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# 4. Feature Modeling – Handout

Software Product Lines | Elias Kuiter, Thomas Thüm, Timo Kehrer | April 24, 2023

# 4. Feature Modeling

## 4a. Feature Models and Configurations

Recap: Software Product Lines

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Specifying Valid Configurations

- Natural Language

- Configuration Map

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## 4b. Transforming Feature Models

## 4c. Analyzing Feature Models

# Recap: Software Product Lines [Lecture 1]

## Software Product Line

[Northrop et al. 2012, p. 5]

“A **software product line** is

- a set of software-intensive systems
- that share a common, managed set of features
- satisfying the specific needs of a particular market segment or mission
- and that are developed from a common set of core assets in a prescribed way.”

[Software Engineering Institute, Carnegie Mellon University]

## Product

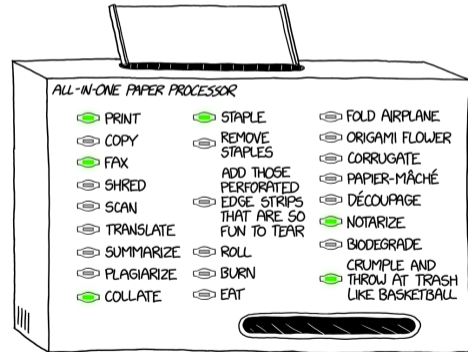
[Apel et al. 2013, p. 19]

“A **product of a product line** is specified by a valid feature selection (a subset of the features of the product line). A feature selection is **valid** if and only if it fulfills all feature dependencies.”

## Feature

[Apel et al. 2013, p. 18]

“A **feature** is a characteristic or end-user-visible behavior of a software system.”



# Features Have Dependencies

## Ordering a Waffle ...



## ... with Sugar



## ... with Cherries



## This is Nice, But ...

- plate and sugar seem to always be included, a fork is only included for some orders  
⇒ limitations seem **arbitrary**
- children get special treatment  
⇒ order process is **unfair**
- what exactly am I paying for?  
⇒ investments are **unclear**

## In This Lecture

1. how to **model and configure** features and their dependencies?
2. how to **store and communicate**?
3. how to **analyze and understand**?



# Specifying Valid Configurations

## Configuration

- a **configuration** over a set of features  $F$  selects and deselects features in  $F$
- formally: a pair  $(S, D)$  such that  $S, D \subseteq F$  and  $S, D$  are disjoint ( $S \cap D = \emptyset$ )
- is **complete** if all features are covered ( $S \cup D = F$ ) and **partial** otherwise
- a complete configuration is **valid** if it “makes sense” in the domain and **invalid** otherwise
- we often abbreviate complete configurations with  $S \equiv (S, F \setminus S)$

Feature set  $F = \{\text{ConfigDB}, \text{Get}, \text{Put}, \text{Delete}, \text{Transactions}, \text{Windows}, \text{Linux}\}$

Examples for **complete** configurations:

- **valid** (read-only database on Windows):  
 $(\{C, G, W\}, \{P, D, T, L\})$
- **valid** (fully functional database on Linux):  
 $(\{C, G, P, D, T, L\}, \{W\})$
- **invalid** ( $\nexists$  no operating system):  
 $(\{C, G\}, \{P, D, T, W, L\})$
- **invalid** (transactions  $\nexists$  read-only database):  
 $(\{C, G, T, L\}, \{P, D, W\})$

Examples for **partial** configurations:

$(\{C, G\}, \{P, D\}), (\emptyset, \emptyset)$

# Specifying Valid Configurations – Natural Language

## Valid Configuration

A complete configuration over  $F$  is valid if it “makes sense” in the domain.  $\rightsquigarrow$  “**makes sense**”?

## Natural Language

- informal description of relationships between features in  $F$
  - a complete configuration  $S$  is **valid** if it conforms to the description
- + succinct
- sometimes ambiguous
  - not machine-readable

“A **configurable database** has an API that allows for at least one of the request types **Get**, **Put**, or **Delete**. Optionally, the database can support **transactions**, provided that the API allows for Put or Delete requests. Also, the database targets a supported operating system, which is either **Windows** or **Linux**.”

# Specifying Valid Configurations – Configuration Map

## Valid Configuration

A complete configuration over  $F$  is valid if it “makes sense” in the domain.  $\rightsquigarrow$  “**makes sense**”?

## Configuration Map

- a **configuration map** over  $F$  is a set of complete configurations  $M \subseteq \mathcal{P}(F)$
  - a complete configuration  $S$  is **valid** if it occurs in the configuration map ( $S \in M$ )
  - also known as product map
- + precise
- not human-readable
  - redundant, explodes in size ( $0 \leq |M| \leq 2^{|F|}$ )

Feature set  $F = \{\text{ConfigDB}, \text{Get}, \text{Put}, \text{Delete}, \text{Transactions}, \text{Windows}, \text{Linux}\}$

Configuration map:

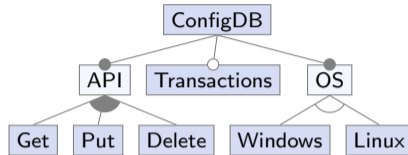
$\{C, G, W\}$	$\{C, G, L\}$
$\{C, P, W\}$	$\{C, P, L\}$
$\{C, G, P, W\}$	$\{C, G, P, L\}$
$\{C, D, W\}$	$\{C, D, L\}$
$\{C, G, D, W\}$	$\{C, G, D, L\}$
$\{C, P, D, W\}$	$\{C, P, D, L\}$
$\{C, G, P, D, W\}$	$\{C, G, P, D, L\}$
$\{C, P, T, W\}$	$\{C, P, T, L\}$
$\{C, G, P, T, W\}$	$\{C, G, P, T, L\}$
$\{C, D, T, W\}$	$\{C, D, T, L\}$
$\{C, G, D, T, W\}$	$\{C, G, D, T, L\}$
$\{C, P, D, T, W\}$	$\{C, P, D, T, L\}$
$\{C, G, P, D, T, W\}$	$\{C, G, P, D, T, L\}$

# Specifying Valid Configurations – Configuration Map in Excel

	A	B	C	D	E	F	G
1	ConfigDB	Get	Put	Delete	Transactions	Windows	Linux
2	x	x				x	
3	x		x			x	
4	x	x	x			x	
5	x			x		x	
6	x	x		x		x	
7	x		x	x		x	
8	x	x	x	x		x	
9	x		x		x	x	
10	x	x	x		x	x	
11	x			x	x	x	
12	x	x		x	x	x	
13	x		x	x	x	x	
14	x	x	x	x	x	x	
15	x	x					x
16	x		x				x
17	x	x	x				x
18	x			x			x
19	x	x		x			x
20	x		x	x			x
21	x	x	x	x			x
22	x		x		x		x
23	x	x	x		x		x
24	x			x	x		x
25	x	x		x	x		x
26	x		x	x	x		x
27	x	x	x	x	x		x

Can we do better?

# Feature Models – Syntax [Apel et al. 2013; Kang et al. 1990, pp. 63–72; Batory 2005]



*Transactions* → *Put* ∨ *Delete*

## Legend:

- Abstract Feature
- Concrete Feature
- Mandatory
- Optional
- ▲ Or Group
- △ Alternative Group

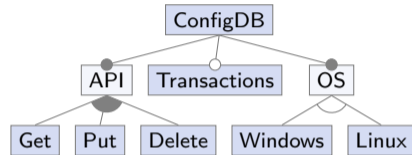
## Feature Model

- hierarchy of features
- dependencies between features modeled by tree and cross-tree constraints
- **tree constraints**: defined by the hierarchy
- **cross-tree constraints**: propositional formulas over features
- **abstract features** are used to group other features
- **concrete features** have an implementation
- also known as feature diagram or feature tree
- notation for **optional/mandatory features** and **or/alternative groups**

# Feature Models – Semantics [Apel et al. 2013; Batory 2005]

## Tree Constraints

- the **root feature** is always required
- each feature requires its parent (aka. **parent-child-relationship**)
- an **optional feature** can be (de-)selected freely when its parent is selected
- a **mandatory feature** is required by its parent
- **or group**: at least one child feature must be selected when the parent is selected
- **alternative group**: exactly one child feature must be selected when the parent is selected

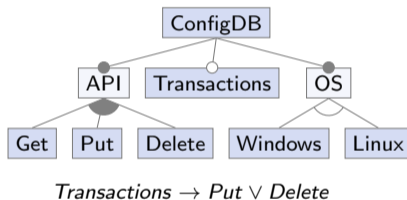


*Transactions*  $\rightarrow$  *Put*  $\vee$  *Delete*

## Cross-Tree Constraints

- a list of **propositional formulas** expressing further dependencies between features
- each cross-tree constraint must be satisfied

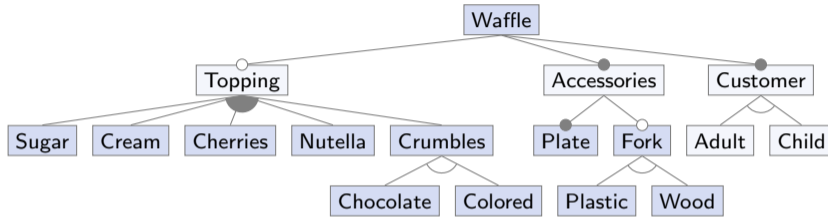
# Feature Models – Examples



## Is This a Valid Configuration?

- **valid** (read-only database on Windows):  
( $\{C, A, G, O, W\}, \{P, D, T, L\}$ )
- **valid** (fully functional database on Linux):  
( $\{C, A, G, P, D, T, O, L\}, \{W\}$ )
- **invalid** ( $\not\leq$  no operating system):  
( $\{C, A, G\}, \{P, D, T, O, W, L\}$ )
- **invalid** (transactions  $\not\leq$  read-only database):  
( $\{C, A, G, T, O, L\}, \{P, D, W\}$ )

# Feature Models – Examples



*Sugar*  
*Cherries*  $\rightarrow$  *Sugar*  $\wedge$  *Fork*  
*Nutella*  $\vee$  *Crumbles*  $\rightarrow$  *Child*  
*Fork*  $\rightarrow$  *Adult*

- abstract and concrete features can be assigned arbitrarily
- groups can be used anywhere
- directly below groups, no optional or mandatory markers are allowed



# Pros and Cons

## Pro: Making Tacit Knowledge Explicit

"I think the best [about feature modeling] is you can see relationships, to actually know what configurations are allowed and what are not allowed. That was also not so easy to express in the past [...] This is from the developer's point of view. But it's also [...] important, because before we noticed that **the same functionality was implemented twice** within the same project, basically they haven't realized that. They implemented the same features." – Interview with Practitioners [Berger et al. 2014]

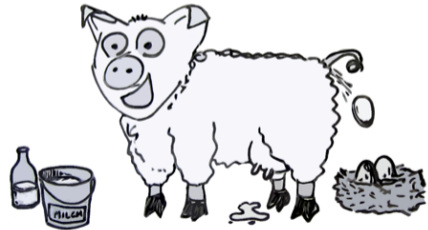
## Pro: Tool Support



, Gears, pure::variants, ...

## Con: Challenges

- **domain scoping**: which features?
- **feature interactions**: which dependencies?
- requires infrastructure, consulting, and training



# Feature Models and Configurations – Summary

## Lessons Learned

- features, dependencies between features, and configurations
- feature models: abstract and concrete features, tree and cross-tree constraints
- tree constraints: optional, mandatory, or group, alternative group

## Further Reading

- Apel et al. 2013, Section 2.3, pp. 26–39 — introduction to feature modeling
- Thorsten Berger et al. (2013): A Survey of Variability Modeling in Industrial Practice
- Damir Nešić et al. (2019): Principles of Feature Modeling

## Practice

1. sketch a feature model with features  $A, B, C, D, E, F$  that has exactly those 5 valid configurations (pen and paper preferred):

$\{A, B\}$                        $\{A, C, E\}$                        $\{A, C, E, F\}$   
 $\{A, B, D\}$                        $\{A, C, F\}$

2. discuss in groups whether your feature models are syntactically correct and specify exactly the above configurations

# 4. Feature Modeling

## 4a. Feature Models and Configurations

### 4b. Transforming Feature Models

Representations and Transformations

UVL, the Universal Variability Language

Propositional Formulas

CNF as a Universal Formula Language

Summary

## 4c. Analyzing Feature Models

# Representations and Transformations

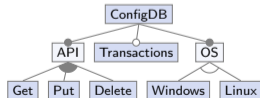
## Natural Language

"A **configurable database** has an API that allows for at least one of the request types **Get**, **Put**, or **Delete**. Optionally, the database can support **transactions**, provided that the API allows for Put or Delete requests. Also, the database targets a supported operating system, which is either **Windows** or **Linux**."

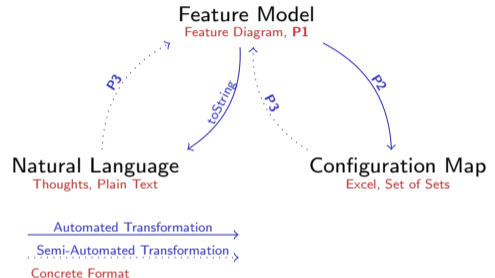
## Configuration Map

{ C, G, W }	{ C, G, L }
⋮	⋮
{ C, G, P, D, T, W }	{ C, G, P, D, T, L }

## Feature Diagram (Graphical Feature Model)



*Transactions* → *Put* ∨ *Delete*



## Problems

- P1 How to express feature models **textually**?
- P2 How to (a) validate configurations and (b) get all valid configurations **automatically**?
- P3 (How to reverse engineer feature models?)

# UVL, the Universal Variability Language [UVL]

## features

ConfigDB

mandatory

API {abstract}

or

Get

Put

Delete

optional

Transactions

mandatory

OS {abstract}

alternative

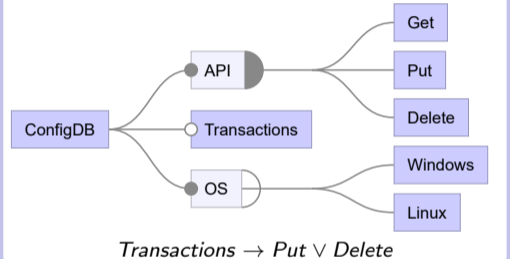
Windows

Linux

## constraints

Transactions => Put | Delete

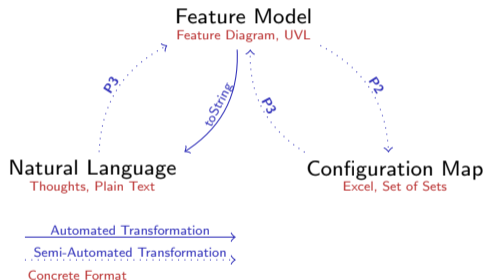
## A Feature Model “Sideways”



## Universal Variability Language (UVL)

- textual language for feature modeling
- adds advanced modeling constructs (e.g., attributes, cardinalities, submodels, ...)

# Representations and Transformations



## Problems

- P1 How to express feature models **textually**?
- P2 How to (a) validate configurations and (b) get all valid configurations **automatically**?
- P3 (How to reverse engineer feature models?)

## Solutions

- P1 Universal Variability Language  $\Rightarrow$  **Syntax**
- P2 **Semantics**?
- P3 –

# Propositional Formulas – Recap

## Syntax of Propositional Formulas

Inductive definition of **propositional formulas**:

- the **Boolean truth values**  $\top$ ,  $\perp$
- any **Boolean variable**  $X$
- any **negation**  $\neg\phi$  of a formula  $\phi$
- any **conjunction**  $(\phi \wedge \psi)$  of formulas  $\phi$  and  $\psi$
- any **disjunction**  $(\phi \vee \psi)$ , **implication**  $(\phi \rightarrow \psi)$ , or **biimplication**  $(\phi \leftrightarrow \psi)$

## Informal Semantics of Propositional Formulas

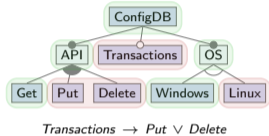
$\top$	}	means	}	“true” (or <b>tautology</b> )
$\perp$				“false” (or <b>contradiction</b> )
$\neg\phi$				“not $\phi$ ”
$\phi \wedge \psi$				“ $\phi$ and $\psi$ ”
$\phi \vee \psi$				“ $\phi$ or $\psi$ ” (inclusive or!)
$\phi \rightarrow \psi$				“if $\phi$ , then $\psi$ ” (and else?)
$\phi \leftrightarrow \psi$	“ $\phi$ if and only if $\psi$ ”			

**Operator Precedence:**  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$

$Transactions \rightarrow (Put \vee Delete)$   
 $\equiv Transactions \rightarrow Put \vee Delete$   
 $\not\equiv (Transactions \rightarrow Put) \vee Delete$

# Propositional Formulas – Example

## A Feature Model $FM$ ...



## ... as a Propositional Formula $\Phi(FM)$

$$\begin{aligned} \Phi(FM) = & ConfigDB \\ & \wedge (API \leftrightarrow ConfigDB) \\ & \wedge (Transactions \rightarrow ConfigDB) \\ & \wedge (OS \leftrightarrow ConfigDB) \\ & \wedge (Get \vee Put \vee Delete \leftrightarrow API) \\ & \wedge (Windows \vee Linux \leftrightarrow OS) \\ & \wedge \neg(Windows \wedge Linux) \\ & \wedge (Transactions \rightarrow Put \vee Delete) \end{aligned}$$

## Is This a Valid Configuration?

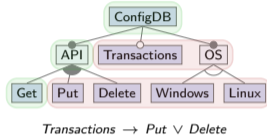
$$\begin{aligned} & \Phi(FM)(\{C, A, G, O, W\}) \\ \equiv & \Phi(FM)(\{\{C, A, G, O, W\}, \{P, D, T, L\}\}) \\ \equiv & C \wedge (A \leftrightarrow C) \wedge (T \rightarrow C) \wedge (O \leftrightarrow C) \\ & \wedge (G \vee P \vee D \leftrightarrow A) \wedge (W \vee L \leftrightarrow O) \\ & \wedge \neg(W \wedge L) \wedge (T \rightarrow P \vee D) \\ \equiv & T \wedge (T \leftrightarrow T) \wedge (\perp \rightarrow T) \wedge (T \leftrightarrow T) \\ & \wedge (T \vee \perp \vee \perp \leftrightarrow T) \wedge (T \vee \perp \leftrightarrow T) \\ & \wedge \neg(T \wedge \perp) \wedge (\perp \rightarrow \perp \vee \perp) \\ \equiv & T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge T \\ \equiv & T \end{aligned}$$

↪ **configuration is valid**  
(read-only database on Windows)



# Propositional Formulas – Example

## A Feature Model $FM$ ...



## ... as a Propositional Formula $\Phi(FM)$

$$\begin{aligned}\Phi(FM) = & ConfigDB \\ & \wedge (API \leftrightarrow ConfigDB) \\ & \wedge (Transactions \rightarrow ConfigDB) \\ & \wedge (OS \leftrightarrow ConfigDB) \\ & \wedge (Get \vee Put \vee Delete \leftrightarrow API) \\ & \wedge (Windows \vee Linux \leftrightarrow OS) \\ & \wedge \neg(Windows \wedge Linux) \\ & \wedge (Transactions \rightarrow Put \vee Delete)\end{aligned}$$

## Is This a Valid Configuration?

$$\begin{aligned}\Phi(FM)(\{C, A, G\}) & \\ \equiv \Phi(FM)(\{\{C, A, G\}, \{P, D, T, O, W, L\}\}) & \\ \equiv C \wedge (A \leftrightarrow C) \wedge (T \rightarrow C) \wedge (O \leftrightarrow C) & \\ \wedge (G \vee P \vee D \leftrightarrow A) \wedge (W \vee L \leftrightarrow O) & \\ \wedge \neg(W \wedge L) \wedge (T \rightarrow P \vee D) & \\ \equiv T \wedge (T \leftrightarrow T) \wedge (\perp \rightarrow T) \wedge (\perp \leftrightarrow T) & \\ \wedge (T \vee \perp \vee \perp \leftrightarrow T) \wedge (\perp \vee \perp \leftrightarrow \perp) & \\ \wedge \neg(\perp \wedge \perp) \wedge (\perp \rightarrow \perp \vee \perp) & \\ \equiv T \wedge T \wedge T \wedge \perp \wedge T \wedge T \wedge T \wedge T & \\ \equiv \perp & \end{aligned}$$

⚡ **configuration is invalid**  
(⚡ no operating system)

# Propositional Formulas – Algorithm

## Algorithm: Translate $FM$ Into $\Phi(FM)$

- translate each tree constraint
  - Root feature:**  $R$  is always required
  - Optional feature:**  $C$  requires  $P$
  - Mandatory feature:**  
Optional +  $P$  requires  $C$
  - Or group:**  
Optional +  $P$  requires at least one  $C_i$
  - Alternative group:**  
Optional +  $P$  requires exactly one  $C_i$
- conjoin translated tree constraints  
 $\Phi(TC) \leftarrow \bigwedge_{tc \in TC} \Phi(tc)$
- conjoin **cross-tree constraints**  
 $\Phi(CTC) \leftarrow \bigwedge_{ctc \in CTC} ctc$
- $\Phi(FM) \leftarrow \Phi(TC) \wedge \Phi(CTC)$

$$\Phi(\text{Root}) = \text{Root}$$

$$\Phi\left(\begin{array}{c} P \\ | \\ \circ \\ | \\ C \end{array}\right) = C \rightarrow P$$

$$\Phi\left(\begin{array}{c} P \\ | \\ \bullet \\ | \\ C \end{array}\right) = C \leftrightarrow P$$

$$\Phi\left(\begin{array}{c} P \\ | \\ \bullet \\ / \quad \backslash \\ C_1 \quad \dots \quad C_n \end{array}\right) = \bigvee_{1 \leq i \leq n} C_i \leftrightarrow P$$

$$\Phi\left(\begin{array}{c} P \\ | \\ \circ \\ / \quad \backslash \\ C_1 \quad \dots \quad C_n \end{array}\right) = \bigvee_{1 \leq i \leq n} C_i \leftrightarrow P$$

$$\wedge \bigwedge_{1 \leq i < j \leq n} \neg(C_i \wedge C_j)$$

# CNF as a Universal Formula Language

## Recap: Conjunctive Normal Form

- a **literal**  $L$  is a variable  $X$  or its negation  $\neg X$
- a **clause**  $C$  is a disjunction of literals  $\bigvee_j L_j$
- a **conjunctive normal form (CNF)** is a conjunction of clauses  $\bigwedge_i C_i = \bigwedge_i \bigvee_j L_j$
- intuitively: a set of “rules” to be satisfied
- any formula  $\phi$  can be transformed into a CNF  $\phi'$  that is logically equivalent ( $\phi \Leftrightarrow \phi'$ )

## Recap: Laws of Propositional Logic

- implication:  $\phi \rightarrow \psi \Leftrightarrow \neg\phi \vee \psi$
- biimplication:  $\phi \leftrightarrow \psi \Leftrightarrow (\neg\phi \vee \psi) \wedge (\neg\psi \vee \phi)$
- De Morgan's laws:  $\neg(\phi \wedge \psi) \Leftrightarrow \neg\phi \vee \neg\psi$
- distributivity:  $(\phi \wedge \psi) \vee \chi \Leftrightarrow (\phi \vee \chi) \wedge (\psi \vee \chi)$

## Transforming Part of $\Phi(FM)$ into $CNF(\Phi(FM))$

$C$	$C$
$\wedge (T \rightarrow C)$	$\wedge (\neg T \vee C)$
$\wedge (O \leftrightarrow C)$	$\wedge (\neg O \vee C) \wedge (\neg C \vee O)$
$\wedge (W \vee L \leftrightarrow O)$	$\wedge (\neg(W \vee L) \vee O)$
$\wedge \neg(W \wedge L)$	$\wedge (\neg O \vee W \vee L)$
	$\wedge \neg(W \wedge L)$

$C$	$C$
$\wedge (\neg T \vee C)$	$\wedge (\neg T \vee C)$
$\wedge (\neg O \vee C) \wedge (\neg C \vee O)$	$\wedge (\neg O \vee C) \wedge (\neg C \vee O)$
$\wedge ((\neg W \wedge \neg L) \vee O)$	$\wedge (\neg W \vee O) \wedge (\neg L \vee O)$
$\wedge (\neg O \vee W \vee L)$	$\wedge (\neg O \vee W \vee L)$
$\wedge (\neg W \vee \neg L)$	$\wedge (\neg W \vee \neg L)$

# CNF as a Universal Formula Language – DIMACS

$C$   
 $\wedge (\neg T \vee C)$   
 $\wedge (\neg O \vee C) \wedge (\neg C \vee O)$   
 $\wedge (\neg W \vee O) \wedge (\neg L \vee O)$   
 $\wedge (\neg O \vee W \vee L)$   
 $\wedge (\neg W \vee \neg L)$

```
c 1 C
c 2 T
c 3 O
c 4 W
c 5 L
p cnf 5 6
1 0
-2 1 0
-3 1 0 -1 3 0
-4 3 0 -5 3 0
-3 4 5 0
-4 5 0
```

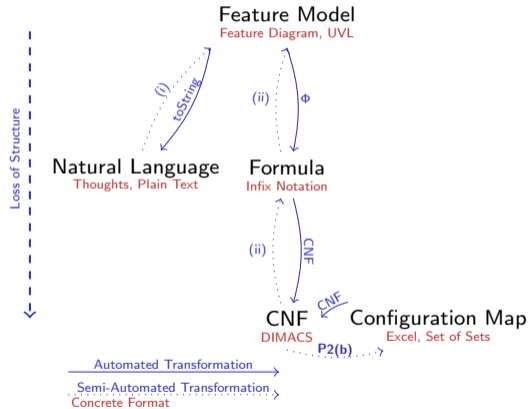
## DIMACS Format

[DIMACS 1993]

- de facto industry standard for storing CNF
- machine-readable, automated analyses, ...
- comments start with c ...
- problem line:  
p cnf #variables #clauses
- clause  $\bigvee_i L_i$  translates to  $L_1 \dots L_n 0$
- intuitively:

$\left. \begin{array}{l} 0 \\ - \\ - \end{array} \right\}$  means  $\left\{ \begin{array}{l} \wedge \\ \vee \\ \neg \end{array} \right.$

# Representations and Transformations



## Problems

- P1 How to express feature models **textually**?
- P2 How to
  - (a) validate configurations and
  - (b) get all valid configurations **automatically**?
- P3 (How to reverse engineer feature models?)

## Solutions

- P1 Universal Variability Language  $\Rightarrow$  **Syntax**
- P2 Propositional Formulas  $\Rightarrow$  **Semantics**
  - (a) evaluate feature-model formula
  - (b) Lecture 4c
- P3 (i) e.g., Bakar et al. 2015
  - (ii) e.g., Czarnecki and Wasowski 2007

# Transforming Feature Models – Summary

## Lessons Learned

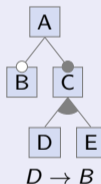
- to understand large configuration spaces, we need formal semantics and machine-readable representations
- propositional formulas satisfy many (though not all) needs for such a representation

## Further Reading

- Don Batory (2005): Feature Models, Grammars, and Propositional Formulas
- UVL — official website for the Universal Variability Language with examples, grammar, literature pointers
- Alexander Knüppel et al. (2017): Is There a Mismatch Between Real-World Feature Models and Product-Line Research?

## Practice

1. translate the following feature diagram into a propositional formula:



2. check formulas of your colleagues

# 4. Feature Modeling

## 4a. Feature Models and Configurations

## 4b. Transforming Feature Models

## 4c. Analyzing Feature Models

Configurators in the Wild

Automated Analysis of Feature Models

SAT, #SAT, and AllSAT

Consistency, Cardinality, and Enumeration

- Feature Model

- Features

- Partial Configurations

Automated Analyses in FeatureIDE


Summary

FAQ

# Configurators in the Wild – Cars


## Configuring a car ...

Customise your wheel




Finish

15" black alloy wheels (5-double-spoke)



Cap colour

Deep Blue Buzz small centre cap



... is complicated!

1. the default configuration
2. we want **black cap**
3. we want **white wheels**
4. black cap unavailable, red selected automatically (not blue!)
5. fine, back to **black wheels**
6. and, back to **black cap**
7. **confirm selection** to continue with selection of the car color
8. we want a **red car**
9. popup dialog: black wheels unavailable (no automatic selection! preview of unavailable wheels!)

what now? back to Step 3?



# Configurators in the Wild – Cars

## Configuring a Car ...

Customize your wheel



Finish

15" black alloy wheels (5-double-spoke)



Cap colour

Black small centre cap



10. cancel the dialog

## with a Weird Price

**Your AYGO**

5 Door Hatchback x-play 1.0 Petrol (69 hp)  
Automatic (Front Wheel Drive - FWD)

Retail price	£11,610.00
Red Pop	
Gleam fabric	
15" black alloy wheels (5-double-spoke)	£500.00
Black small centre cap	
<b>Total</b>	<b>£12,610.00</b>

Summary and save >

## with 8 Wheels!

**Your customisation** [Change customisation](#)

**Colours & wheels**

Red Pop

15" colour-customisable alloy wheels (5 double-spoke)	£500.00
15" black machined-face alloy wheels (5-double-spoke)	£500.00
Black small centre cap	

**Interior trim**

Gleam fabric

**Total**

£12,610.00

- canceling the dialog was not considered and lead to an invalid state (i.e., configuration)
- humans check these configurations, but some errors are only found during production
- many constraints: appear arbitrary, not explained

# Configurators in the Wild – Cars

## Configuring a German Car

[example from Lecture 1]

Configuration Assistant.

► Show instructions

Your most recent action requires your configuration to be adjusted.

### Your choice

	Price
+ Enhanced Bluetooth telephone with USB & Voice Control	+ £ 350.00

### Adding

+ BMW Navigation	£ 0.00
------------------	--------

### Removing

- Enhanced Bluetooth with wireless charging	- £ 395.00
- Navigation system Professional	£ 0.00
- WiFi hotspot preparation	£ 0.00
- Media package - Professional	- £ 900.00
- Online Entertainment	£ 0.00
- Microsoft Office 365	- £ 150.00

Why does the telephone conflict with Microsoft Office?

# Configurators in the Wild – Notebooks

## Configuring a Notebook

### Display

14.0" FHD (1920x1080), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch

SELECTED

14.0" WQHD (2560x1440), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch

+ £91.20

14.0" HDR WQHD (2560x1440) with Dolby Vision™, LED backlight, 500 nits, 16:9 aspect ratio, 1500:1 contrast ratio, 100% gamut, 170° viewing angle, IPS, Touch  
Please note this display is only available with WWAN/mobile broadband.

+ £159.60

### Display

14.0" FHD (1920x1080), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch

Invalid configuration

OK

14.0" WQHD (2560x1440), LED backlight, 300 nits, 16:9 aspect ratio, 700:1 contrast ratio, 72% gamut, 170° viewing angle, IPS, Touch

+ £91.20

14.0" HDR WQHD (2560x1440) with Dolby Vision™, LED backlight, 500 nits, 16:9 aspect ratio, 1500:1 contrast ratio, 100% gamut, 170° viewing angle, IPS, Touch  
Please note this display is only available with WWAN/mobile broadband.

LOADING...

SELECTED

can detect mistakes, but provides no explanations or fixes

# Configurators in the Wild – Notebooks

## Still Configuring a Notebook

### Microsoft Productivity Software

None	SELECTED
Microsoft Office 365 Home	+ £59.99
Microsoft Office 365 Personal	+ £79.99
Microsoft Office Home and Student 2016	+ £119.99
Microsoft Office Home and Business 2016	+ £229.99
Adobe Acrobat Standard 2017 and Microsoft Office Home and Business 2016	+ £399.60
Adobe Acrobat Standard 2017 and Microsoft Office	+ £628.80

### Microsoft Office Not Included

For your best experience, Lenovo recommends selecting a Microsoft Office product with your new purchase.



#### NEED HELP DECIDING?

Roll over each product to get specific details on each Office product

allows some feature combinations and not others, prices are opaque

# Automated Analysis of Feature Models

## Open Questions

- How do such configurators work?
- How to avoid inconsistencies?
- How to provide explanations and fixes?
- How to get all valid configurations automatically? (P2(b))

## Automated Analysis of Feature Models

- up until now: **creation** and **transformation** of feature models
- now: **analysis** of feature models to improve our understanding of a configuration space
- for brevity: product = valid configuration

## Asking Questions About Feature Models

- Is a given configuration valid?
- Is there any product at all?  
How many/which products are there?
- Is a given feature (de-)selectable at all?  
How many/which products include it?
- Is a given partial configuration consistent?  
How many/which products include it?
- (Which features always occur together?)
- (Is a given constraint redundant?)
- (How do two feature model versions differ?)
- (Why is ...? How to fix ...?)

# SAT, #SAT, and AIIISAT

## Recap: Boolean Satisfiability Problem (SAT)

- **decision problem**: is there any assignment  $A$  that satisfies a given formula?
- formally:  $SAT(\phi) \Leftrightarrow \exists A: \phi(A) = \top$
- known to be **NP-complete**:  
in theory, difficult to solve if  $P \neq NP$ ;  
in practice, solvability depends on domain
- answered by **SAT solvers**:  
highly-optimized, off-the-shelf tools;  
competitively developed over several decades;  
takes a CNF in DIMACS format as input

- $X \rightarrow Y$  is satisfiable
- $X \vee \neg X$  is satisfiable (even a tautology)
- $X \wedge \neg X$  is not satisfiable (why?)

## Sharp Satisfiability Problem (#SAT)

- **counting problem**: how many assignments satisfy a given formula?
- $\#SAT(\phi) = |\{A \mid \phi(A) = \top\}|$
- known to be **#P-complete**:  
at least as hard as SAT (probably harder)
- answered by **#SAT solvers**

## Solution Enumeration Problem (AIIISAT)

- **enumeration problem**: which assignments satisfy a given formula?
- $AIIISAT(\phi) = \{A \mid \phi(A) = \top\}$
- at least as hard as #SAT (probably harder)
- answered by **AIIISAT solvers**

# Automated Analysis of Feature Models

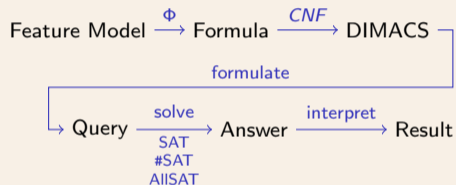
## Asking Questions About Feature Models

- Is a given configuration valid?  $\Rightarrow$  **evaluate**
- **Is** there any valid configuration at all?  
**How many/which** valid configurations are there?
- **Is** a given feature (de-)selectable at all?  
**How many/which** valid configurations include it?
- **Is** a given partial configuration consistent?  
**How many/which** valid configurations include it?

## Choosing the Right Solver

- “is?”  $\approx$  SAT solver query
- “how many?”  $\approx$  #SAT solver query
- “which?”  $\approx$  AIIISAT solver query

## Typical Feature-Model Analysis Process



for brevity, we assume that  $\phi = CNF(\Phi(FM))$   
for a given feature model  $FM$

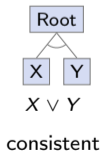
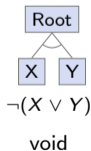
# Consistency, Cardinality, and Enumeration – Feature Model

## Consistency of Feature Models (SAT)

### Void/Consistent Feature Model

- are there grave modeling errors?
- is it possible to configure any product at all?

$$\phi \xrightarrow{\text{SAT}} \perp/\top \begin{cases} \perp & \text{FM is void} \\ \top & \text{FM is consistent} \end{cases}$$



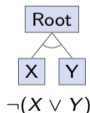
## Cardinality of Feature Models (#SAT)

### How Many Products Are There?

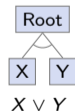
$$\phi \xrightarrow{\#SAT} |C|$$

### Variability Factor: Share of Products?

$$\phi \xrightarrow{\#SAT} |C| \longrightarrow \frac{|C|}{2^{|F|}}$$



0 products, VF 0



2 products, VF  $\frac{2}{8}$



# Consistency, Cardinality, and Enumeration – Feature Model

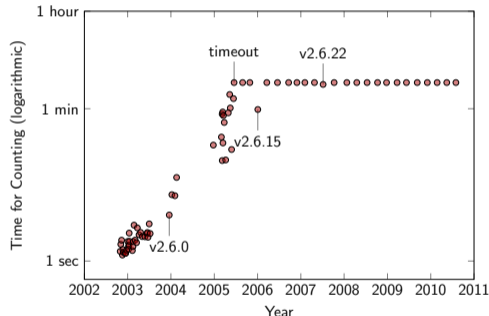
## Feasibility of SAT-Based Analyses

### Is SAT-Based Analysis “Easy”?

- provocative claim: “SAT-based analysis of feature models is easy” [Mendonca et al. 2009]
- easy = performs much better than expected (although NP-complete)
- easy = fast?
  - what about formulating the query? (e.g., CNF transformation)
  - what about many queries? (e.g., what we discuss next)

## Feasibility of #SAT-Based Analyses

### Time to Count Products of Linux



- the solver from 2023 can solve models from 2003
- can we analyze the models from 2023 in 2043?

# Consistency, Cardinality, and Enumeration – Feature Model

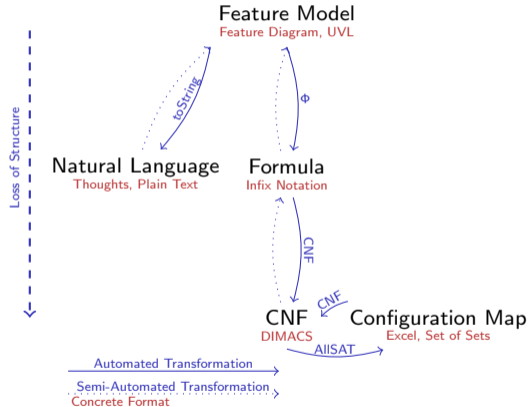
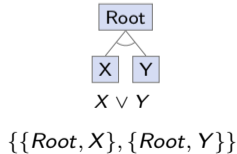
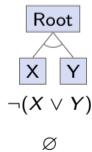
## Enumeration of Feature Models (AIIISAT)

### Which Products Are There?

- **P2(b)**: How to get all products?

$$\phi \xrightarrow{\text{AIIISAT}} C$$

AIIISAT does not scale to realistic feature models!  
50 features, configurations à 1 Byte  $\approx$  1 Petabyte



# Consistency, Cardinality, and Enumeration – Features

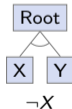
## Consistency of Features (SAT)

### Core/Dead Feature

- can a feature  $F$  be (de-)selected at all?

$$\phi \wedge F \xrightarrow{\text{SAT}} \perp/\top \begin{cases} \perp & F \text{ is dead} \\ \top & F \text{ is not dead} \end{cases}$$

$$\phi \wedge \neg F \xrightarrow{\text{SAT}} \perp/\top \begin{cases} \perp & F \text{ is core} \\ \top & F \text{ is not core} \end{cases}$$



$X$  is dead,  $Root$  and  $Y$  are core

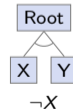
## Cardinality of Features (#SAT)

### How Many Products Include Feature $F$ ?

$$\phi \wedge F \xrightarrow{\#SAT} |\{S \in C \mid F \in S\}|$$

### Commonality: How Dead is This Feature?

$$\phi \wedge F \xrightarrow{\#SAT} |\{S \in C \mid F \in S\}| \rightarrow \frac{|\{S \in C \mid F \in S\}|}{|C|}$$



$X$ : 0 products,  $Root$  and  $Y$ : 1 products

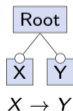
# Consistency, Cardinality, and Enumeration – Partial Configurations

## Consistency of Partial Configurations (SAT)

### Valid Partial Configuration

Is a partial configuration  $C = (S, D)$  consistent with the feature model?

$$\phi \wedge \bigwedge_{s \in S} s \wedge \bigwedge_{d \in D} \neg d \xrightarrow{\text{SAT}} \perp / \top \begin{cases} \perp \rightarrow C \times \\ \top \rightarrow C \checkmark \end{cases}$$

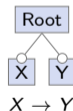


$(\{\text{Root}\}, \{X\}) \checkmark$

## Cardinality of Partial Configurations (#SAT)

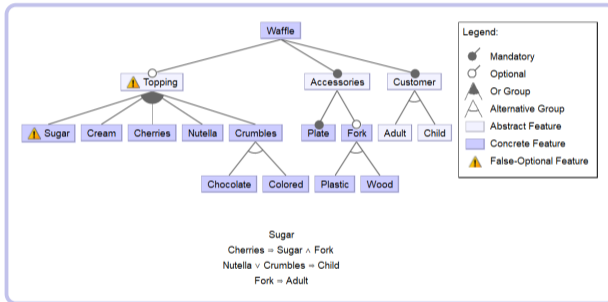
### How Many Products Are Still Configurable for $C$ ?

$$\phi \wedge \dots \xrightarrow{\#SAT} |\{(S', D') \in C \mid S \subseteq S', D \in D'\}|$$



$(\{\text{Root}\}, \{X\})$ : 2 products

# Automated Analyses in FeatureIDE – Feature-Model Editor



The screenshot shows the 'FeatureIDE Statistics' window for a project named 'waffle'. The generation tool is 'Feature Modeling'. The window displays the following statistics:

- Project Name: waffle
- Generation Tool: Feature Modeling
- Statistics of the feature model
  - Syntactical statistics
  - Semantical statistics
    - Feature model is valid (not void): true
    - Number of core features: 6
    - Number of dead features: 0
    - Number of false-optional features: 2
    - Number of atomic sets: (expand to calculate)
    - Number of configurations: (double-click to calculate)
    - Number of program variants: (double-click to calculate)

# Automated Analyses in FeatureIDE – Configuration Editor

The screenshot displays the FeatureIDE Configuration Editor. On the left, a tree view shows a configuration for 'Waffle' with 22 possible configurations. On the right, a similar tree view shows a configuration with 4 possible configurations. A tooltip is visible over the 'Fork' feature, explaining its selection based on constraints.

**invalid, 22 possible configurations**

- Waffle
  - Topping
    - Sugar
    - Cream
    - Cherries
    - Nutella
  - Crumbles
    - Chocolate
    - Colored
  - Accessories
    - Plate
    - Fork
      - Plastic
      - Wood
  - Customer
    - Adult
    - Child

**invalid, 4 possible configurations**

- Waffle
  - Topping
    - Sugar
    - Cream
    - Cherries
    - Nutella
  - Crumbles
    - Chocolate
    - Colored
  - Accessories
    - Plate
    - Fork
      - Plastic
      - Wood
  - Customer
    - Adult
    - Child

**Constraints:**

- Plas • Cherries  $\Rightarrow$  Sugar  $\wedge$  Fork
- Woc • Fork  $\Rightarrow$  Adult

**Open Clauses:**

- $\neg$ Fork  $\vee$  Plastic  $\vee$  Wood

The concrete feature Fork is automatically selected because:

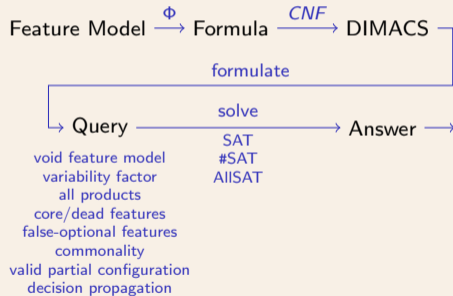
- Cherries  $\Rightarrow$  Sugar  $\wedge$  Fork is a constraint. (2/2)
- Cherries is manually selected. (2/2)

## Decision Propagation

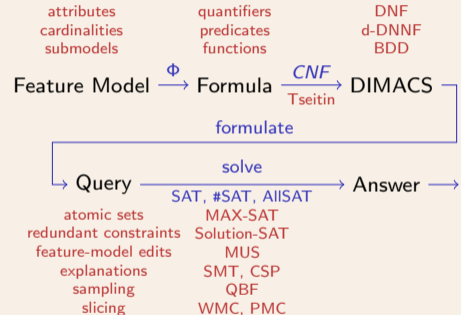
- after each decision (i.e., click) ...
  - ... select features that are now **conditionally core**
  - ... deselect features that are now **conditionally dead**
- this way it is impossible to configure an invalid product
- explanations for all propagated decisions

# Automated Analysis of Feature Models

## The Road So Far ...



## ... and Beyond



- develop new analyses
- improve efficiency of existing analyses
- investigate correctness and compositionality

# Analyzing Feature Models – Summary

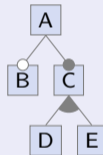
## Lessons Learned

- with solvers, we can build reliable configurators for product lines
- SAT-based analyses: void feature model, core/dead features, decision propagation
- #SAT-based analyses: variability factor, feature commonality

## Further Reading

- Apel et al. 2013, Section 10.1, pp. 244–254  
— introduction to feature-model analysis
- David Benavides et al. (2010): Automated Analysis of Feature Models 20 Years Later: A Literature Review  
— old but extensive literature survey
- Chico Sundermann et al. (2021): Applications of #SAT Solvers on Feature Models  
— experiments on the scalability of #SAT solvers

## Practice



think of a constraint that would make exactly one feature dead



# FAQ – 4. Feature Modeling

## Lecture 4a

- What is feature modeling? When is it needed?
- How can we specify valid combinations of features?
- What is a complete, partial, valid, invalid configuration?
- What are (dis-)advantages of natural language, configuration map, and feature models?
- What is the graphical syntax and semantics of feature models?
- Give an example feature model!

## Lecture 4b

- What representations of feature models are available? Are they equivalent?
- How to represent feature models textually?
- What is UVL (used for)?
- How to identify whether a configuration is valid?
- How to translate feature model into a propositional formula?
- What are DIMACS and KConfig (used for)?
- Would you recommend Excel for feature model? Why (not)?

## Lecture 4c

- Why can configuration become challenging?
- How can we identify problems with feature models and configurations?
- How can feature models be analyzed? What analyses are available?
- What solvers can be used to analyze feature models?
- What is the difference between SAT, #SAT, and ALLSAT?
- Why are solvers useful when creating configurations?