

Expressing Truth directly within a formal system with no need for model theory

Because formal systems of symbolic logic inherently express and represent the deductive inference model formal proofs to theorem consequences can be understood to represent sound deductive inference to deductive conclusions without any need for other representations.

First we lay the foundation of expressing semantic truth directly within a formal system. All of semantic truth has its ultimate ground of being in expressions of language that have been defined to be true.

[https://en.wikipedia.org/wiki/Theory_\(mathematical_logic\)](https://en.wikipedia.org/wiki/Theory_(mathematical_logic))

The construction of a theory begins by specifying a definite non-empty conceptual class E the elements of which are called statements. These initial statements are often called the primitive elements or elementary statements of the theory, to distinguish them from other statements which may be derived from them.

A theory T is a conceptual class consisting of certain of these elementary statements. The elementary statements which belong to T are called the elementary theorems of T and said to be true. In this way, a theory is a way of designating a subset of E which consists entirely of true statements. (Haskell Curry, Foundations of Mathematical Logic, 2010).

From this basis we can infer that every formal proof of theorems in such a (Haskell Curry) formal system would exactly correspond to deriving the conclusion of sound deductive inference. R. B. Braithwaite explains this in depth below:

KURT GÖDEL

Translated by B. MELTZER

Introduction by R. B. BRAITHWAITE

2 INTRODUCTION

In order to show that in a deductive system every theorem follows from the axioms according to the rules of inference it is necessary to consider the formulae which are used to express the axioms and theorems of the system, and to represent the rules of inference by rules Gödel calls them “mechanical” rules, p. 37) according to which from one or more formulae another formula may be obtained by a manipulation of symbols. Such a representation of a deductive system will consist of a sequence of formulae (a calculus) in which the initial formulae express the axioms of the deductive system and each of the other formulae, which express the theorems, are obtained from the initial formulae by a chain of symbolic manipulations. **The chain of symbolic manipulations in the calculus corresponds to and represents the chain of deductions in the deductive system.**

But this correspondence between calculus and deductive system may be viewed in reverse, and by looking at it the other way round Hilbert originated metamathematics. Here a calculus is constructed, independently of any interpretation.

From the above we can see that the formal proof to theorem consequences expressed in symbolic logic represents and expresses sound deductive inference to deductive conclusions. One way to look at this might be that formal proof to theorem consequences corresponds to and expresses the sound deductive inference model.

Since the conclusions of sound deductive inference are understood to be true we can formulate this universal truth predicate:

$\forall F \in \text{Formal_Systems } \forall x \text{ WFF}(F) (\text{True}(F, x) \leftrightarrow (F \vdash x))$