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HW Gould letter

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New Seq p.2

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DEPARTMENT OF MATHEMATICS

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16 Nov. 1973

Dr. Neil J. A. Sloane,
Bell Laboratories,
600 Mountain Avenue,
Murray Hill, N.J. 07974

Dear Dr. Sloane,

I appreciated your letter of 7 November very much. I don't know whether at most 5 elementary transformations will do it, but a finite number will surely suffice! As for handshakes, I don't have the faintest idea. But I do maintain a voluminous correspondence which may help explain why I am only now finally getting around to sending you the table of Fibonomial Catalan numbers. I calculated these by hand (two ways, recursively and every tenth one being checked by direct calculation) while my wife and I watched science fiction (and Dracula, Frankenstein, etc.) Saturday nights here from a Pittsburgh TV channel (sitting up till 3 or 4 A.M.). Makes the TV interesting when the plots are dull. The result is a ten-page table of the first 50 Fibonomial Catalan numbers in factored form. The fiftieth one has 110 distinct prime factors.

Anyway, enclosed is a Xerox copy of the first two pages. I had no precise use for the actual digits in those beyond K(10), so that was the last one I gave that way. You can easily run off the values of others from the factors I give. These first two pages will give you K(n) for n=1(1)22. Be careful to note that where n is given in the left-hand column, then K(n+1) is the value given at the right. I did this to agree with the usual notation I use for ordinary Catalan numbers: $C(n) = \binom{2n}{n}/(n+1)$.

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I want to ask you if you have considered including the following two interesting sequences??? To wit:

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$$S_n = \sum_{k=0}^{n} \frac{n!}{(n-k)!} = 1, 2, 5, 16, 65, 326, 1957, 13700, 109601, 986410, 9864101, 108505112, \ldots.$$

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and

$$S'_n = \sum_{k=0}^{n} k!(n-k)! = 1, 2, 5, 16, 64, 312, 1812, 12288, 95616, 840960, 8254080, 89441280, 1060369920, 13649610240, \ldots$$

where I have given S_n for $n=0(1)11$, and S'_n for $n = 0(1)13$.

You will notice that the two start alike, but begin to differ by 1 unit with $n = 4$, and then the difference grows, but not too remarkably. In fact it is easy to show that

$$S_n \sim e\,n! \quad \text{whereas} \quad S'_n \sim 2\,n! \qquad \text{as} \quad n \to \infty$$

Recurrences:

$$S_n = nS_{n-1} + 1, \qquad S'_n = n! + \frac{n+1}{2} S'_{n-1} \; .$$

These two sequences have recently been confused with one another and I am writing a review for the German Zentralblatt für Mathematik (copy enclosed in fact) that shows how this has happened. I think a table of sequences such as yours would be of great help in just such situations.

Of course, the two sequences above are familiar in other notation. Of course

$$S_n = n! \sum_{k=0}^{n} \frac{1}{k!} \qquad \text{and} \qquad S'_n = n! \sum_{k=0}^{n} \frac{1}{\binom{n}{k}} \; .$$

I came across S'_n one time years ago in calculating resistance of an array of resistors in electrical engineering work.

I would very much like to see your book. How nearly ready is it now? I saw a mention of a prepublication version in a recent paper.

Best regards,

Henry W. Gould

Henry W. Gould

P.S.: I collect peculiar sequences, and might be able to suggest still others you do not have.

P.P.S.: I should remark that you be sure to know that I use F_n with this definition: $F_0 = 0$, $F_1 = 1$, then $F_{n+1} = F_n + F_{n-1}$.

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FIBONOMIAL CATALAN NUMBERS

k	n	F_n	$K(k+1)$, where $K(k) = \frac{1}{F_{k+1}}\left\{\begin{matrix}2k\\k\end{matrix}\right\} = \frac{F_{2k}F_{2k-1}\cdots F_{k+2}}{F_kF_{k-1}\cdots F_1}$.
	0	0	
0	1	1	1
	2	1	
1	3	$\underline{2}$	3
	4	$\underline{3}$	
2	5	$\underline{5}$	$2^2\cdot\underline{5} = 20$
	6	8	
3	7	$\underline{13}$	$2^2\cdot 7\cdot\underline{13} = 364$
	8	21	
4	9	34	$7\cdot 11\cdot 13\cdot 17 = 17017$
	10	55	
5	11	$\underline{89}$	$2\cdot 3^2\cdot 7\cdot 11\cdot 17\cdot\underline{89} = 2097018$
	12	144	
6	13	$\underline{233}$	$2\cdot 3\cdot 11\cdot 17\cdot 29\cdot 89\cdot\underline{233} = 674740506$
	14	377	
7	15	610	$2\cdot 3\cdot 5\cdot 11\cdot 29\cdot 47\cdot 61\cdot 89\cdot 233 = 568965009030$
	16	987	
8	17	$\underline{1597}$	$2^3\cdot 3\cdot 19\cdot 29\cdot 47\cdot 61\cdot 89\cdot 233\cdot\underline{1597} = 1255571292290712$
	18	2584	
9	19	4181	$2^3\cdot 3^2\cdot 19\cdot 29\cdot 37\cdot 41\cdot 47\cdot 61\cdot 113\cdot 233\cdot 1597 = 725498718525054410 4$
	20	6765	
10	21	10946	$13\cdot 19\cdot 29\cdot 37\cdot 41\cdot 47\cdot 61\cdot 113\cdot 199\cdot 233\cdot 421\cdot 1597$
	22	17711	
11	23	$\underline{28657}$	$2\cdot 7\cdot 13\cdot 19\cdot 23\cdot 29\cdot 37\cdot 41\cdot 47\cdot 61\cdot 113\cdot 199\cdot 421\cdot 1597\cdot\underline{28657}$
	24	46368	
12	25	75025	$2\cdot 5^2\cdot 7\cdot 19\cdot 23\cdot 37\cdot 41\cdot 47\cdot 61\cdot 113\cdot 199\cdot 421\cdot 521\cdot 1597\cdot 3001\cdot 28657$

$n = 2k + 1$

<u>Theorem:</u> $k \geq 2$ & F_{2k+1} = prime

$\Longrightarrow F_{2k+1} \mid K(k+1)$.

Calculated by H. W. Gould

k	n	F_n	K(k+1)
	26	121393	
13	27	196418	$2\cdot3\cdot5\cdot7\cdot17\cdot19\cdot23\cdot37\cdot41\cdot47\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot1597\cdot3001\cdot28657$
	28	317811	
14	29	514229	$2^3\cdot5\cdot11\cdot17\cdot19\cdot23\cdot31\cdot37\cdot41\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot1597\cdot3001\cdot28657\cdot514229$
	30	832040	
15	31	1346269	$2^3\cdot5\cdot11\cdot17\cdot19\cdot23\cdot31\cdot37\cdot41\cdot53\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot557\cdot2207\cdot2417\cdot3001\cdot28657\cdot514229$
	32	2178309	
16	33	3524578	$2\cdot5\cdot11\cdot23\cdot31\cdot37\cdot41\cdot53\cdot89\cdot109\cdot113\cdot199\cdot281\cdot421\cdot521\cdot557\cdot2207\cdot2417\cdot3001\cdot3571\cdot19801\cdot28657\cdot514229$
	34	5702887	
17	35	9227465	$2^2\cdot3^3\cdot5^2\cdot11\cdot13\cdot23\cdot31\cdot41\cdot53\cdot89\cdot107\cdot109\cdot199\cdot281\cdot421\cdot521\cdot557\cdot2207\cdot2417\cdot3001\cdot3571\cdot19801\cdot28657\cdot141961\cdot514229$
	36	14930352	
18	37	24157817	$2^2\cdot3^2\cdot5\cdot13\cdot23\cdot31\cdot53\cdot73\cdot89\cdot107\cdot109\cdot149\cdot199\cdot281\cdot421\cdot521\cdot557\cdot2207\cdot2221\cdot2417\cdot3001\cdot3571\cdot9349\cdot19801\cdot28657\cdot141961\cdot514229$
	38	39088169	
19	39	63245986	$2^2\cdot3^2\cdot5\cdot7\cdot23\cdot31\cdot53\cdot73\cdot89\cdot107\cdot109\cdot149\cdot199\cdot233\cdot281\cdot521\cdot557\cdot2161\cdot2207\cdot2221\cdot2417\cdot3001\cdot3571\cdot9349\cdot19801\cdot28657\cdot135721\cdot141961\cdot514229$
	40	102334155	
20	41	165580141	$2^4\cdot3^2\cdot5\cdot7\cdot23\cdot29\cdot31\cdot53\cdot73\cdot107\cdot109\cdot149\cdot211\cdot233\cdot281\cdot521\cdot557\cdot2161\cdot2207\cdot2221\cdot2417\cdot2789\cdot3001\cdot3571\cdot9349\cdot19801\cdot28657\cdot59369\cdot135721\cdot141961\cdot514229$
	42	267914296	
21	43	433494437	$2^4\cdot3^3\cdot5\cdot7\cdot23\cdot29\cdot31\cdot43\cdot53\cdot73\cdot107\cdot109\cdot149\cdot211\cdot233\cdot281\cdot307\cdot521\cdot557\cdot2161\cdot2207\cdot2221\cdot2417\cdot2789\cdot3001\cdot3571\cdot9349\cdot19801\cdot59369\cdot135721\cdot141961\cdot514229\cdot433494437$
	44	701408733	

Calculated by H. W. Gould

PRIME FACTORS OF CATALAN NUMBERS

k	n	$\binom{n}{k} - \binom{n}{k-1}$	Prime factors
0	1	1	
1	3	2	2
2	$\underline{5}$	5	$\underline{5}$
3	$\underline{7}$	14	$2 \cdot \underline{7}$
4	9	42	$2 \cdot 3 \cdot 7$
5	$\underline{11}$	132	$2^2 \cdot 3 \cdot \underline{11}$
6	$\underline{13}$	429	$3 \cdot 11 \cdot \underline{13}$
7	15	1430	$2 \cdot 5 \cdot 11 \cdot 13$
8	$\underline{17}$	4862	$2 \cdot 11 \cdot 13 \cdot \underline{17}$
9	$\underline{19}$	16796	$2^2 \cdot 13 \cdot 17 \cdot \underline{19}$
10	21	58786	$2 \cdot 7 \cdot 13 \cdot 17 \cdot 19$
11	$\underline{23}$	208012	$2^2 \cdot 7 \cdot 17 \cdot 19 \cdot \underline{23}$
12	25	742900	$2^2 \cdot 5^2 \cdot 17 \cdot 19 \cdot 23$
13	27	2674440	$2^3 \cdot 3^2 \cdot 5 \cdot 17 \cdot 19 \cdot 23$
14	$\underline{29}$	9694845	$3^2 \cdot 5 \cdot 17 \cdot 19 \cdot 23 \cdot \underline{29}$
15	$\underline{31}$	35357670	$2 \cdot 3^2 \cdot 5 \cdot 19 \cdot 23 \cdot 29 \cdot \underline{31}$
16	33	129644790	$2 \cdot 3 \cdot 5 \cdot 11 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
17	35	477638700	$2^2 \cdot 3 \cdot 5^2 \cdot 11 \cdot 23 \cdot 29 \cdot 31$
18	$\underline{37}$	1767263190	$2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 23 \cdot 29 \cdot 31 \cdot \underline{37}$
19	39	6564120420	$2^2 \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 23 \cdot 29 \cdot 31 \cdot 37$
20	$\underline{41}$	24466267020	$2^2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot 29 \cdot 31 \cdot 37 \cdot \underline{41}$
21	$\underline{43}$	91482563640	$2^3 \cdot 3 \cdot 5 \cdot 13 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot \underline{43}$
22	45	343059613650	$2 \cdot 3 \cdot 5^2 \cdot 13 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot 43$
23	$\underline{47}$	1289904147324	$2 \cdot 3^2 \cdot 13 \cdot 29 \cdot 31 \cdot 37 \cdot 41 \cdot 43 \cdot \underline{47}$

$$C(k) = \frac{1}{k+1}\binom{2k}{k}$$

$n = 2k+1$, whence $\binom{n}{k} - \binom{n}{k-1} = \binom{2k+1}{k} - \binom{2k+1}{k-1} = \dfrac{(2k+2)!}{(k+1)!(k+2)!} = C(k+1).$

<u>Theorem:</u> $k \geqslant 2$ and $2k+1 =$ prime $\Longrightarrow 2k+1 \mid C(k+1)$.

Calculated by H. W. Gould

REVIEW TO APPEAR IN ZENTRALBLATT FÜR MATHEMATIK:

DA

226218 1972 Wb

Kaucký, Josef:

Some remarks on a combinatorial problem.

Mat. Časopis, Slovensk. Akad. Vied 22, 215-218 (1972).

05

H.W. Gould

Ad

ZENTRALBLATT FÜR MATHEMATIK

Date:

Term: 8 weeks

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PRIMARY CLASSIFICATION CODE 15A15

Secondary classifications 05A19

If full MOS scheme is not available to you, please give the appropriate section number (see reverse) and use keywords of your own choice for finer classification.

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The determinant

$$D_n = \begin{vmatrix} 0! & -\binom{1}{1} & 0 & 0 & \cdots & 0 \\ 1! & \binom{2}{1} & -\binom{2}{2} & 0 & \cdots & 0 \\ 2! & -\binom{3}{1} & \binom{3}{2} & -\binom{3}{3} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ (n-2)! & \cdot & \cdot & \cdot & & -\binom{n-1}{n-1} \\ (n-1)! & (-1)^n\binom{n}{1} & (-1)^{n-1}\binom{n}{2} & & \cdots & \binom{n}{n-1} \end{vmatrix}$$

was the subject of Problem 690 [Math. Magazine, 41(1968),96, 290-291; 42(1969), 154-156] . The author of the present paper claims that the published value of D_n and the proof are incorrect and then gives a fresh derivation of a correct value. This is an interesting situation, because the author quotes the (correct) formula found in the reference cited quite incorrectly. The formula for D_n derived in the present paper is, however, also correct as we shall show below.

Please keep distance between lines

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NOTE TO EDITORS AND PRINTERS: The fractions on this page should be set and printed just as shown because of the fact that the review points out mistakes based on confusion between "a/b" and "ab" in the original paper reviewed. THANK YOU, H. W. Gould

In the first place, the (correct) value given in the published solution is

$$(1) \qquad D_n = \sum_{j=0}^{n-1} S_j , \quad \text{where} \quad S_j = \sum_{i=0}^{j} \frac{j!}{(j-i)!} ,$$

which is incorrectly given in the present paper with

$$(2) \qquad S_j = \sum_{i=0}^{j} i!(j-i)! .$$

This circumstance may be due to the fact that S_j is incorrectly printed in one place in the problem solution as

$$S_j = \sum_{i=0}^{j} i!/(j-i)!$$

and possible confusion of $i!/(j-i)!$ with $i!(j-i)!$.

The first few values (for $j = 0,1,2,\ldots$) of S_j as given by (1) are 1, 2, 5, 16, 65, 326, ... whereas the values given by (2) (as noted by the author) are 1, 2, 5, 16, 64, 312, ... and the correct values of D_n, correspondingly, are 1, 3, 8, 24, 89, 415, ...

In the second place, the derivation of the value of D_n

$$(3) \qquad D_n = \sum_{j=1}^{n} \binom{n}{j} (j-1)!$$

in the present paper seems to be quite correct, and the fact that (3) agrees with (1) as originally set forth in the problem solution is easily seen from the following steps:

$$\sum_{j=0}^{n-1} \sum_{i=0}^{j} \frac{j!}{(j-i)!} = \sum_{i=0}^{n-1} i! \sum_{j=i}^{n-1} \binom{j}{i} = \sum_{i=0}^{n-1} i! \binom{n}{i+1}$$

which is precisely (3) upon replacing i by $j - 1$.

Finally, the recurrence given for (2) is not new.

Henry W. Gould
3. Nov. 1973

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 ATAL ERROR * MISSING ROUTINE   P9     ,MME GEBORT INSERTED AT REFERENCES
 ATAL ERROR * MISSING ROUTINE   M9     ,MME GEBORT INSERTED AT REFERENCES
FATAL ERROR * MISSING ROUTINE   M9     ,MME GEBORT INSERTED AT REFERENCES
FATAL ERROR * MISSI
*#RUNH = (BCD) USER/RAR
 1
 41T   01/31/74    18.19
 2
 5
 16
 64
 312
 1812
 12288
 95616
 840960
 8254080
 89441280
 1060069920
 13649610240
 189550368000
 2824077312000
 44927447040000
 760034451456000
 13622700994560000
 257872110354432000
 5140559166898176000
 107637093007589376000
 2361827297364885504000
 54193944307263602688000
 1297872705574034472960000
 32383555215793434132480000
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50 DO 1 N=2,50
*#RUN = (BCD) USER/T@RAR
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#10

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                              U~#X><>&:0/D
ψ1WK+L1H83P@@<□#
                  F~XX~~
LOGIN: NJS
% DC
13 19 29 37 41 47 61 113 199**********p
     700550979746059
233 421 1597***p
     109744478168199574282739

C
1597 421*233*199*113*61*47*41*37*29*19*13*p
     109744478168199574282739

C
7 13 19 23 29 37 41 47 61 113 199*************p
     225577415478230998
421 1597 28657***p
     4346236474244131564253156182

C
28657 1597*421*199*113*61*46#7*41*37*29*23*19*13*7*2*p
     4346236474244131564253156182
```