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Harvey Dubner Dubner Computer Systems 6 Forest Avenue Paramus, New Jersey 07652

June 10, 1991

N. J. A. Sloane Room 2C-376 AT&T Bell Labs Murray Hill, NJ 07974

Dear Neil,

As you know, Sam Wagstaff asked me if I could send you any additional information concerning repunits, base b, that is numbers of the form,

$$P = \frac{b^{n}-1}{b-1} .$$

I have extended the table as it appeared in Sam's Factorization book. Unfortunately, these results have not yet been published. I plan to submit something in the near future.

I would like to submit two sequences which you might consider for your new edition:

Factorial primes: P = n! + 1, P prime.

n = (1, 2, 3, 11, 27, 37, 41, 73, 77, 116, 154, 320, 340, 399, 427, 872, 1477)

M = P# + 1, P# = 2*3*5*7*11...*P, P,M prime. Primorial primes:

n = 2,3,5,7,11,31,379,1019,1021,2657,3229,4547,4787,11549,13649,18523 4523 4

As with Mersenne primes there are probably an infinite number of these, but this has not been proved.

The references for these sequences are:

H. Dubner, Factorial and Primorial Primes, Journal of Recreational Mathematics, 19:3, pp. 197-203, 1987.

H. Dubner, Factorial and Primorial Primes, Journal of Recreational Mathematics, 21:4, p. 276, 1989.

Please let me know if I can be of any further assistance.

Sincerely,

Harry Dubner

cc: Sam Wagstaff



AT&T Bell Laboratories

600 Mountain Avenue Murray Hill, NJ 07974-2070 908-582-3000

June 19, 1991

Mr. Harvey Dubner Dubner Computer Systems 6 Forest Avenue Paramus, NJ 07652

Dear Harvey:

Thank you very much for those sequences! Beautiful!

I'm not sure if I need to distinguish PRP's from primes. Am I right in assuming that you are sending the PRP's to the team that verifies they are primes? So it is just a matter of time before they become primes (unless a miracle happens).

I hope you will publish them, especially of course the prime repunits that are outside the scope of the Cunningham Project tables.

If you have any copies left of your J.R.M. papers I would love to get them.

These sequences are very impressive.

Best regards,

N. J. A. Sloane

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Harvey Dubner
Dubner Computer Systems
6 Forest Avenue
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June 26, 1991

N. J. A. Sloane Room 2C-376 AT&T Bell Labs Murray Hill, NJ 07974

Dear Neil,

As you requested, I have enclosed copies of my papers about Factorial primes and Primorial primes. I have also enclosed the paper by Williams and Seah which was the original source of the repunits, base b, which were in the Factorizations book.

Now, about PRP's. Until a few years ago it was virtually impossible to prove that a large number that was PRP (greater than perhaps a hundred digits) was truly prime unless a reasonable number of factors of N+1 and/or N-1 could be found. Thus all really large primes are constructed so that this condition will hold. In the early 1980's Adelman, Pomerance, and Rumley developed a new primality test which did not involve factoring. This test was improved by Cohen and Lenstra and is called APRT-CL.

This test is difficult to understand (at least for me) and difficult to program for anyone. Luckily APRT-CL has been implemented in a remarkable public domain program called UBASIC so that 300-digit numbers can be tested for true primality on my 20Mhz 386PC in about 30 hours. for larger numbers you need a faster computer. I have been told that a big supercomputer can test a 1000-digit number in several hours. The test time seems to increase at least as the fourth power of the number of digits of N.

Thus, from a practical point of view it is unlikely that the large base-b PSP's will be tested anytime in the near future. Of course, as you say, it would be a true miracle if any of these PSP's turned out to be composite. However, it is for exactly this reason that you cannot expect people to "waste" precious computer time on a foregone conclusion.

When I started this letter I was planning to say that you should distinguish PSP's from primes. I have changed my mind. With a short explanation at the beginning of your book I think it is technically justified to treat PSP's as primes! Anyone who disagrees is a pompous fool.

Sincerely,

Harry, ex pompous fool

Harvey Dubner