

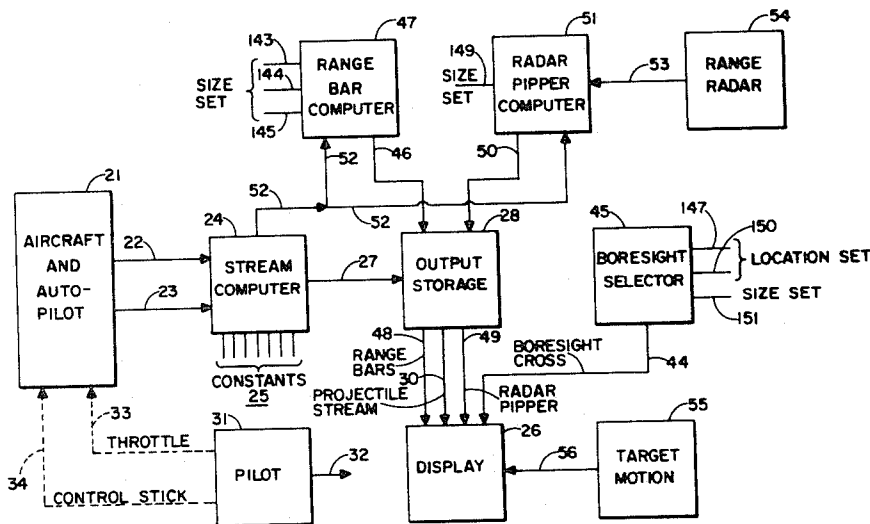
- [54] **PROJECTILE STREAM DISPLAY APPARATUS**  
 [75] Inventors: **Arthur F. Hayes, Fridley; Robert L. Schultz, Edina, both of Minn.**  
 [73] Assignee: **Honeywell, Inc., Minneapolis, Minn.**  
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 [21] Appl. No.: **69,536**  
 [52] U.S. Cl. .... **235/61.5 E**  
 [51] Int. Cl. .... **G06f 15/58**  
 [58] Field of Search ..... **235/61.5 E**

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[57] **ABSTRACT**  
 Digital computer apparatus and computer controlled process for displaying the locus of the present positions of a plurality of projectiles, if discharged sequentially from a freely moving vehicle at known intervals beginning at a predetermined time in the past, as seen from a predetermined point in the vehicle, including computer means for iteratively determining said present positions and for modifying the locus in accordance with changes in parameters of the movement of the vehicle and the projectiles; also included are means for identifying points on the locus at selected distances from the vehicle.

- [56] **References Cited**  
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16 Claims, 13 Drawing Figures



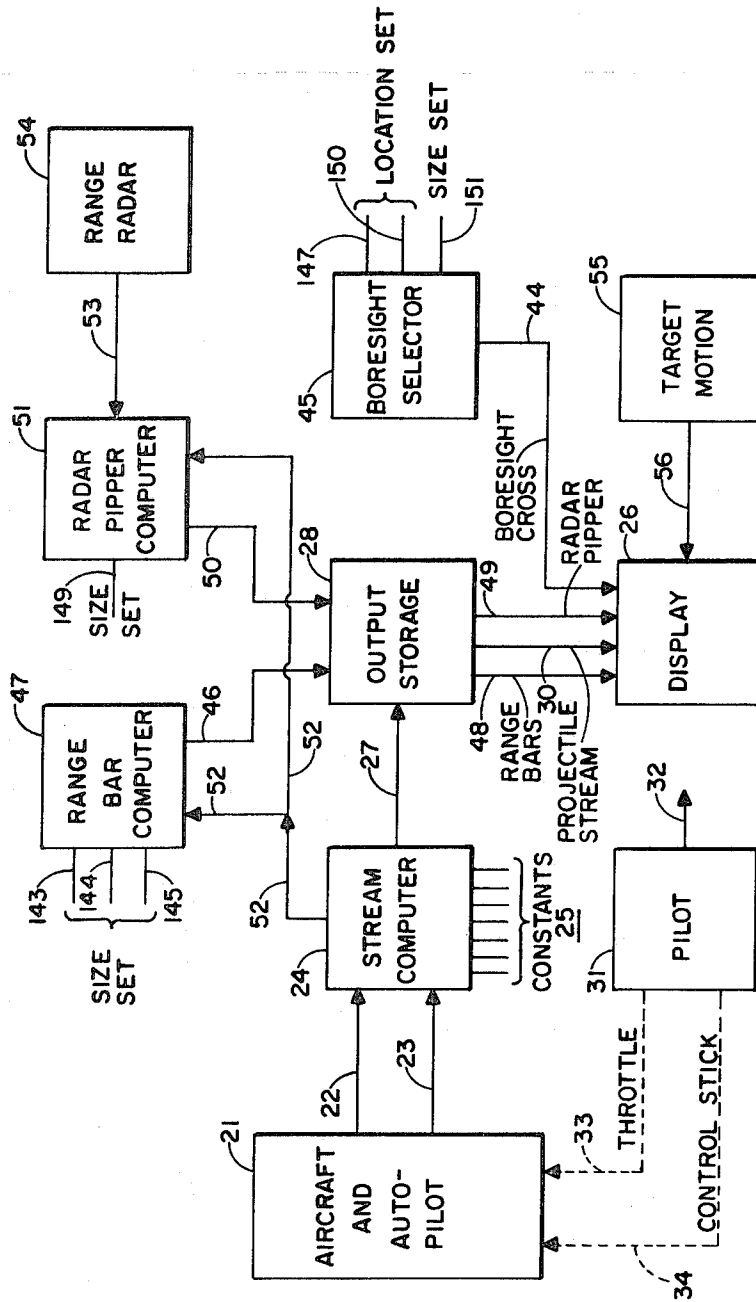


FIG. 1

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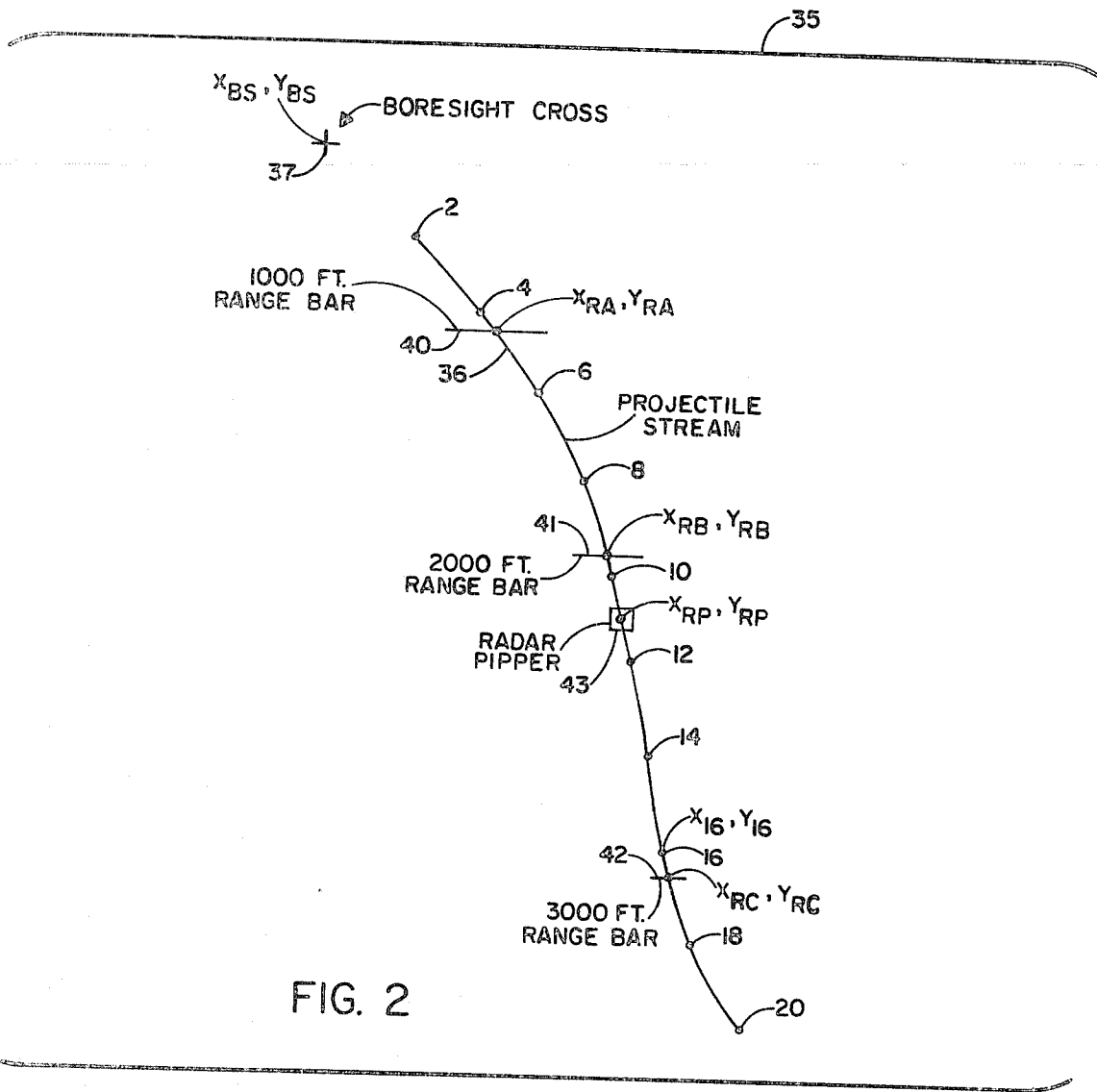


FIG. 2

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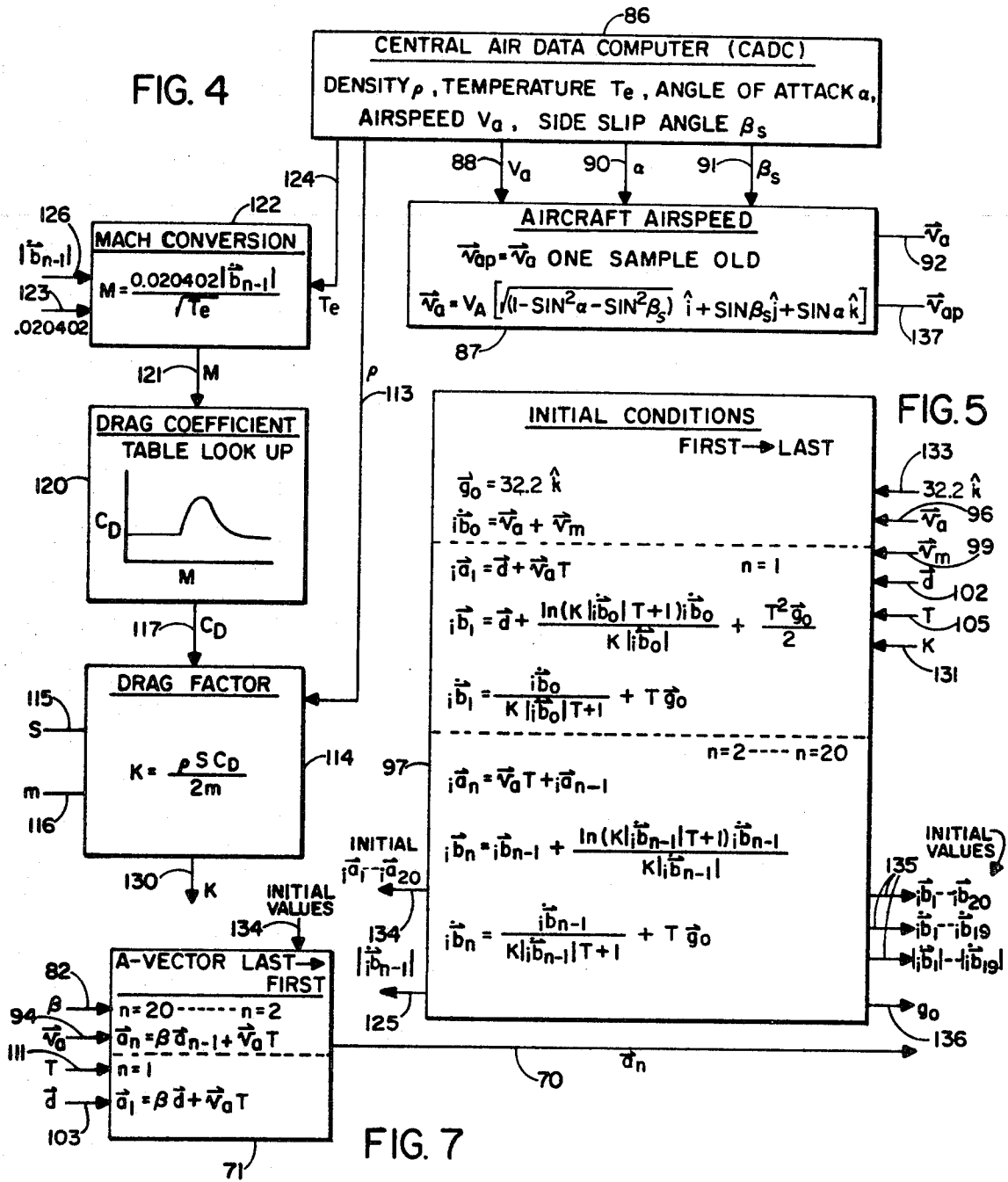


FIG. 7

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FIG. 6

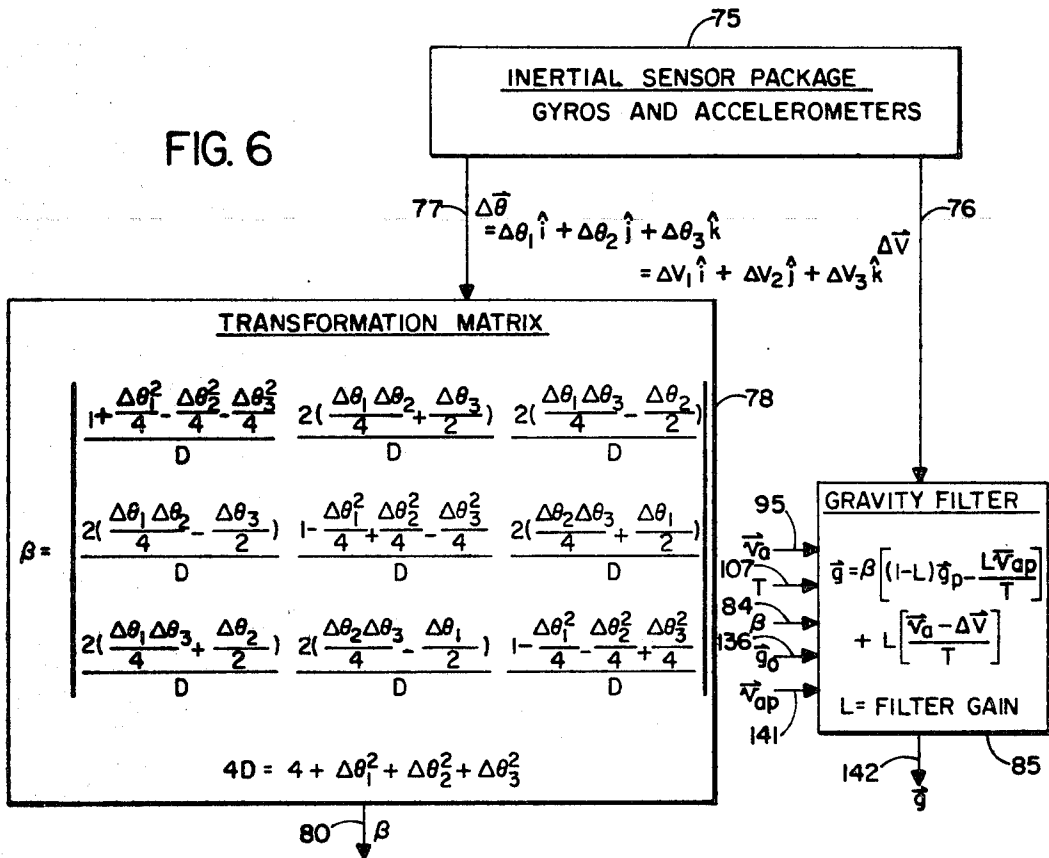
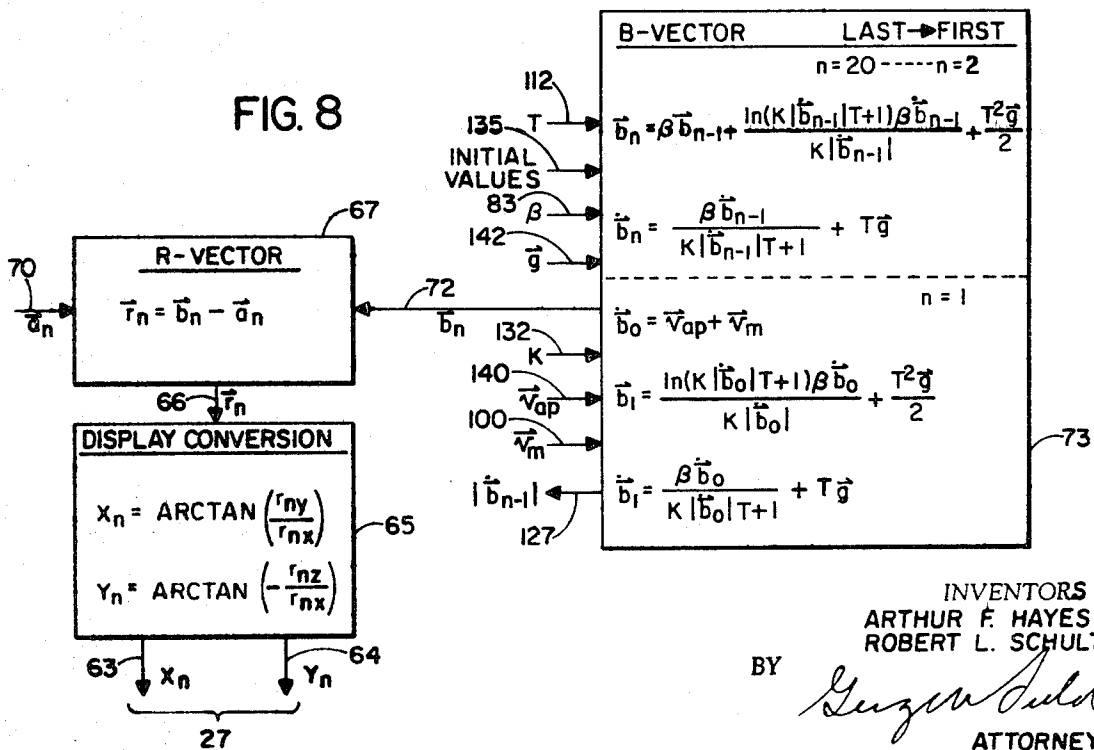
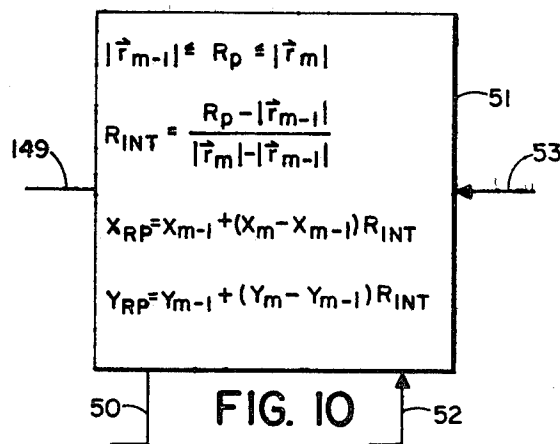
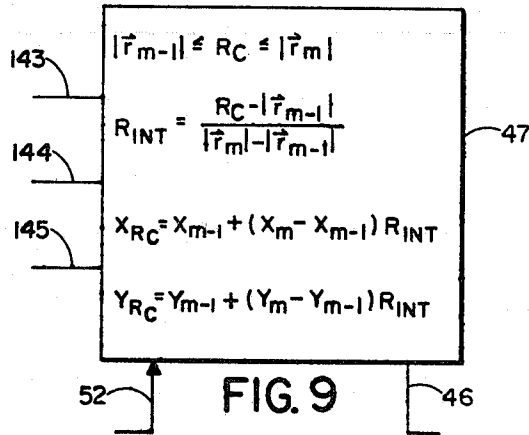


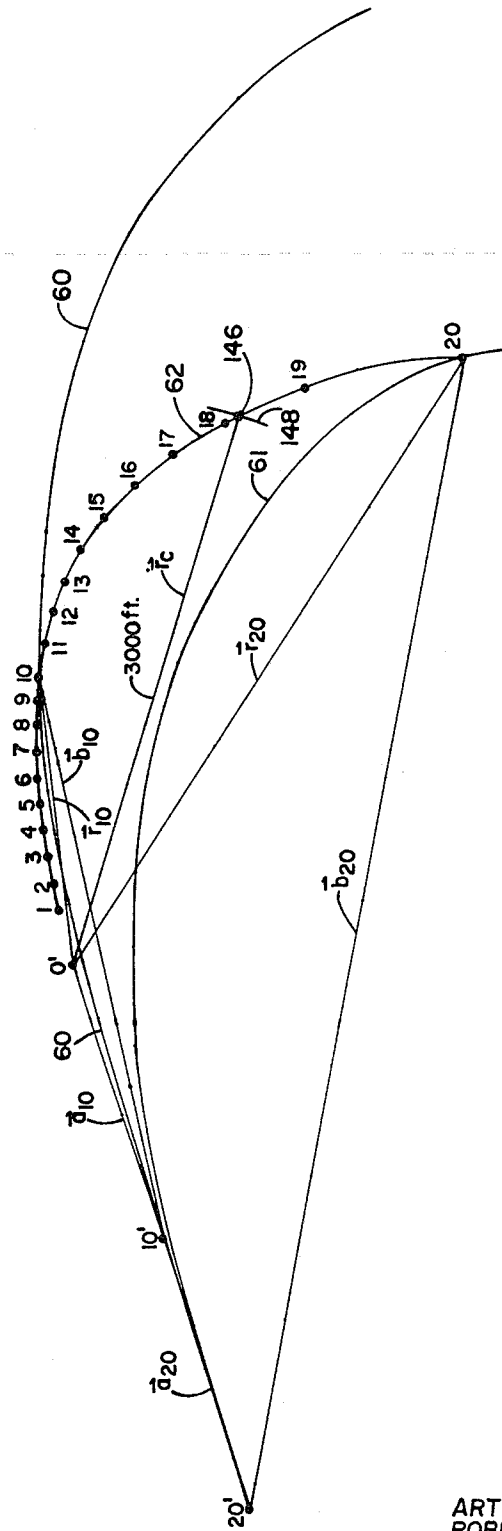
FIG. 8





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FIG. 11



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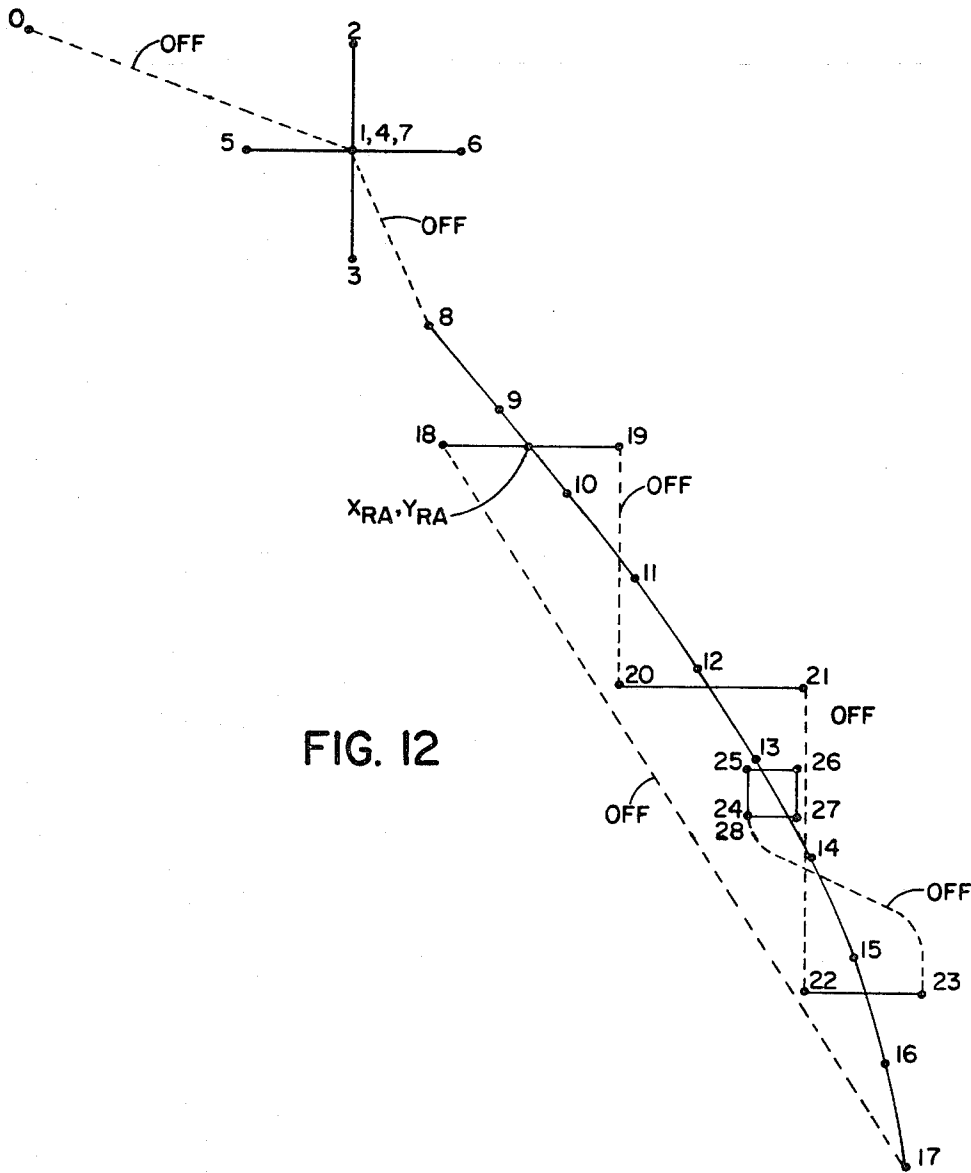


FIG. 12

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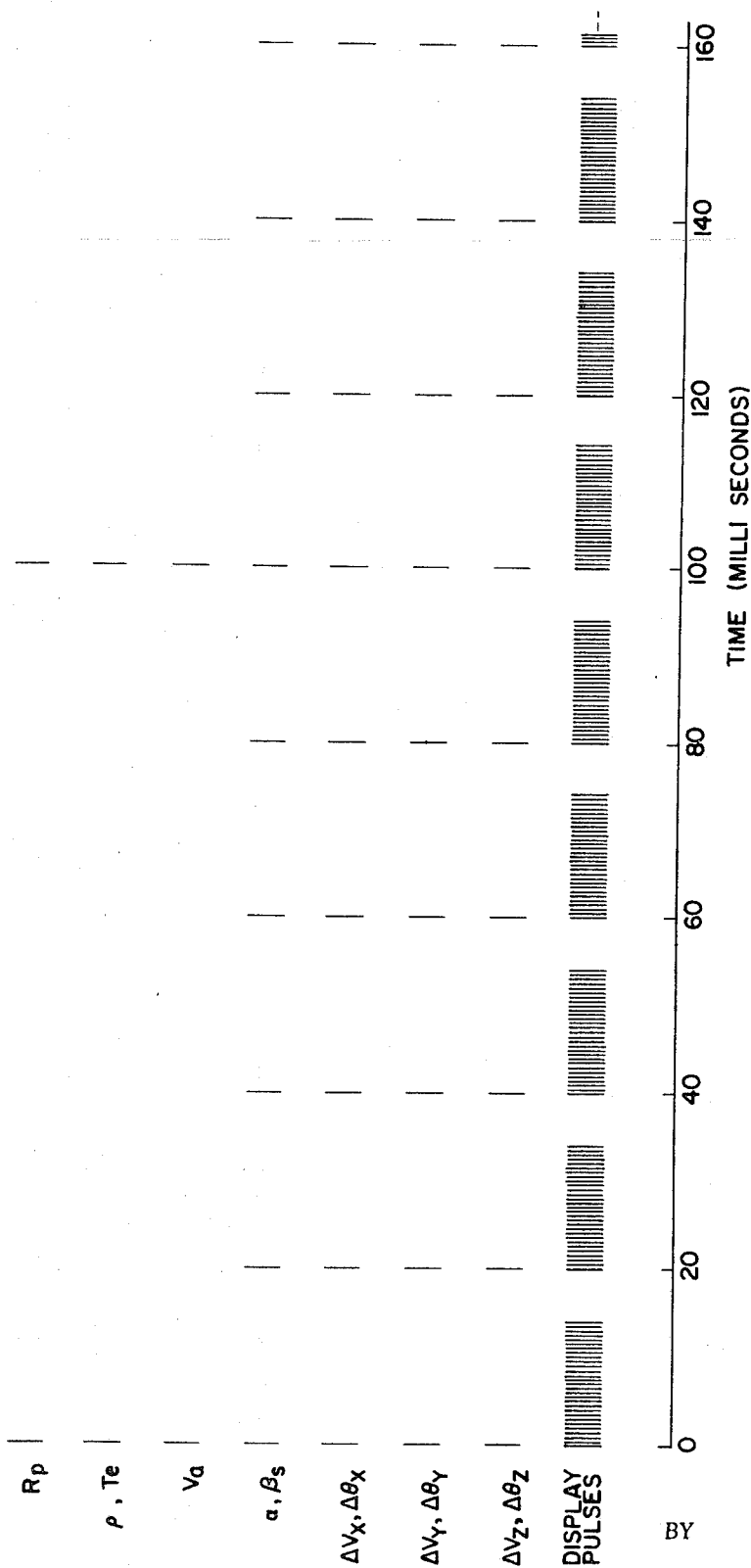


FIG. 13

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**PROJECTILE STREAM DISPLAY APPARATUS****FIELD OF THE INVENTION**

This invention relates to the field of visual display apparatus, and more particularly to apparatus for displaying to the pilot of a fighter aircraft what would be the apparent locus of the present positions of a plurality of projectiles if they had been fired at known recent intervals beginning at a predetermined time in the past. The display may be projected on a combining glass through which the pilot observes target aircraft in evasive action, whereby he may so direct the flight of this aircraft that the target aircraft intersects the displayed projectile stream, at a proper range therealong, triggering his cannon at the appropriate instant to insure that real projectiles are present at the intersection. The invention can equally well be used, however, in aeronautical laboratories, or in training equipment where a student pilot observes directly the face of the instrument producing the display, without the use of a combining glass, the target aircraft being simulated or computer-modeled for the display, and the pursuit aircraft and autopilot being also simulated except for the controls.

**DESCRIPTION OF THE PRIOR ART**

Gunsights for aircraft are not broadly new. It is, of course, understood that the cannon to be fired is fixed to the aircraft, and is aimed by aiming the aircraft as a whole. Initially the only sight necessary was a properly positioned "iron sight" fixed to the aircraft where the pilot could see through it as he maneuvered. As the speed and maneuverability of target and pursuit aircraft increased, it became necessary to design more sophisticated gunsights in which some automatic prediction of the future attitude of the pursuit aircraft and the future position of the target aircraft could be made: exemplary of such instruments are the "Disturbed Reticle Sight" or "Lead Computing Optical Sight," and the "Director Sight."

Target and pursuit aircraft have now reached a stage of sophistication where a new sight is required. Two basic reasons underlie this necessity: the first is that it is now almost impossible to maintain a target in a sight for an interval long enough to allow the sight to "settle," and the second is that the assumption of constant angular velocity of the target, for an interval equal to the time of flight of the projectiles, underlies these sights and is no longer valid.

**SUMMARY OF THE INVENTION**

To overcome the defects mentioned above, applicants have invented a new sight in which prediction of the location of the projectile stream, as modified by aircraft maneuvers, is accomplished by a computer fed with signals representative of appropriate parameters of the pursuit aircraft and the projectiles. The prediction of the target motion is left to the pilot, who is in a better position to perform this function than any computer because of the inherent high "noise" in signals representative of the target, and because of visual cues, such as target attitude, which his special skills make meaningful. The present application comprehends only so much of the overall system as is concerned with generating and presenting a projectile stream display, with range indications, which changes in apparent

shape with changes in the parameters of flight of the projectiles and the violently maneuvering pursuit aircraft.

A principal object of our invention is thus to provide an improved gunsight for aircraft use. Expressed differently, it is a principal object of our invention to provide means for displaying the locus of the present positions of a stream of hypothetical projectiles, as seen by the pilot of an aircraft, in such a way that the display is adapted to be observed concurrently with a real or simulated maneuvering target to give the pilot indications of appropriate instants for initiating the flight of real projectiles with good probability that they will hit the target. Another object is to provide such a display system which is continually corrected for changes in parameters of the movement of the pursuit aircraft, and in parameters of the flight of the projectiles. A still further object of the invention is to provide a data processing method for deriving, from signals representative of aircraft motion, related signals for displaying said positions of said projectiles.

Various other objects, advantages, and features of novelty which characterize our invention are pointed out with particularity in the claims annexed thereto and forming a part hereof. However, for a better understanding of the invention, its advantages, and objects attained by its use, reference should be had to the drawing which forms a further part hereof, and to the accompanying descriptive matter, in which we have illustrated and described a preferred embodiment of the invention.

**BRIEF DESCRIPTION OF THE DRAWINGS**

In the drawing,

FIG. 1 is a block diagram of a display system according to our invention, together with means for adding to the display further inputs representative of the motion of a target aircraft where this is desired;

FIG. 2 is a labeled representation of the display actually observed;

FIG. 3 is a block diagram of the system component identified as STREAM COMPUTER in FIG. 1;

FIGS. 4-10 inclusive, show more details of portions of FIG. 3;

FIG. 11 is a diagram illustrative of considerations underlying our invention;

FIG. 12 is a representation similar to FIG. 2 but modified to show how the locus of the earlier figure is traced; and

FIG. 13 is a diagram illustrative of time relations in the invention.

**DESCRIPTION OF THE PREFERRED EMBODIMENT**

Turning now to FIG 1, block 21 identifies means for producing signals representative of parameters of the motion of a pursuit aircraft, and may comprise either the aircraft and autopilot themselves, or a computer model thereof, depending on whether the invention is to be used in actual flight or in laboratory or instruction work. The autopilot includes an air data computer and an inertial package, and from it a first cable 22 supplies air data computer signals, and a second cable 23 supplies inertial package signals, the signals going to a projectile stream computer 24 having a plurality of means 25 for setting appropriate constants.

A display 26 preferably in the form of a cathode ray oscilloscope is to present data obtained in computer 24 as a line plotted on Cartesian coordinates representative of the angular displacements of various points, measured at the pilot's eye, from a zero or nominal direction, angles in train being represented by X-coordinates and angles in elevation being represented by Y-coordinates. To this end the computed coordinates for each of 20 projectiles are supplied by computer 24 on a cable 27 to a storage device 28 from which they are from time to time supplied on a cable 30 to display 26. It is understood that members 24 and 28 may conveniently comprise functional portions of a single conventional general purpose computer appropriately programmed to perform computations as detailed below, and to store and read out data in a scheduled fashion.

Block 31 is representative of the human pilot, student, or experimental operator of the system: he observes the display as suggested at 32 and operates controls such as the throttle 33 and control stick 34 of the actual or modeled aircraft to produce desired real or simulated aircraft motion, in accordance with which the projectile stream must change appropriately.

Reference should now be made to FIG. 2, which shows the appearance of a typical display as presented by our apparatus. A cathode ray tube 35 may be viewed directly or reflected in a combining glass. If desired a suitable optical system focused at infinity may be used to avoid parallax, as is well known in gun sights. On the screen of the tube there appears a line 36 extending from one to another of a series of points, each point being representative of the instantaneous location of a particular one of a series of hypothetical projectiles fired at intervals of 0.1 second beginning at an instant 2 seconds in the past: for convenience the several projectiles are identified by  $n$  numbers between 1 and 20, the projectile identified by  $n = 1$  being that fired 0.1 second ago and the projectile identified by  $n = 20$  being that fired 2 seconds ago. It will be evident that any projectile fired more than 2 seconds ago no longer appears in the display, so that the  $n$  numbers are not associated exclusively with particular projectiles, but rather with projectiles having particular times of discharge relative to the continually advancing "present." For further convenience, as will be discussed below, only the even-numbered projectiles are represented on the display, at points 2, 4 . . . and 20, although data defining the odd-numbered projectiles is manipulated and stored in the computer. By way of illustration it is shown that the point 16 is identified by the coordinates  $X_{16}$ ,  $Y_{16}$ .

In addition to the line or locus 36, the display presents a "boresight" cross 37, three range markers in the form of bars 40, 41, and 42, and a radar "piper" 43. The center of cross 37 has coordinates  $X_{BS}$ ,  $Y_{BS}$ , and its size is predetermined. The centers of the range bars have coordinates  $X_{RA}$ ,  $Y_{RA}$ ,  $X_{RB}$ ,  $Y_{RB}$ , and  $X_{RC}$ ,  $Y_{RC}$ , and their predetermined lengths are proportional to the angular dimensions, at the pilot's eye, of a target of known size at ranges of 1,000 feet, 2,000 feet, and 3,000 feet, respectively. If desired an adjustment may be provided for selecting different known target sizes: alternatively the range markers may be circles of proper diameters centered at the identified points. The center of radar piper 43 has the coordinates  $X_{RP}$ ,  $Y_{RP}$ : it preferably comprises a hollow square of predetermined size, but may take any other desired configuration.

Returning to FIG. 1, storage device 28 receives signals for determining the positions and sizes of bars 40, 41, and 42 on a cable 46 from a subordinate range bar computer 47 and signals determining the position and size of the radar piper 43 on a cable 50 from a subordinate radar piper computer 51, and supplies these signals on cables 48 and 49 respectively to display 26. For proper coordination with the rest of the apparatus computers 47 and 51 receive signals from stream computer 24 on a cable 52. Computer 51 receives a signal at 53 representative of the output of a real or modeled range radar 54. In actual flight there is often insufficient time for a range radar to lock on to the target, and in this case, piper 43 may not be used.

For completeness a block 55 is shown representing the target which is being observed in terms of its coordinates  $X_T$ ,  $Y_T$  relative to the display: the target may be a visually observed one, in flight use of the apparatus, or its coordinates, size, and attitude may be supplied to display 26 on a cable 56 from the same model which supplies radar range 54.

Referring now to FIG. 11, the basis of operation of the invention will be briefly discussed. FIG. 11 is simplified compared with FIG. 2, for purposes of illustration, by the assumption that the aircraft is in straight flight and has followed in the last 2 seconds a course determined by the line  $20'$ ,  $10'$ ,  $0'$ : the present location of the aircraft is at  $0'$ . If projectiles had been fired at 0.1 second intervals, that fired 0.1 second ago would be at position 1, that fired 0.2 second ago would be at position 2, and so on: the actual trajectory for projectile 10 is shown by curve 60, and that for projectile 20 is shown by curve 61.

A vector  $\vec{r}_n$  is defined to extend from the present position of the aircraft (actually of the pilot's eye) to the present position of projectile  $n$ . A second vector  $\vec{b}_n$  is defined to extend from the position of the aircraft at the time projectile  $n$  was fired to the present position of projectile  $n$ . A third vector  $\vec{a}_n$  is defined to extend from the position of the aircraft at the time projectile  $n$  was fired to the present position of the aircraft. Vectors  $\vec{r}_{10}$ ,  $\vec{b}_{10}$ ,  $\vec{r}_{20}$ , and  $\vec{b}_{20}$  are identified on the figure: vector  $\vec{a}_{10}$  is the line  $10' - 0'$  and vector  $\vec{a}_{20}$  is the line  $20' - 0'$ . The line 62 is the locus of the present positions of projectiles 1-20 on their respective trajectories, and is not, in itself, the trajectory of anything. It is this line, projected on the viewing plane of the pilot, which appears as line 36 in FIG. 2, and a principal purpose of this invention is to represent the directions of the heads of vectors  $\vec{r}_n$  in terms of the angular coordinates  $X_n$  and  $Y_n$ .

FIG. 3 shows projectile stream computer 24 in more detailed block diagram form. The computer is shown functionally: its inherent store, retrieve, and manipulate functions are those of any general purpose computer, and this conventional detail is omitted to keep the specification within reasonable bounds.

In one embodiment of the invention computer 24 was a commercial "Sigma 5" unit supplied by Scientific Data Systems, Inc., El Segundo, Calif., but any general purpose computer may be used.

The output of the computer is a series of 20 values of  $X_n$  on line 63 and a series of corresponding values of  $Y_n$  on line 64: lines 63 and 64 comprise cable 27 in FIG. 1. The values are supplied from a display conversion unit 65 of conventional nature in accordance with a series of 20 vector inputs  $\vec{r}_n$ , each of which is supplied on a

cable 66 from an R-Vector computer 67 in terms of its components aligned with three orthogonal axes  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ . A vector  $\vec{r}_n$  is associated with each projectile computed.

The unit vector  $\hat{i}$  is taken along the longitudinal axis of the aircraft, and is positive in the forward direction: rotation about this line as an axis is identified by the "roll" angle  $\theta_1$ , positive for rotation according to the right hand rule. The unit vector  $\hat{j}$  is normal to the vector  $\hat{i}$  and parallel to the chord of the wings of the aircraft, and is positive to the right, looking forward in the aircraft: rotation about this line as an axis is identified by the "pitch" angle  $\theta_2$ , positive according to the right hand rule. The unit vector  $\hat{k}$  is normal to vectors  $\hat{i}$  and  $\hat{j}$  and hence in straight and level flight is vertical and is then positive in the downward direction: rotation about this axis is identified as the "yaw" angle  $\theta_3$ , positive according to the right hand rule. Angles  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are sometimes referred to in the literature of aerodynamics as  $\phi$ ,  $\theta$ , and  $\Psi$ , and directions along the unit vectors are sometimes identified by lower case subscripts  $x$ ,  $y$ , and  $z$ .

It is to be understood that whenever manipulation of a vector quantity is indicated herein, the computer actually manipulates the orthogonal components and thus must be prepared to work with three related quantities rather than with a single quantity as the simplified equations and drawings herein would suggest. Computer operation with vector quantities is well known in the art, and no new principles or techniques are involved here.

Each vector  $\vec{r}_n$  is expressed in terms of the following identity

$$\vec{r}_n = r_{nx}\hat{i} + r_{ny}\hat{j} + r_{nz}\hat{k} \quad (1)$$

As shown in FIG. 8, the X-coordinate of each projectile is computed from the equation

$$X_n = \text{ARCTAN}(r_{ny}/r_{nx}), \quad (2)$$

and the Y-coordinate of each projectile is computed from the equation

$$Y_n = \text{ARCTAN}[-(r_{nz}/r_{nx})] \quad (3)$$

As is customary in computer technology, the arc tangent is computed by solution of a series:

$$\text{ARCTAN } X = X - (X^3/3) + (X^5/5) - (X^7/7) \quad (4)$$

The R-Vectors are computed according to the equation

$$\vec{r}_n = \vec{b}_n - \vec{a}_n \quad (5)$$

as shown in FIG. 8. Values of  $\vec{a}_n$  are supplied to computer 67 on a cable 70 from an A-Vector computer 71, FIG. 3, and values  $\vec{b}_n$  are supplied to computer 67 on a cable 72 from a B-Vector computer 73. Computers 71 and 73 operate recursively: each value of  $\vec{a}_n$  for example is computed on the basis of a previous value of  $\vec{a}_{n-1}$ . Thus, a new value of  $\vec{a}_{20}$  is computed using the previous value of  $\vec{a}_{19}$  and replaces the previous value of  $\vec{a}_{20}$ , then a new value of  $\vec{a}_{18}$  is computed using the previous value of  $\vec{a}_{18}$  and replaces the previous value of  $\vec{a}_{18}$ , and so on. Non-recursive computation is of course required for  $\vec{a}_1$ , and special means are provided for giving an initial set of values, as will be explained below. It is again remarked that each computation is actually made in terms of orthogonal components, and involves the steps of retrieval from storage, manipulation, and return to

storage of constant quantities, variable input quantities, and computed quantities.

By way of illustration the expansion of equation (5) as used by the computer is

$$r_{nx}\hat{i} + r_{ny}\hat{j} + r_{nz}\hat{k} = b_{nx}\hat{i} + b_{ny}\hat{j} + b_{nz}\hat{k} - a_{nx}\hat{i} - a_{ny}\hat{j} - a_{nz}\hat{k}; \quad (6)$$

this may be simplified to

$$= (b_{nx} - a_{nx})\hat{i} + (b_{ny} - a_{ny})\hat{j} + (b_{nz} - a_{nz})\hat{k} \quad (7)$$

from which the values  $r_{nx}$ ,  $r_{ny}$ , and  $r_{nz}$  are evident by inspection.

As indicated in FIG. 7, after a set of initial values  $\vec{a}_n$  has been supplied, computer 71 uses the initial value of  $\vec{a}_{19}$  to solve for  $\vec{a}_{20}$  the equation

$$\vec{a}_n = \beta \vec{a}_{n-1} + \vec{v}_a T \quad (8)$$

then uses  $\vec{a}_{18}$  to find  $\vec{a}_{19}$ , and so forth to  $\vec{a}_2$ : it then uses the equation

$$\vec{a}_1 = \beta \vec{d} + \vec{v}_a T \quad (9)$$

to find a new value of  $\vec{a}_1$ . This set of computations is performed iteratively, once every 100 milliseconds.

Similarly, as indicated in FIG. 8, after a set of initial values  $\vec{b}_{ni}$  and a related set of initial values  $\vec{b}_{ni}$  have been supplied, computer 73 uses the initial values of  $\vec{b}_{19}$  and  $\vec{b}_{19}$  to solve the equation

$$\vec{b}_n = \beta \vec{b}_{n-1} + \frac{\ln(K|\dot{\vec{b}}_{n-1}|T+1)\beta \dot{\vec{b}}_{n-1} + T^2 \vec{g}}{K|\dot{\vec{b}}_{n-1}|} \quad (10)$$

for a new  $\vec{b}_{20}$ , and to solve the equation

$$\dot{\vec{b}}_n = \frac{\beta \dot{\vec{b}}_{n-1}}{K|\dot{\vec{b}}_{n-1}|T+1} + T \vec{g} \quad (11)$$

for a new  $\dot{\vec{b}}_{20}$ , proceeding then to find  $\vec{b}_{19}$  and  $\dot{\vec{b}}_{19}$ , and so on until it finds  $\vec{b}_2$  and  $\dot{\vec{b}}_2$ . Thereafter, new values of  $\vec{b}_1$  and  $\dot{\vec{b}}_1$  are computed using the equations

$$\vec{b}_0 = \frac{\ln(K|\dot{\vec{b}}_0|T+1)\dot{\vec{b}}_0 + T^2 \vec{g}}{K|\dot{\vec{b}}_0|} \quad (12)$$

(where

$$\dot{\vec{b}}_0 = \vec{v}_{ap} + \vec{v}_m) \quad (13)$$

and

$$\dot{\vec{b}}_1 = \frac{\beta \dot{\vec{b}}_0}{K|\dot{\vec{b}}_0|T+1} + T \vec{g} \quad (14)$$

It will be evident from the above that computer 71 requires as inputs quantities identified as  $\beta$ ,  $T$ ,  $\vec{v}_a$ , and  $\vec{d}$ , as well as a set of initial values  $\vec{a}_n$ . Similarly, computer 73 requires quantities identified as  $\beta$ ,  $T$ ,  $K$ ,  $\vec{v}_{ap}$ ,  $\vec{v}_m$ , and  $\vec{g}$ , as well as a set of initial values  $\vec{b}_n$  and  $\dot{\vec{b}}_n$ . The sources of these quantities will presently be explained.

The inertial package 75 of the aircraft, or its equivalent in the model, supplies two vector signals at 76 and 77 respectively FIGS. 3 and 8, which together make up cable 23 of FIG. 1. The first signal  $\Delta \vec{V}$  is the translation of the aircraft in inertial space, and the second signal  $\Delta \theta$  is the rotation of the aircraft in inertial

space: the axes of rotation and of translation coincide and are as previously defined. Each of the vector signals actually is handled in terms of three orthogonal components: thus

$$\Delta \vec{V} = \Delta V_1 \hat{i} + \Delta V_2 \hat{j} + \Delta V_3 \hat{k} \quad (15)$$

and

$$\Delta \vec{\theta} = \Delta \theta_1 \hat{i} + \Delta \theta_2 \hat{j} + \Delta \theta_3 \hat{k} \quad (16)$$

When  $\Delta \vec{\theta}$  is supplied to a transformation matrix 78, a matrix computation is performed which involves the evaluation of the elements of a matrix  $\beta$  defined by the equation

$$\beta = \begin{bmatrix} 1 + \frac{\Delta \theta_1^2}{4} - \frac{\Delta \theta_2^2}{4} - \frac{\Delta \theta_3^2}{4} & 2 \left( \frac{\Delta \theta_1 \Delta \theta_2}{4} + \frac{\Delta \theta_3}{2} \right) & 2 \left( \frac{\Delta \theta_1 \Delta \theta_3}{4} - \frac{\Delta \theta_2}{2} \right) \\ \frac{D}{D} & \frac{D}{D} & \frac{D}{D} \\ 2 \left( \frac{\Delta \theta_1 \Delta \theta_2}{4} - \frac{\Delta \theta_3}{2} \right) & 1 - \frac{\Delta \theta_1^2}{4} + \frac{\Delta \theta_2^2}{4} - \frac{\Delta \theta_3^2}{4} & 2 \left( \frac{\Delta \theta_2 \Delta \theta_3}{4} + \frac{\Delta \theta_1}{2} \right) \\ \frac{D}{D} & \frac{D}{D} & \frac{D}{D} \\ 2 \left( \frac{\Delta \theta_1 \Delta \theta_3}{4} + \frac{\Delta \theta_2}{2} \right) & 2 \left( \frac{\Delta \theta_2 \Delta \theta_3}{4} - \frac{\Delta \theta_1}{2} \right) & 1 - \frac{\Delta \theta_1^2}{4} - \frac{\Delta \theta_2^2}{4} + \frac{\Delta \theta_3^2}{4} \\ \frac{D}{D} & \frac{D}{D} & \frac{D}{D} \end{bmatrix} \quad (17)$$

to accommodate the transformation of body frame coordinates over an iteration cycle. The matrix elements have a common denominator

$$D = 1 + (\Delta \theta_1^2 + \Delta \theta_2^2 + \Delta \theta_3^2) / 4 \quad (18)$$

It is not asserted herein that computer manipulation of matrices involves anything novel or inventive per se. The matrix quantity  $\beta$  is supplied at 80, 81, and 82, FIG. 3, to computer 71, at 80, 81, and 83 to computer 73, and at 80, 84, to a Gravity Filter 85.

The central air data computer 86 of the aircraft, or its equivalent in the model, supplies five signals. Two of these relate to the ambient air mass;  $\rho$  is the air density and  $T_e$  is the air temperature. The other three signals define the movement of the aircraft with respect to the ambient air mass;  $\alpha$  and  $\beta_s$  are the angle of attack and the side slip angle of the aircraft respectively and  $V_a$  is its air speed. Note that  $\alpha$  and  $\beta_s$  are measured about axes previously defined.

The  $V_a$ ,  $\alpha$ , and  $\beta_s$  signals from air data computer 86 are supplied to an air speed computer 87 at 88, 90, 91 respectively. Computer 87 solves the equation

$$\vec{v}_a = V_a [ \sqrt{1 - \sin^2 \alpha - \sin^2 \beta_s} \hat{i} + \sin \beta_s \hat{j} + \sin \alpha \hat{k} ] \quad (19)$$

A second necessary signal is  $\vec{V}_{ap}$  which is the value of  $\vec{v}_a$  one sample old, and is readily available in the computer memory. The quantity  $\vec{v}_a$  is supplied at 92, 93, and 94 to computer 71, at 92, 93, and 95 to filter 85, and at 92 and 96 to an Initial Condition Computer 97, which also receives at 98 and 99 a vector quantity  $\vec{v}_m$  representative of the muzzle velocity of the projectiles to be used: this quantity is also supplied to computer 73 at 98 and 100. A vector quantity  $\vec{d}$ , representing the line from the pilot's eye to the cannon muzzle, is set into computer 97 at 101 and 102, and into computer 71 at 101 and 103. The quantity  $T$  ( $= 0.1$  second) is set into computer 97 at 104 and 105, into filter 85 at 104, 106, and 107, into computer 71 at 104, 106, 110, and 111, and into computer 73 at 104, 106, 110, and 112.

The  $\rho$  signal from air data computer 86 is supplied at 113 to a drag coefficient computer 114 which also receives at 115 and 116 signals representing the surface area  $S$  and mass  $m$  of a projectile. Computer 114 also receives at 117 a signal representative of drag coefficient  $C_D$  supplied from a look-up table 120 in the com-

puter, in which it is stored as a function of Mach number  $M$ , supplied to table 120 at 121 from a Conversion Computer 122. This computer determines  $M$  for each projectile according to the equation

$$M = (0.020402 | \dot{\vec{b}}_{n-1} |) / \sqrt{T_e} \quad (20)$$

where  $T_e$  is the air temperature and  $b_{n-1}$  is the B-Vector last computer for the next earlier projectile. The expression

$$| \dot{\vec{b}}_{n-1} |$$

is to be interpreted as "the arithmetic value of the rate of change of vector  $\vec{b}_{n-1}$ ." The coefficient 0.020402 is

set into computer 122 at 123, the quantity  $T_e$  is supplied at 124 from air data computer 86, and the vector rates are obtained either from initial condition computer 97, at 125 and 126, or from B-Vector computer 73, at 127 and 126. Conductors 88, 90, 91, 113, and 124 together make up cable 22 of FIG. 1.

The output of computer 114, a drag factor  $K$  derived from the equation

$$K = \rho S C_D / 2m \quad (20a)$$

is supplied to computer 97 at 130 and 131 and to computer 73 at 130 and 132.

Initial condition computer 97 receives another input at 133 representative of the initial value of gravity,  $\vec{g}_0$ , which has the value  $\vec{g}_0 = 32.2 \hat{k}$

$$(21)$$

This computer performs a number of functions. It performs a preliminary computation of  $\dot{\vec{b}}_0$  based on equation (13), and then uses the derivative  $\dot{\vec{b}}_0$  so computed to determine initial values of  $\dot{\vec{a}}_1$ ,  $\dot{\vec{b}}_1$ , and  $\dot{\vec{b}}_1$ , according to the equations

$$\dot{\vec{a}}_1 = \vec{d} - \vec{v}_a T \quad (22)$$

$$\dot{\vec{b}}_1 = \dot{\vec{a}} + \frac{\ln \left( K | \dot{\vec{b}}_0 | T + 1 \right) \dot{\vec{b}}_0 + T^2 \vec{g}_0}{K | \dot{\vec{b}}_0 |} \quad (23)$$

and

$$\dot{\vec{b}}_1 = \frac{\dot{\vec{b}}_0}{K | \dot{\vec{b}}_0 | T + 1} + T \vec{g}_0 \quad (24)$$

The determination of rates of change of vector quantities, and their arithmetic values, is not new in computer technology, and may be performed for example according to the following equations

$$\vec{A} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k} \quad (25)$$

$$\dot{\vec{A}} = \dot{a}_1 \hat{i} + \dot{a}_2 \hat{j} + \dot{a}_3 \hat{k} \quad (26)$$

and

$$\vec{A} = [\dot{a}_1^2 + \dot{a}_2^2 + \dot{a}_3^2]^{1/2} \quad (27)$$

After determining the above quantities, computer 97 proceeds to determine the values of  $\dot{\vec{a}}_2$  to  $\dot{\vec{a}}_{20}$  and of  $\dot{\vec{b}}_2$  to  $\dot{\vec{b}}_{20}$  according to the following equations

$$\dot{\vec{a}}_n = \vec{v}_a T + \dot{\vec{a}}_{n-1} \quad (28)$$

and

$$\dot{\vec{b}}_n = \dot{\vec{b}}_{n-1} + \frac{\ln \left( K | \dot{\vec{b}}_{n-1} | T + 1 \right) \dot{\vec{b}}_{n-1}}{K | \dot{\vec{b}}_{n-1} |} \quad (29)$$

Values for the rates of change of  $\dot{\vec{b}}_2$  to  $\dot{\vec{b}}_{20}$  are also determined according to the equation

$$\dot{\vec{b}}_n = \frac{\dot{\vec{b}}_{n-1}}{K|\dot{\vec{b}}_{n-1}|T+1} + T\vec{g}_0 \quad (30)$$

The initial values  $\vec{a}_n$  are supplied at 134 to A-Vector computer 71, and the initial values  $\vec{b}_n$  and  $|\vec{b}_n|$  are supplied at 135 to B-Vector computer 73. The initial vector  $\vec{g}_0$  is supplied to gravity filter 73 at 136.

It will be appreciated that each computation of  $\vec{b}_n$  and  $\dot{\vec{b}}_n$  must be preceded by recomputation of  $K$  except for  $\vec{b}_1$  and  $\dot{\vec{b}}_1$ .

Air speed computer 97 supplies the quantity  $\vec{v}_{ap}$  to computer 73 at 137 and 140, and to filter 85 at 137 and 141.

In addition to the inputs described above, filter 85 also receives  $\Delta \vec{V}$  inputs at 76 from inertial sensor package 75. It computes the actual vector of gravity,  $\vec{g}$ , according to the equation

$$\vec{g} = \beta \left[ (1-L)\vec{g}_0 - \frac{L\vec{v}_{ap}}{T} \right] + L \left[ \frac{\vec{v}_a - \Delta \vec{V}}{T} \right] \quad (31)$$

where  $L$  is the filter gain and  $\vec{g}_0$  is the previous value of  $\vec{g}$ , starting with  $\vec{g}_0$ . This computation is thus also recursive. The quantity  $\vec{g}$  is supplied by filter 85 to computer 73 at 142.

FIG. 9 gives more details of range bar computer 47, which performs a triplicate function, once each for standard ranges  $R_A = 1,000$  feet,  $R_B = 2,000$  feet, and  $R_C = 3,000$  feet: other values for standard ranges may be obtained by adjustments 143, 144, and 145. The function will be described once for the range  $R_C = 3,000$  feet. Values of  $\vec{r}_n$  are supplied to computer 47 at 52, and the computer first determines a value  $\vec{r}_m$  of  $\vec{r}_n$  such that for example

$$|\vec{r}_{m-1}| \leq R_C \leq |\vec{r}_m| \quad (32)$$

See FIG. 11, where a range of 3,000 feet from the aircraft is shown to lie on an arc 148 (actually a sphere) passing between  $n_{18}$  and  $n_{19}$ : here  $\vec{r}_m$  terminates at point 19 and  $\vec{r}_{m-1}$  terminates at point 18. What is now required is to determine the point where arc 148 intersects the locus 18-19, and for this purpose the locus is assumed to be rectilinear. It will be apparent that coordinates  $X_m$  and  $Y_m$  are associated with vector  $\vec{r}_m$ , and the coordinates  $X_{RC}$  and  $Y_{RC}$  of the head 146 of an interpolation vector  $\vec{r}_c$  having a length  $R_C$  are determined from  $X_m$  and  $Y_m$  respectively as follows.

The interpolation constant  $R_{INT}$  is first computed according to the equation

$$-R_{INT} = \frac{R_C - |\vec{r}_{m-1}|}{|\vec{r}_m| - |\vec{r}_{m-1}|} \quad (33)$$

The desired coordinates are then computed according to equations

$$X_{RC} = X_{m-1} + (X_m - X_{m-1}) R_{INT} \quad (34)$$

and

$$Y_{RC} = Y_{m-1} + (Y_m - Y_{m-1}) R_{INT} \quad (35)$$

and stored at 28 through cable 46. The process is repeated for  $R_B = 2,000$  feet and  $R_A = 1,000$  feet. These coefficients define the centers of range bars 40, 41, and 42, FIG. 2.

FIG. 10 gives more details of the range pipper computer 51. This computer may be provided with a size adjustment 149, and operates like computer 47 except that a single computation is necessary, based on the input  $R_p$  representative of radar range, and the coordinates  $X_{RP}$ ,  $Y_{RP}$  which are supplied at 50 for storage at 28 represent the center of range pipper 43, FIG. 2.

The purpose of boresight selector 45 will now be considered. Reference was earlier made to the "iron sight," a sight fixed to the aircraft which the pilot attempts to bring to bear on his target. In the use of such a sight it is obvious that the pilot must not only lead his target in azimuth, to allow for the time of flight of the projectile, but must also allow for the departure of the projectile, under the force of gravity, from endless flight along the cannon boresight. The boresight cross of the present display corresponds to an "iron sight." It is coordinated with the "harmonization point," a point on the real or modeled boresight of the cannon, if properly aligned, 2,250 feet from the muzzle. A target in line with the boresight cross is on the pilot's line of sight passing through the harmonization point. The apparatus is arranged so that even if there is failure of stream computer 24 or its adjuncts, selector 45 continuously provides the coordinates of the boresight sighting so that the pilot is not completely deprived of sighting aids. The location, on the face of the cathode ray tube, of the center of this cross can be adjusted, in orthogonal directions aligned with the X- and Y-coordinates, by "location set" adjustments 147 and 150, and the lengths of its cross arms may be varied if desired by a further adjustment 151.

The operation of an iterative computing system like the present one is such that a series of computations are performed, some change in the input data is made, if necessary, and the series of computations is repeated. Reference should now be made to FIG. 13, which is a time diagram illustrative of this facet of the invention. From this diagram it will be seen that new values of  $R_p$ ,  $\rho$ ,  $T_e$ , and  $V_a$  are supplied to the computer every 100 milliseconds, while new values of  $\alpha$ ,  $\beta$ ,  $\Delta \vec{V}$ , and  $\Delta \theta$  are provided every 20 milliseconds, if these values change. The computer determines the coordinates for 20 projectiles and those for the range bars and radar pipper, which requires less than 100 milliseconds, waits until the end of 100 milliseconds, when new input values, if any, are supplied, and repeats the computations. As newly computed values for the various coordinates are stored they replace previous values, so that no coordinate in the computer is based on data older than 100 milliseconds.

The function of display unit 26 per se is simply one of nondestructively reading out the coordinates of one point after another, in a predetermined sequence, and deflecting the beam of the CRT to the locations of these points successively, blanking the beam where it is not desired that a visible trace appear. This can be ac-

complied by the same general purpose computer on a time sharing basis, in a conventional fashion, as follows. Although each individual computing step takes only a few microseconds, many such steps have to be performed in sequence in each 100 millisecond interval, and the computer is normally engaged in these computations until near the end of the 100 millisecond period. However, every 0.5 millisecond an interrupt pulse is supplied: the computer completes whatever computing step it was engaged in, and "remembers" where it was, then retrieves from storage the next coordinates and brightness data according to a preset program and supplies them to the display, and then returns to computation, "remembering" also where in the display program it was. If, at the instant when read-out of the coordinates of a particular point to the display is called for, the new values of these coordinates have not yet been computed, the previous values will be used again. This continues until all 29 points have been retrieved, pauses, and then repeats, every 20 milliseconds.

Referring once more to FIG. 13 the lower row of time markings makes it clear that in each iteration period the display receives a single sequence of values for the points to be displayed, after which no further display action occurs until the next iteration period. This is best illustrated in FIG. 12, which shows the manner in which a single trace on the cathode ray tube screen is performed: as mentioned above, we choose to display only the even numbered projectiles, to reduce required computer capacity and display speed requirements.

Initially the beam is deflected to a starting point O and is blanked. After  $0.5\mu s$  the beam is deflected to point 1 and intensified. It is deflected in sequence to points 2, 3, 4, 5, 6, and 7 and then blanked while it is deflected to point 8, and so on. The points are followed in numerical order, deflections during which the beam is blanked being indicated by dashed lines. Note that from point 17 the beam is not deflected to  $X_{RA}$ ,  $Y_{RA}$ , but rather to point 18: the length of each range bar having been previously set, the computer automatically subtracts and adds one-half of this length to  $X_{RA}$  and thus arrives at points 18 and 19 directly. The same principles apply to the other range bars and to the radar piper as well, and could also be applied to the boresight cross. However, for additional security the data for points 1-7 may be stored in a separate memory so that even if the stream computer, and so forth, should fail the cross at least will be kept available. To this end output 44 from selector 45 is shown in FIG. 1 as being supplied directly to display 26, rather than to storage device 28.

The persistence of the oscilloscope screen is long enough that little loss of visual intensity occurs in the 14.5 milliseconds required to trace the pattern of 29 points (including zero) but is also short enough that no trouble is encountered from blurring of the visual image due to change in coordinates of a projectile between one sweep of the pattern and the next, or due to the fact that point 14 for example may be a newly computed point while point 15 is based on an "old" computation.

In practice, particularly for study work and pilot training, the same general purpose computer which determines the various coordinates may also be used to

model the aircraft-and-autopilot and the target-and-radar. By this expedient all information useable by the investigator or student pilot is on the oscilloscope screen and no combining glass is necessary.

The invention here presented, however, pertains to the production of the display representing the projectiles, with the range bars and so forth, as distinguished from apparatus combining the display with a visual target.

A tabulation of the symbols used in this application follows:

#### SYMBOLS USED

- $\rho$  - air density
- $T_e$  - static air temperature
- $V_a$  - true air speed
- $\alpha$  - attack angle
- $\beta_s$  - sideslip angle
- $S$  - missile surface area
- $m$  - missile mass
- $\vec{v}_m$  - missile muzzle velocity
- $\vec{\phantom{a}}$  (superscript) vector quantity
- $\vec{d}$  - vector from pilot's eye location to cannon muzzle
- $\vec{g}$  - gravity vector
- $\vec{a}$  - vector from firing location of vehicle to present location of vehicle
- $\vec{b}$  - vector from firing location of vehicle to present location of missile
- $\vec{r}$  - vector from present location of vehicle to present location of missile
- Numerical subscript indicates number of missile:
  - number 1 is most recently fired
  - subscript  $o$  - preliminary value
  - subscript  $i$  - initial value
  - subscript  $p$  - previous value
- $\hat{i}$  - unit vector along longitudinal axis of vehicle
- $\hat{j}$  - unit vector along transverse axis of vehicle
- $\hat{k}$  - unit vector along axis perpendicular to  $\hat{i}$  and  $\hat{j}$
- $\Delta\vec{\theta}$  - change in attitude vector of vehicle  $=\Delta\theta_1\hat{i} + \Delta\theta_2\hat{j} + \Delta\theta_3\hat{k}$
- $\Delta\vec{V}$  - change in velocity vector of vehicle  $=\Delta V_1\hat{i} + \Delta V_2\hat{j} + \Delta V_3\hat{k}$
- $T$  - interval between missile firings = 0.1 second
- $L$  gain of gravity filter
- $\dot{\phantom{a}}$  (superscript) rate of change of vector quantity
- $|\dot{\phantom{a}}|$  (superscript) arithmetical value of rate of change of vector quantity
- $32.2\hat{k}$  - gravitational constant
- $M$  - Mach number
- $C_D$  - drag coefficient
- $K$  - drag factor
- $\vec{v}_a$  - airspeed vector

$$= V_a \left[ \sqrt{(1 - \sin^2\alpha - \sin^2\beta_s) \hat{i} + \sin\beta_s \hat{j} + \sin\alpha \hat{k}} \right]$$

- $X$  - abscissa on display
- $Y$  - ordinate on display
- $\beta$  - transformation matrix
- $D$  - matrix divisor  $(4 + \Delta\theta_1^2 + \Delta\theta_2^2 + \Delta\theta_3^2) / 4$ .

Numerous objects and advantages of our invention have been set forth in the foregoing description, together with details of the structure and function of the invention, and the novel features thereof are pointed out in the appended claims. The disclosure,



however, is illustrative only, and changes may be made in detail, especially in matters of shape, size, and arrangement of parts, within the principle of the invention, to the full extent indicated by the broad general meaning of the terms in which the appended claims are expressed.

We claim as our invention:

1. In combination:

means programmed for iteratively computing and displaying the locus of the present apparent positions of a plurality of projectiles, if discharged sequentially from a moving vehicle at known intervals beginning at a predetermined time in the past, as seen from a predetermined point in the vehicle; and

means repeatedly modifying the display, at predetermined intervals, in accordance with changes in parameters of the movement of the vehicle.

2. Apparatus according to claim 1 together with means for impressing on the display at least one further indicium definitive of a point therealong at a predetermined range from the vehicle.

3. Apparatus according to claim 1, in which the first named means comprises a general purpose computer to:

1. compute first vectors extending severally from the positions of the aircraft when the projectiles were fired to present position of the aircraft;
2. compute second vectors extending severally from the positions of the aircraft when the projectiles were fired to the present positions of the projectiles;
3. compute from the above vectors third vectors extending severally from the present positions of the aircraft to the present positions of the projectiles;
4. compute from the third vectors the coordinates of the heads thereof in said coordinate system; and means for displaying a line connecting the points defined by the coordinates so computed.

4. A computer according to claim 3 to compute the further coordinates in said Cartesian system of the point defining the direction of the head of an interpolation vector extending from the present position of the aircraft for a known distance and terminating on said locus.

5. Apparatus according to claim 4 together with means for adding to the display an indicium characterized in accordance with said known distance and located in accordance with said further coordinates.

6. The method of computing the coordinates, in a Cartesian system aligned with a selected axis of an aircraft, of the apparent positions of a plurality of projectiles if discharged from a cannon, fixed to the frame of said aircraft and aligned with said axis, at known intervals beginning at a predetermined time in the past, which comprises the steps of:

1. generating first signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of the aircraft;
2. generating second signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of each projectile;

3. utilizing a computer to derive from said signals third signals representative of vectors extending severally from the present position of the aircraft to the present positions of the projectiles; and

4. utilizing said computer to compute from the third signals the coordinates of the projections of the heads of said third vectors in said Cartesian system.

4. utilizing said computer to compute from the third signals the coordinates of the projections of the heads of said third vectors in said Cartesian system.

7. Apparatus according to claim 6 in which the first vectors are determined by vector quantities representative of the rotation of the craft in inertial space,  $\Delta\theta$ , and the displacement of the cannon muzzle from the observer's viewing point,  $\vec{d}$ , and by scalar quantities representative of the interval between projectile firings,  $T$ , the craft speed relative to the ambient air,  $V_a$ , and the side slip angle  $\beta_s$  and the attack angle  $\alpha$  of the craft.

8. Apparatus according to claim 6 in which the second vectors are determined by vector quantities representative of the rotation,  $\Delta\theta$ , and the translation,  $\Delta\vec{V}$ , of the craft in inertial space, the muzzle velocity  $\vec{v}_m$  of the projectiles, and the gravity force  $\vec{g}$ , and by scalar quantities representative of the mass  $m$  and surface area  $S$  of the projectile, the density  $\rho$  and the temperature  $T_e$ , of the ambient air, the interval between firings  $T$ , the craft speed relative to the ambient air,  $V_a$ , and the side slip angle  $\beta_s$  and attack angle  $\alpha$  of the craft.

9. In combination:

computer means programmed for iteratively determining the present apparent positions of a plurality of projectiles, if discharged sequentially from a moving vehicle at known intervals beginning at a predetermined time in the past, as seen from a particular point in the vehicle;

means modifying the operation of the first named means at predetermined intervals in accordance with changes in parameters of the movement of the vehicle; and

means further modifying the operation of the first named means in accordance with changes in parameters of the movements of the projectiles.

10. The method of displaying data, represented by a sequence of vectors radiating from a common origin in directions having a major component aligned with a particular direction, as seen from a point on the side of said origin opposite to said general direction, which comprises, in combination:

programming a computer to resolve each of said vectors into three orthogonal components one of which is aligned with said general direction;

programming said computer to compute the pair of angles subtended at said point by the remaining components of each vector so that each pair of angles may define a display point on a set of Cartesian coordinates representationally aligned with said remaining component; and

displaying a line joining said display points in the order of said sequence.

11. The method of computing the coordinates, in a Cartesian system aligned with a selected axis of an aircraft, of the apparent positions of a plurality of projectiles if discharged from a cannon, fixed to the frame of

the aircraft and aligned with said axis, at known intervals beginning at a predetermined time in the past, which comprises the steps of:

1. generating first signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of the aircraft;
  2. generating second signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of each projectile;
  3. utilizing a computer to derive from said signals third signals representative of vectors extending severally from the present position of the aircraft to the present positions of the projectiles;
  4. utilizing said computer to compute from the third signals the coordinates of the projections of the heads of said third vectors in said Cartesian system; and
  5. displaying a line connecting the points defined by the coordinates so computed.
12. In a computer for repeatedly giving sets of signals representative of families of hypothetical vectors extending from successive points along the path of a moving vehicle, at which successive missiles are fired therefrom, to the locations of the missiles at predetermined intervals thereafter, in combination:
- means for receiving signals representative of characteristics of the motion of the vehicle and of the motion of the missiles;
  - means for computing from said signals one vector of an initial family of said vectors;
  - means for recursively computing, from said signals and said one vector, successive vectors of said family;
  - means for computing, from said signals and said vectors, further vectors making up further families of said vectors; and
  - means giving output signals representative severally of said further vectors.
13. In a computer for repeatedly giving sets of signals representative of families of hypothetical vectors extending from successive points along the path of a moving vehicle, at which successive missiles are fired therefrom, to the present location of the vehicle along the path, in combination:
- means for receiving signals representative of characteristics of the motion of the vehicle and of the motion of the missiles;
  - means for computing from said signals one vector of an initial family of said vectors;
  - means for recursively computing, from said signals and said one vector, successive vectors of said family;
  - means for recursively computing, from said signals and said vectors, further vectors making up further families of said vectors and
  - means giving output signals representative severally of said further vectors.
14. Display apparatus comprising means programmed to compute a family of vectors extending from successive points along the path of a moving vehicle, at which points successive missiles are fired therefrom at predetermined intervals in the past, to the

present locations of the missiles;  
 means for computing a second family of vectors extending from said successive points to the present location of the craft along the path thereof;  
 means for computing from said families of vectors the further family of vectors extending from said present location of the craft to said present locations of said missiles;  
 and means displaying the projections of the heads of said further family of vectors on a plane.

15. In a system for receiving signals representing the movement of a moving vehicle and for using a computer to operate on said signals to produce a display showing the coordinates, in a Cartesian system aligned with a selected axis of an aircraft, of the apparent positions of a plurality of projectiles if discharged from a cannon, fixed to the frame of said aircraft and aligned with said axis, at known intervals beginning at a predetermined time in the past, the method which comprises the steps of:

1. generating first signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of the aircraft;
  2. generating second signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of each projectile;
  3. utilizing a computer to derive from said signals third signals representative of vectors extending severally from the present position of the aircraft to the present positions of the projectiles; and
  4. utilizing said computer to compute from the third signals the coordinates of the projections of the heads of said third vectors in said Cartesian system.
16. A data processing method of automatically producing a visual display of the coordinates, in a Cartesian system aligned with a selected axis of an aircraft, of the apparent positions of a plurality of projectiles if discharged from a cannon, fixed to the frame of the aircraft and aligned with said axis, at known intervals beginning at a predetermined time in the past, said method being performed with a digital computer and comprising the steps of:
1. generating first signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of the aircraft;
  2. generating second signals representative of vectors extending severally from the position of the aircraft when each projectile was fired to the present position of each projectile;
  3. utilizing a computer to derive from said signals third signals representative of vectors extending severally from the present position of the aircraft to the present positions of the projectiles;
  4. utilizing said computer to compute from the third signals the coordinates of the projections of the heads of said third vectors in said Cartesian system; and
  5. displaying a line connecting the points defined by the coordinates so computed.

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