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Robert G. Wilson

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Neil James Alexander Sloane
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Subject: Big Φ

Dear Sir,

Your sequence 111, is Euler's Totient Function, and there is the sequence for the $\sum_1^n \varphi(k)$, sequence 376, or as Graham, Knuth and Patashnik have stated it in their book, Concrete Mathematics, "Big Phi." Its use is in the number of terms minus one of the Farey series of order n. The series reads as follows: 1, 2, 4, 6, 10, 12, 18, ... Also enclosed is a copy of values of Φ to greater height than your handbook would permit.

Sequentially yours,

Robert G. Wilson

4. STOTIENT

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10 ! Sum Euler's TOTIENT or Phi function (n) from L to U.
20 T=0 @ INPUT 'Low &, Up Nbr. ';L,U
30 FOR I=L TO U
40 K=1 @ S=1 @ N=I
50 R=PRIM(N) @ IF R=1 THEN 110
60 P=N/R
70 Q=PRIM(P)
80 IF R=Q THEN K=K+1 @ P=P/R @ GOTO 70
90 S=S*(R-1)*R^(K-1)
100 N=P @ R=Q @ K=1 @ GOTO 50
110 T=T+S @ NEXT I @ DISP T @ STOP
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NOTE: The relative error was computed using
T= ... @ N= ... @ DISP USING "MDP5DX6DXE";(T-3*(N/PI)^2)/T*10^16

Φ (n)	$T=\sum_{n=1}^{n}\varphi(n)$	forward difference	(not asymptotically) ~ 3*(n/Pi)^2	relative error*E5
0	0			
1	1			
10	32		30.40	5011.39033 5308338
100	3,044		3,039.64	143.38011 5961459
200	12,232	9,188	12,158.54	600.53926 5201663
300	27,398	15,166	27,356.72	150.66945 2400911
400	48,678	21,280	48,634.17	90.04447 9390833
500	76,116	27,438	75,990.89	164.37052 4261221
600	109,500	33,384	109,426.88	66.77777 7420280
700	149,018	39,518	148,942.14	50.90663 2597052
800	194,750	45,732	194,536.67	109.53910 4858268
900	246,326	51,576	246,210.48	46.89872 3285083
1000	304,192	57,866	303,963.55	75.10028 9615336
1025	319,764	15,572	319,351.71	128.93706 2116634
2E3	1,216,588	912,396	1,215,854.2037	60.31592 3874536
3E3	2,736,188	1,519,600	2,735,671.9583	18.85987 5742462
4E3	4,863,602	2,127,414	4,863,416.81483	3.80757 2408000
5E3	7,600,458	2,736,856	7,599,088.7731	18.01505 6785619
6E3	10,943,164	3,342,706	10,942,687.83337	4.35127 0140159
7E3	14,895,146	3,951,982	14,894,213.99542	6.25710 2658460
8E3	19,455,782	4,560,636	19,453,667.2593	10.86947 1456597
9E3	24,621,518	5,165,736	24,621,047.62508	1.91042 2062204
1E4	30,397,486	5,775,968	30,396,355.09270	3.72039 7465331
2E4	121,590,396	91,192,910	121,585,420.37080	4.09212 3521560
3E4	273,571,774	151,981,378	273,567,195.83431	1.67347 8817305
4E4	486,345,716	212,773,942	486,341,681.48322	0.82955 7379857
5E4	759,924,264	273,578,548	759,908,877.31753	2.02476 5255648
6E4	1,094,277,506	334,353,242	1,094,268,783.33724	0.79711 6152369
7E4	1,489,425,820	395,148,314	1,489,421,399.54236	0.29678 9378524
8E4	1,945,400,166	455,974,346	1,945,366,725.93288	1.71893 0002126
9E4	2,462,103,830	516,703,664	2,462,104,762.50880	-0.03787 4471274
1E5	3,039,650,754	577,546,924	3,039,635,509.27013	0.50152 8994631

2E5	12,158,598,918	9,118,948,164	12,158,542,037.080
3E5	27,356,748,484	15,198,149,566	27,356,719,583.431
4E5	48,634,207,310	21,277,458,826	48,634,168,148.322
5E5	75,991,039,676	27,356,832,366	75,990,887,731.753
6E5	109,426,976,722	33,435,937,046	109,426,878,333.724
7E5	148,942,189,540	39,515,212,818	148,942,139,954.236
8E5	194,536,947,224	45,594,757,684	194,536,672,593.288
9E5	246,210,582,788	51,673,635,564	246,210,476,250.880
1E6	303,963,552,392	57,752,969,604	303,963,550,927.013
1.1	367,796,225,332	63,832,672,940	367,795,896,621.681
1.2	437,707,790,292	69,911,564,960	437,707,513,334.895
1.3	513,698,482,244	75,990,691,952	513,698,401,066.647
1.4	595,768,940,130	82,070,457,886	595,768,559,816.940
1.5	683,918,138,110	88,149,197,980	683,917,989,585.774
1.6	778,146,879,276	94,228,741,166	778,146,690,373.146
1.7	878,455,036,772	100,308,157,496	878,454,662,179.059
1.8	984,842,349,900	106,387,313,128	984,841,905,003.513
1.9	1097,308,571,350	112,466,221,450	1097,308,418,846.50
2E6	1215,854,699,278	118,546,127,928	

$$\sum_{n=1}^{n} \varphi(n) \sim 3 *(n/Pi)^2 + O (n *\log n)$$

Enc.Brt. p606 and also see Knuth v1p157 and Graham p139.
The O is the Bachmann-Landau O-notation. De Bruijn p3 and Graham p429-435.

PPC ROM p347.
Davidson & Efird
Kennedy
N&Z p48-51.
Neville pxi, He includes both end points so his sums exceed by 1 the figures above. Also see Graham p138.