[HE1]13 Hermite, Oeuvres, IV

ref to MTAC

2103
2639

qui donne immédiatement

$$F_2(k) = \frac{1}{2^4}k^2 + \frac{1}{2^5}k^4.$$

On trouve ensuite

$$\Phi(k) = \frac{1}{2^8}k^4 + \frac{1}{2^8}k^6 + \frac{29}{2^{13}}k^8 + \frac{13}{2^{12}}k^{10}$$

et, en extrayant de nouveau la racine carrée,

$$F_4(k) = \frac{1}{2^4}k^2 + \frac{1}{2^5}k^4 + \frac{21}{2^{10}}k^6 + \frac{31}{2^{11}}k^8.$$

Ce sont les recherches de M. Tisserand sur la libration des petites planètes, où les fonctions elliptiques sont appliquées avec succès à une question importante et difficile, qui ont été l'occasion du travail que j'expose dans cette Note. On les trouvera dans le quatrième volume du beau Traité de Mécanique céleste qui a mis son auteur au premier rang des astronomes de notre époque [1]. J'en avais donné communication à mon éminent Confrère et ami qui a poursuivi les calculs beaucoup plus loin que je ne l'avais fait, en y joignant des remarques intéressantes, comme on le verra dans cette lettre qu'il a bien voulu m'adresser.

Paris, 19 décembre 1895.

... En développant les calculs d'après votre méthode et posant

$$q = a_1 k^2 + a_2 k^4 + \ldots + a_{12} k^{24} + \ldots$$

j'ai trouvé, sans trop de peine,

Please enter

2639 ✓

$$\begin{aligned}
2^4\, a_1 &= 1\\
2^5\, a_2 &= 1\\
2^{10} a_3 &= 21\\
2^{11} a_4 &= 31\\
2^{19} a_5 &= 6\,257\\
2^{20} a_6 &= 10\,293\\
2^{25} a_7 &= 279\,025\\
2^{26} a_8 &= 483\,127\\
2^{36} a_9 &= 435\,506\,703\\
2^{37} a_{10} &= 776\,957\,575\\
2^{42} a_{11} &= 22\,417\,045\,555\\
2^{43} a_{12} &= 40\,784\,671\,953
\end{aligned}$$

OK

(1) *Voir* Chapitre XXV, p. 437.

2103
+
2639

1948

MTAC 3(1948)

8D, log $(K/2\pi) + 2$ log $(1 + \sqrt{k'})$, ROBBINS 2, $k = .4(.01).6(.001).849$
7D, log $[(4K/\pi) \cos \frac{1}{2}\theta (\cos \theta)^{\frac{1}{2}}]$, NEWCOMB 1 (p. 69), $k = .45(.01).75$
7–11D, $K/\ln (4/k')$, DWIGHT 2, $\theta = 86°(1')90°$, Δ
7D, $K/\log (4/k')$, SAMOĬLOVA-ÎAKHONTOVA 1, $k^2 = .95(.001)1$, Δ
6D, $2K/\pi - a \log (90 - \theta)$, $a = (2/\pi) \ln 10 = 1.465871$, HEUMAN 1, $\theta = 65(0.1)90$, Δ
5D, first 8 coefficients in expansions of $K - \frac{1}{2}\pi + \ln k'$ and $K' + (2K/\pi) \ln k - \ln 4$, powers of k^2, TANNERY & MOLK 1 (v. 3, p. 215)
4D, $K - \ln (4/k')$, $\ln (4/k')$, JAHNKE & EMDE 1_3, 1_4, $k^2 = .7(.01)1$, $I\Delta$
4D, $\beta = \log K - \log \ln (4/k')$, HOÜEL 1 (p. 57), $\theta = 50^g(1^g)100^g$
4D, $4E/\pi(1 + k')$, BOLL 1 (p. 348), $\epsilon = (1 - k')/(1 + k') = 0(.01)1$
4D, $4E/\pi(1 + k')$, BOLL 1 (p. 349), $k' = 0(.01)1$

Section III: JACOBI'S NOME q

q is defined as $e^{-\pi K'/K}$. Thus $\ln q = -\pi K'/K$, or $\log q = -\mu\pi K'/K$, where $\mu\pi = \pi \log e = 1.36437\ 63538\ 41841$, so that a table of log q is easily computed from a table of K, if θ or k^2 is the argument. Also

$$\log \log (1/q) = \log (\mu\pi) + \log K' - \log K,$$

where

$$\log (\mu\pi) = 0.13493\ 41839\ 94670\ 6,$$

so that log log $(1/q)$ is very easily computed from a table of log K. It is not essential that a table of log q or of log log $(1/q)$ should extend beyond $\theta = 45°$ or $k^2 = \frac{1}{2}$, for if q' is the complementary nome $e^{-\pi K/K'}$, we have

$$\log q \log q' = \mu^2\pi^2 = 1.86152\ 28349\ 22757$$

and

$$\log \log (1/q) + \log \log (1/q') = 2 \log (\mu\pi) = 0.26986\ 83679\ 89341\ 3.$$

2639 If $q = \Sigma a_n k^{2n}$, the exact values of a_n were calculated for $n = 1(1)12$ by F. TISSERAND, published in HERMITE 1_1, and reproduced in HERMITE 1_2 and in TANNERY & MOLK 1 (v. 4, p. 121).

The most usual way of computing q, other than by using its definition as given above, is to put

$$2\epsilon = (1 - \sqrt{k'})/(1 + \sqrt{k'}),$$

when we have

$$\epsilon = (q + q^9 + q^{25} + \ldots)/(1 + 2q^4 + 2q^{16} + \ldots),$$

which inverts (see WEIERSTRASS & SCHWARZ 1, p. 56) into

2103
$$q = \epsilon + 2\epsilon^5 + 15\epsilon^9 + 150\epsilon^{13} + \ldots.$$

The first 14 terms of the series are given in LOWAN, BLANCH & HORENSTEIN 1.

It should be noted that q and ϵ are called h and $\frac{1}{2}l$ respectively in WEIERSTRASS & SCHWARZ 1 and elsewhere. The small difference $q - \epsilon$ (called $q - \frac{1}{2}l$) is tabulated to 8D for $q = 0(.01).14$ in NAGAOKA 1, and to 8D with Δ for $q = .02(.002).1(.001).15$ in NAGAOKA 2; the second table is reproduced in ROSA & GROVER 1. NAGAOKA & SAKURAI 1 (p. 56) tabulates log 2ϵ (called log l) to 7D with Δ for $k^2 = 0(.001).5$.

For complex k^2, see CAMBI 1 in Section II; also the diagrams in JAHNKE & EMDE 1 (p. 120), 1_3–1_4 (p. 46), giving k^2 as a function of τ, where $q = e^{i\pi\tau}$.

III–A. *q and its powers*

16S, q, SPENCELEY 1, $\theta = 0(1°)90°$
14–15D, q, HIPPISLEY 1, $\theta = 0(5°)80°(1°)90°$
8D, q, GLAISHER 1, $\theta = 0(1°)90°$
7–5D, q, LÁSKA 1, $\theta = 0(1°)90°$
17D, q^n, NBSCL 2, $n = \frac{1}{4}, \frac{1}{2}, 1(1)4(2)8, 9, 12, 16, 20, 25$, $k^2 = 0(.01)1$