The Early Days of RSA -- History and Lessons

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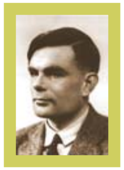
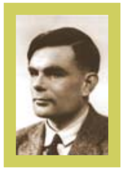
MIT Lab for Computer Science

ACM Turing Award Lecture



Lessons Learned

- Try to solve “real-world” problems
- … using computer science theory
- … and number theory.
- Be optimistic: do the “impossible”.
- Invention of RSA.
- Moore’s Law matters.
- Do cryptography in public.
- Crypto theory matters.
- Organizations matter: ACM, IACR, RSA

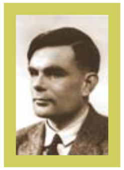


Try to solve real-world problems

- Diffie and Hellman published "New Directions in Cryptography" Nov '76:

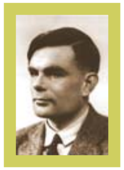
  > "We stand today at the brink of a revolution in cryptography."

- Proposed "*Public-Key Cryptosystem*". (This remarkable idea developed jointly with Merkle.)
- Introduced even more remarkable notion of *digital signatures.*
- *Good cryptography is motivated by applications. (e-commerce, mental poker, voting, auctions, …)*



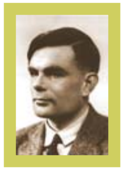
… using computer science theory

- In 1976 “complexity theory” and “algorithms” were just beginning…
- Cryptography is a “theory consumer”: it needs
  - *easy problems* (such as multiplication or prime-finding, for the “good guys”) and
  - *hard problems* (such as factorization, to defeat an adversary).



…and number theory

- Diffie/Hellman used number theory for “key agreement” (two parties agree on a secret key, using exponentiation modulo a prime number).
- Some algebraic structure seemed essential for a PKC; we kept returning to number theory and modular arithmetic…
- Difficulty of factoring not well studied then, but seemed hard…



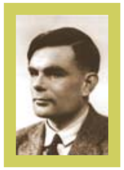
Be optimistic: do the “impossible“

- Diffie and Hellman left open the problem of realizing a PKC:

  $$D(E(M)) = E(D(M)) = M$$

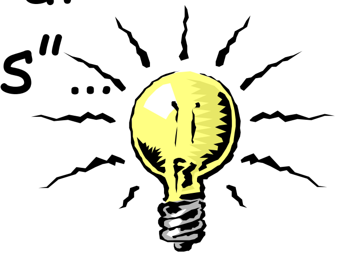
  where E is public, D is private.
- At times, we thought it impossible…
- Since then, we have learned “Meta-theorem of Cryptography“:

  Any apparently contradictory set of requirements can be met using right mathematical approach…

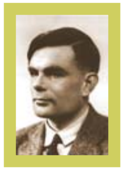


Invention of RSA

- Tried and discarded many approaches, including some “knapsack-based” ones. (Len was great at killing off bad ideas.)
- “Group of unknown size” seemed useful idea… as did “permutation polynomials”…
- After a “seder” at a student’s…
- “RSA” uses $n = pq$ product of primes:



$C = M^e \pmod{n}$ [public key (e,n)]
$M = C^d \pmod{n}$ [private key (d,n)]

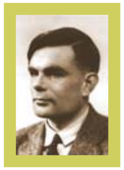


$100 RSA SciAm Challenge

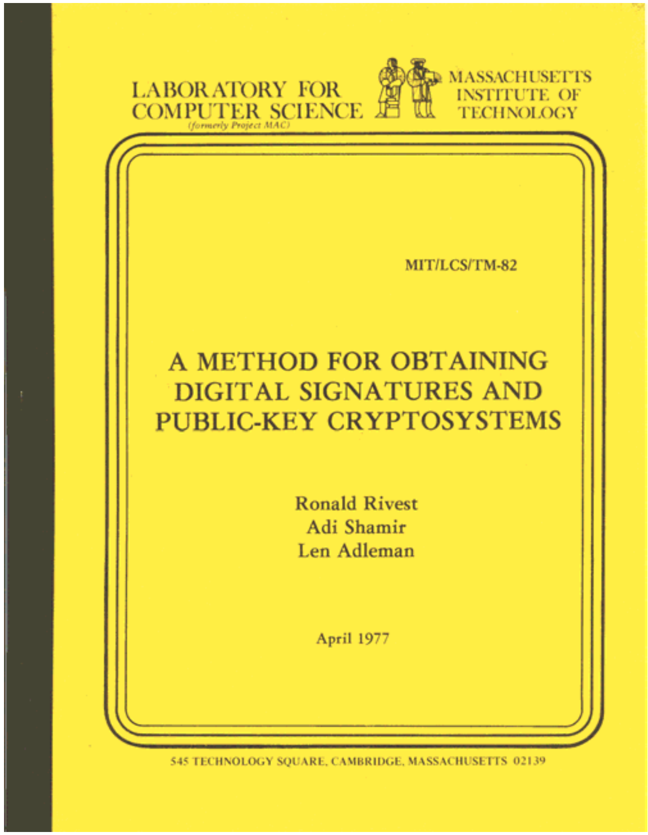
- Martin Gardner publishes *Scientific American* column about RSA in August '77, including our $100 challenge (129 digit n) and our infamous "40 quadrillion years" estimate required to factor

  RSA-129 =
  114,381,625,757,888,867,669,235,779,976,146,612,010,218,296,721,242,362,562,561,842,935,706,935,245,733,897,830,597,123,563,958,705,058,989,075,147,599,290,026,879,543,541
  (129 digits)

  or to decode encrypted message.



TM-82 4/77; CACM 2/78



LABORATORY FOR COMPUTER SCIENCE
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A METHOD FOR OBTAINING DIGITAL SIGNATURES AND PUBLIC-KEY CRYPTOSYSTEMS

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April 1977

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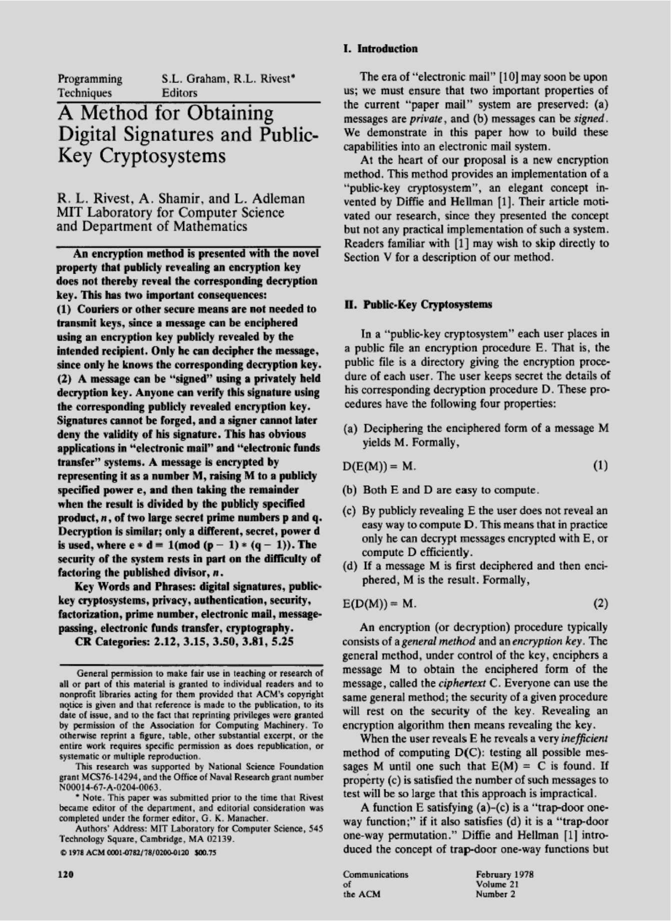

Programming Techniques
S.L. Graham, R.L. Rivest*
Editors

A Method for Obtaining Digital Signatures and Public-Key Cryptosystems

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An encryption method is presented with the novel property that publicly revealing an encryption key does not thereby reveal the corresponding decryption key. This has two important consequences:
(1) Couriers or other secure means are not needed to transmit keys, since a message can be enciphered using an encryption key publicly revealed by the intended recipient. Only he can decipher the message, since only he knows the corresponding decryption key.
(2) A message can be "signed" using a privately held decryption key. Anyone can verify this signature using the corresponding publicly revealed encryption key. Signatures cannot be forged, and a signer cannot later deny the validity of his signature. This has obvious applications in "electronic mail" and "electronic funds transfer" systems. A message is encrypted by representing it as a number M, raising M to a publicly specified power e, and then taking the remainder when the result is divided by the publicly specified product, *n*, of two large secret prime numbers p and q. Decryption is similar; only a different, secret, power d is used, where $e * d = 1 (\bmod (p-1) * (q-1))$. The security of the system rests in part on the difficulty of factoring the published divisor, *n*.

Key Words and Phrases: digital signatures, public-key cryptosystems, privacy, authentication, security, factorization, prime number, electronic mail, message-passing, electronic funds transfer, cryptography.

CR Categories: 2.12, 3.15, 3.50, 3.81, 5.25

I. Introduction

The era of "electronic mail" [10] may soon be upon us; we must ensure that two important properties of the current "paper mail" system are preserved: (a) messages are *private*, and (b) messages can be *signed*. We demonstrate in this paper how to build these capabilities into an electronic mail system.

At the heart of our proposal is a new encryption method. This method provides an implementation of a "public-key cryptosystem", an elegant concept invented by Diffie and Hellman [1]. Their article motivated our research, since they presented the concept but not any practical implementation of such a system. Readers familiar with [1] may wish to skip directly to Section V for a description of our method.

II. Public-Key Cryptosystems

In a "public-key cryptosystem" each user places in a public file an encryption procedure E. That is, the public file is a directory giving the encryption procedure of each user. The user keeps secret the details of his corresponding decryption procedure D. These procedures have the following four properties:

(a) Deciphering the enciphered form of a message M yields M. Formally,

$$D(E(M)) = M. \tag{1}$$

(b) Both E and D are easy to compute.

(c) By publicly revealing E the user does not reveal an easy way to compute D. This means that in practice only he can decrypt messages encrypted with E, or compute D efficiently.

(d) If a message M is first deciphered and then enciphered, M is the result. Formally,

$$E(D(M)) = M. \tag{2}$$

An encryption (or decryption) procedure typically consists of a *general method* and an *encryption key*. The general method, under control of the key, enciphers a message M to obtain the enciphered form of the message, called the *ciphertext* C. Everyone can use the same general method; the security of a given procedure will rest on the security of the key. Revealing an encryption algorithm then means revealing the key.

When the user reveals E he reveals a very *inefficient* method of computing D(C): testing all possible messages M until one such that E(M) = C is found. If property (c) is satisfied the number of such messages to test will be so large that this approach is impractical.

A function E satisfying (a)–(c) is a "trap-door one-way function;" if it also satisfies (d) it is a "trap-door one-way permutation." Diffie and Hellman [1] introduced the concept of trap-door one-way functions but

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* Note. This paper was submitted prior to the time that Rivest became editor of the department, and editorial consideration was completed under the former editor, G. K. Manacher.

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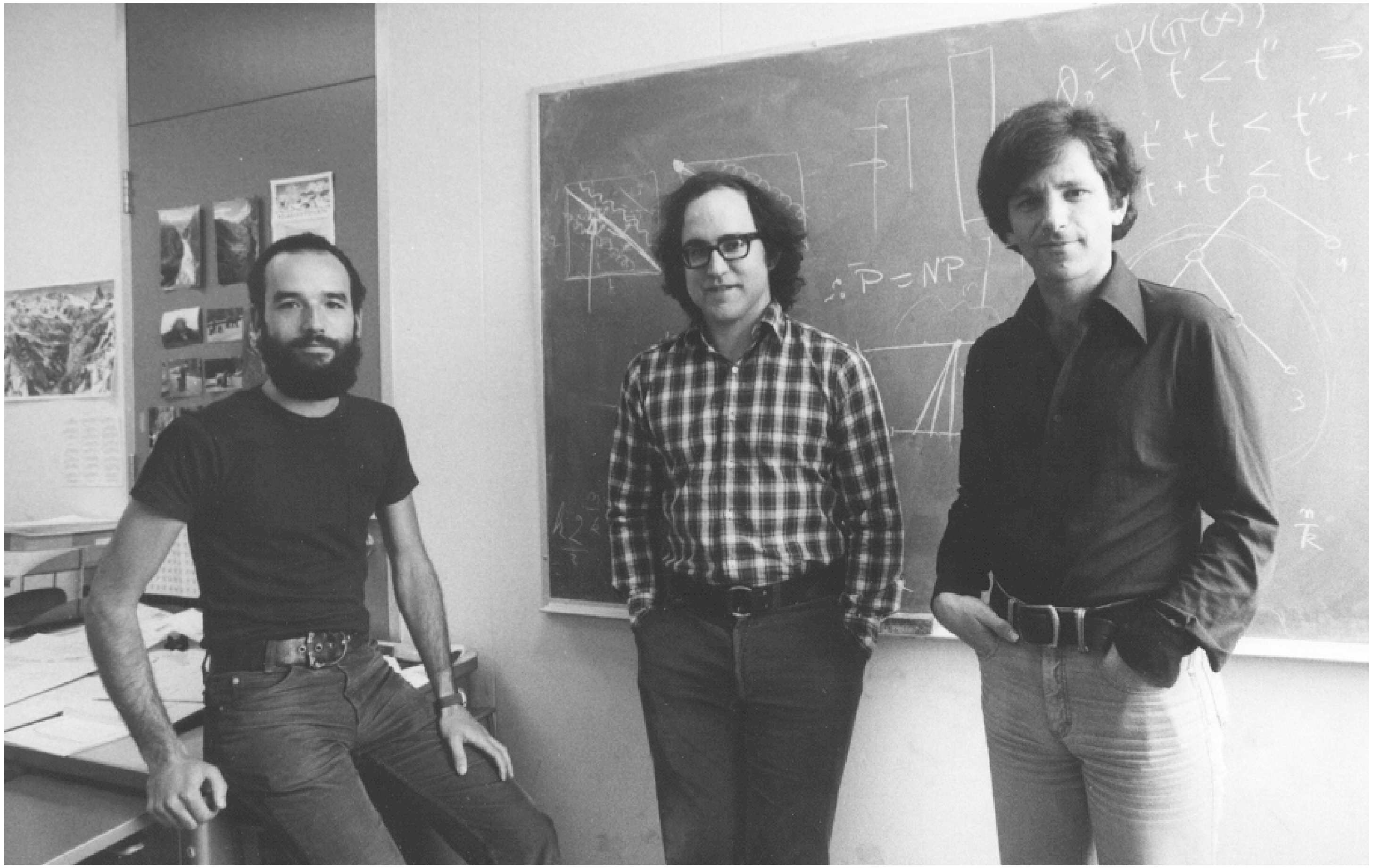
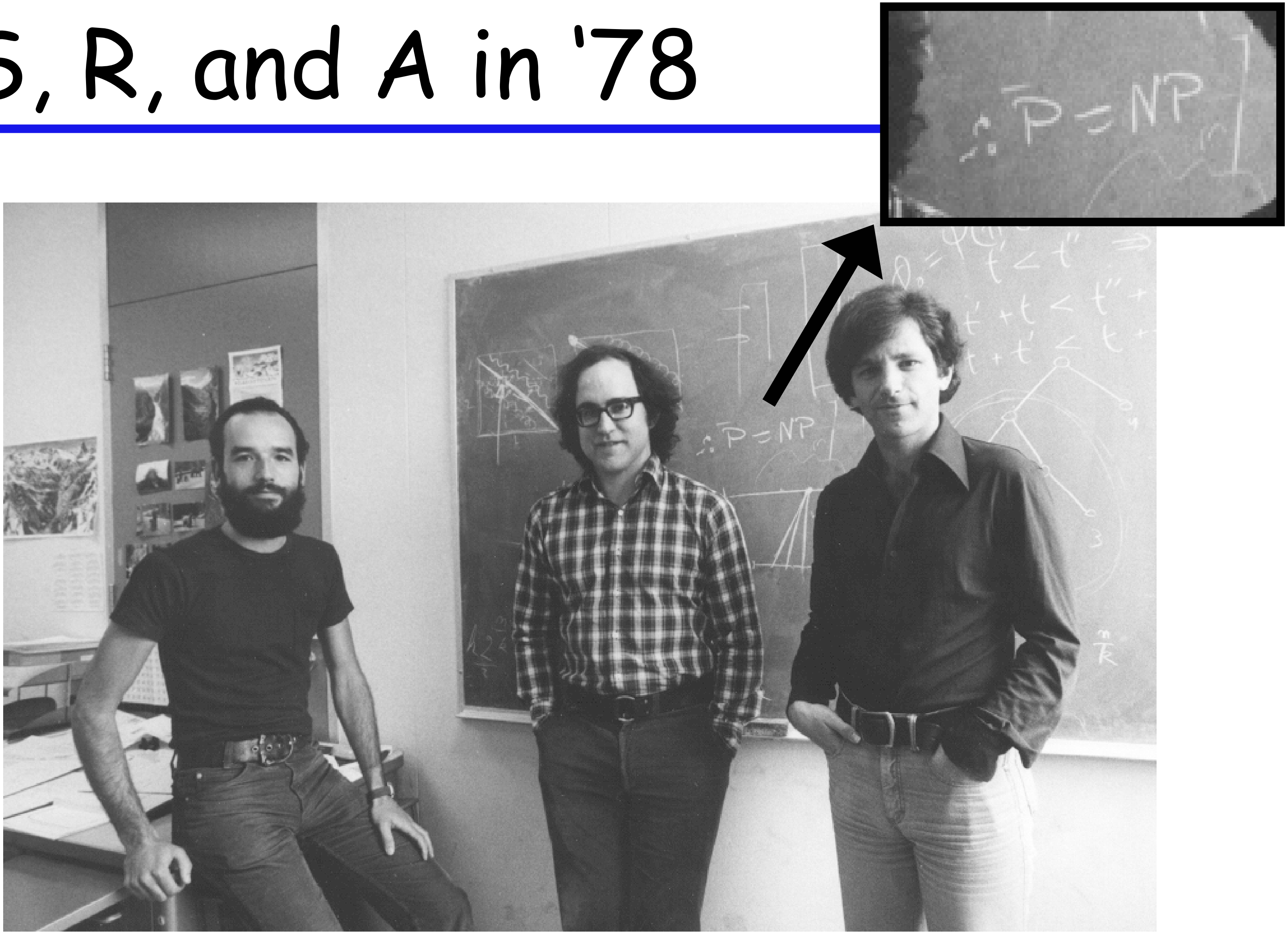
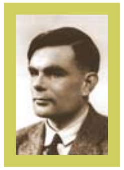
S, R, and *A* in '78



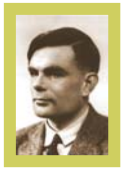
S, R, and *A* in '78





The wonderful Zn*

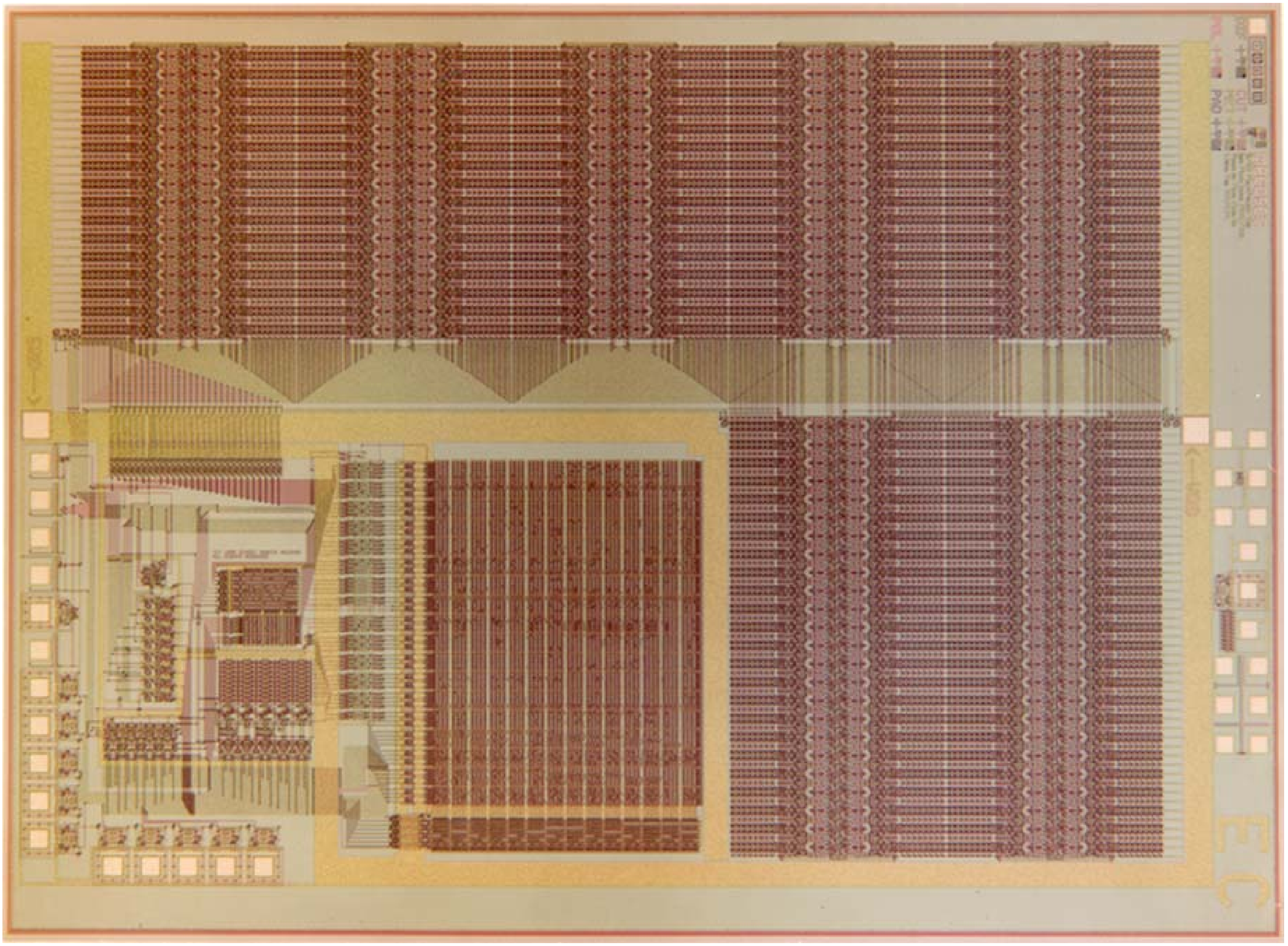
- Zn* = multiplicative group modulo n = pq
- Factoring makes it hard for adversary
  - to compute size of group
  - to compute discrete logs
- Taking e-th roots modulo n is hard ("RSA Assumption")
- Taking e-th roots is hard, where the adversary can pick e>1.
  ("Strong RSA Assumption")

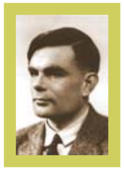


Moore's Law matters.

- Time to do RSA decryption on a 1 MIPS VAX was around 30 seconds (VERY SLOW…)
- IBM PC debuts in 1981
- Still, we worked on efficient special-purpose implementation (e.g. special circuit board, and then the "RSA chip", which did RSA in 0.4 seconds) to prove practicality of RSA.
- Moore's Law to the rescue---software now runs 2000x faster…
- Now software and the Web rule…

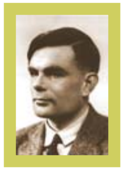
Photo of RSA chip





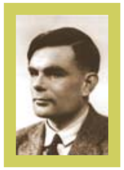
Do cryptography in public.

- Confidence in cryptographic schemes derives from intensive *public review.*
- *Public standards* (e.g. PKCS series)
- Vigorous public research effort results in many new cryptographic proposals, definitions, and attacks



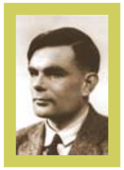
Other PKC proposals

- 1978: Merkle/Hellman (knapsack)
- 1979: Rabin/Williams (factoring)
- 1984: Goldwasser/Micali (QR)
- 1985: El Gamal (DLP)
- 1985: Miller/Koblitz (elliptic curves)
- 1998: Cramer/Shoup
- … many others, too



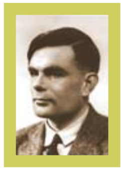
$100 RSA Challenge Met '94

- RSA-129 was factored in 1994, using thousands of computers on Internet. "The magic words are squeamish ossifrage."
- Cheapest purchase of computing time ever!
- Gives credibility to difficulty of factoring, and helps establish key sizes needed for security.



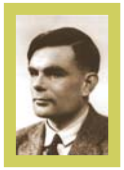
Factoring milestones

- '84: 69D (D = "digits")
  (Sandia; Time magazine)
- '91: 100D
  (Quadratic sieve)
- '94: 129D ($100 challenge number)
  (Distributed QS)
- '99: 155D
  (512-bits; Number field sieve)
- '01: 15 = 3 * 5
  (4 bits; IBM quantum computer!)



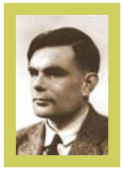
Other attacks on RSA

- Cycling attacks (?)
- Attacks based on “weak keys” (?)
- Attacks based on lack of randomization or improper “padding”
  (use e.g. Bellare/Rogaway’s OAEP ’94)
- Timing analysis, power analysis, fault attacks, …
- See Boneh’s “Twenty Years of Attacks on the RSA Cryptosystem”.



Crypto theory matters

- probabilistic encryption,
- chosen-ciphertext attacks
- GMR digital signatures,
- zero-knowledge protocols,
- concrete complexity of cryptographic reductions; practice-oriented provable security
- …



Organizations matter

- ACM
  - e.g. CACM published RSA paper
- IACR (David Chaum)
  - sponsors CRYPTO conferences
- RSA (Jim Bidzos)
  - sponsors RSA conferences
  - leader in many policy debates
  - helped to set crypto standards

(The End)