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etc

S. Schreiber  
Letters<sup>and</sup> to N.J.A.S.,  
correspondence  
42 pages

Mac & Sloane 2<sup>nd</sup> edit. A6368

Bar-Ilan University  
Ramat-Gan, Israel  
Department of Mathematics  
and Computer Science



אוניברסיטת בר-אילן

A6369 רמת גן

המחלקה למתמטיקה  
הנפקה הניתנת AS651

טלפון 718407, 718408  
Cables UNIBARILAN  
מברקרים

Dr Jessie Mac Williams & Dr N.J.A Sloane  
Math Research Centre, Bell Laboratories,  
600 Mountain Avenue, Murray Hill (Princeton NJ).

May 22 1980

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Dear Jessie & Neil,

just found your letter of April the 24-th awaiting me at the University;  
Thank you very much, that was a load off my young mind. It is still  
a pity that Problem 17 in your Ch.2 will look a bit clumsier in the  
next edition. And, apropos of next editions, is not the sequence at  
the bottom of your p.54 [Ch 2, Problem 8)

1, 1, 2, 3, 5, 9, 32, 56, 44, ... ✓ done 91

a respectable Integer Sequence?

✓ 1.122

R. K. Guy and Hallard Croft plan a book on Problems in Intuitive  
Mathematics. One, [J.H. CONWAY: Unpredictable Iterations, Proc. Number  
Theory Conf. Boulder, Colorado 1972, 49-52] concerns two very "fame"  
sequences, call them  $A(n)$  and  $B(n)$ , with  $A(B(n)) = B(A(n)) = n$ ;

$$(x + 3x^2 + x^3 + 3x^4 + x^5) / ((1-x)^2(1+x)) \Rightarrow 1, 3, 2, 6, 4, 9, 5, 12, 7, 15, 8, 18, 10, 21, 11, \dots$$

6368

$$(x + 3x^2 + 2x^3 + 3x^4 + x^5) / (1-x^3)^2 \Rightarrow 1, 3, 2, 5, 7, 4, 9, 11, 6, 13, 15, 8, 17, 19, 10, 21, \dots$$

6369

whose known cycles ( $n \rightarrow A(n) \rightarrow A(A(n)) \rightarrow \dots$ ) are  $\{1\}$ ,  $\{2, 3\}$ ,  $\{4, 6, 9, 7, 5\}$  and  
 $\{6, 16, 66, 99, 71, 111, 83, 62, 93, 70, 105, 79, 59\}$ . Any use?

Many thanks again and all the best

Shmuel Schechter



## Bell Laboratories

600 Mountain Avenue  
Murray Hill, New Jersey 07974  
Phone (201) 582-3000

June 9, 1980

Dr. Shmuel Schreiber  
Department of Mathematics  
and Computer Science  
Bar-Ilan University  
Ramat-Gan  
ISRAEL

Dear Shmuel:

Thanks for the sequences. As a matter of fact they have arrived at just the right moment - this summer I am writing a second edition of the Handbook of Integer Sequences. So if you come across any others, please send them too.

All the best,

MH-1216-NJAS-mv

N. J. A. Sloane

Bar-Ilan University  
Ramat-Gan, Israel  
Department of Mathematics  
and Computer Science



5651 אוניברסיטת בר-אילן  
5728 רמת-גן  
6370 הנקה גנטיתica →  
6373 הנקה פיזית

טלפון 718407, 718408  
Cables UNIBARILAN מברקים

Dr N.J.A. Sloane  
Math. Research Centre Bell Laboratories  
Murray Hill 07974 N.J.  
U.S.A.

25.6.1980

Dear Neil,

looking forward to the next edition of your Integer Sequences (I may have to look for it at Tel-Aviv U. since Bar-Ilan are either too poor or too stingy to buy even the second half of Error Correcting Codes, - had to borrow this from Tel-Aviv as well). -

In the few pages of offprint Guy has sent me, I have (naturally) met

$$4t + t^2 + 10t^3 + 2t^4 + 16t^5 + 3t^6 + \dots = \frac{4t + t^2 + 2t^3}{(1-t)^4}$$

6370

whose iterations form the well known Syracuse Problem. - Guy & Croft may have some more (they certainly exhibit your sequence 566: 1, 2, 5, 13, 29, 34... in an interesting form).

- The English edition of Comtet (Reidel; Dordrecht & Boston 1974) contains some more sequences. For example

$$\sum_{n \geq 0} A(n)t^n/n! = (1 - \frac{t}{1!})^{-1} \cdot (1 - \frac{t^2}{2!})^{-1} \cdot (1 - \frac{t^3}{3!})^{-1} \dots \quad (\text{sums of multinomial coefficients}),$$

$$\begin{array}{c|ccccccccc} n & 1 & 2 & 3 & 4 & 5 & 6 & \dots \\ \hline A(n) & 1 & 3 & 10 & 47 & 208 & 1002 & \dots \end{array}$$

(p. 126)

✓ 5651

④ Number of different Partial Derivative monomials in  $y_m = f^{(m)}(x)$  if  $f(x,y) = 0$ ,

$a(m)$  being the coefficient of  $t^n u^{n-1}$  in  $\prod_{i,j \geq 0} (1 - t^{i+j})^{-1}$ , where  $(i,j) \neq (0,0), (0,k)$ .

$$\begin{array}{c|ccccccccc} m & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \dots \\ \hline a(m) & 1 & 3 & 9 & 24 & 61 & 145 & 333 & 732 & \dots \end{array} \quad (\text{p. 175}),$$

✓

Before I forget, might you mention in the description of sequence 132: 1, 2, 1, 6, 6, 18... that it also counts primitive polynomials over  $Gf(2)$ ? -



From arithmetic, one might mention the successive length of the Farey sequences

1, 2, 3, 5, 7, 11, 13, 19, 23, 29, 33, 43, 47, 59, 65,

$l(n) = 1 + \sum_{k=1}^n \varphi(k)$ , where  $\varphi$  is Euler's totient (Niven & Zuckerman, Ch. 6).

Next, a pretty ill-behaved sequence:

1, 2, 1, 2, 4, 2, 1, 2, 2, 5, 4, 2, 1, 2, 6, ...

5728

giving the period length of expanding  $\sqrt{D}$  for successive non-square  $D$  into a continued fraction (Davenport, Higher Arithmetic, Ch IV § 10)

From the same source (Ch VI § 5), although he might have copied it from Dickson, one has a sequence beginning with too many 1's, but improving as it goes:

$D =  $	3	4	7	8	11	12	15	16	19	20	23	24	27	28	31	32	35	36	39	40	...
$f =  $	1	1	1	1	1	2	2	2	1	2	3	2	2	2	3	3	2	3	4	2	---

where  $D \equiv 0$  or  $3 \pmod{4}$  and  $f$  counts the reduced binary quadratic forms of discriminant  $-D$ .

Kogbetliantz and Krikorian: Handbook of Complex Prime Numbers

(2 Vols., Gordon & Breach 1971) represent primes  $4N+1 = a^2+b^2$  always with  $a$  odd and  $b$  even. Thus, beside your sequences 169 and 33, one might possibly consider

1, 1, 3, 1, 5, 1, 5, 7, 5, 3, 5, 9, ...  
1, 2, 2, 4, 2, 6, 4, 2, 6, 8, 8, 4, ...

not yet entered

6372

6373

All the best, and my respects to the lady called Jessie

Shmuel.

QA 246. K64