What are Beall and Restall pluralists about?[1]

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In this paper I argue that Beall and Restall's claim that there is one true logic of metaphysical modality is incompatible with the formulation of logical pluralism that they give. I investigate various ways of reconciling their pluralism with this claim, but conclude that none of the options can be made to work.

In their 'Logical Pluralism' J.C. Beall and Greg Restall advocate a pluralistic approach to logical systems. On their view there is not one true logic but many equally viable ones—classical, relevant, and intuitionist logics are not competitors. However Beall and Restall do not count any formal system as a true logic. In their view formal systems which reject the reflexivity or transitivity of consequence are not truly logics, and they also take the view that there is only one correct logic of metaphysical modality—a logic that falls somewhere between S4 and S5 [2000, 489].

In what follows I argue that there is no way of accounting for genuine disagreement between S4 and S5 (or the posited correct logic lying between them) that is compatible with Beall and Restall's assumptions. Any way of characterizing the alethic model logics as in genuine disagreement turns out to also be a way of characterizing classical, relevant, and intuitionist logics as in genuine conflict. Of course the easy way out of this dilemma for Beall and Restall is simply to accept that there is more than one true logic of metaphysical modality—but even if they are inclined to do so, the issue sheds some light on what kinds of disagreement their pluralism can allow.

1. Logical pluralism

On Beall and Restall's account of logical pluralism the central subject matter of Logic is the relation of *logical consequence* [Beall and Restall 2000].[2] The nature of logical consequence is captured in the following principle:

> (V) A conclusion **A** *follows from* premises **Γ** if and only if any case in which each premise in **Γ** is true is also a case in which **A** is true. Or equivalently, there is no case in which each premise in **Γ** is true and **A** is false.

[1] Thanks to the attendees at the Western Canadian Philosophical Association Meetings of 2001 for a helpful discussion of this paper, and also to two anonymous referees for the AJP for their useful comments.

[2] Following Beall and Restall, and subject to the restrictions of grammar, 'Logic' names the discipline, and 'logic' a particular logical system.

However (V) alone does not give us a complete account of logical consequence, since it leaves open the question of what the cases in question are [Beall and Restall 2000, 476]. According to Beall and Restall, one is committed to logical pluralism if one endorses the following three claims:

1. The pretheoretic notion of consequence is given in (V).
2. A *logic* is given by a *specification of the cases* to appear in (V). Such a specification of cases can be seen as a way of spelling out *truth conditions* of the claims expressible in the language in question.
3. There are *at least two* different specifications of cases which may appear in (V) [476-7].

Just to make their point clear, (2) is there to remind the reader that one gets different logics by varying the kinds of cases considered—some of the possibilities are set-theoretic models, possible worlds, situations, constructions, and so on. When this specification of the cases is supplemented by an appropriate definition of truth in a case (model/situation/construction/etc.), the result is a logic. Claim (3) may seem trivial as given, but of course Beall and Restall's claim is that there are at least two equally good ways of spelling out (V), and thus at least two equally good logics. Pluralism follows: there is no one true logic.

We can get a more complete picture of what their pluralism amounts to by considering an example. The consequence relation of classical first-order logic is given by the familiar account of a model $\mathcal{M}$ combined with a recursive specification of truth conditions. Following Beall and Restall let us call this the *Tarskian* account of consequence.

Alternatively, suggest Beall and Restall, we could also spell out the cases in (V) in terms of possible worlds. On this account the definition for truth in a world will read as:

1. ~A is true in *w* iff A is not true in *w*.
2. A ∧ B is true in *w* iff A is true in *w* and B is true in *w*.
3. A ∨ B is true in *w* iff either A is true in *w* or B is true in *w*.

and so on.[3]

Call this the *necessary truth preservation* account of consequence. This account of validity endorses all the consequence claims of classical logic. However it also renders the following claim true:

a *is coloured* is a consequence of a *is red*.

This is because there is no possible world in which a is red but not coloured—that circumstance is impossible.

[3] The clauses for the quantifiers are awkward, but not impossible. Beall and Restall simplify theirs by assuming every object has a unique name in the language.

The pluralist treatment of these two accounts of the consequence relation is as follows. It is true that the *Tarskian* account of consequence is *formal* while the *necessary truth preservation account* is not. However we should be happy to call both accounts logic:

> A pluralist account of disagreement about logical form goes as follows: It is not fruitful to debate which of these things is *logic*. Both are (sic) flesh out (V), so *both* are logic. Given an argument which is necessarily truth preserving but not Tarski-style valid, it is surely more informative to say: yes, there is no possibility in which the premise is true and the conclusion false, but there is a Tarski-style model in which the premise is true and the conclusion false, and this shows the necessary truth preservation is not in virtue of the first-order logical form of the claims involved. That is informative analysis. A debate about which of these is logic adds nothing [Beall and Restall 2000, 480].

These are not the only two specifications of (V) that Beall and Restall countenance of course. They give particular attention to the identification of cases with situations, which unlike possible worlds leave some claims undetermined. The resultant logic will not endorse as cases of consequence some things which classical logic does—e.g. $B \vee \neg B$ is not a consequence of every A on such an account, as there are situations in which neither B nor $\neg B$ are true (Both claims will be undetermined.) Expansion of the domain of cases to include inconsistent results will provide further cases where an inference form is classically valid but relevantly invalid—most notoriously the move from $A \vee B$ and $\neg A$ to B. Beall and Restall comment on the debate surrounding relevant logics and disjunctive syllogism as follows:

> One cause of concern with the rejection of disjunctive syllogism is that disjunctive syllogism is *obviously* valid, and we reason with it all the time—we could not do without it in everyday reasoning. Our pluralist will agree: *Of course* there is a sense in which disjunctive syllogism is valid—and even *obviously* so. In any possible world in which the premises are true, so is the conclusion. In *that* sense — the sense afforded by cases as world like — disjunctive syllogism is valid. The virtue of a pluralist account is that we can enjoy the fruits of relevant consequence as a guide to inference without feeling guilty whenever we make an inference which is not relevantly valid. With classical consequence you know you will not make a step from truth to falsehood. With relevant consequence, the strictures are tighter; you know you will not make a step from one that is true in a situation to something not true in it (but which might be true outside it). This is a tighter canon to guide reasoning [2000, 484].

For Beall and Restall relevant logic is not a *rival* to classical logics but merely other 'important variety of logical consequence' [2000, 485].

2. But not about modal logic

So much for the outline of Beall and Restall's view. Its central features are the dual claims that that every true logic is a filling in of schema (V) and that there are multiple acceptable ways of filling in (V). The open question is whether any way of specifying which cases are to be

considered in our use of (V) will count among the true logics. Every true logic is a specification of (V), but is every specification of (V) a true logic? Some comments made by Beall and Restall suggest that the answer to this question is yes. Discussing the differences between Tarskian consequence and necessary truth preservation they write:

> It is not fruitful to debate which of these things is *logic*. Both flesh out (V) so both of these things are logic [2000, 480].

And later:

> Many appeals to 'Real Validity' are appeals to *real* validity; they are not however, appeals to the *only* real validity. Real validity comes from a specification of cases which appear in (V) [2000, 481].

However, this is not quite all they have to say on the matter. In discussing whether their view leaves any room for genuine rivalry or disagreement between logics they write:

> There are too many modal logics to hold each of them as the logic of broad metaphysical necessity. So given a particular interpretation of each of the symbols in our formalism (including *consequence*) we can admit that there is a great deal of scope for rivalry. For the propositional modal logic of necessary truth preservation, we think that a logic somewhere between S4 and S5 is a candidate for getting things *right*. Anything else *gets it wrong* when it comes to metaphysical necessity. There is scope for rivalry and disagreement, when the meaning of the basic lexicon is settled [2000, 489].

Here is the main source of my puzzlement. Beall and Restall claim that neither classical nor relevant logic gets the logic of the sentential connectives wrong, but that S4 gets the logic of metaphysical necessity wrong. What, one wonders, is the relevant difference?[4]

In one of their concluding statements Beall and Restall sum up pluralism as the view that 'two different logics … are *both* accurate and systematic accounts of … the one notion of logical consequence' [2000, 490]. The question then is why this conclusion applies to relevant and classical logics but not to the various varieties of modal logic. The only clue they give is in a passage in which they compare disagreement between different logics to disagreement between applied geometries:

> [T]his disagreement comes about *simply* by applying the logic to model the validity of real argument. Different formal systems can be equally appropriately used to model the validity of arguments. The analogy with applied geometry becomes appropriate only once the pretheoretic account (V) is fleshed out. Once

[4] This is a version of the problem raised in Goddu 2002—what counts as a specification of the cases? Goddu says this is a problem that Beall and Restall have to address. I take the present paper to be an argument that there is no way to answer this question that leaves classical and relevant logics as both true logics, but allows S4 and S5 to be in conflict.

> you have a specific account of what kind of cases are in use (be they, worlds, constructions, situations) then there is scope for disagreement [2000, 489].

Now, I find parts of this passage particularly dense, but the essence of what they are saying at the end seems to be this: once you specify what kinds of cases you are going to consider, two logics can disagree about what consequence relations obtain given those cases. Now given the way they have laid things out, I'm also fairly sure that the only way this could happen is if the two logics in question disagreed on the truth conditions of the sentences of the language, since a combination of cases and truth conditions just is a logic. The qualifiers in the previous two claims aside, it is perfectly clear that this doesn't apply to the modal logics under discussion—S4 and S5 differ in a precisely analogous way to classical logic and relevant logic. Relevant logic admits situations (e.g. impossible ones) that classical logic doesn't, and S4 admits models (e.g. ones in which the relations between worlds are reflexive, transitive, but not symmetric) that S5 doesn't.

So the reason for rejecting S4 and S5 as acceptable alternative elucidations of metaphysical necessity is not because they agree on the cases that (V) is to be read as quantifying over but disagree over the truth conditions—they don't agree on the cases. There must be another reason for thinking that this disagreement deserves a different treatment by the pluralist than the disagreement between relevant logic and classical logic.

The mention of alternate geometries in this passage brings to mind another view of conflict between logics, as expressed in Prior's 1967 comment:

> We must develop, in fact, alternative tense logics, rather like alternative geometries; though this is not to deny that the question of what sort of time we actually live in …is a real one [Prior 1967, 59].

This view, while perfectly comprehensible, is not one that Beall and Restall can adopt. On Prior's view, alternate tense logics are in conflict if both are taken to be the tense logic of the time we live in. Asking 'what is the logic of real time' is then a (empirical?) question analogous to the question 'what is the geometry of real space'. By analogy, we could then ask 'what is the logic of real metaphysical necessity'. Disagreement on the answer would on this model be counted as actual disagreement.

So why can't Beall and Restall take this stance? Because to do so would undermine their case for the view that necessary truth preservation and classical logics do not conflict. Given the model theoretic emphasis on (V) and a apparent conflict between S4 and S5, the question 'what is the logic of real metaphysical necessity' must be understood as a question about what class of cases is the 'real' one. If that question is meaningful for modal logic, what makes the question illegitimate for first-order logics? Surely the formalist advocate of classical logic and the defender of necessary truth preservation can each claim that their set of cases is the one applicable to real first-order consequence, just as the defender of the correct modal logic can claim that their set of cases is the one applicable to real metaphysical modal consequence. The two claims are on the same footing, and if one is genuine disagreement then the other is also.

3. Logical purposes

Another possible explanation of how S4 and S5 can really be in disagreement is suggested in Beall and Restall's 'Defending Logical Pluralism':

> [T]he purpose of logic, according to the going objection … is that we never be 'led astray'. … Being *led astray* is relative to some fixed direction. With respect to reasoning such 'directions' may be things such as *reasoning constructively*, or perhaps *reasoning relevantly* or *et cetera* [2001, 5].

Now I must quickly say that Beall and Restall do later disavow this picture, reiterating the claim that the goal of logic is to specify *logical consequence*, and rejecting the idea that the purpose has anything to do with avoiding being led astray. However later in the same article they write:

> We hold, of course that classical logic is *not* universally applicable in the sense of dictating what is true in each and every case. Not all situations and constructions are closed under classical consequence. But that does not mean that classical validity is not *validit*y. For it is still true that if the premises of a classically valid argument are true, the conclusion *must* be true too. Of course, classical consequence does not do *every* job required of deductive validity. Other logics are better suited to some of these tasks. That is why we need a plurality of logics [Beall and Restall 2001, 15]

This notion that certain logics have certain purposes—that they each do some of the jobs required of deductive validity—suggests another reason why Beall and Restall might refrain from a pluralist attitude towards the multitude of modal logics. Perhaps no modal logic other than the endorsed midway point between S4 and S5 does a job required of deductive validity.

What would this picture be like? The view would have to be that a logic is a spelling out of (V) that serves a purpose (i.e.: does a job) that we need deductive validity to do. But what are the jobs? One obvious thought would be that this is a version of what Susan Haack calls *local pluralism*—different logics are required for different areas of discourse [Haack 1978, 223]. This may well be a related view to Beall and Restall's, but it is not their view. They tell us so explicitly: 'we agree that … mere contingent or domain restrictions are not appropriate in a logic. Logic applies, come what may. For us, *each* logic applies come what may' [2001, 15]. So the different jobs are not different domains.

The only alternate picture that I can come up with is that we, in our ordinary reasoning practices, have multiple kinds of domain independent reasoning, and associated consequence relations. Anything that counts as a logic must spell out (V) in a way that models one of these actual consequence relations.

One question that this proposal immediately raises is whether this is meant to be a descriptive or a normative claim. Let me begin by clarifying this distinction with some examples. It seems clear that if one wants a descriptive account of the inferential practices of working mathematicians, a classical logic of some sort will be the way to go. Intuitionist logics in contrast had their origins as a reform of mathematical practice—they were normative in nature. Relevant logics have their origins in the observation that in ordinary reasoning we do not license

inferences like P and not P therefore Q, and thus can be seen as partly motivated by descriptive considerations.[5]

Now Beall and Restall seem to be simultaneously concerned with both these issues. They endorse the descriptivist view that the presence of a reasoning practice that does not correspond to an existing logic is a good reason to formulate a new logic. However they do so in such a way as to suggest that the presence of such reasoning is good evidence that one *should* reason in such a way:

> We think that such a defense of a new logic is perfectly appropriate. After all, it seems that if [holding an inconsistent theory does not license one to conclude anything whatsoever] … then there is very likely to be a sense in which arbitrary *B* does *not follow from* arbitrary A ∧ ~A. If this is right, then we claim that … a new logic is needed, or *would be* needed if there weren't one already [2001, 7].[6]

So Beall and Restall's views are compatible with the suggestion that only one of the modal logics is descriptively adequate as an account of metaphysical necessity. But there are problems here too.

As yet I have paid no attention to Beall and Restall's claim that the posited S4.X is the correct logic of *metaphysical* necessity.[7] However, the notion that a logic must model an actual consequence relation from ordinary reasoning suggests that perhaps their view should be understood as allowing that both S4 and S5 are true logics, but neither are the true logic of metaphysical necessity, whereas S4.X is. Of course it is not at all clear that this would give us a case of *disagreement* between logics, but let us put that aside for the moment.

Metaphysical modality is usually explained by pointing out that it affirms not only that it is necessary that Prior is Prior, but also that it is necessary that Prior is not a fish. That is, there are things that are metaphysically necessary without being logical truths of classical first order logic—though of course they are logical truths in the first order logic of necessary truth preservation. On the other hand, metaphysical necessity is contrasted with natural necessity, since the scientific laws that play a role in natural necessity are not taken to be metaphysically necessary. I think we can safely ignore that feature of metaphysical modality. The key difference between S4 and S5 lies in the relations between modalities, not in what modal-free sentences are true in all worlds. So if the emphasis on *metaphysical* modality is doing any work here, it must be because there is some natural set of allowable relations between worlds that is picked out by metaphysical modal notions. The suggestion is that a particular set of cases is built into the notion of metaphysical necessity, so that only one of the modal logics is descriptively adequate.

If this was Beall and Restall's view, they would simply have to give up the idea that there is just one correct modal logic. Even the briefest tour of the history of philosophy makes clear

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[5] This treatment of the distinction owes much to Resnik 1996.

[6] This is an important point, since it licenses the view that multiple reasoning practices for metaphysical modality require multiple modal logics, all of which have equal claim to being true logics of metaphysical modality. (More on this issue below.)

[7] Thanks to Richard Zach for suggesting that I was paying insufficient attention to the adjective.

that (in as far as there are patterns of reasoning at all—more on this in a second) there is more than one pattern of domain independent reasoning regarding metaphysical necessity and possibility (see, for example, Knuuttila 1993 for a discussion of the myriad of conceptions of metaphysical modality found in medieval authors). Perhaps none of these correspond to S4, and so perhaps S4 is not a logic, but it seems highly unlikely that there is just one modal logic.

Furthermore, it is not at all obvious that the patterns of reasoning of ordinary discourse contain even three independent kinds of reasoning, one corresponding to classical logic, one to the logic of necessary truth preservation, and another corresponding to relevant logic. It seems far more accurate to describe the situation this way: our patterns of reasoning contain elements of classical reasoning, reasoning according to necessary truth preservation, and relevant reasoning, and so on. Classical logic, relevant logic, etc., might then be seen as idealizations each of which focus on certain aspects of our reasoning, not as models of pre-existing independent practices.

To put it another way, the kind of pluralism that maintains that each true logic reflects a pattern of actual reasoning practice grounds the existence of logic in a practice we have—it is perhaps a kind of conventionalism.[8] But it is not at all clear that the conventions of reasoning we have can be cleanly divided into unified practices which might be represented by a variety of logics. Rather our conventions of reasoning form an undifferentiated mass, and the logics Beall and Restall identify can at best be described as attempts to model that reasoning that take as primary different aspects of that reasoning.

The upshot seems to be that *if* logics have to meet the additional requirement of modelling some aspect of our actual reasoning practice, debate between relevant logicians and classical logicians over the right account of negation will be just as viable as debate between the modal logicians over the right account of metaphysical necessity. If we countenance the latter as genuine disagreement that pluralists should not dismiss, surely the former is equally genuine disagreement.

In any case, this conventionalist view doesn't seem to be the position Beall and Restall hold. All of their positive statements suggest that any way of filling out the cases in (V) that offers a reasonable domain for the universal quantifier counts as a specification of consequence. For example, in discussing the issue of whether their view confuses true logics with formal systems that have at best instrumental value, they tell us that:

> [T]here is a difference between those ends which have something to do with the *content* of our claims and their *consequences* on the one hand, and those which are foreign to the concerns of deductive logic. Unfortunately we have little to say to make this distinction precise, except to point to our definition of deductive validity. What makes classical, constructive, and relevant logics *logic* is their analysis in terms of the cases in which claims are true. They depend not on extraneous features of the representations of the claims [2001, 13].

Once again, analysis in terms of giving a set of cases over which the quantifier in (V) ranges seems to be a sufficient condition for being a logic.

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[8] See Resnik 1996 for a discussion of various positions on logic, including pluralist conventionalism.

4. Logical disagreement again

Let us turn to the kinds of attitudes Beall and Restall list as possible views to take towards a logic or a disagreement between logics:

1. $\mathcal{L}$ is a logic because it is formally similar to other logics—it models a consequence relation, or it models something analogous to consequence relations.
2. $\mathcal{L}$ and $\mathcal{L}$' agree on a specification of (V), but disagree on whether an informal argument is valid or not. This is one place where genuine disagreement can occur.
3. $\mathcal{L}$ and $\mathcal{L}$' seem to be trying to get at the same kind of thing, but from the perspective of the advocate of $\mathcal{L}$, $\mathcal{L}$' is talking about something else entirely (and perhaps vice versa).
4. $\mathcal{L}$ and $\mathcal{L}$' are different but equally good ways of spelling out (V).

The problem I identified at the beginning of this paper was that Beall and Restall seem to advocate (4) as the appropriate attitude to take towards any spelling out of (V), and yet they also endorse the view that various modal logics are in genuine disagreement. To expand, they seem to reject (1) or at least the second part of it, since something is only a logic on their view if it actually models a consequence relation that is an instance of (V). They seem to think (2) is possible, and the only way I see to spell this out is in terms of a difference of opinion over truth conditions or translation.[9] Disagreement of the type mentioned in (3) seems to be disagreement over whether (V) is an appropriate schema for understanding the consequence relation—the example Beall and Restall give is of the intuitionist and the classical logician disagreeing over whether the consequence relation relates truth or provability. This leaves only option (4) as an approach to disagreement between those who agree on (V) but differ on what cases (V) quantifies over. Modal logics seem to fall into case (4), and so it seems Beall and Restall should be pluralists about them.

It's worth emphasizing that this claim—that only an unspecified logic in between S4 and S5 gets it right for metaphysical necessity, but both necessary truth preservation and classical logics (among others) get it right for the operators of first order logic—is not incidental for Beall and Restall's program. Part of their argument for their pluralism is that it is not indiscriminate 'come-what-may' pluralism—not anything structurally similar to a logic counts as a logic, and there is, as they say, 'scope for rivalry and disagreement' [2000, 15]. If we cannot make sense of what counts as genuine conflict between logics, and we cannot determine when an apparent specification of the cases in (V) is not a legitimate one, then we do not understand Beall and Restall's pluralism.

In short, Beall and Restall's view does not allow for genuine disagreement between modal logics, and yet they claim we should not be pluralists about them. But if not them, what should we be pluralists about?

[9] Again, perhaps my lack of familiarity with the details of alternative geometries is playing havoc with my ability to understand the analogy here.

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