A.P. Bird
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Kronecker, God and the Integers

Natural numbers were created by God, everything else is the work of men — Kronecker (1823–1891).

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"Natural numbers were created by God, everything else is the work of men." Kronecker in a lecture for the Berliner Naturforscher Versammlung (1886).

Leopold Kronecker (1823–1891) was a German mathematician who worked on number theory and algebra. He is considered a pre-intuitionist, being only close to intuitionism because he rejected Cantor's Set Theory. He was, in fact, more radical than the intuitionists. Unlike Poincaré, for example, Kronecker didn't accept the transfinite numbers as valid mathematical entities.

Kronecker is often remembered as the first to cast doubts on non-constructive existence proofs (i.e., proofs that show, by contradiction, that something exists). He carried this debate thru many years against Karl Weierstrass, and he insisted that all mathematics should be reduced to the positive whole numbers. Was he right?

According to O'Conner and Robertson,

> "Kronecker believed that mathematics should deal only with finite numbers and with a finite number of operations. He was the first to doubt the significance of non-constructive existence proofs. It appears that, from the early 1870s, Kronecker was opposed to the use of irrational numbers, upper and lower limits, and the Bolzano-Weierstrass theorem, because of their non-constructive nature. Another consequence of his philosophy of mathematics was that to Kronecker transcendental numbers could not exist." O'Conner, J. and Robertson, E. in *Leopold Kronecker (1999)*.

His most famous phrase, "natural numbers were created by God, everything else is the work of men," summarizes his ideas for the foundation of mathematics, but here he tried to explain it in a less biblical way:

> "So the results of general arithmetic also belong properly to the special, ordinary theory of numbers, and all the results of the profoundest mathematical research must in the end be expressible in the simple forms of the properties of integers." Kronecker in *From Kant to Hilbert: A Source Book in the Foundations Mathematics Vol. II (2007)*.

A great part of the scientific community doesn't agree with him. Says, Boniface,

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> "Kronecker's views on the foundations of mathematics are often reduced to jokes and regarded as an outdated set of ill-assorted ideas. A closer look

> *however shows that they constitute an original and coherent doctrine justified by epistemological convictions." Jacqueline Boniface in Leopold Kronecker's conception of the foundations of mathematics (2005).*

His criticisms, however, can one day be proved right. Mostly because nothing is resolved in the field of the foundations of mathematics…

After his death, according to Chaitin, there were a few scientific results that helped endorse his criticisms. Let's see:

> *"1. The diagonal and probabilistic proofs that reals are uncountable, and*
>
> *2. The diagonal and probabilistic proofs that there are uncomputable reals.*
>
> *(…) In the first case these are the famous Jules Richard paradox (1905), Emile Borel's know-it-all real (1927), and the fact that most reals are unnameable, which was the subject of (…) his last book, published when Borel was 81 years old (…).*
>
> *In the second case the frightening features are the unsolvability of the halting problem (Turing, 1936), the fact that most reals are uncomputable, and last but not least, the halting probability Ω, which is irreducibly complex (algorithmically random), maximally unknowable, and dramatically illustrates the limits of reason." Chaitin, G. in How Real Are Real Numbers? (2006).*

Therefore, these results about real numbers revealed:

> *"Chasms beneath the feet of mathematicians." Chaitin, G. in How Real Are Real Numbers? (2006).*

What to do about his criticisms? I don't know too. Maybe now you can think of a solution.

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