

[54] **FUNCTION GENERATOR**

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 [51] Int. Cl. .... **G06g 7/24**  
 [58] Field of Search ..... **328/144, 145; 307/229, 230; 235/197**

[56] **References Cited**

**UNITED STATES PATENTS**

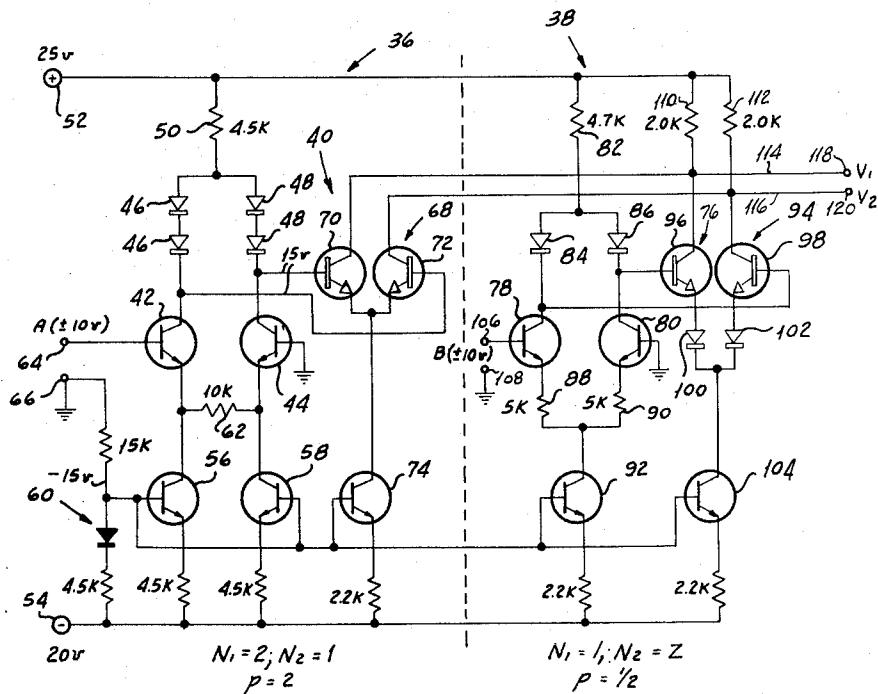
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[57] **ABSTRACT**

A nonlinear generator in which each of a plurality of sections includes two differentially connected devices each of which comprises a first string of elements having the same voltage-current characteristics and through which an input current is passed to provide a voltage which is applied to a second string of a different number of similar elements to produce a current from which there is obtained an output which is proportional to the input raised to a power which is the ratio of the numbers of elements in the respective strings. In making up any function each section produces an output representing a term of the overall approximating equation of the function.

**17 Claims, 3 Drawing Figures**



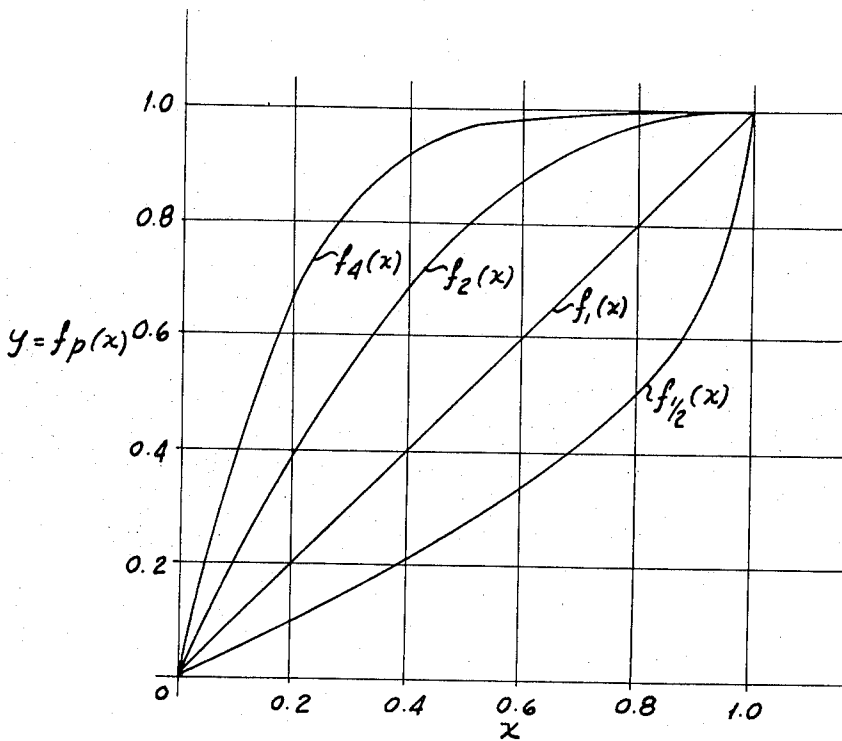
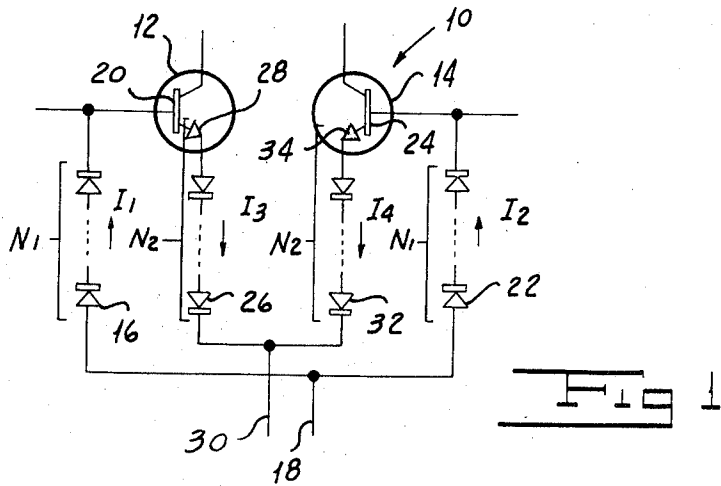
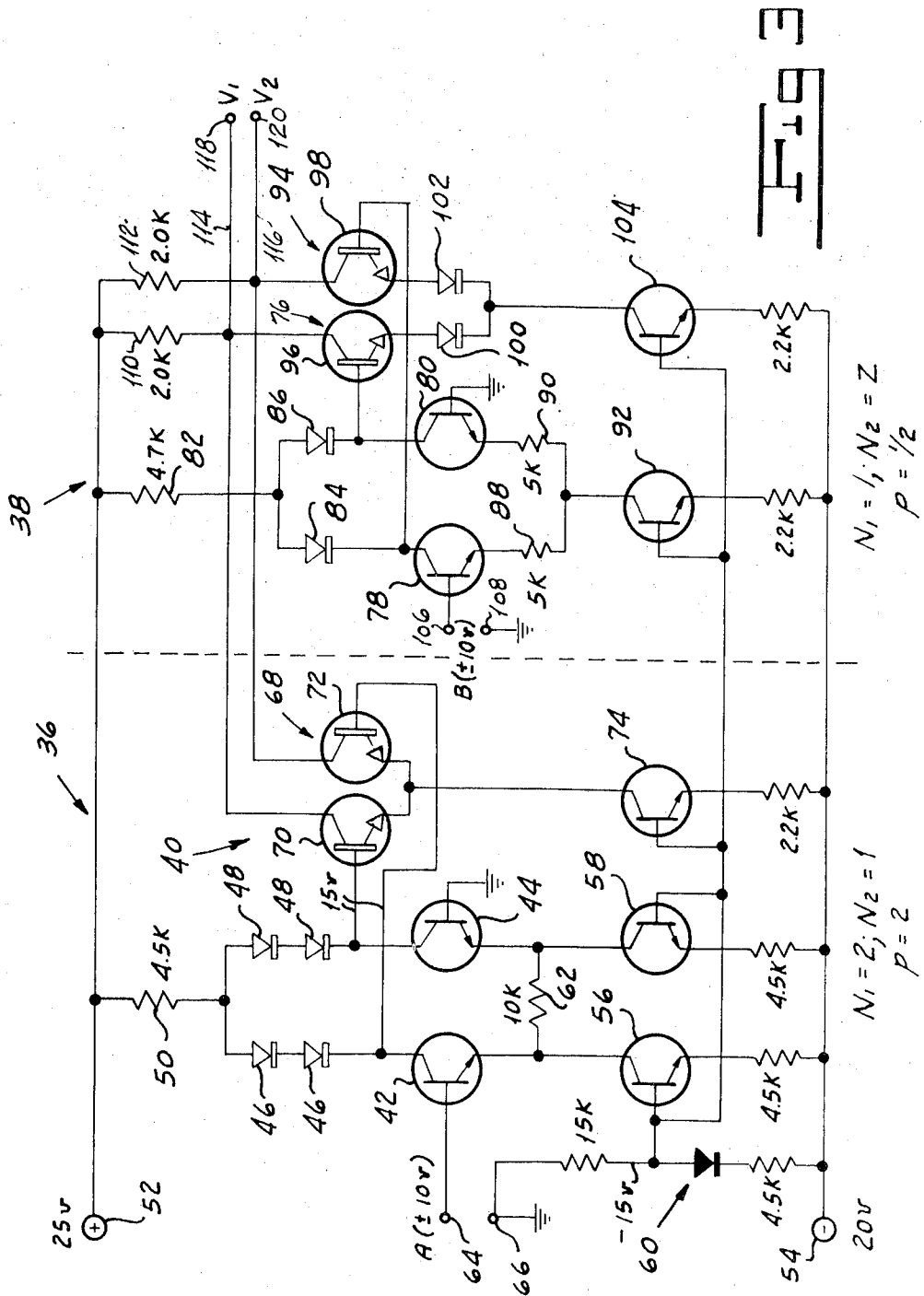


Fig 2

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FUNCTION GENERATOR

BACKGROUND OF THE INVENTION

There are many instances in which it is necessary or desirable to produce an output which is a nonlinear function such for example as the sine, the square or the logarithm of an input. Attempts have been made in the prior art to provide such generators. Systems representing attempts of the prior art to produce such a system variously embody a number of advantages. The speed of operation of many of them is limited. Operation of many of the devices is a function of the ambient temperature. The utility of others of the devices proposed in the prior art is limited in that they will operate only in response to signals of one polarity.

I have invented a function generator for producing a desired nonlinear input-output relationship. My generator functions at a relatively high speed as contrasted with nonlinear generators of the prior art. Operation of my generator is substantially independent of the ambient temperature. My system is capable of operating on both positive and negative inputs.

SUMMARY OF THE INVENTION

One object of my invention is to provide a nonlinear generator for producing a desired nonlinear input-output relationship.

Another object of my invention is to provide a function generator which is versatile in that it is adapted to provide a wide variety of input-output relationships.

A further object of my invention is to provide a function generator which operates at a relatively high speed.

Still another object of my invention is to provide a function generator the operation of which is substantially independent of ambient temperature.

A still further object of my invention is to provide a function generator capable of operating on both positive and negative inputs.

Yet another object of my invention is to provide a function generator which is simple in construction and which is reliable in operation.

Other and further objects of my invention will appear from the following description.

In general my invention contemplates the provision of a function generator each section of which is made up of two differentially connected devices each of which incorporates input and output strings of nonlinear elements all having the same voltage-current characteristics so as to provide an input-output relationship the power of which is determined by the ratio of the numbers of elements in the respective strings. By use of my generator I can build up a system in which each section represents one term of an equation approximating the function it is desired to produce.

BRIEF DESCRIPTION OF THE DRAWINGS

In the accompanying drawings which form part of the instant specification and which are to be read in conjunction therewith and in which like reference numerals are used to indicate like parts in the various views:

FIG. 1 is a schematic view illustrating one section of my function generator.

FIG. 2 is a curve illustrating the various functions which can be produced by a section of my function generator.

FIG. 3 is a schematic view illustrating one embodiment of my function generator for producing a particular input-output relationship.

DESCRIPTION OF THE PREFERRED EMBODIMENT

Referring now to FIG. 1 of the drawings, I have illustrated an arrangement indicated generally by the reference character 10 demonstrating the manner in which my generator operates to produce a nonlinear input-output relationship. The arrangement 10 includes respective NPN-transistors 12 and 14. I connect a first string of diodes 16 between a common line 18

and the base 20 of transistor 12. A second string of diodes 22 is connected between line 18 and the base 24 of transistor 14. I connect a third string of diodes 26 between the emitter 28 of transistor 12 and a common line 30. A fourth string of diodes 32 is connected between the emitter 34 of transistor 14 and the line 30. In the example being considered the base-to-emitter junctions of the respective transistors 12 and 14 comprise diodes of the strings of diodes 26 and 32. All of the diodes are selected to have the same current voltage characteristic.

As is known in the art, the current voltage relationship for any of the diodes 16, 22, 26 and 32 is approximately logarithmic and is given by the relationship:

(1)  $V = k_1 \ln I + k_2$  where  $k_2$  is in millivolts and wherein  $k_1$  is a function of temperature. In the example under consideration, I assume that the transistor beta is infinite although a finite but relatively large beta does not change the approach.

Considering only the transistor 12 and its associated strings of diodes 16 and of diodes 26 and assuming that the strings contain respective numbers  $N_1$  and  $N_2$  of diodes and that an input current  $I_1$  is passed through the diodes 16, and, for purposes of simplicity, that the constant  $K_2$  is zero, the voltage across the string of diodes 16 is given by

(2)  $V = N_1 k_1 \ln I_1$

Since the transistor is connected as an emitter follower, the same voltage exists across the string of diodes 26 and the resultant current  $I_3$  therethrough is given by the relationship

(3)  $I_3 = \text{antiln } V / N_2 k_1$  or

(4)  $I_3 = \text{antiln } N_1 k_1 / N_2 k_1 \ln I_1$

so that we can write:

(5)  $I_3 = I_1 (N_1 / N_2)$

It will thus be seen that the device including transistor 12 and strings of diodes 16 and 32 produces an output current which is related to the input current by an exponent which is the ratio of the numbers of the diodes in the respective strings.

Considering now the relationships over the entire circuit shown in FIG. 1, the currents flowing in the strings of diodes 16, 22, 26 and 32, are designated respectively as  $I_1, I_2, I_3$  and  $I_4$ .

In my arrangement, I select the number  $N_1$  of diodes 16 to be equal to the number of diodes 22 and I select the number  $N_2$  of diodes 26 to be equal to the number of diodes 32. Under these conditions the sum of the voltage drops around the loop including all of the diodes is given by:

(6)  $-N_1 (k_1 \ln I_1 + k_2) - N_2 (k_1 \ln I_3 + k_2) + N_2 (k_1 \ln I_4 + k_2) + N_1 (k_1 \ln I_2 + k_2) = 0$  or

(7)  $-N_1 \ln I_1 - N_2 \ln I_3 + N_2 \ln I_4 + N_1 \ln I_2 = 0$

which reduces to:

(8)  $N_1 / N_2 (\ln I_2 - \ln I_1) = (\ln I_3 - \ln I_4)$

It is to be noted that both constants  $k_1$  and  $k_2$  have dropped out of the relationship so that neither affects the input-output relationship. Thus variations in ambient temperature do not result.

Taking the antilogarithm of both sides of this expression we obtain:

(9)  $(I_2 / I_1)^{N_1 / N_2} (I_4 / I_3) = 1$

For purposes of illustration let the input variable be represented by:

(10)  $x = I_2 - I_1 / I_2 + I_1$  or

(11)  $I_2 / I_1 = I + x / I - x$

Let the output variable  $y$  be represented by

(12)  $y = I_3 - I_4 / I_3 + I_4$  or

(13)  $\frac{I_4}{I_3} \frac{1-y}{1+y}$

Substituting Equations (11) and (13) in Equation (9) we obtain:

(14)  $\left(\frac{1+x}{1-x}\right)^p \left(\frac{1-y}{1+y}\right) = 1$

where  $p = N_1 / N_2$  or

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$$(15) \quad yp(x) = \frac{1 - \left(\frac{1-x}{1+x}\right)^p}{1 + \left(\frac{1-x}{1+x}\right)^p}$$

From the foregoing it can be seen that the output variable is related to the input variable by an expression raised to a power  $p$  which is the ratio of the numbers of nonlinear elements  $N_1$  and  $N_2$ .

Referring to FIG. 2 I have shown various relationships of the input and output variables in accordance with Equation (15). It can be seen that I am able to represent any function made up of convex or concave relationships.

Referring now to FIG. 3 I have shown a system for representing a particular input-output relationship wherein a first section indicated generally by the reference character 36 represents one term of the function depending upon the input variable A. The other section indicated generally by the reference character 38 represents a term of the function of a second variable B. Section 36 includes two differentially connected devices, the first of which is indicated generally by the reference character 40. The device 40 includes as active elements a transistor 70 corresponding to transistor 12 of FIG. 1, a first string of diodes 48 and a second string of diodes comprising only the base-to-emitter junction of transistor 70. The other device 68 of the two differentially connected devices of section 36 includes transistor 68, a first string of diodes 46 and a second string of diodes including only the base-to-emitter junction of transistor 72.

A resistor 50 connects the common terminals of the strings of diodes 46 and 48 to the terminal 52 of a suitable source of positive potential. I connect an input transistor 42 and a balancing transistor 44 respectively between the other terminals of the strings of diodes 46 and 48 and constant current devices 56 and 58. A weighting resistor 62 connects the emitters of transistors 42 and 44. A biasing circuit indicated generally by the reference character 60, connected between a ground terminal 66 and the terminal 54 of a suitable source of negative potential provides the required bias for emitter followers 56 and 58, for an emitter follower 74 connected between the common emitter terminals of transistors 70 and 72 and terminal 54 and for other constant current devices, to be described hereinafter, of the section 38. I apply the input A to a terminal 64 connected to the base of transistor 42.

In the section 36 I provide the same number of diodes 46 as diodes 48 which number is  $N_1$  in the expressions outlined hereinabove. Moreover in the section 36 the base-to-emitter junctions of the transistors 70 and 72 make up the nonlinear elements of the differentially connected devices 40 and 68. In the arrangement shown since only one of these devices is in the path of each of the currents  $I_3$  and  $I_4$ ,  $N_2=1$ . As a result for the section 36  $p=2$ .

In the other section 38 the first one, indicated generally by the reference character 76, of the two differentially connected devices includes the transistor 96, a first diode string including a single diode 86 and a second diode string including the base-to-emitter junction of transistor 96 and a diode 100. The other device, indicated generally by the reference character 94, of the section 38, includes a transistor 98, a first diode string including a single diode 84 and a second diode string including the base-to-emitter junction of transistor 98 and a diode 102.

A resistor 82 connects the common terminal of diodes 84 and 86 to terminal 52. Respective weighting resistors 88 and 90 connect the emitter terminal of input and balancing transistors 78 and 80, connected respectively to diodes 84 and 86, to a constant current emitter follower 92. Another constant current emitter follower 104 connects the common terminal of diodes 100 and 102 to terminal 54. Circuit 60 provides the bias for emitter followers 92 and 104.

I apply the second input variable B between a terminal 106 connected to the base of input transistor 78 and a ground terminal 108. Respective resistors 110 and 112 connect output lines 114 and 116 to terminal 52. Line 114 is common to the

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collectors of transistors 70 and 96. It leads to an output terminal 118. Line 116 is common to the collectors of transistors 72 and 98. It leads to an output terminal 120.

From the foregoing it will be seen that the number  $N_1$  of diodes 84 and of diodes 86 is one while the number  $N_2$  of diodes in each of the emitter circuits of transistors 86 and 98 is two. Thus, for section 38 the output power  $p$  is one-half.

For purposes of clarity in FIG. 3 I have illustrated the active nonlinear elements of the various differential devices by open symbols while the other elements of the system are indicated as being solid. It will readily be appreciated that any non-linearity in those other elements is masked by the resistors associated therewith such as the various resistors indicated in the FIG. as being associated with the emitter circuits of the other transistors.

In FIG. 3 I have indicated nominal values for the various potentials and resistors which will produce the desired relationships. Further, from Equation (12) above it will be apparent that for any section of my system the output current derived will be:

$$(16) \quad I_3 - I_4 = Y(I_3 + I_4)$$

In terms of voltage the output voltage  $V$  is given by:

$$(17) \quad V = Y(I_3 + I_4)R$$

where  $R$  is an output resistor.

For the particular system illustrated in FIG. 3 the voltage between terminals  $V_1$  and  $V_2$  is:

$$(18) \quad V_2 - V_1 = K(y_1 + y_2)$$

where  $K$  is the product of  $I_3$  and  $I_4$  for each section and of the output resistance. In the particular instance illustrated  $K=2 \times 10^{-3}$  amps times  $2 \times 10^3$  ohms or 4 volts.

Substituting for  $y_1$  and for  $y_2$  in Equation (18) we obtain:

$$(19) \quad V_2 - V_1 = K \left[ \frac{1 - \left(\frac{1-x_1}{1+x_1}\right)^{p_1}}{1 + \left(\frac{1-x_1}{1+x_1}\right)^{p_1}} + \frac{1 - \left(\frac{1-x_2}{1+x_2}\right)^{p_2}}{1 + \left(\frac{1-x_2}{1+x_2}\right)^{p_2}} \right]$$

In section 36, owing to the presence of resistor 62,  $x_1=A/10$ , and  $p_1=2$ . In section 38, owing to the presence of resistors 88 and 90,  $x_2=B/10$ , and  $p_2=1/2$ . Substituting these values in Equation (19) the output across terminals  $V_1$  and  $V_2$  is given by:

$$(20) \quad V_2 - V_1 = 4 \left[ \frac{1 - \left(\frac{1-A/10}{1+A/10}\right)^2}{1 + \left(\frac{1-A/10}{1+A/10}\right)^2} + \frac{1 - \left(\frac{1-B/10}{1+B/10}\right)^{1/2}}{1 + \left(\frac{1-B/10}{1+B/10}\right)^{1/2}} \right]$$

It will thus be seen that the output in terms of the inputs for the system of FIG. 3 includes two terms one of which may be termed a convex or square power term of a first variable provided by section 36 and the other of which is a square root term of a second variable provided by section 38.

The operation of my system will readily be apparent from the description thereof given hereinabove. Considering any section thereof it produces an output which is related to the input by an exponent or power which is the ratio of the numbers  $N_1$  and  $N_2$  of nonlinear devices in the section. The section provides one term of an equation approximating the desired function to be generated. By adding sections to the system the number of terms in the approximation may be increased.

It will be seen that I have accomplished the objects of my invention. I have provided a function generator which overcomes the defects of generators of the prior art. It closely approximates any desired function. Its operation is substantially independent of ambient temperature. It will operate on either positive or negative potentials. Its operation is more rapid than is that of generators of the prior art.

It will be understood that certain features and subcombinations are of utility and may be employed without reference to

other features and subcombinations. This is contemplated by and is within the scope of my claims. It is further obvious that various changes may be made in details within the scope of my claims without departing from the spirit of my invention. It is, therefore, to be understood that my invention is not to be limited to the specific details shown and described.

Having thus described my invention, what I claim is:

1. A function generator for producing a desired input-output relationship including in combination, a first number of serially connected elements each having a particular nonlinear voltage-current characteristic, a second number of serially connected elements each having a certain nonlinear voltage-current characteristic, said first and second numbers being unequal, means for passing an input current through the first elements to produce a first voltage thereacross, means for applying said first voltage across the second elements to produce a first current therethrough, and means responsive to said first current for producing an output current related to said input current by an exponent equal to the ratio of the first number to said second number.

2. A function generator as in claim 1 in which said first and second elements are diodes each having the same nonlinear characteristic.

3. A function generator as in claim 1 in which said voltage-current relationship is logarithmic.

4. A function generator as in claim 1 in which said first number is greater than said second number.

5. A function generator as in claim 1 in which said second number is greater than said first number.

6. A function generator as in claim 1 in which said means for applying said first voltage includes a transistor having a base-to-emitter junction comprising one of the second elements.

7. A function generator as in claim 1 in which said elements are diodes having a logarithmic voltage-current characteristic, and in which said means for applying said first voltage comprises a transistor having a base-to-emitter junction comprising one of the second elements.

8. A function generator as in claim 1 in which one of said numbers is unity.

9. A function generator for producing a desired input-output relationship including in combination, a first number of serially connected elements each having a particular nonlinear voltage-current characteristic, a second number of serially connected elements each having a certain nonlinear voltage-current characteristic, said first and second numbers being equal, means for differentially applying an input current to said first and second numbers of elements to produce first voltages thereacross, a third number of serially connected elements having a certain nonlinear voltage-current characteristic, a fourth number of serially connected elements each having a certain nonlinear voltage-current characteristic, said third number being equal to said fourth number, means for differentially applying said first voltages to said third and fourth numbers of elements to produce first currents

therethrough, and means differentially responsive to said first currents for producing an output voltage.

10. A function generator as in claim 9 in which said input current applying means comprises means for maintaining of the currents through said first and second numbers of elements constant.

11. A function generator as in claim 9 in which said first voltage applying means comprises means for maintaining the sum of the currents through said third and fourth numbers of elements constant.

12. A function generator as in claim 9 in which said input current applying means comprises first means for maintaining the sum of the currents through said first and second numbers of elements constant and in which said first voltage applying means comprises means for maintaining the sum of the currents through said third and fourth numbers of elements constant.

13. A function generator as in claim 9 in which said input current applying means comprises a weighting resistance between said first and second strings.

14. A function generator as in claim 9 in which said voltage applying means comprises transistors each having a base-to-emitter junction, each of said junctions forming an element of one of the third and fourth numbers of elements.

15. A function generator as in claim 9 in which said input current applying means comprises a transistor connected to said first number of elements and means for applying an input signal to said transistor.

16. A function generator as in claim 9 in which said input current applying means comprises means for maintaining the sum of the currents through said first and second numbers of elements constant, and a weighting resistance between said first and second numbers of elements, and in which said voltage applying means comprises transistors each having a base-to-emitter junction and in which each of said junctions forms an element of one of the third and fourth numbers of elements and means for maintaining the sum of the currents through said third and fourth numbers of elements constant.

17. A function generator for producing a desired function of one or more input variables including in combination, a first section, a second section, each of said sections including a first device comprising a first string of elements having a particular nonlinear voltage-current characteristic, a second string of a second different number of said elements, and means for applying a voltage produced by a current flow through said first string across said second string, and a second device comprising a third string of said first number of said elements, a fourth string of said second number of elements, and means for applying a voltage produced by a current flow through said first string across said second string, means for applying inputs to said sections to cause currents to flow in said first strings, the second string voltages resulting from said first string currents producing currents in said second strings, and means for producing an output from the combined second string currents.

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**Disclaimer**

3,643,107.—*Dennis R. Gilbreath*, New Haven, Conn. FUNCTION GENERATOR. Patent dated Feb. 15, 1972. Disclaimer filed Nov. 23, 1973, by the assignee, *United Aircraft Corporation*.

Hereby enters this disclaimer to claims 1-8, inclusive, of said patent.

[*Official Gazette May 27, 1975.*]