Remarks on Wittgenstein, Gödel, Chaitin, Incompleteness, Impossiblity and the Psychological Basis of Science and Mathematics

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Abstract

It is commonly thought that such topics as Impossibility, Incompleteness, Paraconsistency, Undecidability, Randomness, Computability, Paradox, Uncertainty and the Limits of Reason are disparate scientific physical or mathematical issues having little or nothing in common. I suggest that they are largely standard philosophical problems (i.e., language games) which were resolved by Wittgenstein over 80 years ago.

Wittgenstein also demonstrated the fatal error in regarding mathematics or language or our behavior in general as a unitary coherent logical 'system,' rather than as a motley of pieces assembled by the random processes of natural selection. "Gödel shows us an unclarity in the concept of 'mathematics', which is indicated by the fact that mathematics is taken to be a system" and we can say (contra nearly everyone) that is all that Gödel and Chaitin show. Wittgenstein commented many times that 'truth' in math means axioms or the theorems derived from axioms, and 'false' means that one made a mistake in using the definitions, and this is utterly different from empirical matters where one applies a test. Wittgenstein often noted that to be acceptable as mathematics in the usual sense, it must be useable in other proofs and it must have real world applications, but neither is the case with Godel's Incompleteness. Since it cannot be proved in a consistent system (here Peano Arithmetic but a much wider arena for Chaitin), it cannot be used in proofs and, unlike all the 'rest' of PA it cannot be used in the real world either. As Rodych notes "…Wittgenstein holds that a formal calculus is only a mathematical calculus (i.e., a mathematical language-game) if it has an extra- systemic application in a system of contingent propositions (e.g., in ordinary counting and measuring or in physics) …" Another way to say this is that one needs a warrant to apply our normal use of words like 'proof', 'proposition', 'true', 'incomplete', 'number', and 'mathematics' to a result in the tangle of games created with 'numbers' and 'plus' and 'minus' signs etc., and with

'Incompleteness' this warrant is lacking. Rodych sums it up admirably. "On Wittgenstein's account, there is no such thing as an incomplete mathematical calculus because 'in mathematics, everything is algorithm [and syntax] and nothing is meaning [semantics]…"

I make some brief remarks which note the similarities of these 'mathematical' issues to economics, physics, game theory, and decision theory.

Those wishing further comments on philosophy and science from a Wittgensteinian two systems of thought viewpoint may consult my other writings -- Talking Monkeys--Philosophy, Psychology, Science, Religion and Politics on a Doomed Planet--Articles and Reviews 2006-2019 3rd ed (2019), The Logical Structure of Philosophy, Psychology, Mind and Language in Ludwig Wittgenstein and John Searle 2nd ed (2019), Suicide by Democracy 4th ed (2019), The Logical Structure of Human Behavior (2019), The Logical Structure of Consciousness (2019, Understanding the Connections between Science, Philosophy, Psychology, Religion, Politics, and Economics and Suicidal Utopian Delusions in the 21st Century 5th ed (2019), Remarks on Impossibility, Incompleteness, Paraconsistency, Undecidability, Randomness, Computability, Paradox, Uncertainty and the Limits of Reason in Chaitin, Wittgenstein, Hofstadter, Wolpert, Doria, da Costa, Godel, Searle, Rodych, Berto, Floyd, Moyal-Sharrock and Yanofsky (2019), and The Logical Structure of Philosophy, Psychology, Sociology, Anthropology, Religion, Politics, Economics, Literature and History (2019).

It is commonly thought that such topics as Impossibility, Incompleteness, Paraconsistency, Undecidability, Randomness, Computability, Paradox, Uncertainty and the Limits of Reason are disparate scientific physical or mathematical issues having little or nothing in common. I suggest that they are largely standard philosophical problems (i.e., language games) which were resolved by Wittgenstein over 80 years ago.

"Philosophers constantly see the method of science before their eyes and are irresistibly tempted to ask and answer questions in the way science does. This tendency is the real source of metaphysics and leads the philosopher into complete darkness." Wittgenstein

“What we are ‘tempted to say’ in such a case is, of course, not philosophy, but it is its raw material. Thus, for example, what a mathematician is inclined to say about the objectivity and reality of mathematical facts, is not a philosophy of mathematics, but something for philosophical treatment.” Wittgenstein PI 234

One might regard all these issues in many contexts as scientism, i.e., as matters for scientific investigation where the facts will provide answers, whereas they can be shown to be philosophical matters of how the language is to be used.

“Thus, we may say of some philosophizing mathematicians that they are obviously not aware of the many different usages of the word “proof; and that they are not clear about the differences between the uses of the word “kind”, when they talk of kinds of numbers, kinds of proof, as though the word “kind” here meant the same thing as in the context “kinds of apples.” Or, we may say, they are not aware of the different meanings of the word “discovery” when in one case we talk of the discovery of the construction of the pentagon and in the other case of the discovery of the South Pole.” BBB p29

“Ought the word “infinite” to be avoided in mathematics? Yes: where it appears to confer a meaning upon the calculus; instead of getting one from it.” RFM revised edition (1978) p141

Horwich has nicely summed up the Wittgensteinian view of scientism in these contexts.

“There must be no attempt to explain our linguistic/conceptual activity (PI 126) as in Frege’s reduction of arithmetic to logic; no attempt to give it epistemological foundations (PI 124) as in meaning based accounts of a priori knowledge; no attempt to characterize idealized forms of it (PI 130) as in sense logics; no attempt to reform it (PI 124, 132) as in Mackie’s error theory or Dummett’s intuitionism; no attempt to streamline it (PI 133) as in Quine’s account of existence; no attempt to make it more consistent (PI 132) as in Tarski’s response to the liar paradoxes; and no attempt to make it more complete (PI 133) as in the settling of questions of personal identity for bizarre hypothetical ‘teleportation’ scenarios.”

“The more narrowly we examine actual language, the sharper becomes the conflict between it and our requirement. (For the crystalline purity of logic was, of course, not a result of investigation: it was a requirement.)” Wittgenstein PI 107

Wittgenstein's remarks on Gödel's famous incompleteness theorems are especially notable as they have until recently been almost universally misunderstood.

"It might justly be asked what importance Gödel's proof has for our work. For a piece of mathematics cannot solve problems of the sort that trouble us. --The answer is that the situation, into which such a proof brings us, is of interest to us. 'What are we to say now?'--That is our theme. However, queer it sounds, my task as far as concerns Gödel's proof seems merely to consist in making clear what such a proposition as: 'Suppose this could be proved' means in mathematics." Wittgenstein "Remarks on the Foundations of Mathematics" p337(1956) (written in 1937).

Here is one of Gödel's own characterizations of his work.

"My theorems only show that the mechanization of mathematics, i.e., the elimination of the mind and of abstract entities, is impossible, if one wants to have a satisfactory foundation and system of mathematics. I have not proved that there are mathematical questions that are undecidable for the human mind, but only that there is no machine (or blind formalism) that can decide all number- theoretic questions, (even of a very special kind) .... It is not the structure itself of the deductive systems which is being threatened with a brakedown, but only a certain interpretation of it, namely its interpretation as a blind formalism." Gödel "Collected Works" Vol 5, p 176-177. (2003)

In my view, it was shown quite convincingly by Wittgenstein in the 1930's (i.e., shortly after Gödel's proof) that the best way to look at this situation is as a typical language game (though a new one for math at the time)—i.e., the "true but unprovable" theorems are "true" in a different sense (since they require new axioms to prove them). They belong to a different system, or as we ought now to say, to a different intentional context. No incompleteness, no loops, no self reference and definitely not strange! Wittgenstein: "Gödel's proposition, which asserts something about itself, does not mention itself" and "Could it be said: Gödel says that one must also be able to trust a mathematical proof when one wants to conceive it practically, as the proof that the propositional pattern can be constructed according to the rules of proof? Or: a mathematical proposition must be capable of being conceived as a proposition of a geometry which is actually applicable to itself. And if one does this it comes out that in certain cases it is not possible to rely on a proof." (RFM p336). These remarks barely give a hint at the depth of W's insights into mathematical intentionality, which began with his first writings in 1912 but was most evident in his writings in the 30's and 40's. Wittgenstein is regarded as a difficult and opaque writer due to his aphoristic, telegraphic style and constant jumping about with seldom any notice that he has changed topics, nor indeed what the topic is, but if one starts with his only textbook style work—the Blue and Brown Books --and understands that he is explaining how our evolved higher order thought works, it will all become clear to the persistent.

W lectured on these issues in the 1930's and this has been documented in several of his books. There are further comments in German in his nachlass (some of it formerly available only on a $1000 cdrom but now, like nearly all his works, on p2p torrents, libgen,io and b-ok.org. Canadian philosopher Victor Rodych has recently written two articles on Wittgenstein and Gödel in the journal Erkenntnis and 4 others on Wittgenstein and math, which I believe constitute a definitive summary of Wittgenstein and the foundations of math. He lays to rest the previously popular notion that Wittgenstein did not understand incompleteness (and much else concerning the psychology of math). In fact, so far as I can see Wittgenstein is one of very few to this day who does (and NOT including Gödel! —though see his penetrating comment quoted above). Related forms of "paradox" which exercise Hofstadter (and countless others) so much was extensively discussed by W with examples in math and language and seems to me a natural consequence of the piecemeal evolution of our symbolic abilities that extends also to music, art, games etc. Those who wish contrary views will find them everywhere and regarding Wittgenstein and math, they may consult Chihara in Philosophical Review V86, p365-81(1977). I have much respect for Chihara (I am one of few who have read his "A Structural Account of Mathematics" cover to cover) but he fails on many basic issues, such as Wittgenstein's explanations of paradoxes as unavoidable and almost always harmless facets of our EP.

Regarding Godel and "incompleteness", since our psychology as expressed in symbolic systems such as math and language is "random" or "incomplete" and full of tasks or situations ("problems") that have been proven impossible (i.e., they have no solution-see below) or whose nature is unclear, it seems unavoidable that everything derived from it—e.g. physics and math) will be "incomplete" also. I believe the first of these in what is now called Social Choice Theory or Decision Theory (which are continuous with the study of logic and reasoning and philosophy) was the famous theorem of Kenneth Arrow over 60 years ago, and there have been many since. Yanofsky notes a recent impossibility or incompleteness proof in two-person game theory. In these cases, a proof shows that what looks like a simple choice stated in plain English has no solution.

A mountain of literature exists on Gödel's two "incompleteness" theorems and Chaitin's more recent work, but I think that Wittgenstein's writings in the 30's and 40's are definitive. Although Shanker, Mancosu, Floyd, Marion, Rodych, Gefwert, Wright and others have done insightful work, it is only recently that Wittgenstein's uniquely penetrating analysis of the language games being played in mathematics have been clarified by Floyd (e.g., 'Wittgenstein's Diagonal Argument-a Variation on Cantor and Turing'), Berto (e.g., 'Godel's Paradox and Wittgenstein's Reasons , and 'Wittgenstein on Incompleteness makes Paraconsistent Sense' and the book 'There's Something about Godel ', and Rodych (e.g., Wittgenstein and Godel: the Newly Published Remarks', 'Misunderstanding Gödel :New Arguments about Wittgenstein', 'New Remarks by Wittgenstein' and his article in the online Stanford Encyclopedia of Philosophy 'Wittgenstein's Philosophy of Mathematics'). Berto is one of the best recent philosophers, and those with time might wish to consult his many other articles and books including the volume he co-edited on paraconsistency (2013). Rodych's work is indispensable, but only two of a dozen or so papers are free online with the usual search but of course it's all free online if one knows where to look (e.g., libgen.io and b- ok.org and probably torrents as well).

Berto notes that Wittgenstein also denied the coherence of metamathematics--i.e., the use by Gödel of a metatheorem to prove his theorem, likely accounting for his "notorious" interpretation of Gödel's theorem as a paradox, and if we accept his argument, I think we are forced to deny the intelligibility of metalanguages, metatheories and meta anything else. How can it be that such concepts (words) as metamathematics and incompleteness, accepted by millions (and even claimed by no less than Penrose, Hawking, Dyson et al to reveal fundamental truths about our mind or the universe) are just simple misunderstandings about how language works? Isn't the proof in this pudding that, like so many "revelatory" philosophical notions (e.g., mind and will as illusions –Dennett, Carruthers, the Churchlands etc.), they have no practical impact whatsoever? Berto sums it up nicely: "Within this framework, it is not possible that the very same sentence…turns out to be expressible, but undecidable, in a formal system… and demonstrably true (under the aforementioned consistency

hypothesis) in a different system (the meta-system). If, as Wittgenstein maintained, the proof establishes the very meaning of the proved sentence, then it is not possible for the same sentence (that is, for a sentence with the same meaning) to be undecidable in a formal system, but decided in a different system (the meta-system) … Wittgenstein had to reject both the idea that a formal system can be syntactically incomplete, and the Platonic consequence that no formal system proving only arithmetical truths can prove all arithmetical truths. If proofs establish the meaning of arithmetical sentences, then there cannot be incomplete systems, just as there cannot be incomplete meanings." And further "Inconsistent arithmetics, i.e., nonclassical arithmetics based on a paraconsistent logic, are nowadays a reality. What is more important, the theoretical features of such theories match precisely with some of the aforementioned Wittgensteinian intuitions…Their inconsistency allows them also to escape from Gödel's First Theorem, and from Church's undecidability result: they are, that is, demonstrably complete and decidable. They therefore fulfil precisely Wittgenstein's request, according to which there cannot be mathematical problems that can be meaningfully formulated within the system, but which the rules of the system cannot decide. Hence, the decidability of paraconsistent arithmatics harmonizes with an opinion Wittgenstein maintained thoughout his philosophical career."

Wittgenstein also demonstrated the fatal error in regarding mathematics or language or our behavior in general as a unitary coherent logical 'system,' rather than as a motley of pieces assembled by the random processes of natural selection. "Gödel shows us an unclarity in the concept of 'mathematics', which is indicated by the fact that mathematics is taken to be a system" and we can say (contra nearly everyone) that is all that Gödel and Chaitin show. Wittgenstein commented many times that 'truth' in math means axioms or the theorems derived from axioms, and 'false' means that one made a mistake in using the definitions, and this is utterly different from empirical matters where one applies a test. Wittgenstein often noted that to be acceptable as mathematics in the usual sense, it must be useable in other proofs and it must have real world applications, but neither is the case with Godel's Incompleteness. Since it cannot be proved in a consistent system (here Peano Arithmetic but a much wider arena for Chaitin), it cannot be used in proofs and, unlike all the 'rest' of PA it cannot be used in the real world either. As Rodych notes "…Wittgenstein holds that a formal calculus is only a mathematical calculus (i.e., a mathematical language-game) if it has an extra- systemic application in a system of contingent propositions (e.g., in ordinary counting and measuring or in physics) …" Another way to say this is that one needs a warrant to apply our normal use of words like 'proof', 'proposition', 'true', 'incomplete', 'number', and 'mathematics' to a result in the tangle of games created with 'numbers' and 'plus' and 'minus' signs etc., and with

'Incompleteness' this warrant is lacking. Rodych sums it up admirably. "On Wittgenstein's account, there is no such thing as an incomplete mathematical calculus because 'in mathematics, everything is algorithm [and syntax] and nothing is meaning [semantics]…"

Wittgenstein has much the same to say of Cantor's diagonalization and set theory. "Consideration of the diagonal procedure shews you that the concept of 'real number' has much less analogy with the concept 'cardinal number' than we, being misled by certain analogies, are inclined to believe" and many other comments (see Rodych and Floyd).

In any case, it would seem that the fact that Gödel's result has had zero impact on math (except to stop people from trying to prove completeness!) should have alerted Hofstadter to its triviality and the "strangeness" of trying to make it a basis for anything. I suggest that it be regarded as another conceptual game that shows us the boundaries of our psychology. Of course, all of math, physics, and human behavior can usefully be taken this way.

By far the most famous (or notorious I would say) remarks on Gödel are those by Douglas Hofstadter in his book Gödel, Escher, Bach: the eternal golden Braid which was subsequently disussed in his book I am a Strange Loop.

Regarding Gödel's famous theorems, in what sense can they be loops? What they are almost universally supposed to show is that certain basic kinds of mathematical systems are incomplete in the sense that there are "true" theorems of the system whose "truth" (the unfortunate word mathematicians commonly substitute for validity) or "falsity (invalidity) cannot be proven in the system. Though Hofstadter does not tell you, these theorems are logically equivalent to Turing's "incompleteness" solution of the famous halting problem for computers performing some arbitrary calculation. He spends a lot of time explaining Gödel's original proof, but fails to mention that others subsequently found vastly shorter and simpler proofs of "incompleteness" in math and proved many related concepts. The one he does briefly mention is that of contemporary mathematician Gregory Chaitin—an originator with Kolmogorov and others of Algorithmic Information Theory-- who has shown that such "incompleteness" or "randomness" (Chaitin's term-- though this is another game), is much more extensive than long thought, but does not tell you that both Gödel's and Turing's results are corollaries to Chaitin's theorem and an instance of "algorithmic randomness". You should refer to Chaitin's more recent writings such as "The Omega Number (2005)", as Hofstadter's only ref. to Chaitin is 20 years old (though Chaitin has no more grasp of the larger issues here –i.e., innate intentionality as the source of the language games in math-- than does H and shares the 'Universe is a Computer" fantasy as well).

Hofstadter takes this “incompleteness” – a language game out of context - to mean that the system is self referential or “loopy” and “strange”. It is not made clear why having theorems that seem to be (or are) true (i.e., valid) in the system, but not provable in it, makes it a loop nor why this qualifies as strange nor why this has any relationship to
anything else.

Hofstadter, in all his writings, follows the common trend and makes much of “paradoxes”, which he regards as self references, recursions or loops, but there are many “inconsistencies” in intentional psychology (math, language, perception, art etc.) and they have no effect, as our psychology evolved to ignore them. Thus, “paradoxes” such as “this sentence is false” only tell us that “this” does not refer to itself or if you prefer that this is one of infinitely many arrangements of words lacking a clear sense. Any symbolic system we have (i.e., language, math, art, music, games etc.) will always have areas of conflict, insoluble or counterintuitive problems or unclear definitions. Hence, we have Gödel’s theorems, the liar’s paradox, inconsistencies in set theory, prisoner’s dilemmas, Schrodinger’s dead/live cat, Newcomb’s problem, Anthropic principles, Bayesian statistics, notes you can’t sound together or colors you can’t mix together and rules that can’t be used in the same game. A set of subindustries within Decision Theory, Behavioral Economics, Game Theory, Philosophy, Psychology and Sociology, Law, Political Science etc. and even the Foundations of Physics and Math (where it is commonly disguised as Philosophy of Science) has arisen which deals with endless variations on “real” (e.g., quantum mechanics) or contrived ((e.g., Newcomb’s problem—see Analysis V64, p187- 89(2004)) situations where our psychology –evolved only to get food, find mates and avoid becoming lunch—gives ambivalent results, or just breaks down.

In the recent book ‘Gödel's Way’ three eminent scientists discuss issues such as undecidability, incompleteness, randomness, computability and paraconsistency. I approach these issues from the Wittgensteinian viewpoint that there are two basic issues which have completely different solutions. There are the scientific or empirical issues, which are facts about the world that need to be investigated observationally and philosophical issues as to how language can be used intelligibly (which include certain questions in mathematics and logic), which need to be decided by looking at how we actually use words in particular contexts. When we get clear about which language game we are playing, these topics are seen to be ordinary scientific and mathematical questions like any others. Wittgenstein’s insights have seldom been equaled and never surpassed and are as pertinent today as they were 80 years ago when he dictated the Blue and Brown Books. In spite of its failings—really a series of notes rather than a finished book—this is a unique source of the work of these three famous scholars who have been working at the bleeding edges of physics, math and philosophy for over half a century. Da Costa and Doria are cited by Wolpert (see below or my articles on Wolpert and my review of Yanofsky's 'The Outer Limits of Reason') since they wrote on universal computation, and among his many accomplishments, Da Costa is a pioneer in paraconsistency.

Chaitin's proof of the algorithmic randomness of math (of which Gödel's results are a corollary) and the Omega number are some of the most famous mathematical results in the last 50 years and he has documented them in many books and articles. His coauthors from Brazil are less well known in spite of their many important contributions. For all the topics here, the best way to get free articles and books on the cutting edge is to visit ArXiv.org, viXra.org, academia.edu, citeseerx.ist.psu.edu, philpapers.org, researchgate.net, libgen.io or b-ok.org etc., where there are millions of preprints/articles/books on every topic (be warned this may use up all your spare time for the rest of your life!).

Chaitin is an American and his many books and articles are well known and easy to find, but Da Costa (who is over 90) and Doria (over 80) are Brazilians and most of Da Costa's work is only in Portuguese, but Doria has many items in English. You can find a partial bibliography for Doria here http://www.math.buffalo.edu/mad/PEEPS2/doria_franciscoA.html and of course see their Wikis.

The best collections of their work are in Chaos, Computers, Games and Time: A quarter century of joint work with Newton da Costa by F. Doria 132p(2011), On the Foundations of Science by da Costa and Doria 294p(2008), and Metamathematics of science by da Costa and Doria 216p(1997), but they were published in Brazil and almost impossible to find. You will likely have to get them through interlibrary loan or as digital files from the authors, but as always try libgen.io and b-ok.org.

There is a nice Festschrift in honor of Newton C.A. Da Costa on the occasion of his seventieth birthday edited by Décio Krause, Steven French, Francisco

Antonio Doria. (2000) which is an issue of Synthese (Dordrecht). Vol. 125, no. 1- 2 (2000), also published as a book, but the book is in only 5 libraries worldwide and not on Amazon.

See also Doria (Ed.), "The Limits Of Mathematical Modeling In The Social

Sciences: The Significance Of Gödel's Incompleteness Phenomenon" (2017) and Wuppuluri and Doria (Eds.), "The Map and the Territory: Exploring the foundations of science, thought and reality" (2018).

Another relevant item is New trends in the foundations of science : papers dedicated to the 80th birthday of Patrick Suppes, presented in Florianópolis, Brazil, April 22-23, 2002 by Jean-Yves Beziau; Décio Krause; Otávio Bueno; Newton C da Costa; Francisco Antonio Doria; Patrick Suppes; (2007), which is vol. 154 # 3 of Synthese, but again the book is in only 2 libraries and not on Amazon.

Brazilian studies in philosophy and history of science: an account of recent works by Decio Krause; Antônio Augusto Passos Videira; has one article by each of them and is an expensive book but cheap on Kindle. Though it is a decade old, some may be interested in "Are the Foundations of Computer Science Logic-dependent?" by Carnielli and Doria, which says that Turing Machine Theory (TMT) can be seen as 'arithmetic in disguise', in particular as the theory of Diophantine Equations in which they formalize it, and conclude that 'Axiomatized Computer Science is Logic-Dependent'. Of course, as Wittgensteinians, we want to look very carefully at the language games (or math games), i.e., the precise Conditions of Satisfaction (truthmakers) resulting from using each of these words (i.e., 'axiomatized', 'computer science', and 'logic-dependent'). Carnielli and Agudello also formalize TMT in terms of paraconsistent logic, creating a model for paraconsistent Turing Machines (PTM's) which has similarities to quantum computing and so with a quantic interpretation of it they create a Quantum Turing Machine model with which they solve the Deutsch and Deutsch-Jozsa problems.

This permits contradictory instructions to be simultaneously executed and stored and each tape cell, when and if halting occurs, may have multiple symbols, each of which represents an output, thus permitting control of unicity versus multiplicity conditions, which simulate quantum algorithms, preserving efficiency.

Doria and Da Costa also proved (1991) that chaos theory is undecidable, and when properly axiomatized within classical set theory, is incomplete in Gödel's sense.

The articles, and especially the group discussion with Chaitin, Fredkin, Wolfram et al at the end of Zenil H. (ed.) 'Randomness through computation' (2011) is a stimulating continuation of many of the topics here, but again lacking awareness of the philosophical issues, and so often missing the point. Chaitin also contributes to 'Causality, Meaningful Complexity and Embodied Cognition' (2010), replete with articles having the usual mixture of scientific insight and philosophical incoherence, and as usual nobody is aware that Wittgenstein provided deep and unsurpassed insights into the issues over half a century ago, including Embodied Cognition (Enactivism).

Since Gödel's theorems are corollaries of Chaitin's theorem showing algorithmic randomness (incompleteness) throughout math (which is just another of our symbolic systems that may result in public testable actions-i.e., if meaningful it has COS), it seems inescapable that thinking (dispositional

behavior having COS) is full of impossible, random or incomplete statements and situations. Since we can view each of these domains as symbolic systems evolved by chance to make our psychology work, perhaps it should be regarded as unsurprising that they are not "complete". For math, Chaitin says this 'randomness' (another group of language games) shows there are limitless theorems that are 'true' but unprovable—i.e., 'true' for no 'reason'. One should then be able to say that there are limitless statements that make perfect "grammatical" sense that do not describe actual situations attainable in that domain. I suggest these puzzles go away if one considers Wittgenstein's views. He wrote many notes on the issue of Godel's Theorems, and the whole of his work concerns the plasticity, "incompleteness" and extreme context sensitivity of language, math and logic, and the recent papers of Rodych, Floyd and Berto are the best introduction I know of to Wittgenstein's remarks on the foundations of mathematics and so to philosophy.

Regarding Gödel and "incompleteness", since our psychology as expressed in symbolic systems such as math and language is "random" or "incomplete" and full of tasks or situations ("problems") that have been proven impossible (i.e., they have no solution-see below) or whose nature is unclear, it seems unavoidable that everything derived from it by using higher order thought (system 2 or S2) to extend our innate axiomatic psychology (System 1 or S1) into complex social interactions such as games, economics, physics and math, will be "incomplete" also.

The first of these in what is now called Social Choice Theory or Decision Theory (which are continuous with the study of logic and reasoning and philosophy) was the famous theorem of Kenneth Arrow 63 years ago, and there have been many since such as the recent impossibility or incompleteness proof by Brandenburger and Kreisel (2006) in two-person game theory. In these cases, a proof shows that what looks like a simple choice stated in plain English has no solution. There are also many famous "paradoxes" such as Sleeping Beauty (dissolved by Rupert Read), Newcomb's problem (dissolved by Wolpert) and Doomsday, where what seems to be a very simple problem either has no one clear answer, or it proves exceptionally hard to find. A mountain of literature exists on Godel's two "incompleteness" theorems and Chaitin's more recent work, but I think that W's writings in the 30's and 40's are definitive. Although Shanker, Mancosu, Floyd, Marion, Rodych, Gefwert, Wright and others have done insightful work in explaining W, it is only recently that W's uniquely penetrating analysis of the language games being played in mathematics and logic have been clarified by Floyd (e.g., 'Wittgenstein's Diagonal Argument-a Variation on Cantor and Turing'), Berto (e.g., 'Godel's Paradox and Wittgenstein's Reasons' , and 'Wittgenstein on Incompleteness makes

Paraconsistent Sense', and Rodych (e.g., 'Wittgenstein and Godel's: the Newly Published Remarks' and 'Misunderstanding Gödel :New Arguments about Wittgenstein and New Remarks by Wittgenstein'). Berto is one of the best recent philosophers, and those with time might wish to consult his many other articles and books including the volume he co- edited on paraconsistency. Rodych's work is indispensable, but only two of a dozen or so papers are free online (but see b-ok.org and also his online Stanford Encyclopedia of Philosophy articles).

Wittgenstein also demonstrated the fatal error in regarding mathematics or language or our behavior in general as a unitary coherent logical 'system,' rather than as a motley of pieces assembled by the random processes of natural selection. "Godel shows us an unclarity in the concept of 'mathematics', which is indicated by the fact that mathematics is taken to be a system" and we can say (contra nearly everyone) that is all that Godel and Chaitin show. Wittgenstein commented many times that 'truth' in math means axioms or the theorems derived from axioms, and 'false' means that one made a mistake in using the definitions (from which results follow necessarily and algorithmically), and this is utterly different from empirical matters where one applies a test (the results of which are unpredictable and debatable). Wittgenstein often noted that to be acceptable as mathematics in the usual sense, it must be useable in other proofs and it must have real world applications, but neither is the case with Godel's Incompleteness. Since it cannot be proved in a consistent system (here Peano Arithmetic but a much wider arena for Chaitin), it cannot be used in proofs and, unlike all the 'rest' of Peano Arithmetic, it cannot be used in the real world either. As Rodych notes "…Wittgenstein holds that a formal calculus is only a mathematical calculus (i.e., a mathematical language-game) if it has an extrasystemic application in a system of contingent propositions (e.g., in ordinary counting and measuring or in physics) …" Another way to say this is that one needs a warrant to apply our normal use of words like 'proof', 'proposition', 'true', 'incomplete', 'number', and 'mathematics' to a result in the tangle of games created with 'numbers' and 'plus' and 'minus' signs etc., and with 'Incompleteness' this warrant is lacking. Rodych sums it up admirably. "On Wittgenstein's account, there is no such thing as an incomplete mathematical calculus because 'in mathematics, everything is algorithm [and syntax] and nothing is meaning [semantics]…"

W has much the same to say of Cantor's diagonalization and set theory. "Consideration of the diagonal procedure shews you that the concept of 'real number' has much less analogy with the concept 'cardinal number' than we, being misled by certain analogies, are inclined to believe" and makes many other penetrating comments (see Rodych and Floyd). Of course, the same remarks apply to all forms of logic and any other symbolic system.

As Rodych, Berto and Priest (another pioneer in paraconsistency) have noted, W was the first (by several decades) to insist on the unavoidability and utility of inconsistency (and debated this issue with Turing during his classes on the Foundations of Mathematics). We now see that the disparaging comments about Wittgenstein's remarks on math made by Godel, Kreisel, Dummett and many others were misconceived. As usual, it is a very bad idea to bet against Wittgenstein. Some may feel we have strayed off the path here—after all in 'Godel's Way' we only want to understand 'science' and 'mathematics' (in quotes because part of the problem is regarding them as 'systems') and why these 'paradoxes' and 'inconsistencies' arise and how to dispose of them. But I claim that is exactly what I have done by pointing to the work of W. Our symbolic systems (language, math, logic, computation) have a clear use in the narrow confines of everyday life, in what we can loosely call the mesoscopic realm--the space and time of normal events we can observe unaided and with certainty (the innate axiomatic bedrock or background as Wittgenstein and later Searle call it). But we leave coherence behind when we enter the realms of particle physics or the cosmos, relativity, math beyond simple addition and subtraction with whole numbers, and language used out of the immediate context of everyday events. The words or whole sentences may be the same, but the meaning is lost (i.e., to use Searle's preferred term, their Conditions of Satisfaction (COS) are changed or opaque). It looks to me like the best way to understand philosophy may be to enter it via Berto, Rodych and Floyd's work on Wittgenstein, so as to understand the subtleties of language as it is used in math and thereafter "metaphysical" issues of all kinds may be dissolved. As Floyd notes "In a sense, Wittgenstein is literalizing Turing’s model, bringing it back down to the everyday and drawing out the anthropomorphic command- aspect of Turing’s metaphors.”

As Wittgenstein noted, most of what people (including many philosophers and most scientists) have to say when philosophizing is not philosophy but its raw material. Chaitin, Doria, and Da Costa join Yanofsky, Hume, Quine, Dummett, Kripke, Dennett, Churchland, Carruthers, Wheeler etc. in repeating the mistakes of the Greeks with elegant philosophical jargon mixed with science. I suggest quick antidotes via my reviews and some Rupert Read such as his books ‘A Wittgensteinian Way with Paradoxes’ and ‘Wittgenstein Among the Sciences’, or go to academia.edu and get his articles , especially ‘Kripke’s Conjuring Trick’ and ‘Against Time Slices’ and then as much of Searle as feasible, but at least his most recent such as ‘Philosophy in a New Century’, ‘Searle’s Philosophy and Chinese Philosophy’, ‘Making the Social World’ and ‘Thinking About the Real World’ (or at least my reviews) and his recent volume on perception. There are also over 100 YouTubes of Searle, which confirm his reputation as the best standup philosopher since Wittgenstein.

A major overlap that now exists (and is expanding rapidly) between game theorists, physicists, economists, mathematicians, philosophers, decision theorists and others, all of whom have been publishing for decades closely related proofs of undecidability, impossibility, uncomputability, and incompleteness. One of the more bizarre is the recent proof by Armando Assis that in the relative state formulation of quantum mechanics one can setup a zero-sum game between the universe and an observer using the Nash Equilibrium, from which follow the Born rule and the collapse of the wave function. Godel was first to demonstrate an impossibility result and (until Chaitin and above all Wolpert— see my article on his work) it is the most far reaching (or just trivial/incoherent), but there have been an avalanche of others. As noted, one of the earliest in decision theory was the famous General Impossibility Theorem (GIT) discovered by Kenneth Arrow in 1951 (for which he got the Nobel Prize in economics in 1972—and five of his students are now Nobel laureates so this is not fringe science). It states roughly that no reasonably consistent and fair voting system (i.e., no method of aggregating individuals’ preferences into group preferences) can give sensible results. The group is either dominated by one person and so GIT is often called the “dictator theorem”, or there are intransitive preferences. Arrow’s original paper was titled "A Difficulty in the Concept of Social Welfare" and can be stated like this:” It is impossible to formulate a social preference ordering that satisfies all of the following conditions: Nondictatorship; Individual Sovereignty; Unanimity; Freedom From Irrelevant Alternatives; Uniqueness of Group Rank.”

Those familiar with modern decision theory accept this and the many related

constraining theorems as their starting points. Those who are not may find it (and all these theorems) incredible and in that case, they need to find a career path that has nothing to do with any of the above disciplines. See "The Arrow Impossibility Theorem"(2014) or "Decision Making and Imperfection"(2013) among legions of publications.

Another recent famous impossibility result is that of Brandenburger and Keisler (2006) for two person games (but of course not limited to "games" and like all these impossibility results it applies broadly to decisions of any kind), which shows that any belief model of a certain kind leads to contradictions. One interpretation of the result is that if the decision analyst's tools (basically just logic) are available to the players in a game, then there are statements or beliefs that the players can write down or 'think about' but cannot actually hold. But note Wittgenstein's characterization of 'thinking' as a potential action with COS, which says they don't really have a meaning (use), like Chaitin's infinity of apparently well-formed formulas that do not actually belong to our system of mathematics. "Ann believes that Bob assumes that Ann believes that Bob's assumption is wrong" seems unexceptionable and multiple layers of 'recursion' (another LG) have been assumed in argumentation, linguistics, philosophy etc., for a century at least, but B&K showed that it is impossible for Ann and Bob to assume these beliefs. And there is a rapidly growing body of such impossibility results for one person or multiplayer decision situations (e.g., they grade into Arrow, Wolpert, Koppel and Rosser etc.). For a good technical paper from among the avalanche on the B&K paradox, get Abramsky and Zvesper's paper from arXiv which takes us back to the liar paradox and Cantor's infinity (as its title notes it is about "interactive forms of diagonalization and self-reference") and thus to Floyd, Rodych, Berto, W and Godel. Many of these papers quote Yanofsky's paper "A universal approach to self- referential paradoxes and fixed points. Bulletin of Symbolic Logic, 9(3):362–386,2003.

Abramsky (a polymath who is among other things a pioneer in quantum computing) is a friend of Y's and so Y contributes a paper to the recent Festschrift to him 'Computation, Logic, Games and Quantum Foundations'(2013). For maybe the best recent (2013) commentary on the BK and related paradoxes see the 165p powerpoint lecture free on the net by Wes Holliday and Eric Pacuit 'Ten Puzzles and Paradoxes about Knowledge and Belief'. For a good multi-author survey see 'Collective Decision Making (2010).

One of the major omissions from all such books is the amazing work of polymath physicist and decision theorist David Wolpert, who proved some stunning impossibility or incompleteness theorems (1992 to 2008-see arxiv.org) on the limits to inference (computation) that are so general they are

independent of the device doing the computation, and even independent of the laws of physics, so they apply across computers, physics, and human behavior, which he summarized thusly: "One cannot build a physical computer that can be assured of correctly processing information faster than the universe does. The results also mean that there cannot exist an infallible, general-purpose observation apparatus, and that there cannot be an infallible, general-purpose control apparatus. These results do not rely on systems that are infinite, and/or non-classical, and/or obey chaotic dynamics. They also hold even if one uses an infinitely fast, infinitely dense computer, with computational powers greater than that of a Turing Machine." He also published what seems to be the first serious work on team or collective intelligence (COIN) which he says puts this subject on a sound scientific footing. Although he has published various versions of these proofs over two decades in some of the most prestigious peer reviewed physics journals (e.g., Physica D 237: 257-81(2008)) as well as in NASA journals and has gotten news items in major science journals, few seem to have noticed, and I have looked in dozens of recent books on physics, math, decision theory and computation without finding a reference.

Wittgenstein's prescient grasp of these issues, including his embrace of strict finitism and paraconsistency, is finally spreading through math, logic and computer science (though rarely with any acknowledgement). Bremer has recently suggested the necessity of a Paraconsistent Lowenheim-Skolem Theorem. "Any mathematical theory presented in first order logic has a finite paraconsistent model." Berto continues: "Of course strict finitism and the insistence on the decidability of any meaningful mathematical question go hand in hand. As Rodych has remarked, the intermediate Wittgenstein's view is dominated by his 'finitism and his view […] of mathematical meaningfulness as algorithmic decidability' according to which '[only] finite logical sums and products (containing only decidable arithmetic predicates) are meaningful because they are algorithmically decidable.'". In modern terms this means they have public conditions of satisfaction (COS)-i.e., can be stated as a proposition that is true or false. And this brings us to W's view that ultimately everything in math and logic rests on our innate (though of course extensible) ability to recognize a valid proof. Berto again: "Wittgenstein believed that the naïve (i.e., the working mathematician's) notion of proof had to be decidable, for lack of decidability meant to him simply lack of mathematical meaning: Wittgenstein believed that everything had to be decidable in mathematics…Of course one can speak against the decidability of the naïve notion of truth on the basis of Godel's results themselves. But one may argue that, in the context, this would beg the question against paraconsistentists-- and against Wittgenstein too. Both Wittgenstein and the paraconsistentists on one side, and the followers of the standard view on the other, agree on the following thesis: the decidability of the notion of proof and

its inconsistency are incompatible. But to infer from this that the naïve notion of proof is not decidable invokes the indispensability of consistency, which is exactly what Wittgenstein and the paraconsistent argument call into question...for as Victor Rodych has forcefully argued, the consistency of the relevant system is precisely what is called into question by Wittgenstein's reasoning." And so: "Therefore the Inconsistent arithmetic avoids Godel's First Incompleteness Theorem. It also avoids the Second Theorem in the sense that its non-triviality can be established within the theory: and Tarski's Theorem too—including its own predicate is not a problem for an inconsistent theory" [As Graham Priest noted over 20 years ago].

This again brings to mind Wittgenstein's famous comment.

"What we are 'tempted to say' in such a case is, of course, not philosophy, but it is its raw material. Thus, for example, what a mathematician is inclined to say about the objectivity and reality of mathematical facts, is not a philosophy of mathematics, but something for philosophical treatment." PI 234

And again, 'decidability' comes down to the ability to recognize a valid proof, which rests on our innate axiomatic psychology, which math and logic have in common with language. And this is not just a remote historical issue but is totally current. I have read much of Chaitin and never seen a hint that he has considered these matters.

Again the work of Douglas Hofstadter also comes to mind. His Godel, Escher, Bach won a Pulitzer prize and a National Book Award for Science, sold millions of copies and continues to get good reviews (e.g. almost 400 mostly 5 star reviews on Amazon to date) but he has no clue about the real issues and repeats the classical philosophical mistakes on nearly every page. His subsequent philosophical writings have not improved (he has chosen Dennett as his muse), but, as these views are vacuous and unconnected to real life, he continues to do excellent science.

Once again note that “infinite”, “compute”, “information” etc., only have meaning in specific human contexts— that is, as Searle has emphasized, they are all observer relative or ascribed vs intrinsically intentional. The universe apart from our psychology is neither finite nor infinite and cannot compute nor process anything. Only in our language games do our laptop or the universe compute.

Yanofsky’s book (The Outer Limits of Reason) is an extended treatment of these issues, but with little philosophical insight. He says math is free of contradictions, yet as noted, it has been well known for over half a century that logic and math are full of them—just google inconsistency in math or search it on Amazon or see the works of Priest, Berto or the article by Weber in the Internet Encyclopedia of Philosophy. W was the first to predict inconsistency or paraconsistency, and if we follow Berto we can interpret this as Wittgenstein’s suggestion to avoid incompleteness. In any event, paraconsistency is now a common feature and a major research program in geometry, set theory, arithmetic, analysis, logic and computer science. Yanofsky on p346 says reason must be free of contradictions, but it is clear that “free of” has different uses and they arise frequently in everyday life, but we have innate mechanisms to contain them. This is true because it was the case in our everyday life long before math and science. Until very recently only Wittgenstein saw that it was unavoidable that our life and all our symbolic systems are paraconsistent and that we get along just fine as we have mechanisms for encapsulating or avoiding it. Wittgenstein tried to explain this to Turing in his lectures on the foundations of mathematics, given at Cambridge at the same time as Turing’s course on the same topic.

Now I will make a few comments on specific items in Yanofsky’s book. As noted on p13, Rice’s Theorem shows the impossibility of a universal antivirus for computers (and perhaps for living organisms as well) and so is, like Turing’s Halting theorem, another alternative statement of Godel’s Theorems, but unlike Turing’s, it is rarely mentioned.

On p33 the discussion of the relation of compressibility, structure, randomness etc. is much better stated in Chaitin’s many other books and papers. Also of fundamental importance is the comment by Weyl on the fact that one can ‘prove’ or ‘derive’ anything from anything else if one permits arbitrarily ‘complex’ ‘equations’ (with arbitrary ‘constants’) but there is little awareness of this among scientists or philosophers. As W said we need to look at the role which any statement, equation, logical or mathematical proof plays in our life in order to discern its meaning since there is no limit on what we can write, say or ‘prove’, but only a tiny subset of these has a use. ‘Chaos’, ‘complexity’, ‘law’, ‘structure’, ‘theorem’, ‘equation’, ‘proof’, ‘result’, ‘randomness’, ‘compressibility’ etc. are all families of language games with meanings (COS) that vary greatly, and one must look at their precise role in the given context.

Likewise, on p54 et seq. it was Wittgenstein who has given us the first and best rationale for paraconsistency, long before anyone actually worked out a paraconsistent logic. Again, as W pointed out many times, it is critical to be aware that not everything is a ‘problem’, ‘question’, ‘answer’, ‘proof’ or a ‘solution’ in the same sense and accepting something as one or the other commits one to an often confused point of view.

In the discussion of physics on p108-9 we must remind ourselves that ‘point’, ‘energy’, ‘space’, ‘time’, ‘infinite’, ‘beginning’, ‘end’, ‘particle’, ’wave’, ‘quantum’ etc. are all typical language games that seduce us into incoherent views of how things are by applying meanings (COS) from one game to a quite different one.

I have read many recent discussions of the limits of computation and the universe as computer, hoping to find some comments on the amazing work of polymath physicist and decision theorist David Wolpert but have not found a single citation and so I present this very brief summary. Wolpert proved some stunning impossibility or incompleteness theorems (1992 to 2008-see arxiv.org) on the limits to inference (computation) that are so general they are independent of the device doing the computation, and even independent of the laws of physics, so they apply across computers, physics, and human behavior. They make use of Cantor's diagonalization, the liar paradox and worldlines to provide what may be the ultimate theorem in Turing Machine Theory, and seemingly provide insights into impossibility, incompleteness, the limits of computation, and the universe as computer, in all possible universes and all beings or mechanisms, generating, among other things, a non- quantum mechanical uncertainty principle and a proof of monotheism. There are obvious connections to the classic work of Chaitin, Solomonoff, Komolgarov and Wittgenstein and to the notion that no program (and thus no device) can generate a sequence (or device) with greater complexity than it possesses. One might say this body of work implies atheism since there cannot be any entity more complex than the physical universe and from the Wittgensteinian viewpoint, ‘more complex’ is meaningless (has no conditions of satisfaction, i.e., truth-maker or test). Even a ‘God’ (i.e., a ‘device’with limitless time/space and energy) cannot determine whether a given ‘number’ is ‘random’, nor find a certain way to show that a given ‘formula’, ‘theorem’ or ‘sentence’ or ‘device’ (all these being complex language games) is part of a particular ‘system’.

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It is most unfortunate that almost nobody is aware of Wolpert, since his work can be seen as the ultimate extension of computing, thinking, inference, incompleteness, and undecidability, which he achieves (like many proofs in Turing machine theory) by extending the liar paradox and Cantors diagonalization to include all possible universes and all beings or mechanisms and thus may be seen as the last word not only on computation, but on cosmology or even deities. He achieves this extreme generality by partitioning the inferring universe using worldlines (i.e., in terms of what it does and not how it does it) so that his mathematical proofs are independent of any particular physical laws or computational structures in establishing the physical limits of inference for past, present and future and all possible calculation, observation and control. He notes that even in a classical universe Laplace was wrong about being able to perfectly predict the future (or even perfectly depict the past or present) and that his impossibility results can be viewed as a "non-quantum

mechanical uncertainty principle" (i.e., there cannot be an infallible observation or control device). Any universal physical device must be infinite, it can only be so at one moment in time, and no reality can have more than one (the "monotheism theorem"). Since space and time do not appear in the definition, the device can even be the entire universe across all time. It can be viewed as a physical analog of incompleteness with two inference devices rather than one self-referential device. As he says, "either the Hamiltonian of our universe proscribes a certain type of computation, or prediction complexity is unique (unlike algorithmic information complexity) in that there is one and only one version of it that can be applicable throughout our universe." Another way to say this is that one cannot have two physical inference devices (computers) both capable of being asked arbitrary questions about the output of the other, or that the universe cannot contain a computer to which one can pose any arbitrary computational task, or that for any pair of physical inference engines, there are always binary valued questions about the state of the universe that cannot even be posed to at least one of them. One cannot build a computer that can predict an arbitrary future condition of a physical system before it occurs, even if the condition is from a restricted set of tasks that can be posed to it— that is, it cannot process information (though this is a vexed phrase, as many including John Searle and Rupert Read note) faster than the universe.

The computer and the arbitrary physical system it is computing do not have to be physically coupled and it holds regardless of the laws of physics, chaos, quantum mechanics, causality or light cones and even for an infinite speed of light. The inference device does not have to be spatially localized but can be nonlocal dynamical processes occurring across the entire universe. He is well aware that this puts the speculations of Wolfram, Landauer, Fredkin, Lloyd etc., concerning the universe as computer or the limits of "information processing", in a new light (though the indices of their writings make no reference to him and another remarkable omission is that none of the above are mentioned by Yanofsky in his recent comprehensive book 'The Outer Limits of Reason' (see my review). Wolpert says he shows that 'the universe' cannot contain an inference device that can 'process information' as fast as it can, and since he shows you cannot have a perfect memory nor perfect control, its past, present or future state can never be perfectly or completely depicted, characterized, known or copied. He also proved that no combination of computers with error correcting codes can overcome these limitations. Wolpert also notes the critical importance of the observer ("the liar") and this connects us to the familiar conundrums of physics, math and language. As noted in my other articles I think that definitive comments on many relevant issues here (completeness, certainty, the nature of computation etc.) were made long ago by Ludwig Wittgenstein and here is one relevant comment of Juliet Floyd on Wittgenstein:

"He is articulating in other words a generalized form of diagonalization. The argument is thus generally applicable, not only to decimal expansions, but to any purported listing or rule-governed expression of them; it does not rely on any particular notational device or preferred spatial arrangements of signs. In that sense, Wittgenstein's argument appeals to no picture and it is not essentially diagrammatical or representational, though it may be diagrammed and insofar as it is a logical argument, its logic may be represented formally). Like Turing's arguments, it is free of a direct tie to any particular formalism. Unlike Turing's arguments, it explicitly invokes the notion of a language-game and applies to (and presupposes) an everyday conception of the notions of rules and of the humans who follow them. Every line in the diagonal presentation above is conceived as an instruction or command, analogous to an order given to a human being..." The parallels to Wolpert are obvious.

K and R's second theorem shows possible nonconvergence for Bayesian (probabilistic) forecasting in infinite- dimensional space. The third shows the impossibility of a computer perfectly forecasting an economy with agents knowing its forecasting program. The astute will notice that these theorems can be seen as versions of the liar paradox, and the fact that we are caught in impossibilities when we try to calculate a system that includes ourselves has been noted by Wolpert, Koppl, Rosser and others in these contexts and again we have circled back to the puzzles of physics when the observer is involved. K&R conclude "Thus, economic order is partly the product of something other than calculative rationality".

Bounded rationality is now a major field in itself, the subject of thousands of papers and hundreds of books. And this seemingly abstruse work of Wolpert's may have implications for all rationality. Of course, one must keep in mind that (as Wittgenstein noted) math and logic are all syntax and no semantics and they have nothing to tell us until connected to our life by language (i.e., by psychology) and so it is easy to do this in ways that are useful (meaningful or having COS) or not (no clear COS).

Finally, one might say that many of Wolpert's comments are restatements of the idea that no program (and thus no device) can generate a sequence (or device) with greater complexity than it possesses. There are obvious connections to the classic work of Chaitin, Solomonoff, Komolgarov and Wittgenstein and to the notion that no program (and thus no device) can generate a sequence (or device) with greater complexity than it possesses. One might say this body of work implies atheism since there cannot be any entity more complex than the physical universe and from the Wittgensteinian viewpoint, 'more complex' is meaningless (has no conditions of satisfaction, i.e., truth-maker or test). Even a 'God' (i.e., a 'device' with limitless time/space and energy) cannot determine whether a given 'number' is 'random' nor can find a certain way to show that a given 'formula', 'theorem' or 'sentence' or 'device' (all these being complex language games) is part of a particular 'system'.

Regarding "incompleteness" or "randomness" in math, Y's failure to mention the work of Gregory Chaitin is truly amazing, as he must know of his work, and Chaitin's proof of the algorithmic randomness of math (of which Godel's results are a corollary) and the Omega number are some of the most famous mathematical results in the last 50 years.

A mountain of literature exists on Godel's two "incompleteness" theorems and Chaitin's more recent work, but I think that W's writings in the 30's and 40's are definitive. Although Shanker, Mancosu, Floyd, Marion, Rodych, Gefwert, Wright and others have done insightful work, it is only recently that W's uniquely penetrating analysis of the language games being played in mathematics have been clarified by Floyd (e.g., 'Wittgenstein's Diagonal Argument-a Variation on Cantor and Turing'), Berto (e.g., 'Godel's Paradox and Wittgenstein's Reasons , and 'Wittgenstein on Incompleteness makes Paraconsistent Sense' and the book 'There's Something about Godel ', and Rodych (e.g., Wittgenstein and Godel: the Newly Published Remarks', 'Misunderstanding Gödel : New Arguments about Wittgenstein', 'New Remarks by Wittgenstein' and his article in the online Stanford Encyclopedia of Philosophy 'Wittgenstein's Philosophy of Mathematics'). Berto is one of the best recent philosophers, and those with time might wish to consult his many other articles and books including the volume he co-edited on paraconsistency (2013). Rodych's work is indispensable, but only two of a dozen or so papers are free online with the usual search but it's probably all free online if one knows where to look.

Wittgenstein pointed out how in math, we are caught in more LG's (Language Games) where it is not clear what "true", "complete", "follows from", "provable", "number", "infinite", etc. mean (i.e., what are their COS or truthmakers in THIS context), and hence what significance to attach to 'incompleteness' and likewise for Chaitin's "algorithmic randomness". As Wittgenstein noted frequently, do the "inconsistencies" of math or the counterintuitive results of metaphysics cause any real problems in math, physics or life? The apparently more serious cases of contradictory statements –e.g., in set theory---have long been known but math goes on anyway. Likewise for the countless liar (self-referencing) paradoxes in language which Yanofsky discusses, but he does not really understand their basis, and fails to make clear that self-referencing is involved in the "incompleteness" and “inconsistency” (groups of complex language games) of mathematics as well.

Yanofsky mentions the famous impossibility result of Brandenburger and Keisler (2006) for two person games (but of course not limited to “games” and like all these impossibility results it applies broadly to decisions of any kind) which shows that any belief model of a certain kind leads to contradictions. One interpretation of the result is that if the decision analyst’s tools (basically just logic) are available to the players in a game, then there are statements or beliefs that the players can write down or ‘think about’ but cannot actually hold. “Ann believes that Bob assumes that Ann believes that Bob’s assumption is wrong” seems unexceptionable and ‘recursion’ (another language game) has been assumed in argumentation, linguistics, philosophy etc., for a century at least, but they showed that it is impossible for Ann and Bob to assume these beliefs. And there is a rapidly growing body of such impossibility results for 1 or multiplayer decision situations (e.g., it grades into Arrow, Wolpert, Koppel and Rosser etc). For a good technical paper from among the avalanche on the B&K paradox, get Abramsky and Zvesper’s paper from arXiv which takes us back to the liar paradox and Cantor’s infinity (as its title notes it is about “interactive forms of diagonalization and self-reference”) and thus to Floyd, Rodych, Berto, Wittgenstein and Godel. Many of these papers quote Yanofsky’s paper “A universal approach to self- referential paradoxes and fixed points. Bulletin of Symbolic Logic, 9(3):362–386, 2003. Abramsky (a polymath who is among other things a pioneer in quantum computing) is a friend of Yanofsky’s and so Yanofsky contributes a paper to the recent Festschrift to him ‘Computation, Logic, Games and Quantum Foundations’ (2013). For maybe the best recent (2013) commentary on the BK and related paradoxes see the 165p powerpoint lecture free on the net by Wes Holliday and Eric Pacuit ’Ten Puzzles and Paradoxes about Knowledge and Belief’. For a good multi-author survey see ’Collective Decision Making (2010).

One of the major omissions from all such books is the amazing work of polymath physicist and decision theorist David Wolpert, who proved some stunning impossibility or incompleteness theorems (1992 to 2008-see arxiv.org) on the limits to inference (computation) that are so general they are independent of the device doing the computation, and even independent of the laws of physics, so they apply across computers, physics, and human behavior, which he summarized thusly: “One cannot build a physical computer that can be assured of correctly processing information faster than the universe does. The results also mean that there cannot exist an infallible, general-purpose observation apparatus, and that there cannot be an infallible, general-purpose control apparatus. These results do not rely on systems that are infinite, and/or non-classical, and/or obey chaotic dynamics. They also hold even if one uses an infinitely fast, infinitely dense computer, with computational powers greater than that of a Turing Machine."

He also published what seems to be the first serious work on team or collective intelligence (COIN) which he says puts this subject on a sound scientific footing. Although he has published various versions of these over two decades in some of the most prestigious peer reviewed physics journals (e.g., Physica D 237: 257-81(2008)) as well as in NASA journals and has gotten news items in major science journals, few seem to have noticed and I have looked in dozens of recent books on physics, math, decision theory and computation without finding a reference.

It is most unfortunate that Yanofsky and others have no awareness of Wolpert, since his work is the ultimate extension of computing, thinking, inference, incompleteness, and undecidability, which he achieves (like many proofs in Turing machine theory) by extending the liar paradox and Cantors diagonalization to include all possible universes and all beings or mechanisms and thus may be seen as the last word not only on computation, but on cosmology or even deities. He achieves this extreme generality by partitioning the inferring universe using worldlines (i.e., in terms of what it does and not how it does it) so that his mathematical proofs are independent of any particular physical laws or computational structures in establishing the physical limits of inference for past, present and future and all possible calculation, observation and control. He notes that even in a classical universe Laplace was wrong about being able to perfectly predict the future (or even perfectly depict the past or present) and that his impossibility results can be viewed as a "non-quantum mechanical uncertainty principle" (i.e., there cannot be an infallible observation or control device). Any universal physical device must be infinite, it can only be so at one moment in time, and no reality can have more than one (the "monotheism theorem").

Since space and time do not appear in the definition, the device can even be the entire universe across all time. It can be viewed as a physical analog of incompleteness with two inference devices rather than one self-referential device. As he says, "either the Hamiltonian of our universe proscribes a certain type of computation, or prediction complexity is unique (unlike algorithmic information complexity) in that there is one and only one version of it that can be applicable throughout our universe." Another way to say this is that one cannot have two physical inference devices (computers) both capable of being asked arbitrary questions about the output of the other, or that the universe cannot contain a computer to which one can pose any arbitrary computational task, or that for any pair of physical inference engines, there are always binary valued questions about the state of the universe that cannot even be posed to at least one of them. One cannot build a computer that can predict an arbitrary future condition of a physical system before it occurs, even if the condition is

from a restricted set of tasks that can be posed to it— that is, it cannot process information (though this is a vexed phrase as Searle and Read and others note) faster than the universe. The computer and the arbitrary physical system it is computing do not have to be physically coupled and it holds regardless of the laws of physics, chaos, quantum mechanics, causality or light cones and even for an infinite speed of light. The inference device does not have to be spatially localized but can be nonlocal dynamical processes occurring across the entire universe. He is well aware that this puts the speculations of Wolfram, Landauer, Fredkin, Lloyd etc., concerning the universe as computer or the limits of ″information processing″, in a new light (though the indices of their writings make no reference to him and another remarkable omission is that none of the above are mentioned by Yanofsky either).

Wolpert says it shows that the universe cannot contain an inference device that can process information as fast as it can, and since he shows you cannot have a perfect memory nor perfect control, its past, present or future state can never be perfectly or completely depicted, characterized, known or copied. He also proved that no combination of computers with error correcting codes can overcome these limitations. Wolpert also notes the critical importance of the observer ("the liar") and this connects us to the familiar conundrums of physics, math and language that concern Yanofsky. Again cf. Floyd on Wittgenstein: ″He is articulating in other words a generalized form of diagonalization. The argument is thus generally applicable, not only to decimal expansions, but to any purported listing or rule-governed expression of them; it does not rely on any particular notational device or preferred spatial arrangements of signs. In that sense, Wittgenstein's argument appeals to no picture and it is not essentially diagrammatical or representational, though it may be diagrammed and insofar as it is a logical argument, its logic may be represented formally). Like Turing's arguments, it is free of a direct tie to any particular formalism. [The parallels to Wolpert are obvious.] Unlike Turing's arguments, it explicitly invokes the notion of a language-game and applies to (and presupposes) an everyday conception of the notions of rules and of the humans who follow them. Every line in the diagonal presentation above is conceived as an instruction or command, analogous to an order given to a human being...″

Wittgenstein's prescient viewpoint of these issues, including his embrace of strict finitism and paraconsistency, is finally spreading through math, logic and computer science (though rarely with any acknowledgement). Bremer has recently suggested the necessity of a Paraconsistent Lowenheim-Skolem Theorem. "Any mathematical theory presented in first order logic has a finite paraconsistent model." Berto continues: "Of course strict finitism and the insistence on the decidability of any meaningful mathematical question go hand in hand. As Rodych has remarked, the intermediate Wittgenstein's view is dominated by his 'finitism and his view […] of mathematical meaningfulness as algorithmic decidability' according to which '[only] finite logical sums and products (containing only decidable arithmetic predicates) are meaningful because they are algorithmically decidable.'" In modern terms this means they have public conditions of satisfaction-i.e., can be stated as a proposition that is true or false. And this brings us to Wittgenstein's view that ultimately everything in math and logic rests on our innate (though of course extensible) ability to recognize a valid proof. Berto again: "Wittgenstein believed that the naïve (i.e., the working mathematicians) notion of proof had to be decidable, for lack of decidability meant to him simply lack of mathematical meaning: Wittgenstein believed that everything had to be decidable in mathematics…Of course one can speak against the decidability of the naïve notion of truth on the basis of Godel's results themselves. But one may argue that, in the context, this would beg the question against paraconsistentists-- and against Wittgenstein too. Both Wittgenstein and the paraconsistentists on one side, and the followers of the standard view on the other, agree on the following thesis: the decidability of the notion of proof and its inconsistency are incompatible. But to infer from this that the naïve notion of proof is not decidable invokes the indispensability of consistency, which is exactly what Wittgenstein and the paraconsistent argument call into question...for as Victor Rodych has forcefully argued, the consistency of the relevant system is precisely what is called into question by Wittgenstein's reasoning." And so: "Therefore the Inconsistent arithmetic avoids Gödel's First Incompleteness Theorem. It also avoids the Second Theorem in the sense that its non-triviality can be established within the theory: and Tarski's Theorem too—including its own predicate is not a problem for an inconsistent theory "[As Priest noted over 20 years ago]. Prof. Rodych thinks my comments reasonably represent his views, but notes that the issues are quite complex and there are many differences between he, Berto and Floyd.

And again, 'decidability' comes down to the ability to recognize a valid proof, which rests on our innate axiomatic psychology, which math and logic have in common with language. And this is not just a remote historical issue but is totally current. I have read much of Chaitin and never seen a hint that he has considered these matters. The work of Douglas Hofstadter also comes to mind. His Godel, Escher, Bach won a Pulitzer prize and a National Book Award for Science, sold millions of copies and continues to get good reviews (e.g. almost 400 mostly 5 star reviews on Amazon to date) but he has no clue about the real issues and repeats the classical philosophical mistakes on nearly every page. His subsequent philosophical writings have not improved (he has chosen Dennett as his muse), but, as these views are vacuous and unconnected to real life, he continues to do excellent science.

However once again note that "infinite", "compute", "information" etc., only have meaning in specific human contexts—that is, as Searle has emphasized, they are all observer relative or ascribed vs intrinsically intentional. The universe apart from our psychology is neither finite nor infinite and cannot compute nor process anything. Only in our language games do our laptop or the universe compute.

However not everyone is oblivious to Wolpert. Well known econometricians Koppl and Rosser in their famous 2002 paper "All that I have to say has already crossed your mind" give three theorems on the limits to rationality, prediction and control in economics. The first uses Wolpert's theorem on the limits to computability to show some logical limits to forecasting the future. Wolpert notes that it can be viewed as the physical analog of Gödel's incompleteness theorem and K and R say that their variant can be viewed as its social science analog, though Wolpert is well aware of the social implications. Since Gödel's are corollaries of Chaitin's theorem showing algorithmic randomness (incompleteness) throughout math (which is just another of our symbolic systems), it seems inescapable that thinking (behavior) is full of impossible, random or incomplete statements and situations. Since we can view each of these domains as symbolic systems evolved by chance to make our psychology work, perhaps it should be regarded as unsurprising that they are not "complete". For math, Chaitin says this 'randomness' (again a group of language games) shows there are limitless theorems that are true but unprovable—i.e., true for no reason. One should then be able to say that there are limitless statements that make perfect "grammatical" sense that do not describe actual situations attainable in that domain. I suggest these puzzles go away if one considers Wittgenstein's views. He wrote many notes on the issue of Godel's Theorems, and the whole of his work concerns the plasticity, "incompleteness" and extreme context sensitivity of language, math and logic, and the recent papers of Rodych, Floyd and Berto are the best introduction I know of to Wittgenstein's remarks on the foundations of mathematics and so to philosophy.
Koppl and Rosser's second theorem shows possible nonconvergence for Bayesian (probabilistic) forecasting in infinite- dimensional space. The third shows the impossibility of a computer perfectly forecasting an economy with agents

knowing its forecasting program. The astute will notice that these theorems can be seen as versions of the liar paradox and the fact that we are caught in impossibilities when we try to calculate a system that includes ourselves has been noted by Wolpert, Koppl, Rosser and others in these contexts and again we have circled back to the puzzles of physics when the observer is involved. K&R conclude "Thus, economic order is partly the product of something other than calculative rationality". Bounded rationality is now a major field in itself, the subject of thousands of papers and hundreds of books.

On p19 of his book Yanofsky says math is free of contradictions, yet as noted, it has been well known for over half a century that logic and math (and physics) are full of them—just google inconsistency in math or search it on Amazon or see the works of Priest, Berto or the article by Weber in the Internet Encyclopedia of Philosophy. W was the first to predict inconsistency or paraconsistency, and if we follow Berto we can interpret this as Wittgenstein's suggestion to avoid incompleteness. In any event, paraconsistency is now a common feature and a major research program in geometry, set theory, arithmetic, analysis, logic and computer science. Yanofsky returns to this issue other places such as on p346 where he says reason must be free of contradictions, but it is clear that "free of" has different uses and they arise frequently in everyday life but we have innate mechanisms to contain them. This is true because it was the case in our everyday life long before math and science

On p347 of Yanofsky's book, what we discovered about irrational numbers that gave them a meaning is that they can be given a use or clear COS in certain contexts and at the bottom of the page our "intuitions" about objects, places, times, length are not mistaken- rather we began using these words in new contexts where the COS of sentences in which they are used were utterly different. This may seem a small point to some, but I suggest it is the whole point. Some "particle" which can "be in two places" at once is just not an object and/or is not "being in places" in the same sense as a soccer ball, i.e., like so many terms its language games have clear COS in our mesoscopic realm but lack them (or have different and commonly unstated ones) in the macro or micro realms.

In sum, I suggest that what appear to be scientific issues to be investigated factually, are often issues for philosophical treatment, and can only be understood by looking at the language in context, and that science and mathematics are parts of higher order thought (System 2 in the modern two systems framework) as Wittgenstein realized some 80 years ago.