### Controls

EC 607, Set 06

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# Prologue

### Schedule

#### Last time

The conditional independence assumption:  $\{Y_{0i}, Y_{1i}\} \perp D_i | X_i$ I.e., conditional on some controls  $(X_i)$ , treatment is as-good-as random.

#### Today

- Omitted variable bias
- Good vs. bad controls

#### **Upcoming**

- DAGS
- Matching estimators

#### Revisiting an old friend

Let's start where we left off: Returns to schooling.

We have two linear, population models

$$Y_i = \alpha + \rho s_i + \eta_i \tag{1}$$

$$Y_i = \alpha + \rho s_i + X_i' \gamma + \nu_i \tag{2}$$

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For model (2), we can interpret  $\hat{\rho}$  causally **if**  $\mathbf{Y}_{si} \perp \mathbf{I}_{si} | \mathbf{X}_i$  (CIA).

In other words, the CIA says that our observable vector  $X_i$  must explain all of correlation between  $s_i$  and  $\eta_i$ .

#### The OVB formula

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What happens if we can't get data on  $A_i$  and opt for (1)?

$$rac{ ext{Cov}( ext{Y}_i,\, ext{s}_i)}{ ext{Var}( ext{s}_i)} = 
ho + \gamma' \delta_{As}$$

where  $\delta_{As}$  are coefficients from regressing  $A_i$  on  $s_i$ .

#### Interpretation

Our two regressions

$$Y_i = \alpha + \beta s_i + v_i \tag{1}$$

$$\mathbf{Y}_i = \pi + \rho \mathbf{s}_i + \mathbf{A}_i' \gamma + e_i$$
 (2)

will yield the same estimates for the returns to schooling

$$rac{ ext{Cov}( ext{Y}_i,\, ext{s}_i)}{ ext{Var}( ext{s}_i)} = 
ho + \gamma' \delta_{As}$$

if (**a**) schooling is uncorrelated with ability ( $\delta_{As} = 0$ ) or (**b**) ability is uncorrelated with earnings, conditional on schooling ( $\gamma = 0$ ).

#### Example

Table 3.2.1, The returns to schooling

	1	2	3	4
Schooling	0.132	0.131	0.114	0.087
	(0.007)	(0.007)	(0.007)	(0.009)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT

Here we have four specifications of controls for a regression of log wages on years of schooling (from the NLSY).

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**Column 1** (no control variables) suggests a 13.2% increase in wages for an additional year of schooling.

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**Column 2** (age dummies) suggests a 13.1% increase in wages for an additional year of schooling.

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**Column 3** (column 2 controls plus parents' ed. and self demographics) suggests a 11.4% increase in wages for an additional year of schooling.

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**Column 4** (column 3 controls plus AFQT<sup>†</sup> score) suggests a 8.7% increase in wages for an additional year of schooling.

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As we ratchet up controls, the estimated returns to schooling drop by 4.5 percentage points (34% drop in the coefficient) from **Column 1** to **Column 4**.

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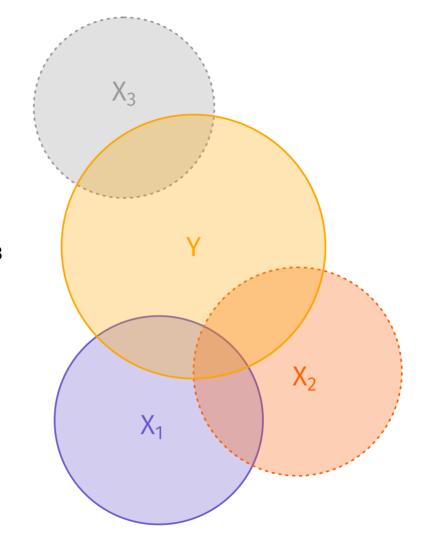
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$$rac{\mathrm{Cov}(\mathrm{Y}_i,\,\mathrm{s}_i)}{\mathrm{Var}(\mathrm{s}_i)} = 
ho + \gamma' \delta_{As}$$

If we think ability positively affects wages, then it looks like we also have positive selection into schooling.



Omitted:  $X_2$  and  $X_3$ 

#### Note

This OVB formula **does not** require either of the models to be causal.

The formula compares the regression coefficient in a **short model** to the regression coefficient on the same variable in a **long model**.<sup>†</sup>

#### The OVB formula and the CIA<sup>†</sup>

In addition to helping us think through and sign OVB, the formula

$$rac{ ext{Cov}( ext{Y}_i,\, ext{s}_i)}{ ext{Var}( ext{s}_i)} = 
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- **Q** When is the CIA plausible?
- A Two potential answers
  - 1. Randomized experiments
  - 2. Programs with arbitrary cutoffs/lotteries

Control variables play an enormous role in our quest for causality (the CIA).
Q Are "more controls" always better (or at least never worse)?

A No. There are such things as...

#### Defined

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Hint It's a flavor of selection bias.

Let's consider an example...

#### Example

Suppose we want to know the effect of college graduation on wages.

- 1. There are only two types of jobs: blue collar and white collar.
- 2. White-collar jobs, on average, pay more than blue-collar jobs.
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- A No. Imagine college degrees are randomly assigned. When we condition on occupation, we compare degree-earners who chose blue-collar jobs to non-degree-earners who chose blue-collar jobs. Our assumption of random degrees says **nothing** about random job selection.

Bad controls can undo valid randomizations.

#### Formal-ish derivation

More formally, let

- $W_i$  be a dummy for whether i has a white-collar job
- Y<sub>i</sub> denote i's earnings
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$$egin{aligned} \mathbf{Y}_i &= \mathbf{C}_i \mathbf{Y}_{1i} + \left(1 - \mathbf{C}_i\right) \mathbf{Y}_{0i} \ \mathbf{W}_i &= \mathbf{C}_i \mathbf{W}_{1i} + \left(1 - \mathbf{C}_i\right) \mathbf{W}_{0i} \end{aligned}$$

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$$\mathbf{Y}_i = \mathbf{C}_i \mathbf{Y}_{1i} + (1 - \mathbf{C}_i) \mathbf{Y}_{0i}$$
 $\mathbf{W}_i = \mathbf{C}_i \mathbf{W}_{1i} + (1 - \mathbf{C}_i) \mathbf{W}_{0i}$ 

Because we've assumed  $C_i$  is randomly assigned, differences in means yield causal estimates, *i.e.*,

$$E[Y_i \mid C_i = 1] - E[Y_i \mid C_i = 0] = E[Y_{1i} - Y_{0i}]$$
  
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$$E[Y_i \mid W_i = 1, \frac{C_i = 1}{1}] - E[Y_i \mid W_i = 1, \frac{C_i = 0}{1}]$$

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Specifically, the selection bias term

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Note Even if the causal effect is zero, this selection bias need not be zero.

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What's the answer?

# Proxy variables

Angrist and Pischke bring up an interesting scenario that intersects omitted-variable bias and bad controls.

- We want to estimate the returns to education.
- Ability is omitted.
- We have a proxy for ability—a test taken after schooling finishes.

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We're a bit stuck.

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With some math/luck, we can bound the true effect with these estimates.

#### Example

Returning to our OVB-motivated example, we control for occupation.

	1	2	3	4	5
Schooling	0.132	0.131	0.114	0.087	0.066
	(0.007)	(0.007)	(0.007)	(0.009)	(0.010)
Controls	None	Age Dum.	2 + Add'l	3 + AFQT	4 + Occupation

Schooling likely affects occupation; how do we interpret the new results?

#### Conclusion

Timing matters.

The right controls can help tremendously, but bad controls hurt.

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