

(19)
(12)

(KR)
(B1)

(51) 。 Int. Cl.⁷
G06F 7/52

(45)
(11)
(24)

2004 11 26
10-0458031
2004 11 11

(21) 10-2003-0016100
(22) 2003 03 14

(65)
(43)

10-2004-0081581
2004 09 22

(73) 416

(72) 113-902

2 198-23 201

834-18

(74)
:

(54)

가 $A \cdot B \cdot R - 1 \pmod N$ (n , $R=4^{m+2}$) (A) (B) / $(M+2)$ (,

, 가 2 가 (Booth recording) , .

1		1	
2	1		
3	1		가 (CSA)1
4	1		
5	1	CSA2	
6	1	가 (FA)	
7		2	
8	7		
9	7	CSA1	
10	7		
11	7	CSA2	
12	7	FA	
13			

(small - sized computer) (smart card) (mobile device),

가 가

RSA(Rivest - Shamir - Adleman), ElGamal, Schnorr

al Standard Organization)/IEC(International Electrotechnical Commission) 9796
 Gamal DSA(Digital Signature Standard)가 가 (GOSSTA
 NDART: GOST) KC-DSA가
 PKCS(Public Key Cryptography Standard)
 $m^e \bmod N$, $A \cdot B \bmod N$

RSA

, R. L. Rivest et al, 'A method for obtaining digital signatures and public-key cryptosystems,' Communications of the ACM, Vol. 21, pp. 120-126, 1978, P. L. Montgomery, 'Modular Multiplication without Trial Division,' Math. Of Comp., Vol. 44, No. 170, pp. 519-521, 1985, S. R. Dusse and B. S. Kaliski Jr., 'A cryptographic library for the Motorola DSP56000, ' Proc. Eurocrypt'90, pp. 230-244, 1990. Springer - Verlag, A. Bosselaers, R. Govaerts and J. Vandewalle, 'Comparison of three modular reduction functions,' Advances in Cryptology - CRYPTO'93, pp. 175-186, 1993

(Montgomery)

Cryptography, CRC Press, 1995

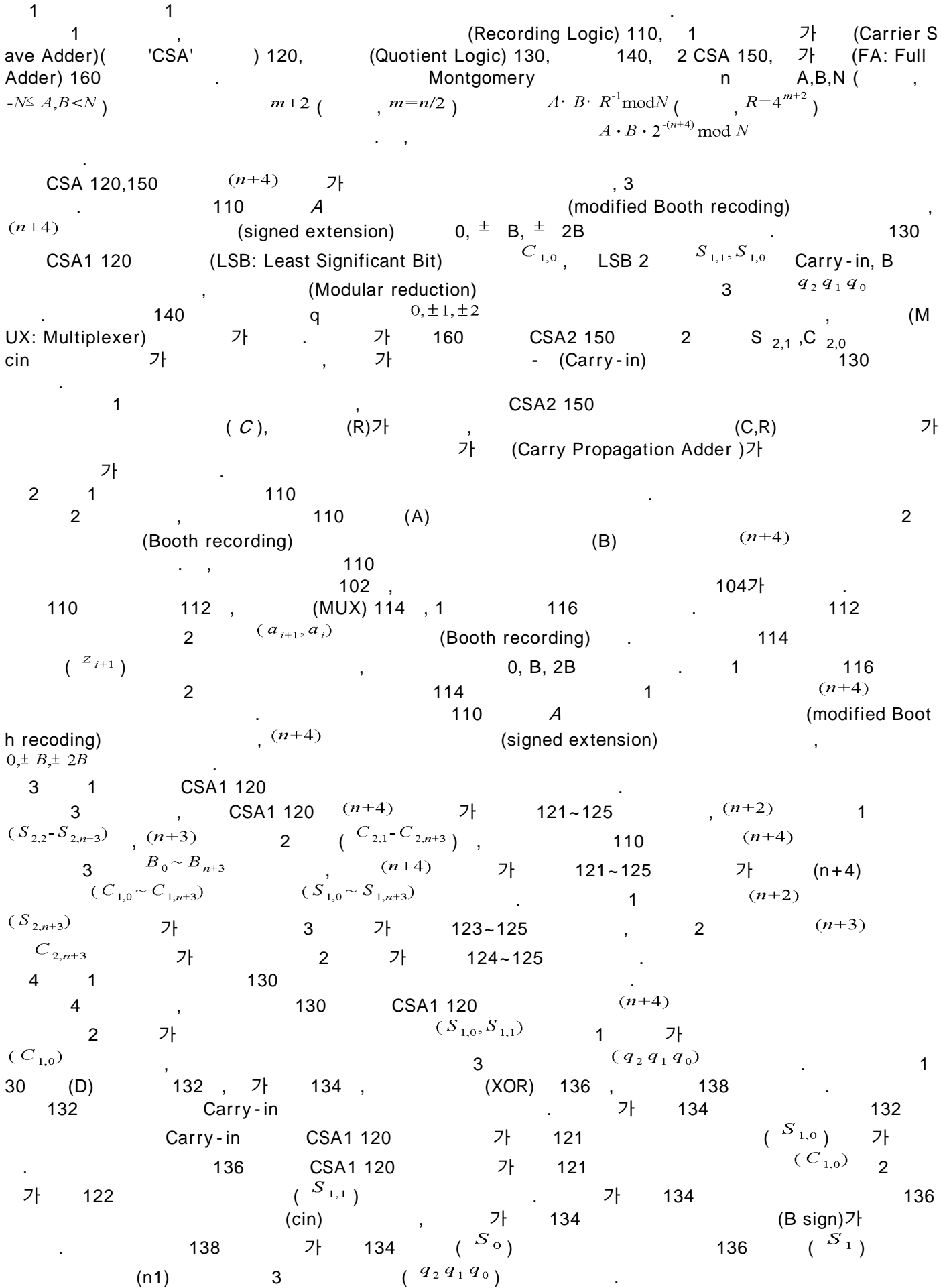
가 가
 US Patent No. 6,185,596

D. R. Stinson,
 Montgomery

(Gate)

가 가

B-1.



5 1 CSA2 150
 5 , CSA2 150 (n+4) 가 151~156 (N: N₀~N_{n+3}) 1 CSA2 150
 140 (n+4) (n+3) , CSA1 12
 0 (n+4) (n+3) (C_{1,0}~C_{1,n+2})
 2 (S_{1,1}~S_{1,n+3}) 3 (n+4) 가 151~156 (n+4)
 (C_{2,0}~C_{2,n+3}) (S_{2,0}~S_{2,n+3}) 1 (n+4) 가
 가 151 2 (n+3) 가 2 가
 152 , 3 (n+3) 가 2 가 152
 (S₀) , a_{i,2} , (N) 가 151 (N₀)가 130 가 134
 6 1 가 160
 6 , 가 160 CSA2 150 가 151
 (C_{2,0}) 2 가 152 (S_{2,0}) 가 (Carry-in)
 가 160 가 (cin) , 가
 (Carry-in) (Carry-in) 130

B-2.

n A,B,N (-N ≤ A,B < N) m+2 (, m=n/2)
 $A \cdot B \cdot R^{-1} \pmod N$ (, $R=4^{m+2}$)
 3가 (A) (B) 가
 Montgomery 가
 < Number Representation >
 tation) , n A , B (A) (B) (Signed binary represen
 1 (n+4)
 < Booth's Recording >
 ing) (Modified Booth Recording) (Booth Record
 가 A
 2 z_i (, 0 ≤ i ≤ m+1) a_{n+4} = a_{n+3}, a₋₁ = 0 가 <
 1>

[1]

a _{i+1}	a _i	a _{i-1}	z _{i+1}
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	2
1	0	0	-2
1	0	1	-1
1	1	0	-1
1	1	1	0

< Booth's Recoding Radix-4 Montgomery Algorithm >
 radix-4 Montgomery
 Montgomery modulus N 가 modulus N

Input: $N, -N \leq A, B < N$ ¹
 Output: $S = A \cdot B \cdot 4^{-m-2} \bmod N, -N < S < N$

$$S = 0 \tag{1}$$

$$\text{for } i = 0 \text{ to } \left\lceil \frac{n+1}{2} \right\rceil \tag{2}$$

$$S = S + A_i \times B \tag{3}$$

$$q_{i(2,1,0)} = f(s_1, s_0, n_1, n_0) \tag{4}$$

$$S = S + q_i \times N \tag{5}$$

$$S = S / 2^2 \tag{6}$$

$$\text{endfor} \tag{7}$$

(4) $\langle 2 \rangle$ (5) $\langle 3 \rangle$ A_i $-2 < A_i < 2$
 (4) LSB(Least Significant Bit) 가 '0'
 s_1, s_0, n_1, n_0 $\langle 4 \rangle$
 (modular reduction) q_i MSB(Most Significant Bit) q_{i2}, q_i
 $\{0, \pm 1, 2\}$ q_i $\langle 3 \rangle$

$$q_0 = s_0 \tag{2}$$

$$q_1 = \overline{s_0 s_1}$$

$$q_2 = s_0 s_1 n_1 + s_0 s_1 n_1$$

[2]

s_0	s_1	n_1	q_2	$q_1 q_0$
0	0	0	0	00
0	0	1	0	00
0	1	0	0	10
0	1	1	0	10
1	0	0	1	01
1	0	1	0	01
1	1	0	0	01
1	1	1	1	01

B-3.

$A \cdot B \cdot R^{-1} \bmod N$ ($n, A, B, N (-N \leq A, B < N), R = 4^{m+2}, m+2 (m = n/2)$)
 $A \cdot B \cdot R^{-1} \bmod N$ ($n, A, B, N (-N \leq A, B < N), R = 4^{m+2}, m+2 (m = n/2)$)
 a) $(m+2)$
 b)~h) i) b)~h)

[3]

i	A_i	CSA 1 out S C	B-sign	Carry in	$S_1 S_0$	C
i	0	0000.0000.0000.0000 0.0000.0000.0000.000	0	0	00	0
0	-2	1111.0100.0111.1001 0.0000.0000.0000.000	1	0	10	1
1	0	1111.0010.0010.1010 0.0001.0000.0010.100	0	1	11	0
2	0	1111.0011.0000.0000 0.0001.0000.0010.100	0	1	01	0
3	1	1111.1000.1111.0000 0.0000.1011.0000.011	0	1	11	0
4	1	1111.1110.1000.0000 0.0000.1010.1101.011	0	1	11	0
5	-2	0000.1110.1001.0010 1.1110.1010.1101.001	1	1	10	1
6	1	1111.1110.1011.0110 0.0000.1010.1001.001	0	1	01	0
7	0	1111.1111.0011.1011 0.0000.0000.0000.000	0	1	00	1

[4]

i	A_i	$S_1 S_0$	C	$q_2 q_1 q_0$	CSA 2 out S C	Carry in
i	0	00	0	000	0000.0000.0000.0000 0.0000.0000.0000.000	0
0	-2	10	1	010	(11).1110.0000.1100.1010 (0)0.0010.1000.0110.000	1
1	0	11	0	001	(11).1110.1000.0101.0010 (0)0.0010.0100.0101.001	1
2	0	01	0	101	(00).0001.0110.1000.1110 (1)1.1110.0010.0100.001	1
3	1	11	0	001	(11).1111.1001.1010.1110 (0)0.0001.0100.1010.001	1
4	1	11	0	001	(11).1111.1110.0000.1110 (0)0.0001.0101.1010.001	1
5	-2	10	1	010	(11).1111.0000.1111.0010 (0)0.0001.1101.0010.010	1
6	1	01	0	101	(00).0000.0001.1000.0010 (1)1.1111.1101.0110.111	1
7	0	00	1	000		1

1111.1111.1011.1010
0.0000.0000.0000.000

$A \cdot B \bmod N$

- 1) $P = 2^{2(n+4)} \bmod N$
- 2) $C = A \cdot B \cdot 2^{-(n+4)} \bmod N$
- 3) $P \cdot C \cdot 2^{-(n+4)} \bmod N = A \cdot B \bmod N$

RSA

$m^e \bmod N$

- 1) e ()
- 2) C modulus N
- 3) C S 0
- 4) Montgomery $m' = f_m(m, P, N) = m \cdot P \cdot R^{-1} \bmod N$
 $R = 2^{n+4}$

P

- 5) m' B
- 6) B Montgomery
- 7) A B
- 8) e MSB(Most Significant Bit) 1
- 9) e 가 0 1 4) -5)

- 10) e 가 1 B 9) 4) -5)
- 11) e 8) - 10) 4)
- 1) - 11) B 1
- C S
- modulus N modulus N $m^e \bmod N$

CPA(Carry Propagation Adder)

B-4.

$A \cdot B \cdot 2^{-(n+4)} \bmod N$
 $A \cdot B \bmod N$ $A \cdot B \bmod N$

IC

, NIST -DSS, RSA, ElGamal, Schnorr

C. 2

C-1.

7 7 2 (Recording Logic) 210, 1 가 (Carrier S
ave Adder)('CSA') 220, (Quotient Logic) 230, 240, 2 CSA 250, (AN
D) 260 Montgomery n A,B,N (
-N ≤ A,B < N) m+2 (, m=n/2) $A \cdot B \cdot R^{-1} \bmod N$ (, $R=4^{m+2}$)
 $A \cdot B \cdot 2^{-(n+4)} \bmod N$

CSA 220,250 (n+4) 가 , 3
(n+3) 0,B,2B,3B 210 A (modified Booth recoding)
st Significant Bit) $C_{1,0}$, LSB 2 $S_{1,1}, S_{1,0}$ 230 CSA1 220 (LSB: Lea
(Modular reduction) 2 $q_1 q_0$ 240
q 0,N,2N,3N (MUX: Multiplexer) 가
260 Carry-in CSA2 250 2 $S_{2,1}, C_{2,0}$
230

7 (C), (R)가 , CSA2 250 (C,R) 가
 가 (Carry Propagation Adder)가

8 7 210 (A) 2
 8 (Booth recording) (B) (n+3)
 210
 202 , 204가 210
 (MUX) 212 212 2 (a_{i+1}, a_i)
 , 0,B,2B,3B , (n+3) 210 A
 (modified Booth recoding) , (n+3) 0,B,2B,3B

9 7 CSA1 220
 9 CSA1 220 (n+4) 가 221~225 , (n+1) 1
 (S_{2,2}~S_{2,n+2}) , (n+2) 2 (C_{2,1}~C_{2,n+2}) , 210 (n+3)
 3 (B₀~B_{n+2}) , (n+3) 가 221~225 가 (n+3)
 (C_{1,0}~C_{1,n+3}) (S_{1,0}~S_{1,n+3}) 1 2 CSA2 250
 (S_{2,n+3}) , 3 210 1
 가 3 가 223 , 2
 C_{2,n+3} 가 2 가 224 . 가 가 225
 1 2 '0' , 2 가 224 1 가 '0' , CSA1 22
 0 , (n+2) 가 221 (n+1) 가 223 (n+1) 1 S가
 0 가 224 (n+3) 가 225 '0' 1 가 , CSA1 22
 가 , (n+3) 가 221 (n+2) 가 224 (n+2) 2 (C_{2,1}~C_{2,n+2})
 가 221 (n+1) 가 223 (n+3) 3 (B₀~B_{n+2}) 가 .

10 7 230
 10 230 CSA1 220 (n+3)
 (C_{1,0}) 2 가 (S_{1,0}, S_{1,1}) 1 가
 (D) 232 , 가 (HA: Half Adder) 234 , (XOR) 236 , 238 230
 . 232 (AND) 260 Carry-in
 가 234 232 Carry-in CSA1 220 가
 221 (S_{1,0}) 가 . 236 CSA1 220 가
 221 (C_{1,0}) 2 가 222 (S_{1,1})
 . 238 가 134 (S₀) 236 (S₁)
 (n1) 2 (q₁ q₀) .

11 7 CSA2 250
 11 CSA2 250 (n+3) 가 251~256 . CSA2 250
 240 (n+3) (N: N₀~N_{n+3}) 1 , CSA1
 220 (n+3) (n+3) (n+2) (n+2)
 C_{1,0}~C_{1,n+2}) 2 , (n+3) (n+3) 가 251~256 (n+3)
 (S_{1,1}~S_{1,n+3}) 3 , (n+3) 가 251~256 (n+3)
 (C_{2,0}~C_{2,n+2}) (S_{2,0}~S_{2,n+2}) . 1 (n+3)
 가 가 251 , 2 (n+2) 가
 2 가 252 , 3 (n+2) 가 2
 가 252 가 가 251 230 가 234
 (S₀) , (AND) 260 Carry-in .

12 7 260
 12 260 CSA2 250 가 251 (n+3)
 C_{2,0}) 2 가 252 (S_{2,1}) 가 (Carry-in)

C-2. (Carry-in) 230

$A \cdot B \cdot R^{-1} \pmod N$ (, $-N \leq A, B < N$) $m+2$ (, $m=n/2$)

3가 (A) 가 (B) 가

< 2bit scanning > (A) LSB (scanning)((B) (shifting)) α_i {0, 1, 2, 3}

Montgomery (B) CSA1 220

< Radix-4 Montgomery algorithm > radix-4 Montgomery modulus N modulus N

< 8> Montgomery 가

[5]

Input: $N, 0 \leq A, B < N$

Output: $S = A \cdot B \cdot 4^{-m-2} \pmod N, 0 < S < N$

$$S = 0 \tag{1}$$

$$\text{for } i = 0 \text{ to } \left\lceil \frac{n+1}{2} \right\rceil \tag{2}$$

$$S = S + A_i \times B \tag{3}$$

$$q_{i(1,0)} = f(s_1, s_0, n_1, n_0) \tag{4}$$

$$S = S + q_i \times N \tag{5}$$

$$S = S / 2^2 \tag{6}$$

$$\text{endfor} \tag{7}$$

< 8>

(3) A_i A (4) (5)

LSB(Least Significant Bit) 가 '0' (4)

s_1, s_0, n_1, n_0

, Montgomery modular

N

(modular reduction)

n_0 1

< 10>

q_i {0, 1, 2, 3}

q_i < 9>

4

$$q_0 = s_0$$

$$q_1 = s_0 s_1 n_1 + s_0 s_1 + s_1 n_1$$

[6]

s_0	s_1	n_1	q_1	q_0
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	0

1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

C-3.

$A \cdot B \cdot R^{-1} \pmod N$ (n , A, B, N ($0 \leq A, B < N$) , $R=4^{m+2}$) , $m+2$ ($m=n/2$)
 , 7 , 7 , $A \cdot B \cdot R^{-1} \pmod N$ ($R=4^{m+2}$) , 7 a)
 , b)~h) , i) b)~h) ($m+2$)
 a) $2B, 3B$ 가 (n) A, B, N () , $n+2$, A, B 8
 202, 204 , $2B$ $3B$, A
 202 가 2 A , B B , A
 () C S()가 0 7 CSA2 150
 b) 202, 204 , 110 A 202 LSB
 A 202 LSB 0, $B, 2B, 3B$ 110 MUX 212 B 204 ,
 CSA1 220 3
 c) CSA1 220 $n+3$ 2 (binary unsigned number) 3 . CSA1 2
 20 $n+3$ 가 221~225
 d) 230 CSA1 220 $S_{1,1}, C_{1,0}, S_{1,0}$, 260 Carry-in
 , 가 234 236 S_1, S_0
 e) 230 238 d) S_1, S_0 , $< 10 >$
 2 q $< 10 >$ 가 q
 f) CSA2 250 c) CSA1 120 , e) q LSB
 0, $N, 2N, 3N$ $n+3$ $n+3$
 CSA2 250 CSA1 220 가 $n+3$ 가 251~256
 가 251~256 가 LSB 가 251 Carry_in
 가
 g) (AND) 260 CSA2 250 $S_{2,1}, C_{2,0}$ Carry-in
 h) CSA2 250 MSB ($n+1$) ($n+2$) CSA1 220
 CSA 1 '0' 2 가
 $S_{2,n+2}$ CSA1 220 3 가 223 , 2 가 224, 225 '0'
 CSA2 250 가 256 $C_{2,n+2}$ CSA1 220 2 가
 224 , 가 225 '0'
 i) ($m+2$) b)~h) , CPA(Carry Propag
 ation Adder)() CSA2 150 가
 , A, B, N $< 11 >$ 12 Montgomery
 $< 12 >$ $< 13 >$

FinalResult: 0111.1100.0111(0x7C7)+0010.1000.0000(0x280)+1=1010.0100.1000(0x448)

5

N=000.1010.0101.1001 (0xA59)

B=000.0101.1100.0011 (0x5C3)

2N=001.0100.1011.0010 (0x13B2)

2B=000.1011.1000.0110 (0xB86)

3N=001.1111.0000.1011 (0x1F0B)

3B=001.0001.0100.1001 (0x1149)

A=000.1001.0011.1110 (0x93E)

[7]

i	A_i	CSA 1 out' S C	Carry_in	$S_1 S_0$
i	0	000.0000.0000.0000 0000.0000.0000.0000	0	00
0	2	000.1011.1000.0110 0000.0000.0000.0000	0	10
1	3	001.0110.1100.0101 0000.0010.1001.001	0	11
2	3	001.0111.1010.0010 0000.0010.1001.001	1	01
3	0	000.1001.0100.1111 0000.0101.0000.000	1	00
4	1	000.0110.0101.0000 0000.0011.0000.011	1	11
5	2	000.1001.0110.1101 0000.0111.0000.010	1	10
6	0	000.0100.0010.0100 0000.0101.0010.010	1	01
7	0	000.0101.0001.0000 0000.0101.0000.010	1	01

[8]

i	A_i	$S_1 S_0$	$q_1 q_0$	CSA 2 out S C	Carry in
i	0	00	00	000.0000.0000.0000 0000.0000.0000.000	0
0	2	10	10	(0.0).001.1111.0011.0100 (0).0000.0001.0000.010	0
1	3	11	01	(0.0)001.1110.0000.1110 (0).0000.0101.1010.001	1
2	3	01	11	(0.0).000.1010.0011.1010 (0).0010.1111.0000.011	1
3	0	00	00	(0.0)000.1100.0100.1110	1

				(0).0000.0010.0000.001	
4	1	11	01	(0.0)000.1111.0000.1110 (0).0000.0100.1010.001	1
5	2	10	10	(0.0)001.1010.1101.1010 (0).0000.1010.0100.101	1
6	0	01	11	(0.0)001.1110.0000.1010 (0).0000.1010.0100.101	1
7	0	01	11	(0.0)001.1111.0001.1110 (0).0000.1010.0000.001	1

$$A \cdot B \text{ mod } N$$

- 1) $P=2^{2(n+4)} \text{ mod } N$
- 2) $C=A \cdot B \cdot 2^{-(n+4)} \text{ mod } N$
- 3) $P \cdot C \cdot 2^{-(n+4)} \text{ mod } N=A \cdot B \text{ mod } N$

RSA

$$m^e \text{ mod } N$$

- 1) e ()
- 2) N modulus N
- 3) C S 0
- 4) Montgomery $m'=fm(m,P,N)=m \cdot P \cdot R^{-1} \text{ mod } N$
 $R=2^{n+4}$

P

- 5) m' B
- 6) B Montgomery
- 7) A B , radix-4 Recoding

- 8) e MSB(Most Significant Bit) 1 9) - 10)
- 9) e 가 0 1 4) -5)
- 10) e 가 1 9) , 4) -5)
- 11) e B 8) - 10) 4)
- 1) - 11) C S CPA(Carry Propagation Adder)

$$m^e \text{ mod } N$$

C-4.

$$A \cdot B \cdot 2^{-(n+4)} \text{ mod } N$$

$$A \cdot B \text{ mod } N$$

$$A \cdot B \text{ mod } N$$

IC

NIST -DSS, RSA, ElGamal, Schnorr

D.

13

IC

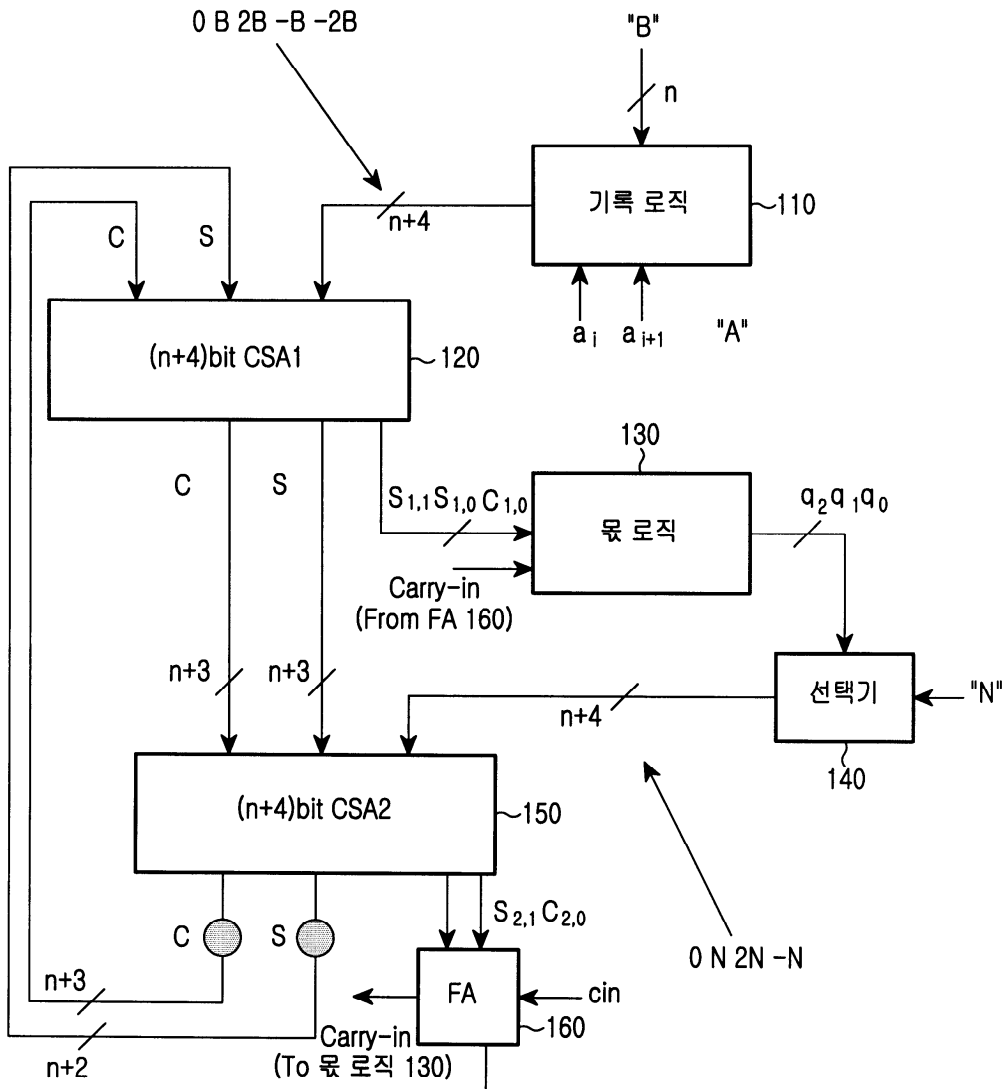
13 , (CPU: Central Processing Unit) 310 , 가
(modular arithmetic coprocessor) 330
(ROM: Read Only Memory) 350 (key)
(security module)

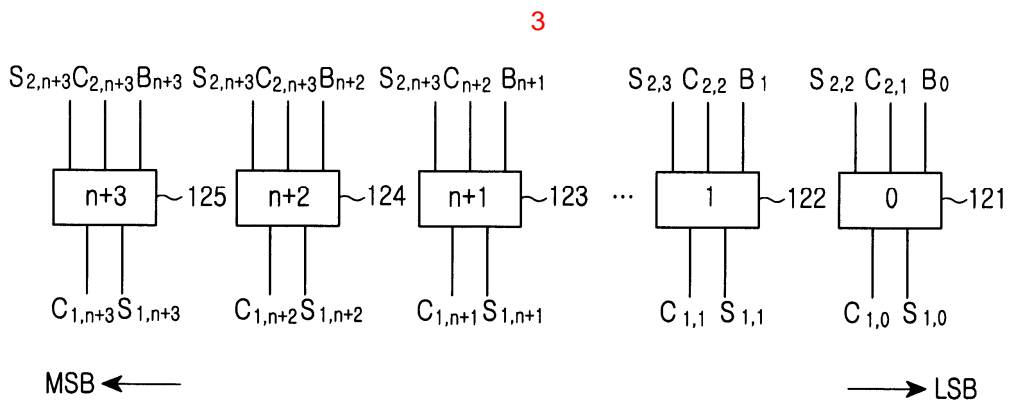
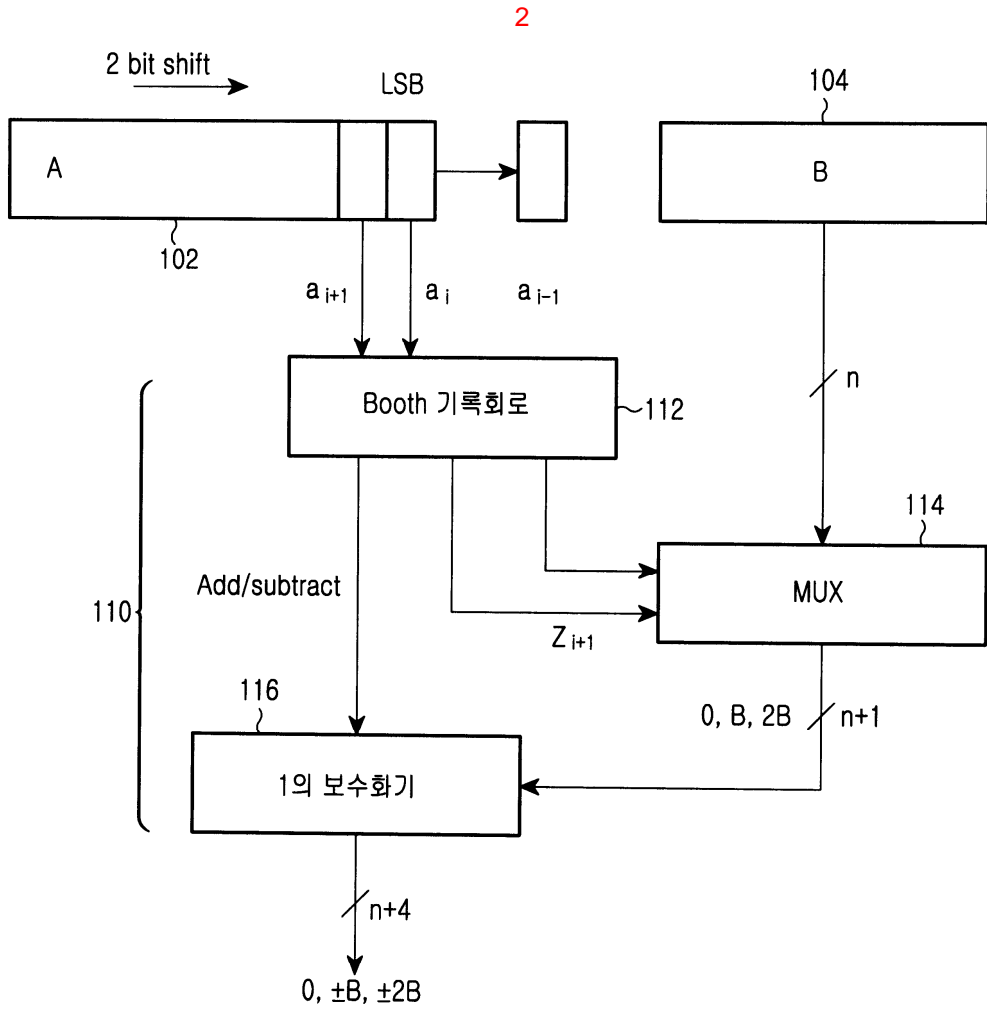
가 가

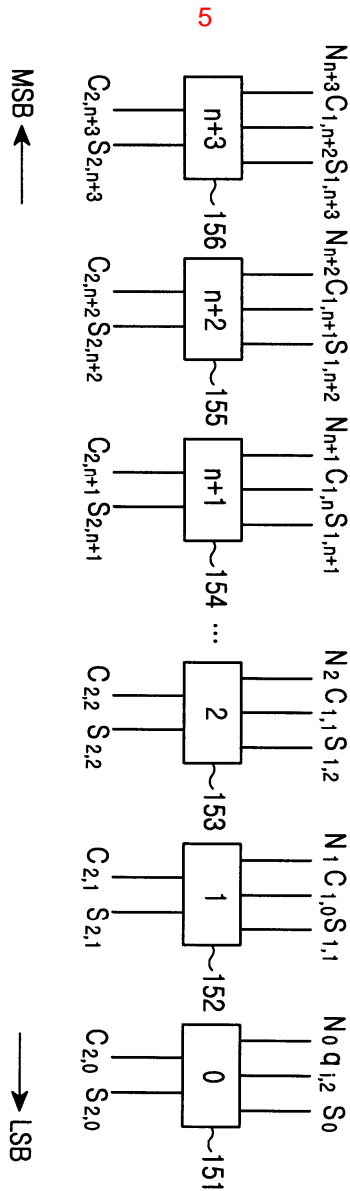
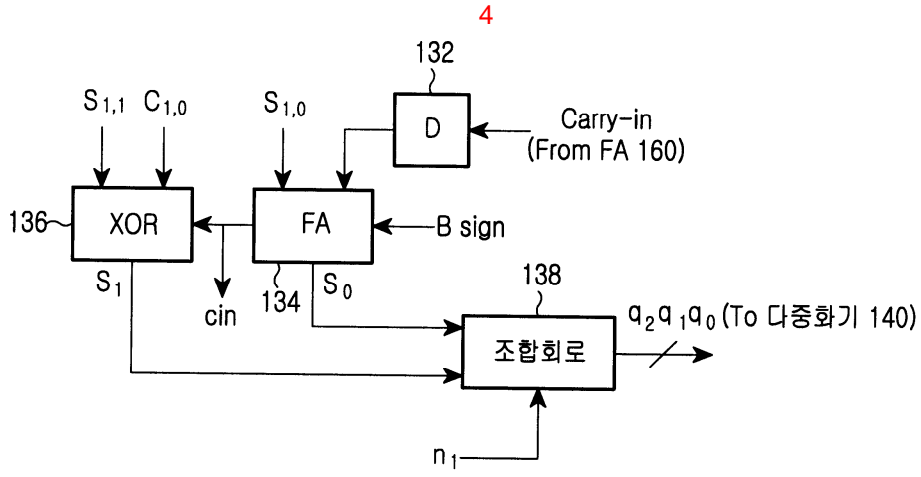
- 4 6. , 가
- 6 7. , 가 가
- 4 8. , 가 2 CSA 가
9. (A) (B) $m+2$ (, $m=n/2$) $A \cdot B \cdot R^{-1} \bmod N$ (, $R=4^{m+2}$)
 , (Booth recording)
 $(n+3)$ 가 , $(n+1)$ 1 , $(n+2)$ 2 ,
 $(n+4)$ 3 가 , 1 $(n+1)$ 가
 가 3 가 , 2 가 $(n+2)$ 가 2 가 ,
 가 가 2 가 '0' 1 가 1 $(n+3)$ 가
 $(n+3)$ 1 가 (CSA) ,
 1 CSA $(n+3)$ 2 가 2
 1 가 , 2
 $(n+3)$ 가 , $(n+3)$ 1 ,
 1 CSA $(n+3)$ $(n+2)$
 2 , $(n+3)$ $(n+2)$
 3 , 1 $(n+3)$ 가 가
 , 2 $(n+2)$ 가 가 2 가 , 3
 $(n+2)$ 가 2 가 , $(n+3)$ 가
 $(n+4)$ 2 가 (CSA) ,
 2 CSA 가 2 가
 2 CSA 가 가 (CPA)
- 9 10. , ,
- 9 11. , , 2
- 9 12. , ,
 1 CSA 1 CSA 가 2 가 가 가 ,
 가 , 2
- 12 13. , .
- 12 14. , 가 2 CSA 가
- 15.

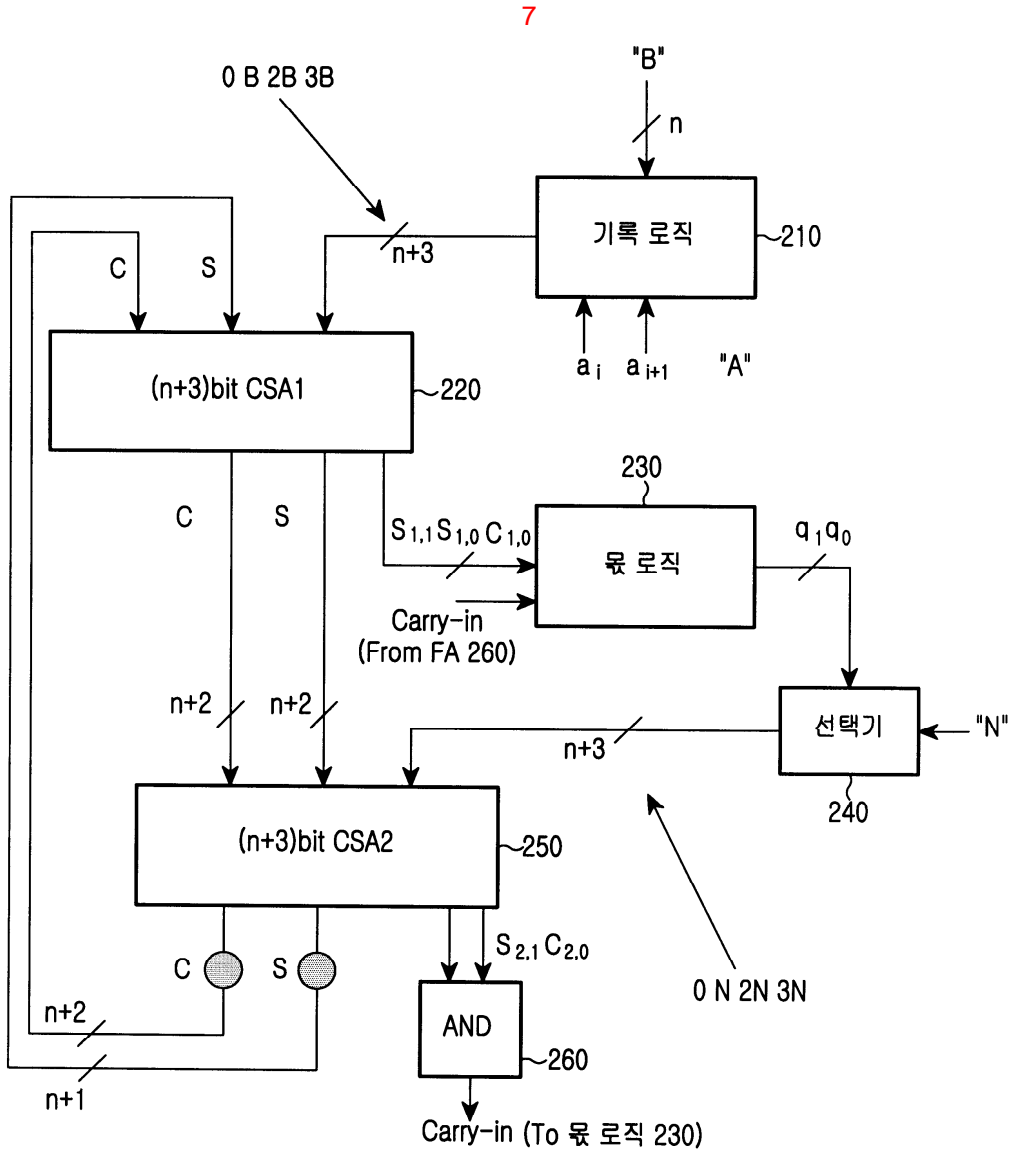
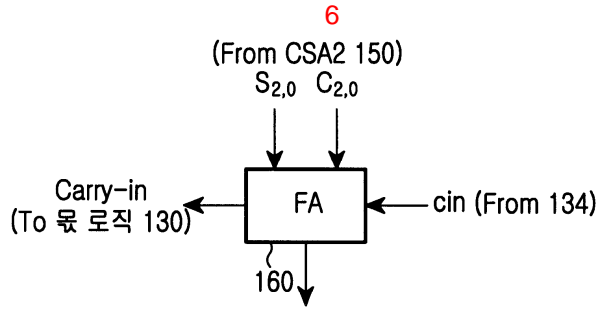
, $(n+3)$ 2 가 , 가 2 가 '0' 1
 1 CSA 가 $(n+3)$ 2 가 , 2
 1 가 , 2
 ,
 $(n+3)$ 가 2 가 (CSA) $(n+3)$
 1 , 1 CSA $(n+3)$
 $(n+2)$ 2 , $(n+3)$
 $(n+2)$ 3 , 1 $(n+3)$ 가
 가 , 2 $(n+2)$ 가 2 가 2 가
 , 3 , $(n+2)$ 가 2 가
 , $(n+3)$ 가 $(n+4)$,
 2 CSA 가 2 가 ,
 2 CSA 가 ,
22.
 21 , 2 1 CSA 가 , 가 ,
 1 CSA 가 2 가 ,
 가 , 2
23.
 22 , .

1

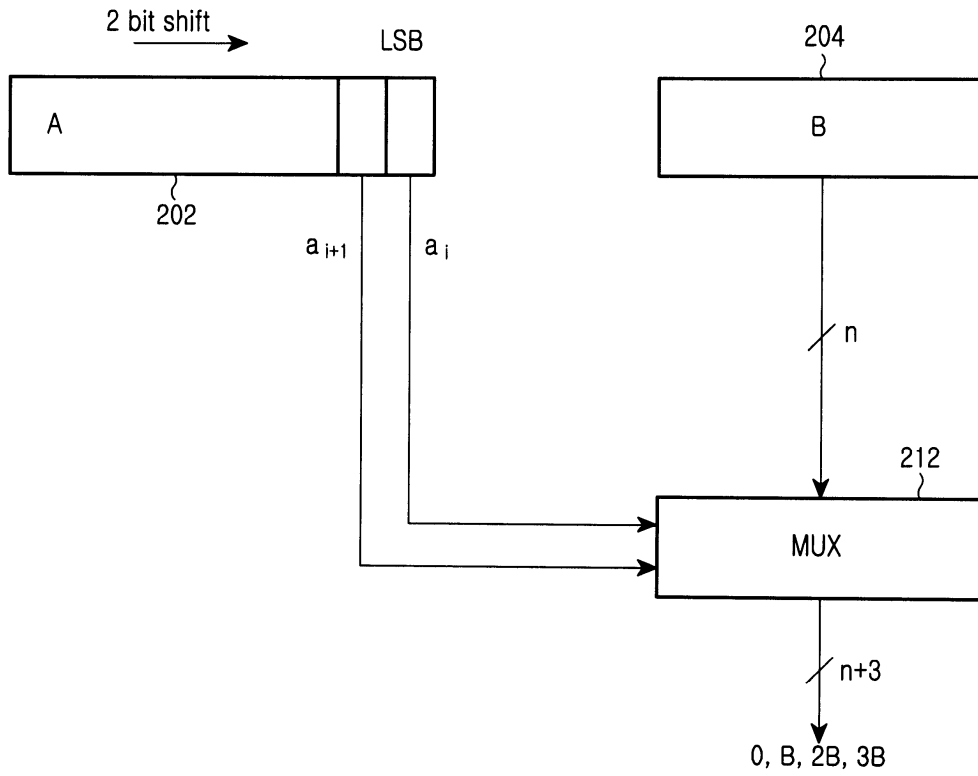




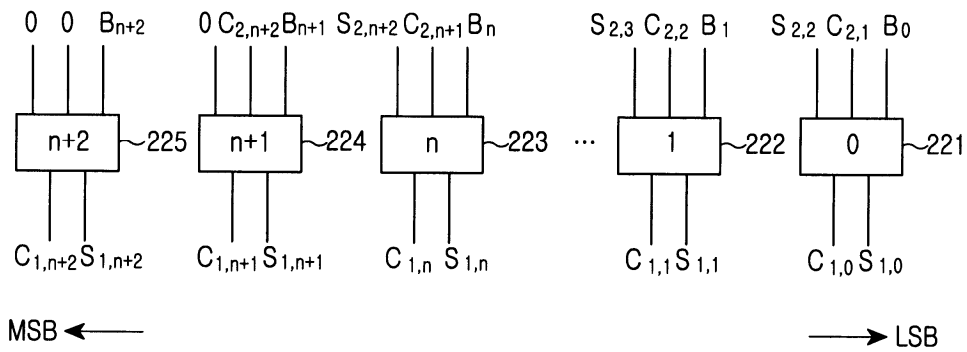




8



9



10

