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A.P. Bird

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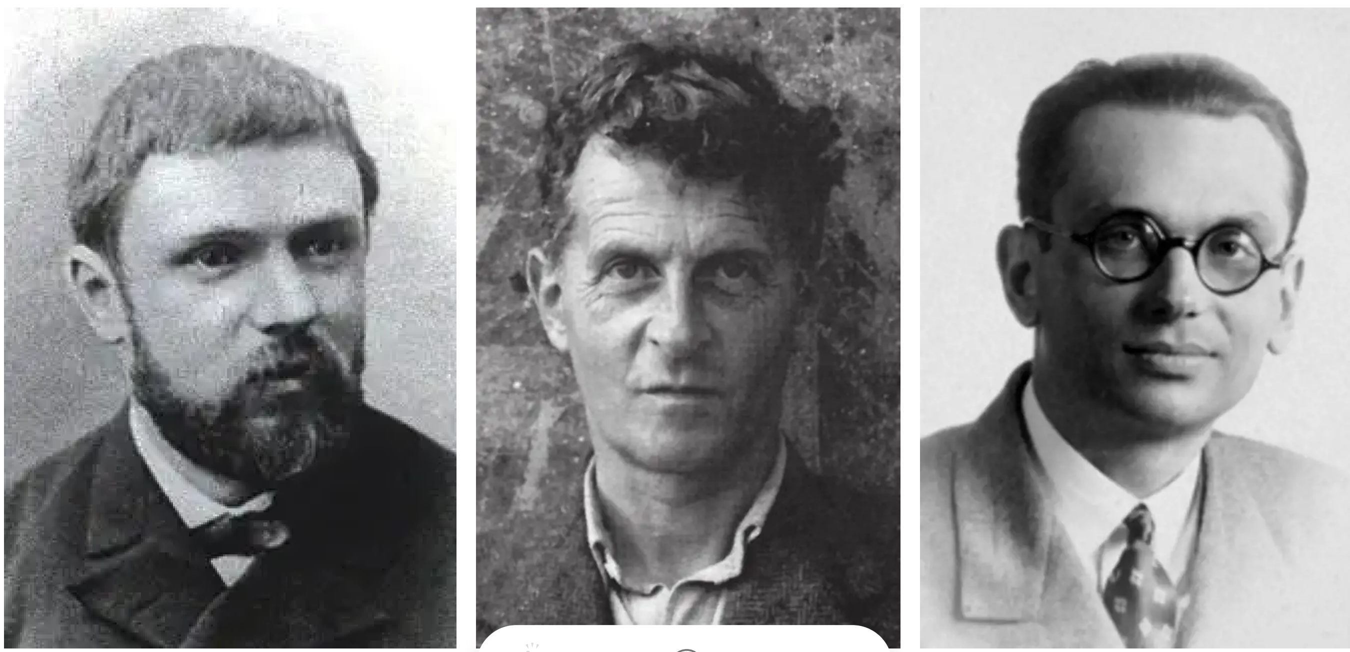
Incomplete In What Sense?

Mutable and immutable systems, and the differences between Platonic completeness and pragmatic completeness.



(1) Portrait of Henr… …ein. (3) Kurt Gödel.

Poincaré's "logic of infinity":

Poincaré differentiates between two types of classification: immutable (predictive) and mutable (non-predicative, which deals with potential infinite). He says,

> *Let us suppose for example that we classify the integers into two families according to their size. We can recognize whether a number is greater or less than 10 without having to consider the relations of this number with the set of the other integers. Presumably, after the first 100 numbers have been defined, we shall know which among them are less than and which are greater than 10. When we then introduce the number 101, or any one of the numbers which follow, those among the first 100 integers which were less than 10 will remain less than 10, those which were greater will remain greater; the classification is predicative.*
> *On the contrary let us imagine that we want to classify the points in space and that we differentiate between those which can be defined in a finite number of words and those which cannot. Among the possible sentences, there will be some which will refer to the entire collection, that is, to space or else to some portions of space. When we introduce new points in space, these sentences will change in meaning, they will no longer define the same point; or they will lose all meaning; or else they will acquire a meaning although they did not have any previously. And then points which were not definable will become capable of being defined; others which were definable will cease to be definable. They*

> *will have to change from one category into another. The classification will not be predicative (Poincaré, 1963, Chapter: Logic of Infinity, p.47–48).*

These two types of classifications (predicative and non-predicative), according to him, get even easier to understand when applied to two types of infinite. We can describe these two types of infinite as follows:

- Regular, or complete infinite (which supports predicative immutable classifications), in which all elements of the infinite series can be previously determined.
- Irregular, or incomplete, contingent infinite (which deals with non-predicative classifications), in which all elements of the infinite series **can't** be previously determined.

But the entire mathematics, or even any possible language system, belongs to which realm? Regular complete infinite, or irregular incomplete contingent infinite? For Poincaré, mathematics belongs to the second and logic to regular complete infinite (which I think is a mistake). Mathematics, indeed, can deal with chaos, and many types of variables, and no one has proved its foundational set of rules to be consistent; but logic too has no consistent foundational set of rules, and isn't a completed science either (I may write about it in another article).

Nevertheless, Poincaré’s conclusion is that the entire mathematics cannot be classified predicatively.

Thus, no mathematical system can receive “immutable logical classifications,” or, as Gödel prefers, it can’t consistently prove its own completeness (or incompleteness).

Is there a solution to Gödel’s incompleteness, or to Poincaré’s non-predicative classifications?

Wittgenstein got pretty close to figuring a way out of Gödel’s ideas, and Floyd and Putnam, in 2000, also grasped something meaningful from Wittgenstein’s ideas.

- Here is what Floyd and Putnam wrote:

> ***Gödel theorem* shows** *that (1) there is a welll-defined notion of “mathematical truth” applicable to every formula of PM (the closed formal system of Russell and Whitehead); and (2) that,* if PM is consistent, then some “mathematical truths” in that sense are undecidable in PM, is **not a mathematical result but a metaphysical claim. (…)** *Wittgenstein wants simply to deny the metaphysical claim; rather, he wants us to see how little sense we have succeeded in giving it (Floyd and Putnam, 2000: p.632).*

- Here is what Wittgenstein wrote:

> *"may there not be true propositions which are written in this (Gödelian) symbolism, but are not provable in Russell's system?" And Wittgenstein replies, "why should not propositions — of physics, e.g. — be written in Russell's symbolism?" (adapted from: Wittgenstein, 1998: p.83, §7).*

For me, it's like Wittgenstein tried to point out, in response to Gödel's work, that we must test propositions in "reality" if we want to determine whether they are true, false, or indeterminate. But the right answer to his question (if we can apply Gödel ideas to physics) is both "yes and no." It depends on which meaning of the concept of *completeness* we are using. There can coexist, I think, two different notions of completeness: a platonic one and a pragmatic one.

Incomplete in a Platonic sense:

Gödel tried to prove later at Princeton's Institute for Advanced Studies that biological evolution wouldn't be defined by chance. Maybe, it's like he was trying to fight the "indeterminacy" he discovered with his theorems. Or maybe he wasn't. Maybe he wasn't worried about the incompleteness of everything because he knew the philosophy of mathematics would advance more on the subject of the foundations of mathematics (which, unfortunately, is happening very very slowly).

But, honestly, I think it's more likely he wasn't worried about indeterminism because his notion of incompleteness was considered by him as a universal Platonic statement. For him, mathematics can always try to be absolutely, universally complete. In this sense, the search for completeness is *almost* like an attempt to reach the horizon. We walk four steps, the horizon walks four steps. The philosophers of mathematics walk four steps, completeness *may* also walk four steps.

Incomplete in a pragmatic sense:

Let's suppose all the rules of physics will change, but, before the change, we finally figured out everything there was to be figured out about physics. This means that we achieved pragmatic completeness at that point. It's not a universal Platonic completeness, but everything there was to be expressed about the physics at that moment was expressed.

This kind of completeness, the pragmatic one, can be applied to smaller examples as well. If there are only three types of mammals, and so far, there only exist or ever existed three types of mammals, then the expression “there are only three types of mammals” is complete, or pragmatically complete.

It’s like Wittgenstein was about to conclude that there can be completeness in an irregular, incomplete, infinite series, but only before the appearance of an irregular element. It’s true irregular elements may ever appear, but Wittgenstein could say “what we cannot complete, complete it is.” A sentence very similar to one of his most famous phrases: “whereof one cannot speak, thereof one must be silent” (Wittgenstein, 1999: p.171).

A Note on Wittgenstein’s “Notorious Paragraph” about the Gödel Theorem (2000). By Juliet Floyd and Hilary Putnam in *The Journal of Philosophy*, Vol. 97, No 11 (2000), pp. 624–632.

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