88. Supersymmetry, Part I (Theory)

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88.1 Introduction

Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa [1]. The existence of such a non-trivial extension of the Poincaré symmetry of ordinary quantum field theory was initially surprising, and its form is highly constrained by theoretical principles [2]. SUSY also provides a framework for the unification of particle physics and gravity [3–5] at the Planck energy scale, $M_{\rm P} \sim 10^{19}$ GeV, where the gravitational interactions become comparable in strength to the gauge interactions. Moreover, supersymmetry can stabilize the hierarchy between the energy scale that characterizes electroweak symmetry breaking, $M_{\rm EW} \sim 100$ GeV, and the Planck scale [6–9] against large radiative corrections. The stability of this large gauge hierarchy with respect to radiative quantum corrections is not possible to maintain in the Standard Model (SM) without an unnatural fine-tuning of the parameters of the fundamental theory at the Planck scale. In contrast, in a supersymmetric extension of the SM, it is possible to maintain the gauge hierarchy while providing a natural framework for elementary scalar fields.

If supersymmetry were an exact symmetry of nature, then particles and their superpartners, which differ in spin by half a unit, would be degenerate in mass. Since superpartners have not (yet) been observed, supersymmetry must be a broken symmetry. Nevertheless, the stability of the gauge hierarchy can still be maintained if the SUSY breaking is soft [10,11], and the corresponding SUSY-breaking mass parameters are no larger than a few TeV. Whether this is still plausible in light of recent SUSY searches at the LHC (see Sec. 89) will be discussed in Sec. 88.7.

In particular, soft-SUSY-breaking terms of the Lagrangian involve combinations of fields with total mass dimension of three or less, with some restrictions on the dimension-three terms as elucidated in Ref. [10]. The impact of the soft terms becomes negligible at energy scales much larger than the size of the SUSY-breaking masses. Thus, a theory of weak-scale supersymmetry, where the effective scale of supersymmetry breaking is tied to the scale of electroweak symmetry breaking, provides a natural framework for the origin and the stability of the gauge hierarchy [6–9].

At present, there is no unambiguous experimental evidence for the breakdown of the SM at or below the TeV scale. The expectations for new TeV-scale physics beyond the SM are based primarily on three theoretical arguments. First, in a theory with an elementary scalar field of mass m and interaction strength λ (*e.g.*, a quartic scalar self-coupling, the square of a gauge coupling or the square of a Yukawa coupling), the stability with respect to quantum corrections requires the existence of an energy cutoff roughly of order $(16\pi^2/\lambda)^{1/2}m$, beyond which new physics must enter [12]. A significantly larger energy cutoff would require an unnatural fine-tuning of parameters that govern the effective low-energy theory. Applying this argument to the SM leads to an expectation of new physics at the TeV scale [9].

Second, the unification of the three SM gauge couplings at a very high energy close to the Planck scale is possible if new physics beyond the SM (which modifies the running of the gauge couplings above the electroweak scale) is present. The minimal supersymmetric extension of the SM, where superpartner masses lie below a few TeV, provides an example of successful gauge coupling unification [13].

Third, the existence of dark matter that makes up approximately one quarter of the energy density of the universe, cannot be explained within the SM of particle physics [14]. Remarkably, a stable weakly-interacting massive particle (WIMP) whose mass and interaction rate are governed by new physics associated with the TeV-scale can be consistent with the observed density of dark matter (this is the so-called WIMP miracle, which is reviewed in Ref. [15]). The lightest supersymmetric particle (LSP), if stable, is a promising (although not the unique) candidate for the dark matter [16–20]. Further aspects of dark matter can be found in Sec. 27.

88.2 Structure of the MSSM

The minimal supersymmetric extension of the SM (MSSM) consists of the fields of the two-Higgs-doublet extension of the SM and the corresponding superpartner fields [21–25]. A particle and its superpartner together form a supermultiplet. The corresponding field content of the supermultiplets of the MSSM and their gauge quantum numbers are shown in Table 88.1. The electric charge $Q = T_3 + \frac{1}{2}Y$ is determined in terms of the third component of the weak isospin (T_3) and the U(1) weak hypercharge (Y).

The gauge supermultiplets consist of the gluons and their gluino fermionic superpartners and the SU(2)×U(1) gauge bosons and their gaugino fermionic superpartners. The matter supermultiplets consist of three generations of left-handed quarks and leptons and their scalar superpartners

Table 88.1: The fields of the MSSM and their representations under the SU(3)×SU(2)×U(1) gauge group are listed. For simplicity, only one generation of quarks and leptons is exhibited. For each lepton, quark, and Higgs supermultiplet (each denoted by a hatted upper-case letter), there is a corresponding antiparticle multiplet of charge-conjugated fermions and their associated scalar partners [26].

Field Content of the MSSM						
Super-multiplets	Super-field	Bosonic fields	Fermionic partners	SU(3)	SU(2)	U(1)
gluon/gluino	$\hat{V}_8$	g	$\widetilde{g}$	8	1	0
gauge boson/ gaugino	$\hat{V}$	$W^\pm$, W^0	$\widetilde{W}^\pm$, $\widetilde{W}^0$	1	3	0
	$\hat{V}'$	B	$\widetilde{B}$	1	1	0
slepton/ lepton	$\hat{L}$	$(\widetilde{\nu}_L, \widetilde{e}^-_L)$	$(\nu, e^-)_L$	1	2	-1
	$\hat{E}^c$	$\tilde{e}^+_R$	e^c_L	1	1	2
squark/ quark	$\hat{Q}$	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$	3	2	$1/3$
	$\hat{U}^c$	$\widetilde{u}^*_R$	u^c_L	$\bar{3}$	1	$-4/3$
	$\hat{D}^c$	$\widetilde{d}^*_R$	d^c_L	$\bar{3}$	1	$2/3$
Higgs boson/ higgsino	$\hat{H}_d$	$(H^0_d\,,\, H^-_d)$	$(\widetilde{H}^0_d, \widetilde{H}^-_d)$	1	2	-1
	$\hat{H}_u$	$(H^+_u\,,\, H^0_u)$	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$	1	2	1

(squarks and sleptons, collectively referred to as sfermions), and the corresponding antiparticles. The Higgs supermultiplets consist of two complex Higgs doublets, their higgsino fermionic superpartners, and the corresponding antiparticles. The enlarged Higgs sector of the MSSM constitutes the minimal structure needed to guarantee the cancellation of gauge anomalies [27] generated by the higgsino superpartners that can appear as internal lines in triangle diagrams with three external electroweak gauge bosons. Moreover, without a second Higgs doublet, one cannot generate mass for both "up"-type and "down"-type quarks (and charged leptons) in a way consistent with the underlying SUSY [28–30].

In the most elegant treatment of SUSY, spacetime is extended to superspace which consists of the spacetime coordinates and new anticommuting fermionic coordinates θ and $\theta^\dagger$ [25,31,32]. Each supermultiplet is represented by a superfield that is a function of the superspace coordinates. The fields of a given supermultiplet (which are functions of the spacetime coordinates) are coefficients of the θ and $\theta^\dagger$ expansion of the corresponding superfield.

Vector superfields contain the gauge-boson fields and their gaugino partners. Chiral superfields contain the spin-0 and spin-1/2 fields of the matter or Higgs supermultiplets. A general supersymmetric Lagrangian is determined by three functions of the chiral superfields [4]: the superpotential, the Kähler potential, and the gauge kinetic function (which can be appropriately generalized to accommodate higher derivative terms [33]). Minimal forms for the Kähler potential and gauge kinetic function, which generate canonical kinetic energy terms for all the fields, are required for renormalizable globally supersymmetric theories. A renormalizable superpotential, which is at most cubic in the chiral superfields, yields supersymmetric Yukawa couplings and mass terms. A combination of gauge invariance and SUSY produces couplings of gaugino fields to matter (or Higgs) fields and their corresponding superpartners. The (renormalizable) MSSM Lagrangian is then constructed by including all possible supersymmetric interaction terms (of dimension four or less) that satisfy SU(3)×SU(2)×U(1) gauge invariance and B–L conservation (where $B =$ baryon number and $L =$ lepton number). Finally, the most general soft-supersymmetry-breaking terms consistent with these symmetries are added [10, 11, 34].

Although the MSSM is the focus of much of this review, there is some motivation for considering non-minimal supersymmetric extensions of the SM [35]. For example, extra structure is needed to generate non-zero neutrino masses as discussed in Sec. 88.8. In addition, in order to address some theoretical issues and tensions associated with the MSSM, it has been fruitful to introduce one additional singlet Higgs superfield. The resulting next-to-minimal supersymmetric extension of the Standard Model (NMSSM) [36] is briefly considered in Sec. 88.4–88.7 and 88.9. Finally, one is always free to add additional fields to the SM along with the corresponding superpartners. However, only certain choices for the new fields (*e.g.*, the addition of complete SU(5) multiplets) will preserve the successful gauge coupling unification of the MSSM. Some examples will be briefly mentioned in Sec. 88.9.

88.2.1 *R-parity and the lightest supersymmetric particle*

The (renormalizable) SM Lagrangian possesses an accidental global B–L symmetry due to the fact that B and L-violating operators composed of SM fields must have dimension $d = 5$ or larger [37]. Consequently, B and L-violating effects are suppressed by $(M_{\rm EW}/M)^{d-4}$, where M is the characteristic mass scale of the physics that generates the corresponding higher dimensional operators. Indeed, values of M of order the grand unification scale or larger may be responsible for the observed (approximate) stability of the proton and suppression of neutrino masses. Unfortunately, these results are not guaranteed in a generic supersymmetric extension of the SM. For example, it is possible to construct gauge invariant supersymmetric dimension-four B and L-violating operators made up of fields of SM particles and their superpartners. Such operators, if simultaneously present in the theory, would typically yield a proton decay rate many orders of magnitude larger than the current experimental bound. It is for this reason that B–L conservation is *imposed* on the supersymmetric Lagrangian when defining the MSSM, which is sufficient for eliminating all B and L-violating operators of dimension $d \leq 4$.

As a consequence of the B–L symmetry, the MSSM possesses a multiplicative R-parity invariance, where $R = (-1)^{3(B-L)+2S}$ for a particle of spin S [38]. This implies that all the particles of the SM have even R-parity, whereas the corresponding superpartners have odd R-parity. The conservation of R-parity in scattering and decay processes has a critical impact on supersymmetric phenomenology. For example, any initial state in a scattering experiment will involve ordinary (R-even) particles. Consequently, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay into lighter states. Moreover, R-parity invariance also implies that the LSP is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle. In order to be consistent with cosmological constraints, a stable LSP is almost certainly electrically and color neutral [18]. Consequently, the LSP in an R-parity-conserving theory is weakly interacting with ordinary matter, *i.e.*, it behaves like a stable heavy neutrino and will escape collider detectors without being directly observed. Thus, the canonical signature for conventional R-parity-conserving supersymmetric theories is missing (transverse) momentum, due to the escape of the LSP. Moreover, as noted in Sec. 88.1 and reviewed in Refs. [19] and [20], the stability of the LSP in R-parity-conserving SUSY makes it a promising candidate for dark matter.

The possibility of relaxing the R-parity invariance of the MSSM (which would generate new B and/or L-violating interactions) will be addressed in Sec. 88.8.2. However, note that in R-parity violating (RPV) models, the LSP is no longer stable and thus would not be a viable candidate for the dark matter (unless its lifetime was significantly longer than the age of the universe). In such scenarios, one has to look elsewhere to explain the origin of dark matter.

88.2.2 *The goldstino and gravitino*

In the MSSM, SUSY breaking is accomplished by including the most general renormalizable soft-SUSY-breaking terms consistent with the SU(3)×SU(2)×U(1) gauge symmetry and R-parity invariance. These terms parameterize our ignorance of the fundamental mechanism of supersymmetry breaking. If supersymmetry breaking occurs spontaneously, then a massless Goldstone fermion called the goldstino ($\widetilde{G}_{1/2}$) must exist. The goldstino would then be the LSP, and could play an important role in supersymmetric phenomenology [39].

However, the goldstino degrees of freedom are physical only in models of spontaneously-broken global SUSY. If SUSY is a local symmetry, then the theory must incorporate gravity; the resulting theory is called supergravity [5, 40]. In models of spontaneously-broken supergravity, the goldstino is "absorbed" by the gravitino ($\widetilde{G}$), the spin-3/2 superpartner of the graviton, via the super-Higgs mechanism [41]. Consequently, the goldstino is removed from the physical spectrum and the gravitino acquires a mass (denoted by $m_{3/2}$). If $m_{3/2}$ is smaller than the mass of the lightest superpartner of the SM particles, then the gravitino is the LSP.

In processes with center-of-mass energy $E \gg m_{3/2}$, one can employ the goldstino–gravitino equivalence theorem [42], which implies that the interactions of the helicity $\pm\frac{1}{2}$ gravitino (whose properties approximate those of the goldstino) dominate those of the helicity $\pm\frac{3}{2}$ gravitino. The interactions of gravitinos with other light fields can be described by a low-energy effective Lagrangian that is determined by fundamental principles [43].

88.2.3 *Hidden sectors and the structure of SUSY breaking*

It is very difficult (perhaps impossible) to construct a realistic model of spontaneously-broken weak-scale supersymmetry where the supersymmetry breaking arises solely as a consequence of the interactions of the particles of the MSSM. A more successful scheme posits a theory with at least two distinct sectors: a visible sector consisting of the particles of the MSSM [34] and a sector where SUSY breaking is generated. It is often (but not always) assumed that particles of the hidden sector are neutral with respect to the SM gauge group. The effects of the hidden sector supersymmetry breaking are then transmitted to the MSSM by some mechanism (often involving the mediation by particles that comprise an additional messenger sector). Two theoretical scenarios that exhibit this structure are gravity-mediated and gauge-mediated SUSY breaking.

Supergravity models provide a natural mechanism for transmitting the SUSY breaking of the hidden sector to the particle spectrum of the MSSM. In models of gravity-mediated SUSY breaking, gravity is the messenger of supersymmetry breaking [44–48]. More precisely, supersymmetry breaking is mediated by effects of gravitational strength (*i.e.* suppressed by inverse powers of the Planck mass). The soft-SUSY-breaking parameters with dimensions of mass arise as model-dependent multiples of the gravitino mass $m_{3/2}$. In this scenario, $m_{3/2}$ is of order the electroweak-symmetry-breaking scale, while the gravitino couplings are roughly gravitational in strength [3, 49].[1]

Under certain theoretical assumptions that govern the structure of the Kähler potential (the so-called sequestered form introduced in Ref. [51]), SUSY breaking is due entirely to the super-conformal (super-Weyl) anomaly, which is common to all supergravity models [51]. In particular, gaugino masses are radiatively generated at one-loop, and squark and slepton squared-mass matrices are flavor-diagonal. In sequestered scenarios, sfermion squared-masses arise at two-loops, which implies that gluino and sfermion masses are of the same order of magnitude. This approach is called anomaly-mediated SUSY breaking (AMSB). Indeed, anomaly mediation is more generic than originally conceived, and provides a ubiquitous source of SUSY breaking [52]. However in

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[1] However, such a gravitino typically plays no direct role in supersymmetric phenomenology at colliders (except perhaps indirectly in the case where the gravitino is the LSP [50]).

the simplest formulation of AMSB as applied to the MSSM, the squared-masses of the sleptons are negative (known as the tachyonic slepton problem). It may be possible to cure this otherwise fatal flaw in non-minimal extensions of the MSSM [53]. Alternatively, one can assert that anomaly mediation is not the sole source of SUSY breaking in the sfermion sector. In non-sequestered scenarios, sfermion squared-masses can arise at tree-level, in which case squark masses would be parametrically larger than the loop-suppressed gaugino masses [54].

In gauge-mediated supersymmetry breaking (GMSB), gauge forces transmit the supersymmetry breaking to the MSSM. A typical structure of such models involves a hidden sector where SUSY is broken, a messenger sector consisting of particles (messengers) with nontrivial SU(3)×SU(2)×U(1) quantum numbers, and the visible sector consisting of the fields of the MSSM [55–58]. The direct coupling of the messengers to the hidden sector generates a supersymmetry-breaking spectrum in the messenger sector. Supersymmetry breaking is then transmitted to the MSSM via the virtual exchange of the messenger fields. In models of direct gauge mediation, there is no separate hidden sector. In particular, the sector in which the SUSY breaking originates includes fields that carry nontrivial SM quantum numbers, which allows for the direct transmission of SUSY breaking to the MSSM [59].

In models of gauge-mediated SUSY breaking with a minimal Kähler potential, the gravitino is the LSP [16], as its mass can range from a few eV (in the case of low SUSY breaking scales) up to a few GeV (in the case of high SUSY breaking scales). In particular, the gravitino is a potential dark matter candidate (for a review and guide to the literature, see Ref. [20]). The couplings of the helicity $\pm\frac{1}{2}$ components of $\widetilde{G}$ to the particles of the MSSM (which approximate those of the goldstino as previously noted in Sec. 88.2.2) are significantly stronger than gravitational strength and amenable to experimental collider analyses.

The mass ranges of the gravitino in either gravity-mediated or gauge-mediated SUSY breaking are further constrained by cosmological considerations [60, 61]. In particular, there is a danger of over-abundance of gravitinos if it is the dark matter or, if it decays before nucleosynthesis, modifications to the successful predictions of light element abundances. Avoiding these cosmological gravitino problems imposes strong constraints on gravity-mediated and gauge-mediated SUSY breaking models.

The concept of a hidden sector is more general than SUSY. Hidden valley models [62] posit the existence of a hidden sector of new particles and interactions that are very weakly coupled to particles of the SM. The impact of a hidden valley on supersymmetric phenomenology at colliders can be significant if the LSP lies in the hidden sector [63].

88.2.4 *SUSY and extra dimensions*

Approaches to SUSY breaking have also been developed in the context of theories in which the number of spatial dimensions is greater than three. In particular, a number of SUSY-breaking mechanisms have been proposed that are inherently extra-dimensional [64]. The size of the extra dimensions can be significantly larger than $M_{\rm P}^{-1}$; in some cases of order $({\rm TeV})^{-1}$ or even larger (see, *e.g.*, Sec. 85 and Ref. [65]).

For example, in one approach the fields of the MSSM live on some brane (a lower-dimensional manifold embedded in a higher-dimensional spacetime), while the sector of the theory that breaks SUSY lives on a second spatially-separated brane. Two examples of this approach are AMSB [51] and gaugino-mediated SUSY breaking [66]. In both cases, SUSY breaking is transmitted through fields that live in the bulk (the higher-dimensional space between the two branes). This setup has some features in common with both gravity-mediated and gauge-mediated SUSY breaking (*e.g.*, hidden and visible sectors and messengers).

Since a higher dimensional theory must be compactified to four spacetime dimensions, one can also generate a source of SUSY breaking by employing boundary conditions on the compactified space that distinguish between fermions and bosons. This is the so-called Scherk-Schwarz mechanism [67]. The phenomenology of such models can be strikingly different from that of the usual MSSM [68].

88.2.5 *Split-SUSY*

If SUSY is not connected with the origin of the electroweak scale, it may still be possible that some remnant of the superparticle spectrum survives down to the TeV-scale or below. This is the idea of split-SUSY [69, 70], in which scalar superpartners of the quarks and leptons are significantly heavier than 1 TeV, whereas the fermionic superpartners of the gauge and Higgs bosons have masses on the order of 1 TeV or below. With the exception of a single light neutral scalar whose properties are practically indistinguishable from those of the SM Higgs boson, all other Higgs bosons are also assumed to be very heavy. Among the supersymmetric particles, only the fermionic superpartners may be kinematically accessible at the LHC.

In models of split SUSY, the top squark masses cannot be arbitrarily large, as these parameters enter in the radiative corrections to the mass of the observed Higgs boson [71–73]. In the MSSM, a Higgs boson mass of 125 GeV (see Sec. 11) implies an upper bound on the top squark mass scale in the range of 10 to 10^8 TeV [74–76], depending on the value of the ratio of the two neutral Higgs field vacuum expectation values, although this mass range can be somewhat extended by varying other relevant MSSM parameters. In some approaches, gaugino masses are one-loop suppressed relative to the sfermion masses, corresponding to the so-called mini-split SUSY spectrum [72, 77]. The higgsino mass scale may or may not be likewise suppressed depending on the details of the model [78].

The SUSY breaking required to produce such a split-SUSY spectrum would destabilize the gauge hierarchy, and thus would not provide an explanation for the scale of electroweak symmetry breaking. Nevertheless, models of split-SUSY can account for the dark matter (which is assumed to be the LSP gaugino or higgsino) and gauge coupling unification, thereby preserving two of the desirable features of weak-scale SUSY. Finally, as a consequence of the very large squark and slepton masses, neutral flavor changing and CP-violating effects, which can be problematic in models with TeV-scale SUSY-breaking masses, are sufficiently reduced to avoid conflict with experimental observations.

88.3 Parameters of the MSSM

The parameters of the MSSM are conveniently described by considering separately the supersymmetry-conserving and the supersymmetry-breaking sectors. A careful discussion of the conventions used here in defining the tree-level MSSM parameters can be found in Refs. [25, 79, 80]. For simplicity, consider first the case of one generation of quarks, leptons, and their scalar superpartners.

88.3.1 *The SUSY-conserving parameters*

The parameters of the supersymmetry-conserving sector consist of: (i) gauge couplings, g_s, g, and g', corresponding to the SM gauge group SU(3)×SU(2)×U(1) respectively; (ii) a supersymmetry-conserving higgsino mass parameter μ; and (iii) Higgs-fermion Yukawa couplings, λ_u, λ_d, and λ_e, of one generation of left- and right-handed quarks and leptons, and their superpartners to the Higgs bosons and higgsinos. Because there is no right-handed neutrino/sneutrino in the MSSM as defined here, a Yukawa coupling λ_ν is not included. The complex μ parameter and Yukawa couplings enter via the most general renormalizable R-parity-conserving superpotential,

$$W_{\rm MSSM} = \lambda_d \hat{H}_d \hat{Q} \hat{D}^c - \lambda_u \hat{H}_u \hat{Q} \hat{U}^c + \lambda_e \hat{H}_d \hat{L} \hat{E}^c + \mu \hat{H}_u \hat{H}_d \,, \tag{88.1}$$

where the superfields are defined in Table 1 and the gauge group indices are suppressed. More explicitly, the so-called "μ-term" can be written out as $\mu\epsilon^{ab}(\hat{H}_u)_a(\hat{H}_d)_b$ with an implicit sum over repeated indices, where ϵ^{ab} is used to tie together the SU(2) weak isospin indices $a, b \in \{1, 2\}$ in a gauge-invariant way (where $\epsilon^{12} = -\epsilon^{21} = 1$ and $\epsilon^{11} = \epsilon^{22} = 0$). Likewise, the term $\hat{H}_u\hat{Q}\hat{U}^c$ can be written out as $\epsilon^{ab}(\hat{H}_u)_a\hat{Q}_{ib}(\hat{U}^c)^i$, where $i \in \{1, 2, 3\}$ is the SU(3) color index. Finally, $\hat{H}_d\hat{Q}\hat{D}^c$ can be written out as $\epsilon^{ab}(\hat{H}_d)_a\hat{Q}_{ib}(\hat{D}^c)^i$, with an analogous expression for $\hat{H}_d\hat{L}\hat{E}^c$.

88.3.2 *The SUSY-breaking parameters*

The supersymmetry-breaking sector contains the following sets of parameters: (i) three complex gaugino Majorana mass parameters, M_3, M_2, and M_1, associated with the SU(3), SU(2), and U(1) subgroups of the SM; (ii) five sfermion squared-mass parameters, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$, corresponding to the five electroweak gauge multiplets, *i.e.*, superpartners of the left-handed fields $(u, d)_L$, u^c_L, d^c_L, $(\nu,\ e^-)_L$, and e^c_L, where the superscript c indicates a charge-conjugated fermion field [26]; and (iii) three Higgs-squark-squark and Higgs-slepton-slepton trilinear interaction terms, with complex coefficients $T_U \equiv \lambda_u A_U$, $T_D \equiv \lambda_d A_D$, and $T_E \equiv \lambda_e A_E$ (which define the "A-parameters"), following the notation employed in Ref. [80]. It is conventional to separate out the factors of the Yukawa couplings in defining the A-parameters [3, 25] (originally motivated by a simple class of gravity-mediated SUSY-breaking models). If the A-parameters are parametrically of the same order (or smaller) relative to other SUSY-breaking mass parameters, then in most cases only the third generation A-parameters will be phenomenologically relevant.

Finally, we have (iv) two real squared-mass parameters, $m^2_{H_d}$ and $m^2_{H_u}$ (also called m^2_1 and m^2_2, respectively, in the literature), and one complex squared-mass parameter, $m^2_{12} \equiv \mu B$ (the latter defines the "B-parameter"), which appear in the MSSM tree-level scalar Higgs potential [30],

$$\begin{aligned} V = {} & (m^2_{H_d} + |\mu|^2)H^\dagger_d H_d + (m^2_{H_u} + |\mu|^2)H^\dagger_u H_u + (m^2_{12} H_u H_d + \text{h.c.}) \\ & + \frac{1}{8}(g^2 + g'^2)(H^\dagger_d H_d - H^\dagger_u H_u)^2 + \frac{1}{2}g^2|H^\dagger_d H_u|^2\,, \end{aligned} \tag{88.2}$$

where the SU(2)-invariant combination of the complex doublet scalar fields H_u and H_d that appears in Eq. (88.2) is given by $H_u H_d \equiv \epsilon^{ab}(H_u)_a(H_d)_b = H^+_u H^-_d - H^0_u H^0_d$. Note that the quartic Higgs couplings are related to the gauge couplings g and g' as a consequence of SUSY. The breaking of the SU(2)×U(1) electroweak symmetry group to $\text{U}(1)_{\text{EM}}$ is only possible after incorporating the SUSY-breaking Higgs squared-mass parameters $m^2_{H_d}$, $m^2_{H_u}$ (which can be negative) and m^2_{12}. After minimizing the Higgs scalar potential, these three squared-mass parameters can be re-expressed in terms of the two Higgs vacuum expectation values, $\langle H^0_d\rangle \equiv v_d/\sqrt{2}$ and $\langle H^0_u\rangle \equiv v_u/\sqrt{2}$, and the CP-odd Higgs mass m_A [cf. Eqs. (88.4) and (88.5) below]. One is always free to rephase the Higgs doublet fields such that v_d and v_u (also called v_1 and v_2, respectively, in the literature) are both real and positive.

The quantity, $v^2_d + v^2_u = 4m^2_W/g^2 = (2G^2_F)^{-1/2} \simeq (246\ \text{GeV})^2$, is fixed by the Fermi constant, G_F, whereas the ratio

$$\tan\beta = v_u/v_d \tag{88.3}$$

is a free parameter such that $0 < \beta < \pi/2$. By employing the tree-level conditions resulting from the minimization of the scalar potential, one can eliminate the diagonal and off-diagonal Higgs squared-masses in favor of $m^2_Z = \frac{1}{4}(g^2 + g'^2)(v^2_d + v^2_u)$, the CP-odd Higgs mass m_A and the parameter $\tan\beta$,

$$\sin 2\beta = \frac{2m^2_{12}}{m^2_{H_d} + m^2_{H_u} + 2|\mu|^2} = \frac{2m^2_{12}}{m^2_A}\,, \tag{88.4}$$

$$\frac{1}{2}m^2_Z = -|\mu|^2 + \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1}\,. \tag{88.5}$$

One must also guard against the existence of charge and/or color breaking global minima due to non-zero vacuum expectation values for the squark and charged slepton fields. This possibility can be avoided if the A-parameters are not unduly large [45, 81, 82]. Additional constraints must also be respected to avoid the possibility of directions in scalar field space in which the full tree-level scalar potential can become unbounded from below [82]. A computer program has been developed to calculate vacuum stability bounds in general models at the one-loop level [83], and has been applied to the MSSM in Ref. [84].

Note that SUSY-breaking mass terms for the fermionic superpartners of the scalar fields and non-holomorphic trilinear scalar interactions (*i.e.*, interactions that mix scalar fields and their complex conjugates) have not been included above in the soft-SUSY-breaking sector. These terms can potentially destabilize the gauge hierarchy [10] in models with a gauge-singlet superfield. The latter is not present in the MSSM; hence as noted in Ref. [11], these so-called non-standard soft-SUSY-breaking terms are benign. The phenomenological impact of non-holomorphic soft SUSY-breaking terms has been reconsidered in Refs. [85–87]. However, in the most common approaches to constructing a fundamental theory of SUSY-breaking, the coefficients of these terms (which have dimensions of mass) are significantly suppressed compared to the TeV-scale [88]. Consequently, we follow the usual approach and omit these terms from further consideration.

88.3.3 *MSSM-124*

The total number of independent physical parameters that define the MSSM (in its most general form) is quite large, primarily due to the soft-supersymmetry-breaking sector. In particular, in the case of three generations of quarks, leptons, and their superpartners, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$ are hermitian 3×3 matrices, and A_U, A_D, and A_E are complex 3×3 matrices. In addition, M_1, M_2, M_3, B, and μ are in general complex parameters. Finally, as in the SM, the Higgs-fermion Yukawa couplings, λ_f ($f = u$, d, and e), are complex 3×3 matrices that are related to the quark and lepton mass matrices via: $M_f = \lambda_f v_f / \sqrt{2}$, where $v_e = v_d$ [with v_u and v_d as defined above Eq. (88.3)].

However, not all these parameters are physical. Some of the MSSM parameters can be eliminated by expressing interaction eigenstates in terms of the mass eigenstates, with an appropriate redefinition of the MSSM fields to remove unphysical degrees of freedom. The analysis of Ref. [89] shows that the MSSM possesses 124 independent real degrees of freedom. Of these, 18 correspond to SM parameters (including the QCD vacuum angle θ_{QCD}), one corresponds to a Higgs sector parameter (the analogue of the SM Higgs mass), and 105 are genuinely new parameters of the model. The latter include: five real parameters and three CP-violating phases in the gaugino/higgsino sector, 21 squark and slepton (sfermion) masses, 36 real mixing angles to define the sfermion mass eigenstates, and 40 CP-violating phases that can appear in sfermion interactions. The most general parameterization of the R-parity-conserving MSSM (without additional theoretical assumptions) will be denoted henceforth as MSSM-124 [90].

88.4 The supersymmetric-particle spectrum

The supersymmetric particles (sparticles) differ in spin by half a unit from their SM partners. The superpartners of the gauge and Higgs bosons are fermions, whose names are obtained by appending "ino" to the end of the corresponding SM particle name. The gluino is the color-octet Majorana fermion partner of the gluon with mass $M_{\widetilde{g}} = |M_3|$. The superpartners of the electroweak gauge and Higgs bosons (the gauginos and higgsinos) can mix due to SU(2)×U(1) breaking effects. As a result, the physical states of definite mass are parameter dependent linear combinations of the charged or neutral gauginos and higgsinos, called charginos and neutralinos, respectively (sometimes collectively called electroweakinos). The neutralinos are Majorana fermions, which can lead to some distinctive phenomenological signatures [91, 92]. The superpartners of the quarks and leptons are spin-zero bosons: the squarks, charged sleptons, and sneutrinos, respectively. A complete set of Feynman rules for the sparticles of the MSSM can be found in Refs. [93,94]. The MSSM Feynman rules also are implicitly contained in a number of amplitude generation and Feynman diagram software packages (see *e.g.*, Refs. [95–97]).

It should be noted that all mass formulae quoted below in this Section are tree-level results. Radiative loop corrections will modify these results and must be included in any precision study of supersymmetric phenomenology [98]. Beyond tree level, the definition of the supersymmetric parameters becomes convention-dependent. For example, one can define physical couplings or running couplings, which differ beyond the tree level. This provides a challenge to any effort that attempts to extract supersymmetric parameters from data. The SUSY Les Houches Accord (SLHA) [80,99] has been adopted, which establishes a set of conventions for specifying generic file structures for supersymmetric model specifications and input parameters, supersymmetric mass and coupling spectra, and decay tables. These provide a universal interface between spectrum calculation programs, decay packages, and high energy physics event generators.

88.4.1 *The charginos and neutralinos*

The mixing of the charged gauginos ($\widetilde{W}^{\pm}$) and charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) is described (at tree-level) by a 2×2 complex mass matrix [100,101],

$$M_C \equiv \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g v_u \\ \frac{1}{\sqrt{2}} g v_d & \mu \end{pmatrix} . \tag{88.6}$$

To determine the physical chargino states and their masses, one must perform a singular value decomposition [102] of the complex matrix M_C [25,103]:

$$U^* M_C V^{-1} = \mathrm{diag}(M_{\tilde{\chi}_1^+}\,,\, M_{\tilde{\chi}_2^+})\,, \tag{88.7}$$

where U and V are unitary matrices, and the right-hand side of Eq. (88.7) is the diagonal matrix of (real non-negative) chargino masses. Explicit formulae for the singular value decomposition of M_C can be found in Ref. [104]. The physical chargino states are denoted by $\tilde{\chi}_1^{\pm}$ and $\tilde{\chi}_2^{\pm}$. These are linear combinations of the charged gaugino and higgsino states determined by the matrix elements of U and V [100,101] The chargino masses correspond to the singular values [102] of M_C, *i.e.*, the positive square roots of the eigenvalues of $M_C^\dagger M_C$:

$$M^2_{\tilde{\chi}_1^+,\tilde{\chi}_2^+} = \frac{1}{2}\left\{ |\mu|^2 + |M_2|^2 + 2m_W^2 \mp \sqrt{\left(|\mu|^2 + |M_2|^2 + 2m_W^2\right)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2} \right\}, \tag{88.8}$$

in a convention where v_u and v_d are real and positive, and where the states are ordered such that $M_{\tilde{\chi}_1^+} \leq M_{\tilde{\chi}_2^+}$. The relative phase of μ^* and M_2 is physical and potentially observable [105].

The mixing of the neutral gauginos ($\widetilde{B}$ and $\widetilde{W}^0$) and neutral higgsinos ($\widetilde{H}_d^0$ and $\widetilde{H}_u^0$) is described (at tree-level) by a 4×4 complex symmetric mass matrix [100,101],

$$M_N \equiv \begin{pmatrix} M_1 & 0 & -\frac{1}{2} g' v_d & \frac{1}{2} g' v_u \\ 0 & M_2 & \frac{1}{2} g v_d & -\frac{1}{2} g v_u \\ -\frac{1}{2} g' v_d & \frac{1}{2} g v_d & 0 & -\mu \\ \frac{1}{2} g' v_u & -\frac{1}{2} g v_u & -\mu & 0 \end{pmatrix} . \tag{88.9}$$

To determine the physical neutralino states and their masses, one must perform an Autonne-Takagi factorization [102,106] (also called Takagi diagonalization in Refs. [25,103,107]) of the complex symmetric matrix M_N:

$$W^T M_N W = \mathrm{diag}(M_{\widetilde{\chi}_1^0}\,, M_{\widetilde{\chi}_2^0}\,, M_{\widetilde{\chi}_3^0}\,, M_{\widetilde{\chi}_4^0})\,, \tag{88.10}$$

where W is a unitary matrix (which is called N^{-1} in Refs. [21, 30]) and the right-hand side of Eq. (88.10) is the diagonal matrix of (real non-negative) neutralino masses. The physical neutralino states are denoted by $\widetilde{\chi}_i^0$ (for $i = 1, \ldots, 4$), where the states are ordered such that $M_{\widetilde{\chi}_1^0} \le M_{\widetilde{\chi}_2^0} \le M_{\widetilde{\chi}_3^0} \le M_{\widetilde{\chi}_4^0}$. The $\widetilde{\chi}_i^0$ are the linear combinations of the neutral gaugino and higgsino states determined by the matrix elements of W. The neutralino masses correspond to the singular values of M_N, *i.e.*, the positive square roots of the eigenvalues of $M_N^\dagger M_N$. Exact formulae for these masses can be found in Refs. [108, 109]. A numerical algorithm for determining the mixing matrix W has been given in Ref. [110].

If a chargino or neutralino state approximates a particular gaugino or higgsino state, it is convenient to employ the corresponding nomenclature. Specifically, if $|M_1|$ and $|M_2|$ are small compared to m_Z and $|\mu|$, then the lightest neutralino $\widetilde{\chi}_1^0$ would be nearly a pure photino, $\widetilde{\gamma}$, the superpartner of the photon. If $|M_1|$ and m_Z are small compared to $|M_2|$ and $|\mu|$, then the lightest neutralino would be nearly a pure bino, $\widetilde{B}$, the superpartner of the weak hypercharge gauge boson. If $|M_2|$ and m_Z are small compared to $|M_1|$ and $|\mu|$, then the lightest chargino pair and neutralino would constitute a triplet of roughly mass-degenerate pure winos, $\widetilde{W}^\pm$, and $\widetilde{W}_3^0$, the superpartners of the weak SU(2) gauge bosons. Finally, if $|\mu|$ and m_Z are small compared to $|M_1|$ and $|M_2|$, then the lightest chargino pair and neutralino would be nearly pure higgsino states, the superpartners of the Higgs bosons. Each of the above cases leads to a strikingly different phenomenology.

In the NMSSM, an additional Higgs singlet superfield is added to the MSSM. This superfield comprises two real Higgs scalar degrees of freedom and an associated neutral higgsino degree of freedom. Consequently, there are five neutralino mass eigenstates that are obtained by a Takagi-diagonalization of the 5 × 5 neutralino mass matrix. In many cases, the fifth neutralino state is dominated by its SU(2)×U(1) singlet component, and thus is very weakly coupled to the SM particles and their superpartners.

88.4.2 *The squarks and sleptons*

For a given Dirac fermion f, there are two superpartners, $\widetilde{f}_L$ and $\widetilde{f}_R$, where the L and R subscripts simply identify the scalar partners that are related by SUSY to the left-handed and right-handed fermions, $f_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)f$, respectively. (There is no $\widetilde{\nu}_R$ in the MSSM.) However, $\widetilde{f}_L$–$\widetilde{f}_R$ mixing is possible, in which case $\widetilde{f}_L$ and $\widetilde{f}_R$ are not mass eigenstates. For three generations of squarks, one must diagonalize 6×6 matrices corresponding to the basis $(\widetilde{q}_{iL}, \widetilde{q}_{iR})$, where $i = 1, 2, 3$ are the generation labels. For simplicity, only the one-generation case is illustrated in detail below.

Using the notation of the third family, the one-generation tree-level squark squared-mass matrix is given by [25, 111],

$$\mathcal{M}^2 = \begin{pmatrix} M_{\widetilde{Q}}^2 + m_q^2 + L_q & m_q X_q^* \\ m_q X_q & M_{\widetilde{R}}^2 + m_q^2 + R_q \end{pmatrix}\,, \tag{88.11}$$

where

$$X_q \equiv A_q - \mu^* (\cot\beta)^{2T_{3q}}\,, \tag{88.12}$$

and $T_{3q} = \frac{1}{2}$ $[-\frac{1}{2}]$ for $q = t$ $[b]$. The diagonal squared-masses are governed by soft-SUSY-breaking squared-masses $M_{\widetilde{Q}}^2$ and $M_{\widetilde{R}}^2 \equiv M_{\widetilde{U}}^2$ $[M_{\widetilde{D}}^2]$ for $q = t$ $[b]$, the corresponding quark masses m_t $[m_b]$ and the electroweak correction terms:

$$\begin{aligned} L_q &\equiv (T_{3q} - e_q \sin^2\theta_W) m_Z^2 \cos 2\beta\,, \\ R_q &\equiv e_q \sin^2\theta_W\, m_Z^2 \cos 2\beta\,, \end{aligned} \tag{88.13}$$

where $e_q = \frac{2}{3}$ $[-\frac{1}{3}]$ for $q = t$ $[b]$. The off-diagonal squark squared-masses are proportional to the corresponding quark masses and depend on $\tan\beta$, the soft-SUSY-breaking A-parameters and the higgsino mass parameter μ. Assuming that the A-parameters are parametrically of the same order (or smaller) relative to other SUSY-breaking mass parameters, it then follows that the first and second generation $\widetilde{q}_L$–$\widetilde{q}_R$ mixing is smaller than that of the third generation where mixing can be enhanced by factors of m_t and $m_b \tan\beta$.

In the case of third generation $\widetilde{q}_L$–$\widetilde{q}_R$ mixing, the squark mass eigenstates (usually denoted by $\widetilde{q}_1$ and $\widetilde{q}_2$, with $m_{\tilde{q}_1} < m_{\tilde{q}_2}$) are determined by diagonalizing the 2×2 matrix $\mathcal{M}^2$ given by Eq. (88.11). The corresponding squared-masses and mixing angle are given by [111]:

$$\begin{aligned} m^2_{\tilde{q}_{1,2}} &= \frac{1}{2}\left[\operatorname{Tr}\mathcal{M}^2 \mp \sqrt{(\operatorname{Tr}\mathcal{M}^2)^2 - 4\det\mathcal{M}^2}\,\right], \\ \sin 2\theta_{\tilde{q}} &= \frac{2m_q|X_q|}{m^2_{\tilde{q}_2} - m^2_{\tilde{q}_1}}\,. \end{aligned} \tag{88.14}$$

The one-generation results above also apply to the charged sleptons, with the obvious substitutions: $q \to \ell$ with $T_{3\ell} = -\frac{1}{2}$ and $e_\ell = -1$, and the replacement of the SUSY-breaking parameters: $M^2_{\widetilde{Q}} \to M^2_{\widetilde{L}}$, $M^2_{\widetilde{D}} \to M^2_{\widetilde{E}}$, and $A_q \to A_\tau$. For the neutral sleptons, $\widetilde{\nu}_R$ does not exist in the MSSM, so $\widetilde{\nu}_L$ is a mass eigenstate.

In the case of three generations, the SUSY-breaking scalar-squared masses [$M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$] and the A-parameters [A_U, A_D, and A_E] are now 3×3 matrices as noted in Sec. 88.3.3. The diagonalization of the 6×6 squark mass matrices yields $\widetilde{f}_{iL}$–$\widetilde{f}_{jR}$ mixing. In practice, since the $\widetilde{f}_L$–$\widetilde{f}_R$ mixing is appreciable only for the third generation, this additional complication can often be neglected (although see Ref. [112] for examples in which the mixing between the second and third generation squarks is relevant).

88.5 The supersymmetric Higgs sector

Consider first the MSSM Higgs sector [29, 30, 113]. Despite the large number of potential CP-violating phases among the MSSM-124 parameters, the tree-level MSSM Higgs potential given by Eq. (88.2) is automatically CP-conserving. This follows from the fact that the only potentially complex parameter (m^2_{12}) of the MSSM Higgs potential can be chosen real and positive by re-phasing the Higgs fields, in which case $\tan\beta$ is a real positive parameter. Consequently, the physical neutral Higgs scalars are CP-eigenstates (at tree-level). The MSSM Higgs sector contains five physical spin-zero particles: a charged Higgs boson pair ($H^\pm$), two CP-even neutral Higgs bosons (denoted by h^0 and H^0 where $m_h < m_H$), and one CP-odd neutral Higgs boson (A^0). The discovery of a SM-like Higgs boson at the LHC with a mass of 125 GeV (see Sec. 11) strongly suggests that this state should be identified with h^0, although the possibility that the 125 GeV state should be identified with H^0 cannot yet be completely ruled out [114].

In the NMSSM [36], the scalar component of the singlet Higgs superfield adds two additional neutral states to the Higgs sector. In this model, the tree-level Higgs sector can exhibit explicit CP-violation. If CP is conserved, then the two extra neutral scalar states are CP-even and CP-odd, respectively. These states can potentially mix with the neutral Higgs states of the MSSM. If scalar states exist that are dominantly singlet, then they are weakly coupled to SM gauge bosons and fermions through their small mixing with the MSSM Higgs scalars. Consequently, it is possible that one (or both) of the singlet-dominated states is considerably lighter than the Higgs boson that was observed at the LHC.

88.5.1 *The tree-level Higgs sector*

The tree-level properties of the Higgs sector are determined by the Higgs potential given by Eq. (88.2). The quartic interaction terms are manifestly supersymmetric (although these are modified by SUSY-breaking effects at the loop level). In general, the quartic couplings arise from two sources: (i) the supersymmetric generalization of the scalar potential (the so-called "F-terms"), and (ii) interaction terms related by SUSY to the coupling of the scalar fields and the gauge fields, whose coefficients are proportional to the corresponding gauge couplings (the so-called "D-terms").

In the MSSM, F-term contributions to the quartic Higgs self-couplings are absent. As a result, the strengths of the MSSM quartic Higgs interactions are fixed in terms of the gauge couplings, as noted below Eq. (88.2). Consequently, all the tree-level MSSM Higgs-sector parameters depend only on two quantities: $\tan\beta$ [defined in Eq. (88.3)] and one Higgs mass usually taken to be m_A. For example, the tree-level squared mass of the charged Higgs boson is given by

$$m^2_{H^\pm} = m^2_A + m^2_W\,, \tag{88.15}$$

where $m^2_A = m^2_{H_d} + m^2_{H_u} + 2|\mu|^2$ [cf. Eq. (88.5)] and

$$H^\pm = H^\pm_d \sin\beta + H^\pm_u \cos\beta\,, \qquad A = \sqrt{2}\left(\operatorname{Im} H^0_d \sin\beta + \operatorname{Im} H^0_u \cos\beta\right)\,. \tag{88.16}$$

The CP-even scalar mass eigenstate fields h and H are identified by diagonalizing the 2×2 squared-mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} m^2_A \sin^2\beta + m^2_Z \cos^2\beta & -(m^2_A + m^2_Z)\sin\beta\cos\beta \\ -(m^2_A + m^2_Z)\sin\beta\cos\beta & m^2_A\cos^2\beta + m^2_Z\sin^2\beta \end{pmatrix}\,. \tag{88.17}$$

In particular,

$$h = -(\sqrt{2}\operatorname{Re} H^0_d - v_d)\sin\alpha + (\sqrt{2}\operatorname{Re} H^0_u - v_u)\cos\alpha\,, \tag{88.18}$$

$$H = (\sqrt{2}\operatorname{Re} H^0_d - v_d)\cos\alpha + (\sqrt{2}\operatorname{Re} H^0_u - v_u)\sin\alpha\,, \tag{88.19}$$

with corresponding tree-level squared masses,

$$m^2_{H,h} = \frac{1}{2}\left(m^2_A + m^2_Z \pm \sqrt{(m^2_A + m^2_Z)^2 - 4m^2_Z m^2_A \cos^2 2\beta}\right)\,, \tag{88.20}$$

and mixing angle α given by

$$\cos\alpha = \sqrt{\frac{m^2_A\sin^2\beta + m^2_Z\cos^2\beta - m^2_h}{m^2_H - m^2_h}}\,, \tag{88.21}$$

in a convention where $|\alpha| \le \pi/2$. However, because the off-diagonal elements of $\mathcal{M}^2$ are negative, it follows that $-\pi/2 \le \alpha \le 0$ [104]. In light of Eq. (88.20), the tree-level mass of the lighter CP-even Higgs boson is bounded [29, 30],

$$m_h \le m_Z|\cos 2\beta| \le m_Z\,. \tag{88.22}$$

This bound can be substantially modified when radiative corrections are included, as discussed in Sec. 88.5.2.

In the NMSSM, we set $\mu = 0$ in Eq. 88.1 and then add two additional terms to the superpotential,

$$W_{\rm NMSSM} \supset \lambda \hat{H}_u \hat{H}_d \hat{S} + \frac{1}{3}\kappa \hat{S}^3\,, \tag{88.23}$$

where $\hat{S}$ is a singlet Higgs superfield. In the NMSSM as defined here, all terms in $W_{\rm NMSSM}$ are cubic in the superfields due to the presence of a discrete $\mathbb{Z}_3$ symmetry. An effective μ-term is generated, $\mu_{\text{eff}} = \lambda\langle S\rangle$, where $\langle S\rangle$ is the vacuum expectation value of the scalar field component of $\hat{S}$. Moreover, due to the term proportional to λ in Eq. 88.23, there is now an F-term contribution to the quartic Higgs self-couplings. Consequently, the tree-level bound for the mass of the lightest CP-even MSSM Higgs boson is modified [115],

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta\,, \tag{88.24}$$

where $v \equiv (v_u^2 + v_d^2)^{1/2} = 246$ GeV. By requiring that λ remain finite after renormalization-group evolution up to the Planck scale, one finds that λ is constrained to lie below about 0.7–0.8 at the electroweak scale [36] (although larger values of λ have also been considered in Ref. [116]).

The tree-level Higgs couplings to gauge bosons and the Higgs boson self-couplings are governed by the electroweak gauge couplings and the parameter $\cos(\beta-\alpha)$. Explicitly,

$$\cos(\beta-\alpha) = \frac{m_Z^2 \sin 2\beta \cos 2\beta}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - m_Z^2\cos^2 2\beta)}}\,. \tag{88.25}$$

Note that $\cos(\beta-\alpha) \to 0$ in the limit of $m_H \gg m_h, m_Z$. In this *decoupling limit* [117], the properties of h coincide with those of the SM Higgs boson. In light of the LHC Higgs data [118, 119], which are compatible with the SM predictions (see Sec. 11), one can conclude that if h is identified with the observed Higgs boson then H, A and $H^{\pm}$ must be substantially heavier (most likely of order 500 GeV or larger [120]).

The tree-level Higgs-quark and Higgs-lepton interactions of the MSSM are derived from the superpotential given in Eq. (88.1). The corresponding Higgs-fermion Yukawa couplings can be expressed in terms of the fermion masses and the separate parameters $\cos(\beta-\alpha)$ and $\tan\beta$. In particular, the Higgs sector of the MSSM is a Type-II two-Higgs doublet model [121], in which one Higgs doublet (H_d) couples exclusively to the right-handed down-type quark (or lepton) fields and the second Higgs doublet (H_u) couples exclusively to the right-handed up-type quark fields. Consequently, the diagonalization of the fermion mass matrices simultaneously diagonalizes the matrix of Yukawa couplings, resulting in flavor-diagonal tree-level couplings of the neutral Higgs bosons h^0, H^0 and A^0 to quark and lepton pairs. One can again check that in the decoupling limit where $\cos(\beta-\alpha) \to 0$, the couplings of h reduce to those of the SM.

88.5.2 *The radiatively-corrected Higgs sector*

When radiative corrections are incorporated, additional parameters of the supersymmetric model enter via virtual supersymmetric particles that appear in loops. The impact of these corrections can be significant [122]. The qualitative behavior of these radiative corrections can be most easily seen in the large top-squark mass limit, where in addition, both the splitting of the two diagonal entries and the off-diagonal entries of the top-squark squared-mass matrix [Eq. (88.11)] are small in comparison to the geometric mean of the two top-squark squared-masses, $M_{\mathrm{S}}^2 \equiv M_{\widetilde{t}_1} M_{\widetilde{t}_2}$. In this case (assuming $\mathrm{m}_A > m_Z$), the predicted upper bound for m_h is approximately given by [123]

$$m_h^2 \lesssim m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_{\mathrm{S}}^2}{m_t^2}\right) + \frac{X_t^2}{M_{\mathrm{S}}^2}\left(1 - \frac{X_t^2}{12 M_{\mathrm{S}}^2}\right)\right], \tag{88.26}$$

where $X_t \equiv A_t - \mu\cot\beta$ [cf. Eq. (88.12)] is proportional to the off-diagonal entry of the top-squark squared-mass matrix (where for simplicity, A_t and μ are taken to be real). The Higgs mass upper limit specified by Eq. (88.26) is saturated when $\tan\beta$ is large (*i.e.*, $\cos^2 2\beta \sim 1$) and $X_t = \sqrt{6}\,M_S$, which defines the so-called maximal mixing scenario.

In applying the radiatively corrected MSSM Higgs sector to the analysis of LHC Higgs data, the authors of Refs. [124,125] suggested that a reasonable approximation would consist of retaining the leading corrections employed in deriving Eq. (88.26), while discarding all other subleading terms. This procedure was implemented by simply replacing $\mathcal{M}^2_{22} \to \mathcal{M}^2_{22} + \Delta\mathcal{M}^2_{22}$ in the 22 element of the CP-even Higgs squared-mass matrix given in Eq. (88.17), since $\Delta\mathcal{M}^2_{22}$ contains the leading contributions that govern the Higgs mass radiative corrections. One can now re-diagonalize the CP-even Higgs squared-mass matrix and determine $\Delta\mathcal{M}^2_{22}$ in terms of the parameters m_A, $\tan\beta$ and the *measured* Higgs mass (*e.g.*, $m_h \simeq 125$ GeV, if h is identified with the observed Higgs boson). This framework was dubbed the hMSSM in Ref. [124].

Although the hMSSM can be readily applied to LHC data to derive interesting constraints on the MSSM Higgs sector, it can lead to results that are not robust in a more general MSSM parameter scan. Indeed, a more complete treatment of the radiative corrections can yield results that cannot be accounted for by the hMSSM framework. Examples of benchmark points of the MSSM parameter space that cannot be reproduced by the hMSSM analysis are discussed in 11.6.1.1.

The set of approximations employed in obtaining Eq. (88.26) somewhat overestimates the value of m_h. The most complete treatment of the MSSM Higgs mass radiative corrections, which incorporate renormalization group improvement, the two loop, and the leading three-loop contributions [73,126], yields a predicted value of m_h shown in Fig. 88.1, as a function of X_t (assumed for simplicity to be real).

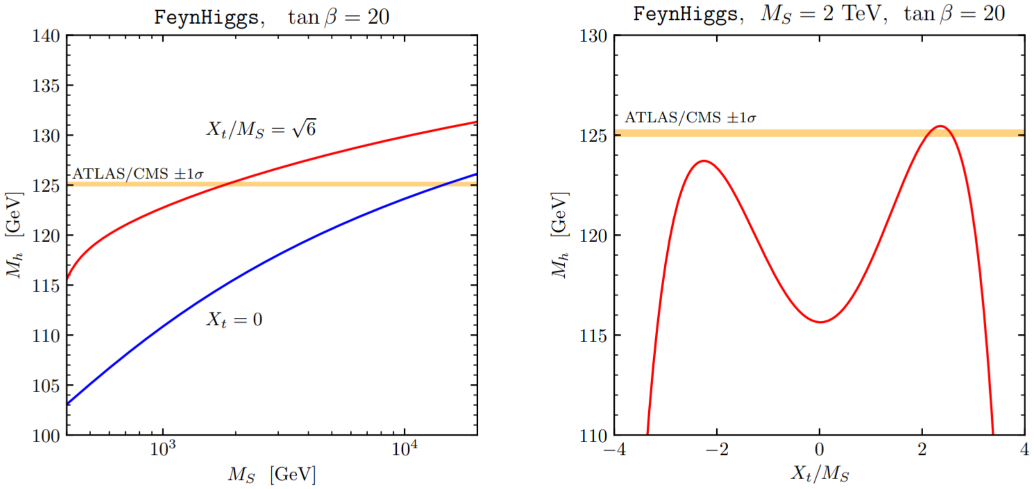

Figure 88.1: The lighter CP-even Higgs mass in the MSSM as a function of a common SUSY mass parameter M_S and of the stop mixing parameter X_t (normalized to M_S). Figure taken from Ref. [73].

In addition, one-loop radiative corrections can introduce CP-violating effects in the Higgs sector that depend on some of the CP-violating phases among the MSSM-124 parameters [127]. This phenomenon is most easily understood in a scenario where $m_A \ll M_S$ (i.e., all five physical Higgs states are significantly lighter than the SUSY breaking scale). In this case, one can integrate out the heavy superpartners to obtain a low-energy effective theory with two Higgs doublets. The resulting effective two-Higgs doublet model will now contain all possible Higgs self-interaction terms (both CP-conserving and CP-violating) and Higgs-fermion interactions (beyond those of Type-II) that are consistent with electroweak gauge invariance [128].

In the NMSSM with $m_h \simeq 125$ GeV, the dominant radiative correction to Eq. (88.24) is the same as the one given in Eq. (88.26). However, in contrast to the MSSM, one does not need as large a boost from radiative corrections to achieve a Higgs mass of 125 GeV in certain regimes of the NMSSM parameter space (*e.g.*, $\tan\beta \sim 2$ and $\lambda \sim 0.7$ [129]).

88.6 Restricting the MSSM parameter freedom

In Sections 88.4 and 88.5, we surveyed the parameters that comprise the MSSM-124. However, the MSSM-124 is not a phenomenologically viable theory over much of its parameter space. In particular, a generic point of the MSSM-124 parameter space exhibits: (i) no conservation of the separate lepton numbers L_e, L_μ, and L_τ; (ii) unsuppressed flavor-changing neutral currents (FCNCs); and (iii) new sources of CP violation that are inconsistent with the experimental bounds.

For example, the MSSM contains new sources of CP violation [130]. Indeed, for TeV-scale sfermion and gaugino masses, some combinations of the complex phases of the gaugino-mass parameters, the A-parameters, and μ must be less than about 10^{-2}–10^{-3} to avoid generating electric dipole moments for the neutron, electron, and atoms [131–133] in conflict with observed data [134, 135]. The rarity of FCNCs [136–138] places additional constraints on the off-diagonal matrix elements of the squark and slepton soft-SUSY-breaking squared-masses and A-parameters (see Sec. 88.3.3).

The MSSM-124 is also theoretically incomplete as it provides no explanation for the fundamental origin of the supersymmetry-breaking parameters. The successful unification of the MSSM gauge couplings at a grand unified mass scale $M_{\rm GUT}$, close to the Planck scale [7, 70, 139–141],

$$g_s(M_{\rm GUT}) = g(M_{\rm GUT}) = \sqrt{\frac{5}{3}}\, g'(M_{\rm GUT}), \tag{88.27}$$

suggests that the high-energy structure of the theory may be considerably simpler than its low-energy realization.[2] In a top-down approach, the dynamics that governs the theory at high energies is used to derive the effective broken-supersymmetric theory at the TeV scale.

In this Section, we examine a number of theoretical frameworks that potentially yield phenomenologically viable regions of the MSSM-124 parameter space. The resulting supersymmetric particle spectrum is then a function of a relatively small number of input parameters. This is accomplished by imposing a simple structure on the soft SUSY-breaking parameters at a common high-energy scale M_X (typically chosen to be the Planck scale, M_P, the grand unified theory scale, $M_{\rm GUT}$, or the messenger scale, $M_{\rm mess}$). These serve as initial conditions for the MSSM renormalization group equations (RGEs), which are given in the two-loop approximation in Ref. [142]. An automated program to compute RGEs for the MSSM and other supersymmetric models of new physics has been developed in Ref. [143]. Solving these equations numerically, one can then derive the low-energy MSSM parameters relevant for phenomenology. A number of software packages exist that numerically calculate the spectrum of supersymmetric particles, consistent with theoretical conditions on SUSY breaking at high energies and some experimental data at low energies [73, 144].

Examples of viable frameworks are provided by models of gravity-mediated, anomaly-mediated, and gauge-mediated SUSY breaking. In some of these approaches, one of the diagonal Higgs squared-mass parameters is driven negative by renormalization group evolution [145]. In such models, electroweak symmetry breaking is generated radiatively, and the resulting electroweak symmetry-breaking scale is intimately tied to the scale of low-energy SUSY breaking.

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[2] Generically, the normalization of the U(1) hypercharges exhibited in Table 88.1 is a matter of convention. In particular, the U(1) hypercharges can be rescaled by absorbing the scaling factor into a redefinition of the hypercharge gauge coupling g'. However, the embedding of the hypercharge U(1) generator into the Lie algebra of a grand unified simple gauge group fixes the normalization of the U(1) hypercharges and results in the rescaled hypercharge gauge coupling shown in Eq. (88.27).

88.6.1 *Gaugino mass relations*

One prediction of many supersymmetric grand unified models is the unification of the (tree-level) gaugino mass parameters[3] at some high-energy scale, M_X,

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}\,. \tag{88.28}$$

Due to renormalization group running, in the one-loop approximation the effective low-energy gaugino mass parameters (at the electroweak scale) are related,

$$M_3 = (g_s^2/g^2)M_2 \simeq 3.5M_2\ , \qquad M_1 = (5g'^2/3g^2)M_2 \simeq 0.5M_2. \tag{88.29}$$

Eq. (88.29) can arise more generally in gauge-mediated SUSY-breaking models where the gaugino masses are generated at the messenger scale M_{mess} (which typically lies significantly below the unification scale where the gauge couplings unify). In this case, the gaugino mass parameters are proportional to the corresponding squared gauge couplings at the messenger scale.

When Eq. (88.29) is satisfied, the chargino and neutralino masses and mixing angles depend only on three unknown parameters: the gluino mass, μ, and $\tan\beta$. It then follows that the lightest neutralino must be heavier than 46 GeV due to the non-observation of charginos at LEP [147]. If in addition $|\mu| \gg |M_1| \gtrsim m_Z$, then the lightest neutralino is nearly a pure bino, an assumption often made in supersymmetric particle searches at colliders. Although Eq. (88.29) is often assumed in many phenomenological studies, a truly model-independent approach would take the gaugino mass parameters M_1, M_2, and M_3 to be independent parameters to be determined by experiment. Indeed, an approximately massless neutralino *cannot* be ruled out at present by a model-independent analysis [148].

It is possible that the tree-level masses of the gauginos are zero. In this case, the gaugino mass parameters arise at one-loop and do not satisfy Eq. (88.29). For example, the gaugino masses in AMSB models arise entirely from a model-independent contribution derived from the superconformal anomaly [51, 149]. In this case, Eq. (88.29) is replaced (in the one-loop approximation) by:

$$M_i \simeq \frac{b_i g_i^2}{16\pi^2} m_{3/2}\,, \tag{88.30}$$

where $m_{3/2}$ is the gravitino mass and the b_i are the coefficients of the MSSM gauge beta-functions corresponding to the corresponding U(1), SU(2), and SU(3) gauge groups, $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$. Eq. (88.30) yields $M_1 \simeq 2.8M_2$ and $M_3 \simeq -8.3M_2$, which implies that the lightest chargino pair and neutralino comprise a nearly mass-degenerate triplet of winos, $\widetilde{W}^\pm$, $\widetilde{W}^0$ (cf. Table 1), over most of the MSSM parameter space. For example, if $|\mu| \gg m_Z$, $|M_2|$, then Eq. (88.30) implies that $M_{\tilde{\chi}^\pm} \simeq M_{\tilde{\chi}_1^0} \simeq M_2$ [150]. Alternatively, one can construct an AMSB model where $|\mu|, m_Z \ll M_2$, which yields an LSP that is an approximate higgsino state [151]. In both cases, the corresponding supersymmetric phenomenology differs significantly from the standard phenomenology based on Eq. (88.29) [152, 153].

Finally, it should be noted that the unification of gaugino masses (and scalar masses) can be accidental. In particular, the energy scale where unification takes place may not be directly related to any physical scale. One version of this phenomenon has been called mirage unification and can occur in certain theories of fundamental SUSY breaking [154].

88.6.2 *Constrained versions of the MSSM: mSUGRA, CMSSM, etc.*

In the minimal supergravity (mSUGRA) framework [3–5, 25, 44–46], the minimal form of the Kähler potential is employed, which yields standard kinetic energy terms for the MSSM fields

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[3] Non-universal gaugino mass parameters can also be a viable option in grand unified models with non-minimal gauge kinetic functions [146].

[48]. As a result, the soft supersymmetry-breaking parameters at the high-energy scale M_X take a particularly simple form in which the scalar squared-masses and the A-parameters are flavor-diagonal and universal [46]:

$$\begin{aligned}
&M^2_{\widetilde{Q}}(M_X) = M^2_{\widetilde{U}}(M_X) = M^2_{\widetilde{D}}(M_X) = m_0^2\mathbf{1}\,,\\
&M^2_{\widetilde{L}}(M_X) = M^2_{\widetilde{E}}(M_X) = m_0^2\mathbf{1}\,,\\
&m_1^2(M_X) = m_2^2(M_X) = m_0^2\,,\\
&A_U(M_X) = A_D(M_X) = A_E(M_X) = A_0\mathbf{1}\,,
\end{aligned} \tag{88.31}$$

where $\mathbf{1}$ is a 3×3 identity matrix in generation space. As in the SM, this approach exhibits minimal flavor violation (see *e.g.* Refs. [155, 156]), whose unique source is the nontrivial flavor structure of the Higgs-fermion Yukawa couplings. The gaugino masses are also unified according to Eq. (88.28).

Renormalization group evolution is then used to derive the values of the supersymmetric parameters at the low-energy (electroweak) scale. For example, to compute squark masses, one should use the low-energy values for $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, and $M^2_{\widetilde{D}}$ in Eq. (88.11). Through the renormalization group running with boundary conditions specified in Eq. (88.29) and Eq. (88.31), one can show that the low-energy values of $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, and $M^2_{\widetilde{D}}$ depend primarily on m_0^2 and $m^2_{1/2}$. A number of useful approximate analytic expressions for superpartner masses in terms of the mSUGRA parameters can be found in Ref. [157].

One can count the number of independent parameters in the mSUGRA framework. In addition to 18 SM parameters (excluding the Higgs mass), one must specify m_0, $m_{1/2}$, A_0, the Planck-scale values for μ and B-parameters (denoted by μ_0 and B_0), and the gravitino mass $m_{3/2}$. Without additional model assumptions, $m_{3/2}$ is independent of the parameters that govern the mass spectrum of the superpartners of the SM [46]. In principle, A_0, B_0, μ_0, and $m_{3/2}$ can be complex, although in the mSUGRA approach, these parameters are taken to be real for simplicity.

As previously noted, renormalization group evolution is used to compute the low-energy values of the mSUGRA parameters, which then fixes all the parameters of the low-energy MSSM. In particular, the two Higgs vacuum expectation values (or equivalently, m_Z and $\tan\beta$) can be expressed as a function of the Planck-scale supergravity parameters. In light of Eq. (88.4) and Eq. (88.5), a common procedure is to determine μ_0 and B_0 in terms of m_Z and $\tan\beta$ [the sign of μ_0, denoted $\mathrm{sgn}(\mu_0)$ below, is not fixed in this process]. In this case, the MSSM spectrum and its interaction strengths are fixed by five parameters:

$$m_0\,,\ A_0\,,\ m_{1/2}\,,\ \tan\beta\,,\ \text{and}\ \mathrm{sgn}(\mu_0)\,, \tag{88.32}$$

and an independent gravitino mass $m_{3/2}$ (in addition to the 18 parameters of the SM). In Ref. [158], this framework was dubbed the constrained minimal supersymmetric extension of the SM (CMSSM). Additional relations such as $B_0 = A_0 - m_0$ and $m_{3/2} = m_0$ comprise the original mSUGRA proposal [44, 48, 159].

One can also relax the universality of scalar masses by decoupling the squared-masses of the Higgs bosons and the squarks/sleptons. This leads to the non-universal Higgs mass models (NUHMs), thereby adding one or two new parameters to the CMSSM depending on whether the diagonal Higgs scalar squared-mass parameters ($m^2_{H_d}$ and $m^2_{H_u}$) are set equal (NUHM1 [160]) or taken to be independent (NUHM2 [161]) at the high energy scale M_X. Clearly, this modification preserves the minimal flavor violation of the mSUGRA approach. Nevertheless, the mSUGRA approach and its NUHM generalizations are probably too simplistic. Theoretical considerations suggest that the universality of Planck-scale soft SUSY-breaking parameters is not generic [162].

In particular, effective operators at the Planck scale exist that do not respect flavor universality, and it is difficult to find a theoretical principle that would forbid them.

In the framework of supergravity, if anomaly mediation is the sole source of SUSY breaking, then the gaugino mass parameters, diagonal scalar squared-mass parameters, and the SUSY-breaking trilinear scalar interaction terms (proportional to $\lambda_f A_F$) are determined in terms of the beta functions of the gauge and Yukawa couplings and the anomalous dimensions of the squark and slepton fields [51, 149, 153]. As noted in Sec. 88.2.3, this approach yields tachyonic sleptons in the MSSM unless additional sources of SUSY breaking are present. In the minimal AMSB (mAMSB) scenario, a universal squared-mass parameter, m_0^2, is added to the AMSB expressions for the diagonal scalar squared-masses [153]. Thus, the mAMSB spectrum and its interaction strengths are determined by four parameters, m_0^2, $m_{3/2}$, $\tan\beta$ and $\mathrm{sgn}(\mu_0)$.

The mAMSB scenario appears to be ruled out based on the observed value of the Higgs boson mass, assuming an upper limit on M_S of a few TeV, since the mAMSB constraint on A_F implies that the maximal mixing scenario cannot be achieved [cf. Eq. (88.26)]. Indeed, under the stated assumptions, the mAMSB Higgs mass upper bound lies below the observed Higgs mass value [163]. Thus within the AMSB scenario, either an additional SUSY-breaking contribution to $\lambda_f A_F$ and/or new ingredients beyond the MSSM are required.

88.6.3 *Gauge-mediated SUSY breaking*

In contrast to models of gravity-mediated SUSY breaking, the flavor universality of the fundamental soft SUSY-breaking squark and slepton squared-mass parameters is guaranteed in gauge-mediated SUSY breaking (GMSB) because the supersymmetry breaking is communicated to the sector of MSSM fields via gauge interactions [56, 58]. In GMSB models, the mass scale of the messenger sector (or its equivalent) is sufficiently below the Planck scale such that the additional SUSY-breaking effects mediated by supergravity can be neglected.

In the minimal GMSB approach, there is one effective mass scale, Λ, that determines all low-energy scalar and gaugino mass parameters through loop effects, while the resulting A-parameters are suppressed. In addition, the minimal form of the Kähler potential is employed. In order that the resulting superpartner masses be of order 1 TeV, one must have $\Lambda \sim \mathcal{O}(100~\text{TeV})$. The origin of the μ and B-parameters is model-dependent, and lies somewhat outside the purview of gauge-mediated SUSY breaking [164].

The simplest GMSB models appear to be ruled out based on the observed value of the Higgs boson mass. Due to suppressed A parameters, it is difficult to boost the contributions of the radiative corrections in Eq. (88.26) to obtain a Higgs mass as large as 125 GeV. However, this conflict can be alleviated in more complicated GMSB models [165]. To analyze these generalized GMSB models, it has been especially fruitful to develop model-independent techniques that encompass all known GMSB models [166]. These techniques are well-suited for a comprehensive analysis [167] of the phenomenological profile of gauge-mediated SUSY breaking.

The gravitino is the LSP in minimal GMSB models, as noted in Sec. 88.2.3. As a result, the next-to-lightest supersymmetric particle (NLSP) now plays a crucial role in the phenomenology of supersymmetric particle production and decays. Note that unlike the LSP, the NLSP can be charged. In GMSB models, the most likely candidates for the NLSP are $\tilde{\chi}_1^0$ and $\widetilde{\tau}_R^\pm$. The NLSP will decay into its superpartner plus a gravitino (*e.g.*, $\tilde{\chi}_1^0 \to \gamma\widetilde{G}$, $\tilde{\chi}_1^0 \to Z\widetilde{G}$, $\tilde{\chi}_1^0 \to h^0\widetilde{G}$ or $\widetilde{\tau}_1^\pm \to \tau^\pm\widetilde{G}$), with lifetimes and branching ratios that depend on the model parameters. There are also GMSB scenarios in which there are several nearly degenerate co-NLSPs, any one of which can be produced at the penultimate step of a supersymmetric decay chain [168]. For example, in the slepton co-NLSP case, all three right-handed sleptons are close enough in mass and thus can each play the role of the NLSP.

Different choices for the identity of the NLSP and its decay rate lead to a variety of distinctive supersymmetric phenomenologies [58,169]. For example, a long-lived $\tilde{\chi}_1^0$-NLSP that decays outside collider detectors leads to supersymmetric decay chains with missing energy in association with leptons and/or hadronic jets (this case is indistinguishable from the standard phenomenology of the $\tilde{\chi}_1^0$-LSP). On the other hand, if $\tilde{\chi}_1^0 \to \gamma\widetilde{G}$ is the dominant decay mode, and the decay occurs inside the detector, then nearly *all* supersymmetric particle decay chains would produce a photon. In contrast, in the case of a $\widetilde{\tau}_1^\pm$-NLSP, the $\widetilde{\tau}_1^\pm$ would either be long-lived or would decay inside the detector into a τ-lepton plus missing energy.

A number of attempts have been made to address the origins of the μ and B-parameters in GMSB models based on the field content of the MSSM (see, *e.g.*, Refs. [164,170]). An alternative approach is to consider GMSB models based on the NMSSM [171]. The vacuum expectation value of the additional singlet Higgs superfield can be used to generate effective μ and B-parameters [172]. Such models provide an alternative GMSB framework for achieving a Higgs mass of 125 GeV, while still being consistent with LHC bounds on supersymmetric particle masses.

88.6.4 *The phenomenological MSSM*

Any of the theoretical assumptions described in the previous three subsections must be tested experimentally and could turn out to be wrong. To facilitate the exploration of MSSM phenomena in a more model-independent way while respecting the constraints noted at the beginning of this Section, the phenomenological MSSM (pMSSM) has been introduced [173].

The pMSSM is governed by 19 independent real supersymmetric parameters: the three gaugino mass parameters M_1, M_2 and M_3, the Higgs sector parameters m_A and $\tan\beta$, the Higgsino mass parameter μ, five sfermion squared-mass parameters for the degenerate first and second generations ($M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$ and $M^2_{\widetilde{E}}$), the five corresponding sfermion squared-mass parameters for the third generation, and three third-generation A-parameters (A_t, A_b and A_τ). The first and second generation A-parameters are typically neglected in pMSSM studies, as their phenomenological consequences are negligible in most applications. One counterexample arises when considering the A_μ dependence of the anomalous magnetic moment of the muon, which can be as significant as other contributions due to superpartner mediated radiative corrections [174]. Since its initial proposal, the pMSSM approach has been extended to include a 20th parameter, A_μ [175]. It also has been further extended to include CP-violating SUSY-breaking parameters in Ref. [176].

The 19-parameter pMSSM is often further constrained to expedite scans over the parameter space. For example, in Ref. [177], the number of pMSSM parameters is reduced to ten by assuming one common squark squared-mass parameter for the first two generations, a second common squark squared-mass parameter for the third generation, a common (charged) slepton squared-mass parameter and a common third generation A parameter. In Ref. [178] an eleven parameter pMSSM is defined by allowing for a different stau squared-mass parameter from that of the first two generation charged sleptons. Other applications of the pMSSM approach (with a reduced pMSSM parameter space) to supersymmetric particle searches, and a discussion of the implications for past and future LHC and dark matter studies can be found in Refs. [177,179,180].

88.6.5 *Simplified models*

As Sec. 89 demonstrates, experiments present their searches for supersymmetric particles primarily in terms of simplified models. Simplified models for supersymmetric searches [181] are defined mostly by the empirical objects and kinematic variables involved in the search. Their interpretation by an experimental collaboration usually involves only a small number of supersymmetric particles (often two or three). Other supersymmetric particles are assumed to play no role (this may happen by virtue of them being too heavy to be produced). Experimental bounds from non-observation of a signal are usually presented in terms of the physical masses of the supersymmetric particles involved. Bounds may be presented on the relevant supersymmetric particle masses assuming certain values for the branching ratio of certain supersymmetric particle decays, or as an upper bound on the signal production cross-section as a function of the relevant supersymmetric particle masses.

For example, consider a search for hadronic jets plus missing transverse momentum. One can match such a search to the simplified model of squark pair production followed by the subsequent decay of each squark into a quark (which appears as a jet) and a neutralino LSP that produces the missing transverse momentum, *i.e.* $\tilde{q}\tilde{q} \to (q\tilde{\chi}_1^0)(q\tilde{\chi}_1^0)$. Excluded cross-sections resulting from the non-observation of a signal (which in this case could consist of some specified minimum value of missing transverse momentum and at least two hard jets) may be exhibited in the squark mass versus LSP mass plane.

Simplified models have the apparent advantage that they have fewer free parameters than more complete supersymmetric models, whose greater number of free parameters makes it difficult to present excluded regions in any generality. If limits are quoted on supersymmetric particle masses without reference to the signal production cross-section from a simplified model analysis, then there is a potential pitfall—namely, mass limits can differ from those obtained in full models because there may be contributions to the signal coming from processes involving supersymmetric particles other than those assumed. For example, in the $\tilde{q}\tilde{q} \to q\tilde{\chi}_1^0 q\tilde{\chi}_1^0$ process mentioned above, the simplified model analysis does not account for the interference with tree-level t-channel gluino contributions. Nevertheless, simplified model bounds quoted purely in terms of supersymmetric particle masses may still approximately hold over sizable regions of parameter space of more complete models, within which the simplified model is embedded. When simplified model limits are phrased as bounds on signal cross-sections, the aforementioned pitfall is sidestepped. Simplified models thus remain an efficient tool for organizing and presenting the results of supersymmetric particle searches. A comparison between supersymmetric particle search constraints in the context of simplified models and the corresponding constraints obtained in the more complete pMSSM can be found in Ref. [182].

88.7 Experimental data confronts the MSSM

At present, there is no significant evidence for weak-scale SUSY from the data analyzed by the LHC experiments. Recent LHC data have been employed in ruling out the existence of colored supersymmetric particles (primarily the gluino and the first generation of squarks) with masses below about 2 TeV (see Fig. 89.14). Moreover, given that the mass of the observed Higgs boson is 125 GeV, the results exhibited in Fig. 88.1 tend to favor a mass scale of the top squarks somewhat above 2 TeV. However, the precise mass limits are very model dependent. For example, as Fig. 89.13 demonstrates, regions of the pMSSM parameter space can be identified in which lighter squarks and gluinos below 1 TeV cannot be definitely ruled out. Under the assumption of gaugino mass unification [cf. Eq. (88.29)], LHC searches result in a lower bound on neutralino and chargino masses of roughly 200 GeV. It is difficult to place general bounds on neutralino and chargino masses, since the limit in terms of masses from direct searches tends to be particularly model dependent. Nevertheless, one must confront the tension that exists between the theoretical expectations for the magnitude of the SUSY-breaking parameters and the non-observation of supersymmetric phenomena at colliders.

88.7.1 *Naturalness constraints and the little hierarchy*

In Sec. 88.1, weak-scale SUSY was motivated as a natural solution to the hierarchy problem, which could provide an understanding of the origin of the electroweak symmetry-breaking scale without a significant fine-tuning of the fundamental parameters that govern the MSSM. In this context, the weak scale soft supersymmetry-breaking masses must be generally of the order of

1 TeV or below [183]. This requirement is most easily seen in the determination of m_Z by the scalar potential minimum condition. In light of Eq. (88.5), to avoid the fine-tuning of MSSM parameters, the soft SUSY-breaking squared-masses $m^2_{H_d}$ and $m^2_{H_u}$ and the higgsino squared-mass $|\mu|^2$ should all be roughly of $\mathcal{O}(m_Z^2)$. Many authors have proposed quantitative measures of fine-tuning [183–188]. One of the simplest measures is the one advocated by Barbieri and Giudice [183] (which was also introduced previously in Ref. [184]),

$$\varDelta_i \equiv \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right| , \qquad \varDelta \equiv \max\, \varDelta_i\,, \tag{88.33}$$

where the p_i are the MSSM parameters at the high-energy scale M_X, which are set by the fundamental SUSY-breaking dynamics. The theory is more fine-tuned as $\varDelta$ becomes larger. However, different measures of fine-tuning yield quantitatively different results; in particular, calculating minimal fine-tuning based on the high-scale parameters [as defined in Eq. (88.33)] yields a difference by a factor ~ 10 to fine-tuning based on TeV-scale parameters [189, 190].

One can apply the fine-tuning measure to any explicit model of SUSY breaking. For example, in the approaches discussed in Sec. 88.6, the p_i are parameters of the model at the energy scale M_X where the soft SUSY-breaking operators are generated by the dynamics of SUSY breaking. Renormalization group evolution then determines the values of the parameters appearing in Eq. (88.5) at the electroweak scale. In this way, $\varDelta$ is sensitive to all the SUSY-breaking parameters of the model (see *e.g.* Ref. [191]). The computation of $\varDelta$ is often based on Eq. (88.5), which is a tree-level condition. However, the fine-tuning measure obtained at tree level can be somewhat reduced in value when loop corrections are included while remaining consistent with all experimental constraints [87, 192].

One way of taking fine-tuning into account in fits to data using Bayesian statistics is to have a prior probability distribution proportional to $1/\varDelta$ [193] so that fine-tuning is balanced against the fit to empirical data. In such a Bayesian approach, it is important to choose the prior probability distribution carefully, since prior probability densities that are flat in one set of variables may not be flat in another, more fundamental set. One can in fact derive a different measure of fine-tuning resulting from a Jacobian factor when transforming to other variables.[4] By comparing the results of several Bayesian fits with different (but reasonable) prior probability distributions, one can assess the robustness of the fit with respect to their variation, mitigating for subjectivity in the interpretation of the fine-tuning measure.

As anticipated, there is a tension between the present experimental lower limits on the masses of colored supersymmetric particles [195, 196] and the expectation that supersymmetry-breaking is associated with the electroweak symmetry-breaking scale. Moreover, this tension is exacerbated [197] by the observed value of the Higgs mass ($m_h \simeq 125$ GeV), which is not far from the MSSM upper bound ($m_h \lesssim 135$ GeV) [which depends on the top-squark mass and mixing as noted in Sec. 88.5.2]. If M_{SUSY} characterizes the scale of supersymmetric particle masses, then one would crudely expect $\varDelta \sim M^2_{\text{SUSY}}/m_Z^2$. For example, if $M_{\text{SUSY}} \sim 1$ TeV then one expects a $\varDelta^{-1} \sim 1\%$ fine-tuning of the MSSM parameters to achieve the observed value of m_Z. This separation of the electroweak symmetry-breaking and SUSY-breaking scales is an example of the little hierarchy problem [198, 199].

The fine-tuning parameter $\varDelta$ can depend quite sensitively on the structure of the SUSY-breaking dynamics, such as the value of M_X and relations among SUSY-breaking parameters in the fundamental high energy theory [200]. For example, in so-called focus point SUSY models [187, 201], all

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[4] For example, one may consider the parameters μ and m^2_{12} to be more fundamental than $\tan\beta$ and M_Z. In this case, one would choose a flat prior probability distribution in μ and m^2_{12} rather than in $\tan\beta$ and M_Z [186, 194]. The Jacobian factor is then obtained from Eq. (88.4) and Eq. (88.5).

squark masses can be as heavy as 5 TeV *without* significant fine-tuning. This can be attributed to a focusing behavior of the renormalization group evolution when certain relations hold among the high-energy values of the scalar squared-mass SUSY-breaking parameters. Although the focus point region of the CMSSM still yields an uncomfortably high value of Δ due to the observed Higgs mass of 125 GeV, one can achieve moderate values of Δ in models with NUHM2 boundary conditions for the scalar masses [197].

Among the colored superpartners, the third generation squarks typically have the most significant impact on the naturalness constraints [202], while their masses are the least constrained by the LHC data. Hence, in the absence of any relation between third generation squarks and those of the first two generations, the naturalness constraints due to present LHC data can be considerably weaker than those obtained in the CMSSM. Indeed, models with first and second generation squark masses in the multi-TeV range do not necessarily require significant fine tuning. Such models have the added benefit that undesirable FCNCs mediated by squark exchange are naturally suppressed [203]. Other MSSM mass spectra that are compatible with moderate fine tuning have been considered in Refs. [200] and [204].

The lower bounds on squark and gluino masses may not be as large as suggested by the experimental analyses based on the CMSSM or simplified models. For example, mass bounds for the gluino and the first and second generation squarks based on the CMSSM can often be evaded in alternative or extended MSSM models, *e.g.*, compressed SUSY [205] and stealth SUSY [206]. Moreover, the experimental upper limits for the third generation squark masses (which have a more direct impact on the fine-tuning measure) are weaker than the corresponding mass limits for other colored supersymmetric states.

Among the uncolored superpartners, the higgsinos are typically the most impacted by the naturalness constraints. Eq. (88.5) suggests that the masses of the two neutral higgsinos and charged higgsino pair (which are governed by $|\mu|$) should not be significantly larger than m_Z to avoid an unnatural fine-tuning of the supersymmetric parameters, which would imply the existence of light higgsinos (whose masses are not well constrained, as they are difficult to detect directly at the LHC due to their soft decay products). However, it may be possible to avoid the conclusion that $\mu \sim \mathcal{O}(m_Z)$ if additional correlations among the SUSY breaking mass parameters and μ are present. Such a scenario can be realized in models in which the boundary conditions for SUSY breaking are generated by approximately conformal strong dynamics. For example, in the so-called scalar-sequestering model of Ref. [207], values of $|\mu| > 1$ TeV can be achieved while naturally maintaining the observed value of m_Z.

Finally, one can also consider extensions of the MSSM in which the degree of fine-tuning is relaxed. For example, it has already been noted in Sec. 88.5 that it is possible to accommodate the observed Higgs mass more easily in the NMSSM due to contributions to m_h^2 proportional to the parameter λ^2. This means that one does not have to rely on a large contribution from radiative corrections to boost the Higgs mass sufficiently above its tree-level bound. This allows for smaller top squark masses, which are more consistent with the demands of naturalness. The reduction of the fine-tuning in various NMSSM models was initially advocated in Ref. [208], and subsequently treated in more detail in Refs. [116, 209]. Naturalness can also be relaxed in extended supersymmetric models with vector-like quarks [210] and in gauge extensions of the MSSM [211].

The experimental absence of any new physics beyond the SM at the LHC suggests that the principle of naturalness is presently under significant stress [212]. Nevertheless, one must be very cautious when drawing conclusions about the viability of weak-scale SUSY to explain the origin of electroweak symmetry breaking, since different measures of fine-tuning noted above can lead to different assessments [189, 190]. Moreover, the maximal value of Δ that determines whether weak-scale SUSY is a fine-tuned model (should it be $\Delta \sim 10$? 100? 1000?) is ultimately subjective.

Thus, it is premature to conclude that weak-scale SUSY is on the verge of exclusion. It might be possible to sharpen the upper bounds on superpartner masses based on naturalness arguments, which ultimately will either confirm or refute the weak scale SUSY hypothesis [213]. Of course, if evidence for supersymmetric phenomena in the multi-TeV regime were to be established at a future collider facility (with an energy reach beyond the LHC [214]), it would be viewed as a spectacularly successful explanation of the large gauge hierarchy between the (multi-)TeV scale and Planck scale. In this case, the remaining little hierarchy, characterized by the somewhat large value of the fine-tuning parameter $\varDelta$ discussed above, would be regarded as a less pressing issue.

88.7.2 *Indirect constraints on supersymmetric models*

While direct empirical searches for supersymmetric particles provide various limits on their properties, indirect constraints can depend more sensitively on details of the whole model. The cold dark matter relic density inferred from cosmological fits to observational data is one such example of an indirect constraint. In supersymmetric models where the LSP is stable (and thus is a dark matter candidate), its thermally-produced relic density depends upon the scattering of various supersymmetric particles into dark matter particles and SM particles. The resulting relic density can depend sensitively on the masses of the non-LSP supersymmetric particles as well as on the mass of the LSP. In a typical model, an appreciable region of the parameter space is ruled out because it yields an overabundance of dark matter (see for example Ref. [215] for a fit to a seven parameter version of the pMSSM). However, subsequent tweaks to the supersymmetric model that yield an unstable LSP, such as the introduction of R-parity violating effects, can mean that the relic density no longer constrains the parameter space.

There are a number of indirect constraints based on low-energy measurements that are sensitive to the effects of new physics via supersymmetric loop effects. For example, the virtual exchange of supersymmetric particles can contribute to the muon anomalous magnetic moment, $a_\mu \equiv \frac{1}{2}(g-2)_\mu$, as reviewed in Ref. [216]. The SM prediction for a_μ, which employs dispersion relations and low energy e^+e^- scattering data to determine the hadronic corrections [217], exhibits a deviation of 5σ from the experimentally observed value [218]. The deviation of Ref. [217] from the measured value of a_μ [218] is difficult to accommodate in the constrained models of Sec. 88.6.2 and 88.6.3 given the present sparticle mass bounds [196]. Nevertheless, such a deviation can be consistent in the context of a more general version of the MSSM [219].

However, the Budapest-Marseille-Wupertal (BMW) lattice determination of the leading hadronic contribution to the muon magnetic moment obtained in Ref. [220] yields a significantly smaller deviation from the SM prediction [221]. Indeed, there is a tension of almost 4σ between the BMW lattice calculation and the corresponding dispersive estimate based on the same e^+e^- data, as stated in Ref. [222] where the relevant references are cited. Moreover, Ref. [222] notes that crucial parts of the BMW calculation have recently been cross checked by other lattice groups. In addition, various tensions exist among the relevant sets of e^+e^- scattering data that are used in the dispersive estimate, some of which have appeared after the publication of Ref. [217]. Thus, the deviation of the measured value of a_μ from the corresponding SM prediction still needs to be clarified.

The precision of the measured value of a_μ is not sensitive to the experimental error associated with the measured value of the fine structure constant, α. In contrast, the comparison of the SM prediction with the experimental measurement of the anomalous magnetic moment of the electron, a_e, depends critically on the value of α. Using the experimentally determined value of α given in Ref. [223] yields a SM prediction for a_e that is 2.4σ above its measured value [224]. However, this previous determination of α is in tension at the 5σ level with a more recent measurement of the fine structure constant [225]. The latter yields a SM prediction for a_e that is 1.6σ below its measured value [225].

Measurements of the fine structure constant, a_e and a_μ jointly constrain the pMSSM parameter space [226] due to shifts originating from supersymmetric loop effects. In particular, if the supersymmetric interpretation of the deviation in the measured value of a_μ from its SM prediction is combined with the experimental limits on the electron electric dipole moment [135], then the resulting upper bounds on MSSM CP-violating phases are even more constraining [227] than previously noted at the beginning of Sec. 88.6.

Flavor transitions in radiative, leptonic and semi-leptonic b quark decays [228] provide a fertile ground for physics beyond the SM. For example, the rare inclusive decay $b \to s\gamma$ is a sensitive probe of the virtual effects of new physics beyond the SM. The experimental measurements of $B \to X_s + \gamma$ [229] are in agreement with the theoretical SM predictions of Ref. [230]. Since supersymmetric loop corrections can contribute an observable shift from the SM predictions, the absence of any significant deviation places useful constraints on the MSSM parameter space [231].

The rare decays $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$ are especially sensitive to supersymmetric loop effects, with some loop contributions scaling as $\tan^6\beta$ when $\tan\beta \gg 1$ [232]. At present, a combination [233] of the measurements of these rare decay modes [234] is in slight tension at the 1.6σ level with the predicted SM rates [235]. Such a tension can be resolved by the aforementioned supersymmetric loop effects [232].

Several tensions exist between SM predictions and measurements of some other experimental observables that probe $b \to s\mu^+\mu^-$ transitions, although the level of tension depends upon the theoretical treatment of the SM analysis. In a certain angular distribution parameter (denoted by P_5') extracted from $B^0 \to K^{*0}\mu^+\mu^-$ decays, the tension is around the 4σ level [236]. An even larger discrepancy is observed in a combination of angular distributions and rates derived from $B^\pm \to K^\pm\mu^+\mu^-$ and $B^0 \to K^0\mu^+\mu^-$ [237]. Finally, there is a 3.6σ deviation in the branching ratio of $B_s \to \phi\mu^+\mu^-$ for di-muon invariant mass squared values between 1.1 GeV^2 and 6.0 GeV^2 [238]. Finally, a recent measurement of $B^+ \to K^+\nu\bar{\nu}$ by the Belle II Collaboration [239] obtained a branching fraction that is roughly four times larger than the SM prediction [240], corresponding to 2.7σ above the SM expectation. However, it is unlikely that this result can be attributed to new contributions from the MSSM [241], as the latter are expected to be negligible in comparison with the SM.

The decays $B^\pm \to \tau^\pm\nu_\tau$ and $\overline{B} \to D^{(*)}\tau^-\overline{\nu}_\tau$ are noteworthy, since in models with extended Higgs sectors such as the MSSM, these processes possess tree-level charged Higgs exchange contributions that can compete with the dominant W-exchange. As Sec. 72 shows, experimental measurements of $B^\pm \to \tau^\pm\nu_\tau$ are currently consistent with SM expectations [242]. The BaBar Collaboration measured values of the rates for $\overline{B} \to D\tau^-\overline{\nu}_\tau$ and $\overline{B} \to D^*\tau^-\overline{\nu}_\tau$ [243] which exhibited a combined 3.4σ discrepancy from the SM predictions, which was also not compatible with the Type-II Higgs Yukawa couplings employed by the MSSM. Subsequent measurements by the LHCb and Belle Collaborations were compatible with the BaBar measurements although they displayed less deviation from the SM expectations; the combined difference between the measured values of the $\overline{B} \to D\tau^-\overline{\nu}_\tau$ and $\overline{B} \to D^*\tau^-\overline{\nu}_\tau$ decay rates relative to the corresponding SM values has a significance of 3.3 standard deviations [244].

In summary, although there are a few hints of possible deviations from the SM in B decays, none of the discrepancies by themselves are significant enough to conclusively imply the existence of new physics beyond the SM. Moreover, the absence of evidence for sizable deviations in other B-physics observables from their SM predictions can place useful constraints on the MSSM parameter space [138, 195, 245, 246].

The CDF Collaboration has reported a measurement of the W boson mass [247] that is 7σ above the SM prediction. Supersymmetric models have been shown to be able to accommodate the measured value by the CDF Collaboration at the 2σ level [248]. However, other measurements, including a more recent determination by ATLAS Collaboration [249], are consistent with SM predictions for m_W (see Sec. 10 for further details).

88.8 Massive neutrinos in weak-scale SUSY

In the minimal version of the SM and its supersymmetric extension, there are no right-handed neutrinos, and Majorana mass terms for the left-handed neutrinos are absent. However, given the overwhelming evidence for neutrino masses and mixing (see Sec. 14 and [250]), any viable model of the fundamental particles must provide a mechanism for generating neutrino masses [251]. In extended supersymmetric models, various mechanisms exist for producing massive neutrinos [252]. Although one can devise models for generating massive Dirac neutrinos [253], the most common approaches for incorporating neutrino masses are based on L-violating supersymmetric extensions of the MSSM, which generate massive Majorana neutrinos. Two classes of L-violating supersymmetric models will now be considered.

88.8.1 *The supersymmetric seesaw*

Neutrino masses can be incorporated into the SM by introducing SU(3)×SU(2)×U(1) singlet right-handed neutrinos (ν_R) whose mass parameters are very large, typically near the grand unification scale. In addition, one must also include a standard Yukawa couplings between the lepton doublets, the Higgs doublet, and ν_R. The Higgs vacuum expectation value then induces an off-diagonal ν_L–ν_R mass on the order of the electroweak scale. Diagonalizing the neutrino mass matrix (in the three-generation model) yields three superheavy neutrino states, and three very light neutrino states that are identified with the light neutrinos observed in nature. This is the seesaw mechanism [254].

It is straightforward to construct a supersymmetric generalization of the seesaw model of neutrino masses [255,256] by promoting the right-handed neutrino field to a superfield $\widehat{N}^c = (\widetilde{\nu}_R\,;\,\nu_R)$. Integrating out the heavy right-handed neutrino supermultiplet yields a new term in the superpotential [cf. Eq. (88.1)] of the form

$$W_{\rm seesaw} = \frac{f}{M_R}(\widehat{H}_U\widehat{L})(\widehat{H}_U\widehat{L})\,, \tag{88.34}$$

where M_R is the mass scale of the right-handed neutrino sector and f is a dimensionless constant. Note that lepton number is broken by two units by Eq. (88.34), which implies that R-parity invariance is preserved. The supersymmetric analogue of the Majorana neutrino mass term in the sneutrino sector leads to sneutrino–antisneutrino mixing phenomena [256, 257]. In addition, new Higgs-slepton interaction terms can probe the structure of the supersymmetric seesaw model [258]. The right-handed sneutrino that resides in $\widehat{L}$ also provides an intriguing dark matter candidate [259].

The SUSY Les Houches Accord [80, 99], mentioned at the end of the introduction to Sec. 88.4, has been extended to the supersymmetric seesaw (and other extensions of the MSSM) in Ref. [260].

88.8.2 *R-parity-violating SUSY*

It is possible to incorporate massive neutrinos in renormalizable supersymmetric models while retaining the minimal particle content of the MSSM by relaxing the assumption of R-parity invariance. The most general R-parity-violating model involving the MSSM spectrum introduces many new parameters to both the SUSY-conserving and the SUSY-breaking sectors [80, 261]. Each new interaction term violates either B or L conservation. For example, starting from the MSSM superpotential given in Eq. (88.1) [suitably generalized to three generations of quarks, leptons and their superpartners], consider the effect of adding the following new terms:

$$W_{\rm RPV} =(\lambda_L)_{pmn}\widehat{L}_p\widehat{L}_m\widehat{E}^c_n + (\lambda'_L)_{pmn}\widehat{L}_p\widehat{Q}_m\widehat{D}^c_n + (\lambda_B)_{pmn}\widehat{U}^c_p\widehat{D}^c_m\widehat{D}^c_n + (\mu_L)_p\widehat{H}_u\widehat{L}_p\,, \tag{88.35}$$

where p, m, and n are generation indices, and gauge group indices are suppressed. Eq. (88.35) yields new scalar-fermion Yukawa couplings consisting of all possible combinations involving two SM fermions and one scalar superpartner.

Note that the term in Eq. (88.35) proportional to λ_B violates B, while the other three terms violate L. The L-violating term in Eq. (88.35) proportional to μ_L is the RPV analog of the $\mu\widehat{H}_u\widehat{H}_d$ term of the MSSM superpotential, in which the $Y = -1$ Higgs/higgsino supermultiplet $\widehat{H}_d$ is replaced by the slepton/lepton supermultiplet $\widehat{L}_p$.

Phenomenological constraints derived from data on various low-energy B- and L-violating processes can be used to establish limits on each of the coefficients $(\lambda_L)_{pmn}$, $(\lambda'_L)_{pmn}$, and $(\lambda_B)_{pmn}$ taken one at a time [261, 262]. If more than one coefficient is simultaneously non-zero, then the limits are in general more complicated [263]. All possible RPV terms cannot be simultaneously present and unsuppressed; otherwise the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose B or L invariance (either one alone would suffice). Otherwise, one must accept the requirement that certain RPV coefficients must be extremely suppressed.

One particularly interesting class of RPV models is one in which B is conserved, but L is violated. It is possible to enforce baryon number conservation (and the stability of the proton), while allowing for lepton-number-violating interactions by imposing a discrete $\mathbb{Z}_3$ baryon triality symmetry on the low-energy theory [264], in place of the standard $\mathbb{Z}_2$ R-parity. Since the distinction between the Higgs and matter supermultiplets is lost in RPV models where L is violated, the mixing of sleptons and Higgs bosons, the mixing of neutrinos and neutralinos, and the mixing of charged leptons and charginos are now possible, leading to more complicated mass matrices and mass eigenstates than in the MSSM. The treatment of neutrino masses and mixing in this framework can be found, *e.g.*, in Ref. [265].

Alternatively, one can consider imposing a lepton parity such that all lepton superfields are odd [264, 266]. In this case, only the B-violating term in Eq. (88.35) survives, and L is conserved. Models of this type have been considered in Ref. [267]. Since L is conserved in these models, the mixing of the lepton and Higgs superfields is forbidden. Moreover, neutrino masses (and mixing) are not generated if lepton parity is an exact symmetry. However, one expects that lepton parity cannot be exact due to quantum gravity effects. Remarkably, the standard $\mathbb{Z}_2$ R-parity and the $\mathbb{Z}_3$ baryon triality are stable with respect to quantum gravity effects, as they can be identified as residual discrete symmetries that arise from spontaneously broken non-anomalous gauge symmetries [264].

The symmetries employed above to either remove or suppress R-parity violating operators were flavor independent. In contrast, there exist a number of motivated scenarios based on flavor symmetries that can also yield the suppression as required by the experimental data (*e.g.*, see Ref. [268]).

The supersymmetric phenomenology of the RPV models exhibits features that are distinct from that of the MSSM [261]. The LSP is no longer stable, which implies that not all supersymmetric decay chains must yield missing-energy events at colliders. A comprehensive examination of the phenomenology of the MSSM extended by a single R-parity violating coupling at the unification scale and its implications for LHC searches has been given in Ref. [269]. As an example, the sparticle mass bounds obtained in searches for R-parity-conserving SUSY can be considerably relaxed in certain RPV models due to the absence of large missing transverse momentum signatures [270]. This can alleviate some of the tension with naturalness (introduced in Sec. 88.7.1).

Nevertheless, the loss of the missing-energy signature is often compensated by other striking signals (which depend on which R-parity-violating parameters are dominant). For example, supersymmetric particles in RPV models can be singly produced (in contrast to R-parity-conserving models where supersymmetric particles must be produced in pairs). The phenomenology of pair-produced supersymmetric particles is also modified in RPV models due to new decay chains not present in R-parity-conserving SUSY models [261].

In RPV models with lepton number violation (these include weak-scale SUSY models with baryon triality mentioned above), both $\Delta L = 1$ and $\Delta L = 2$ phenomena are allowed, leading to neutrino masses and mixing [271], neutrinoless double-beta decay [272], sneutrino-antisneutrino mixing [273], and resonant s-channel production of sneutrinos in e^+e^- collisions [274] and in charged sleptons in $p\bar{p}$ and pp collisions [275], respectively.

88.9 Extensions beyond the MSSM

Extensions of the MSSM have been proposed to solve a variety of theoretical problems [35]. One such problem involves the μ parameter of the MSSM. Although μ is a SUSY-*preserving* parameter, it must be of order the effective SUSY-breaking scale of the MSSM to yield a consistent supersymmetric phenomenology [276]. Any natural solution to the so-called μ-problem must incorporate a symmetry that enforces $\mu = 0$ and a small symmetry-breaking parameter that generates a value of μ that is not parametrically larger than the effective SUSY-breaking scale [277]. A number of proposed mechanisms in the literature (*e.g.*, see Ref. [276–278]) provide concrete examples of a natural solution to the μ-problem of the MSSM.

In extensions of the MSSM, other compelling solutions to the μ-problem are possible. For example, one can replace μ by the vacuum expectation value of a new SU(3)×SU(2)×U(1) singlet scalar field, as noted below Eq. 88.23. This is the NMSSM, which yields phenomena that were briefly discussed in Sections 88.4–88.7. The NMSSM superpotential consists only of trilinear terms whose coefficients are dimensionless. There are some advantages to extending the NMSSM further to the 'USSM' [107] by adding a new broken U(1) gauge symmetry [279], under which the singlet field is charged.

Alternatively, one can consider a generalized version of the NMSSM (called the GNMSSM in Ref. [209]), where all possible renormalizable terms in the superpotential are allowed, which yield new supersymmetric mass terms (analogous to the μ term of the MSSM). A discussion of the parameters of the GNMSSM can be found in Ref. [80]. Although the GNMSSM does not solve the μ-problem, it does exhibit regions of parameter space in which the degree of fine-tuning is relaxed, as discussed in Sec. 88.7.1.

The generation of the μ-term may be connected with the solution to the strong CP problem [280]. Models of this type, which include new gauge singlet fields that are charged under the Peccei-Quinn (PQ) symmetry [281], were first proposed in Ref. [276]. The breaking of the PQ symmetry is thus intimately tied to SUSY breaking, while naturally yielding a value of μ that is of order the electroweak symmetry breaking scale [282].

All supersymmetric models discussed so far in this review possess self-conjugate fermions—the Majorana gluinos and neutralinos. However, it is possible to add additional chiral superfields in the adjoint representation. The spin-1/2 components of these new superfields can pair up with the gauginos to form Dirac gauginos [283, 284]. Such states appear in models of so-called supersoft SUSY breaking [285], in some generalized GMSB models [286], and in R-symmetric SUSY models [287, 288]. Such approaches often lead to improved naturalness and/or significantly relaxed flavor constraints. The implications of models of Dirac gauginos on the observed Higgs boson mass and its properties are addressed in Ref. [289].

For completeness, we briefly note other MSSM extensions considered in the literature. These include an enlarged electroweak gauge group beyond SU(2)×U(1) [290]; the addition of new Higgs supermultiplets beyond the doublets and singlets of the MSSM/NMSSM [291]; and/or the addition of new (possibly exotic) matter supermultiplets [210,292,293] such as vector-like fermions and their superpartners.

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