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A. L. Brown
& NJAB

"Correspondence,
1974,"
and notes
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together!)

5820

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

March 13, 1974

Professor Alan L. Brown
4800 Fillmore Street
Alexandria, Virginia 22311

Dear Professor Brown:

Rather a long time ago you published an article in Scripta Mathematica on multiply perfect numbers, so I wonder if you could possibly answer a question about these numbers. Let a_n be the n^{th} triply perfect number ($\sigma(n) = 3n$). How far is a_n rigorously known? The best answer I can find for the literature is that Carmichael's list was complete to 10^9, but this was in 1907! It appears that $a_1 = 120$, $a_2 = 672$, $a_3 = 523776$. Is it known that the next one is $b = 459818240$? Or could there be another between a_3 and b_7 etc.? Also, are there infinitely many triply perfect numbers? Any light you can throw on this problem would be much appreciated.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

[redacted] Alan L. Brown
4800 Fillmore Ave.
Alexandria, Va. 22311

591 5820 3/15/74

Dear Dr. Sloane,

It was a great pleasure to me to get your letter! If you are really interested welcome to the club; there are only 13-14 of us down through the centuries, and I can assure you that they embody the greatest pleasure; it is true in my case at least, and the cases of my friends Paul Poulet (deceased) and Prof Garcia and B. Franqui of Puerto Rico.

In regard to what I call the "Triperfects" ($\sigma(n) = 3n$) it is most unlikely that any more than the six already known will ever be discovered: they are so ridiculously easy to find, and just as easy to determine that nothing will work. Of course 120 was found first (in 1631 I think, in Paris by Marin Mersenne) and they were all discovered by that first great group of French mathematicians. That is in the 17th century.

I also state my strong belief that no more "Quadriperfects" will ever be found; quite otherwise where ($\sigma(n) = 5n$) as I found a few myself, and there are almost surely a few more.

A good approximation – ($\sigma(n)=4n$) twice as hard as ($\sigma(n)=3n$); ($\sigma(n)=5n$) twice as hard as ($\sigma(n)=4n$), etc. I am <u>convinced</u> that ($\sigma(n)=9n$) exist, and all the way up to ($\sigma(n)=\infty n$), but none of these will <u>ever</u> be found, in my opinion. I was scolded by Dr. Lehmer for my view, but I still hold it! mathematical or not.

Also that there really are no odd perfect or multiperfect numbers. The closest is the aliquot suite (one of many to be sure) found in France by the computers, and why are the French so good at this?

$$\begin{tikzcd} 25158165 \arrow[r] & 29902635 \arrow[d] \\ 29971755 \arrow[u] & 30853845 \arrow[l] \end{tikzcd}$$

Very sincerely yours,

Alan L. Brown

[redacted] Alan L. Brown
4800 Fillmore Ave.
Alexandria, Va. 22311

3/16/74

Dear Dr. Sloane

Sorry I ran out of envelopes - it is typical!

I thought after I mailed my letter that perhaps you would like to see the six ~~multiper~~ triperfects, so I worked them out again and I enclose my worksheet. 120, 672, - - - - - - 51001180160 given also in factored form (the best way for these numbers, in fact, no one would bother to multiply out the big ones)

Please let me call two things to your attention: ① #1 and #4 contain all factors in the first term; e.g. $\frac{3 \cdot 5}{2^3}$ and $\frac{3 \cdot 11 \cdot 31}{2^9}$ contain all the terms in the factored form of those 2 numbers, and this is (without proof) unique!!! ② #3 and #6 when multiplied by 3 will give quadriperfects. This is not unique as you can see by my article for instance; several 6-perfects are obtained from 5-perfects on multiplication by 5.

One of my great ambitions in this work was to obtain a 7-perfect not divisible by 7 giving an automatic

There are about 550 multiperfects known.

8-perfect. I don't think any one will ever know if one of these exist!
None of this is helped along by computers; the programming on them is by no means easy.
Paul Poulet was by far the best (Garcia and Franqui and I agree). He had so many "gimmicks" to help the work along. Garcia and Franqui are "next best" --- it is really more of an art than a science.
The best ever written is "La Chasse aux Nombres" by Poulet - Vol I. It is in most Libraries - 1929.
Quite the other way, the "perfect numbers" are ideal for computers - being of identical form - I am sure you know all this.
When I began my work (in Basking Ridge, N.J.) the "Bell Labs" land was a tree nursery, and nearly all I did was in South Orange. I knew a minimum of Math - calculus went in one ear and out the other in 1912, but I did have enthusiasm from the beginning. I still work (on trivia) in numbers; I would not be happy if I did not.

Very sincerely yours,
Alan L. Brown

$$\frac{\cancel{3\cdot 5}}{\cancel{2^3}} \times \frac{\cancel{2^2}}{\cancel{3}} \times \frac{\cancel{2}\cdot 3}{\cancel{5}} = 3 \quad — \quad 2^3\cdot 3\cdot 5 = \underline{120}$$

$$\frac{\cancel{7}\cdot 3}{\cancel{2^5}} \times \frac{\cancel{2^2}}{\cancel{3}} \times \frac{\cancel{2^3}}{\cancel{7}} = 3 \quad — \quad 2^5\cdot 3\cdot 7 = \underline{672}$$

$$\frac{\cancel{7\cdot 73}}{\cancel{2^8}} \times \frac{\cancel{2^3}}{\cancel{7}} \times \frac{\cancel{2\cdot 37}}{\cancel{73}} \times \frac{\cancel{2\cdot 19}}{\cancel{37}} \; \frac{\cancel{2^2\cdot 5}}{\cancel{19}} \; \frac{\cancel{2}\cdot 3}{\cancel{5}} = 3$$

$$\boxed{2^8\cdot 7\cdot 73\cdot 37\cdot 19\cdot 5 = \underline{459818240}}$$

$$\frac{\cancel{3\cdot 11\cdot 31}}{\cancel{2^9}} \times \frac{\cancel{2^2}}{\cancel{3}} \times \frac{\cancel{2^5}}{\cancel{31}} \times \frac{\cancel{2^2}\cdot 3}{\cancel{11}} = 3 \quad — \quad \boxed{2^9\cdot 3\cdot 31\cdot 11 = \underline{523776}}$$

$$\frac{\cancel{3\cdot 43\cdot 127}}{\cancel{2^{13}}} \times \frac{\cancel{2^2}}{\cancel{3}} \times \frac{\cancel{2^2\cdot 11}}{\cancel{43}} \times \frac{\cancel{2^7}}{\cancel{127}} \times \frac{\cancel{2^2}\cdot 3}{\cancel{11}} = 3$$

$$\boxed{2^{13}\times 3\times 43\times 127\times 11 = \underline{1476304896}}$$

$$\frac{\cancel{7\cdot 31\cdot 151}}{\cancel{2^{14}}} \times \frac{\cancel{2^3}}{\cancel{7}} \times \frac{\cancel{2^5}}{\cancel{31}} \times \frac{\cancel{2^3\cdot 19}}{\cancel{151}} \times \frac{\cancel{2^2\cdot 5}}{\cancel{19}} \times \frac{\cancel{2}\cdot 3}{\cancel{5}} = 3$$

$$\boxed{2^{14}\times 7\times 31\times 151\times 19\times 5 = \underline{51001180160}}$$

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Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

March 26, 1974

Professor Alan L. Brown
4800 Fillmore Avenue
Alexandria, Virginia 22311

Dear Professor Brown:

Thank you for your two very interesting letters. Certainly your calculations make it very clear how to compute the first 6 triperfect numbers. I take it that there are no theoretical results concerning the size of the 7th triperfect number, if it exists? I was disappointed to discover that this is probably a finite sequence - as you can see from the blue enclosure, I am mostly interested in <u>infinite</u> sequences. But the smallest number of persistence n (small white enclosure) is another sequence which seems to stop after a while.

It was interesting to hear from someone who remembers when Bell Labs was a tree nursery. You would not recognize the place now - two new wings have just been added.

Thank you again for your letters.

With best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

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Dear Dr. Sloane,

Thank you very much indeed for your letter of Mar. 26, and I am looking forward with a whole lot of pleasure to putting my mind on the enclosures. For some reason my life here does not seem to be that of a retired person; in fact it is necessary to choose which important thing I should neglect. I give no reason because I don't know any; I merely state facts. Perhaps it is a good thing.

I have a few puzzles myself which are not too bad; I will try to think of as many as I can for this letter. First let me figure a few unusual multiperfects to give a little background

$$\frac{7}{2^2} \times \frac{13}{3^2} \times \frac{2\cdot 3}{5} \times \frac{2\cdot 7}{13} \times \frac{3\cdot 19}{7^2} \times \frac{2^2\cdot 5}{19} = 4$$

The only one formed from a power of 2 giving also a perfect number $\left(\frac{7}{2^2} \times \frac{2^3}{7}\right)$ (And here what else is unique about the number 28?)

$$\frac{3^2\cdot 7}{2^5} \times \frac{11^2}{3^4} \times \frac{3\cdot 19}{7^2}\ \frac{7\cdot 19}{11^2}\ \frac{3\cdot 127}{19^2} \times \frac{2^7}{127} = 4$$ (over)

(TWIN of the next one)

$$\frac{3\cdot 11}{2^5}\times\frac{11}{3^4}\times\frac{3\cdot 19}{7^2}\times\frac{7\cdot 19}{11^2}\times\frac{151\cdot 911}{19^4}\times\frac{2^3\cdot 19}{151}\times\frac{2^4\cdot 3\cdot 19}{911}=4$$

(The above is of very frequent occurrance)

Another one -- $\frac{3\cdot 5\cdot 17}{2^7}\times\frac{1093}{3^6}\times\frac{2\cdot 3}{5}\times\frac{2\cdot 3^2}{17}\times\frac{2\cdot 3\cdot 23}{137}\times\frac{2^2\cdot 137}{547}\times\frac{2\cdot 547}{1093}\times\frac{2^3\cdot 3}{23}=4$

and $\frac{3\cdot 5\cdot 17}{2^7}\times\frac{23\cdot 3851}{3^{10}}\times\frac{2\cdot 3}{5}\times\frac{2\cdot 3^2}{17}\times\frac{2^3\cdot 3}{23}\times\frac{2^2\cdot 3^3}{107}\times\frac{2^2\cdot 3^2\cdot 107}{3851}$ also $=4$.

There are many other substitutions in this work, unfortunately easy to overlook. In fact a great part of finding the multiperfects is needfully close to skillful use of substitutions.

So I have wandered! But why do I call that same $2^2\cdot 7=28$ the "lonely number"? For the same reason I call 220 - 284 the ideal amicable pair, and both of these statements are true, but not subject to rigorous proof, and I am willing to be justly criticized for these and other statements! In this category --- there are no odd perfect numbers, --- (O Boy!)

I am sure you recognize two ϕ functions [$\phi(6)=12$; $\phi_0(6)=6$. $(=1+2+3)$]. We use the latter in designating the perfect numbers, the amicable pairs and the sociable groups; The ~~latter~~ former for multiperfects, etc. Both of these functions exist for every integer! I give you $\phi(1)=1$, $\phi_0(1)=0$, $\phi(p)=p+1$, $\phi_0(p)=1$, $\phi_0(p^2)$ or $\phi_0(2p^2)=(p+1)^2$, $\phi_0(p+1)^2=p^2$, etc., etc.

These give rise to "Aliquot suites" where $\phi_0(n)=n_1$, $\phi_0(n_1)=n_2$ etc. I am going to give you [illegible] suite for 88 which is the smallest of what I call "Orphan Numbers" (no parent) and all "a suites" begin only with these. (I was not quite accurate, 2 and 5 are the only "Orphan primes") 88, 92, 76, 64, 63, 41, 1, 0 – End (I take back — 52, then 88, then 96, then 120 – this was all done years ago.) So here is 120 — 120, 240, 504, 1056, 1968, 3240, 7650, 14112, 32571, 27333, 12161, 1, 0. (I checked this one)

Now these suites can end in many ways – prime, 1, 0; or they can keep growing – faster and faster and faster and apparently never stop. Or they can become perfect numbers (except for 28 which is also orphan) or an amicable pair (except for 220, 284) or they can become "sociable groups" like the one I sent you. Only a very few "sociables" are known and there must be billions of them – they are not easy to find! I already sent you the only odd one — computer – discovered.

$$\begin{tikzcd}
3^3 \cdot 5 \cdot 7 \cdot 83 \cdot 359 \arrow[r] & 3^3 \cdot 5 \cdot 7 \cdot 31643 \arrow[d] \\
3^3 \cdot 5 \cdot 11 \cdot 20183 \arrow[u] & 3^3 \cdot 5 \cdot 11 \cdot 79 \cdot 263 \arrow[l]
\end{tikzcd}$$

My latest amusement (in Florida this winter where I had more time) Find products of 2 primes of form $100k^2+1$ and I got about 125 of them. My largest was 2401 4900 2501 sum of 4 squares in an interesting way. My neatest was 1010201 × 9901 $[illegible]010^2 + 1^2 = 9799^2 + 19999^2 = 510051^2 - 500150^2$ = 1040401.

My very best regards, and I will write again soon.

Sincerely, Alan L. Brown

F91

Multiply Perfect Nos. #5820

Alan L Brown
Scripta Math 20 1954 p 103 –106
Multiperfect Numbers

Early history Dickson History I p 33; [illegible]
Gives no triply perfect nos

Franqui & Garcia

$P_3^{(1)} = 120$ Fermat

$P_3^{(2)} = 672$

$P_3^{(3)} = 523776$

$P_3^{(4)} = 1476{,}304896$

$P_3^{(5)} = 4598{,}18240$

$P_3^{(6)} = 5100{,}118{,}0160$

these seem to be the only ones known

Lehmer, Guide, p5, says

Carmichael, A table of multiply perfect nos
Bull Am Math Soc
13 (1907) 383–386

is complete to $< 10^9$

Poulet 2 is most complete to date

Carmichael & Mason 1 give 251 nos

Carmichael & Mason
Note on multiply perfect nos ...
Indiana Acad Sci., Proc., 21 1911 pp 257–270
Ref Lehmer, Guide to Tables, p 89