Photo: Hannah Theile (OVGU)

Tseitin or not Tseitin?
The Impact of CNF Transformations on Feature-Model Analyses

ASE 2022 — October 10–14 — Rochester, Michigan

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OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

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Implementing Configurable Software Systems

A Configurable Graph

```
class Node {
  #ifdef LABELED
    std::string label;
  #endif
  #ifdef COLORED
    std::string color;
  #endif
};

class Edge {
  #ifdef DIRECTED
    Node from, to;
  #elif UNDIRECTED && HYPER
    std::set<Node> nodes;
  #endif
};
```

Product Line Implementation
(here: C++ with C preprocessor)

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A Labeled Directed Graph

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Product Implementation

Implementing Configurable Software Systems

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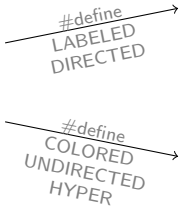

Configuration

A Labeled Directed Graph

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class Node {
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};

class Edge {
  Node from, to;
};
```

A Colored Undirected Hypergraph

```
class Node {
  std::string color;
};

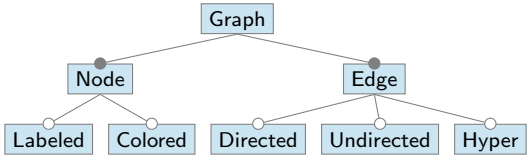
class Edge {
  std::set<Node> nodes;
};
```

Product Implementation

Modeling Features and their Dependencies

Feature Models

- tree models **features**
- cross-tree **constraints** model dependencies
- solver-based **analyses** can be used to understand the configuration space better



$\neg(\mathit{Directed} \wedge \mathit{Undirected})$ **feature conflict**

$\mathit{Hyper} \rightarrow \mathit{Undirected}$ **missing implementation**

$\mathit{Directed} \nleftrightarrow (\mathit{Undirected} \wedge \mathit{Hyper})$ **well-defined #ifdef**

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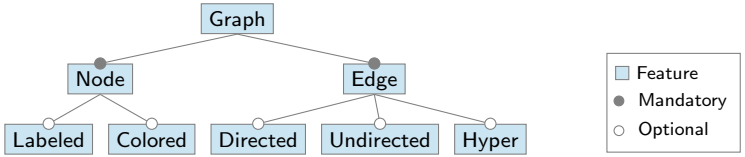


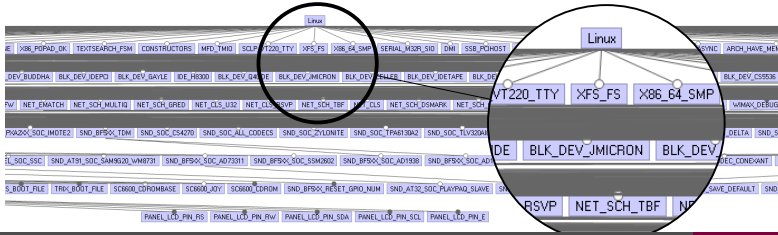
$\neg(Directed \wedge Undirected)$ **feature conflict**

$Hyper \rightarrow Undirected$ **missing implementation**

$Directed \nleftrightarrow (Undirected \wedge Hyper)$ **well-defined #ifdef**

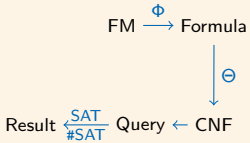
The Linux Kernel

- $>$ 12000 features [2016]
- $> 10^{5000}$ products [2016]
- 114 dead features [2013]
- 151 reverse dependency bugs [2019]



Analyzing Feature Models with SAT and #SAT Solvers

Feature-Model Analysis



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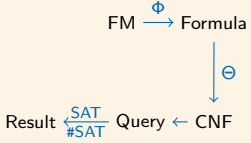
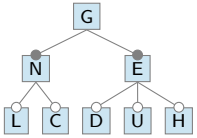


A Feature Model *FM*



$\neg(D \wedge U)$
$H \rightarrow U$
$D \not\leftrightarrow (U \wedge H)$

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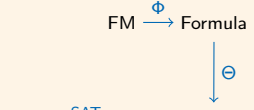

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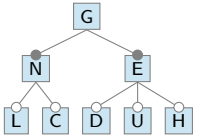

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$\xrightarrow{\Phi}$

As a Formula $\Phi(FM)$

$$
\begin{aligned}
& G \\
\wedge\ & (N \leftrightarrow G) \wedge (E \leftrightarrow G) \\
\wedge\ & ((L \vee C) \rightarrow N) \\
\wedge\ & ((D \vee U \vee H) \rightarrow E) \\
\wedge\ & \neg(D \wedge U) \wedge (H \rightarrow U) \\
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Analyzing Feature Models with SAT and #SAT Solvers

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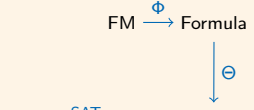

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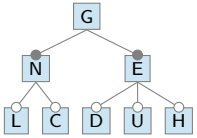

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\end{aligned}
$$

$\downarrow \Theta$

As a CNF $\Theta(\Phi(FM))$

$$
\begin{aligned}
& \{\{G\}, \{\neg N, G\}, \{N, \neg G\}, \\
& \{\neg E, G\}, \{E, \neg G\}, \{\neg L, N\}, \\
& \{\neg C, N\}, \{\neg D, E\}, \{\neg U, E\}, \\
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Analyzing Feature Models with SAT and #SAT Solvers

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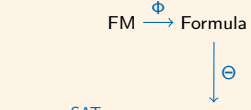

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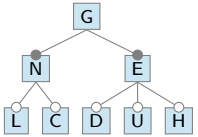

Φ →

As a Formula Φ(*FM*)

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\end{aligned}
$$

↓ Θ

As a CNF Θ(Φ(*FM*))

$$\{\{G\}, \{\neg N, G\}, \{N, \neg G\}, \{\neg E, G\}, \{E, \neg G\}, \{\neg L, N\}, \{\neg C, N\}, \{\neg D, E\}, \{\neg U, E\}, \{\neg H, E\}, \{\neg D, \neg U\}, \{\neg H, U\}, \{\{D, U\}, \{D, H\}, \{\neg D, \neg U, \neg H\}\}\}$$

←

Core Feature *F*?

$$SAT(\Theta(\Phi(FM)) \wedge \neg F)$$

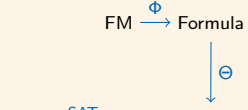
←

Core Features

$$\{G, N, E\}$$

Analyzing Feature Models with SAT and #SAT Solvers

Feature-Model Analysis



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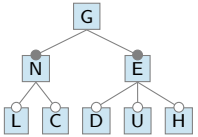

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←

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←

Core Features

$$\{G, N, E\}$$

←

Products in *FM*?

$$\#SAT(\Theta(\Phi(FM)))$$

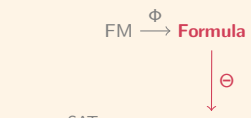
←

Feature Model Cardinality

8

Often Overlooked: Conjunctive Normal Form (CNF)

Feature-Model Analysis



From Formula ...

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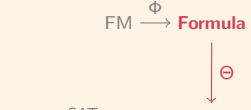

Conjunctive Normal Form

- **conjunction** $\wedge$ of **disjunctions** $\vee$ of **literals** X, $\neg X$
- here: a set of **clauses**, which are sets of literals
- used by almost all solvers

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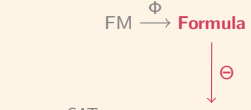

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Our Goal: Raise Awareness for CNF Transformations

- how to **transform** feature-model formulas **into CNF**?
  $\Rightarrow$ describe and classify CNF transformations
- does this **impact** the work of **practitioners and researchers**?
  $\Rightarrow$ evaluate efficiency and correctness on feature models

CNF Transformations

Distributive $\Theta = D$

apply laws of logic (**De Morgan's laws** and **distributivity**)

$$D \not\leftrightarrow (U \wedge H)$$
$$\xrightarrow{D} (D \vee (U \wedge H)) \wedge (\neg D \vee \neg (U \wedge H))$$
$$\xrightarrow{D} \{\{D, U\}, \{D, H\}, \{\neg D, \neg U, \neg H\}\}$$

✓ **equivalence**
SAT ✓, $\#SAT = 4$

✓ easy to implement

✗ **exponential** complexity

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Tseitin $\Theta = T$ ['83]

abbreviate a subformula ϕ with an auxiliary variable $x_\phi \leftrightarrow \phi$

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Plaisted-Greenbaum $\Theta = PG$ ['86]

abbreviate a subformula ϕ with an auxiliary variable $x_\phi \rightarrow \phi$

$$D \not\Leftrightarrow (U \wedge H)$$
$$\xrightarrow{PG} (D \not\Leftrightarrow x) \wedge x \rightarrow (U \wedge H)$$
$$\xrightarrow{D} \{\{D, x\}, \{\neg D, \neg x\}, \{\neg x, U\}, \{\neg x, H\}\}$$

- ✓ **equi-assignability** SAT ✓
- ✓ **linear** complexity $< T$
- ✗ **equi-countability** $\#SAT = 5$

Evaluation

Research Questions

RQ 1 **efficiency** of CNF transformations?

RQ 2 CNF transformation → **efficiency** of analyses?

RQ 3 CNF transformation → **correctness** of analyses?

Evaluation

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Experimental Setup

- 22 configurable software systems
- 3 CNF transformation tools
- 23 SAT and #SAT solvers

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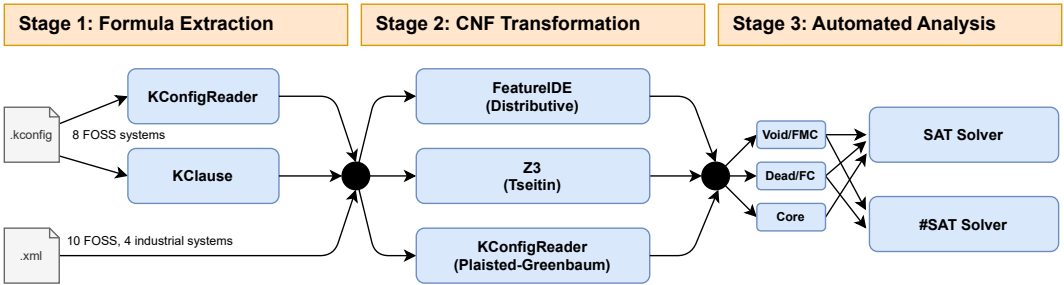
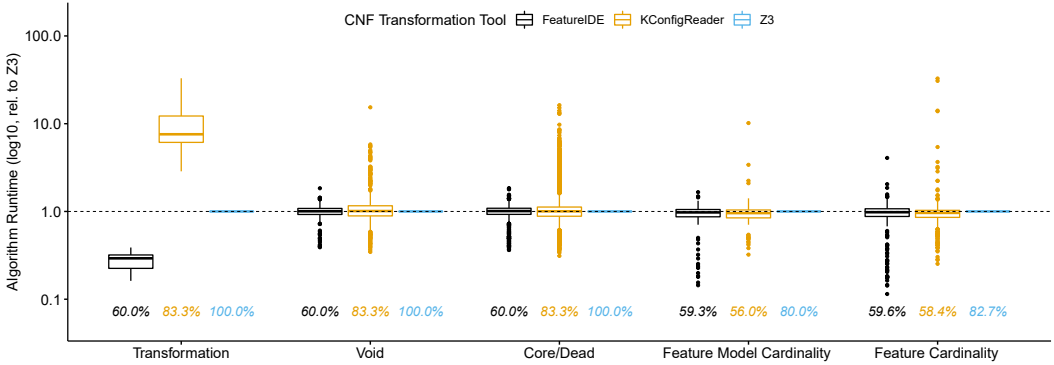


Efficiency of CNF Transformations (RQ 1) and Analyses (RQ 2)



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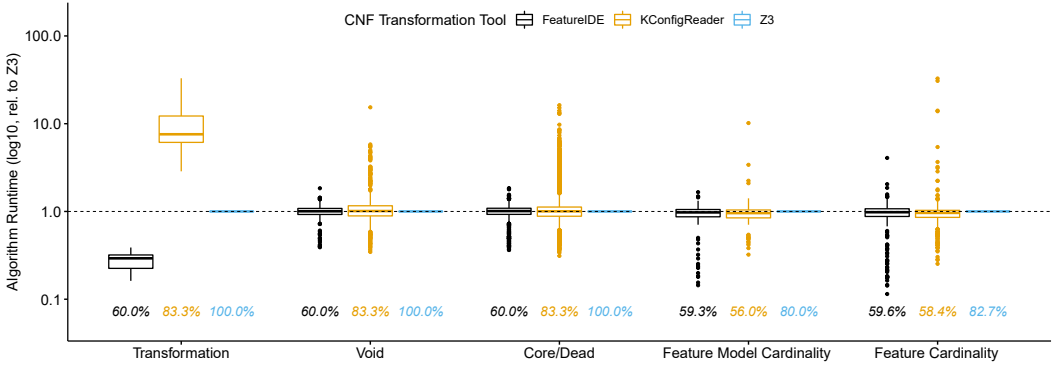

RQ 1: *D* **often fails**, tools **differ significantly**

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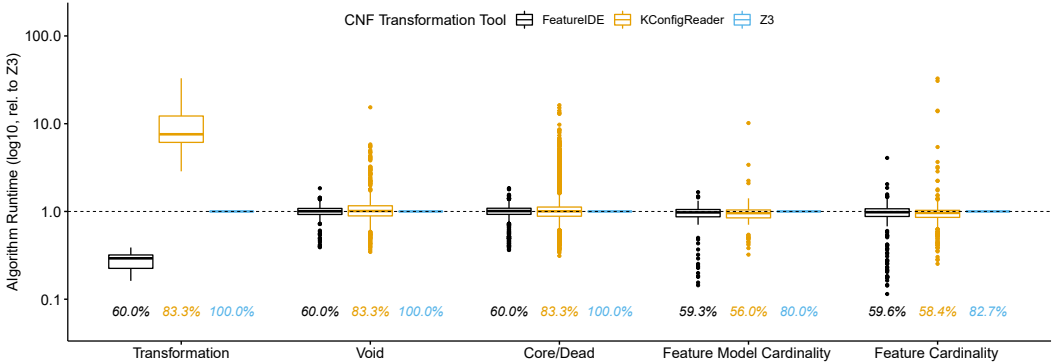

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RQ 2 (SAT): almost all calls succeed, **solve time varies** by factor 0.31–16.27

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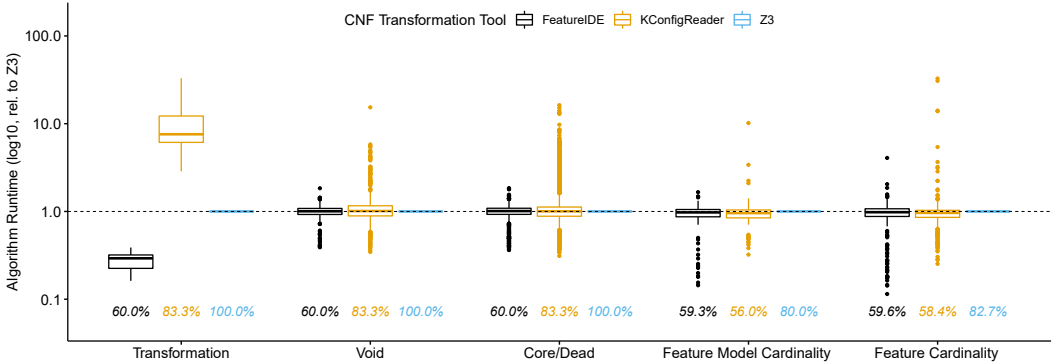
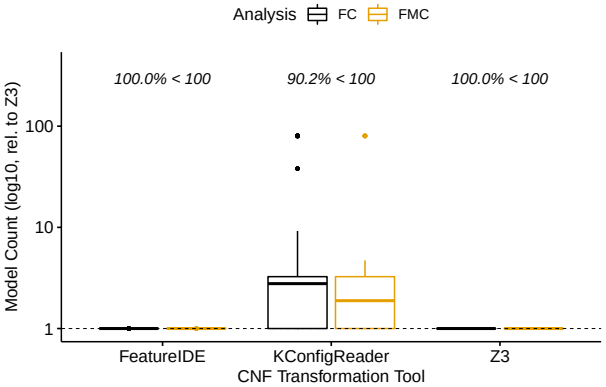


RQ 1: *D* **often fails**, tools **differ significantly**

RQ 2 (SAT): almost all calls succeed, **solve time varies** by factor 0.31–16.27

RQ 2 (#SAT): 81.6% of calls succeed, **solve time varies** by factor 0.11–32.7

Correctness of #SAT-Based Analyses (RQ 3)



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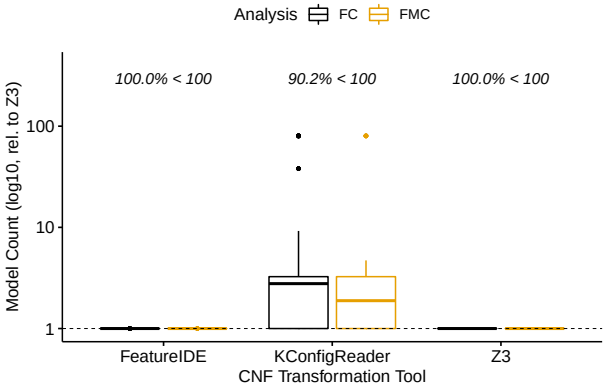
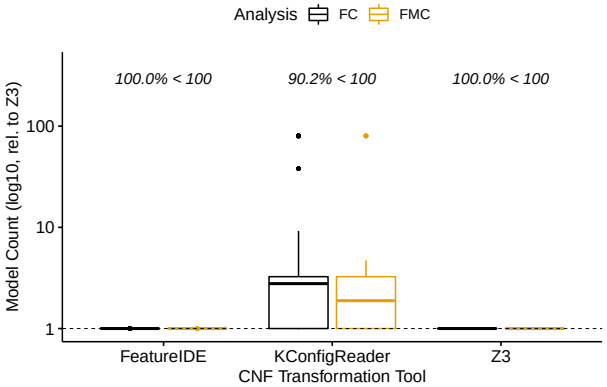


RQ 3

- with PG, $\approx$ 70% of #SAT calls return **incorrect results**
- incorrect by factor $\approx$ 3 (median)
- incorrect by factor $\approx 10^{77}$ (worst)

Correctness of #SAT-Based Analyses (RQ 3)



RQ 3

- with PG, $\approx$ 70% of #SAT calls return **incorrect results**
- incorrect by factor $\approx$ 3 (median)
- incorrect by factor $\approx 10^{77}$ (worst)

Our Recommendations

RQ 1	D for small, T for large models
RQ 2	largely depends on the model $\Rightarrow$ future work
RQ 3	do not use PG for #SAT

Conclusion

The Impact of CNF Transformations on Feature-Model Analyses

Distributive	Tseitin	Plaisted-Greenbaum
apply laws of logic	abbreviate ϕ with $x_\phi \leftrightarrow \phi$	abbreviate ϕ with $x_\phi \rightarrow \phi$
✓ **equivalence** ✓ easy to implement ✗ **exponential** complexity	✓ **quasi-equivalence** ✓ **linear** complexity ✗ take care of new variables	✓ **equi-assignability** ✓ **linear** complexity ✗ **equi-countability**
FeatureIDE	**Z3**	**KConfigReader**
often fails on large models	succeeds correctly on all models	often incorrect for #SAT calls

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Tseitin or not Tseitin?

Conclusion

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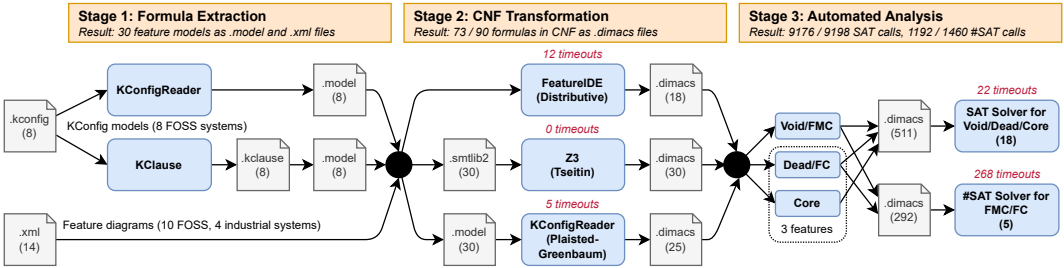
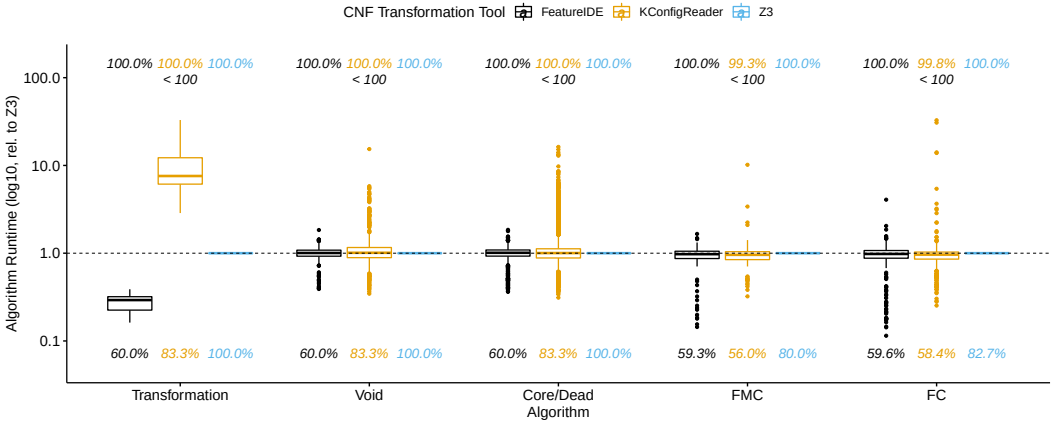
Tseitin or not Tseitin? $\Rightarrow$ Yes!

find out more:

https://github.com/ekuiter/tseitin-or-not-tseitin


Stage 1: Formula Extraction
Result: 30 feature models as .model and .xml files
Stage 2: CNF Transformation
Result: 73 / 90 formulas in CNF as .dimacs files
Stage 3: Automated Analysis
Result: 9176 / 9198 SAT calls, 1192 / 1460 #SAT calls
.kconfig (8)
KConfig models (8 FOSS systems)
KConfigReader
KClause
.model (8)
.kclause (8)
.model (8)
.xml (14)
Feature diagrams (10 FOSS, 4 industrial systems)
12 timeouts
FeatureIDE (Distributive)
.dimacs (18)
0 timeouts
.smtlib2 (30)
Z3 (Tseitin)
.dimacs (30)
5 timeouts
.model (30)
KConfigReader (Plaisted-Greenbaum)
.dimacs (25)
Void/FMC
Dead/FC
Core
3 features
.dimacs (511)
22 timeouts
SAT Solver for Void/Dead/Core (18)
.dimacs (292)
268 timeouts
#SAT Solver for FMC/FC (5)


CNF Transformation Tool
FeatureIDE
KConfigReader
Z3
Algorithm Runtime (log10, rel. to Z3)
100.0
10.0
1.0
0.1
100.0% 100.0% 100.0%
< 100
100.0% 100.0% 100.0%
< 100
100.0% 100.0% 100.0%
< 100
100.0% 99.3% 100.0%
< 100
100.0% 99.8% 100.0%
< 100
60.0% 83.3% 100.0%
60.0% 83.3% 100.0%
60.0% 83.3% 100.0%
59.3% 56.0% 80.0%
59.6% 58.4% 82.7%
Transformation
Void
Core/Dead
FMC
FC
Algorithm