



# Comparing CNF Transformations for Feature-Model Analysis

Elias Kuitert | FOSD Meeting 2022

w/ Sebastian Krieter, Chico Sundermann, Thomas Thüm, Gunter Saake

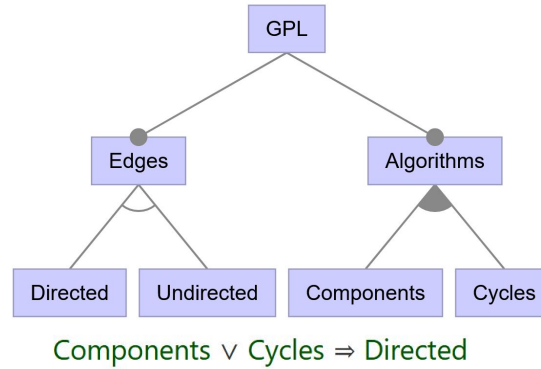


Databases  
and  
Software  
Engineering

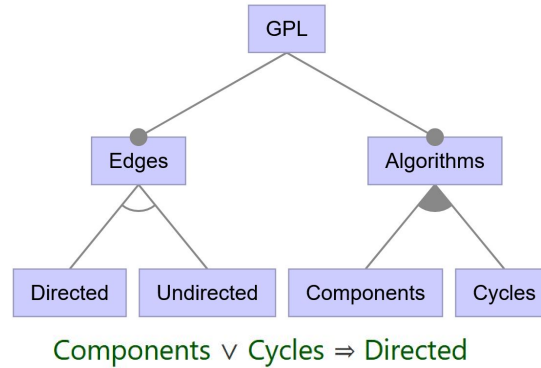


universität  
**uulm**

# SAT-Based Analysis of Feature Models



# SAT-Based Analysis of Feature Models



$$\begin{aligned}\phi_{GPL} = & GPL \wedge Edges \wedge Algorithms \wedge (Directed \vee Undirected) \\ & \wedge (\neg Directed \vee \neg Undirected) \wedge (Components \vee Cycles) \\ & \wedge (\neg(Components \vee Cycles) \vee Directed)\end{aligned}$$

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void, dead, AS, DP... ..... SAT( $\phi_{GPL}$ )?



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void, dead, AS, DP... .....  $SAT(\phi_{GPL})?$



**conjunctive normal form (CNF)**

conjunction ( $\wedge$ ) of  
disjunction ( $\vee$ ) of  
literals ( $x, \neg x$ )

**How to transform feature  
model formulas into CNF?**



**Is this a threat to validity  
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# (#)SAT-Based Analysis of Feature Models

void, dead, AS, DP... SAT( $\phi_{GPL}$ )? #SAT( $\phi_{GPL}$ )? URS, F. Prio...



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## Distributive Transformation

**Idea:** Apply *distributive* and *De Morgan's laws*

**Example:**

$$\dots \wedge (C_o \vee C_y) \wedge (\neg(C_o \vee C_y) \vee D)$$

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**Properties:**

- preserves equivalence
- easy to implement
- exponential space complexity

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## Tseitin Transformation

**Idea:** Use  $\leftrightarrow$  to define variables ("shortcuts") for subformulas

**Example:**  $\Box \phi \wedge \dots$

$$\wedge (\Box \phi \leftrightarrow \dots \wedge \Box C_0 \vee C_y \wedge \Box (\neg C_0 \wedge \neg C_y) \vee D))$$

$$\wedge (\Box C_0 \vee C_y \leftrightarrow (C_0 \vee C_y))$$

$$\wedge (\Box (\neg C_0 \wedge \neg C_y) \vee D \leftrightarrow (\Box (\neg C_0 \wedge \neg C_y) \vee D))$$

$$\wedge (\Box \neg C_0 \wedge \neg C_y \leftrightarrow (\neg C_0 \wedge \neg C_y))$$

**Properties:**

- preserves assignments + count
- introduces artificial variables
- linear space complexity

# How to transform into CNF?

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## Tseitin Transformation

## Plaisted-Greenbaum Trans.

often mixed up

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**Properties:**

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- introduces artificial variables
- linear space complexity

**Idea:** Like Tseitin, but using  $\rightarrow$  ("half-definitions")

**Example:**  $\Box \phi \wedge \dots$

$$\begin{aligned} & \wedge (\Box \phi \rightarrow \dots \wedge \Box C_0 \vee C_y \wedge \Box (\neg C_0 \wedge \neg C_y) \vee D)) \\ & \wedge (\Box C_0 \vee C_y \rightarrow (C_0 \vee C_y)) \\ & \wedge (\Box (\neg C_0 \wedge \neg C_y) \vee D \rightarrow (\Box (\neg C_0 \wedge \neg C_y) \vee D)) \\ & \wedge (\Box \neg C_0 \wedge \neg C_y \rightarrow (\neg C_0 \wedge \neg C_y)) \end{aligned}$$

**Properties:**

- preserves assignments
- does not preserve count
- requires less space than Tseitin

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# Evaluation

Is this a threat to validity?



# Research Questions



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1. Does the CNF transformation influence the performance of SAT and #SAT-based analyses?



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2. Does this influence depend on the ...
  - a. size of the feature model (do larger models imply larger performance differences)?
  - b. size increase of the formula (is it costly to introduce variables/literals)?



# Research Questions

1. Does the CNF transformation influence the **performance of SAT and #SAT-based analyses**?
2. Does this influence depend on the ...
  - a. **size of the feature model** (do larger models imply larger performance differences)?
  - b. **size increase of the formula** (is it costly to introduce variables/literals)?
3. Does the CNF transformation affect the **correctness of #SAT-based analyses**?





# Subject Systems





# Subject Systems

*Berger et al. 2013*

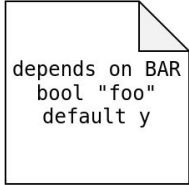
## **Kconfig-based feature models**

Current versions of 9 FOSS systems:

*axTLS, Buildroot, BusyBox, EmbToolkit, Fiasco, Freetz-NG, Linux, toybox, uClibc-ng*

## **A Study of Variability Models and Languages in the Systems Software Domain**

Thorsten Berger, Steven She, Rafael Lotufo,  
Andrzej Wasowski, *Member, IEEE*, and Krzysztof Czarnecki



```
depends on BAR
bool "foo"
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*Knüppel et al. 2017*

## Large feature models (w/ tree)

4 automotive and 8 FOSS systems:

*axTLS, BusyBox, eCos/CDL (3), EmbToolkit, Linux, uClibc, uClinux-base/distribution*

## Is There a Mismatch between Real-World Feature Models and Product-Line Research?

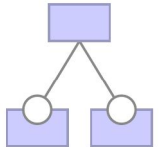
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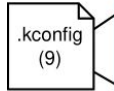
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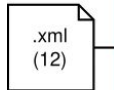




# Experimental Setup



.kconfig  
(9)

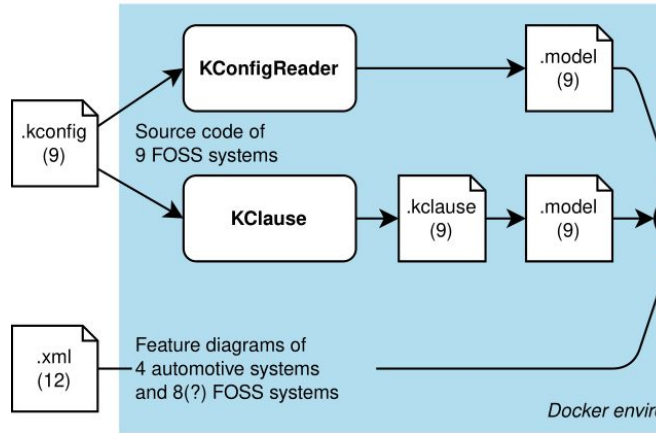


.xml  
(12)

# Experimental Setup

## Stage 1: Extraction

Result: 30 feature models as .model and .xml files



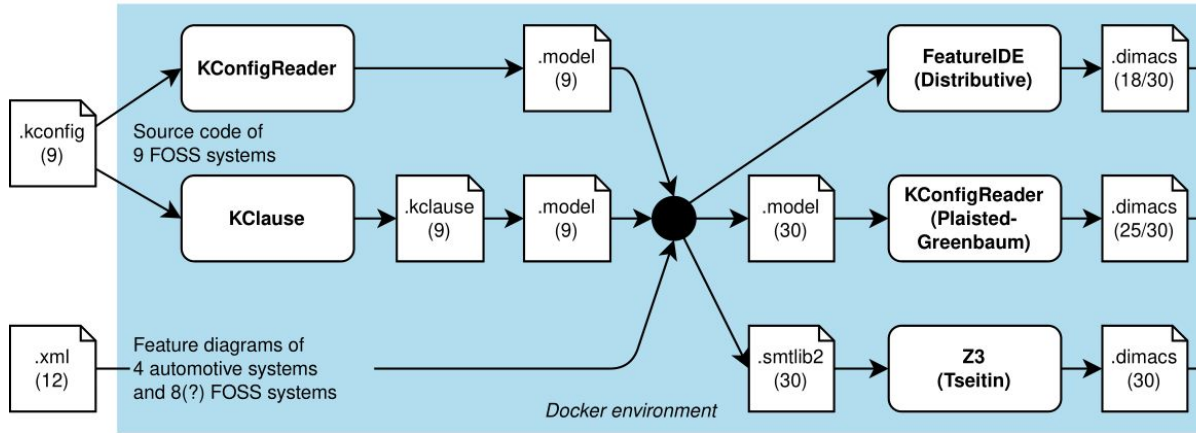
# Experimental Setup

## Stage 1: Extraction

Result: 30 feature models as .model and .xml files

## Stage 2: Transformation

Result: 73 formulas in CNF as .dimacs files



# Experimental Setup

3 iterations  
180s timeout

## Stage 1: Extraction

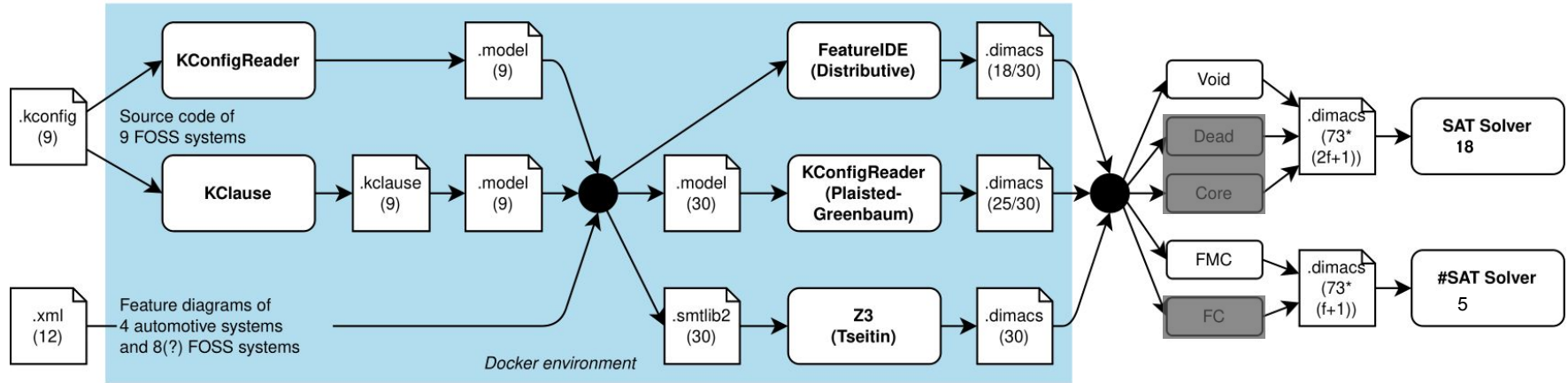
Result: 30 feature models as .model and .xml files

## Stage 2: Transformation

Result: 73 formulas in CNF as .dimacs files

## Stage 3: Analysis

Result: 1769 successful (#)SAT solver calls





# SAT and #SAT Solvers

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# SAT and #SAT Solvers

[www.satcompetition.org](http://www.satcompetition.org), *SAT Heritage*

## International SAT Competition

18 SAT solvers (winners in main track 2002-2021):

*zchaff* (2), *Forklift*, *SatELiteGTI*, *MiniSat*, *RSat*,  
*precosat*, *CryptoMiniSat*, *glucose* (2),  
*lingeling* (2), *Maple* (4), *Kissat* (2)

## SAT Competition 2020 <sup>☆,☆☆</sup>

Nils Froleyks <sup>a</sup>, Marijn Heule <sup>b</sup>, Markus Iser <sup>c</sup>, Matti Järvisalo <sup>d,\*</sup>, Martin Suda <sup>e</sup>

<sup>a</sup> Institute for Formal Models and Verification, Johannes Kepler University, Austria

<sup>b</sup> Computer Science Department, Carnegie Mellon University, USA

<sup>c</sup> Department of Informatics, Karlsruhe Institute of Technology, Germany

<sup>d</sup> HILT, Department of Computer Science, University of Helsinki, Finland

<sup>e</sup> Czech Technical University in Prague, Czech Republic



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---

*Sundermann et al. 2020*

## Evaluating #SAT solvers

5 fastest exact #SAT solvers:

*countAntom*, *d4*, *dSharp*, *GANAK*, *sharpSAT*

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## Evaluating #SAT Solvers on Industrial Feature Models

Chico Sundermann  
Technische Universität Braunschweig

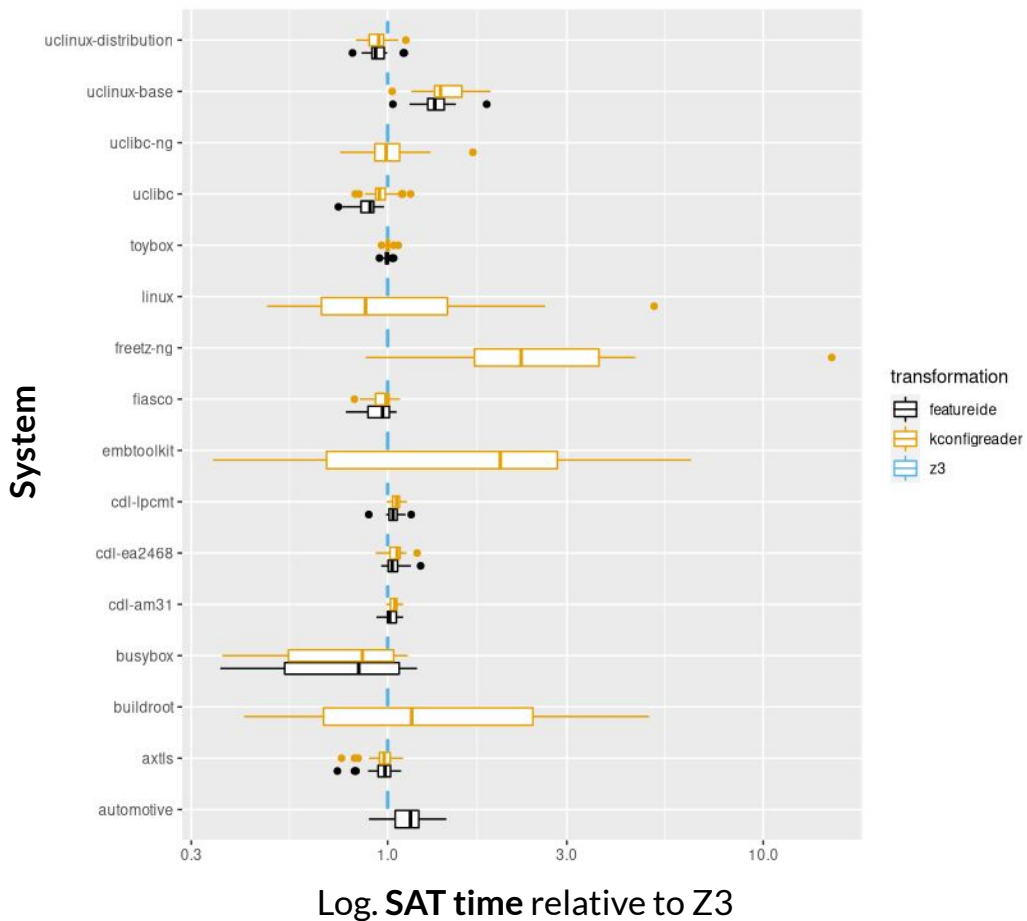
Thomas Thüm  
University of Ulm

Ina Schaefer  
Technische Universität Braunschweig

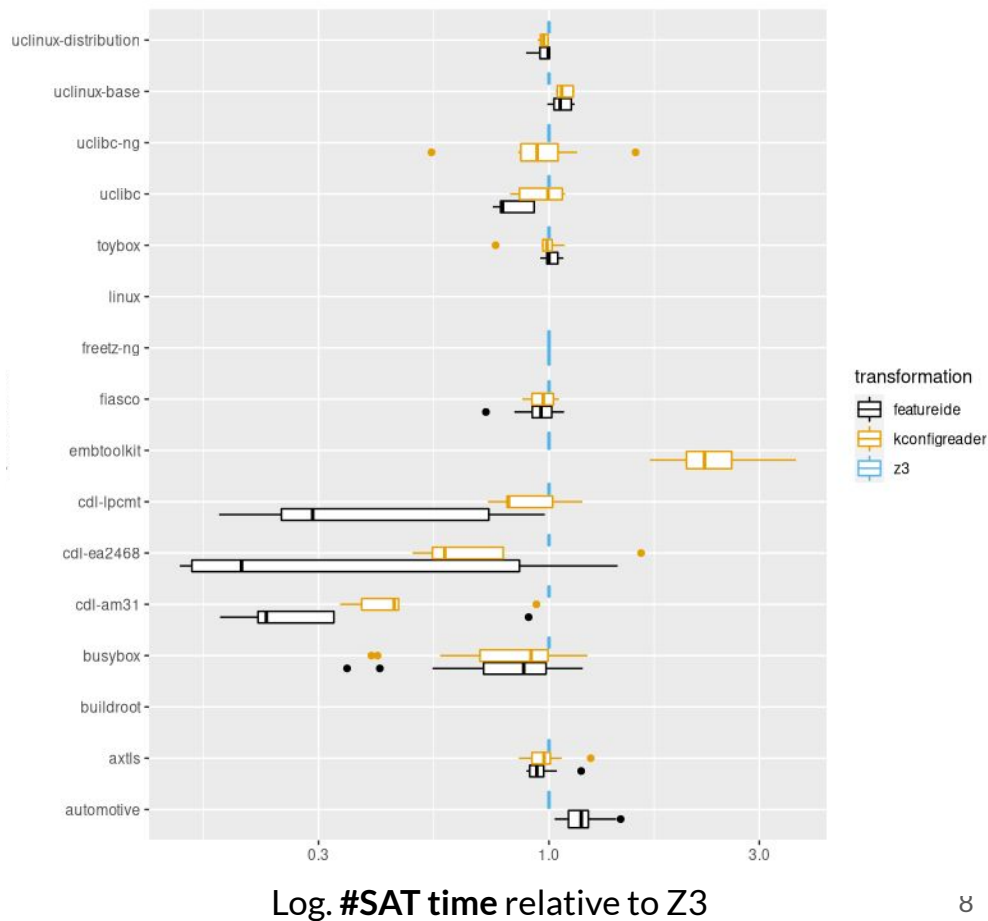
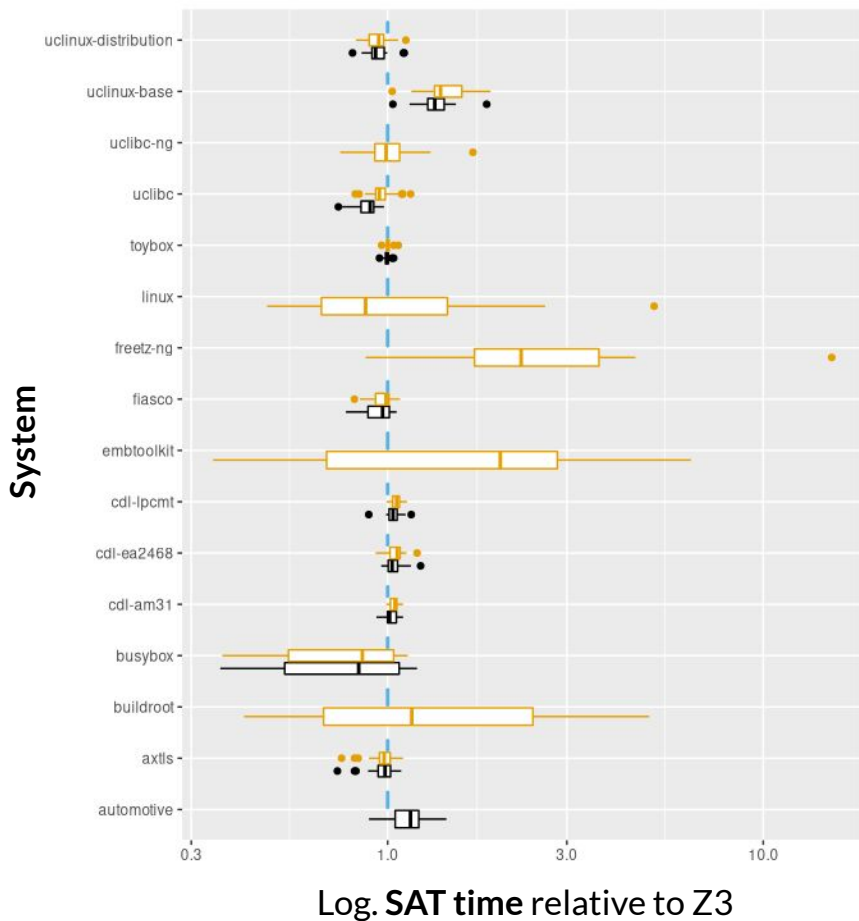
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# Preliminary Results

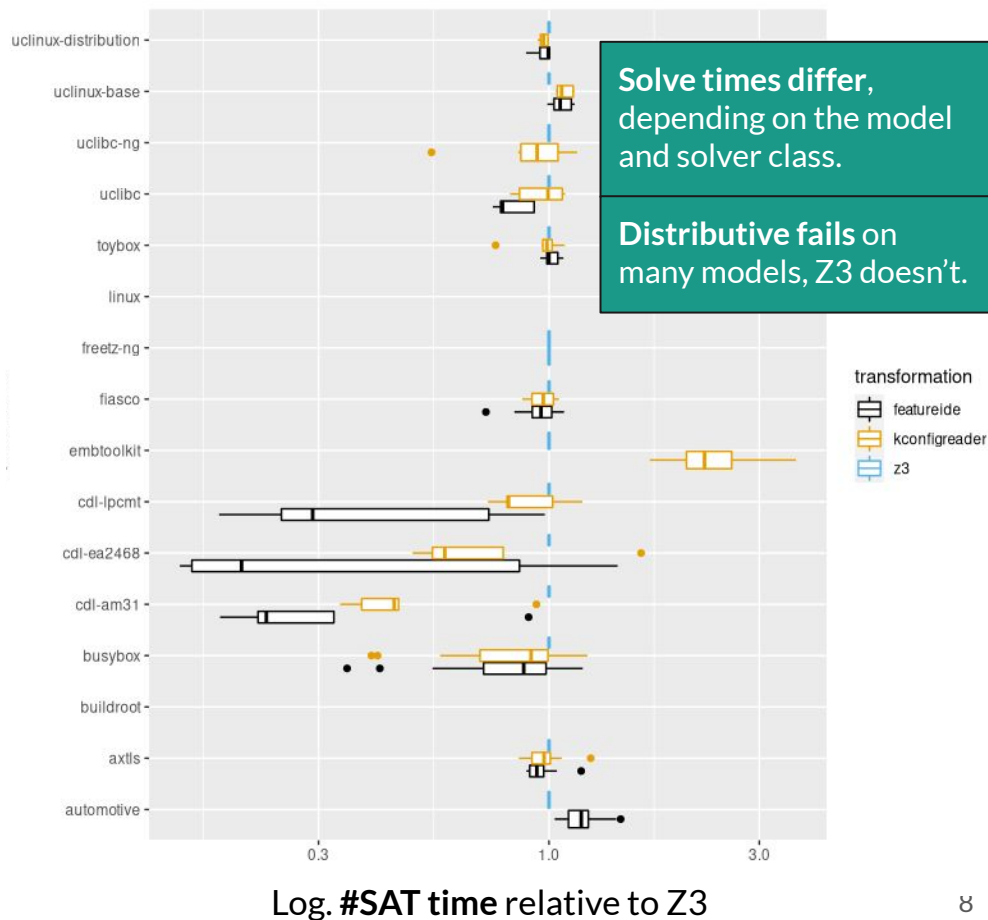
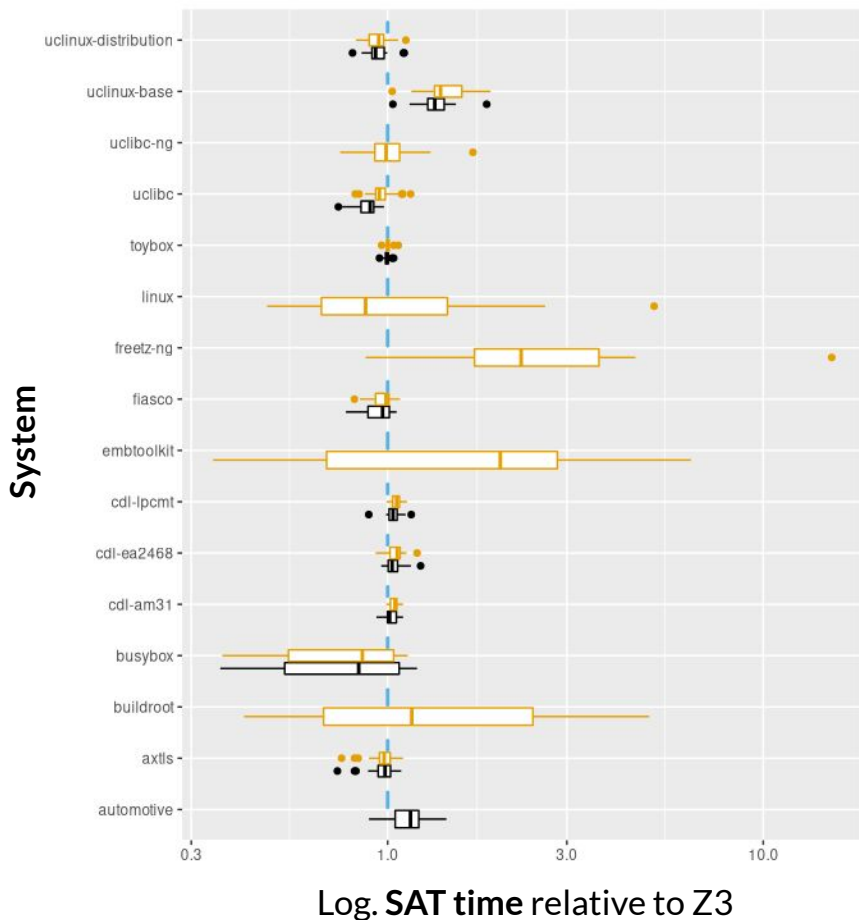
# RQ1: Performance of SAT and #SAT



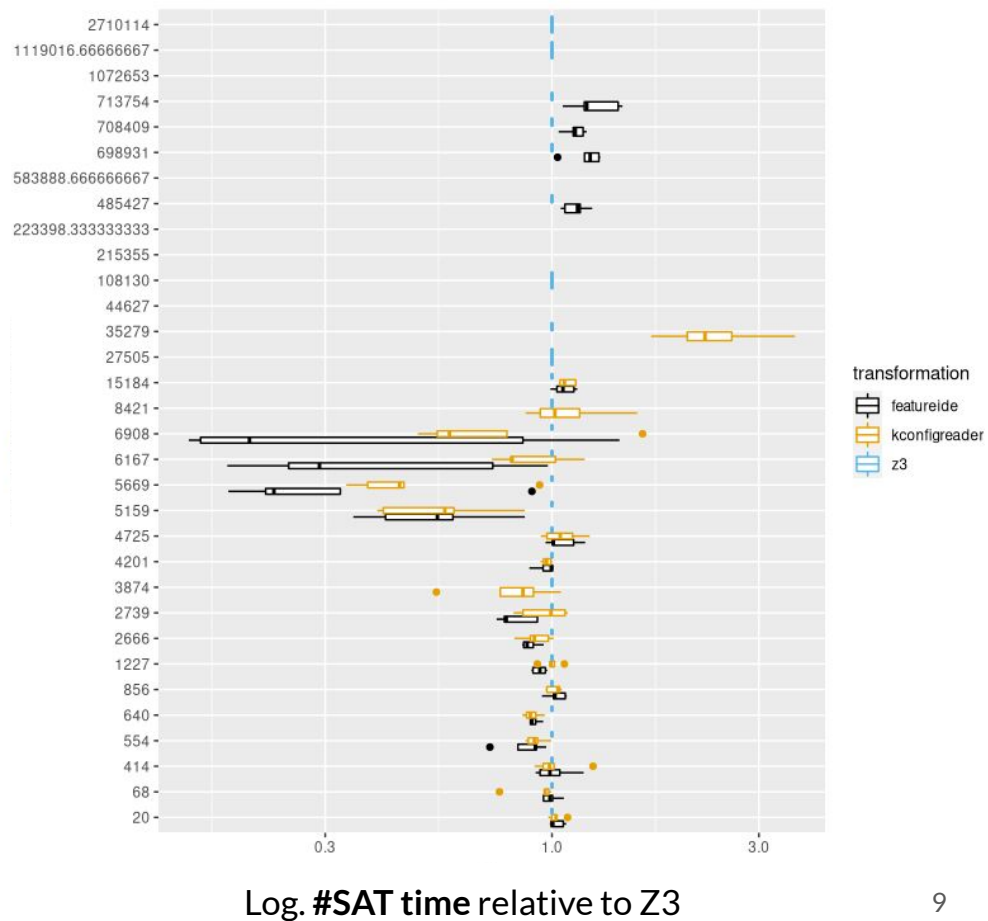
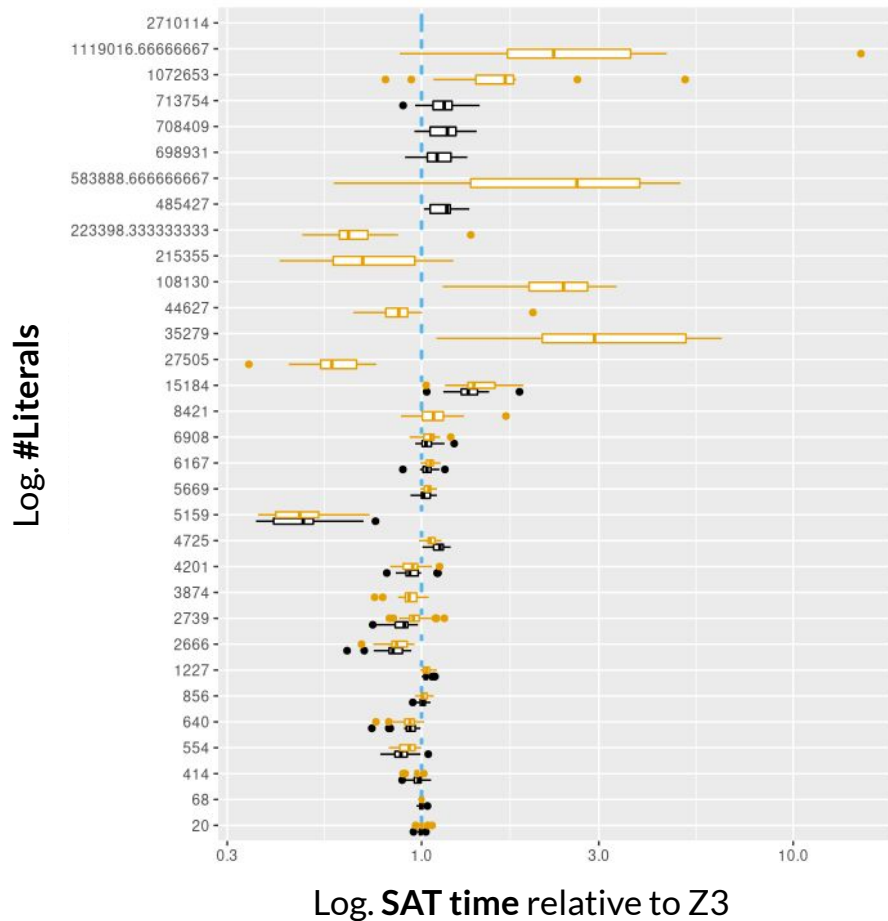
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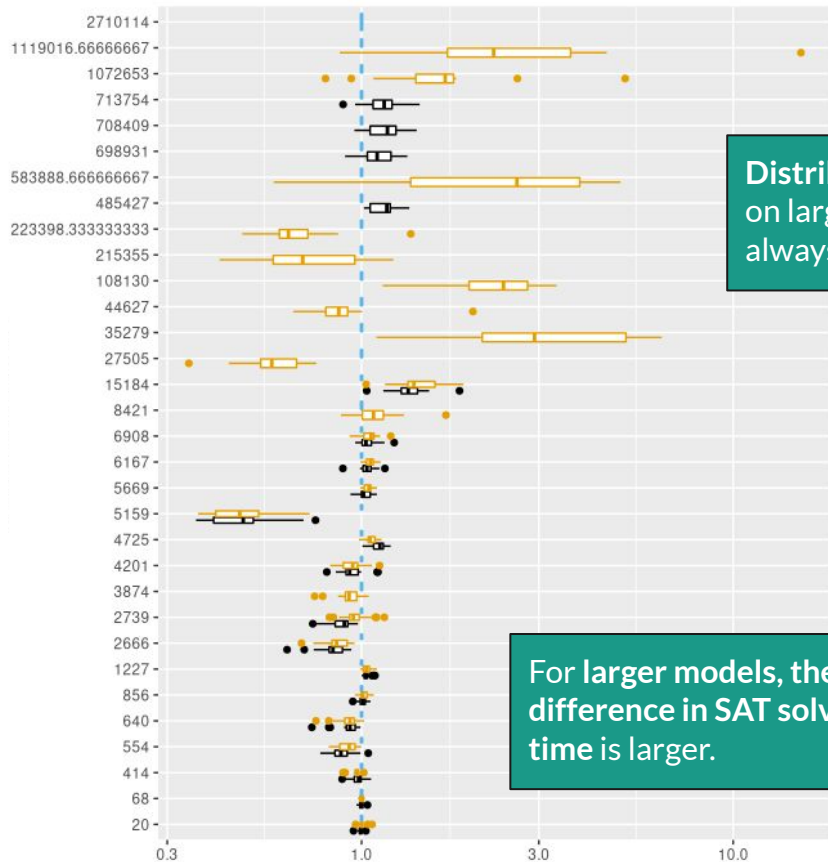


# RQ2a: Feature Model Size



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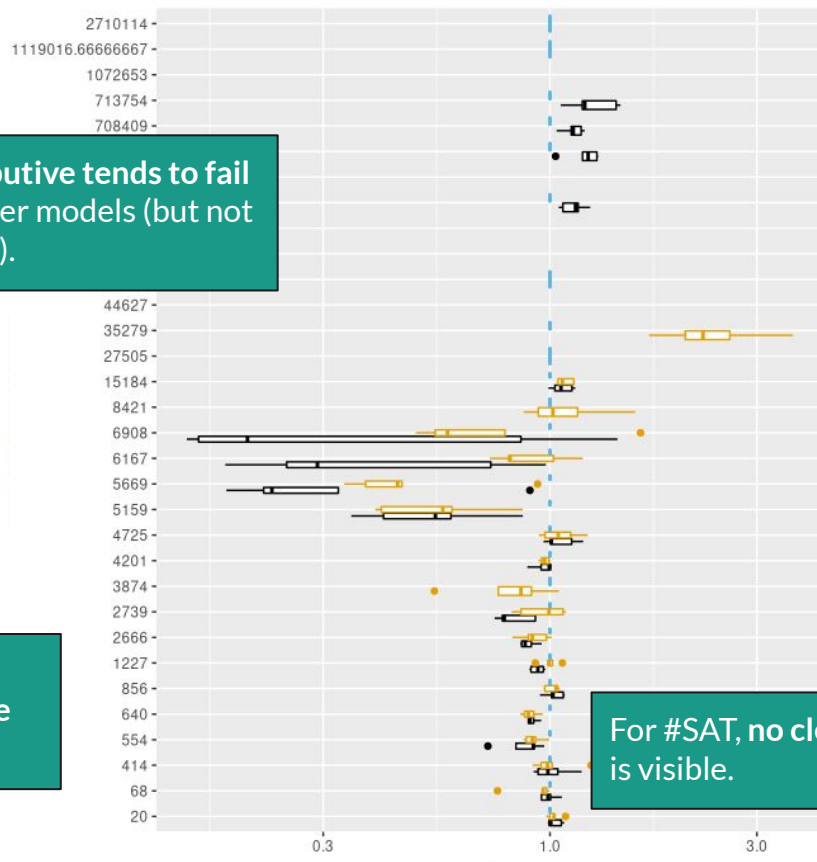
Log. #Literals



Distributive tends to fail on larger models (but not always).

For larger models, the difference in SAT solve time is larger.

Log. SAT time relative to Z3

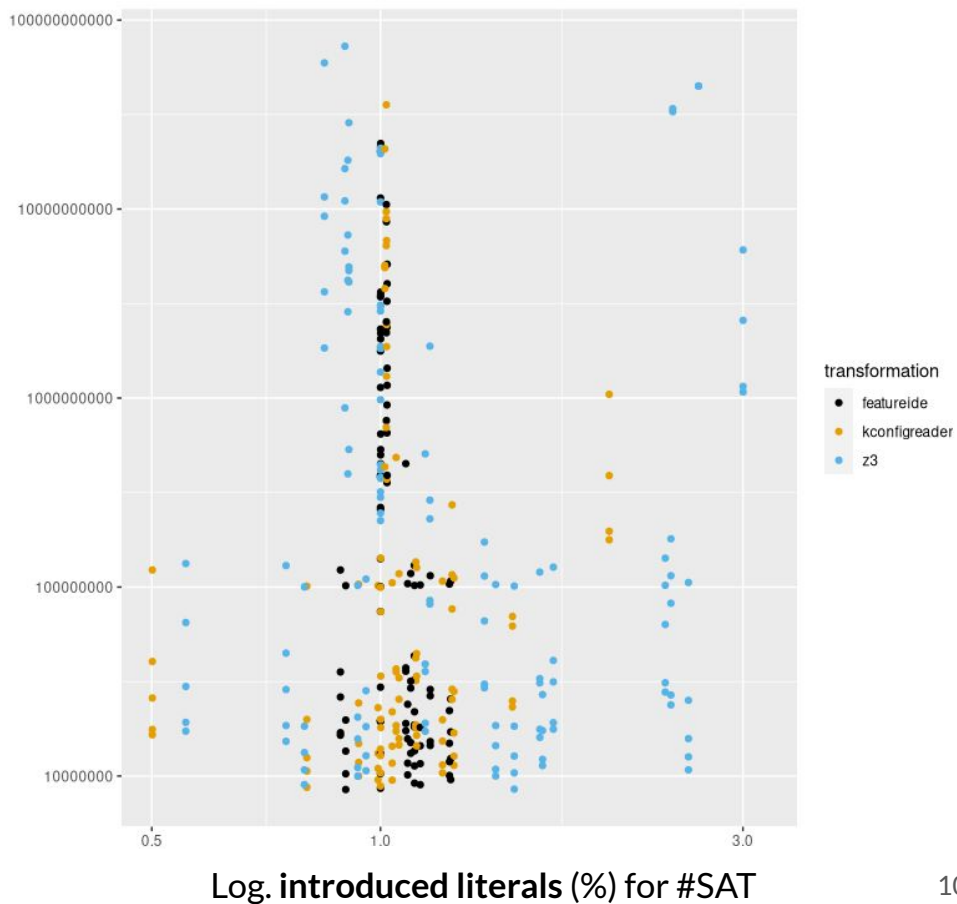
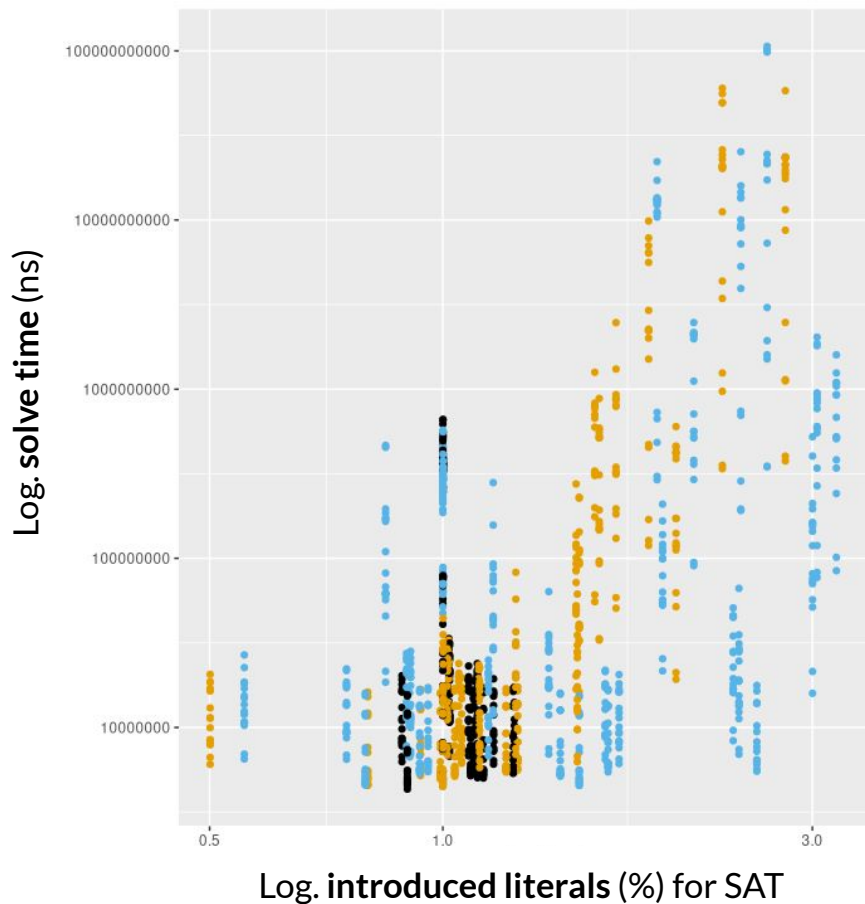


For #SAT, no clear trend is visible.

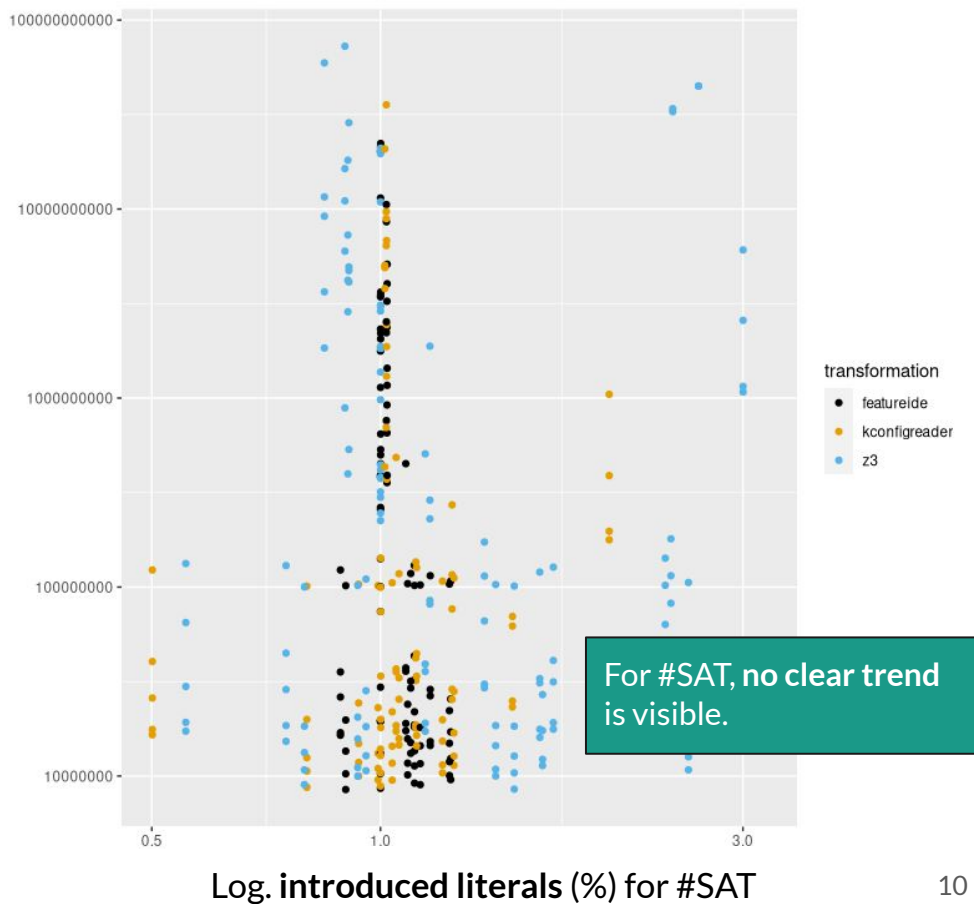
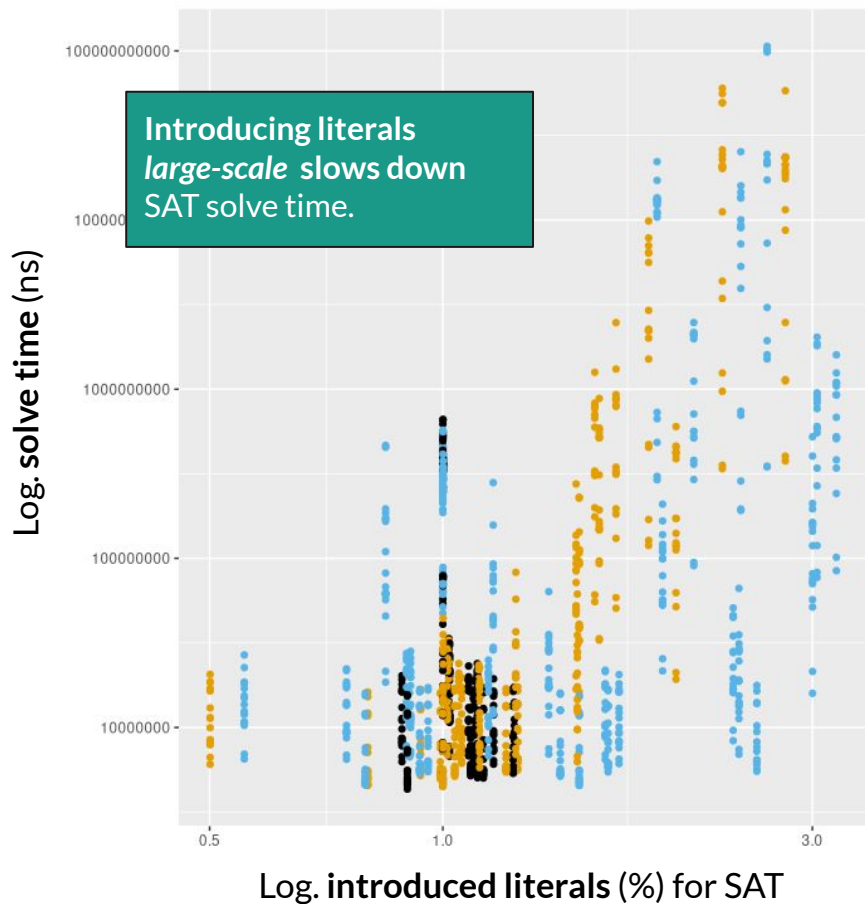
Log. #SAT time relative to Z3



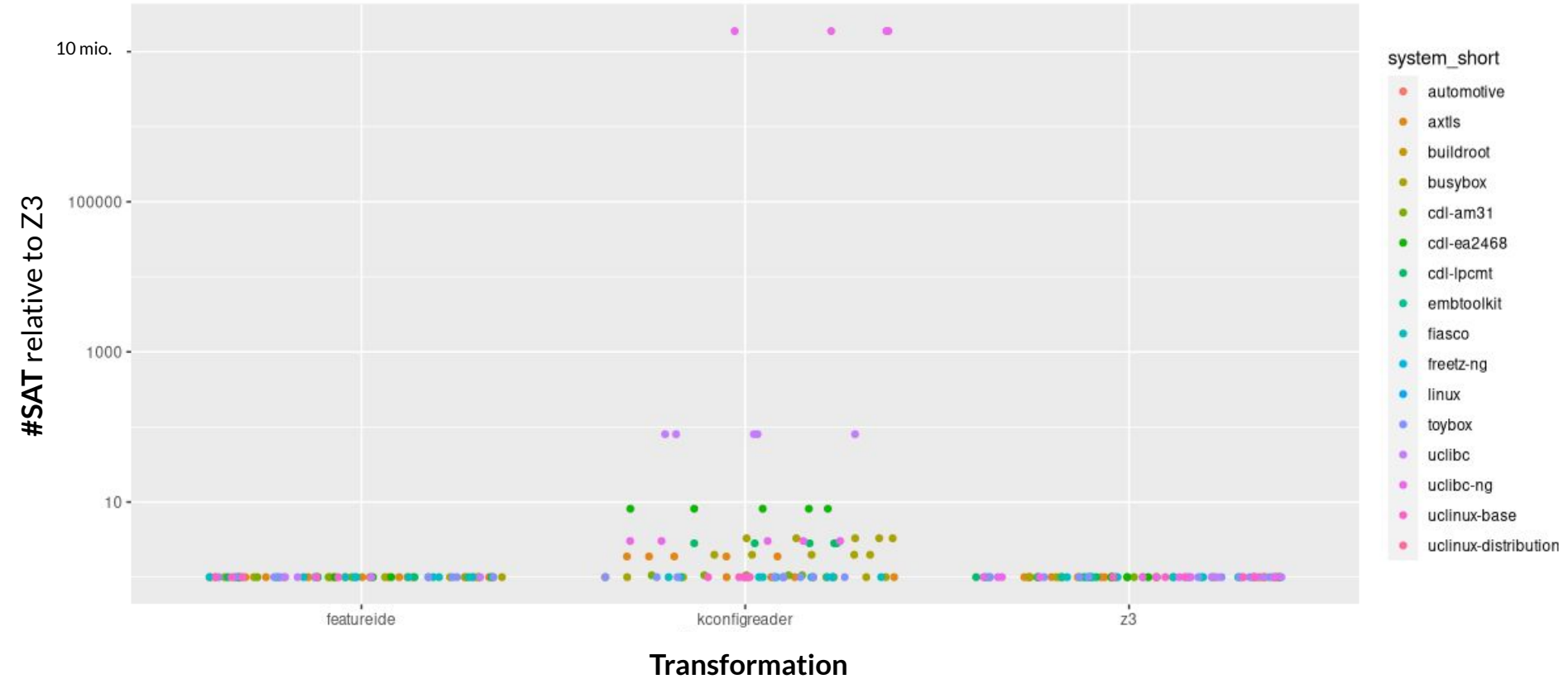
# RQ2b: Formula Size Increase



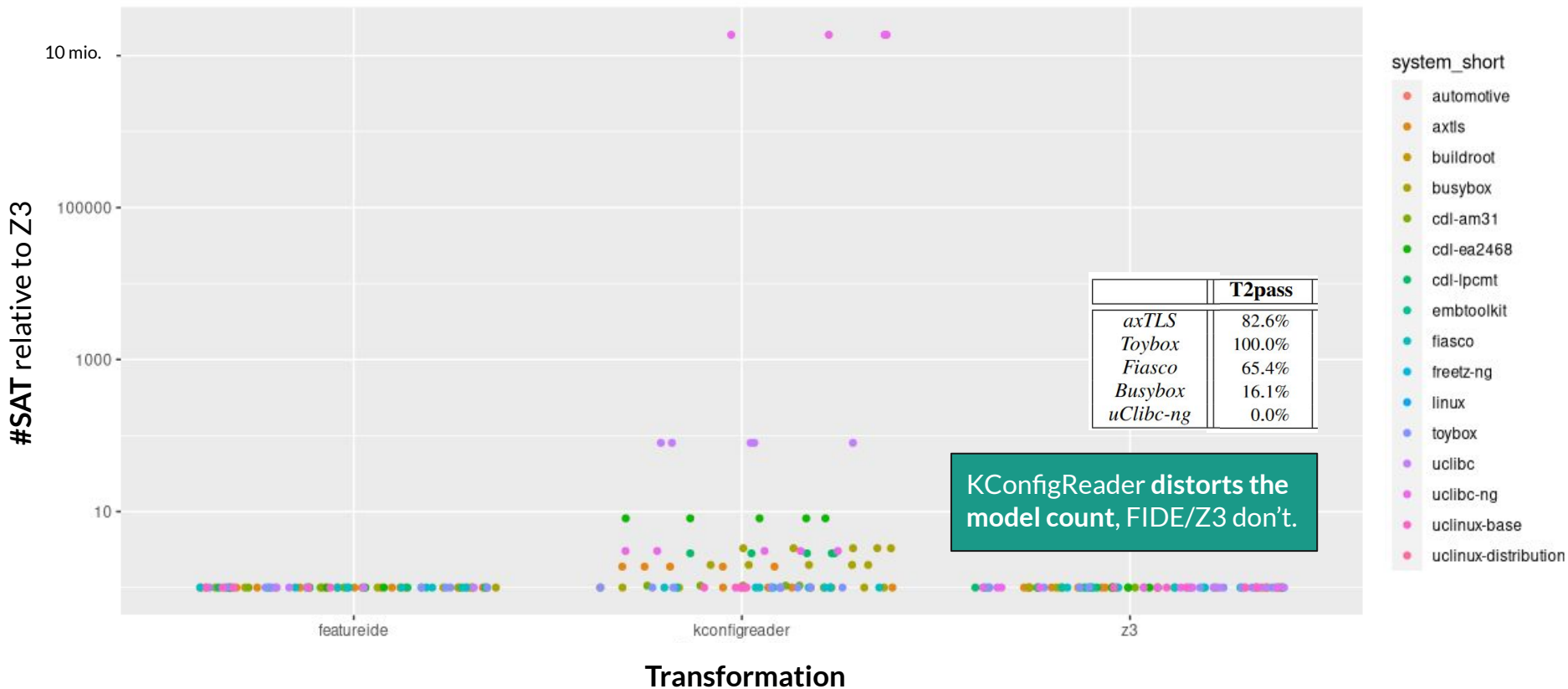
# RQ2b: Formula Size Increase



# RQ3: Correctness of #SAT



# RQ3: Correctness of #SAT



# Perspective



- **External threats to validity**
  - We only evaluated **specific implementations**  
→ cannot draw conclusions about transformations themselves
  - We chose **specific systems**/extractors/transformations/solvers
  - We do not account for **non-Boolean** (e.g., numeric) variability

# Perspective



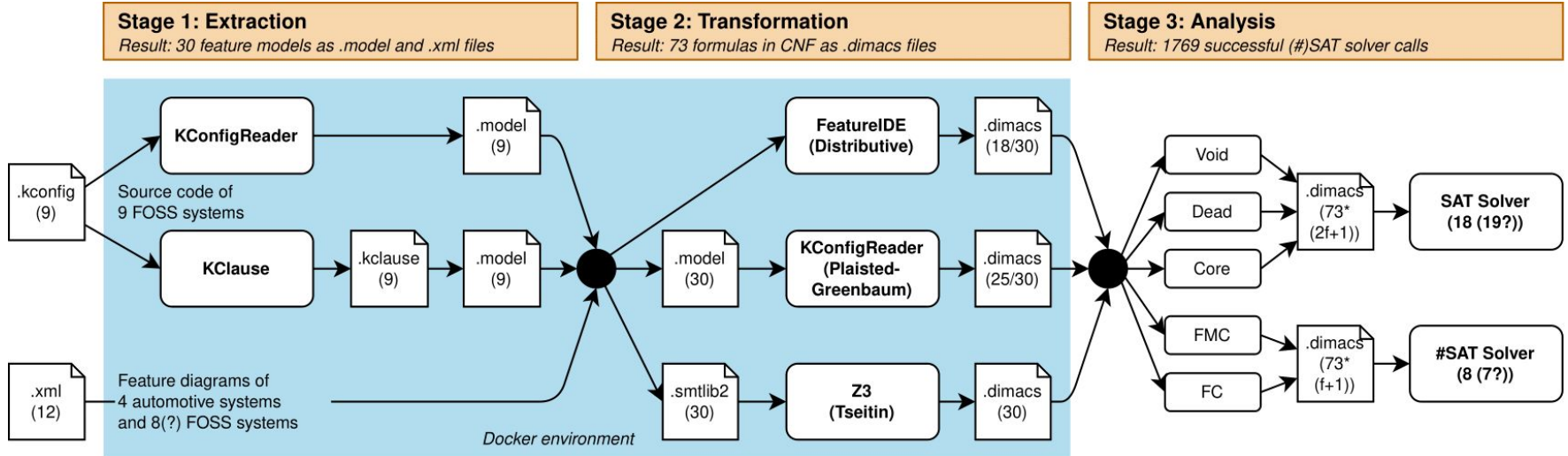
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- **Future work**
  - **Controlled, parametrized** evaluation of all three transformations by implementing them all in Z3 and KConfigReader
    - make **recommendations** for which transformation to choose

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- **Future work**
  - **Controlled, parametrized** evaluation of all three transformations by implementing them all in Z3 and KConfigReader
    - make **recommendations** for which transformation to choose
- **Artifact:** Reproducible model extraction pipeline (VM-based, Docker-based WIP)  
<https://github.com/ekuiter/feature-model-repository-pipeline>

# Conclusion



<https://github.com/ekuiter/feature-model-repository-pipeline>







# Results: Differences between Solvers

