May 16, 1967 M. V. SCHNEIDER 3,320,556

IMPEDANCE TRANSFORMER

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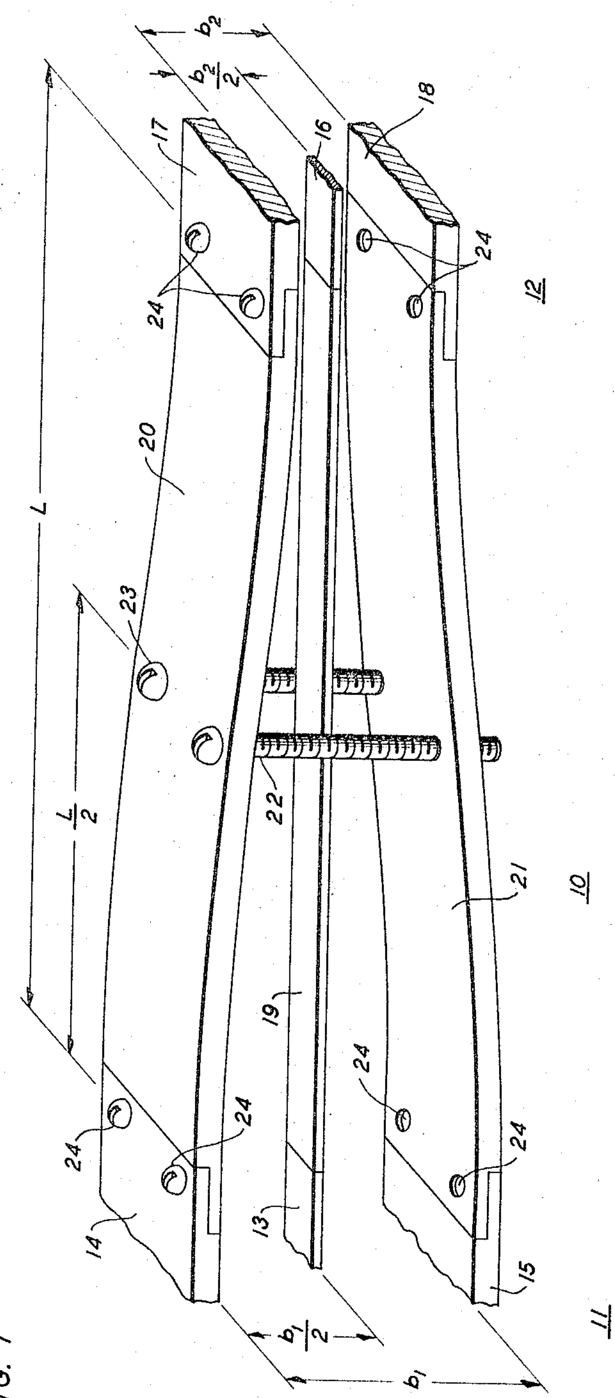

FIG. 1

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IMPEDANCE TRANSFORMER

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FIG. 2

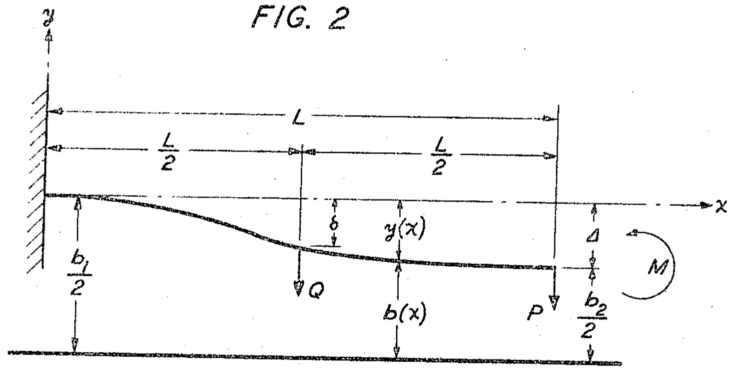

FIG. 3

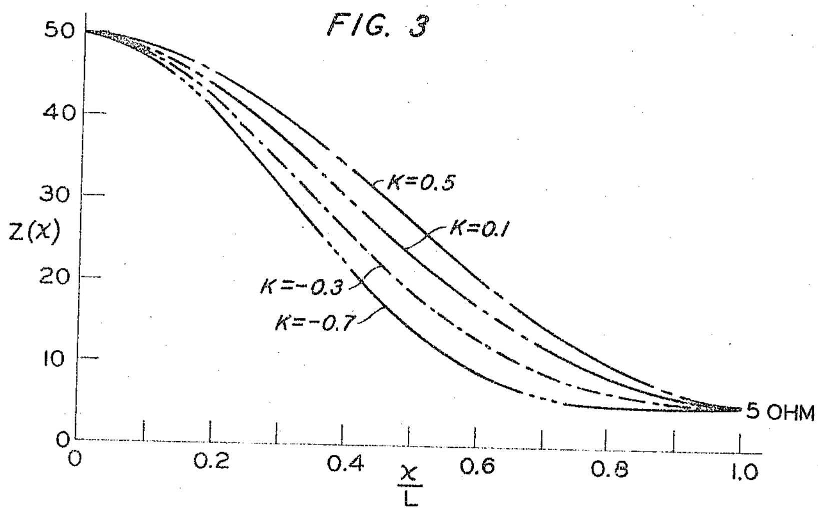

FIG. 4

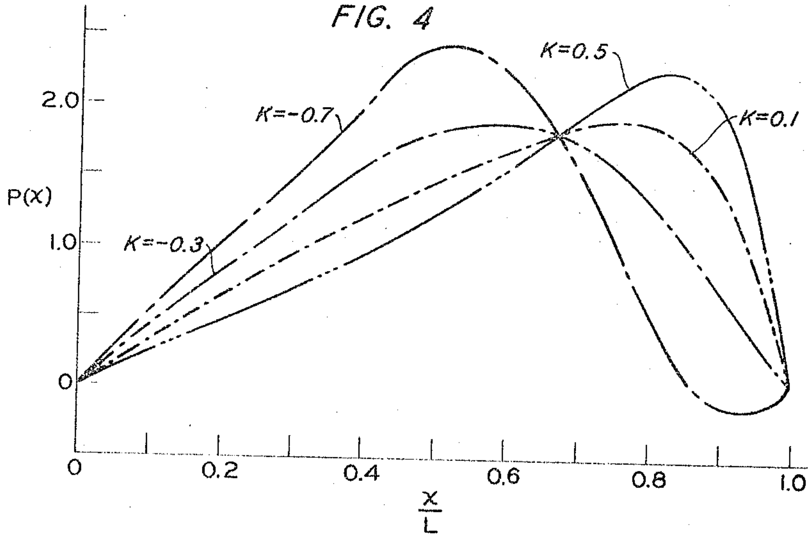

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FIG. 5

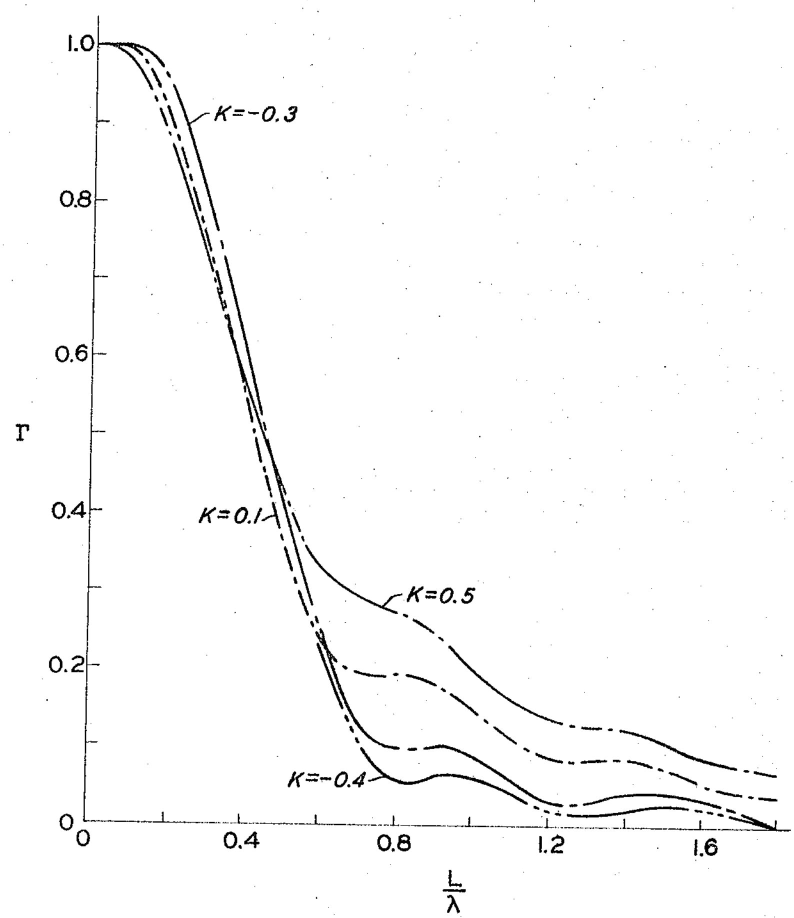


FIG. 6

K=0.5

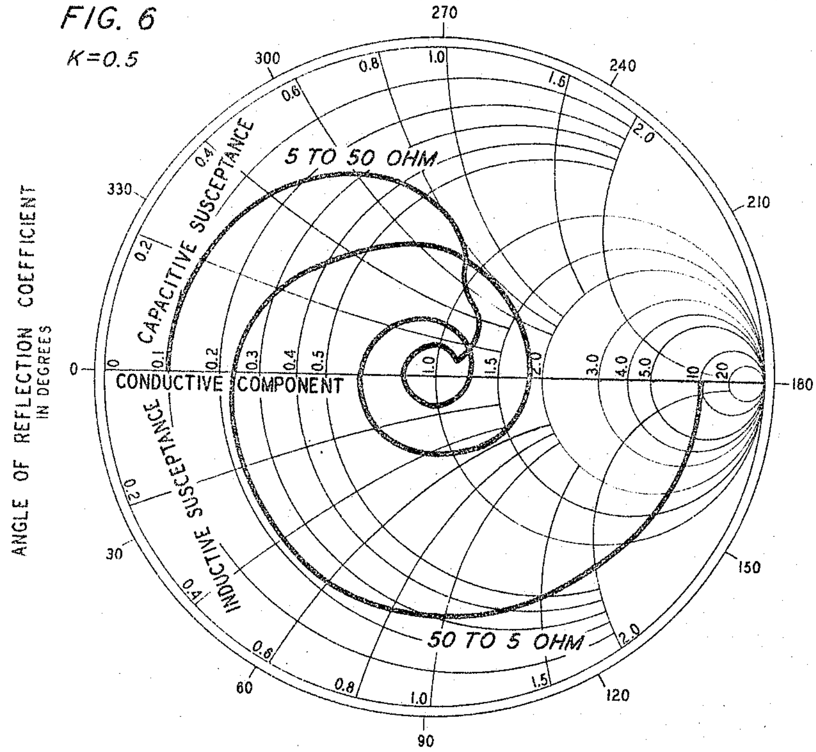
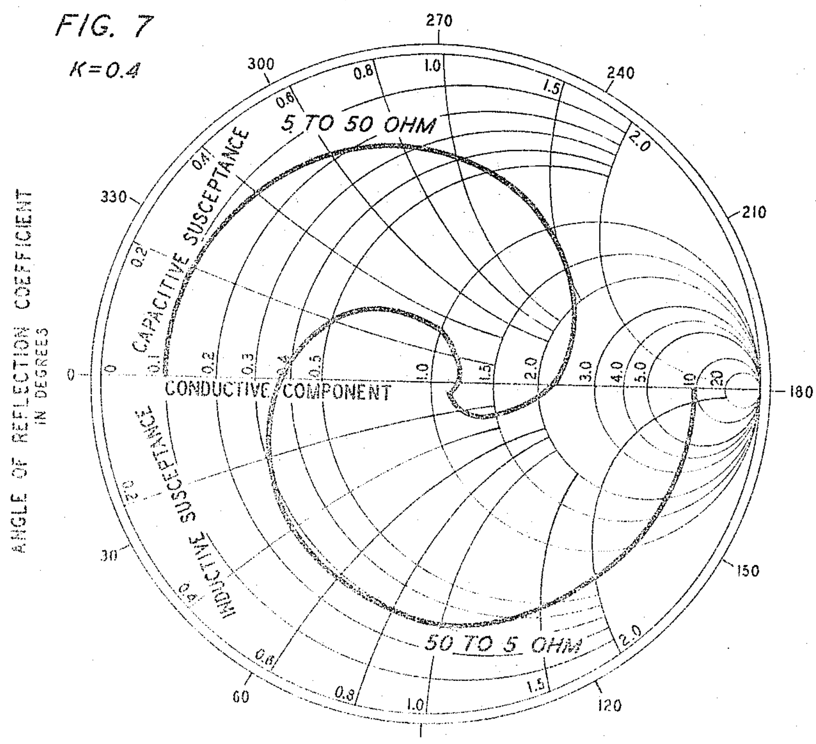


FIG. 7

K=0.4



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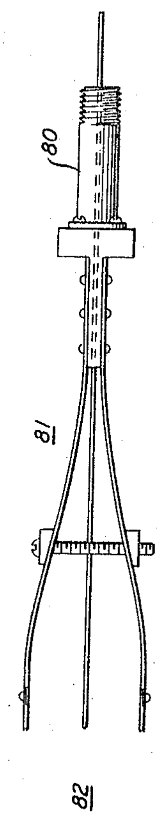

FIG. 8

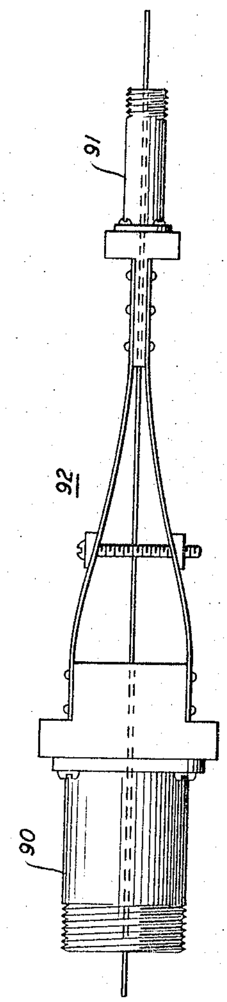

FIG. 9

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IMPEDANCE TRANSFORMER

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Filed May 23, 1963, Ser. No. 282,610
7 Claims. (Cl. 333—34)

This invention relates to impedance matching transformers and, more particularly, to tapered sections for interconnecting sections of balanced strip transmission line having different ground plane spacings.

Gradually tapered, inhomogeneous transmission lines are often used for interconnecting transmission lines operating at different impedance levels. Since the reflection coefficient of a transmission line taper has a high-pass characteristic, it is well suited for the transmission of wave energy over wide frequency bands. While solutions of the line equations have been found for certain special cases (see, for example, "A Transmission Line Taper of Improved Design," by R. W. Klopfenstein, Proceedings of the I.R.E., January 1956, pages 31–35 and "The Optimum Tapered Transmission Line Matching Section," by R. E. Collins, Proceedings of the I.R.E., April 1956, pages 539–548), the variety of tapers for which this can be done is limited. A further difficulty in the design and construction of impedance transformers arises when the solution obtained is to be translated into a piece of hardware. Typically, a mandrel is built and the transformer electroformed on the mandrel. Obviously, this is an expensive and time consuming procedure which must be repeated if the original design does not perform as anticipated or whenever the circuit conditions have been changed.

It is, accordingly, an object of this invention to simplify the design and construction of impedance transformers for use with balanced strip transmission lines.

A further object of this invention is an adjustable impedance transformer.

When impedance transformers are used in conjunction with active circuit elements, such as tunnel diodes, the phase of the reflection coefficient, as well as its amplitude, assumes considerable importance. It is, in such cases, highly desirable, from the stability point of view, to have a reflection coefficient for which the phase angle changes gradually with frequency.

Therefore, a further object of this invention is an impedance transformer whose reflection coefficient has a phase angle which varies gradually as a function of frequency.

In accordance with the invention the impedance along a section of balanced strip transmission line is caused to vary as a function of the deflection produced along the outer conductors of the line where the deflection of each conductor is that of a beam under the influence of concentrated loads. In particular, the outer conductors or ground planes of a section of strip transmission line are fastened at both ends to supporting members having different transverse dimensions. Thus, the spacing of the transformer ground planes at each end is fixed and is equal to that of the respective supporting member. In addition, the transformer ground planes are parallel to each other and to the center conductor at both ends.

In the region intermediate the ends, the transformer ground conductors assume a tapered configuration corresponding to that of a beam under the influence of concentrated loads in accordance with the laws of mechanics. It has been found that the taper thus produced has a reflection coefficient which compares favorably with the best prior art tapers. Further improvements in the operating characteristics of the transformer are obtained by modifying the shape of the taper by the application of additional transverse forces along the ground conductors.

It is an advantage of such a transformer that its operating characteristics can be continuously varied thus making it a relatively simple matter to adjust precisely the transformer at any particular operating frequency.

These and other objects and advantages, the nature of the present invention, and its various features, will appear more fully upon consideration of the various illustrative embodiments now to be described in detail in connection with the accompanying drawings, in which:

FIG. 1 shows a beam impedance transformer in accordance with the invention;

FIG. 2, given for the purposes of explanation, shows the forces acting upon each of the ground conductors of the transformer illustrated in FIG. 1 and the deflection produced by these forces;

FIG. 3 shows the variation in the impedance along a beam transformer for different values of transverse loading;

FIG. 4 shows the distribution of reflections along a beam transformer for different values of transverse loading;

FIG. 5 shows the variation in the reflection coefficient from a beam transformer as a function of length for different values of transverse loading;

FIGS. 6 and 7 are admittance Smith charts for a beam transformer having a ten to one impedance transformation for two different values of transverse loading; and

FIGS. 8 and 9 show, respectively, a beam impedance transformer for connecting a circular coaxial cable to a strip transmission line of a different impedance, and a beam impedance transformer for connecting two circular coaxial cables of difference impedance.

Referring to FIG. 1, there is shown an impedance transformer **10**, of length L, connecting a first strip transmission line **11** to a second strip transmission line **12**. Line **11** comprises an inner conductor **13** and a pair of parallel, planar, outer conductive members, or ground planes, **14** and **15** equally spaced a distance $b_{1/2}$ from the inner conductor **13**. As indicated in the drawings, the total spacing between outer conductors **14** and **15** is b_1. Line **12** similarly comprises an inner conductor **16** and a pair of parallel, planar, outer conductive members **17** and **18** equally spaced a distance $b_{2/2}$ from inner conductor **16**. The total spacing between outer conductors **17** and **18** is b_2, where b_2 is less than b_1. Suitable means, not shown, are provided for maintaining the spaced relationship between the several conductors of lines **11** and **12** in a manner well known in the art.

Because of the unequal spacing between the outer conductors of the two lines, their respective characteristic impedances are different. Means must therefore be provided to interconnect the two lines with an impedance transformer which, preferably, has a low reflection coefficient over the range of operating frequencies. In accordance with the invention, this coupling is achieved by means of the beam transformer **10** which comprises an inner conductor **19** which connects conductor **13** to conductor **16**, and the outer, tapered conductive members **20** and **21** which connect outer conductors **14** and **15** to outer conductors **17** and **18**, respectively. The inner conductor **19** can be a separate element which connects to the adjacent inner conductors or, alternatively, conductors **13**, **19** and **16** can be a single, continuous member.

The taper produced in transformer **10** is the natural deformation assumed by the planar conductors **20** and **21** when they are fastened at both ends to the adjacent outer conductors of lines **11** and **12**. More particularly, members **20** and **21** are fastened such that the distances between them at the respective ends of the transformer are equal to the ground plane spacings of lines **11** and **12**. In addition, members **20** and **21** initially extend parallel to each other and to the center conductors **13**, **16** and **19** at both ends. That is, the angle between the tangent to the inner surface of members **20** and **21** and the inner conductors at each of the transformer ends is zero.

One simple way of making the above-described connection is to notch the ends of the outer conductors of lines **11** and **12** and of transformer **10**, as shown in FIG. 1, and to fasten them together with screws **24**.

Located midway along the transformer are the two additional screws **22** and **23** spaced transversely across the transformer, each of which passes through member **20** and threads into member **21**. Screws **22** and **23** are used to modify the taper of members **20** and **21** for reasons which will be explained hereinbelow.

The configuration assumed by the members **20** and **21**, when mounted as described above, can be computed by considering a beam, as shown in FIG. 2, secured at one end and acted upon by a force P and a twisting moment M at the other, free end. In addition, there is a second force Q at the center of the beam produced by the action of screws **22** and **23**.

The deflection y at any point x along the beam is given by the second order differential equation

$$\frac{d^2y}{dx^2}=\frac{M}{EI} \qquad (1)$$

where M is the total bending moment at any point x along the beam,
E is the modulus of elasticity of the beam, and
I is the moment of inertia of the beam.

Solving Equation 1 for the conditions depicted in FIG. 2 gives

$$y=\frac{1}{EI}\left[-\frac{P+Q}{6}x^3+\left(\frac{PL}{2}+\frac{QL}{4}-\frac{M}{2}\right)x^2\right]$$

for

$$0\leq x\leq\frac{L}{2} \qquad (2)$$

and

$$y=\frac{1}{EI}\left[-\frac{P}{6}x^3+\left(\frac{PL}{2}-\frac{M}{2}\right)x^2+\frac{QL^2}{8}x-\frac{QL^3}{48}\right]$$

for

$$\frac{L}{2}\leq x\leq L \qquad (3)$$

Since we require that the derivative dy/dx vanish at $x=O$ and $x=L$, we further get that

$$M=\frac{PL}{2}+\frac{QL}{8} \qquad (4)$$

As no generality is lost, we put $L=1$ and $EI=1$. Doing this and substituting for M the value given by Equation 4, we get

$$y=-\frac{1}{6}(P+Q)x^3\left(\frac{P}{4}+\frac{3Q}{16}\right)x^2 \qquad (5)$$

for

$$0\leq x\leq\frac{1}{2}$$

and

$$y=-\frac{P}{6}x^3+\left(\frac{P}{4}-\frac{Q}{16}\right)x^2+\frac{Q}{8}x-\frac{Q}{48} \qquad (6)$$

$$\frac{1}{2}\leq x\leq 1$$

The deflection at $x=1$, from Equation 6, is given by

$$y_{x=1}=\Delta=\frac{2P+Q}{24} \qquad (7)$$

The impedance of a balanced strip transmission line can be calculated from the formula given by S. B. Cohn in his paper "Problems in Strip Transmission Lines," published in the I.R.E. Transactions on Microwave Theory and Techniques, MTT-3, pages 119–126, 1955, as follows:

$$Z=\frac{1}{4}\sqrt{\frac{\mu}{\epsilon}}\frac{1}{\frac{w}{b-t}+\frac{C_f}{0.885\epsilon_r}} \qquad (8)$$

where μ is the permeability of the dielectric material between the center conductor and the outer conductors,
ϵ is the dielectric constant of this dielectric,
w is the width of the center conductor,
b is the ground plane spacing,
t is the thickness of the inner conductor,
C_f is the fringe field capacitance per unit length of line, and
ϵ_r is the relative dielectric constant of the dielectric.

Assuming that C_f is negligible and that t is much smaller than b, Equation 8 reduces to

$$Z=\frac{1}{4}\sqrt{\frac{\mu}{\epsilon}}\cdot\frac{b}{w} \qquad (9)$$

from which it is seen that the impedance is proportional to the ground plane spacing b. Because of this linear relationship, we can more generally express this relationship as a function of the distance x along the taper as

$$X(x)=c\cdot b(x) \qquad (10)$$

where

$$c=\frac{1}{4}\sqrt{\frac{\mu}{\epsilon}}\cdot\frac{1}{w}$$

If Z_1 and Z_2 are the impedances at the ends of the transformer for ground plane spacing b_1 and b_2, respectively, we may write, in view of Equation 9, that

$$\frac{Z_1-Z(x)}{Z_1-Z_2}=\frac{b_1-b(x)}{b_1-b_2} \qquad (11)$$

or that

$$Z(x)=Z_1-\frac{b_1-b(x)}{b_1-b_2}(Z_1-Z_2) \qquad (12)$$

From FIG. 2 it follows that

$$\left.\begin{array}{l}2y(x)=b_1-b(x)\\2\Delta=b_1-b_2\end{array}\right\} \qquad (13)$$

and

$$Z(x)=Z_1\frac{y(x)}{\Delta}(Z_2-Z_1) \qquad (14)$$

If the transformer is tuned by changing the center deflection δ by means of tuning screws **22** and **23**, the forces P and Q change. However, because of the end restraints, the deflection Δ at the end $x=1$ remains constant. From Equation 7 we then get that

$$2P+Q=24\Delta=C \qquad (15)$$

Rewriting Equation 15 as

$$2\frac{P}{C}+\frac{Q}{C}=1 \qquad (16)$$

we obtain the following tabulation which is useful for selecting the optimum taper arrangement:

TABLE I

k=P/C	Q/C	Δ/δ
0.5	0.0	2.00
0.4	0.2	1.90
0.3	0.4	1.82
0.2	0.6	1.74
0.1	0.8	1.67
0.0	1.0	1.60
−0.1	1.2	1.54
−0.2	1.4	1.48
−0.3	1.6	1.43
−0.4	1.8	1.38
−0.5	2.0	1.33
−0.6	2.2	1.29
−0.7	2.4	1.25

where δ is the deflection at the center of the beam

$$\left(x=\frac{1}{2}\right)$$

and is given by

$$\delta=\frac{1}{24}\left(P+\frac{5}{8}Q\right) \quad (17)$$

and

$$\frac{\Delta}{\delta}=\frac{2P+Q}{P+\frac{5}{8}Q}=\frac{8}{5-2k} \quad (18)$$

The choice of k depends upon the type of transformer needed for a particular application. It will be shown hereinbelow that transformers with k in the range of 0.5 have a reflection coefficient whose phase characteristic is particularly useful for active circuits requiring a positive susceptance and a nearly constant conductance over an extended frequency range. On the other hand, different values of k are needed for tapers having a minimum length L/λ for a specified reflection coefficient. Such tapers are referred to as "optimum" tapers. It is a common feature of optimum tapers (i.e., cosine square, Gaussian, and Dolph-Chetycheff tapers) that the impedance Z_C at the center is the geometric mean of the end impedances. Hence, if we require that

$$Z_C=\sqrt{Z_1Z_2} \quad (19)$$

for the beam transformer, we obtain

$$\frac{\Delta}{\delta}=\frac{Z_1-Z_2}{Z_1-\sqrt{Z_1Z_2}} \quad (20)$$

from which we derive that

$$k_{opt}\approx\frac{8\sqrt{\frac{Z_1}{Z_2}}-3\frac{Z_1}{Z_2}-5}{\frac{Z_1}{Z_2}-1} \quad (21)$$

The deflection of the taper for any value of k is obtained by the substitution of $P=kC$ and $Q=(1-2k)C$ in Equations 5 and 6).

The reflection coefficient of any beam transformer can be calculated by means of methods described by F. Bolinder ("Fourier Transformers in the Theory of Inhomogeneous Transmission Lines," Acta Polytechnica 85, pages 1–83, 1951), S. I. Orlov ("Concerning the Theory of Nonuniform Transmission Lines," Soviet Physics 1, pages 2284–2294, 1956), C. B. Sharpe ("An Alternative Derivation of Orlov's Synthesis Formula for Nonuniform Lines," Proceedings of the I.E.E., Monograph No. 483E, November 1961) and others. A large variety of such transformers with different ratios of Z_1 to Z_2 have been calculated and measured. For purposes of discussion and illustration, however, one particular taper with a ratio $Z_1/Z_2=10$ will be discussed in detail below.

A beam transformer from 50 ohms to 5 ohms was constructed with the following dimensions:

Balanced Strip Transmission Line	High Impedance Side (50 ohms)	Low Impedance Side (5 ohms)
Ground plane spacing b	b^1=300 mils	b^2=26.5 mils.
Width of center conductor w	427 mils	427 mils.
Thickness of center conductor t	2 mils	2 mils.

The taper had a length L of 6.70 cm. and the center conductor was supported by a dielectric spacer on the low impedance side.

Table II shows a comparison of the calculated reflection coefficient and the measured reflection coefficient for $k=0.5$ (without a centered tuning load).

TABLE II

L/λ (2 to 3.8 Kmc./sec.)	Computed Reflection Coefficient	Measured Reflection Coefficient
0.446	0.552	0.56
0.492	0.485	0.48
0.536	0.410	0.405
0.580	0.387	0.38
0.625	0.344	0.345
0.680	0.310	0.315
0 715	0.296	0.30
0.760	0.281	0.285
0.805	0.270	0.27
0.850	0.258	0.25

The results for beam transformers with nonvanishing tuning load ($Q\neq0$) also show a high degree of aggreement between the computed and the measured coefficient of reflection.

The impedance characteristic of a beam transformer for different tuning loads is shown in FIG. 3. The curve $k=0.5$ is obtained for $Q=0$ as pointed out previously. The function $P(x)$ shown in FIG. 4 represents the distribution of reflection along the line and is included to point out that $P(x)$ is unsymmetrical for $k=0.5$, having a maximum close to the low impedance side of the transformer. The case $k=-0.7$ cannot be realized physically but is included for completeness.

FIG. 5 is a plot of the reflection coefficient for different values of k. As can be seen from these curves, the optimum value of k is -0.4. That is, for a given coefficient of reflection, the shortest transformer is obtained with a loading force, Q, such that $k=-0.4$. It is interesting to note that for $k=-0.4$ a return loss of 20 decibels is obtained with a beam transformer of length $L/\lambda=0.72$. A cosine square taper of the same length and the same impedance transformation, on the other hand, has a return loss of only 14 decibels.

The phase of the reflection coefficient is plotted on an admittance Smith chart in FIGS. 6 and 7. As indicated hereinabove, the phase characteristic can be very important from the stability standpoint in the design of active circuits. In this regard it is highly desirable that the phase of the reflection coefficient change slowly or, when plotted on a Smith chart, spiral slowly. A typical example of this type of behavior is obtained for $k=0.5$ for the transformer as viewed from the 5 ohm side as shown in FIG. 6. It will be noted, however, that the coefficient of reflection is quite different, as viewed from the 50 ohm side, in that it spirals more rapidly. This is due to the highly unsymmetrical charcteristic of $P(x)$ for $k=0.5$ as shown in FIG. 4. $P(x)$, on the other hand, is much more symmetrical for $k=-0.4$, which gives rise to the more symmetrical curves of FIG. 7.

The beam transformer described above utilizes a centered tuning adjustment. It is understood, however, that additional tunning adjustments, longitudinally distributed along the taper, can be used to more particularly adjust the shape of the transformer.

While the embodiment of FIG. 1 is shown interconnecting two strip transmission lines, it is recognized that a beam transformer can also be used to interconnect a circular coaxial cable of a given impedance to either a strip transmission line of a different impedance or to another coaxial cable of a different impedance. The former of these arrangements is illustrated in FIG. 8 which shows a coaxial fitting **80** connected at one end of transformer **81** and a strip transmission line **82** connected at the other end. FIG. 9 shows a first coaxial fitting **90** connected at one end of transformer **92**, and a second coaxial fitting **91** connected at the other end. Thus, in all cases it is understood that the above-described arrangements are illustrative of a small number of the many possible specific embodiments which can represent application of the principles of the invention. Numerous and varied other arrangements can readily be devised in accordance with these principles by those skilled in the art without departing from the spirit and scope of the invention.

What is claimed is:

1. A tapered section of strip transmission line comprising an inner conductor and a pair of transversely spaced, outer conductors, said section having a first spacing between outer conductors at one end and a second different spacing at the other end, characterized in that the shape of said taper between said ends is that of a beam under the influence of concentrated loads.

2. A tapered section of strip transmission line comprising an inner conductor and a pair of transversely spaced outer conductors characterized in that the deflection y of said outer conductors at any point x along their respective lengths is given by the second order differential equation

$$\frac{d^2y}{dx^2}=\frac{M}{EI}$$

where M is the total bending moment at a point x along said outer conductors,

E is the modulus of elasticity of said outer conductors, and

I is the moment of inertia of said outer conductors.

3. The section according to claim **2** wherein the first derivative dy/dx equals zero at both ends of said section.

4. An impedance transformer comprising an inner conductor and a pair of outer conductors, said transformer having a first transverse spacing between said outer conductors at one end and a second different transverse spacing at the other end, said spacing varying gradually in the region between said ends in accordance with the natural deformation produced in planar conductors under concentrated loads, and means for varying said spacing at at least one point along said region.

5. The transformer according to claim **4** including a first strip transmission line having a first impedance connected to one end of said transformer and a second strip transmission line having a different impedance connected to the other end of said transformer.

6. The transformer according to claim **4** including a first coaxial cable having a first impedance connected to one end of said transformer and a second coaxial cable having a different impedance connected to the other end of said transformer.

7. The transformer according to claim **4** including a coaxial cable having a first impedance connected to one end of said transformer and a strip transmission line having a different impedance connected to the other end of said transformer.

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