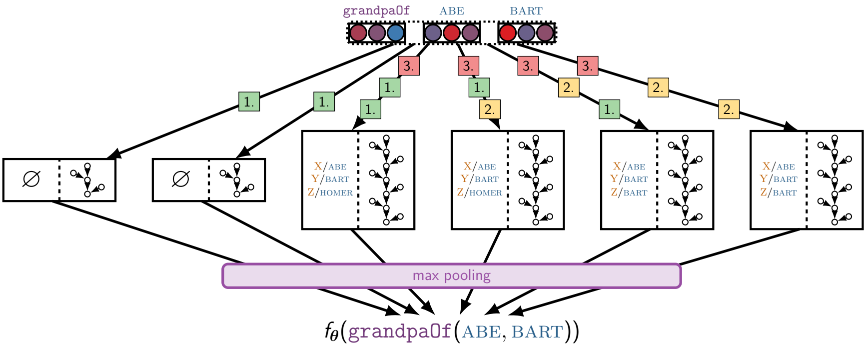

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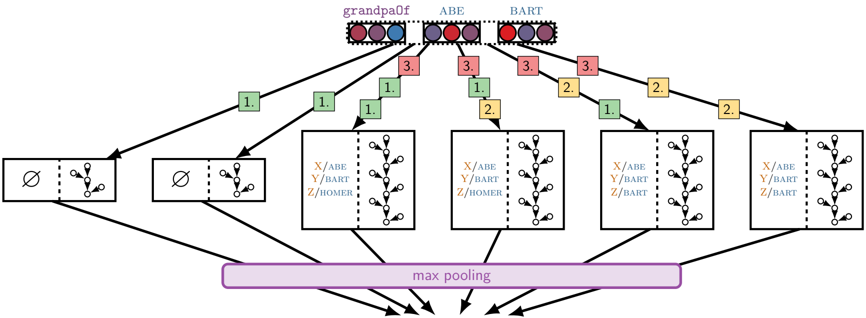
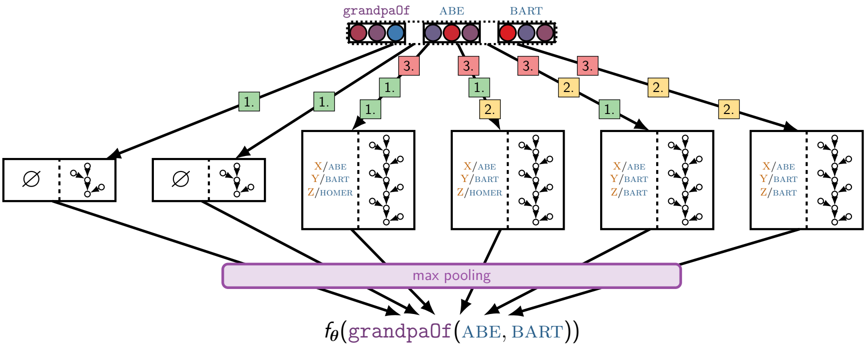


$f_\theta(\texttt{grandpaOf}(\text{ABE}, \text{BART}))$

- Loss: negative log-likelihood w.r.t. target proof success

Training Objective



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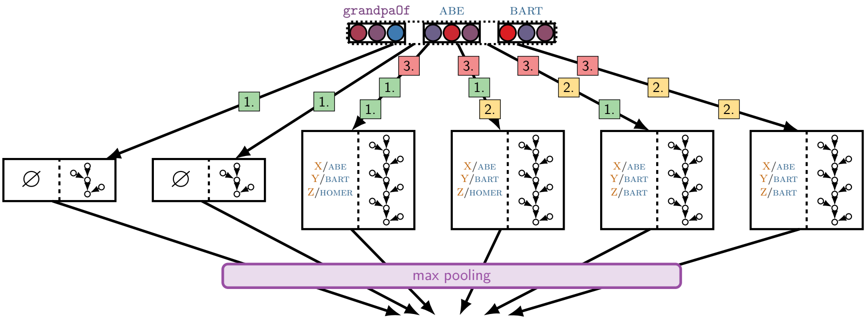

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- Loss: negative log-likelihood w.r.t. target proof success
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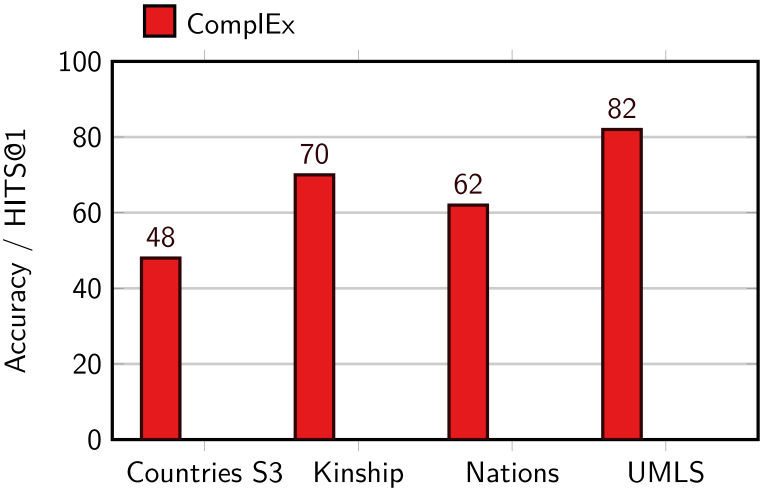
- Vectors are **learned such that proof success is high for known facts** and low for sampled negative facts

Results


ComplEx
Accuracy / HITS@1
100
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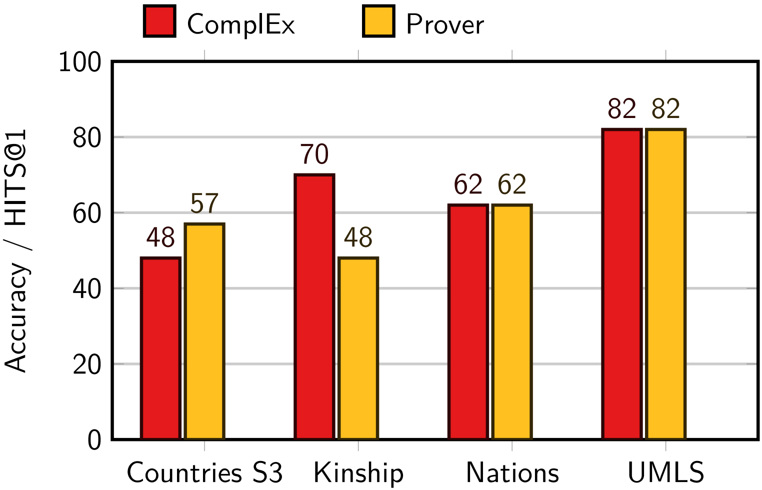
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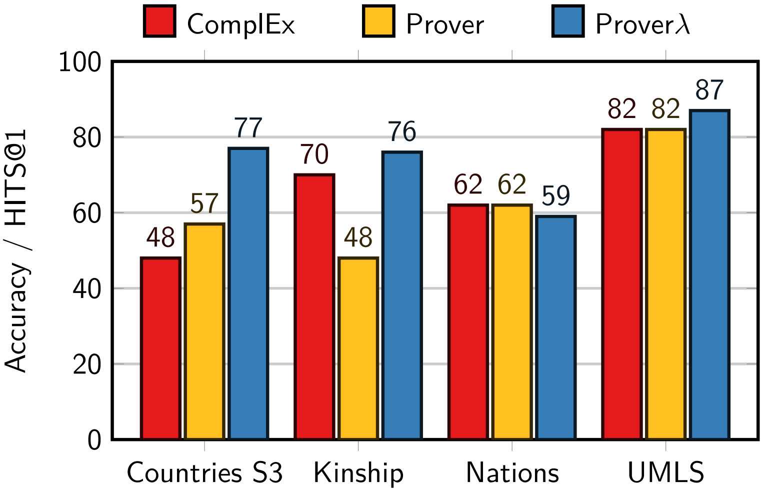
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Examples of Induced Rules

$\texttt{locatedIn}(X, Y) \coloneq \texttt{locatedIn}(X, Z), \texttt{locatedIn}(Z, Y).$

$\texttt{interacts\_with}(X, Y) \coloneq \texttt{interacts\_with}(X, Z), \texttt{interacts\_with}(Z, Y).$

$\texttt{derivative\_of}(X, Y) \coloneq \texttt{derivative\_of}(X, Z), \texttt{derivative\_of}(Z, Y).$

Summary

- We used Prolog's backward chaining as recipe for recursively constructing a neural network to prove queries to a knowledge base

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- **Learns vector representations of symbols** from data via gradient descent

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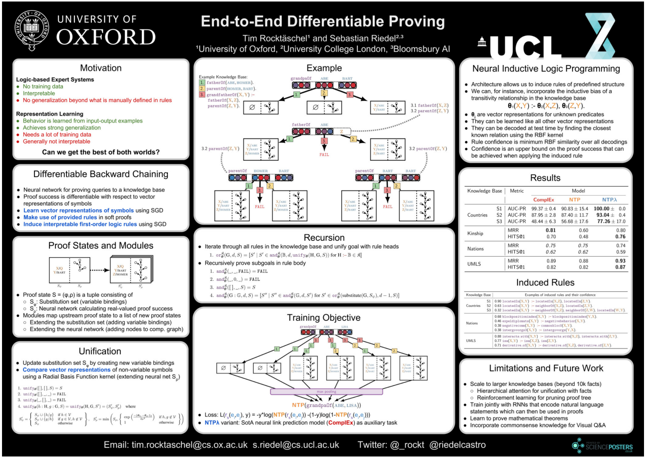
Thank you!

Poster: Today 6:30-10:30pm, Pacific Ballroom #128



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