Oct. 19, 1948. H. WALLMAN 2,451,893

STAGGER TUNED AMPLIFIER

Filed Aug. 30, 1945

FIG. 1

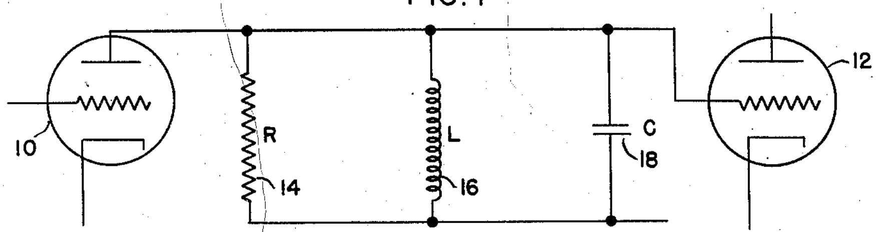
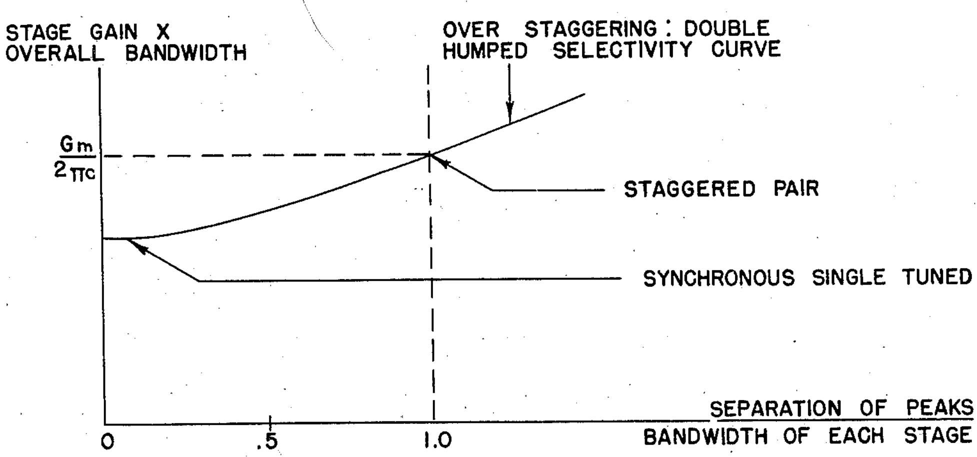


FIG. 2



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Patented Oct. 19, 1948

2,451,893

UNITED STATES PATENT OFFICE

2,451,893

STAGGER TUNED AMPLIFIER

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Application August 30, 1945, Serial No. 613,605

7 Claims. (Cl. 179—171)

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The present invention relates generally to a communications method and more particularly to a method of tuning successive stages of a single-tuned electronic amplifier.

In an electronic amplifier stage using a given type of vacuum tube and coupling circuit, the gain and bandwidth are dependent in such a manner that if one is increased, the other is decreased, the product of the two being substantially constant. It is also well known that if several amplifier stages are connected in cascade, the resulting product of overall bandwidth and stage gain of the combination is much less than that for a single stage. A special case of this is a synchronous single-tuned IF amplifier, that is, one whose stages are all tuned to the same center frequency. The product of bandwidth and stage gain of a receiver in which such an amplifier is used decreases rapidly as the number of IF stages increases.

In certain types of radio object locating systems, it is necessary to adapt a receiver to pass pulses having a duration of the order of a fraction of a microsecond. To preserve the character of such a pulse the receiver must pass a wide band of frequencies with minimum attenuation. Since the receivers of these systems must also possess high gain, a relatively large number of IF stages are generally necessary, and hence the problem of simultaneously maintaining this broad bandwidth and high gain becomes important.

It is therefore an object of the present invention to provide a method of obtaining a broad bandwidth in an IF amplifier, and simultaneously maintaining the gain per stage and overall amplification. It is another object of the invention to provide means for producing in an electronic amplifier employing single-tuned circuits a selectivity characteristic having the maximum possible bandwidth while maintaining a single-peaked upper portion. It is a further object to provide criteria by means of which an amplifier of a given number of stages may be designed to pass a band of frequencies having given limits.

The invention in general contemplates tuning successive single-tuned stages of an IF amplifier to different center frequencies according to criteria determined by the desired overall bandwidth, the overall center frequency, and the number of amplifier stages to be jointly employed.

Other objects, features, and advantages of this invention will suggest themselves to those skilled in the art and will become apparent from the following description of the invention taken in connection with the accompanying drawing in which:

Fig. 1 is a schematic diagram of the A.-C. equivalent circuit for a single-tuned IF amplifier stage; and

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Fig. 2 is a curve showing the variation of the product of stage gain and overall bandwidth as a function of the amount of separation between the frequencies to which the two stages of a staggered pair as tuned.

Reference is now made more particularly to Fig. 1 in which there is shown the A.-C. equivalent coupling circuit between two successive stages of a single-tuned amplifier employing vacuum tubes 10 and 12 respectively, the circuit elements shown representing lumped constants. Resistor 14 of value R represents the parallel combination of the plate load resistance, the plate resistance of vacuum tube 10, the input resistance of vacuum tube 12, and the equivalent shunt resistances of the coil and capacitor of the tuned circuit. Inductance 16 of value L is the tuning coil inductance, and capacitor 18 of value C represents the total circuit capacitance made up of the output capacitance of vacuum tube 10, the input capacitance of vacuum tube 12, and the stray wiring capacitance of the coupling circuit.

The parallel impedance (Z) of this circuit, which is a function of frequency, can be shown to vary according to the equation

$$Z(\omega)=\frac{R}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

where ω is the frequency expressed in radians per second,

$$Q=\omega_0 RC, \text{ and } \omega_0=\frac{1}{\sqrt{LC}}$$

If the transconductance of vacuum tube 10 is g_m, the voltage gain (A) from the grid of this tube to the grid of vacuum tube 12 is $g_m R$ at the resonant frequency ω_0, and

$$A=g_m R\frac{1}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)} \qquad (1)$$

as a function of frequency.

The complex gain function expressed by Equation 1 displays "geometric symmetry"; that is, for any two frequencies ω and

$$\frac{\omega_0^2}{\omega}$$

having ω_0 as their geometric mean, the absolute values of A are equal, and the phase angles are opposite in sign.

In this discussion "bandwidth" means "3*db* bandwidth," i. e., the bandwidth included between the two half-power or .707 voltage points on the gain characteristic. It can be shown that the bandwidth for this single stage is

$$\Delta\omega=\frac{1}{RC}$$

in radians per second, and that the Q of the circuit as defined previously is equal to the ratio of center frequency to bandwidth.

An approximation may be made for high Q circuits; namely,

$$A=g_mR\frac{1}{1+j2RC(\omega-\omega_0)} \qquad (2)$$

Equation 2 displays "arithmetic symmetry"; that is, for any two frequencies, ω and $2\omega_0-\omega$, having ω_0 as their arithmetic means, the absolute values of A are equal and the phase angles are opposite in sign.

Using the high Q form of the gain function shown by Equation 2, the absolute value of the gain can be seen to be proportional to the quantity

$$\frac{1}{\sqrt{1+x^2}}$$

which is the absolute value of

$$\frac{1}{1+jx}$$

where x is a function of frequency.

A figure of merit for a one-stage amplifier is the product of voltage gain at band center and bandwidth. If the stage is single tuned, then since the bandwidth in cycles per second is

$$\Delta f=\frac{1}{2\pi RC}$$

and the center frequency gain is g_mR, this figure of merit becomes

$$\frac{g_m}{2\pi C}$$

which is independent of R and frequency.

When dealing with an n-stage amplifier the appropriate figure of merit is

(overall gain)$^{1/n}\times$overall bandwidth

If we denote (overall gain)$^{/1n}$ by "mean stage gain," the figure of merit becomes

mean stage gain$\times$overall bandwidth

and this can be shown to decrease quite rapidly as n increases for synchronous single-tuned stages.

It can be shown mathematically that by tuning successive single-tuned stages of an amplifier to different frequencies or, as it will be referred to in this application, stagger tuning, one may obtain an amplifier the gain of which has an absolute value proportional to the gain factor

$$\frac{1}{\sqrt{1+x^{2n}}}$$

where x is a function of frequency as before, and n is the number of stages. It can be shown further that the product of mean stage gain and overall bandwidth for such an amplifier can be kept the same as for a single stage, rather than decreasing with the number of stages, as is the case with a synchronous single-tuned amplifier as mentioned above. The selectivity curve resulting from the gain factor above has the maximum possible bandwith, if single-tuned stages are employed, while maintaining a flat top, the flat portion becoming wider as n increases.

The proper frequencies to which the various stages should be tuned and the individual stage bandwidths to obtain the results mentioned above may be computed mathematically by two methods. The choice of method depends on whether or not the high-Q approximation with arithmetic symmetry Equation 2 may be used.

The results obtained in the high Q case are summarized in Table 1 for various numbers of staggered stages from two (staggered pair) to 9 (staggered nonuple).

The results obtained in the exact case, taking into account geometric symmetry, are summarized in Table 2 for all cases from a staggered pair to a staggered quintuple. The advantage of using the approximations of Table 1 is that of much greater simplicity of computation, as can be seen by comparison to the two tables.

TABLE I.—STAGGERED n-UPLES

High-Q case: Arithmetic symmetry

[Center-frequency=f_0, overall bandwidth=Δf, and $\Delta f/f_0$ small[1]]

	n	component single-tuned stages
2	staggered-pair	two stages of bandwidth .71Δf, staggered at $f_0\pm$.35Δf.
3	staggered triple	two stages of bandwidth .5Δf, staggered at $f_0\pm$.43Δf. one stage of bandwidth Δf, centered at f_0.
4	staggered-quadruple	two stages of bandwidth .38Δf, staggered at $f_0\pm$.46Δf. two stages of bandwidth .92Δf, staggered at $f_0\pm$.19Δf.
5	staggered-quintuple	two stages of bandwidth .31Δf, staggered at $f_0\pm$.48Δf. two stages of bandwidth .81Δf, staggered at $f_0\pm$.29Δf. one stage of bandwidth Δf, centered at f_0.
6	staggered-sextuple	two stages of bandwidth .26Δf, staggered at $f_0\pm$.48Δf. two stages of bandwidth .71Δf, staggered at $f_0\pm$.35Δf. two stages of bandwidth .97Δf, staggered at $f_0\pm$.13Δf.
7	staggered-septuple	two stages of bandwidth .22Δf, staggered at $f_0\pm$.49Δf. two stages of bandwidth .62Δf, staggered at $f_0\pm$.39Δf. two stages of bandwidth .90Δf, staggered at $f_0\pm$.22Δf. one stage of bandwidth Δf, centered at f_0.
8	staggered-octuple	two stages of bandwidth .20Δf, staggered at $f_0\pm$.49Δf. two stages of bandwidth .56Δf, staggered at $f_0\pm$.42Δf. two stages of bandwidth .83Δf, staggered at $f_0\pm$.28Δf. two stages of bandwidth .98Δf, staggered at $f_0\pm$.10Δf.
9	staggered-nonuple	two stages of bandwidth .17Δf, staggered at $f_0\pm$.49Δf. two stages of bandwidth .5Δf, staggered at $f_0\pm$.43Δf. two stages of bandwidth .77Δf, staggered at $f_0\pm$.32Δf. two stages of bandwidth .94Δf, staggered at $f_0\pm$.17Δf. one stage of bandwidth Δf, centered at f_0.

[1] Less than .05.

TABLE II.—STAGGERED n-UPLES

Exact case: Geometric symmetry

[Center frequency=f_0, overall bandwidth=Δf, and $\Delta f/f_0=\delta$]

	n	component single-tuned stages
2	staggered-pair	two stages of dissipation factor[1] d, staggered at $f_0\alpha$ and f_0/α, where $d^2=\frac{4+\delta^2-\sqrt{16+\delta^4}}{2}$ and $\left(\alpha-\frac{1}{\alpha}\right)^2+d^2=\delta^2$
3	staggered-triple	two stages of dissipation factor d, staggered at $f_0\alpha$ and f_0/α, one stage of dissipation factor δ, centered at f_0, where $d^2=\frac{4+\delta^2-\sqrt{16+4\delta^2+\delta^4}}{2}$ and $\left(\alpha-\frac{1}{\alpha}\right)^2+d^2=\delta^2$
4	staggered-quadruple	two stages of dissipation factor d_1, staggered at $f_0\alpha_1$ and f_0/α_1, two stages of dissipation factor d_3, staggered at $f_0\alpha_3$ and f_0/α_3, where $d_1^2=\frac{4+\delta^2-\sqrt{16+5.656\delta^2+\delta^4}}{2}$ and $\left(\alpha_1-\frac{1}{\alpha_1}\right)^2+d_1^2=\delta^2$ $d_{32}=\frac{4+\delta^2-\sqrt{16-5.656\delta^2+\delta^4}}{2}$ and $\left(\alpha_3-\frac{1}{\alpha_3}\right)^2+d_3^2=\delta^2$
5	staggered-quintuple	two stages of dissipation factor d_1, staggered at $f_0\alpha_1$ and f_0/α_1, two stages of dissipation factor d_3, staggered at $f_0\alpha_3$ and f_0/α_3. One stage of dissipation factor δ, centered at f_0, where $d_1^2=\frac{4+\delta^2-\sqrt{16+6.472\delta^2+\delta^4}}{2}$ and $\left(\alpha_1-\frac{1}{\alpha_1}\right)^2+d_1^2=\delta^2$ $d_3^2=\frac{4+\delta^2-\sqrt{16-2.472\delta^2+\delta^4}}{2}$ and $\left(\alpha_3-\frac{1}{\alpha_3}\right)^2+d_3^2=\delta^2$

[1] Dissipation factor of a single-tuned circuit=1/Q=bandwidth/resonant frequency.

A plurality of staggered n-uples obtained from the information in the tables may be serially connected just as are single stages to form a complete amplifier. Since the selectivity curves of these n-uples are more square-topped than those of single stages, the overall bandwidth goes down more slowly for an increasing number of n-uples than for the same number of synchronous single-tuned stages.

The cases of stagger tuning so far described produce selectivity curves which are flat-topped. The case sometimes arises in which one requires a wide IF bandpass in order to obtain non-critical tuning but is not too much concerned with maintaining a flat-topped selectivity curve. It is then possible to increase the amount of staggering, so that it exceeds the values listed in the tables and thereby realize a useful increase in the product of stage gain and overall bandwidth. Fig. 2 shows the variation of this product as a function of the amount of staggering in the case of a staggered pair. The ratio of the separation of the peak frequencies of the individual stages to the bandwidth of each stage is plotted along the horizontal axis, and the product of stage gain and overall bandwidth is plotted along the vertical axis. The right-hand region of the curve in which the separation of the individual resonance peaks exceeds the bandwidth of the individual circuits represents an "over-staggered" condition. In this region the overall selectivity curve displays a double hump, which is not a serious deficiency in many cases. This method of overstaggering may be applied as well in the case of more than two stages, and a similar increase in the product of stage gain and overall bandwidth is obtained.

The equations for d_k^2 and δ^2 are derived as follows: Suppose for the purpose of this derivation that a single tuned stage peaked at a frequency "α" is followed by a single tuned stage peaked at "$1/\alpha$" so that $f=1$ is the geometric mean of the two resonant frequencies. Suppose further that the two stages have the same Q hence the same d. The complex gain function of the combination is

$$\frac{1/\alpha}{d+j(f/\alpha-\alpha/f)}\cdot\frac{\alpha}{d+j(f\alpha-1/f\alpha)} \qquad \text{(I)}$$

Multiplying the denominators yields

$$d^2+df\alpha-jd/f\alpha+jdf/\alpha-f^2+1/\alpha^2-jd\alpha/f+\alpha^2-1/f^2$$

which equals

$$d^2+\alpha^2+1/\alpha^2+jd\alpha(f-1/f)+jd/\alpha(f-1/f)-(f^2+1/f^2) \qquad \text{(II)}$$

Now

$$\alpha^2+1/\alpha^2=(\alpha-1/\alpha)^2+2$$

and

$$-(f^2+1/f^2)=-(f-1/f)^2-2$$

hence Equation II can be written

$$[d^2+(\alpha-1/\alpha)^2]+j(f-1/f)[d(\alpha+1/\alpha)]+[j(f-1/f)]^2$$

and now we have

$$\frac{1/\alpha}{d+j(f/\alpha-\alpha/f)}\cdot\frac{\alpha}{d+j(f\alpha-1/f\alpha)}$$

equals

$$\frac{1}{[d^2+(\alpha-1/\alpha)^2]+j(f-1/f)[d(\alpha+1/\alpha)]+[j(f-1/f)]^2} \qquad \text{(III)}$$

It can be shown that the complex impedance of the amplifier must have the absolute value of

$$\frac{1}{\sqrt{\delta^{2n}+(f-1/f)^{2n}}} \qquad \text{(IV)}$$

where

$$\delta=\frac{\text{overall bandwidth}}{\text{center frequency}}=\frac{\Delta f}{f_0}$$

It can also be shown that Equation IV above is the absolute value of

$$\frac{1}{\left[\delta^2+j(f-1/f)2\delta\sin\frac{\pi}{2n}+[j(f-1/f)]^2\right]\left[\delta^2+j(f-1/f)2\delta\sin\frac{(n-1)\pi}{2n}+[j(f-1/f)]^2\right]}$$

for n equal to an even integer, or

$$\frac{1}{\left[\delta^2+j(f-1/f)2\sin\frac{\pi}{2n}+[j(f-1/f)]^2\right][\delta+j(f-1/f)]} \qquad \text{(V)}$$

for n equal to an odd integer.

We can now synthesize the absolute value of Equation IV by equating the coefficients of $j(f-1/f)$ in Equations III and V for the first factor of Equation V. For example, we need to know the dissipation factor d_1 and the resonant frequency α_1 satisfying

$$d_1^2+(\alpha_1+1/\alpha_1)^2=\delta \qquad \text{(VI)}$$

$$d_1(\alpha_1+1/\alpha_1)=2\delta\sin\frac{\pi}{2n} \qquad \text{(VII)}$$

squaring Equation VII and replacing $(\alpha_1+1/\alpha_1)^2$ by $(\alpha_1-1/\alpha)^2$ by $(\alpha_1-1/\alpha)^2+4$, we get

$$d_1^2(\alpha_1-1/\alpha_1)^2+4d_1^2=4\delta^2\sin^2\frac{\pi}{2n}$$

Inserting (VI)

$$d_1^4-d_1^2(4+\delta^2)+4\delta^2\sin^2\frac{\pi}{2n}=0$$

Solving

$$d_1^2=\frac{4+\delta^2-\sqrt{16+8\delta^2+\delta^4-16\delta^2\sin^2\frac{\pi}{2n}}}{2}$$

and finally using the double angle formula

$$1-2\sin^2\frac{\pi}{2n}=\cos\frac{\pi}{n}$$

we have

$$d_1^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos\frac{\pi}{n}+\delta^4}}{2} \qquad \text{(VIII)}$$

Proceeding similarly with the other factors of Equation V we see that

$$\frac{1}{\sqrt{\delta^{2n}+(f-1/f)^{2n}}}$$

is the absolute value of

$$\left[\frac{1/\alpha}{d_1+j(f/\alpha_1-\alpha_1/f)}\cdot\frac{\alpha}{d_1+j(f\alpha_1-1/f\alpha_1)}\right]^A\left[\frac{1/\alpha_3}{d_3+j(f/\alpha_3-\alpha_3/f)}\cdot\frac{\alpha_3}{d_3+j(f\alpha_3-1/f\alpha_3)}\right]^B----$$

$$----\left[\frac{1/\alpha_{(n-1)}}{d_{(n-1)}+j(f/\alpha_{(n-1)}-\alpha_{(n-1)}/f)}\cdot\frac{\alpha_{(n-1)}}{d_{(n-1)}+j(f\alpha_{(n-1)}-1/f\,\alpha_{(n-1)})}\right]$$

for n equal to an even integer, or where M and N respectively are the terms under brackets with exponents A and B respectively;

$$[M]^A[N]^B----\left[\frac{1}{\delta+j(f-1/f)}\right] \qquad \text{(IX)}$$

for n equal to an odd integer.

We may now synthesize Equation IX by providing a series of amplifiers in cascade, an amplifier or a pair of amplifiers representing each term of Equation IX.

By analogy to Equation VIII

$$d_k^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos k\pi/n+\delta^4}}{2} \qquad \text{(X)}$$

and

$$\delta^2=(\alpha_k-1/\alpha_k)^2+d_k^2 \qquad \text{(XI)}$$

where $k=1, 3, 5, \ldots$

It will be recognized that Equations X and XI represent single tuned stages of dissipation factor d_k and resonant in pairs at frequencies α_k and $1/\alpha_k$.

While there has been described what is at present considered to be the preferred embodiment of this invention, it will be obvious to those skilled in the art that various changes and modifications may be made therein without departing from the scope of the invention as set forth in the appended claims.

The invention claimed is:

1. In a method of tuning an electronic amplifier including n serially connected stages, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the desired overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the desired maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being equal to a value δ, the steps including: combining a plurality of said stages into pairs, assigning to each of said pairs a number k, where k is a different value for each pair, said values being a series of successive odd integers beginning with one, assigning to each of said pairs a set of parameters d_k and α_k, where k is as defined above, determining values for d_k and α_k for each pair from the equations:

$$d_k^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos\frac{k\pi}{n}+\delta^4}}{2}$$

and

$$\left(\alpha_k-\frac{1}{\alpha_k}\right)^2+d_k^2=\delta^2$$

adjusting the circuit constants of each stage so that d_k is the approximate ratio of bandwidth to resonant frequency for each individual stage of each of said pairs, and so that the resonant frequency of the individual stages of each of said pairs are approximately $f_0\alpha_k$ and

$$\frac{f_0}{\alpha_k}$$

respectively.

2. In an electronic amplifier, a plurality of n

serially connected stages, n being an odd integer, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being equal to a value δ, one of said stages having an individual resonant frequency f_0 and a ratio of stage bandwidth to resonant frequency of value δ, the remainder of said stages being combined into pairs numbered 1, 3, . . . (n—2), each of said pairs having parameters d_k and α_k, where k is the number assigned to that pair, d_k being the approximate ratio of bandwidth to resonant frequency for each individual stage of each of said pairs, the resonant frequencies of the individual stages of each said pairs being approximately $f_0\alpha_k$ and

$$\frac{f_0}{\alpha_k}$$

respectively, d_k for each of said pairs being given by the relation:

$$d_k^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos\frac{k\pi}{n}+\delta^4}}{2}$$

and α_k for each of said pairs being given by the relation:

$$\left(\alpha_k-\frac{1}{\alpha_k}\right)^2+d_k^2=\delta^2$$

3. In an electronic amplifier, a plurality of n serially connected stages, n being an even integer, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being equal to a value δ, said stages being combined into pairs numbered 1, 3, . . . (n—1), each of said pairs having parameters d_k and α_k, where k is the number assigned to that pair, d_k being the approximate ratio of bandwidth to resonant frequency for each individual stage of each of said pairs, the resonant frequencies of the individual stages of each of said pairs being approximately $f_0\alpha_k$ and

$$\frac{f_0}{\alpha_k}$$

respectively, d_k for each stage of each of said pairs being given by the relation:

$$d_k^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos\frac{k\pi}{n}+\delta^4}}{2}$$

and α_k for each of said pairs being given by the relation:

$$\left(\alpha_k-\frac{1}{\alpha_k}\right)^2+d_k^2=\delta^2$$

4. In an electronic amplifier, a plurality of n serially connected stages, n being an odd integer, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being small as compared with unity, one of said stages having an individual bandwidth of Δf and a resonant frequency f_0, the remainder of said stages being divided into pairs, each of said pairs of stages being assigned a number k, of a different value for each pair, said values being a series of successive odd integers beginning with one, the individual bandwidths of the two stages of each of said pairs being approximately equal in cycles per second to the value

$$\left(\sin\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair to which the stage belongs, and the resonant frequency of the two stages of each of said pairs being respectively above and below frequency f_0 by an amount approximately equal in cycles per second to the value

$$\left(\cos\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair.

5. In an electronic amplifier, a plurality of n serially connected stages, n being an even integer, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being small as compared with unity, said stages being divided into pairs, each of said pairs being assigned a number k, of a different value for each pair, said values being a series of successive odd integers beginning with one, the individual bandwidths of the two stages of each of said pairs being approximately equal in cycles per second to the value

$$\left(\sin\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair to which the stage belongs, and the resonant frequencies of the two stages of each of said pairs being respectively above and below frequency f_0 by an amount approximately equal in cycles per second to the value

$$\left(\cos\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair.

6. An electronic amplifier including a plurality of pairs of serially connected stages, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having a bandwidth Δf between points where the absolute value of the ordinate is approximately .707

of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being equal to a value δ, each of said pairs of stages being assigned a number k, of a different value for each pair, said values being a series of successive odd integers beginning with one, each of said pairs having parameters d_k and α_k, where k is the number assigned to that pair, d_k being the approximate ratio of bandwidth to resonant frequency for each individual stage of each of said pairs, the resonant frequencies of the individual stages of each of said pairs being approximately $f_0\alpha_k$ and

$$\frac{f_0}{\alpha_k}$$

respectively, the total number of individual stages being n, d_k for each stage of each of said pairs being given by the relation:

$$d_k^2=\frac{4+\delta^2-\sqrt{16+8\delta^2\cos\frac{k\pi}{n}+\delta^4}}{2}$$

and α_k for each of said pairs being given by the relation:

$$\left(\alpha_k-\frac{1}{\alpha_k}\right)^2+d_k^2=\delta^2$$

7. An electronic amplifier including a plurality of pairs of serially connected stages, the total number of individual stages being n, each of said stages being tuned to a resonant frequency by means of a single resonant circuit, the overall selectivity characteristic of said amplifier having

a bandwidth Δf between points where the absolute value of the ordinate is approximately .707 of the value of said ordinate at frequency f_0, the maximum point of said characteristic, the ratio

$$\frac{\Delta f}{f_0}$$

being small as compared with unity, each of said pairs of stages being assigned a number k, of a different value for each pair, said values being a series of successive odd integers beginning with one, the individual bandwidths of the two stages of each of said pairs being approximately equal in cycles per second to the value

$$\left(\sin\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair to which the stage belongs, and the resonant frequency of the two stages of each of said pairs being respectively above and below frequency f_0 by an amount approximately equal in cycles per second to the value

$$\left(\cos\frac{k\pi}{2n}\right)\Delta f$$

where k is the number assigned to the pair.

HENRY WALLMAN.

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