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(54) Title: TRANSPORTATION MANAGEMENT SYSTEM AND METHOD FOR SHIPMENT PLANNING OPTIMIZATION
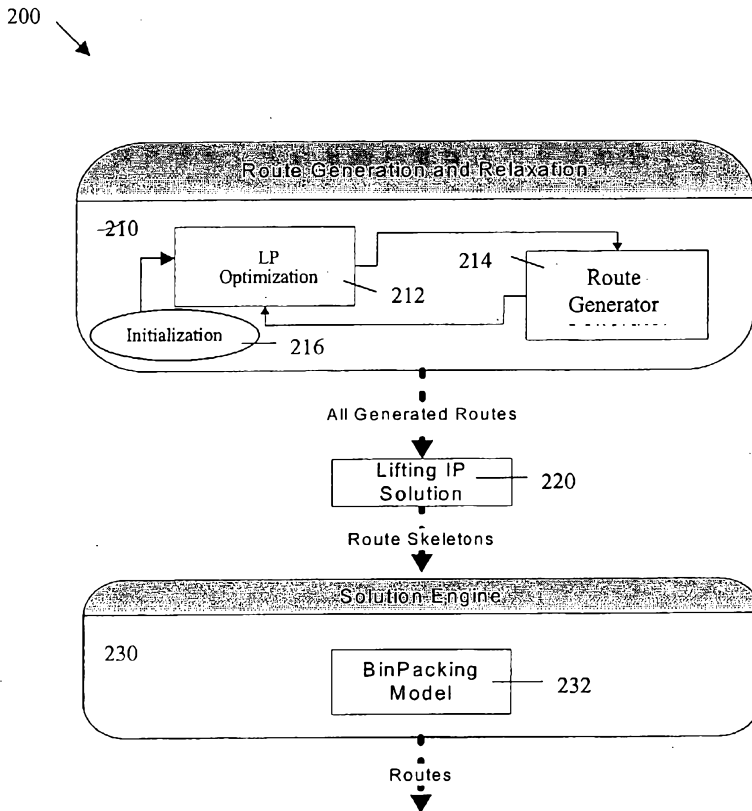
(57) Abstract: System and method for planning transportation shipments for delivery and pickup of goods. The system may plan shipments based on such factors as requested goods to be picked up and delivered, while minimizing the cost of the shipments planned. Constraints can be placed on the transportation resources and the goods to be moved that will restrict the possible shipments considered by the planning method. The method may be capable of considering all possible locations through which goods can be moved by shipments. The method may also be capable of quickly solving problems with a large number of potentially varying goods to be transported.

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TRANSPORTATION MANAGEMENT SYSTEM AND METHOD FOR SHIPMENT PLANNING OPTIMIZATION

Field

The invention relates to a computer-implemented method for shipment planning

5 optimization and a computer system for generating planned routes.

Background

Transportation Management Systems (TMS) have been addressing the problem of shipment planning optimization in one form or another for years. Numerous algorithms and approaches to this class of problems have been proposed. However, each of these

10 approaches suffers certain drawbacks. One notable drawback has been that many systems and methods, due to inherent complexities and other factors, have been unable to consider all desired variables in determining a solution. The present invention seeks to address certain of these and other shortcomings of known solutions or at least provide a useful alternative.

15 ## Summary

In accordance with the present invention there is provided a computer-implemented method, for shipment planning optimization, comprising:

accessing a master optimization program to which, in an initialization step, a set of initial routes is provided, each initial route starting at a pickup location and stopping at a

20 delivery location, said set of initial routes being represented by a set of initial variables in the master optimization program;

using a computer to generate another set of routes through an iteration of the master optimization program including said set of initial variables, said another set of routes including at least one new route based on the pickup and delivery locations of one of the

25 initial routes and on an additional pickup/dropoff location, wherein said another set of routes is represented by another set of variables;

using a computer to select or reject at least one of the another set of routes based on a selection criterion;

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iteratively using a computer, until an end condition is met, to generate further sets of routes by iterations of the master optimization program, and to select or reject at least one of the further routes in each iteration based on said selection criterion;

using a computer to add lifting inequalities to the master optimization program to
5　discard one or more of the selected routes based on selected constraints;

using a computer to create route skeletons, each route skeleton including a sequence of stops with no orders assigned, based on the routes generated by the master optimization program following the adding of the lifting inequalities; and

using a computer to assign individual orders to the route skeletons based on
10　optimization of an objective function to develop a shipment planning solution.

The present invention also provides a computer system, for generating planned routes, comprising: means for performing the above method.

Exemplary embodiments of the invention use a route generation algorithm to solve large-scale consolidation and routing problems. The transportation network optimized may be
15　formed by pickup locations, consolidation centers ("center-points") and delivery locations, among others. Typically, a route starts at a pickup location, loads some or all orders at this location and, if the route is multi-stop, may continue to one or more additional pickup/dropoff locations. The final stop may be, for example, a consolidation center or delivery location. Multiple deliveries to delivery locations are allowed in some routes if
20　desired and/or determined to be optimal/preferred. Figure 1 provides an overview illustration 100 of possible routes from origins (O), potentially through center-points (CP), to destinations (D).

Shipment plans generated by embodiments of the invention may be used to dispatch transportation resources, e.g., common carriers, private fleets, etc. The shipments may
25　provide information and directions for designated transportation resources to perform the physical transportation of the orders - i.e. the execution of the shipment - among other goals. Such planning may be useful at various levels of a supply chain, between trading partners, or other possible entities. For example, a supplier may utilize various aspects of embodiments of the invention to schedule delivery of goods from a manufacturer and/or

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delivery of goods to a retailer, etc. The invention may relate to shipments with respect to a single location, or may be used with respect to a vast network of locations spread across a wide area, depending on a particular implementation.

Decisions to be made may include, but are not limited to:

5   • Which pickup locations to visit in a route and/or in what order

• Which location (e.g., destination or center-point) is the final stop

• Which orders are assigned to which routes

• Which truck types to assign to which routes

• What driver types to assign to which routes

10   • How many routes to send to a center-point

• The timing - stops, rests, waiting - of each event on a route

• When an order should be routed by itself

• When an order should be routed together with other orders

• Others as desired, depending on a particular problem to be solved

15   In embodiments of the invention, an effective global (e.g., in the optimization sense) consolidation system is provided that is able to consider some or all of these variables and/or others simultaneously, seeking to optimize a global metric, often total cost or time, or other variables. Various known methods have been proposed that include dividing such a process into sequential stages (e.g., assigning consolidation centers to orders and then

20   performing the routing), often obscuring important consolidation opportunities that might otherwise lower a cost or other relevant variable associated with a solution. Thus, embodiments of the present invention seek to provide an improved system and method, the details of which are provided herein.

Embodiments of the present invention provide a computer system, for generating planned

25   routes, comprising:

means for providing initialization information to a master optimization program, wherein said information comprises a set of routes represented by a set of variables;

means for generating further routes through an iteration of the master optimization program including said set of variables, wherein said further routes are represented by a

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further set of variables and said further set of variables is added to the master optimization program as relaxed parameters;

 means, functioning in combination with the means for generating further routes, for selecting a plurality of optimal route skeletons based on lifting inequalities; and

5   means for determining an assignment of individual orders to the route skeletons based on optimisation of an objective function.

Brief Description of the Drawings

 Preferred embodiments of the present invention are hereinafter further described, by way of non-limiting example only, with reference to the accompanying drawings, in

10 which:

 Figure 1 illustrates an overview of possible routes considered in an embodiment of the present invention;

 Figure 2 is a flow diagram illustrating a solution approach in accordance with an embodiment of the present invention;

15 **Figure 3** is a flow diagram illustrating an embodiment of a method for route generation in accordance with an embodiment of the present invention;

 Figure 4 is a flow diagram illustrating an embodiment of a method for route generation in accordance with an embodiment of the present invention; and

 Figure 5 illustrates an overview of possible routes considered in an embodiment of

20 the present invention.

Detailed Description of Embodiments of the Invention

In one embodiment, a global approach is used in solving a shipment-planning problem (also known as consolidation, or route planning problem) based on route generation techniques. This class of problem is widely recognized, and is

sometimes called the Vehicle Routing Problem (VRP), which is itself a variation on the

Traveling Sales-Person problem (TSP). Specifically, these problems relate to methods for

solving problems such as designing transportation routes for vehicles, using such variables

as vehicle capacities, required delivery pick-up and/or delivery locations, etc. The routes

may be solved with an aim to achieve such goals as minimizing the total cost of the

transportation involved in moving the orders, minimizing overall delivery time, or a

combination of these or other desired outcomes.

Large optimization problems involving large numbers of variables (in some cases, a

million or more) may require specialized solution techniques. In particular, problems in

which those variables represent combinations of decisions (e.g., combinations of truck,

driver, center-points, etc.) may be especially large, often ranging in the billions of variables

or more. Generation methods may be used to trim those variables, such as by looking only

for relevant combinations (in one embodiment, routes) that are more likely to lead to some

improvement in the solution quality. In one embodiment, the generation method of the

invention works by adding new stops to promising routes at each iteration of the process.

The process may end when, for example, no more promising routes are generated, when a

maximum number of stops per route is achieved, etc.

In addition to route generation, the invention may generate priorities to rely more

heavily on certain factors and/or disregard others, to help speed up the solution of the

optimization model. In one embodiment, a master optimization engine of the invention is

an integer-programming (IP) model. After a generation phase is complete, a set of "lifting

inequalities" may be added. This set of lifting inequalities, referred to herein as "cuts," uses

generally recognized methods to discard one or more non-optimal solutions to the problem,

often large groups of solutions at a time, potentially greatly expediting the solution process.

Due to such factors as restrictions in the physical memory, speed and/or processing

power (among other qualities) of many computers, the method of the invention may be

practiced in phases. In one embodiment, three phases are used. For illustrative purposes,

Figure 2 provides an overview of a three-phase solution approach 200 in accordance with

an embodiment of the invention.

Figure 2 shows a solution approach **200** having a route generation and relaxation

phase **210**, a lifting solution phase **220** and a bin-packing phase **230**. The route generation

and relaxation phase **210** includes an LP Optimization portion **212** and a route generator

214. An Initialization portion **216** may provide any needed initialization information. The

lifting solution phase **220** may implement a lifting integer programming (IP) solution. Bin-

packing phase **230**, as shown, may include a bin-packing model **232**. Additional details are

provided herein.

In one embodiment, such aspects of the invention are implemented purely in

software or similar modules, and may be supported on any of a variety of devices, such as

on a mainframe or by a stand-alone or networked processor, etc. For example, in a three-

phase implementation discussed below, individual phases may be implemented as discrete

software modules, embodied in a computer-readable medium. Databases or other record

structures may be variously incorporated as well. The items of data considered by an

embodiment of the invention may be generated and/or received locally, or may be

transmitted over vast distances, such as over a communication network, e.g., the Internet or

others.

Additional detail is provided below through a discussion of embodiments of the

invention, including exemplary constraints, assumptions, calculations, etc. For example, in

one such embodiment, a solution is implemented as follows:

- **Phase 1: Generation and relaxation.** In this phase, the method creates new routes

    using a relaxed version of a master linear program (described below), i.e., certain

    requirements may be relaxed or eliminated. Aggregated quantities derived from the

    orders for Origin-Destination (O-D) pairs may be used instead of individual orders.

    The invention may generate and optimize routes that finish at center-points. The

    dual prices obtained by solving the problem defined herein as a linear programming

(LP) solution may be used in the generation of direct routes to delivery locations. Routes with multiple deliveries to delivery locations may then be generated in this stage and added to the master linear program.

- **Phase 2: Lifted solution.** After the route generation routine is finished, lifting inequalities may be added to the master linear program, such as to strengthen the relaxation. If time and problem size allow, this stage can be solved as an integer programming model using well-established techniques. An integer programming model is a combinatorial problem that determines optimal values (where the values for the variables are often required to be integers) for multiple variables to maximize an objective function (such as cost) while meeting multiple constraints on those variables. Exemplary constraints are provided below. There are many tools and packages available that can be used to solve a general IP problem. The solution of this stage may be saved as route skeletons. In one embodiment, these route skeletons do not yet have specific orders assigned to them, only a sequence of stops.

- **Phase 3: Bin-Pack solution.** As will be appreciated by one skilled in the art, when a problem is formulated as a bin pack problem, the problem generally becomes one of determining how to put the most objects in the least number of fixed space "bins". More formally, it may be desirable to find a partition assignment of a set of objects such that a constraint is satisfied or an objective function is minimized (or maximized). There are many variants, such as, 3D, 2D, linear, pack by volume, pack by weight, minimize volume, maximize value, fixed shape objects, etc. In one embodiment, the "bins" are the route skeletons. These route skeletons may be used in a fully detailed model that routes some or all individual orders using their individual characteristics, time windows, travel time, etc.

In various embodiments of the invention, several advantages may be realized as compared with certain known solutions. For example, the present invention may:

- Be capable of quickly solving problems of large sizes, e.g., problems with tens of thousands of orders or more, without having to split, or "decompose" the problem into independent, smaller problems. Certain known solution attempts have taken such a decomposition approach to solving large problem sizes, and this has been found to sacrifice solution quality under some circumstances. For example, after such decomposition, the best possible solution has very often been found to be worse than the overall best solution.

- Be capable of considering many or all possible locations as options through which an order could be moved on its way from origin to destination (e.g. cross dock locations, pool point locations, etc.). Certain known solutions artificially restrict these possible "center-points" for each order as a way of reducing problem complexity. This simplification, however, has been found to sacrifice solution quality in a manner that the present invention seeks to avoid.

- Offer explicit optimization using lane-based rates (e.g., rates that differ based on such factors as the origin and destination of the route to be taken) for route generation. This has been found to improve solution quality versus many known solutions, such as those that make an assumption that the rates to be used are the same regardless of where the routes start and end. In many real-world situations, this is not a correct assumption, and may lead to degraded solution quality.

- Offer a unique heuristic method for route generation. This heuristic method seeks to enable faster and better quality solutions over such known solutions as a route generation approach.

- Offer an innovative solution formulation where route generation need not be dependent on explicit orders and/or the optimization need not depend on set covering. That is, in one embodiment for example, each order in a problem formulation need not be put on exactly one route. This approach seeks to increase a number of potential solution alternatives that can be examined and/or increase a

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speed in which they can be examined, versus such known methods as the set

covering approach. The Bin-Pack solution is described in greater detail below.

- Provide lifting constraints in such a formulation as to obtain near-optimal or optimal

  solutions (examples provided below). The lifting constraint approach generally is

  an established method for speeding solution of IP problems by adding additional

  variables to the solution having the effect of simplifying the structure of the

  problem.

- Provide lifting constraints during route generation (examples provided below).

Additional exemplary details of various embodiments of the present invention will

now be provided. In that regard, various variables and parameters associated with

following description are as follows:

z_r is the fractional number of skeletons needed to cover this route. Solution Output.

$x_{od,r}$ is the fraction of orders from pickup-destination pair od assigned to route r. Solution

Output.

φ_{od} is the fraction of orders from pickup-destination pair od using the base-line mode. Solution

Output.

p_{od} is the aggregate base-line cost for pickup-destination pair od. Preprocessed input.

$h_{od,cp}$ is the handling cost of pickup-destination pair od assigned to center-point cp. Problem

input.

c_r is the cost of route r. Calculated during route generation.

V_{od} is the total volume of the pickup-destination pair. Preprocessed input.

W_{od} is the total weight of the pickup-destination pair. Preprocessed input.

TV is the representative volume capacity. Problem input.

TW is the representative weight capacity. Problem input.

$\delta_{cp,r}$ is one if route r serves center-point cp, 0 otherwise. Calculated during route generation.

C_{cp} is the loads capacity at the cp. Preprocessed input.

$\beta_{d,r}$ is one if route r uses driver type d, 0 otherwise. Calculated during route generation.

ND_d is the number of drivers of type d available. Problem input.

$\gamma_{t,r}$ is one if route r uses truck t, 0 otherwise. Calculated during route generation.

NT_t is the number of trucks of type t available. Problem input.

ω_{od} is the size factor of the origin-destination pair in on route r. Preprocessed input.

$$\omega_{od} = \max\left[\frac{V_{od}}{\min_{o \in O(od)} V_o}, \frac{W_{od}}{\min_{o \in O(od)} W_o}\right]$$

ε_r is a proportional factor for route r, about .01. Problem parameter.

In one embodiment, the first and second phases of the invention solution process are strategic while the third phase is tactical. For example, the first two modules may merely create routes, and need not consider the specific orders to be assigned. Specifically, they may explore the universe of feasible routes that cover the aggregate demand of the planning problem while disregarding the issues of individual orders meeting their individual target pick up and delivery times. In such an embodiment, the Bin-Pack phase of the present

invention may then be utilized to address tactical issues, including the time dimension that may be ignored by the strategic part of the solution engine, among others.

As noted above, in one aspect the present invention may seek to simplify certain factors considered in generating a transportation plan. As an example, certain values may be considered in the aggregate, rather than discretely. In one embodiment, such aggregation may be applied to one or more of: volume and/or weight of orders, center-point capacity, baseline cost, among others. Specifically, in an implementation utilizing three phases as described above, beneficial results have been observed upon applying such aggregations to at least phases 1 and 2. Examples of such aggregations are now provided. Of course, numerous variations on such will be readily apparent to one skilled in the art upon consideration of the present disclosure.

Phase one and two order aggregation

Volume and weight may be aggregated by origin-destination pair for some or all orders in the consolidation run, as follows:

$$V_{od} = \sum_{o \in O(od)} V_o \quad \forall od$$

$$W_{od} = \sum_{o \in O(od)} W_o \quad \forall od$$

Phase one and two center-point capacity aggregation

The Center-Point capacities used in the strategic part of the model may be, or may be based on, the aggregated capacities of individual periods. The center-point capacity is a limit that may be set on the volume of orders that can be sent through a particular center-point.

$$C_{cp} = \sum_{p} C_{p,cp} \quad \forall cp$$

Phase one and two baseline cost aggregation

An aggregated value of the base-line cost may be used to help bound the dual prices in the linear program. The baseline cost represents the cost of moving an order by itself. The solution seeks to move the order more cheaply by consolidating the order onto routes with other orders.

$$p_{od} = \sum_{o \in O(od)} p_o \qquad \forall od$$

In accordance with the present invention, routes may be generated in an iterative way. In a three-phase embodiment, a system of the invention may start in phase 1 with an initial set of one-stop routes to various center-points, if any, and may add new stops to some of these routes as desired, such as at every new iteration. The new routes are then represented in the master linear program by adding new variables and new constraints. The process can continue until some predefined maximum number of iterations has been reached, until no new routes are found, or until another predetermined condition has been achieved.

LP Modeling - Generation LP. In a three-phase embodiment, as described above, the first phase may include an LP model. This LP model defines the transportation problem that is to be solved in mathematical terms. This formulation in intended to ensure that the solutions obtained during each iteration are feasible solutions in that they take into account all the necessary business rules. These business rules may be described by any of a variety of constraints. Exemplary constraints and other features, any or all of which may be used in any particular implementation, among others, are described below. Throughout this disclosure, parenthetical notations may be included with the exemplary features as a source of additional information.

Objective function

Depending on a particular embodiment or implementation, an objective or objectives of this phase of the invention may be to minimize the cost of routing the aggregated order volume, the sum of the cost when routing orders by themselves, the cost when handling orders at each center point, and/or the cost when routing orders together, among other possibilities.

$$\min \sum_{od} p_{od} \varphi_{od} + \sum_{od} \sum_{r} h_{od,r} x_{od,r} + \sum_{r} c_r z_r$$

Volume and Weight constraints

These represent limits that may be applied to ensure that the total volume and weight of a shipment do not exceed a maximum capacity.

$$\sum_{od \in OD(r)} V_{od} \, x_{od,r} \leq TV \, z_r \qquad \forall r$$

$$\sum_{od \in OD(r)} W_{od} \, x_{od,r} \leq TW \, z_r \qquad \forall r$$

Center-point capacity constraints

These represent limits that may be applied to ensure that the number of routes to a center-point does not exceed the capacity of the center-point.

$$\sum_{r} \delta_{cp,r} \, z_r \leq C_{cp} \qquad \forall cp$$

Truck availability constraints

These represent limits that may be applied to ensure that the number of routes generated does not exceed the number of available truck units.

$$\sum_r \gamma_{t,r} z_r \le NT_t \qquad \forall t \ in \ truck \ types.$$

Driver availability constraints

These represent limits that may be applied to ensure that the number of routes generated does not exceed the number of available driver units.

$$\sum_r \beta_{d,r} z_r \le ND_d \qquad \forall d \ in \ driver \ types.$$

Location tie constraints (an implicit constraint that may be applied to ensure that locations are not visited more than once)

$$\sum_{od \in OD(r,l)} \omega_{od} x_{od,r} \ge \varepsilon_r z_r \qquad \forall r, l : l \in stops(r)$$

Cover constraints (may be applied to ensure that the aggregate order volume is completely placed on some combination of routes)

$$\varphi_{od} + \sum_{r \in R(od)} x_{od,r} = 1$$

Variable domain

$$z_r \ge 0 \qquad \forall r$$
$$x_{od,r} \ge 0, \quad x_{od,r} \in R, \quad \forall r, od \in OD(r)$$
$$\varphi_{od} \ge 0 \qquad \forall od$$

Once appropriate constraints are determined, a route generation algorithm of the present invention may be applied. In one embodiment, the following procedures are

utilized in solving the model established in Phase 1, as described above. For illustrative

purposes, implementations involving problems both of 1) a center-point route generation

and 2) direct routes and multiple deliveries to destinations, are described herein.

Center-Point route generation

Initialization: In one embodiment, center-point route generations begin with creating a set

of all feasible combinations of pickup locations to center-point legs, truck types and drivers.

This set may represent all feasible one-stop routes, and is referred to herein as G_0, with r_0

representing the number of routes in the set G_0. Variables z_r $r = 0,...,r_0 - 1$ may then be

assigned to represent each one of these routes in the master linear program, with a

generation counter being initialized to $g = 0$ and a route set to $G = G_0$. Initialization

portion **216** in Figure 2 is an exemplary implementation.

Re-Optimization: Solve the relaxed version of the master linear program and obtain the

LP solution values for $\tilde{z}_r$ $r = 0,...,r_k$. LP Optimization portion **212** in Figure 2 is an

exemplary implementation.

Generation step (k iterations): Route Generator **214** in Figure 2 is an exemplary

implementation. Let the generation counter be $g = k$. In one embodiment, a generation

$k + 1$ is achieved in accordance with the following method **300** illustrated in Figure 3:

1. **LP Solution (310):** Retrieve the incumbent LP solution for all previous iterations
   $\tilde{z}_r$ $r = 0,...,r_k$; initialize the new generation route set $G_k = \varnothing$.

2. **Iterate (320):** Begin iterating over all $r = r_k,...,r_k - 1$ such that $\tilde{z}_r > 0$.

3. **First Stop (330):** Let s_r be the first stop of route r. Let Q_r the set of pickup
   locations defined by $Q_r = \{q \mid (q,s_r) \in N\}$ where N is the set of all valid network
   legs. Iterate over all $q \in Q_r$. Iterate by increasing distance (q,s_r).

4. **Temporal route (340):** Create the temporal route $\hat{r}$ by appending q as the first stop of route r. Find the cost $\hat{c}_{\hat{r}}$ of the temporal route using its new length and the applicable lane rate.

5. **Selection Route (350):** From the set of routes r' such that $0 \le r' \le r_k$, $\tilde{z}_{r'} > 0$ and the first stop of r' is q, find r^* where

   $r^* = \arg\max\{c_{r'}\cdot(Ceil(\tilde{z}_{r'}) - \tilde{z}_{r'}) - c_{\hat{r}}\cdot(Ceil(\tilde{z}_{r'} + \tilde{z}_r) - \tilde{z}_{r'} - \tilde{z}_r)\}$. This is the route with origin in q that will be used in the selection criterion. Calculate the pseudo-value:
   $z_{\hat{r}} = 1 - Ceil(\tilde{z}_{r^*} + \tilde{z}_r) + \tilde{z}_{r^*} + \tilde{z}_r$.

6. **Selection Criterion (360):** If
   $c_{\hat{r}}\cdot(Ceil(\tilde{z}_{\hat{r}}) - \tilde{z}_{\hat{r}}) \le c_r\cdot(Ceil(\tilde{z}_r) - \tilde{z}_r) + c_{r^*}\cdot(Ceil(\tilde{z}_{r^*}) - \tilde{z}_{r^*})$, then the new route is a candidate for optimization. Make $G_k = G_k \cup \{\hat{r}\}$ and loop back to 2. If the criterion is not met, reject route $\hat{r}$ and continue the loop on 3.

Generation Termination Criterion: If the set G_k is empty, no more routes will be found and the generation method may be halted. Otherwise the algorithm should proceed through another iteration of route generation.

Generation of Destination direct routes and multiple deliveries to Destinations

The invention disclosed herein can be used to solve a transportation routing problem for many different transportation networks. In addition to the above-described Centerpoint (CP) route types, other route types may of course be addressed. For purposes of further illustration, a discussion will now be provided for an embodiment that may be used to solve a network having routes defined by the following terminology:

- "V-V-D" - pickup at one or more origins (V) and delivery at one destination (D).

- "V-V-D-D" - pickup at one or more origins (V) and delivery at one or more destinations (D).

This section describes an embodiment to solve such networks, and illustrates the flexibility of the invention as a utility and approach to transportation networks in general.

Candidate Origin-Destination pairs in a route: For illustration, a V-V-D-D route is assumed to have one or more pick-ups followed by one or more deliveries. In one embodiment, the Origin-Destination (OD) pairs that are candidates to be considered as part of the route are required to conform to the following conditions:

- Only OD pairs that have both their pick-up location and their delivery location as part of the route.

- Only OD pairs that conform to First-In-Last-Out ("FILO") are considered as candidates in the route. For example, it may be required that any two OD pairs conform to FILO only if their pick-up and delivery sequences meet the condition:

$$sequence(PickUp(od_i)) \le sequence(PickUp(od_j)) \Leftrightarrow$$
$$sequence(Delivery(od_i)) \ge sequence(Delivery(od_j))$$

Initialization: After the CP route generation has finished, dual values of the cover constraints may be obtained. These are referred to herein as q_{od}, and they represent the cost in the relaxed model of delivering all the orders in the origin destination pair od. The route set is initialized as $H = \varnothing$.

Acceptance criterion rule: For every route generated at any step of this algorithm, the route r may be accepted and added to the route set if it meets the criterion:

$$c_r \max\left(\frac{\sum\limits_{od \in \{od\,candidates\,in\,r\}} V_{od}}{TV_r}, \frac{\sum\limits_{od \in \{od\,candidates\,in\,r\}} W_{od}}{TW_r} \right) \le (1 + \eta) \sum\limits_{od \in \{od\,candidates\,in\,r\}} q_{od}$$

where η is a fixed positive parameter that will set a tolerance on the selection criterion.

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In one embodiment, the generation step may be performed in accordance with the following method **400** illustrated in Figure 4:

1. **Outer loop (410):** For all the origin-destination pairs *od*, iterate by decreasing value of q_{od}.

2. **One stop route initialization (420):** Create the one stop route $r^{1,1}$ that starts at the pickup location of pair *od* and ends at the corresponding destination.

3. **One stop route acceptance criterion (430):** If the acceptance criterion for the single stop route is met, add the route to the route set $H = H \cup \{r^{1,1}\}$.

4. **New stop loop (440):** At any iteration of this loop, the route $r^{i,j}$ has *i* pick-up stops and *j* delivery stops.

5. **New pick-up stop (450):** For all the pickup locations that are not along route $r^{i,j}$, loop by increasing distance from the current last pick-up. Create the new route $r^{i+1,j}$ by inserting the new pickup stop after the last pick-up and before the first delivery. If it meets the feasibility criterion (time-windows, location, etc.), continue to 6. Otherwise, loop 5 again.

6. **Acceptance criteria for new pick-up stop (460):** If there is an OD pair with non-negative volume and weight from the last pick-up to the first delivery of this route, use the acceptance criterion rule. If it meets the criterion: $H = H \cup \{r^{i+1,j}\}$.

7. **New delivery stop loop (470):** For all the destinations that are not part of route $r^{i+1,j}$, loop by increasing distance from the current first delivery. Create the new route $r^{i+1,j+1}$ by inserting the new destination stop after the last pick-up and before the first delivery. If it meets the feasibility criterion (time-windows, truck-location, etc.), continue to 8. Otherwise, loop on 7 again.

8. **Acceptance criteria for new delivery stop (480):** If there is an OD pair with non-negative volume and weight from the last pick-up to the first delivery of this route, use the acceptance criterion rule. If it meets the criterion: $H = H \cup \{r^{i+1,j+1}\}$. Loop on 5.

9. Loop on 4 (490).

Following completion of the generation phase 1, in accordance with one

embodiment of the present invention, phase 2 may involve the addition of lifting

constraints, as described below. Route skeletons to be passed to the Bin-Pack model may

be selected during this phase. A formulation for this phase in accordance with one

embodiment of the invention is as follows:

Lifting constraints

$$\sum_{r \in R(l)} z_r \geq \lambda_l \qquad \forall \ l \in VendorLocation,$$

where the lower bound λ_l is minimum number of routes needed to serve this location.

$$\lambda_l = \max\left(\left\lceil \frac{\sum_{o \in O(l)} V_o}{\max_{r \in R(l)} TV_r} \right\rceil, \left\lceil \frac{\sum_{o \in O(l)} W_o}{\max_{r \in R(l)} TW_r} \right\rceil\right)$$

Certain routes may then be utilized in subsequent phases of the invention. For

example, the routes with non-zero solution may be passed as route-skeletons to the Bin-

Pack solver. In one embodiment, the Bin-Pack solver assigns individual orders to route

skeletons. It may also calculate optimal arrival and departure times to the pickup locations.

Orders that do not fit one of the candidate routes at their location may be assigned to a route

on their own, where the cost may be assumed to be the baseline cost. As described in the

previous phases, in one embodiment, the route skeletons passed to Bin-Pack are all non-

negative solutions of the master problem that has been "lifted" (using the lifting constraint

approach) as described in phase 2, along with all non-negative solutions of all the phase 1

route generation LPs for V-V-CP, V-V-D and V-V-D-D.

To illustrate an implementation of the Bin-Pack solution in accordance with an

embodiment of the present invention, a number of potential constraints, exceptions, functions,

etc., that may be used are hereinafter provided. One skilled in the art, however, will appreciate

that any or all of the following or other features may be applied in any particular implementation

of the invention.

Various variables and parameters associated with the following description are as

follows:

Variable domain

$z_{r,t} \in \{0,1\}$ $\forall r$

$x_{o,r} \in \{0,1\}$ $\forall o, r : o \in O(r)$

$ltl_o \in \{0,1\}$ $\forall o : o \in Orders$

$0 \leq ta_{s,r} \leq \overline{T}$ $\forall r, s : s \in stops(r)$

$0 \leq td_{s,r} \leq \overline{T}$ $\forall r, s : s \in stops(r)$

$u_{p,s,r} \in \{0,1\}$ $\forall r, s \in stops(r), p \in P(s,r)$

$v_{p,s,r} \in \{0,1\}$ $\forall r, s \in stops(r), p \in P(s,r)$

$\gamma_{q,r} \in \{0,1\}$ $\forall r, \forall q \in \{Product\ classes\}$

Variable and parameter definition

$z_{r,t}$ is one if route r is selected using trailer type t. 0 otherwise. Solution Output.

$x_{o,r}$ is one if order o is transported in route r. Solution Output.

ltl_o is one if order o is sent by itself. 0 otherwise. Solution Output.

$u_{p,s,r}$ is one if route r reaches stop s at period p. Solution Output.

$v_{p,s,r}$ is one if route r leaves stop s at period p. Solution Output.

$\gamma_{q,r}$ is one if route r carries product class q. Solution Output.

$ta_{s,r}$ is the arrival time of route r to stop s. Solution Output.

$td_{s,r}$ is the departure time of route r to stop s. Solution Output.

Ω is a positive constant. Problem parameter.

$\tau_{i,j}$ is the travel time between locations i and j. Problem input.

WT_s is the maximum idle time at stop s. Problem parameter.

$\overline{T}$ is the length of the planning horizon. Problem parameter.

a_o is the Pick-Up start time for order o. Problem input.

b_o is the Pick-Up end time for order o. Problem input.

a'_o is the Delivery start time for order o. Problem input.

b'_o is the Delivery end time for order o. Problem input.

$\alpha_{p,s,r}$ is the location open time for stop s in route r during period p. Problem input.

$\beta_{p,r,s}$ is the location close time for stop s in route r during period p. Problem input.

$C_{p,cp}$ is the center-point cp capacity in planning period p. Problem input.

$MC_{p,cp}$ is the minimum number of routes that should get to center-point cp in planning period p. Problem Input.

TV_t is the volume capacity of equipment t. Problem Input.

TW_t is the weight capacity of equipment t. Problem Input.

θ_o is the loading time of order o. Problem input.

ϑ_o is the unloading time of order o. Problem Input.

p_o is the penalty (baseline cost) of order o. Problem input.

$h_{o,r}$ is the total handling cost of processing order o at the center-point where route r ends. If route r doesn't deliver order o at a center-point, the handling cost is zero. Problem input.

$\delta_{cp,r}$ is one if route r finishes at center-point cp. Generated Input.

$$n_{q,r} = Card\left(\{Pr\,oduct\,class\ q\} \cap \left\{ \begin{matrix} orders\ eligible \\ in\ route\ r \end{matrix} \right\} \right)$$ number of orders of product class q,

that can travel in route r. Generated Input.

$HT_{o,cp}$ handling time of order o at center-point cp. Problem Input.

$\bar{\tau}_{o,cp}$ travel time of order o from center-point cp to its final destination. Problem Input.

FLT_s fixed loading time at stop s. Problem Input.

FUT_s fixed unloading time at stop s. Problem Input.

FHT_{cp} fixed handling time at center-point cp. Problem Input.

m_r number of stops in route r. Generated Input.

$$\chi_{r,s} = loading\_rate_s \bullet \min\left[V_r, \sum_{o \in Orders(s)} v_o \right]$$ average dwelling time of route r at stop s.

Problem Input.

DT Driver allowed duty time. Problem Input.

RT Driver allowed resting time. Problem Input.

$P(s,r)$ is the set of applicable time periods to stop s in route r. It includes the periods

between the earliest pick-up of the orders eligible to go in that route to the latest pick-up.

ε_u value of the perturbation for the u variables, default value 0.0001. Problem Input.

ε_v value of the perturbation for the v variables, default value 0.0001. Problem Input.

Bin-Pack route preprocessing

In one embodiment, it may be desirable that, for example, routes that are more

expensive than the sum of the cost of sending every possible order that could potentially

travel on the route by itself, not be passed to the Bin-Pack model. That is, if it is possible to assign orders to a route in such a way that the cost of having sent those orders by themselves is more than the cost of the route, it may be desired that the route not be considered in the bin-pack solution. Therefore, the condition to be satisfied may be defined by:

$$\sum_{o \in \left\{ \substack{orders\ that\ can \\ travel\ in\ route\ r} \right\}} p_o \geq c_r$$

In one embodiment, the bin-pack problem is now used to compute the solution to the transportation problem by using the following formulation. This formulation takes into account the required business rules, and each exemplary constraint below models one of those business constraints.

Bin-Pack objective function

Depending on a particular embodiment or implementation, an objective or objectives of this phase of the invention may be to minimize the cost of routing orders while simultaneously minimizing the duration of each route and/or spreading out the arrival times of routes at each facility, among other possibilities. In one embodiment, the cost routing the orders may be defined as the sum of the cost when routing orders by themselves (baseline), the cost when handling an order at the center point, and the cost of each route when orders are routed together.

$$\min \sum_{o \in Orders} p_o ltl_o + \sum_{o \in Orders} \sum_{r \in R(o)} h_{o,r} x_{o,r} + \sum_r \sum_{t \in Trucks(r)} c_{r,t} z_{r,t} + \Omega \sum_r \sum_{s \in stops(r)} ta_{s,r}$$
$$+ \varepsilon_u \sum_r \sum_{s \in stops(r)} \sum_{p \in P(r,s)} p u_{p,s,r} + \varepsilon_v \sum_r \sum_{s \in stops(r)} \sum_{p \in P(r,s)} p v_{p,s,r}$$

Route-Vehicle assignment (each vehicle can only be assigned to one route)

$$\sum_{t \in Trucks(r)} z_{r,t} \leq 1 \qquad \forall r$$

Volume and weight constraints

$$\sum_{o \in O(r)} V_o x_{o,r} \leq \sum_{t \in Trucks(r)} TV_t z_{r,t} \qquad \forall r$$

$$\sum_{o \in O(r)} W_o x_{o,r} \leq \sum_{t \in Trucks(r)} TW_t z_{r,t} \qquad \forall r$$

Center-point capacity constraints

$$MC_{p,cp} \leq \sum_r \delta_{cp,r} \, u_{p,cp,r} \leq C_{p,cp} \qquad \forall cp, p$$

Truck availability constraint

$$\sum_r z_{r,t} \leq NT_t \qquad \forall t \text{ in truck types}$$

Driver availability constraints

$$\sum_r \sum_{t \in Trucks(r)} \beta_{d,r} \, z_{r,t} \leq ND_d \qquad \forall d \text{ in driver types.}$$

Location-tie constraints (an implicit modeling constraint that may be applied to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{o\in O(r,l)} x_{o,r} \geq \sum_{t\in Trucks(r)} z_{r,t} \qquad \forall r,l:l \in stops(r)$$

Bin-Pack Lifting (a solution strategy that may be intended to speed the solution by simplifying the solution space)

$$ltl_o + \sum_{r\in\left\{\substack{routes\ that\ stop\ at\\ the\ location\ of\ order\ o}\right\}} \sum_{t\in Trucks(r)} z_{r,t} \geq 1 \qquad \forall o \in Orders$$

Bin-Pack Facet Lifting constraints (a variation on the lifting constraint solution strategy that may be intended to speed the solution by simplifying the solution space)

$$x_{o,r} \leq \sum_{t\in trucks(r)} z_{r,t} \qquad \forall o,r \in R(o)$$

Order cover constraints (all orders must be on one route in the solution)

$$ltl_o + \sum_{r\in R(o)} x_{o,r} = 1 \qquad \forall o \in Orders$$

Loading time constraints (time to load orders, where order overlapping not enforced)

$$ta_{s,r} + FLT_s \sum_{t\in Trucks(r)} z_{r,t} + \sum_{o\in O(r,s)} \theta_o x_{o,r} \leq td_{s,r} \qquad \forall r,s:s \in stops(r)$$

Loading time constraints (time to load orders, where order overlapping enforced)

$$ta_{s,r} + FLT_s \sum_{t\in Trucks(r)} z_{r,t} + \sum_{o\in O(r,s)} \theta_o x_{o,r} = td_{s,r} \qquad \forall r,s:s \in stops(r)$$

Travel time constraints (time to travel between stops on the route)

$$td_{s,r} + \tau_{s,s+1} \sum_{t \in Trucks(r)} z_{r,t} \le ta_{s+1,r} \qquad \forall r,s : s \in stops(r)$$

Idle time while loading (first stop only, use only if order overlapping is not enforced)

$$td_{0,r} - ta_{0,r} - FLT_s \sum_{t \in Trucks(r)} z_{r,t} - \sum_{o \in O(r,s)} \theta_o x_{o,r} \le WT_0 \sum_{t \in Trucks(r)} z_{r,t} + \overline{T}\left(1 - \sum_{t \in Trucks(r)} z_{r,t}\right) \quad \forall r$$

Waiting time before a stop and idle time before a pick-up (may be used to take into account the case where the travel time between stops is less than the elapsed time between the open hours (time) of each stop)

$$td_{s+1,r} - td_{s,r} - \tau_{s,s+1} \sum_{t \in Trucks(r)} z_{r,t} - FLT_s \sum_{t \in Trucks(r)} z_{r,t} - \sum_{o \in O(r,s+1)} \theta_o x_{o,r}$$

$$\le WT_{s+1} \sum_{t \in Trucks(r)} z_{r,t} + \overline{T}\left(1 - \sum_{t \in Trucks(r)} z_{r,t}\right) \qquad \forall r,s : s \in stops(r)$$

Waiting time before a Center-Point (may be used to take into account the time waiting for a CP to open)

$$ta_{cp,r} - td_{\hat{s},r} - \tau_{\hat{s},cp} \sum_{t \in Trucks(r)} z_{r,t} \le WT_{cp} \sum_{t \in Trucks(r)} z_{r,t} + \overline{T}\left(1 - \sum_{t \in Trucks(r)} z_{r,t}\right) \quad \forall r, \hat{s} \text{ is the stop before the cp}$$

Unloading time constraints (v-v-d-d routes only) (time to unload)

$$ta_{s,r} + FUT_s \sum_{t \in Trucks(r)} z_{r,t} + \sum_{o \in O(r,s)} \vartheta_o x_{o,r} \le td_{s,r} \qquad \forall r,s : s \in stops(r)$$

Waiting time before a stop and idle time before a delivery (v-v-d-d routes only)

$$td_{s+1,r} - td_{s,r} - \tau_{s,s+1} \sum_{t \in Trucks(r)} z_{r,t} - FUT_s \sum_{t \in Trucks(r)} z_{r,t} - \sum_{o \in O(r,s+1)} \vartheta_o x_{o,r}$$

$$\leq WT_{s+1} \sum_{t \in Trucks(r)} z_{r,t} + \overline{T}\left(1 - \sum_{t \in Trucks(r)} z_{r,t}\right) \qquad \forall r, s : s \in stops(r)$$

Pick-Up start time constraint (cannot pick up before this time)

$$td_{s,r} \geq \left(a_o + \theta_o\right) x_{o,r} \qquad \forall r, s : s \in stops(r) \quad o \in Orders(s)$$

Pick-Up end time constraint (must pick up before this time)

$$ta_{s,r} \leq b_o x_{o,r} + \overline{T}\left(1 - x_{o,r}\right) \qquad \forall r, s : s \in stops(r) \quad o \in Orders(s)$$

Delivery start time constraint (cannot deliver before this time)

$$\left(a_o + \vartheta_o\right) x_{o,r} \leq td_{s,r} \qquad \forall r, s : s \in stops(r) \quad o \in Orders(s)$$

Delivery end time constraint (must deliver before this time)

$$ta_{s',r} \leq b'_o x_{o,r} + \overline{T}\left(1 - x_{o,r}\right) \qquad \forall r, s' = LastStop\ (r) \quad o \in Orders\ (s')$$

Delivery time end constraint at a center-point (must deliver at CP before this time)

$$ta_{cp,r} \leq -FHT_{cp} \sum_{t \in Trucks(r)} z_{r,t} - \left(b'_o - HT_{o,cp} - \overline{\tau}_{o,cp}\right) x_{o,r} + \overline{T}\left(1 - x_{o,r}\right)$$

$$\forall o, \forall cp, \forall r \in R(o) : r\ arrives\ to\ cp$$

Stop-Time arrival period cover (cannot exceed trailer unit capacity during the arrival period)

$$\sum_{p \in P(s,r)} u_{p,s,r} = \sum_{t \in Trucks(r)} z_{r,t} \qquad \forall r, s \in stops(r)$$

Stop-Time departure period cover (cannot exceed trailer unit capacity during the delivery period)

$$\sum_{p \in P(s,r)} v_{p,s,r} = \sum_{t \in Trucks(r)} z_{r,t} \qquad \forall r, s \in stops(r)$$

Stop arrival time window (must arrive within window)

$$\alpha_{p,s,r} u_{p,r,s} \le ta_{s,r} \le \beta_{p,s,r} u_{p,r,s} + \overline{T}(1 - u_{p,r,s}) \qquad \forall r, s \in stops(r) \qquad p \in P(s,r)$$

Stop departure time window (must depart within window)

$$\alpha_{p,s,r} v_{p,r,s} \le td_{s,r} \le \beta_{p,s,r} v_{p,r,s} + \overline{T}(1 - v_{p,r,s}) \qquad \forall r, s \in stops(r) \qquad p \in P(s,r)$$

Duty Time Constraint (route cannot violate the driver's duty time allowance)

$$td_{last\_stop,r} - ta_{first\_stop,r} \ge$$

$$\sum_{s \in \{Stops\ in\ route\ r\}} \tau_{s,s+1} + m_r \chi + RT\ floor \left[\left[\frac{\sum_{s \in \{Stops\ in\ route\ r\}} \tau_{s,s+1} + m_r \chi - \left(\sum_{s \in \{Stops\ in\ route\ r\}} \tau_{s,s+1} + m_r \chi \right) \bmod DT}{DT} \right] \right] \qquad \forall r$$

Product class in a route (may be used to ensure that there are not more orders of a specific product class than are feasible for each route)

$$\sum_{\substack{o \in \{Product\ class\ q\} \\ \cap \{orders\ eligible\ in\ route\ r\}}} x_{o,r} \le n_{q,r} \gamma_{q,r} \qquad \forall r, \forall q$$

Order to order exceptions (incompatible product classes)

For all incompatible pairs of product classes:

$$\gamma_{q_i,r} + \gamma_{q_j,r} \leq 1 \qquad \forall r, q_i \text{ and } q_j \text{ incompatible product clases.}$$

Order to route exception

Note: Order to route exception may include: order to center-point exception, order to truck exception and/or order to driver exception, among others. In one embodiment, these exceptions are handled by not generating the variable $x_{o,r}$ that may assign that order to that route; otherwise, the following constraint may be used:

$$x_{o,r} = 0.$$

Truck to driver exception

This constraint need not be modeled explicitly. During route generation it may be checked that no route is created that has a driver type incompatible with the truck type.

Truck to location exception

This constraint need not be modeled explicitly. During route generation it may be checked that no route is created that visits a location that is incompatible with the truck type.

Order forced through a particular CP

Note: In one embodiment, this constraint is handled by not generating $x_{o,r}$ for a route that will visit a center-point different from the one this order should be routed through; otherwise, the following constraint may be used:

$$\sum_{r \in \{routes \ that \ don't \ visit \ the \ assigned \ CP\}} x_{o,r} = 0$$

Objective Function Perturbation

In order to strengthen the LP relaxation of the Bin-Pack, it may be desirable to perturb the objective function of repeated routes. Repeated routes may be defined as those that have the same stops, the same time windows and the same set of potential orders, among other potential definitions. In one embodiment, the cost of the repeated routes are perturbed by adding a small amount to the handling cost and transit cost of each one of the copies.

The following pseudo code provides an example:

For all routes *r*:

    For all copies *i* of route *r*:

        Transit_cost_*r_i*:= Transit_cost_*r_i* + *i* * transit_perturb_value

        For all orders *o* that can travel in route *r*

            Handling_cost_*o_r_i*:= Handling_cost_*o_r_i* + *i* *

handling_perturb_value

        Next *o*

    Next *i*

Next *r*

Multi-Temp Bin-Pack

The following is an additional set of constraints that may be added to the Bin-Pack when it is desired that temperature storage requirements are part of the optimization process. An associated variable domain and variable definitions are also provided. In order to model temperatures in the trailer, it is may be desirable to group the orders by temperature class, stop and route. Again, like other such lists of constraints, definitions,

etc. herein, the following are by way of example only, as one skilled in the art would readily envision much variation upon review of the present disclosure.

In one embodiment, the sets are defined as follows:

$$T(r, s, temp(i)) = \{orders\ eligible\ in\ route\ r\ at\ stop\ s\ of\ temperature\ temp(i)\}$$

Multi-Temp Variable Domain

$$\varepsilon_{\theta, o, r} \in \{0,1\} \qquad \forall r, \theta \in \{Compartments\ in\ r\},\ o \in \{Orders\ in\ r\}$$

$$y_{r, s, temp(i)} \in \{0,1\} \qquad \forall r, s \in stops(r),\ \forall temp(i)$$

Multi-Temp Variable definition

$\varepsilon_{\theta, o, r}$ is 1 if order o travels in route r in multi-temp compartment θ. Solution Output.

$y_{r, s, temp(i)}$ is 1 if route r leaves stop s at temperature temp(i), 0 otherwise. Solution Output.

N_r Upper bound on the number of orders in the route (could be equal to the total number of orders). Parameter.

k_o is the size of the order in the multi-temp compartment (usually in pallets). Problem Input.

$K_{\theta, r}$ is the capacity of multi-temp compartment p in route r (usually in pallets). Problem Input.

NK is the upper bound in the number of multi-temp compartments in a route. Problem Input.

Multi-Temp compartment assignment

$$\sum_{\theta \in \{Compartments\ in\ r\}} \varepsilon_{\theta, o, r} = x_{o, r} \qquad \forall r, o \in \{Orders\ eligible\ in\ route\ r\}$$

Multi-Temp compartment capacity

$$\sum_{o \in \{Orders\ eligcble\ in\ route\ r\}} k_o\, \varepsilon_{\theta,o,r} \leq K_{\theta,r} \qquad \forall r, \theta \in \{Compartments\ in\ r\}$$

Multi-Temp compartment compatibility

$$N_r\, NK\, \left(1 - \varepsilon_{\theta,o,r}\right) \geq \sum_{\substack{o' \in \{Orders\ eligeble\ in\ route\ r\} \\ o' \neq o}} \sum_{\substack{\theta' \in \{Compartments\ in\ r\} \\ \theta' \neq \theta}} \varepsilon_{\theta',o',r}$$

$$\forall r, o \in \{Orders\ eligeble\ in\ route\ r\}, \theta \in \{Compartments\ in\ r\}$$

Multi-Temp FILO

$$N_r\left(1 - y_{r,s,temp(i)}\right) \geq \sum_{s'>s} \sum_{temp(k)<temp(i)} \sum_{o \in T(r,s',temp(k))} x_{o,r} \qquad \forall r, s \in stops(r),\ \forall temp(i)$$

Multi-Temp FILO-2

$$N_r \sum_{temp(k)\geq temp(i)} y_{r,s,temp(k)} \geq \sum_{o \in T(r,s,temp(i))} x_{o,r} \qquad \forall r, s \in stops(r),\ \forall temp(i)$$

Multi-Temp Temperature Cover

$$\sum_{temp(i)} y_{r,s,temp(i)} = \sum_{t \in Trucks(r)} z_{r,t} \qquad \forall r, s \in stops(r)$$

Bi-Temp Bin-Pack

For purposes of still further illustration, the following is an additional set of constraints that may be added to Bin-Pack in order to model Bi-Temperature trailers. It is typical for there to be only two compartments on a Bi-Temperature trailer, one in the front

and the other in the back of the trailer, with the front being colder than the back. To enforce FILO, colder orders are commonly picked-up first. Given that there are a limited number of positions in which the wall separating the compartments can be set, it is desirable to model the configurations of the trailer. The sets of low temperature and high temperature orders may be defined at each stop of the route.

$Low(r,s) = \{orders\ eligible\ in\ route\ r\ at\ stop\ s\ that\ should\ go\ in\ the\ front\ compartment\}$

$High(r,s) = \{orders\ eligible\ in\ route\ r\ at\ stop\ s\ that\ should\ go\ in\ the\ back\ compartment\}$

Bi-Temp Variable definition

$y_{r,s}$ is 1 if route r leaves stop s on high temperature, 0 otherwise.

$\eta_{j,r}$ is one if the wall in the trailer is set to configuration j, 0 otherwise.

k_o is the size of the order o, usually in pallets.

$LK_{j,r}$ is the capacity of the low temperature compartment (front compartment) for configuration j in route r, usually in pallets.

$HK_{j,r}$ is the capacity of the high temperature compartment (back compartment) for configuration j in route r, usually in pallets.

Bi-Temp FILO

$$N_r y_{r,s} \geq \sum_{o \in High(r,s)} x_{o,r} \qquad \forall r, s \in stops(r)$$

Bi-Temp FILO-2

$$N_r (1 - y_{r,s}) \geq \sum_{s' > s} \sum_{o \in Low(r,s)} x_{o,r} \qquad \forall r, s \in stops(r)$$

Bi-Temp Compartment Capacity

$$\sum_{s\in stops(r)}\sum_{o\in Low(r,s)}k_o x_{o,r} \leq \sum_{j\in Config(r)}LK_{j,r}\eta_{j,r} \qquad \forall r$$

$$\sum_{s\in stops(r)}\sum_{o\in High(r,s)}k_o x_{o,r} \leq \sum_{j\in Config(r)}HK_{j,r}\eta_{j,r} \qquad \forall r$$

Bi-Temp Compartment Assignment

$$\sum_{j\in Config(r)}\eta_{j,r} \leq \sum_{t\in Trucks(r)}z_{r,t} \qquad \forall r$$

Bi-Temp Compartment Assignment 2

$$\sum_{\substack{j\in Config(r) \\ \cap \{Compatible Configurations with type t\}}}\eta_{j,r} \geq z_{r,t} \qquad \forall r,t$$

Bi-Temp Variable Domain

$$y_{r,s} \in \{0,1\} \qquad \forall r,s \in stops(r)$$

$$\eta_{j,r} \in \{0,1\} \qquad \forall r,j \in Config(r)$$

Driving Rules

An additional factor that may be taken into account in practicing the present invention is driving time. For example, drivers are often restricted in how long they can drive before they must rest. The DOT currently mandates 10 hours rest for every 11 hours of driving. This rule may be incorporated into a planned route by modifying the travel time between stops to add rest time wherever is it needed. To do that, total driving time may be tracked, with 10 hours of rest being added to a leg that goes over 11 hours. Considering that these times may change, they may be parameterized using the pseudo-code variables maxDrivingTime, maxDutyTime and restTime. The following pseudo code is provided as exemplary:

Initialize: A route with stops 1, 2..., n and travel times between stops $\tau_{1,2}, \tau_{2,3}, \ldots, \tau_{n-1,n}$.

Let drivingTime=0. Let dutyTime= $\chi_{r,s}$.

Loop: For s=1 to n-1 Step +1

   Let drivingTime:= drivingTime + $\tau_{s,s+1}$

   Let dutyTime:= drivingTime + $\chi_{r,s}$

   If drivingTime >= maxDrivingTime Then

      Let r= drivingTime Mod maxDrivingTime

      Let n= (drivingTime – r) / maxDrivingTime

      Let $\tau_{s,s+1} = \tau_{s,s+1}$ + restTime*n

      Let drivingTime= r

      Let dutyTime= r

   Else

      If dutyTime >= maxDutyTime Then

         Let r0= dutyTime Mod maxDutyTime

         Let n0= (dutyTime – r) / maxDutyTime

         Let $\tau_{s,s+1} = \tau_{s,s+1}$ + restTime*n0

         Let drivingTime= r0

         Let dutyTime= r0

      End If

   End If

   Next s

The modified travel times $\tau_{i,j}$ may thus be passed to the Bin-Pack model and used

in optimization.

Inbound-Outbound Algorithm

For at least the reason that numerous variations on the above-described embodiments are contemplated, an embodiment of the present invention that may be used to implement simultaneous inbound-outbound multi-stop routing through cross-docks and potentially additionally allowing by-pass routes, will now be described. Exemplary route types (Inbound, Outbound, Bypass) from origins (Orig), potentially through a Cross-Dock (XD) to Destinations (Dest) are shown in an overview illustration **500** in Figure 5. This section is related to the route generation algorithm described above. While the above algorithm may be more suited to a problem of planning inbound delivery of goods, the following algorithm may be more suited to an inbound-outbound problem, depending on a particular implementation, desired results, etc., among other variables.

The following section describes an alternative process that may be implemented in accordance with the present invention to solve problems with specific transportation network types, or possible order routings. In such an embodiment, the network may consider some or all of the following types of order routing, or others:

- Order routed by itself, direct from origin to destination.
- Order routed by itself from origin to cross-dock, routed consolidated (single or multi-stop) to destination.
- Routed consolidated from origin to cross-dock, order routed by itself to destination.
- Order routed by itself from origin to cross-dock, order routed by itself to destination.
- Routed consolidated from origin to cross-dock, routed consolidated to destination.
- Routed consolidated from origin bypassing cross-docks to destination.

Decisions that may be considered by this model include, but are not limited to:

- Which routed consolidated routes are generated and used.

- What is the best path (i.e. through a cross-dock or direct) for each order.

- Which is the best cross-dock for each order considering inbound cost, labor cost and outbound cost.

- Level of usage at cross-docks.

- Others.

Phases of the Algorithm

In one embodiment, the Inbound-Outbound algorithm is implemented in three phases, which again may in certain aspects be analogous or similar to those described above with respect to the generation algorithm above. Reference may therefore again be made to Figure 2.

If an element of simplification is desired in accordance with an aspect of the present invention, order aggregation by OD pair and cross-dock capacity aggregation may be accomplished in this embodiment in a manner comparable to that described above, among others. Reference may therefore be made to any or all associated aggregations described herein, and/or others apparent to one skilled in the art upon consideration of the present disclosure.

In accordance with an embodiment of the invention, an acceptable way to formulate the LP for this phase will now be described. The solution to this LP may be route skeletons that may be used as input to the subsequent bin-pack phase. As in the inbound embodiment, this phase may use an aggregated, strategic approach intended to quickly find a route skeleton solution to the problem. Even though order volumes may be aggregated in this phase, the business rules may be modeled in this LP using the mathematical constraints listed below, among countless other possibilities and/or variations.

Variable Domain

$z_r \geq 0 \qquad \forall r \in \{Inbound\ routes\}$

$z'_{r'} \geq 0 \qquad \forall r' \in \{Outbound\ routes\}$

$zd_{rd} \geq 0 \quad \forall rd \in \{Bypass\ routes\}$

$x_{od,r} \geq 0 \qquad \forall od, \forall r \in \{Inbound\ routes\} \cup \{Bypass\ routes\}$

$iltl_{od,cp} \geq 0 \quad \forall od, \forall cp \in \{Cross - Docks\}$

$oltl_{od,cp} \geq 0 \quad \forall od, \forall cp \in \{Cross - Docks\}$

$ltl_{od} \geq 0 \qquad \forall od$

Variable and parameter definition

z_r is how many times rout r is selected. Solution Output.

$z'_{r'}$ is one if outbound route r' is selected. Solution Output.

zd_{rd} is one if bypass route rd is selected. Solution Output.

$x_{od,r}$ is the proportion of od pair sent routed consolidated from its origin in route r.

Solution Output.

$y_{od,r'}$ is the proportion of od pair sent routed consolidated from the center-point in route r'.

Solution Output.

$xd_{od,rd}$ is the proportion of od pair sent routed consolidated in by-pass route rd. Solution

Output.

$iltl_{od,cp}$ is the proportion of od pair that travels routed by itself from its origin to cross-dock

cp. Solution Output.

$oltl_{od,cp}$ is the proportion of od pair that travels routed by itself from cross-dock cp to its

destination. Solution Output.

ltl_{od} is the proportion of od par that travels routed by itself from its origin to its destination.

Solution Output.

$h_{od,r}$ handling cost of od pair traveling in route r. This value depends of the type route and

where it finish. Input Parameter.

$c_r, c'_{r'}, cd_{rd}$ are respectively the transit costs for inbound route r, outbound route r' and

bypass route rd. Calculated during route generation.

$ip_{od,cp}$ is the penalty of sending od pair routed by itself from its origin to center-point cp.

Input parameter.

$op_{od,cp}$ is the penalty of sending od pair routed by itself) from center-point cp to its

destination. Input parameter.

p_{od} is the penalty of sending od pair routed by itself from its origin to its final destination.

Input parameter.

V_{od} aggregated volume of od pair. Input parameter.

W_{od} aggregated weight of od pair. Input parameter.

TV volume capacity of the representative vehicle used in route r. Input parameter.

TW weight capacity of the representative vehicle used in route r. Input parameter.

C_{cp} it's the maximum aggregated throughput capacity of cross-dock cp. Input parameter.

MC_{cp} it's the minimum aggregated throughput capacity of cross-dock cp. Input parameter.

$\delta_{cp,r}$ is one if route r finishes at cross-dock cp. Generated Input.

ω_{od} is the size factor of pair od in route r, computed as:

$$\omega_{od} = \max\left[Ceil\left(\frac{V_{od}}{TV}\right), Ceil\left(\frac{W_{od}}{TW}\right)\right]$$

ε route factor appox. 0.001. *Input.*

$$\min \sum_{od} \sum_{r \in \{Inbound\ routes\}} h_{od,r} x_{od,r} + \sum_{r \in \{Inbound\ routes\}} c_r z_r$$

$$+ \sum_{r' \in \{Outbound\ routes\}} c'_{r'} z'_{r'} + \sum_{rd \in \{Bypass\ routes\}} cd_{rd} zd_{rd}$$

$$+ \sum_{od} \sum_{cp \in \{Cross-Docks\}} ip_{od,cp}\ iltl_{od,cp} + \sum_{od} \sum_{cp \in \{Cross-Docks\}} op_{od,cp}\ oltl_{od,cp} + \sum_{od} p_{od}\ ltl_{od}$$

Cross-Dock Period Capacity (the number of routes that can visit the cross-dock during a given period)

$$MC_{cp} \leq \sum_{r \in \left\{ \begin{subarray}{l} Inbound\ Routes \\ Through\ XD\ r \end{subarray} \right\}} \delta_{cp,r}\, z_{r,t} \leq C_{cp} \qquad \forall cp,$$

Inbound Cover Constraints (all orders must be on an inbound route)

$$ltl_{od} + \sum_{cp \in \{Cross-Docks\}} iltl_{od,cp} + \sum_{r \in \{Inbound\ routes\}} x_{od,r} + \sum_{rd \in \{Bypass\ routes\}} xd_{od,rd} = 1 \qquad \forall od$$

Outbound Cover Constraints (all orders must be on an outbound routes)

$$ltl_{od} + \sum_{cp \in \{Cross-Docks\}} oltl_{od,cp} + \sum_{r' \in \{outbound\ routes\}} y_{od,r'} + \sum_{rd \in \{Bypass\ routes\}} xd_{od,rd} = 1 \qquad \forall od$$

Cross-dock continuity (all orders that are routed into a cross-dock must be routed out of the cross dock)

$$iltl_{od,cp} + \sum_{r \in \{inbound\ routes\ at\ cp\}} x_{od,r} = oltl_{od,cp} + \sum_{r' \in \{outbound\ routes\ at\ cp\}} y_{od,r} \qquad \forall od, \forall cp \in \{Cross-Docks\}$$

Inbound Volume

$$\sum_{od} V_{od} x_{od,r} \leq TV z_r \qquad \forall r$$

Inbound Weight

$$\sum_{od} W_{od} x_{od,r} \leq TW z_r \qquad \forall r$$

Outbound Volume

$$\sum_{od} V_{od} y_{od,r'} \le TV z'_{r'} \qquad \forall r'$$

Outbound Weight

$$\sum_{od} W_{od} y_{od,r'} \le TW z'_{r'} \qquad \forall r'$$

Bypass Route Volume

$$\sum_{od} V_{od} x d_{od,rd} \le TV z d_{rd} \qquad \forall rd$$

Bypass Route Weight

$$\sum_{od} W_{od} x d_{od,rd} \le TW z d_{rd,t} \qquad \forall rd$$

Inbound Location Tie (an implicit modeling constraint that may be applied to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{od} \omega_{od} x_{od,r} \ge \varepsilon z_r \qquad \forall r, l:l \in stops(r)$$

Outbound Location Tie (an implicit modeling constraint that may be used to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{od} \omega_{od} y_{od,r'} \ge \varepsilon z'_{r'} \qquad \forall r', l:l \in stops(r')$$

Bypass Route Location Tie (an implicit modeling constraint that may be used to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{od} \omega_{od} \cdot x d_{od,rd} \geq \varepsilon \ z d_{rd} \qquad \forall rd, l : l \in stops(rd)$$

Lifting Constraints Inbound

$$\sum_{od \in \left\{ \substack{od \ from \\ location \ l} \right\}} \max\left[Ceil\left(\frac{V_{od}}{TV}\right), Ceil\left(\frac{W_{od}}{TW}\right)\right]\left(ltl_{od} + \sum_{cp \in \{Cross-Docks\}} iltl_{od,cp} \right) + \sum_{r \in \left\{ \substack{Inbound \ routes \\ through \ l} \right\}} z_r +$$

$$\sum_{rd \in \left\{ \substack{Bypass \ routes \\ through \ l} \right\}} z d_{rd} \geq \max\left[Ceil\left(\frac{\sum_{od \in \{od \ pairs \ through \ l\}} V_{od}}{TV}\right), Ceil\left(\frac{\sum_{od \in \{od \ pairs \ through \ l\}} W_{od}}{TW}\right)\right] \qquad \forall l \in \{Origin \ location\}$$

Lifting Constraints Outbound

$$\sum_{od \in \left\{ \substack{od \ to \\ location \ l} \right\}} \max\left[Ceil\left(\frac{V_{od}}{TV}\right), Ceil\left(\frac{W_{od}}{TW}\right)\right]\left(ltl_{od} + \sum_{cp \in \{Cross-Docks\}} oltl_{od,cp} \right) + \sum_{r \in \left\{ \substack{Outbound \ routes \\ to \ l} \right\}} y_r +$$

$$\sum_{rd \in \left\{ \substack{Bypass \ routes \\ to \ l} \right\}} z d_{rd} \geq \max\left[Ceil\left(\frac{\sum_{od \in \{od \ pairs \ to \ l\}} V_{od}}{TV}\right), Ceil\left(\frac{\sum_{od \in \{od \ pairs \ to \ l\}} W_{od}}{TW}\right)\right] \qquad \forall l \in \{Destination \ location\}$$

Route generation

Route generation may be done following the same algorithms described in the above section.

- Inbound Route Generation: Use the results of $x_{od,r}, z_r$.

- Outbound Route Generation: Use the results of $y_{od,r'}, z'_{r'}$.

- Bypass Route Generation: Use the results of $xd_{od,rd}$ zd_r

The final phase of this embodiment of the invention may then apply explicit order

volumes in a bin-pack approach on order to explicitly assign orders to routes. The input to

this phase will be the route skeletons obtained in the previous phase. At this point, many

business rules are explicitly modeled using the constraints and variables listed below.

Variable Domain

$z_{r,t} \in \{0,1\}$ $\forall r \in \{Inbound\ routes\}$ $t \in Trucks(r)$

$z'_{r',t} \in \{0,1\}$ $\forall r' \in \{Outbound\ routes\}$ $t \in Trucks(r)$

$zd_{rd,t} \in \{0,1\}$ $\forall rd \in \{Bypass\ routes\}$ $t \in Trucks(r)$

$x_{o,r} \in \{0,1\}$ $\forall o \in \{orders\}, \forall r \in \{Inbound\ routes\} \cup \{Bypass\ routes\}$

$iltl_{o,cp} \in \{0,1\}$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

$oltl_{o,cp} \in \{0,1\}$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

$ltl_o \in \{0,1\}$ $\forall o \in \{orders\}$

$ta_{s,r} \in [0,\overline{T}]$ $\forall r \in \{All\ routes\}, \forall s \in stops(r)$

$td_{s,r} \in [0,\overline{T}]$ $\forall r \in \{All\ routes\}, \forall s \in stops(r)$

$u_{p,r,s} \in \{0,1\}$ $\forall r \in \{All\ routes\}, \forall s \in stops(r), p \in P(r,s)$

$v_{p,r,s} \in \{0,1\}$ $\forall r \in \{All\ routes\}, \forall s \in stops(r), p \in P(r,s)$

$tid_{o,cp} \in [0,\overline{T}]$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

$tia_{o,cp} \in [0,\overline{T}]$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

$tod_{o,cp} \in [0,\overline{T}]$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

$toa_{o,cp} \in [0,\overline{T}]$ $\forall o \in \{orders\}, \forall cp \in \{Cross - Docks\}$

Variable and parameter definition

$z_{r,t}$ is one if inbound route r is selected using truck t, 0 otherwise. Solution Output.

$z'_{r',t}$ is one if outbound route r' is selected using truck t, 0 otherwise. Solution Output.

$zd_{rd,t}$ is one if bypass route rd is selected using truck t, 0 otherwise. Solution Output.

$x_{o,r}$ is one if order o is sent routed consolidated from its origin in route r, 0 otherwise.

Solution Output.

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$y_{o,r'}$ is one if order o is sent routed consolidated from the center-point in route r', 0

otherwise. Solution Output.

$xd_{o,rd}$ is one if order o is sent routed consolidated in by-pass route rd, 0 otherwise.

Solution Output.

$iltl_{o,cp}$ is one if order o travels routed by itself from its origin to cross-dock cp, 0 otherwise.

Solution Output.

$oltl_{o,cp}$ is one if order o travels routed by itself from cross-dock cp to its destination, 0

otherwise. Solution Output.

ltl_o is one if order o travels routed by itself from its origin to its destination, 0 otherwise.

Solution Output.

$ta_{s,r}$ is the arrival time of route r to stop s. Solution Output.

$td_{s,r}$ is the departure time of route r from stop s. Solution Output.

$u_{p,r,s}$ is one if route r reaches stop s on period p. Solution Output.

$v_{p,r,s}$ is one if route r reaches stop s on period p. Solution Output.

$tid_{o,cp}$ is the time order o departs from the vendor to cross-dock cp when routed by itself.

Solution Output.

$tia_{o,cp}$ is the time order o arrives to cross-dock cp when routed by itself. Solution Output.

$tod_{o,cp}$ is the time order o leaves from cross-dock cp when routed by itself. Solution

Output.

$toa_{o,cp}$ is the time order o arrives to its final destination from cross-dock cp when routed by

itself. Solution output.

$h_{o,r}$ handling cost of order o traveling in route r. This value depends of the type route and

where it finish. Input Parameter.

$c_r, c'_{r'}, cd_{rd}$ are respectively the transit costs for inbound route r, outbound route r' and

bypass route rd. Calculated during route generation.

$ip_{o,cp}$ cost of sending order o routed by itself from its origin to center-point cp. Input

parameter.

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$op_{o,cp}$ cost of sending order o routed by itself from center-point cp to its destination. Input

parameter.

p_o cost of sending order o routed by itself from its origin to its final destination. Input

parameter.

V_o volume of order o. Input parameter.

W_o weight of order o. Input parameter.

TV_r volume capacity of the representative vehicle used in route r. Input parameter.

TW_r weight capacity of the representative vehicle used in route r. Input parameter.

$HT_{o,cp}$ handing time required for order o at center-point cp. Input parameter.

$\overline{T}$ length of the planning horizon. Preprocessed input.

θ_o loading time of order o. Input parameter.

ϑ unloading time of order o. Input parameter.

$\tau_{s,s'}$ travel time between location s and location s'. Input parameter.

a_o earliest pick-up time for order o. Input parameter.

b_o latest delivery time for order o. Input parameter.

$C_{p,cp}$ it's the maximum throughput capacity of cross-dock cp during period p. Input

parameter.

$MC_{p,cp}$ it's the minimum throughput capacity of cross-dock cp during period p. Input

parameter.

$\delta_{cp,r}$ is one if route r finishes at cross-dock cp. Generated Input.

WT_s maximum waiting time before stop s. Input parameter.

Objective Function

Depending on an embodiment or particular implementation of the invention, an

objective or objectives of this phase of the invention may be to minimize the cost of routing

and/or handling of orders, while potentially also simultaneously minimizing some of the business constraints related to timing in the problem.

$$
\begin{aligned}
\min \quad & \sum_{o\in\{orders\}}\sum_{r\in\{Inbound\ routes\}} h_{o,r} x_{o,r} + \sum_{r\in\{Inbound\ routes\}}\sum_{t\in Trucks(r)} c_{r,t} z_{r,t} \\
& + \sum_{r'\in\{Outbound\ routes\}}\sum_{t\in Trucks(r')} c'_{r',t}\, z'_{r',t} + \sum_{rd\in\{Bypass\ routes\}}\sum_{t\in Trucks(rd)} cd_{rd,t}\, zd_{rd,t} \\
& + \sum_{o\in\{orders\}}\sum_{cp\in\{Cross-Docks\}} ip_{o,cp}\, iltl_{o,cp} + \sum_{o\in\{orders\}}\sum_{cp\in\{Cross-Docks\}} op_{o,cp}\, oltl_{o,cp} + \sum_{o\in\{orders\}} p_o\, ltl_o \\
& + \Omega \sum_{r\in\{All\ routes\}}\sum_{s\in stops(r)} ta_{s,r} + \Omega' \sum_{r\in\{All\ routes\}}\sum_{s\in stops(r)} td_{s,r} \\
& + \Omega \sum_{o\in\{orders\}}\sum_{cp\in\{Cross-Docks\}}\left(tia_{o,cp} + toa_{o,cp}\right) + \Omega' \sum_{o\in\{orders\}}\sum_{cp\in\{Cross-Docks\}}\left(tid_{o,cp} + tod_{o,cp}\right)
\end{aligned}
$$

Cross-Dock Period Capacity (number of routes visiting cross-dock cannot exceed capacity for that period)

$$
MC_{p,cp} \leq \sum_{r\in\left\{\begin{array}{l}Inbound\ Routes\\Through\ XD\ r\end{array}\right\}} \delta_{cp,r}\, u_{p,r,s} \leq C_{p,cp} \qquad \forall cp, p
$$

Route-Vehicle Assignment Inbound

$$
\sum_{t\in Trucks(r)} z_{r,t} \leq 1 \qquad \forall r
$$

Route-Vehicle Assignment Outbound

$$
\sum_{t\in Trucks(r')} z'_{r',t} \leq 1 \qquad \forall r'
$$

Route-Vehicle Assignment Bypass

$$
\sum_{t\in Trucks(rd)} zd_{rd,t} \leq 1 \qquad \forall rd
$$

Inbound Bin-Pack Lifting

$$ltl_o + \sum_{cp \in \{Cross-Docks\}} iltl_{o,cp} + \sum_{r \in \left\{ \begin{smallmatrix} inbound\,routes\,stoping\,at \\ location\,of\,order\,o \end{smallmatrix} \right\}} \sum_{t \in Trucks(r)} z_{r,t} + \sum_{rd \in \left\{ \begin{smallmatrix} Bypass\,routes\,stopping \\ at\,location\,of\,order\,o \end{smallmatrix} \right\}} \sum_{t \in Trucks(rd)} zd_{rd,t} \geq 1$$

Outbound Bin-pack Lifting

$$ltl_o + \sum_{cp \in \{Cross-Docks\}} oltl_{o,cp} + \sum_{r' \in \left\{ \begin{smallmatrix} outbound\,routes\,stoping\,at \\ location\,of\,order\,o \end{smallmatrix} \right\}} \sum_{t \in Trucks(r')} z_{r',t} + \sum_{rd \in \left\{ \begin{smallmatrix} Bypass\,routes\,delivering \\ at\,location\,of\,order\,o \end{smallmatrix} \right\}} \sum_{t \in Trucks(rd)} zd_{rd,t} \geq 1$$

Inbound Cover Constraints (all orders may be required to be routed inbound)

$$ltl_o + \sum_{cp \in \{Cross-Docks\}} iltl_{o,cp} + \sum_{r \in \{Inbound\,routes\}} x_{o,r} + \sum_{rd \in \{Bypass\,routes\}} xd_{o,rd} = 1 \qquad \forall o \in \{orders\}$$

Outbound Cover Constraints (all orders may be required to be routed outbound)

$$ltl_o + \sum_{cp \in \{Cross-Docks\}} oltl_{o,cp} + \sum_{r' \in \{outbound\,routes\}} y_{o,r'} + \sum_{rd \in \{Bypass\,routes\}} xd_{o,rd} = 1 \qquad \forall o \in \{orders\}$$

Cross-dock continuity (all orders routed into a cross-dock may be required to be routed out of the cross-dock)

$$iltl_{o,cp} + \sum_{r \in \{inbound\,routes\,at\,cp\}} x_{o,r} = oltl_{o,cp} + \sum_{r' \in \{outbound\,routes\,at\,cp\}} y_{o,r} \qquad \forall o \in \{orders\}, \forall cp \in \{Cross-Docks\}$$

Inbound Volume

$$\sum_{o \in \left\{ \begin{smallmatrix} Orders\,at\,locations \\ visited\,by\,route\cdots r \end{smallmatrix} \right\}} V_o x_{o,r} \leq \sum_{t \in Trucks(t)} TV_t\, z_{r,t} \qquad \forall r$$

Inbound Weight

$$\sum_{o \in \left\{ \begin{smallmatrix} Orders\,at\,locations \\ visited\,by\,route\cdots r \end{smallmatrix} \right\}} W_o x_{o,r} \leq \sum_{t \in Trucks(r)} TW_t\, z_{r,t} \qquad \forall r$$

Outbound Volume

$$\sum_{o \in \left\{ \substack{Orders\ at\ locations \\ visited\ by\ route \cdots r'} \right\}} V_o y_{o,r'} \le \sum_{t \in Trucks(r')} TV_t\, z'_{r',t} \qquad \forall r'$$

Outbound Weight

$$\sum_{o \in \left\{ \substack{Orders\ at\ locations \\ visited\ by\ route \cdots r'} \right\}} W_o y_{o,r'} \le \sum_{t \in Trucks(r')} TW_r\, z'_{r',t} \qquad \forall r'$$

Bypass Route Volume

$$\sum_{o \in \left\{ \substack{Orders\ for\ pick-up\ at \\ locations\ visited\ by\ route\ rd} \right\}} V_o xd_{o,rd} \le \sum_{t \in Trucks(rd)} TV_{rd}\, zd_{rd,t} \qquad \forall rd$$

Bypass Route Weight

$$\sum_{o \in \left\{ \substack{Orders\ for\ pick-up\ at \\ locations\ visited\ by\ route\ rd} \right\}} W_o xd_{o,rd} \le \sum_{t \in Trucks(rd)} TW_{rd}\, zd_{rd,t} \qquad \forall rd$$

Inbound Location Tie (an implicit modeling constraint that may be applied to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{o \in O(r,l)} x_{o,r} \ge \sum_{t \in Trucks(r)} z_{r,t} \qquad \forall r, l : l \in stops(r)$$

Outbound Location Tie (an implicit modeling constraint that may be applied to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{o \in O(r,l)} y_{o,r'} \ge \sum_{t \in Trucks(r')} z'_{r',t} \qquad \forall r', l : l \in stops(r')$$

Bypass Route Location Tie (an implicit modeling constraint that may be applied to ensure that the model does not inadvertently assign more routes to trucks at a location than it has assigned to orders at that location)

$$\sum_{o \in O(rd,l)} xd_{o,rd} \geq \sum_{t \in Trucks(rd)} zd_{rd,t} \qquad \forall rd, l : l \in stops(rd)$$

Cross-Dock Time Feasibility (may be applied to ensure that orders traveling through a cross dock will be able to meet their timing constraints and requirements)

$$ta_{cp,r} + HT_{o,cp}\left(x_{o,r} + y_{o,r'} - 1\right) \leq td_{cp,r'} + 2\overline{T}\left(1 - x_{o,r}\right) + 2\overline{T}\left(1 - y_{o,r'}\right) \quad \forall o, cp, r \in R(o), r' \in R'(o)$$

Cross-Dock Time Feasibility Inbound order routed by itself

$$tia_{o,cp} + HT_{o,cp}\left(iltl_{o,cp} + y_{o,r'} - 1\right) \leq td_{cp,r'} + 2\overline{T}\left(1 - iltl_{o,cp}\right) + 2\overline{T}\left(1 - y_{o,r'}\right) \quad \forall o, cp, r' \in R'(o)$$

Cross-Dock Time Feasibility Outbound order routed by itself

$$ta_{cp,r} + HT_{o,cp}\left(x_{o,r} + oltl_{o,cp} - 1\right) \leq tod_{o,cp} + 2\overline{T}\left(1 - x_{o,r}\right) + 2\overline{T}\left(1 - oltl_{o,cp}\right) \qquad \forall o, cp, r \in R(o)$$

Inbound Loading Time Constraints

$$ta_{s,r} + \sum_{o \in O(r,s)} \theta_o \, x_{o,r} \leq td_{s,r} \qquad \forall r, s : s \in stops(r)$$

Outbound Unloading Time Constraints

$$ta_{s',r'} + \sum_{o \in \{orders\ delivered\ at\ s'\}} \vartheta_o \, y_{o,r'} \leq td_{s',r'} \qquad \forall r', s' : s' \in delivery\ stops(r')$$

Inbound Travel Time Constraints

$$td_{s,r} + \tau_{s,s+1} z_r \leq ta_{s+1,r} \qquad \forall r, s : s \in stops(r)$$

Outbound Travel Time Constraints

$$td_{s',r'} + \tau_{s',s'+1} z'_{r'} \leq ta_{s'+1,r'} \qquad \forall r', s' : s' \in stops(r')$$

Inbound Pick-Up Start Time Constraint

$$td_{s,r} \geq (a_o + \theta_o) x_{o,r} \qquad \forall r, s : s \in stops(r) \quad o \in Orders(s)$$

Inbound Pick-Up End Time Constraint

$$ta_{s,r} \leq b_o x_{o,r} + \overline{T}(1 - x_{o,r}) \qquad \forall r, s : s \in stops(r) \quad o \in Orders(s)$$

Outbound Delivery Time Constraint

$$ta_{s',r'} \leq b'_o y_{o,r'} + \overline{T}(1 - y_{o,r'}) \qquad \forall r', s' = LastStop(r') \quad o \in Orders(s')$$

Inbound Loading Time Constraints

$$ta_{s,r} + \sum_{o \in O(r,s)} \theta_o x_{o,r} \leq td_{s,r} \qquad \forall r, s : s \in stops(r)$$

Outbound Unloading Time Constraints

$$ta_{s',r'} + \sum_{o \in \{orders\ delivered\ at\ s'\}} \vartheta_o y_{o,r'} \leq td_{s',r'} \qquad \forall r', s' : s' \in delivery\ stops(r')$$

Bypass Loading Time Constraints

$$ta_{s,rd} + \sum_{o \in O(rd,s)} \theta_o x_{o,rd} \leq td_{s,rd} \qquad \forall rd, s : s \in stops(rd)$$

Bypass Unloading Time Constraints

$$ta_{s,rd} + \sum_{o \in \{orders\ delivered\ at\ s\}} \vartheta_o y_{o,rd} \leq td_{s,rd} \qquad \forall rd, s : s \in delivery\ stops(rd)$$

Stop-Time arrival period cover (may be used to ensure that for each arrival time period defined, the model assigns the appropriate number of trucks)

$$\sum_{p \in P(s,r)} u_{p,s,r} = \sum_{t \in Trucks(r)} z_{r,t}$$

$$\forall r \in \{Inbound\ routes\} \cup \{Outbound\ routes\} \cup \{Bypass\ routes\}, s \in stops(r)$$

Stop departure period cover (may be used to ensure that for each departure time period defined, the model assigns the appropriate number of trucks)

$$\sum_{p \in P(s,r)} v_{p,s,r} = \sum_{t \in Trucks(r)} z_{r,t}$$

$\forall r \in \{Inbound\ routes\} \cup \{Outbound\ routes\} \cup \{Bypass\ routes\}, s \in stops(r)$

Stop arrival time window

$$\alpha_{p,r,s} u_{p,r,s} \le ta_{s,r} \le \beta_{p,r,s} u_{p,r,s} + \overline{T}(1 - u_{p,r,s})$$

$\forall r \in \{Inbound\ routes\} \cup \{Outbound\ routes\} \cup \{Bypass\ routes\}, s \in stops(r), p \in P(r,s)$

Stop departure time window

$$\alpha_{p,r,s} v_{p,r,s} \le ta_{s,r} \le \beta_{p,r,s} v_{p,r,s} + \overline{T}(1 - v_{p,r,s})$$

$\forall r \in \{Inbound\ routes\} \cup \{Outbound\ routes\} \cup \{Bypass\ routes\}, s \in stops(r), p \in P(r,s)$

Waiting time before a stop - inbound routes

$$ta_{s+1,r} - td_{s,r} - \tau_{s,s+1} \sum_{t \in Trucks(r)} z_{r,t} \le WT_{s+1} \sum_{t \in Trucks(r)} z_{r,t} + \overline{T}\left(1 - \sum_{t \in Trucks(r)} z_{r,t}\right) \qquad \forall r, s \in stops(r)$$

Waiting time before a stop - outbound routes

$$ta_{s+1,r'} - td_{s,r'} - \tau_{s,s+1} \sum_{t \in Trucks(r')} z'_{r',t} \le WT_{s+1} \sum_{t \in Trucks(r')} z'_{r',t} + \overline{T}\left(1 - \sum_{t \in Trucks(r')} z'_{r',t}\right) \qquad \forall r', s \in stops(r')$$

Waiting time before a stop - by-pass routes

$$ta_{s+1,rd} - td_{s,rd} - \tau_{s,s+1} \sum_{t \in Trucks(rd)} zd_{rd,t} \le WT_{s+1} \sum_{t \in Trucks(rd)} zd_{rd,t} + \overline{T}\left(1 - \sum_{t \in Trucks(rd)} zd_{rd,t}\right) \qquad \forall rd, s \in stops(rd)$$

In accordance with the above description and discussions, the present invention may enable one who practices it to quickly solve shipping or related problems of large sizes without having to sacrifice a quality of a result. It should be noted that, as discussed above, the methods disclosed herein in accordance with the invention need not include all

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disclosed steps or necessarily be practiced in a described order. For example, various constraints and related features of the above processes are exemplary, and solutions may be obtained in accordance with the invention through the use of a portion of the disclosed features, variations thereon, or others. In addition, it is contemplated that various method steps disclosed in one example or embodiment may be combined with one or more other steps in one or more other examples or embodiments, to achieve a method in accordance with the invention. For these and other reasons, the inventions disclosed should not be limited to embodiments presented herein, but rather are defined more generally, as by the appended claims.

Throughout this specification and claims which follow, unless the context requires otherwise, the word "comprise", and variations such as "comprises" and "comprising", will be understood to imply the inclusion of a stated integer or step or group of integers or steps but not the exclusion of any other integer or step or group of integers or steps.

The reference in this specification to any prior publication (or information derived from it), or to any matter which is known, is not, and should not be taken as an acknowledgment or admission or any form of suggestion that that prior publication (or information derived from it) or known matter forms part of the common general knowledge in the field of endeavour to which this specification relates.

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THE CLAIMS DEFINING THE INVENTION ARE AS FOLLOWS:-

1.   A computer-implemented method, for shipment planning optimization, comprising:

accessing a master optimization program to which, in an initialization step, a set of

5   initial routes is provided, each initial route starting at a pickup location and stopping at a

delivery location, said set of initial routes being represented by a set of initial variables in

the master optimization program;

using a computer to generate another set of routes through an iteration of the master

optimization program including said set of initial variables, said another set of routes

10   including at least one new route based on the pickup and delivery locations of one of the

initial routes and on an additional pickup/dropoff location, wherein said another set of

routes is represented by another set of variables;

using a computer to select or reject at least one of the another set of routes based on

a selection criterion;

15   iteratively using a computer, until an end condition is met, to generate further sets

of routes by iterations of the master optimization program, and to select or reject at least

one of the further routes in each iteration based on said selection criterion;

using a computer to add lifting inequalities to the master optimization program to

discard one or more of the selected routes based on selected constraints;

20   using a computer to create route skeletons, each route skeleton including a

sequence of stops with no orders assigned, based on the routes generated by the master

optimization program following the adding of the lifting inequalities; and

using a computer to assign individual orders to the route skeletons based on

optimization of an objective function to develop a shipment planning solution.

25

2.   The method of claim 1, wherein the end condition is that no more new routes are

generated that fulfil selected criteria, or that a maximum number of stops per route is

reached.

30   3.   The method of claim 1 or 2, wherein the assigning the individual orders to the route

skeletons is performed based on pickup and/or delivery times of the pickup locations, the

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delivery locations and/or the orders.

4.   The method of any one of claims 1–3, including assigning an order that does not fit one of the route skeletons to a route on its own.

5

5.   The method of any one of claims 1–4, wherein the adding of the lifting inequalities is to discard one of the selected routes if the one route is more expensive than the sum of the costs of sending every possible order that could potentially travel on the one route by itself.

10

6.   The method of any one of claims 1–5, wherein the generating of the another set of routes is based on aggregated quantities of orders on the initial routes.

7.   The method of claim 6, wherein the aggregated quantities include aggregated

15   volume, weight, cross-point capacity, or baseline cost associated with the orders on each initial route.

8.   The method of any one of claims 1–7, wherein the selection criterion is based on a cost of the another set of routes.

20

9.   A computer system, for generating planned routes, comprising:
        means for performing the method of any one of claims 1–8.

10.   The system of claim 9, wherein said system includes means for solving an

25   optimization problem.

11.   A computer-implemented method substantially as hereinbefore described with reference to the accompanying drawings.

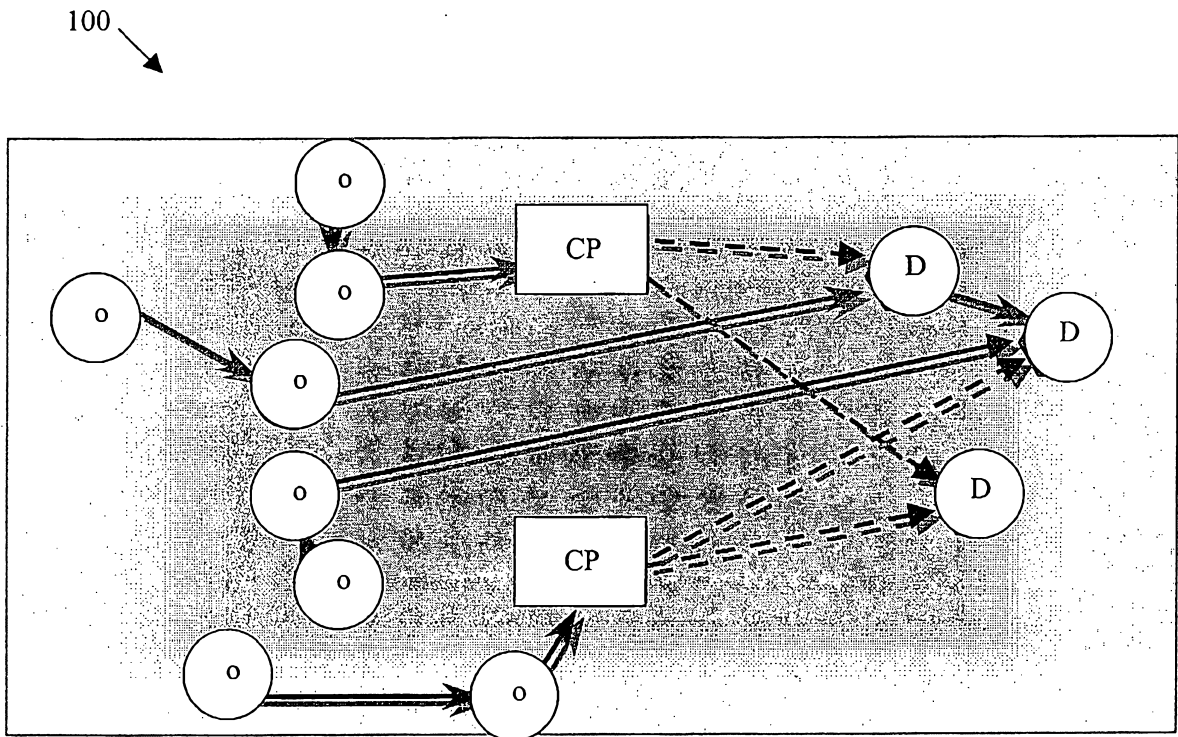30   12.   A computer system substantially as hereinbefore described with reference to the accompanying drawings.
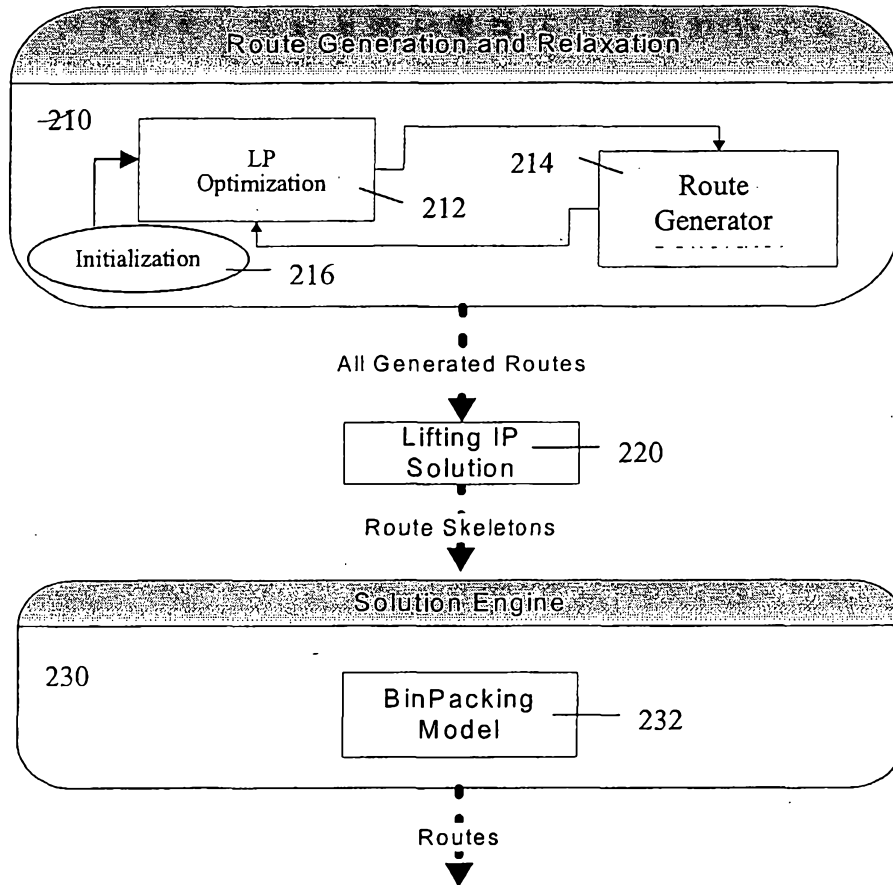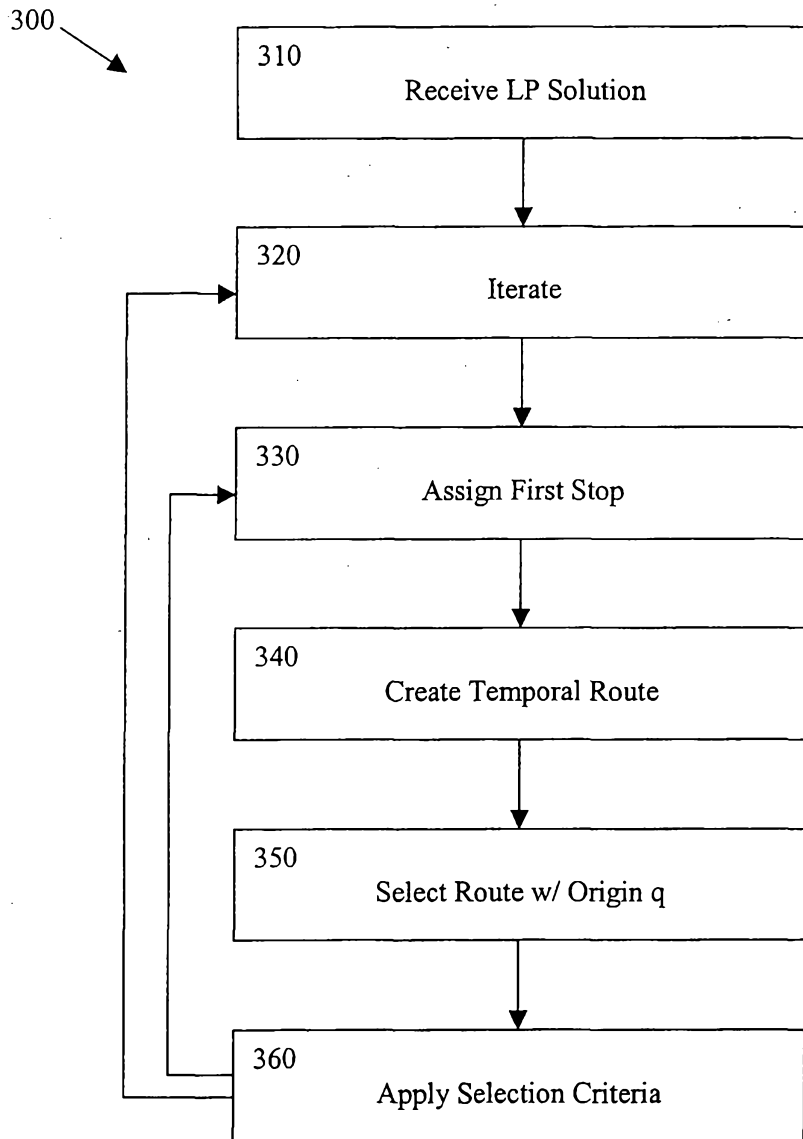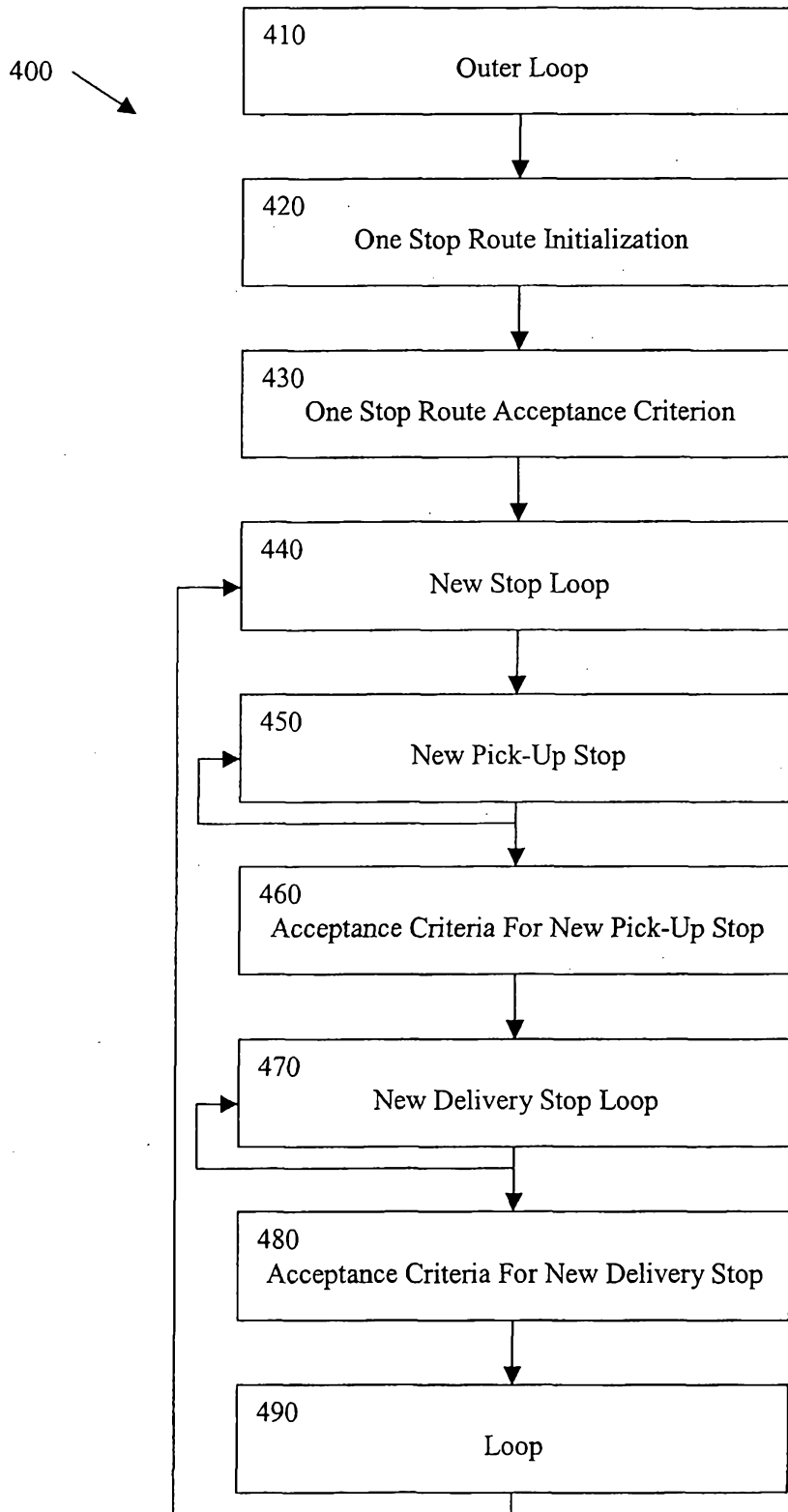
FIGURE 1

200

FIGURE 2

300

310
Receive LP Solution

320
Iterate

330
Assign First Stop

340
Create Temporal Route

350
Select Route w/ Origin q

360
Apply Selection Criteria

FIGURE 3

400

| 410 |
| Outer Loop |

| 420 |
| One Stop Route Initialization |

| 430 |
| One Stop Route Acceptance Criterion |

| 440 |
| New Stop Loop |

| 450 |
| New Pick-Up Stop |

| 460 |
| Acceptance Criteria For New Pick-Up Stop |

| 470 |
| New Delivery Stop Loop |

| 480 |
| Acceptance Criteria For New Delivery Stop |

| 490 |
| Loop |

FIGURE 4

FIGURE 5