BELL TELEPHONE LABORATORIES
INCORPORATED



# COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE— Some Observations on the Converse of Fermat's Theorem

MM 72— 1271 — 9

CASE CHARGED— 39199

FILING CASES— 39199-11

DATE— October 3, 1973

AUTHOR | LOC. | EXT.
Robert Morris | MH 2C-524 | 3878

FILING SUBJECTS— Number Theory

## ABSTRACT

A composite pseudoprime is defined as a number that obeys the congruence $2^{p-1} \equiv 1 \pmod{p}$ and yet is not prime. The divisibility and congruence properties of the first 1500 of these numbers reveal some unexpected regularities. Some problems and conjectures are suggested whose solution might bear on the problem of rapid factorization of integers.

NO. OF PAGES— 12

NO. OF REFERENCES— 1

NO. OF TABLES—

NO. OF FIGURES—



SEE REVERSE SIDE FOR DISTRIBUTION LIST

Bell Laboratories

Some Observations on the
Converse of Fermat's Theorem
Case 39399-11

October 3, 1972

Robert Morris

MM 72-1271-9

MEMORANDUM FOR FILE

## INTRODUCTION

Fermat's theorem states that if p is a prime and a is relatively prime to p, then $a^{p-1} \equiv 1 \pmod{p}$.

A possible converse to Fermat's theorem, which was long believed to be true, is the statement that if $2^{p-1} \equiv 1 \pmod{p}$, then p must be prime. The smallest counterexample to this proposition is the number $341 = 31 \cdot 11$, and there are only 245 such numbers less than one million. Therefore, although this converse to Fermat's theorem is false, there are only a relatively tiny proportion of numbers that disobey it.

A number n that satisfies the congruence $2^{n-1} \equiv 1 \pmod{n}$ is called a pseudoprime (or by some authors, a pseudoprime base 2). The exceptional numbers which disobey the converse can be described as composite pseudoprimes.

Historically, a great deal of interest has been focused on these composite pseudoprimes for the following reasons. First, they are extremely sparse in the integers and even in the pseudoprimes and all of them less than 10 million will fit on one ordinary printed page. Further, it is a much easier matter to test a number for pseudoprimality than to test it for primality. A number n can be tested for pseudoprimality by performing only $\log_2(n)$ multiplications modulo n whereas testing for primality may require the equivalent of $\pi(\sqrt{n})$ divisions. $\pi(k)$ is the number of primes less than k. Of course economies are possible for both methods, but they do not come near to closing the gap in computing times.

For example, a number less than 10 million can be tested for primality by first checking it for pseudoprimality. If it is not a pseudoprime, then by Fermat's theorem it cannot be prime. If it is a pseudoprime then it need only be compared with the one page list of composite pseudoprimes less than 10 million. If it is not on this list, then it is prime.

Before the advent of electronic computers, this scheme was an attractive alternative method, flawed only by the fact that the only existing table of composite pseudoprimes [1] is too inaccurate to be of much use.

More recently, the technique has become less inviting, since on a modern computer, primality can be detected in a matter of milliseconds for numbers as large as $10^{15}$ and this is far beyond

the range of any table of composite pseudoprimes that could possibly be generated with current techniques.

On the other hand, if restrictions could be put on the possible values of composite pseudoprimes or on their possible divisors, then the pseudoprime approach might again become interesting.

In summary, any theorem about pseudoprimes offers the potential of increasing the speed of testing large numbers for primality. The remainder of this paper is concerned with some empirical observations which lead to the conclusion that there are very strong restrictions on the values of numbers which can be composite pseudoprimes.

## A DIGRESSION

An even stronger converse of Fermat's theorem is also false, namely that if $a^{p-1} \equiv 1 \pmod{p}$ for every value of a which is relatively prime to p, then p is a prime. The smallest counterexample to this statement is $561 = 3 \cdot 11 \cdot 17$. On the other hand, a great deal is known about such numbers. They are square-free [2,p.119] and they are the product of at least three primes [1].

## NEARLY ALL COMPOSITE PSEUDOPRIMES ARE SQUARE FREE

A check of the first 1500 pseudoprimes showed that all were square-free except for two numbers which were themselves square,

$1093^2$ and $3511^2$. The numbers 1093 and 3511 are prime and they are the only known primes which satisfy the congruence $2^{p-1} \equiv 1 \pmod{p^2}$. Primes of this sort are of importance in the study of Fermat's Last Theorem. The square of any such prime is a pseudoprime. It is not known whether there are infinitely many such numbers. There are known to be no other such $p < 100{,}000$ [3].

It is relatively easy to prove [4] that if a pseudoprime is divisible by the square of a prime p, then $p^2$ is a pseudoprime. There is apparently no other reason to believe that square-free composite pseudoprimes should be particularly sparse or that the only pseudoprimes divisible by a square are themselves square. Among the integers, the square-free integers represent a proportion asymptotically equal to $6/\pi^2$.

Conjecture 1.: Does the observed excess of square-free pseudoprimes also hold asymptotically?

Conjecture 2.: Are all pseudoprimes either square-free or square?

## CONGRUENCE PROPERTIES OF PSEUDOPRIMES

The distribution of the first 1500 pseudoprimes among the residue classes mod n was tested for a variety of small n with the following results:

Residue Classes mod 3

| | |
|---|---|
| = 1 | 1202 |
| = 2 | 202 |
| = 0 | 96 |

Residue Classes mod 4

| | |
|---|---|
| = 1 | 1295 |
| = 3 | 205 |

Residue Classes mod 5

| | |
|---|---|
| = 1 | 789 |
| = 2 | 211 |
| = 3 | 190 |
| = 4 | 163 |
| = 0 | 147 |

Residue Classes mod 7

| | |
|---|---|
| = 1 | 552 |
| = 2 | 155 |
| = 3 | 172 |
| = 4 | 130 |
| = 5 | 160 |
| = 6 | 155 |
| = 0 | 176 |

Residue Classes mod 8

| | |
|---|---|
| = 1 | 827 |
| = 3 | 97 |
| = 5 | 468 |
| = 7 | 108 |

Residue Classes mod 9

| | |
|---|---|
| = 1 | 722 |
| = 2 | 68 |
| = 3 | 47 |
| = 4 | 243 |
| = 5 | 69 |
| = 6 | 49 |
| = 7 | 237 |
| = 8 | 65 |

Residue Classes mod 11

| | |
|---|---|
| = 1 | 336 |
| = 2 | 110 |
| = 3 | 112 |
| = 4 | 108 |
| = 5 | 132 |
| = 6 | 111 |
| = 7 | 100 |
| = 8 | 123 |
| = 9 | 110 |
| = 10 | 126 |
| = 0 | 132 |

Residue Classes mod 12

| | |
|---|---|
| = 1 | 1034 |
| = 3 | 5 |
| = 5 | 170 |
| = 7 | 168 |
| = 9 | 91 |
| = 11 | 32 |

Residue Classes mod 13

| | |
|---|---|
| = 1 | 265 |
| = 2 | 106 |
| = 3 | 98 |
| = 4 | 65 |
| = 5 | 110 |
| = 6 | 110 |
| = 7 | 116 |
| = 8 | 95 |
| = 9 | 89 |
| = 10 | 98 |
| = 11 | 81 |
| = 12 | 84 |
| = 0 | 183 |

The distribution among these residue classes is most irregular and to each of these moduli there is a large and unexpected surplus of pseudoprimes which have residue 1. The moduli up to 13 have been checked and in each case there is a significant excess of pseudoprimes with residue 1.

The pseudoprimes are all odd numbers by construction but there does not appear to be any other reason why the distribution among residue classes should be irregular. The primes, for example, are known to be distributed evenly among the possible residue classes mod n for every value of n.

Conjecture 3: Does the observed excess of composite pseudoprimes which leave a remainder of 1 for small n hold for all n?

Conjecture 4: Does the observed excess of composite pseudoprimes which leave a remainder of 1 mod n also hold asymptotically as the number of pseudoprimes increases?

Conjecture 5: Is there any significance to the fact that the number of these pseudoprimes in residue class 1 for prime moduli is very nearly 2.5 times what would be expected?

Robert Morris

MH-1271-RHM-pdp

Attached
References
Appendix

[1] Poulet, Table des nombres composés vérifiant le théorème du Fermat pour le module 2 jusqu'á 100.000.000
Sphinx 8 (March 1937) 42-52

[2] Shanks, Solved and Unsolved Problems in Number Theory, Vol. I
Spartan, 1962.

[3] Kravitz, The congruence $2^{p-1} = 1 \pmod{p^2}$ for $p<100,000$
Math. Comp. 14 (1960) 378

[4] Lehmer, D. H. On the Converse of Fermat's Theorem
Am. Math. Monthly 43 (1936) 347-354

[5] Hardy & Wright, An Introduction to the Theory of Numbers, 3rd edition
Oxford, 1954

composite pseudoprimes (base 2)

| 1 | 51 | 101 | 151 | 201 | 251 |
|---|---|---|---|---|---|
| 341 | 41665 | 181901 | 396271 | 665333 | 1033669 |
| 561 | 42799 | 188057 | 399001 | 665401 | 1050985 |
| 645 | 46657 | 188461 | 401401 | 670033 | 1052503 |
| 1105 | 49141 | 194221 | 410041 | 672487 | 1052929 |
| 1387 | 49981 | 196021 | 422659 | 679729 | 1053761 |
| 1729 | 52633 | 196093 | 423793 | 680627 | 1064053 |
| 1905 | 55245 | 204001 | 427233 | 683761 | 1073021 |
| 2047 | 57421 | 206601 | 435671 | 688213 | 1082401 |
| 2465 | 60701 | 208465 | 443719 | 710533 | 1082809 |
| 2701 | 60787 | 212421 | 448921 | 711361 | 1092547 |
| 2821 | 62745 | 215265 | 449065 | 721801 | 1093417 |
| 3277 | 63973 | 215749 | 451905 | 722201 | 1104349 |
| 4033 | 65077 | 219781 | 452051 | 722261 | 1106785 |
| 4369 | 65281 | 220729 | 458989 | 729061 | 1109461 |
| 4371 | 68101 | 223345 | 464185 | 738541 | 1128121 |
| 4681 | 72885 | 226801 | 476971 | 741751 | 1132657 |
| 5461 | 74665 | 228241 | 481573 | 742813 | 1139281 |
| 6601 | 75361 | 233017 | 486737 | 743665 | 1141141 |
| 7957 | 80581 | 241001 | 488881 | 745889 | 1145257 |
| 8321 | 83333 | 249841 | 489997 | 748657 | 1152271 |
| 8481 | 83665 | 252601 | 493697 | 757945 | 1157689 |
| 8911 | 85489 | 253241 | 493885 | 769567 | 1168513 |
| 10261 | 87249 | 256999 | 512461 | 769757 | 1193221 |
| 10585 | 88357 | 258511 | 513629 | 786961 | 1194649 |
| 11305 | 88561 | 264773 | 514447 | 800605 | 1207361 |
| 12801 | 90751 | 266305 | 526593 | 818201 | 1246785 |
| 13741 | 91001 | 271951 | 530881 | 825265 | 1251949 |
| 13747 | 93961 | 272251 | 534061 | 831405 | 1252697 |
| 13981 | 101101 | 275887 | 552721 | 838201 | 1275681 |
| 14491 | 104653 | 276013 | 556169 | 838861 | 1277179 |
| 15709 | 107185 | 278545 | 563473 | 841681 | 1293337 |
| 15841 | 113201 | 280601 | 574561 | 847261 | 1302451 |
| 16705 | 115921 | 282133 | 574861 | 852481 | 1306801 |
| 18705 | 121465 | 284581 | 580337 | 852841 | 1325843 |
| 18721 | 123251 | 285541 | 582289 | 873181 | 1333333 |
| 19951 | 126217 | 289941 | 587861 | 875161 | 1357441 |
| 23001 | 129889 | 294271 | 588745 | 877099 | 1357621 |
| 23377 | 129921 | 294409 | 604117 | 898705 | 1373653 |
| 25761 | 130561 | 314821 | 611701 | 915981 | 1394185 |
| 29341 | 137149 | 318361 | 617093 | 916327 | 1397419 |
| 30121 | 149281 | 323713 | 622909 | 934021 | 1398101 |
| 30889 | 150851 | 332949 | 625921 | 950797 | 1419607 |
| 31417 | 154101 | 334153 | 635401 | 976873 | 1433407 |
| 31609 | 157641 | 340561 | 642001 | 983401 | 1441091 |
| 31621 | 158369 | 341497 | 647089 | 997633 | 1457773 |
| 33153 | 162193 | 348161 | 653333 | 1004653 | 1459927 |
| 34945 | 162401 | 357761 | 656601 | 1016801 | 1461241 |
| 35333 | 164737 | 367081 | 657901 | 1018921 | 1463749 |
| 39865 | 172081 | 387731 | 658801 | 1023121 | 1472065 |
| 41041 | 176149 | 390937 | 665281 | 1024651 | 1472353 |

composite pseudoprimes (base 2)

| 301 | 351 | 401 | 451 | 501 | 551 |
|---|---|---|---|---|---|
| 1472505 | 1969417 | 2513841 | 3186821 | 4181921 | 5131589 |
| 1485177 | 1987021 | 2528921 | 3224065 | 4188889 | 5133201 |
| 1489665 | 1993537 | 2531845 | 3225601 | 4209661 | 5148001 |
| 1493857 | 1994689 | 2537641 | 3235699 | 4229601 | 5173169 |
| 1500661 | 2004403 | 2603381 | 3316951 | 4259905 | 5173601 |
| 1507561 | 2008597 | 2609581 | 3336319 | 4314967 | 5176153 |
| 1507963 | 2035153 | 2615977 | 3337849 | 4335241 | 5187637 |
| 1509709 | 2077545 | 2617451 | 3345773 | 4360621 | 5193721 |
| 1520905 | 2081713 | 2626177 | 3363121 | 4361389 | 5250421 |
| 1529185 | 2085301 | 2628073 | 3370641 | 4363261 | 5256091 |
| 1530787 | 2089297 | 2649029 | 3375041 | 4371445 | 5258701 |
| 1533601 | 2100901 | 2649361 | 3375487 | 4415251 | 5271841 |
| 1533961 | 2113665 | 2670361 | 3400013 | 4463641 | 5284333 |
| 1534541 | 2113921 | 2704801 | 3413533 | 4469471 | 5310721 |
| 1537381 | 2121301 | 2719981 | 3429037 | 4480477 | 5351537 |
| 1549411 | 2134277 | 2722681 | 3435565 | 4502485 | 5400489 |
| 1569457 | 2142141 | 2746477 | 3471071 | 4504501 | 5423713 |
| 1579249 | 2144521 | 2746589 | 3539101 | 4513841 | 5444489 |
| 1584133 | 2162721 | 2748023 | 3542533 | 4535805 | 5456881 |
| 1608465 | 2163001 | 2757241 | 3567481 | 4567837 | 5481451 |
| 1615681 | 2165801 | 2773981 | 3568661 | 4613665 | 5489121 |
| 1620385 | 2171401 | 2780731 | 3581761 | 4650049 | 5489641 |
| 1643665 | 2181961 | 2793351 | 3605429 | 4670029 | 5524693 |
| 1678541 | 2184571 | 2797921 | 3656449 | 4682833 | 5529745 |
| 1690501 | 2205967 | 2811271 | 3664585 | 4698001 | 5545145 |
| 1711381 | 2213121 | 2827801 | 3679201 | 4706821 | 5551201 |
| 1719601 | 2232865 | 2867221 | 3726541 | 4714201 | 5560809 |
| 1730977 | 2233441 | 2880361 | 3746289 | 4767841 | 5575501 |
| 1735841 | 2261953 | 2882265 | 3755521 | 4806061 | 5590621 |
| 1746289 | 2264369 | 2899801 | 3763801 | 4827613 | 5599765 |
| 1755001 | 2269093 | 2909197 | 3779185 | 4835209 | 5632705 |
| 1773289 | 2284453 | 2921161 | 3814357 | 4863127 | 5672041 |
| 1801969 | 2288661 | 2940337 | 3828001 | 4864501 | 5681809 |
| 1809697 | 2290641 | 2944261 | 3898129 | 4868701 | 5733649 |
| 1811573 | 2299081 | 2953711 | 3911197 | 4869313 | 5758273 |
| 1815465 | 2304167 | 2976487 | 3916261 | 4877641 | 5766001 |
| 1826203 | 2313697 | 2977217 | 3936691 | 4895065 | 5804821 |
| 1827001 | 2327041 | 2987167 | 3985921 | 4903921 | 5859031 |
| 1830985 | 2350141 | 3020361 | 4005001 | 4909177 | 5872361 |
| 1837381 | 2387797 | 3048841 | 4014361 | 4917331 | 5919187 |
| 1839817 | 2414001 | 3057601 | 4025905 | 4917781 | 5968261 |
| 1840357 | 2419385 | 3059101 | 4038673 | 4922413 | 5968873 |
| 1857241 | 2433601 | 3073357 | 4069297 | 4974971 | 5977153 |
| 1876393 | 2434651 | 3090091 | 4072729 | 4984001 | 6027193 |
| 1892185 | 2455921 | 3094273 | 4082653 | 5016191 | 6049681 |
| 1896961 | 2487941 | 3116107 | 4097791 | 5031181 | 6054985 |
| 1907851 | 2491637 | 3125281 | 4101637 | 5034601 | 6118141 |
| 1908985 | 2503501 | 3146221 | 4151869 | 5044033 | 6122551 |
| 1909001 | 2508013 | 3165961 | 4154161 | 5049001 | 6135585 |
| 1937881 | 2510569 | 3181465 | 4154977 | 5095177 | 6140161 |

composite pseudoprimes (base 2)

| 601 | 651 | 701 | 751 | 801 | 851 |
|---|---|---|---|---|---|
| 6159301 | 7306261 | 8719921 | 10004681 | 11328409 | 13421773 |
| 6183601 | 7306561 | 8725753 | 10024561 | 11335501 | 13446253 |
| 6189121 | 7414333 | 8727391 | 10031653 | 11367137 | 13448593 |
| 6212361 | 7416289 | 8745277 | 10033777 | 11433301 | 13500313 |
| 6226193 | 7428421 | 8812273 | 10079521 | 11473885 | 13554781 |
| 6233977 | 7429117 | 8830801 | 10084177 | 11541307 | 13635289 |
| 6235345 | 7455709 | 8902741 | 10134601 | 11585293 | 13635649 |
| 6236257 | 7462001 | 8916251 | 10185841 | 11592397 | 13694761 |
| 6236473 | 7516153 | 8927101 | 10226161 | 11644921 | 13696033 |
| 6242685 | 7519441 | 8992201 | 10239985 | 11767861 | 13747361 |
| 6255341 | 7546981 | 9006401 | 10251473 | 11776465 | 13757653 |
| 6278533 | 7656721 | 9037729 | 10266001 | 11777599 | 13773061 |
| 6309901 | 7674967 | 9040013 | 10267951 | 11875821 | 13823601 |
| 6313681 | 7693401 | 9046297 | 10275685 | 11921001 | 13838569 |
| 6334351 | 7724305 | 9056501 | 10317601 | 11972017 | 13856417 |
| 6350941 | 7725901 | 9063105 | 10323769 | 12032021 | 13899565 |
| 6368689 | 7759937 | 9069229 | 10331141 | 12067705 | 13942081 |
| 6386993 | 7803769 | 9073513 | 10386241 | 12096613 | 13971841 |
| 6474691 | 7808593 | 9084223 | 10393201 | 12261061 | 13991647 |
| 6539527 | 7814401 | 9106141 | 10402237 | 12262321 | 13992265 |
| 6617929 | 7820201 | 9131401 | 10402561 | 12263131 | 13996951 |
| 6628385 | 7883731 | 9143821 | 10403641 | 12273769 | 14012797 |
| 6631549 | 7995169 | 9223401 | 10425511 | 12322133 | 14026897 |
| 6658669 | 8012845 | 9224391 | 10505701 | 12327121 | 14154337 |
| 6732817 | 8036033 | 9273547 | 10513261 | 12376813 | 14179537 |
| 6733693 | 8041345 | 9345541 | 10545991 | 12407011 | 14282143 |
| 6749021 | 8043841 | 9371251 | 10606681 | 12490201 | 14324473 |
| 6779137 | 8095447 | 9439201 | 10610063 | 12498061 | 14469841 |
| 6787327 | 8134561 | 9480461 | 10635751 | 12584251 | 14556081 |
| 6836233 | 8137585 | 9494101 | 10655905 | 12599233 | 14589901 |
| 6840001 | 8137633 | 9533701 | 10680265 | 12643381 | 14609401 |
| 6868261 | 8180461 | 9564169 | 10700761 | 12659989 | 14671801 |
| 6886321 | 8209657 | 9567673 | 10712857 | 12702145 | 14676481 |
| 6912079 | 8231653 | 9582145 | 10763653 | 12711007 | 14684209 |
| 6952037 | 8239477 | 9585541 | 10802017 | 12757361 | 14709241 |
| 6955541 | 8280229 | 9588151 | 10818505 | 12783811 | 14794081 |
| 6973057 | 8321671 | 9591661 | 10837321 | 12854437 | 14796289 |
| 6973063 | 8322945 | 9613297 | 10877581 | 12932989 | 14865121 |
| 6998881 | 8341201 | 9638785 | 10956673 | 12936763 | 14870801 |
| 7008001 | 8355841 | 9692453 | 10958221 | 12939121 | 14885697 |
| 7017193 | 8362201 | 9724177 | 10974385 | 12945745 | 14892153 |
| 7040001 | 8384513 | 9729301 | 10974881 | 12979765 | 14898631 |
| 7177105 | 8388607 | 9774181 | 11034365 | 13057787 | 14899751 |
| 7207201 | 8462233 | 9816465 | 11075857 | 13073941 | 14913991 |
| 7215481 | 8534233 | 9834781 | 11081459 | 13187665 | 14980411 |
| 7232321 | 8640661 | 9863461 | 11115037 | 13216141 | 15082901 |
| 7233265 | 8646121 | 9890881 | 11119105 | 13295281 | 15101893 |
| 7259161 | 8650951 | 9908921 | 11157721 | 13333441 | 15124969 |
| 7273267 | 8656705 | 9920401 | 11205601 | 13338371 | 15139199 |
| 7295851 | 8719309 | 9995671 | 11242465 | 13357981 | 15162941 |

composite pseudoprimes (base 2)

| 901 | 951 | 1001 | 1051 | 1101 | 1151 |
|---|---|---|---|---|---|
| 15188557 | 16973393 | 19020191 | 21814417 | 24158641 | 27062101 |
| 15207361 | 16998961 | 19054933 | 21880801 | 24161905 | 27108397 |
| 15220951 | 17020201 | 19092921 | 21907009 | 24214051 | 27118601 |
| 15247621 | 17098369 | 19149571 | 22066201 | 24356377 | 27128201 |
| 15248773 | 17116837 | 19260865 | 22075579 | 24726773 | 27168337 |
| 15268501 | 17134043 | 19328653 | 22087477 | 24776557 | 27218269 |
| 15403285 | 17208601 | 19384289 | 22137809 | 24904153 | 27219697 |
| 15472441 | 17236801 | 19404139 | 22203181 | 24913681 | 27271151 |
| 15479777 | 17316001 | 19471033 | 22215961 | 24929281 | 27279409 |
| 15510041 | 17327773 | 19523505 | 22269745 | 25080101 | 27331921 |
| 15525241 | 17375249 | 19569265 | 22351249 | 25150501 | 27336673 |
| 15560461 | 17405537 | 19607561 | 22369621 | 25266745 | 27380831 |
| 15583153 | 17429861 | 19683001 | 22397497 | 25270105 | 27392041 |
| 15603391 | 17450569 | 19734157 | 22432201 | 25276421 | 27401401 |
| 15621409 | 17509501 | 19781763 | 22480381 | 25326001 | 27402481 |
| 15698431 | 17585969 | 19985269 | 22487101 | 25457833 | 27409541 |
| 15700301 | 17586361 | 20081953 | 22509691 | 25520833 | 27476641 |
| 15716041 | 17590957 | 20099017 | 22513457 | 25540291 | 27491237 |
| 15732721 | 17641207 | 20117467 | 22564081 | 25557121 | 27492581 |
| 15757741 | 17698241 | 20140129 | 22591301 | 25603201 | 27509653 |
| 15802681 | 17759681 | 20202481 | 22665505 | 25629913 | 27600001 |
| 15829633 | 17777191 | 20234341 | 22669501 | 25640641 | 27664033 |
| 15888313 | 17812081 | 20261251 | 22711873 | 25696133 | 27700609 |
| 15913261 | 17870561 | 20417311 | 22848541 | 25768261 | 27714961 |
| 15976747 | 17895697 | 20489239 | 22849481 | 25831585 | 27736345 |
| 15978007 | 18003349 | 20494401 | 22885129 | 25840081 | 27798461 |
| 16046641 | 18007345 | 20626165 | 22899097 | 25846913 | 27808463 |
| 16053193 | 18067501 | 20647621 | 22953673 | 25873381 | 27846721 |
| 16070429 | 18073817 | 20770621 | 23054601 | 25909453 | 27966709 |
| 16132321 | 18137505 | 20964961 | 23247901 | 25947959 | 27986421 |
| 16149169 | 18151861 | 20968501 | 23261713 | 26254801 | 28011001 |
| 16149601 | 18162001 | 21042001 | 23283037 | 26280073 | 28029001 |
| 16153633 | 18300241 | 21224401 | 23286781 | 26296401 | 28071121 |
| 16158331 | 18307381 | 21303343 | 23315977 | 26377921 | 28172629 |
| 16263105 | 18366937 | 21306157 | 23382529 | 26465089 | 28175001 |
| 16324001 | 18443701 | 21355951 | 23386441 | 26470501 | 28312921 |
| 16349477 | 18454921 | 21359521 | 23405341 | 26474581 | 28325881 |
| 16360381 | 18468901 | 21397381 | 23464033 | 26553241 | 28406953 |
| 16435747 | 18487267 | 21400481 | 23517985 | 26634301 | 28449961 |
| 16539601 | 18490381 | 21414169 | 23577497 | 26719701 | 28527049 |
| 16666651 | 18535177 | 21417991 | 23634181 | 26758057 | 28572961 |
| 16705021 | 18541441 | 21459361 | 23734901 | 26813221 | 28629613 |
| 16717061 | 18595801 | 21474181 | 23736385 | 26821601 | 28717483 |
| 16773121 | 18607009 | 21559741 | 23808721 | 26840269 | 28787185 |
| 16778881 | 18653353 | 21584305 | 23822329 | 26877421 | 29020321 |
| 16818877 | 18736381 | 21585313 | 23828017 | 26886817 | 29111881 |
| 16822081 | 18740971 | 21623659 | 23872213 | 26921089 | 29137021 |
| 16843009 | 18779761 | 21654533 | 23963869 | 26932081 | 29143633 |
| 16853077 | 18900973 | 21715681 | 23966011 | 26977001 | 29214541 |
| 16879501 | 18985627 | 21789901 | 24037021 | 27032545 | 29581501 |

composite pseudoprimes (base 2)

| 1201 | 1251 | 1301 | 1351 | 1401 | 1451 |
|---|---|---|---|---|---|
| 29593159 | 32368609 | 36121345 | 38801089 | 41590297 | 44963029 |
| 29732221 | 32428045 | 36168265 | 38903287 | 41604109 | 45100177 |
| 29878381 | 32497921 | 36255451 | 38971661 | 41607721 | 45175201 |
| 30022129 | 32676481 | 36291193 | 39016741 | 41642681 | 45219329 |
| 30058381 | 32701297 | 36307981 | 39052333 | 41662297 | 45318561 |
| 30069721 | 32756581 | 36338653 | 39117439 | 41840809 | 45393601 |
| 30090817 | 32899201 | 36354449 | 39126313 | 41866001 | 45414433 |
| 30185569 | 32914441 | 36448387 | 39353665 | 41987111 | 45485881 |
| 30219757 | 33018985 | 36507801 | 39465091 | 42009217 | 45563027 |
| 30295141 | 33146717 | 36721021 | 39467377 | 42344609 | 45593065 |
| 30296761 | 33193117 | 36724591 | 39512773 | 42485119 | 45769645 |
| 30338593 | 33298337 | 36765901 | 39573073 | 42490801 | 45819541 |
| 30388753 | 33302401 | 36852481 | 39655153 | 42623017 | 45830161 |
| 30411201 | 33408145 | 36861901 | 39684157 | 42689305 | 45877861 |
| 30418957 | 33596641 | 36919681 | 39789841 | 42694279 | 45879941 |
| 30529693 | 33600533 | 36942157 | 40094341 | 42697873 | 45890209 |
| 30576151 | 33627301 | 36974341 | 40160737 | 42702661 | 46045117 |
| 30653245 | 33704101 | 36981601 | 40165093 | 42709591 | 46055851 |
| 30662497 | 33840397 | 37109467 | 40238797 | 42763501 | 46094401 |
| 30718441 | 33848311 | 37167361 | 40280065 | 42984589 | 46104697 |
| 30739969 | 33872593 | 37280881 | 40315441 | 42998901 | 46256489 |
| 30740417 | 33965261 | 37354465 | 40325041 | 43039501 | 46282405 |
| 30881551 | 34003061 | 37376509 | 40361197 | 43136821 | 46325029 |
| 30894307 | 34043101 | 37439201 | 40374901 | 43224397 | 46386589 |
| 30951181 | 34100821 | 37469701 | 40430401 | 43235641 | 46469809 |
| 30958201 | 34111441 | 37491301 | 40622401 | 43286881 | 46483633 |
| 30971161 | 34124641 | 37695505 | 40629601 | 43331401 | 46517857 |
| 30992401 | 34196401 | 37727341 | 40778989 | 43363601 | 46647745 |
| 30996001 | 34386121 | 37769887 | 40782589 | 43397551 | 46657181 |
| 31040833 | 34487601 | 37938901 | 40801861 | 43584481 | 46679761 |
| 31118221 | 34540801 | 37962541 | 40827473 | 43620409 | 46860001 |
| 31146661 | 34581457 | 37964809 | 40841821 | 43661257 | 46878601 |
| 31150351 | 34603041 | 37988497 | 40886241 | 43798457 | 47006785 |
| 31166803 | 34657141 | 38010307 | 40917241 | 43914949 | 47063611 |
| 31198693 | 34856167 | 38046817 | 40928701 | 44070841 | 47220367 |
| 31405501 | 34890481 | 38118763 | 40933705 | 44081101 | 47253781 |
| 31436123 | 34901461 | 38151361 | 40987201 | 44238481 | 47349373 |
| 31692805 | 34944001 | 38171953 | 41017681 | 44314129 | 47356171 |
| 31735621 | 35428141 | 38210323 | 41067665 | 44347381 | 47647117 |
| 31759121 | 35498467 | 38239741 | 41073241 | 44465221 | 47734141 |
| 31766983 | 35571601 | 38342071 | 41102601 | 44472001 | 47744209 |
| 31794241 | 35576599 | 38404501 | 41121433 | 44482901 | 47759041 |
| 31880577 | 35626501 | 38439523 | 41262073 | 44521301 | 47903701 |
| 32080651 | 35703361 | 38453151 | 41298985 | 44671001 | 47918581 |
| 32091781 | 35820937 | 38560861 | 41341321 | 44695211 | 47930023 |
| 32095057 | 35851037 | 38584801 | 41396921 | 44731051 | 47953621 |
| 32158621 | 35926801 | 38624041 | 41424801 | 44823241 | 48064021 |
| 32168117 | 35932441 | 38637361 | 41471521 | 44824501 | 48191653 |
| 32264029 | 35967921 | 38789913 | 41541241 | 44912701 | 48269761 |
| 32285041 | 35976721 | 38790753 | 41568101 | 44953441 | 48277081 |