FOCUS Coil-to-Coil Separation Objective Function

Thomas Kruger

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General

FOCUS can optimize for the distance between coils through the coil-to-coil separation objective function given in (1). This objective function uses penalty functions to constrain the minimum coil-to-coil separation. A coil is parameterized as $\boldsymbol{r}(\zeta)$ with $\zeta \in [0, 2\pi]$.

$$f_{cc} = \frac{2}{N_c(N_c-1)} \sum_{i=1}^{N_c-1} \sum_{j=i+1}^{N_c} \int_0^{2\pi} \int_0^{2\pi} p_{1,2}(\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|, r_\Delta)\, d\zeta_i\, d\zeta_j \tag{1}$$

$$p_1(\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|, r_\Delta) = H_{-r_\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|)\left(\cosh\left(\alpha\left(r_\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|\right)\right) - 1\right)^2 \tag{2}$$

$$\alpha \geq 0 \qquad r_\Delta \geq 0$$

$$p_2(\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|, r_\Delta) = H_{-r_\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|)\left(\alpha\left(r_\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|\right)\right)^\beta \tag{3}$$

$$\alpha \geq 0 \qquad \beta \geq 2 \qquad r_\Delta \geq 0$$

$$H_{-r_\Delta}(-\left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|) \equiv H(r_\Delta - \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right|) = \begin{cases} 1, & \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right| < r_\Delta \\ \frac{1}{2}, & \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right| = r_\Delta \\ 0, & \left|\boldsymbol{r_i} - \boldsymbol{r_j}\right| > r_\Delta \end{cases} \tag{4}$$

First Derivatives

First derivatives of the coil-to-coil separation objective function, (1) are now given. Let λ_k be an optimization variable that defines the position of the k-th coil, $\boldsymbol{r_k}$ where $k \in \{1, 2, ..., N_c\}$.

$$\delta f_{cc} = \sum_{\substack{i=1 \\ i\neq k}}^{N_c} \int_0^{2\pi} \frac{\delta f_{cc}}{\delta \boldsymbol{r_k}} \cdot \delta \boldsymbol{r_k}\, d\zeta_k \tag{5}$$

$$\frac{\delta f_{cc}}{\delta \boldsymbol{r_k}} = \frac{2}{N_c(N_c-1)} \int_0^{2\pi} \frac{\partial p_{1,2}}{\partial \boldsymbol{r_k}}\, d\zeta_i \tag{6}$$

$$\frac{\partial p_1}{\partial \boldsymbol{r_k}} = -H_{-r_\Delta}(-\left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|)\, 2\alpha\left(\cosh\left(\alpha\left(r_\Delta - \left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|\right)\right) - 1\right)\sinh\left(\alpha\left(r_\Delta - \left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|\right)\right)\left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|^{-1}(\boldsymbol{r_k} - \boldsymbol{r_i}) \tag{7}$$

$$\frac{\partial p_2}{\partial \boldsymbol{r_k}} = -H_{-r_\Delta}(-\left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|)\beta\alpha\left(\alpha\left(r_\Delta - \left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|\right)\right)^{\beta-1}\left|\boldsymbol{r_k} - \boldsymbol{r_i}\right|^{-1}(\boldsymbol{r_k} - \boldsymbol{r_i}) \tag{8}$$

$$\delta f_{cc} = \frac{2}{N_c(N_c-1)} \sum_{\substack{i=1 \\ i\neq k}}^{N_c} \int_0^{2\pi} \int_0^{2\pi} \frac{\partial p_{1,2}}{\partial \boldsymbol{r_k}} \cdot \boldsymbol{\delta r_k} \, d\zeta_k \, d\zeta_i \tag{9}$$

$$\frac{\partial f_{cc}}{\partial \lambda_k} = \frac{2}{N_c(N_c-1)} \sum_{\substack{i=1 \\ i\neq k}}^{N_c} \int_0^{2\pi} \int_0^{2\pi} \frac{\partial p_{1,2}}{\partial \boldsymbol{r_k}} \cdot \frac{\partial \boldsymbol{r_k}}{\partial \lambda_k} \, d\zeta_k \, d\zeta_i \tag{10}$$

Notes

This objective function is parameterization dependent. The integrals should be over the arc-length and not the parameterizing variable, and the integrals should be normalized by the coil length. In practice this parameterization dependent objective function worked well and the math/implementation is much easier.

How to Use

To use this coil-to-coil separation objective functions, a weight, ”weight_ccsep”, needs to be set. This variable and all other variables in this section are set in the ”*.input” file. No changes to the ”*.focus” file are necessary. An integer named ”penfun_ccsep” can be set to 1 or 2 and determines which penalty function is used. The value of r_Δ is set by the variable ”r_delta”. The values of α and β are set by the variables ”ccsep_alpha” and ”ccsep_beta”.