June 29, 1943. P. S. CARTER 2,323,201

TUNED CIRCUIT AND ASSOCIATED DEVICES THEREFOR

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Fig. 1

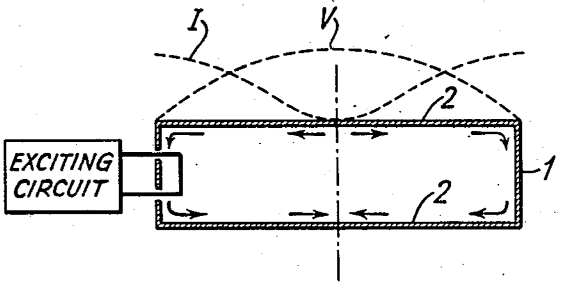


Fig. 2

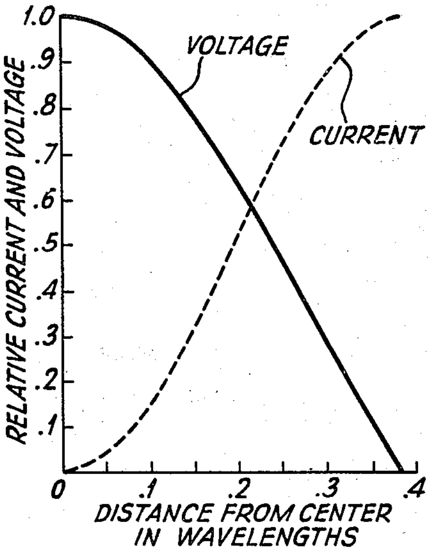


Fig. 1a

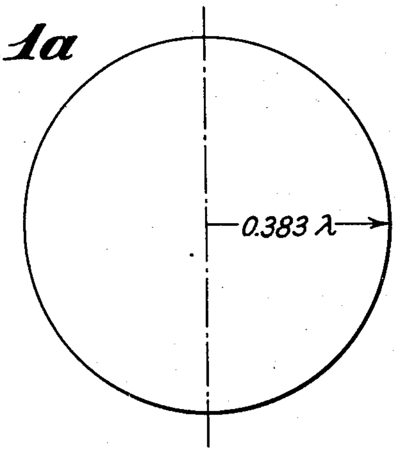


Fig. 2a

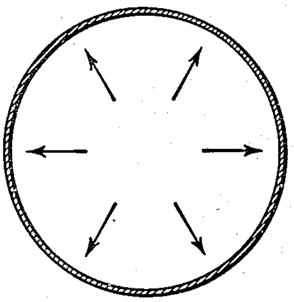
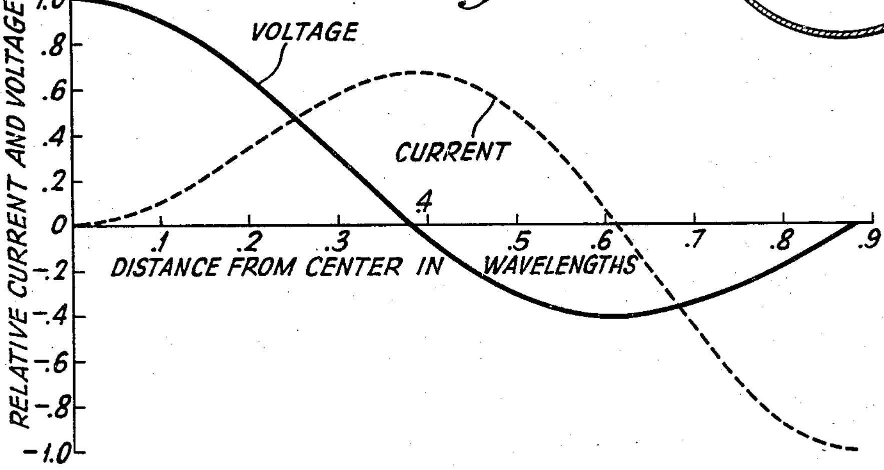


Fig. 3



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TUNED CIRCUIT AND ASSOCIATED DEVICES THEREFOR

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Fig. 4

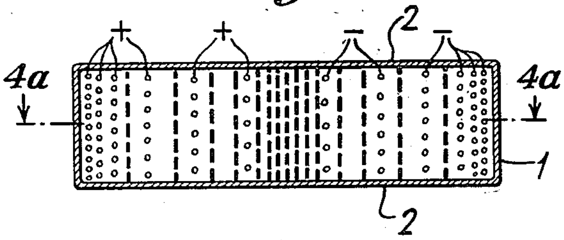


Fig. 5

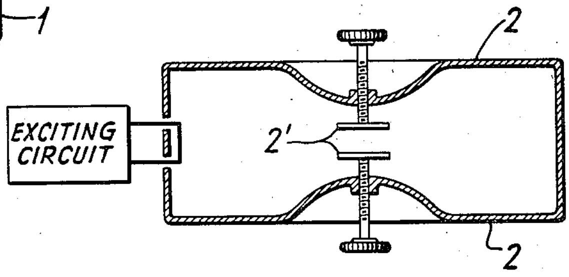


Fig. 4a

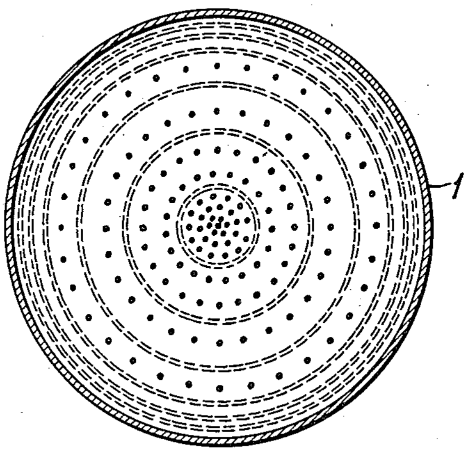


Fig. 6

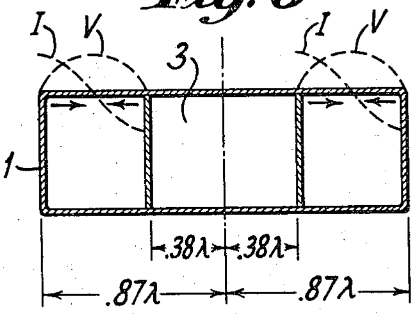


Fig. 6a

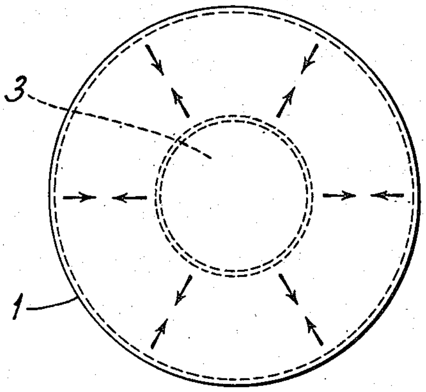
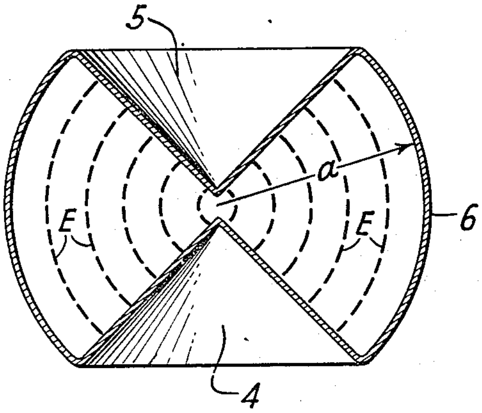


Fig. 7



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TUNED CIRCUIT AND ASSOCIATED DEVICES THEREFOR

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Fig. 8

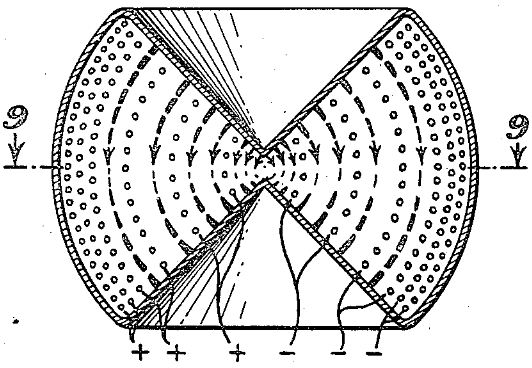


Fig. 9

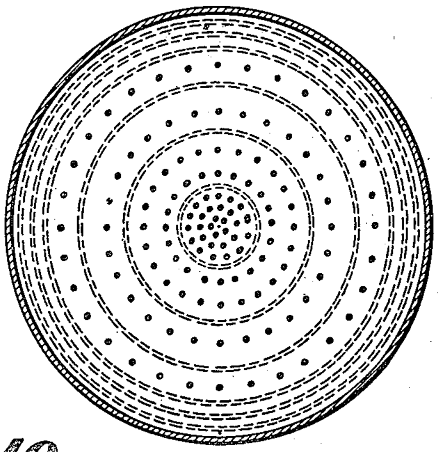


Fig. 10

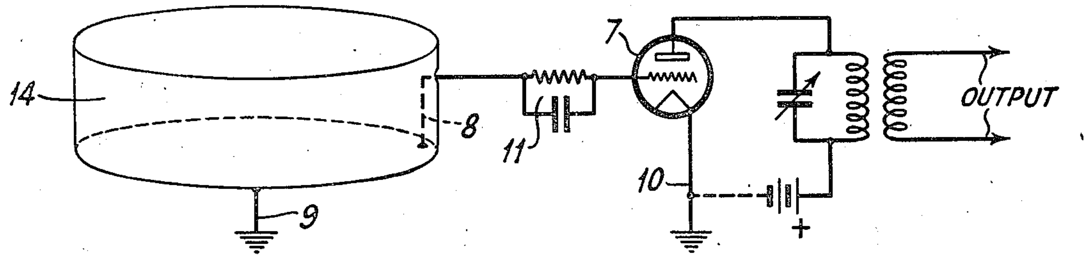


Fig. 11

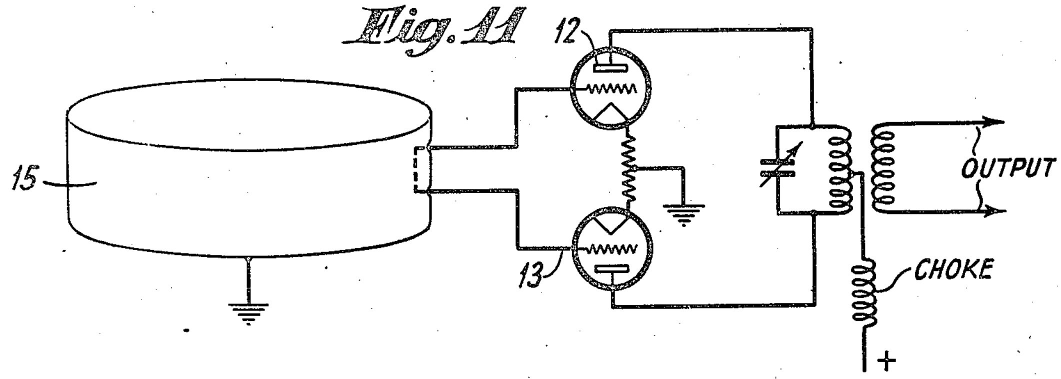
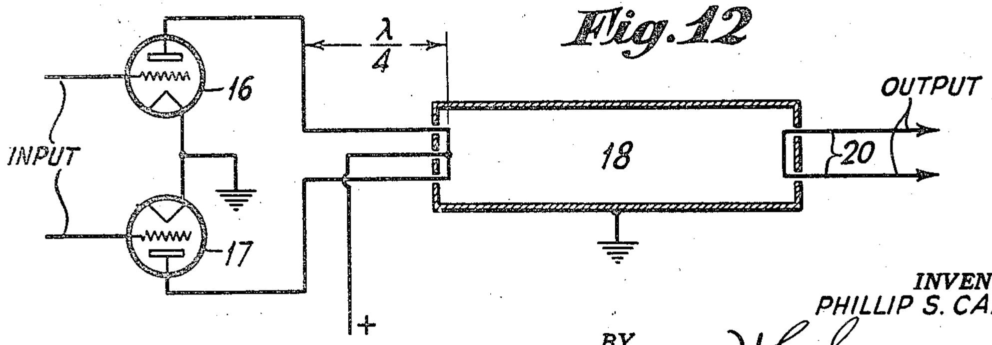


Fig. 12



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TUNED CIRCUIT AND ASSOCIATED DEVICES THEREFOR

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Fig. 13

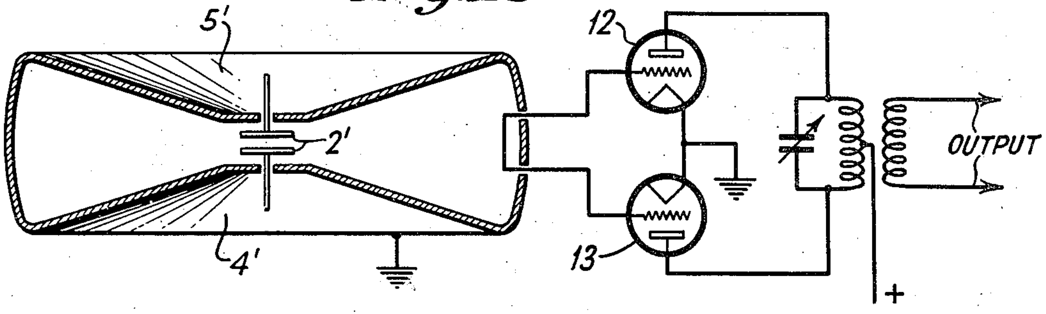


Fig. 14

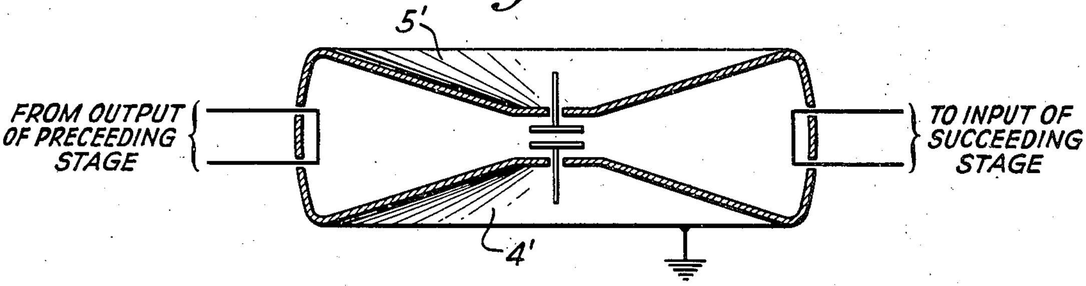


Fig. 15

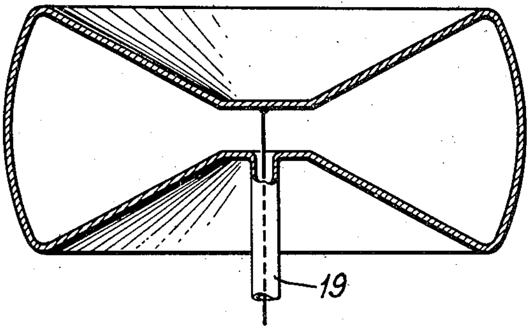
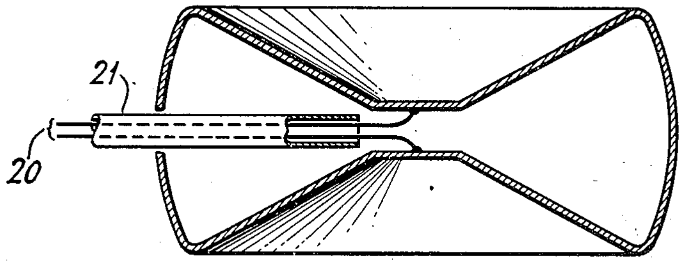


Fig. 15a



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Patented June 29, 1943

2,323,201

UNITED STATES PATENT OFFICE

2,323,201

TUNED CIRCUIT AND ASSOCIATED DEVICES THEREFOR

Philip S. Carter, Port Jefferson, N. Y., assignor to Radio Corporation of America, a corporation of Delaware

Application January 7, 1939, Serial No. 249,711

2 Claims. (Cl. 178—44)

The present invention relates to improvements in high frequency tuned circuits, and has for its principal object to provide a novel type of high frequency tuned circuit which is highly efficient, of extremely low loss, and substantially radiationless. A more specific object is to provide a high frequency tuned circuit which employs a different type of electromagnetic wave phenomenon within the circuit than heretofore employed. A further object is to provide a tuned circuit which has a series of natural modes of oscillation which differ from the modes of oscillation of known types of resonators of uniformly distributed constants.

The conventional types of tuned circuits used for high frequency work include the mechanically vibrating crystal and the section of concentric transmission line, the latter of which is known as the resonant line. With the crystal, the energy is stored in the form of mechanical vibration, while in the resonant line the energy is in the form of electrical oscillations. Both of these known types of tuned circuits have certain disadvantages. The crystal is fragile and does not lend itself as readily as might be desired for use with extremely high frequencies below five meters. In the concentric resonant line the natural modes of oscillation are harmonically related, thus enabling in certain circumstances the existence of undesired harmonic frequencies. Within this resonant line standing waves are produced, and the wave energy expands between the two conductors in a single direction, on the style of a planar wave, the electric field being parallel to the end plates or flat end surfaces. For a more detailed description of the concentric resonant line, reference may be had to the following literature: An article by Clarence W. Hansell and Philip S. Carter entitled "Frequency control by low power factor line circuits," published in "Proceedings of the Institute of Radio Engineers," April, 1936; an article by Clarence W. Hansell entitled "Resonant lines for frequency control," published in "Electrical Engineering," August, 1935, pages 852 et seq.; United States Patent No. 2,077,800, granted April 20, 1937, to Fred H. Kroger; United States Patent No. 2,121,158, granted June 21, 1938, to Nils E. Lindenblad; and United States Patent No. 2,120,518, granted June 14, 1938, to John F. Dreyer, Jr.

The foregoing disadvantages, as well as others, of known tuned circuits are overcome by the present invention which, in brief, provides an extremely low loss tuned circuit having standing waves produced therein, and which employs a type of electromagnetic wave expanding in a plurality of dimensions. By means of the tuned circuit of the invention, it is possible to obtain a series of natural modes of oscillation whose frequency ratios are not rational numbers, thus tending to eliminate undesired harmonic frequencies.

One embodiment of the present invention comprises a hollow enclosed cylinder wherein the electromagnetic wave energy is in the form of a cylindrical wave which expands in two dimensions, the direction of the electric field being perpendicular to the flat end surfaces of the circuit. Such a tuned circuit has a series of natural modes of oscillation which are inharmonic (musically speaking); that is, the frequency ratios are not rational numbers.

Another embodiment of the present invention takes the form of an hour glass composed of two cones disposed coaxially with their apices adjacent but suitably separated from each other, the bases of the cones terminating in a spherical conducting surface, wherein the electromagnetic wave energy has a spherical wave front.

A better understanding of the invention may be had by referring to the following description which is accompanied by drawings, wherein:

Figs. 1, 5 and 6 illustrate certain embodiments of my invention taking the form of enclosed cylinders, wherein there is utilized a cylindrical electromagnetic wave expanding in two dimensions;

Fig. 1*a* is a plan view of the tuned circuit of Fig. 1;

Fig. 2 is a graph of the calculated voltage and current curves of a tuned circuit, such as shown in Fig. 1;

Fig. 2*a* diagrammatically illustrates, by means of arrows, the directions of the current flow in one of the flat surfaces of the tuned circuit of Fig. 1;

Fig. 3 shows graphically the current and voltage distribution of the circuit of Fig. 1 when oscillating at its second natural mode, when the radius is 0.879 wavelength;

Fig. 4 is a vertical section of the tuned circuit of Fig. 1, showing possible electric and magnetic fields within the tuned circuit;

Fig. 4*a* is a horizontal section of Fig. 4, along the lines 4*a*—4*a*, showing a plan view of the electric and magnetic fields illustrated in Fig. 4;

Fig. 6*a* is a plan view of the tuned circuit of Fig. 6 and shows by means of arrows the directions of current flow in one of the flat surfaces;

Fig. 7 illustrates another embodiment of a tuned circuit in accordance with the invention, wherein there is utilized a spherical electromagnetic wave. This embodiment employs a pair of coaxially arranged cones disposed with their apices adjacent one another and having their bases terminating in a spherical conducting surface;

Fig. 8 is a vertical section of the tuned circuit of Fig. 7, showing possible electric and magnetic fields inside the spherical surface;

Fig. 9 is a horizontal section along the lines **9—9** of Fig. 8, showing a plan view of the electric and magnetic fields illustrated in Fig. 8;

Figs. 10 and 11 illustrate the manner in which tuned circuits of the type of Figs. 1 and 4 can be employed as the frequency determining element of single ended and push pull electron discharge device circuits, respectively;

Fig. 12 illustrates how a tuned circuit such as shown in Figs. 1, 5 and 6 can be used as an impedance coupling element between stages of radio frequency transmitting or receiving apparatus; and

Figs. 13, 14 and 15 illustrate practical arrangements of the tuned circuit of Fig. 7, in connection with suitable utilization devices.

Although the tuned circuits of the invention will be described with particular reference to certain utilization circuits, it should be distinctly understood that the illustrations given in the drawings are merely illustrative and not by way of limitation, since the tuned circuits of the invention may be employed wherever there is need for a tuned circuit; for example, as a frequency determining element in an oscillator, as a tank circuit for an amplifier, as a filter, and as an impedance coupling element between stages of a receiver or transmitter.

Referring to Fig. 1 in more detail, this embodiment of my novel tuned circuit comprises a hollow metallic cylinder **1** enclosed by flat surfaces or end plates **2, 2**, the size and shape of the enclosure determining its resonant frequency. The instantaneous current in one of the flat surfaces end plates **2** will be from the center out toward the cylinder, in the manner shown in Fig. 2*a*, while the current in the other end plate **2** will be from the cylindrical surface in toward the center. The directions of the arrows indicate the manner in which these currents may flow. The current will be substantially zero at the centers of the flat surfaces **2, 2** and substantially maximum at the periphery thereof, while the voltage will be maximum at the center of the flat surfaces **2, 2** and minimum at the periphery. The dotted curves labeled I and V illustrate graphically, by way of example only, the total current on the flat surfaces **2, 2**, and the voltage between these surfaces, respectively.

Fig. 2 graphically illustrates calculated current and voltage distributions of the resonant circuit of Fig. 1 measured from the center toward the periphery of the end plate, when the tuned circuit is used at its lowest mode of oscillation. The current shown in this figure is the total current flowing in the face **2** at a particular distance given on the figure. It should be noted that the current is large only at surfaces having large areas; in other words, large currents flow only through low resistance portions of the circuit. It will thus be obvious that the tuned circuit of Fig. 1 is a low loss circuit and has considerably less loss than a concentric resonant line which has less area over which large currents flow.

By calculation, I have found that a tuned circuit of the type shown in Fig. 1 is resonant when the radius multiplied by $2\pi/\lambda$ is a root of the zero order Bessel function of the first kind, where λ is the length of the operating wave. The current distribution is in proportion to the radial distance r multiplied by the first order Bessel function of the first kind of $2\pi r/\lambda$; i. e., $r.J_1(2\pi r/\lambda)$.

The following table shows the dimensions of the tuned circuit of Fig. 1 for modes of oscillation up to the fifth, and the frequency ratio of the higher modes with respect to the lowest mode. It should be noted that the lowest mode of vibration has not been referred to as the fundamental, because it is desired to avoid giving the impression that the modes are in harmonic relation. The frequency ratios of the modes are not rational numbers, and consequently the tendency toward the presence of undesired harmonic frequencies existing in previous types of resonators is overcome by the present invention.

Mode	Radius in wavelengths	f_n/f_1
1	0.383	1
2	0.879	2.27
3	1.376	3.59
4	1.875	4.90
5	2.375	6.20

Fig. 4 illustrates a vertical section of Fig. 1 with possible electric and magnetic fields inside the circular cylinder. The central portion of the cylindrical tank circuit has a maximum number of electric lines of force extending perpendicular to the flat surfaces **2, 2**, while the portions more remote from the center have less lines of electric force. The small dot-like circles marked on one side of the center — and on the other side of the center + indicate a possible distribution of the magnetic field. The magnetic field is of maximum density near the cylinder **1** and of minimum density near the center of the cylinder, being the reverse of the condition of the electric field.

Fig. 4*a* is a cross section of Fig. 4 along the line **4*a*, 4*a***, showing the same condition of Fig. 4. In Fig. 4*a* the small dot-like circles represent the electric field, while the circular lines represent the magnetic field.

One important feature of my tuned circuit lies in the fact that the wave energy within the tuned circuit is a cylindrical wave extending in two dimensions, thus differing from other types of known resonators employing a planar or one dimension type of wave.

Fig. 3 shows the current and voltage distribution for the tuned circuit of Fig. 1 when oscillating at its second natural mode; that is, when its radius is equal to 0.879 wavelength. It should be observed that the voltage and current distributions have some similarity to a three-quarter wave transmission line short circuited at one end.

From what has gone before, it will be apparent that the tuned circuit of the invention has considerably less power loss than that of the concentric line. It should also be noted that my tuned circuit requires no insulation whatever except possibly small insulators where the exciting circuit enters the interior of the enclosure. Thus, one of the advantages of the present invention lies in the elimination of insulator losses. The exciting circuit shown in Fig. 1, in conventional box form, is merely given by way of illustration to show one method by which the tuned circuit can be excited. The source of excitation may be a suitable radio frequency transmitter, or an electron discharge device oscillator. The methods of exciting the tuned circuits of the invention are described in more detail later.

Fig. 5 shows a modification of the tuned circuit of Fig. 1, wherein the spacing of the flat ends **2**, **2** is gradually reduced to a lower value at the center. Since the dimensions of the tuned circuit of Fig. 5 depart from those mentioned above in connection with Fig. 1, the natural frequency of the tuned circuit of Fig. 5 must be found by experiment. In most cases, such a modification tends to increase losses and thus would not ordinarily be recommended. In order to change the natural frequency of the tuned circuit, I have shown, in Fig. 5, a pair of spaced variable condenser plates **2'** whose spacing can be varied for tuning purposes. If desired, such plates can also be used in Fig. 1 for the same purpose. In a practical application, it may be advisable to completely coat the inner surfaces of the tuned circuits of Figs. 1 and 4 with highly electrically conducting metal, such as silver, in order to reduce the losses of the circuit to the lowest possible value. Where this is done, however, caution should be taken that condensation on the inside surfaces is prevented, which can be done by making the enclosed tuned circuit airtight and sealing it permanently after filling with a dry gas.

Fig. 6 is a modification of the circuit of Fig. 1 and shows a tuned circuit utilizing an inner conductor **3** within the cylinder **1** located between the flat end surfaces. This tuned circuit also utilizes a cylindrical wave expanding in two directions in the interior of the enclosure, and is especially useful for push-pull types of electron discharge device circuits. It should be noted that the radius of the inner conductor **3** is 0.38 wavelength, while the radius of the cylinder **1** is 0.87 wavelength. Consequently, the tuned circuit bears a similarity to the tuned circuit of Fig. 1 when oscillating at its second mode, as will be apparent from an inspection of the table previously given. The electric field of this tuned circuit, like the electric field of Fig. 1, is at right angles to the flat end plates enclosing the cylinder **1**, and the directions of the currents in the end plates, as indicated by the arrows. The dotted lines I and V respectively indicate the current and voltage distributions on the flat end surfaces. In effect, the dimensions of the tuned circuit of Fig. 6 between the cylinder **1** and the inner conductor **3** are roughly, though not exactly, one-half wavelength, being the difference between two successive roots of the Bessel function of $2\pi r/\lambda$, and the tuned circuit corresponds in a way to a one-half wave-length transmission line resonator.

Fig. 6*a* is a plan view of Fig. 6 showing the directions of the currents in the end plates.

Fig. 7 shows a different and a preferred form of the invention, wherein there is employed a spherical surface and a spherical type of wave. The tuned circuit of Fig. 7 is in the form of an hour-glass electrical resonator consisting of two cones **4** and **5** coaxially arranged with their apices adjacent to but not in contact with one another and whose bases or larger ends terminate in a spherical surface **6**. Such a resonator, I have found, give a higher Q than any other type of electrical wave resonator having approximately the same dimensions. The radius *a* of the spherical surface **6** is, in the theoretical case, one-quarter of a wavelength at the operating frequency, although in a practical case, as shown in some of the figures to be described later, the radius *a* may be greater or less, though normally less than one-quarter wavelength depending upon such factors as the amount of capacity inserted at the center between the apices, or, putting it another way, the electrical length of the radius is always one-quarter wavelength regardless of compromises made in practical construction. The electric field vectors and the general directions they assume, are shown by the light curved lines within the spherical surface extending between the cones and designated by E.

The type of wave existing between the surfaces of the two cones **4** and **5** in a resonator of the type shown in Fig. 7 is what may be called a Q_0 type of spherical wave, where Q_0 is the Legendre function of the second kind, or the zonal surface harmonic of the second kind. The mathematical theory of this function is given in Byerlys Fourier's Series & Spherical Harmonics, and defined also briefly in Jahne-Emde Funktionentafeln. This function is that from which the electric and magnetic fields in a spherical wave front may be derived. If we define the characteristic impedance for a spherical wave between the conical surfaces shown, as the ratio of total voltage between one conical surface and the other taken along a spherical wave front to the total current flowing along the surface of one cone, the characteristic or surge impedance for such a wave can be shown to be 120 $Q_0(\cos\alpha)$. Here α is the angle of revolution of the cones and $Q_0(\cos\alpha)$ the function already defined. Such a Q_0 type of wave can only exist between conical surfaces and is one in which the characteristic impedance is a constant and does not vary with distance from the apices. The mathematical theory of such a wave is quite complicated and will not be given in detail herein, except insofar as certain relations are concerned which may be helpful in an understanding of the invention.

The voltage along a spherical wave front between the cones of the hour-glass embodiment of my invention, is proportional to $Q_0(\cos\alpha)$, where Q_0 is the zero order of the Legendre function of the second kind, or the zontal surface harmonic of the second kind, and α is the angle of revolution of the cone. This voltage is also proportional to

$$\frac{\cos\frac{(2\pi r)}{\lambda}}{r}$$

where *r* is the distance from the center of the sphere (i. e., the theoretical junction of the apices of the cones) to the reference point in the enclosed space. Thus, the voltage theoretically becomes infinity at the center of the hour-glass (disregarding losses in the circuit) between the apices of the cones. The practical structure departs somewhat from the theoretical by rounding off the apices of the cones in the manner illustrated in Figs. 13, 14 and 15.

The total current flowing along the surface of each cone is proportional to

$$\sin\frac{(2\pi r)}{\lambda}$$

The current density along each cone, however, is proportional to

$$\frac{\sin\frac{(2\pi r)}{\lambda}}{r}$$

The current density in the spherical surface is proportional to cosecant θ, where θ is the angle of the reference position on the spherical surface to the common axis of the cones.

The electric field vector of the hour-glass is perpendicular to the cones and lies in a spherical wave front. The electric intensity is proportional to

$$\frac{\cos\frac{(2\pi r)}{\lambda}}{r}\cdot\text{cosec }\theta$$

Putting it another way, the variation of electric intensity in a wave front is proportional to the cosecant of the angle between the common axis and the reference point. The electric intensity is also inversely proportional to the distance from the center of the hour-glass, and proportional to the cosine of the angular function of this same distance.

The magnetic field vector is parallel to the cones and forms a system of circles lying in any spherical wave front. The law governing the variation in intensity of the magnetic field vector within any wave front is the same as that given for the electric vector. The magnetic vector is inversely proportional to the distance from the center of the hour-glass and is proportional to the sine of the angular function of the same distance. In order words, the magnetic intensity is proportional to

$$\frac{\sin\frac{(2\pi r)}{\lambda}}{r}\cdot\text{cosec }\theta$$

Fig. 8 shows possible electric and magnetic fields inside the spherical surface of my hour-glass type of electrical tuned circuit. The lines (bearing arrows) between the two cones represent the electric field vectors which lie in a spherical wave front. The heaviness of the lines indicates the relative intensity of the electric field within a particular wave front. It should be noted that these lines are more numerous near the apices of the cones, thus indicating that there is greater intensity of electric field near these apices than near the larger surfaces of the cones. The small dot-like circles to the left and right of the apices, marked + and —, respectively, show the magnetic field whose vectors form a system of circles lying in any spherical wave front. These circles are more numerous near the spherical surface than near the apices of the cone, thus showing that the magnetic field is more intense near the spherical surface than near the apices.

Fig. 9, which is a cross section of Fig. 8 along the lines **9—9** thereof, shows the magnetic field vectors as circles parallel to the axis of the cones, and the electric vectors as small dots perpendicular to the cones.

Fig. 10 shows, by way of example, how a tuned circuit of the type shown in Figs. 1 and 5 can be used as a frequency determining element of an electron discharge device oscillator. In this figure, the oscillator comprises an evacuated electron discharge device **7** having an anode, cathode and grid, to the grid of which is connected a lead **8** extending within the interior of a tuned circuit **14**. Lead **8** is parallel to the cylinder and inductively coupled thereto along the length of the portion within the container **14**. The cylinder is shown grounded at **9** and the cathode grounded at **10**. In the grid lead there is shown the usual form of grid leak and condenser combination **11**. By virtue of the inductive coupling between lead **8** and the cylinder of the tuned circuit, the resonator will be excited to stabilize or control the frequency of the oscillator. The output circuit may be any suitable type of conventional arrangement, as indicated in the drawings.

Fig. 11 shows how the tuned circuit of the invention can be employed to control the frequency of a pair of electron discharge devices **12** and **13** connected in push-pull relation. The tuned circuit **15** is excited by these vacuum tubes and impresses potentials of opposite instantaneous polarities on the grids of the electron discharge devices. The oscillator circuit per se is well known in the art, and is merely shown in conventional form.

Fig. 12 illustrates how the tuned circuit of the invention shown in Fig. 1 can be employed as an impedance coupling element between two stages of a multi-stage radio frequency transmitter or receiver. In this figure, vacuum tubes **16** and **17** may comprise amplifier tubes or oscillator tubes whose output excites the tank circuit at a low impedance point. In order to match the high impedance output of vacuum tubes **16** and **17** to the low impedance connection of the tank circuit **18**, there is provided a quarter wavelength line, as shown. The output from the tank **18** may be delivered to the input circuit of another vacuum tube, such as an amplifier stage, and is also shown coupled to the tank circuit **18** in such a way as to provide voltages of opposite instantaneous polarities on the leads **20** of the output circuit. It will be obvious that the tank circuit **18** may form part of a filter circuit in an arrangement very much like that of Fig. 12.

Fig. 13 shows a practical embodiment of an hour-glass tuned circuit comprising a pair of cones **4'**, **5'** arranged in the same manner as Fig. 7, but differing therefrom in that the apices are cut off or rounded out to provide additional capacity therebetween. For tuning purposes, there is provided a condenser comprising a pair of plates **2'** which are variable relative to one another to vary the tuning of the hour-glass. For exciting the tuned circuit, there are shown a pair of evacuated electron discharge device oscillators **12** and **13** coupled to the spherical surface of the hour-glass inductively in substantially the same manner as the vacuum tubes of Fig. 11 are coupled to the resonator **15** of Fig. 11. It will be obvious that out of phase potentials are applied to the grids of vacuum tubes **12**, **13** of Fig. 12 to maintain the vacuum tubes in push-pull relation.

Fig. 14 illustrates the manner in which an hour-glass tuned circuit comprising a pair of cones **4'**, **5'** can be used as an interstage impedance coupling element between the output of one stage and the input of a succeeding stage, in a manner very generally like that illustrated in Fig. 12 for a different form of tuned circuit of my invention. The leads extending from the output of the preceding stage and the input of the succeding stage to the interior of the tuned circuit are as parallel as possible to the spherical surfaces for enabling an inductive coupling therebetween. The points of coupling between the hour-glass and these leads are points of relatively low impedance.

Fig. 15 shows one manner in which the practical form of hour-glass resonator of the invention can be connected at high impedance points to an external circuit. This figure shows how a coaxial line **19** can be connected to the apices of the cones. Fig. 15*a* is similar to Fig. 15 and shows how a pair of conductors **20** enclosed within an outer sheath **21** can also be coupled to the apices.

It should be distinctly understood that the figures of the drawings are merely given by way of illustration, since various modifications can be made without departing from the spirit and scope of the invention. The tuned circuits of the invention can be used either as an input, or an output circuit of an electron discharge device, or used both in the input and output circuits of an electron discharge device circuit. Where several of my novel tuned circuits are used in a single radio frequency apparatus (such as in a transmitter or a receiver) either as impedance coupling elements between stages, or otherwise, they may be stacked side by side, or one above the other with their longest dimensions parallel to one another in order to conserve space and/or minimize lead lengths between the tuned circuits and the electrodes of the tubes. Where one or more of the hour-glass tuned circuits are employed in a single piece of radio frequency apparatus, the electron discharge device tubes may be located in the interior of the cones to conserve space, and if condensers are provided between the apices of the cones, said condensers may be unicontrolled in any suitable fashion.

What is claimed is:

1. A high frequency electrical resonator comprising a pair of conical conducting surfaces of revolution arranged along the same axis with their apices adjacent each other to simulate the general shape of an hour-glass, the bases of said cones terminating in a spherical conducting surface extending from one cone to the other, said apices being flattened to increase the capacity therebetween and being provided with means for varying said capacity, and means for exciting said resonator to produce waves in the interior of said spherical conducting surface but outside the cones formed by said conical surfaces.

2. A high frequency electrical resonator comprising a pair of conical conducting surfaces of revolution arranged along the same axis with their apices adjacent each other to simulate the general shape of an hour-glass, the bases of said cones terminating in a spherical conducting surface extending from one cone to the other, whereby spherical standing waves are produced within the interior of said spherical conducting surface but outside said conical surfaces when the same is excited by high frequency electromagnetic energy, and means for varying the effective capacity between the apices of said pair of conical surfaces of revolution.

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