Artistic Mediation in Mathematized Phenomenology

Robert Prentner[1,2] and Shanna Dobson[3,4]

1 Munich Center for Mathematical Philosophy, LMU Munich, Germany.

2 Association for Mathematical Consciousness Science, Munich, Germany.

3 Department of Mathematics, University of California, Riverside, CA.

4 Department of Mathematics, California State University, Los Angeles, CA.

robert.prentner@amcs.science

shanna.dobson@email.ucr.edu

Abstract

Mathematics has a long track record of refining the concepts by which we make sense of the world. For example, mathematics allows one to speak about different senses of "sameness", depending on the larger context.

Phenomenology is the name of a philosophical discipline that tries to systematically investigate the first-personal perspective on reality and how it is constituted. Together, mathematics and phenomenology seem to be a good fit to derive statements about our experience that are, at the same time, well-defined, precise, and significant to our inner lives.

However, there are difficulties stemming from the fact that phenomenology deals with inherently subjective things. Phenomenological investigations seem to lose their appeal when trying to approach them in the clear-cut way afforded by mathematics. How to overcome this obstacle?

We argue that the Arts play a special role in mediating between the precise statements of mathematics and the sometimes fuzzy nature of our experience. Mathematics and art are complementary ways to come to a comprehensive understanding of reality.

1 Introduction

In one sense, a painting always stays the same, but in another sense, it really becomes a different painting, depending on who is looking at it. Picasso's Guernica is differently real for the Spanish Nationalist, for the spectator at the Paris world exhibition, or for us living in the 21st century. But we often assume that the painting (qua physical object) really is the same throughout. Well, is it? If not, how could that be understood? We think that mathematics might provide us with the tools to more carefully look at these questions.

Let us start this investigation by attacking one of the elephants in the room – the problematic status of mathematics vis-à-vis a qualitative experience such as viewing a painting. From the outset they seem at odds, and so does the idea that mathematics, the exact and quantitative discipline, has anything to offer (or to gain from) artistic forms of expression. After all, could we not split up the world into an objective part which is expressible in terms of mathematics and a subjective part which is not? Yet, on closer inspection this apparent opposition is ill-conceived.

As any mathematician knows, quantification is only one part of mathematics, perhaps even only relevant when mathematics is applied to the study of specific objects (physical or otherwise). And for exactness: artistic expression can be highly exact, though perhaps in a different way than in mathematics. The more pertinent questions are thus whether mathematics can explicitly acknowledge different forms of exactness – different ways in which things can be equivalent – and whether, a forteriori, it helps us to approach the subjective (rather than, or in addition to, the objective). We answer "yes" to both questions, with the addition that neither "the subjective" nor "the objective" pick out fundamental categories of being. We start by answering the first question, leading us to the second question. We then end with some final speculations about the non-dual nature of being.

2 Mathematizing different notions of equivalence

Mathematics offers a method of classifying and systematizing the notions of "sameness" and "equivalence". To wit, mathematics provides a way to analyze the notion of sameness from multiple perspectives. That is, mathematics provides the very language needed to allow different forms of exactness to obtain. What does it truly mean to be the same? In a topological space, the notion of sameness is encapsulated in the notion of homeomorphism.

Two objects are topologically indistinguishable if they are invariant under homeomorphism. In this regard, sameness is a topological property. Contrariwise, in group theory, the notion of equivalence can be encapsulated in the notion of a bijective homomorphism. Two groups are "the same" if they are related by such a structure-preserving homomorphism. Further, a famous result in linear algebra states that vector spaces having the same dimension are isomorphic and therefore "equivalent". So, in mathematics the notions of sameness and equivalence are not meaningful by themselves. Rather, it is always "up to" some mathematical concept. This means that, for example, vector spaces are "equivalent" only up to isomorphism. Thus, any mathematical definition of sameness is dependent upon the ambient mathematical space, encapsulated in the "up to" phraseology.

We can easily generalize this. Category theory offers notions of equivalence encapsulating an entire category of objects, in its functorial interpretations. By definition [1]:

> "The concept of *equivalence of categories* is the correct category theoretic notion of 'sameness' of categories.
>
> Concretely, an equivalence between two categories is a pair of functors between them which are inverse to each other up to natural isomorphism of functors (inverse functors).
>
> This is like an isomorphism, but weakened such as to accomodate for the fact that the correct ambient context for categories is not itself a 1-category, but is the 2-category Cat of all categories. Hence abstractly an equivalence of categories is just the special case of an equivalence in a 2-category specialized to Cat."

So, mathematics entertains an ambient space in which to define the very notion of sameness and equivalence. To wit, in having a multi-perspective dependency built into the definition of sameness itself, mathematics in advance acknowledges different forms of exactness.

3 Mathematics and Subjectivity

At this point it becomes instructive to look at the philosophical study of subjective, first-personal experience, known as "phenomenology" [2]. Phenomenology has been conceived in the early 20th century and influenced some of the most important philosophers of the time, mostly on the European Continent (in particular in Germany and France). Among those thinkers were philosophers such as Edmund Husserl, Karl Jaspers, Martin Heidegger, Jean-Paul Sartre, or Maurice Merleau-Ponty. Other important thinkers such as Henri Bergson, Gilles Deleuze, or Jacques Derrida are not phenomenologists themselves but share many of the important insights that phenomenologists put forward. More recently, phenomenology has been influential also outside of a purely philosophical discourse such as in "naturalized phenomenology" [3] or psychopathology [4].

Phenomenology, narrowly conceived, amounts to uncovering the invariant structures of conscious experience. More broadly construed, phenomenologists investigate how the notion of "reality" is constituted from a first-person perspective. The aforementioned structures together define the objective counterparts that figure in our experiences, that is, their "intentional contents". In a further step that makes all the difference, phenomenologists try to show how these objects are related to the way we engage with the world and with others.

Prototypical phenomenological analyses pertain to, for example, the temporal structure of experience, the aspectual shape of perception, or the notion of a pre-reflective self – that is, they pertain to the myriad ways how experiences could unfold and how reality's (seeming) "inner" aspect relates to its (seeming) "outer" one. Phenomenology describes how we come to experience objects in space and time in the first place.

It is here where phenomenology metaphysically distinguishes itself by suspending ontological judgments. In phenomenological jargon, this is known as the "epoché". For example, phenomenologists do not assume the existence of an outer, objective (e.g. physical) world, although they also do not dispute it. They just do not engage (at least on a first pass) with these sorts of questions:

> "I disconnect them all, I make absolutely no use of their standards, I do not appropriate a single one of the propositions that enter into their systems... I take none of them, no one of them serves me for a foundation" [5]

In particular, this is true for the propositions of science and mathematics. The reason is that phenomenologists want to study the things that really matter, meaning the very (purest) way of experiencing. Although phenomenologists sometimes sound as if they would want to cleanly (and sometimes: less cleanly) separate "subjective" and "objective" components of our experience, the very idea that there really are objects "out there" viewed by subjects "in there" is a typical metaphysical assumption that phenomenologists would bracket out:

> "It is precisely the apparently so obvious thought, that everything given is either physical or psychical that must be abandoned." [6]

But if we cannot reduce mental phenomena to physical ones, nor the other way around, what is left to do? Here we note a flavor of phenomenology that renders it quite distinct from other scientific projects, for example from studying the mind in terms of what the brain does. A ("transcendental") variety of phenomenology asks what makes it possible that the world could appear to us in the first place. These "conditions of possibility" cannot ultimately be found in brain activity, since brains are objects that are themselves grounded in experiences, which need to be grounded in conditions of possibility... Such a move is called "transcendental" (after the philosopher Immanuel Kant). In particular, phenomenologists engage in what is called a "transcendental reduction", that is, a reduction of worldly objects to experiences, thereby showing how "the world is constituted" in pure experience.

But rather than positing an external realm as a source of this transcendental activity, phenomenologists regard them as being rooted in the lived experience of an agent – "transcendence in immanence" [7]. The twist is that neither the objective nor the subjective moments involved here can claim any independent reality status (which most of us intuitively, but mistakenly, assume). Viewed from different perspectives, both "objects" and "subjects" stem from the same source [8]. There exists a sense in which they are equivalent to each other (though not on the same footing): a phenomenological analysis would lead to a form of non-duality.

Mathematized phenomenology is a specific approach that seeks to formulate this in the language of mathematics. Mathematized phenomenologists should thus come up with mathematical models that fulfill two tasks: (i) they capture the essences of all possible experience (such as the temporal structure of experience), and (ii) the models can also be grounded in experience (suggesting, for example, that the conditions of possibility of mathematical thoughts are encoded within the mathematics itself). Mathematized phenomenology thus seeks to formalize the conditions underlying the emergence of reality from the interplay between the (seemingly) subjective and objective aspects of consciousness, given through the lense of first-personal access.

4 Mathematics and the Creation of Concepts

Mathematics is esteemed for its rich array of concepts that have been developed to offer precision to pre-existing concepts suffering from unclarity and vagueness in order to solve problems, as epitomized in the ancient Greek mathēmatikos which means "inclined to learn." (Whether mathematics is being created or discovered in a Platonic sense, is an ongoing debate that we will not pursue here). Whereas the poet could be said to *worship* infinity, the mathematician seeks to *harness* infinity, give it perspective, classify it, and use this to solve her problems. Mathematics is fearless at attempting to conceptualize anything that lies beyond even our own subjective experience (which is but a tiny fragment of a vast sea of non-dual consciousness).

In the mathematician's pursuit of classification, there is no loss of respect or curiosity for the object itself. For example, the set of symmetries of the famous Monster Group are realized in an object having $196,883$ dimensions [9]. Mathematicians ask why that number and not another number, and these questions lead to further explorations and conceptual advances.

Whereas in most fields, concepts are taken for granted, mathematicians and philosophers try to reveal where concepts break down, for example by using the technique of paradox [10]. Russell's paradox is a famous example of showing the pitfalls of working in naive set theory and has led to new perspectives in the foundations of mathematics. The Banach-Tarski paradox draws attention to the need of making the concept of size/measure more robust, by highlighting the mathematical and philosophical pathologies that can result when a set is a non-measurable set whose size is not well-defined. It could be argued that mathematics is the best language to describe phenomenological experience because it provides quantitative and qualitative precision to whatever system it finds itself the investigation of. For instance, the size or measure of an ordinary object might be intuitively equated with the "size it appears to have." After all, everyone knows what the size of a butterfly is. Or do we? But we can see there are issues with such reasoning.

Terry Tao encapsulates this very conundrum in what is called the "problem of measure" [11]. Tao elucidates the problem by way of explaining that any attempt to mathematically measure the size of a solid body quickly becomes intractable. For example, consider measuring the size of a throw pillow. Conventionally, size would be interpreted in terms of dimensions, volume, and mass, which are all mathematical entities. So, let us try to be as precise as possible in measuring this pillow. Firstly, the pillow can be approximated as a solid body described by Euclidean geometry. Secondly, let us use the Lebesgue measure, where all singletons and countable sets have measure zero. Alarmingly, we see that the pillow consists of infinitely many point-like objects (perhaps even uncountably many of them), and each of them have measure zero. The resulting product of infinity times zero is an indeterminate form. Thus the size of the pillow is indeterminate. But, again, everyone knows what the size of a pillow is. Or do they? It is more precise to say that everyone has an *idea* of what the size of a pillow would be, given a certain context in which measure is defined (e.g. via the Lebesgue measure), which is a very different statement after all.

Perhaps looking at the physical domain is helpful here. Yet, according to our best theory, namely quantum mechanics, the notion of size is also peculiar in physics. Recall that in quantum mechanics there exists a measurement boundary of ca. 10^{-35}m, after which our current laws of physics (and perhaps even our conventional mathematical notions) no longer make sense, and no measurement can be made. This means we have no concept of what an electron looks like or behaves like on a spatial scale smaller than 10^{-35}m. There is a similar imposed cutoff in the temporal dimension in the form of Planck time, which is ca. 10^{-42} seconds, and is defined as the time it takes a photon to travel one Planck length. Mathematics can make sense of these cutoffs by stating that the quantum world does not have the archimedean property [12]. Space and time in quantum mechanics are non-archimedean. This means the quantum world can be approximated by the mathematics of the p-adic numbers, surreal numbers, or any other number system without the archimdean property, which sharply contrasts with the non-quantum world which has the archimedean property and is approximated by the mathematics of real analysis.

In these examples, we see that mathematics curates the concept of size relative to the scale of observation. In this sense, reality seems to teeter between our mathematical definitions, a viewpoint which is encapsulated in Max Tegmark's Multiverse Level 4 [13], which states that mathematical existence = physical existence. Absent the concepts of non-archimedean time and non-archimedean space, we might not have the ability to mathematically distinguish physics in the quantum world from physics in the non-quantum world.

Mathematics offers precision to vague notions, such as infinity and "world". For example, the mathematician Georg Cantor developed an intricate classification of the different sizes of infinity and of the ordinal numbers, which are an enumeration of infinities. Even the notion of "reality" itself is further clarified and strengthened by mathematics, which can offer concepts such as number systems, class field theory, supersymmetries, rigid analytic varieties, and formal spectra as probes of the infinitesimal. But whether such concepts are appropriate depends on what we want to achieve with them. For example, to a company wishing to secure its sensitive networks, mathematics can offer supersingular elliptic curve cryptography, as a robust interpretation of securing systems and a solution to the problem of non-secure systems.

Similarly, we argue that mathematics offers rich concepts along multiple perspectives of precision that can be applied to the study of phenomenal experience. However, there seems to be a fundamental obstacle:

> "[T]he inquiry of phenomenology into Pure Consciousness sets itself and needs set itself no other task than that of making such descriptive analyses as can be resolved into pure intuition" [5]

At first sight, it may seem that resolving any mathematical description into this "pure intuition" is out of reach – a category error of sorts. It is here where the Arts become most relevant from the perspective of a mathematical phenomenologist: through the practices of art can abstract, mathematical knowledge about the invariant structures of experience be turned into the "pure intuition" of our sensitive natures.

But first, let us distinguish a "naïve approach" to mathematizing phenomenology from a more refined one. The naïve approach is probably the target that most people have in mind when they are from the outset critical of the idea of mathematically describing consciousness. According to the naïve approach, one should just search the mathematician's toolkit for models that "fit" the presumed essential features of subjective experience. This would amount to describing phenomenological processes, without paying attention to grounding these descriptions in experience. For example, from a psychological point of view, it seems that our (visual) perception largely mirrors the structure of projective geometry [14]. Hence, we could straightforwardly apply the mathematical findings from projective geometry and – et voilà – we are doing mathematized phenomenology. However, such an approach faces two criticisms. First, it is not clear why anything that holds in projective geometry, also holds in our experience. In other words: why is it that mathematics should be "unreasonably effective" in the phenomenological domain? Second, why is it that projective geometry has anything to do with the phenomenal quality of our experience (sometimes referred to as the "what-it-is-likeness" [15] of experience)?

Conceptually, it seems straightforward to implement a projective perceptual model in a robot. Would that endow the robot with a form of experience? If not, then why is it that a robot cannot be programmed to experience anything? Perhaps the mathematics being programmed is merely too simple to cause an experience. Would a robot programmed with perceptual representations that resemble infinity categories undergo experience? What about programming perceptual representations to lie in the very large endomorphism rings of supersingular elliptic curves? Is there a complexity/threshold argument that specifies the emergence of experience? Or perhaps it is a question about implementation. Would a robot with perceptual representation implemented via generative adversarial networks undergo experiences? Do we need feedback? A transformer architecture [16]? Biology? It is not clear how a robot would entertain subjective experiences, but it is also not clear how humans entertain subjective experiences either!

It could be argued that it is absurd to assume that there exists any intimate connection between mathematics and phenomenal experience, which seems consistent with a general claim made by Husserl:

> "[T]he precise aims and methods [of mathematical disciplines]... should in principle be unsuited for the sphere of experience."[5]

However, we take this to hold true for the naïve approach only. By contrast, the mathematical models of a "non-naïve" approach to mathematized phenomenology could avoid this, but they need to be grounded in phenomenology before they can "reflect back" on the thing that grounded them in the first place. Not any mathematics that fits the phenomena could do the trick, but only mathematics that seems to be intrinsically related to experience. Of course, it is not obvious what approaches fall under this category.

A first pass at this question is to look at those mathematical models that embody the notions of connectivity, (dis)similarity, or (different forms of) equivalence. A concrete proposal proceeds from topology [17], which allows one to speak of "regions" or "geometries" (Phenomenologically speaking: topology is "the condition of possibility" to define a perspectival geometry or temporal succession between events in the first place). Another, related, candidate is measurability [18], allowing us to to assign probabilities to different instances of experiences. The most abstract framework appears to be that of category theory [19], particularly featuring categories that are endowed with a topological structure, for example, a Grothendieck topology that allows the definitions of sheaves on sites [20]. (Phenomenologically speaking: sheaves allow one to precisely talk about an unknown space of experience: to "probe" a seemingly inaccessible representation of subjective experience with the help of mappings from known "test spaces".) A special case is the theory of symmetric monoidal categories that focuses on the composition between processes that represent the ever-changing actions of consciousness [21].

Mathematics has been used to capture the symmetries or canonical invariances of an object, being more interested in the "essence" of an object (rather than its overt appearance). What does this mean for phenomenology? In particular, what does it mean for phenomenology when mathematics is differentiating between different notions of sameness and equivalence? When it is stated that an "object is unknowable", does that mean the object itself lives outside the realm of construct classification? Does it merely mean that, if only given an appropriate definition, the object would become knowable? Or does it dare mean that not even an infinity category could make the object "recognizable" and thus knowable? We have argued that mathematics can systematize any concept so that the concept is not lost to vagaries. But the question is whether this systematization applies to the object "itself" or just to the concept *as attached* to an object?

And, perhaps most importantly, how to show "empirically" that these and similar constructions indeed capture phenomenological insights (we have just outlined some theoretical heuristics in the previous paragraph)? Husserl has argued that our mathematical descriptions, in the end, have to "resolve into pure intuition". And we have previously argued that merely endowing, e.g., a robot with such structures is not enough, since it is not clear that this would be accompanied by any kind of (phenomenal) experience at all, so that would not by itself answer Husserl's request.

5 The role of the Arts

An important role of art in this context is to demonstrate how any proposed mathematical structure can be grounded in the reality of experience. But what is "reality"? There are several aspects of the notion of reality that can be distinguished. We will see that art has important things to say about every one of them.

1. *Reality is what can be seen, heard, or touched.*

   Such a definition would leave out many entities, such as mathematical ones (I have never seen the number 5 in front of my eyes, nor any infinity-category). This also leaves out much of what could happen when confronted with good art. Take as an example the song "Stairway to Heaven" by the British band Led Zeppelin. Listening to Stairway to Heaven evokes a feeling of pleasant sadness, certain images, and overall confusion. But the only things that would be real, according to our definition, are the specific sounds played, but not the objects and feelings those sounds evoke.

Upon gazing into Van Gogh's Winter, a viewer can feel that vastly forlorn fallen snow, immortalized in thick white paint, which makes it fall forlorn forever, while simultaneously also feeling peace given the soft emerald light which blankets the snow. In reality only white dots? That would be a strange reality indeed!

A peculiar finding of phenomenologists is that we are only ever aware of objects that transcend the specific sensory impressions, even though they are consistent with them. For example, phenomenologists would argue that one is aware of a "melody" and not of a mere succession of sounds. So, either there is some (objective) reality which can be measured and quantified and our minds create "less-real" and subjective images of this "real reality", or reality is instead populated by the things that we merely sort into subjective and objective categories.

2. *Reality is that which affects and withstands us.*
   Some things affect us and have consequences. For example, the developments of the stock market might have very real consequences for our lives. But the stock market is a social construct. It is real because we believe it to be, though we cannot wish it away. But the example of the stock market might divert our attention. It is not "social constructionism" vs. "natural scientism" that matters here. Art presents us with artifacts that are neither natural nor socially constructed but have an effect on us. I am buying tickets to a concert because of the way the music moves me when performed well. I am visiting the museum store after I saw an exhibition of Picasso's work that I found uncanny, yet magnificently executed. I get another book from the same author whose work previously inspired me. What counts is that "reality" is a mix of objective and subjective aspects that affect us.

3. *Reality is what is there when no one is there.*
   Conceivably, reality has its properties regardless of whether it is seen, heard, or measured. But reality is not the same as objectivity. Philosophically speaking, the

notion of "reality" pertains to the domain of "ontology", that is, to the way of being. "Objectivity", by contrast, pertains to the domain of "epistemology", that is, to the conditions underlying our knowledge and the access to reality. Reality is not merely exhausted by facts about the objects in front of us. It contains an objective side but crucially also a subjective element, the way how an object appears to us or the co-relation between the objective properties of the things we consider to be "really there outside" and the subjective experiences that we believe happen "inside of us". These are facts about objects-to-subjects. If we believe that there are only facts about objects (and that the relation of objects-to-subject is unimportant for matters of reality), we conflate the way we describe reality (namely in terms of collections of objects) with the being of reality. But this seems to put the cart before the horse. The physicist Niels Bohr found that physical properties are real only relative to a measurement context, an intellectual line which has been continued by Wheeler [22] and quantum Bayesianism [23]. The parallels between QBism and transcendental phenomenology have recently been investigated [24], and point into a similar direction as our general picture.

4. *Reality is what grounds everything else.*

   Perhaps, sounds, songs, stock markets, novels, and paintings are all real in some sense. But surely there is something that grounds all of these ideas. Wouldn't this be "really real" then? Let us entertain that thought. Neuroscientists probe what they believe to be the ultimate source of our conscious experience: the human brain. In some way or another, they are still stuck. In a curious turn of events, ever more people begin to give up the belief that the brain truly grounds our experience. But the alternative is still up for grabs.

   An interesting thesis stems from the French social scientist Bruno Latour: "Nothing is, by itself, either reducible or irreducible to anything else" [25]. While the correct interpretation of this statement is disputed, it seems to be consistent with our general approach to reality: rather than being able to draw neat distinctions into "really real", "almost real", and only "subjectively real", we need perhaps to acknowledge that no such thing exists as a substantial (=for itself) bedrock principle of reality. By the same token, all things depend on other things. Reality consists of nothing but relations between subjective$\sim$objective entities (which are themselves nothing

> but relations between subjective$\sim$objective entities on yet another level). Could mathematics categorify this ad infinitum layers of relations? Mathematics would then describe how those networks of subjective$\sim$objective relations unfold. Art could demonstrate how this is resolved into intuition.

Sometimes it is said that art provides you with a new perspective on reality. On further scrutiny, however, one should rather make the stronger claim that art creates new realities out of old ones. Guernica as seen by the Spanish Nationalist is neither the same nor different to Guernica as seen by us in the 21st century. They are only weakly equivalent "up to" a certain context.

6 Conclusion

Mathematics creates concepts to make tractable what is otherwise termed as unknowable, while art is a technology for the creation of new experiences.

We have applied categorical thinking to the systematic study of consciousness (phenomenology), in particular to the question whether our subjective experience finds analysis in a formal setting. We emphasized the need for artistic forms of expression to fulfill the requirement that such meditations need to be brought back to our concrete intuition. In this respect, *mathematics and art are complementary systems*, rather than ways to explore fundamentally opposed "objective" or "subjective" domains of reality.

We have mostly speculated on the direction of approaching reality from the angle of mathematics and then evoke art (mathematics $\rightarrow$ art). Now, we want to briefly ponder the other route (art $\rightarrow$ mathematics). Suppose that artistic expression allows us to induce novel experiences. It seems that we would need mathematics to harness and classify this experience so we can hold onto it. After all, can we say that anything is novel without cross referencing it (which is a mathematical process) to our database of previous lived experience? How could we make new (precise) concepts based on such novel experiences without the use of mathematics?

Regardless of how one approaches reality, it seems that mathematics and art form two necessary poles, analytic and synthetic, both with rigor but without fear. Only together do they give rise to a more comprehensive understanding of the non-dual nature of being.

References

[1] nLab authors. equivalence of categories. Available online: `http://ncatlab.org/nlab/show/equivalence%20of%20categories` (last accessed on July 20, 2022).

[2] S. Gallagher and D. Zahavi. *The Phenomenological Mind*, Routledge, London, 2002.

[3] J. Petitot, F.J. Varela, B. Pachoud, and J-M. Roy. *Naturalizing Phenomenology. Issues in Contemporary Phenomenology and Cognitive Science*, Stanford University Press, Stanford, CA, 1999.

[4] T. Fuchs, G.P. Messab, and G. Stanghellini (eds.). Phenomenological Approaches to Psychotherapy, *Psychpathology* **52**:2, 2019.

[5] E. Husserl. Ideas pertaining to a pure phenomenology and to a phenomenological philosophy. Routledge, London, 2012.

[6] E. Husserl. Einleitung in die Logik und Erkenntnistheorie. Vorlesungen 1906/07. Martinus Nijhoff, Dordrecht; quoted after D. Zahavi. "Being Someone". *Psyche* **11**(5), 2005.

[7] J. Patočka. *An Introduction to Husserl's Phenomenology*, Open Court, Chicago, IL, 1999.

[8] H. Atmanspacher and D. Rickles. *Dual-aspect Monism and the Deep Structure of Meaning.* Routledge, London, 2022.

[9] R.E. Borcherds. What is the Monster? *Notices of the AMS* 49(9): 1076-1077, 2002.

[10] A. Cantini and R. Bruni, "Paradoxes and Contemporary Logic", *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition), edited by E.N. Zalta (ed.). Available online: `https://plato.stanford.edu/archives/fall2021/entries/paradoxes-contemporary-logic/` (last accessed on July 20, 2022).

[11] T. Tao. 245A, prologue: The problem of measure. Available online: `https://terrytao.wordpress.com/2010/09/04/245a-prologue-the-problem-of-measure/` (last accessed July 20, 2022)

[12] nLab authors. archimedean field. Avalaible online: `http://ncatlab.org/nlab/show/archimedean%20field` (last accessed on July 20, 2022)

[13] M. Tegmark. *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality.* Knopf Doubleday Publishing Group, New York, 2014.

[14] E. Morsella. Can Projective Geometry Help Explain Consciousness? *Psychology Today*, April 7, 2022.

[15] T. Nagel. What Is It Like to Be a Bat? *Philosophical Review*, 83 (4): 435–450, 1974.

[16] G. Marcus. Nonsense on Stilts. Available online: `https://garymarcus.substack.com/p/nonsense-on-stilts` (last accessed on July 20, 2022).

[17] R. Prentner. Consciousness and Topologically Structured Phenomenal Spaces. *Consciousness and Cognition* 70: 25-38, 2019.

[18] D.D. Hoffman and C. Prakash. Objects of Consciousness, Frontiers in Psychology 5: 577, 2014.

[19] N. Tsuchiya and H. Saigo. A relational approach to consciousness: categories of level and contents of consciousness. *Neuroscience of Consciousness*, 2: niab034, 2021.

[20] S. Dobson and R. Prentner. *Perfectoid Diamonds and n-Awareness. A Meta-Model of Subjective Experience.* Available online: `https://arxiv.org/abs/2102.07620` (last accessed on July 20, 2022)

[21] C.M. Signorelli, Q. Wang, and B. Coecke. Reasoning about Conscious Experience with Axiomatic and Graphical Models, *Consciousness and Cognition* 95: 103168, 2021.

[22] J. A. Wheeler. *At Home in the Universe.* American Institute of Physics, New York, 1994.

[23] C. Fuchs. *QBism, the perimeter of quantum Bayesianism.* Accessible at `https://arxiv.org/abs/1003.5209` (last accessed on July 20, 2022).

[24] L. de la Tremblaye and M. Bitbol. Towards a Phenomenological Constitution of Quantum Mechanics: A QBist Approach. *Mind and Matter* **20**(1):35-62, 2022.

[25] B. Latour. *The Pasteurization of France.* Harvard University Press, Cambridge, MA, 1988.