THE LOGIC DIAGRAM

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INTRODUCTION

The logic diagram was introduced in 1761[1] and was, from the time of Hamilton to the publishing of the Principia Mathematica, a center of controversy among logicians. Some of the major logical problems of our time are crystallized and clarified, though not solved, in these diagrams.[2] More important, the history of nineteenth-century logic, with the important exception of Boole, can be traced in the development of these diagrams. Yet there has been little work done in our own time or in the past on this fascinating branch of logic.

Sir William Hamilton and John Venn both attempted to collect what was known about diagrams in their own time but Hamilton's analysis is prejudiced as we shall see, and Venn's is sketchy. C.I. Lewis introduced only geometric diagrams in A Survey of Symbolic Logic and used them with presumptions which must be examined. Martin Gardner collected several systems of logic diagrams in Logic Machines and Diagrams but treated them, with the exception of Venn's, as interesting curiosities. Some work has been done on the diagrams of individual logicians[3] but in the interest of

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[1]For qualifications of this statement see chapter I section 1 and chapter II section 3 part (a) and chapter II section 4 parts (a) and (b). Hereafter cross-references to this thesis will be abbreviated. The above reference would be written thus: I 1, II 3 (a) and 4 (a) and (b).

[2]III 4 (a) and (b).

[3]E.g. D.D. Roberts, The Existential Graphs of C.S. Peirce, Urbana: University of Illinois, 1963, unpublished thesis.

completeness this needs the context of the complete development of logic diagrams.

On the other hand, we need only pick up a textbook on elementary logic, set theory, switching circuits or even arithmetic if it is the "new math" to be faced with a wide variety of logic diagrams. Euler, Venn, Marquand, Lambert and Carroll are all represented, sometimes in the same book. That these systems are incompatible[4] seems of little importance to their users.

It would seem therefore, that there is a necessity for an examination of logic diagrams, qua logic diagrams, which will endeavour to discover what they are and what characteristics they must possess if they are to function as their users intend. Since the diagrams were first introduced in logic, and since the mathematician and electrical engineer can hardly be expected to perform such an analysis, and since, moreover, Aristotelian and nineteenth-century symbolic logic would seem to be the most adequate tools to be used in such an analysis, we may drop this work in the logicians' lap. The purpose of this paper is to lay the foundation for such an analysis. It will, of course, be impossible to examine any single problem extensively but will be within our purpose to locate those areas in which problems of a logical or philosophical nature should be raised. Our work will be divided into three parts, the purposes of which will be as follows:

I. to give a chronological résumé of the history of the diagram in logic from 1761 to 1910 with emphasis on those logicians who contributed

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[4]See the comparative portions of II.

to its development and their relevant writings.

II. to establish a simple classification for such diagrams and to describe and compare the various systems of diagrams within this classification.

III. in the light of the foregoing, to describe the uses and assess the worth of the logic diagram in our own time and to indicate some relevant problems raised by the diagrams.

If the diagrams are to be used they ought to be used well. This can only be done if they are systematically examined.

All of the diagrams mentioned in this work will be found in numerical order in the first appendix.

I

THE LIFE HISTORY OF THE LOGIC DIAGRAM (1763 - 1910)

1. Its Conception (prior to 1763)

The first diagrams used by logicians are in all probability now lost. We have, nevertheless, many early Mediaeval diagrams which represent the individual valid arguments of Aristotelian logic. These are not actual logic diagrams: they illustrate the argument after it has been solved and are not primarily intended as aids to reasoning as more recent systems are. Hamilton, whose scholarship we shall have reason to question[1], traces these as far back as the fifth century A.D. Giordano Bruno incorporates three of these Mediaeval diagrams in one diagram.[2] (Diagram I).

Gardner is fascinated by the life and works of Raymon Lull.[3] The Ars Magna was more mechanical than diagrammatic and more metaphysical than logical. We may, therefore, safely and thankfully ignore Lull's incredibly obscure system for our purposes in this chapter and turn to

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[1]Hamilton's scholarship will be found to be questionable in his discussions of all prior logicians, particularly Euler and Maass. II 2 (a) and (b).

[2]Martin Gardner, Logic Machines and Diagrams, Toronto: McGraw-Hill, 1958, p. 30. I find these diagrams confusing and will not treat them in this work. For more information see John Venn, Symbolic Logic, 2nd ed., London: Macmillan, 1894, pp. 504 ff.

[3]Gardner, op. cit., pp. 1 ff.

more modern sources.[4]

Hamilton attributed the geometric diagram to Christian Weise in Nucleous Logicae Weisiania (1712).[5] This was a mistake for, as Venn pointed out, Weise did not write this book. The author was, in fact, Johann Christian Lange.[6] Hamilton did not seem to recognize that Lange used the diagram to represent propositions but not syllogisms for he equates Lange's diagrams with Euler's. Johann Christoph Strum in Universalia Euclides (1661) and Leibniz both used circles to represent propositions prior to Lange.[7] This would seem to indicate further that Hamilton's historical research was not as thorough as he thought.

In A Survey of Symbolic Logic, C. I. Lewis translates two brief portions of Leibniz which indicate that Leibniz understood the principles of the linear diagram[8] almost a century before Lambert. These fragments are taken from Gerhardt's text, Die Philosophischen Schriften von G. W. Leibniz, Band VII, "Scientia Generalis. Characteristica," XIX and XX.[9] Hamilton, with his usual historical scholarship, attributes the linear diagram to J. H. Alsted in his Logic (1614). Venn pointed out that there

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[4]An example of Lull's system will be described in II 4 (b).

[5]This book was not available to the author of this thesis.

[6]Venn, op. cit., p. 509.

[7]See Gardner, op. cit., p. 31, where he cites Church. The Strum book was unavailable but we discuss Leibniz briefly, from what information is available, in II 3 (a).

[8]See II 1 for a definition of "linear" and II 3 for descriptions of linear systems.

[9]C. I. Lewis, A Survey of Symbolic Logic, New York: Dover, 1960, pp. 291 ff.

were no diagrams in the book.[10]

Thus, although there were diagrams in logic prior to 1663 they either represented propositions and gave little or no aid in drawing conclusions or, in Lull's case, were so obscure as to be useless. The exception to this is a few diagrams used by Leibniz which we shall compare to Lambert's.[11] The actual birth of the logic diagram, its entrance into logic as a major force, awaited the work of Leonard Euler.[12]

2. Its Birth (1763 - 1807)

The modern logic diagram was born of Aristotelian logic in 1763. Its father was unknown but the midwife who brought it into the world was Leonard Euler. Euler, like most of the great eighteenth century figures, was a man of many talents. He is best remembered as a mathematician and logician but he was no mean philosopher and moralist and was known as a political counsellor to most of the thrones of Europe. Because of his great reputation for learning in the sciences Euler was commissioned by Frederick II of Prussia as tutor to his neice, the Princess d'Anhalt Dessau. His correspondence with the princess was published in 1772 as Lettres a une Princess d'Allemagne. It was in this work that Euler

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[10] Venn, op. cit., p. 507.

[11] We refer here to such authors as Reimarus and Vives whose works are unavailable but who, as described by Venn, op. cit., p. 504 ff., seem to contribute nothing to the logic diagram. Although their systems are different from either the mediaeval or the modern systems, it is, at best, difficult to understand what they mean and there seems to be no reason why they should be accepted either on pragmatic or on iconic principles.

[12] With the exception of Leibniz.

introduced the logic diagram to the world.[13]

Euler's diagrams were geometrical,[14] circular in fact, and were intended to be an aid to the student in understanding the structure of the various syllogisms in respect to the relationships between the three terms involved.[15] On the ground of Euler's diagrams much of the work of Hamilton and almost all of the work of Jevons, Venn and the later inventors of geometrical logic diagrams was based. His influence is still felt as many introductory logic texts, especially those written by scholastic logicians, make extensive and exclusive use of Euler's system of diagrams.[16]

In the year following the writing of Letters to a German Princess Johann Heinrick Lambert published Neues Organon (1764) in which he introduced a form of notation which was actually a form of linear diagram performing the same function as Euler's geometric diagrams.[17] He apparently struck on this system independently for it has obvious disadvantages which he would have attempted to correct had he been familiar with Euler's system.

The final figure of this period was J.G.E. Maass. In his Grundriss

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[13]Letters CII to CVIII, dated February 14, 1761 to March 7, 1761 pp. 450 - 485 in the Hunter translation.

[14]See II 1 for a definition and II 2 for a description of geometric diagrams.

[15]See II 2 (a) for an exact description of Euler's intention and his system.

[16]E.g. Celestine N. Bittle, The Science of Correct Thinking, Milwaukee: The Bruce Publishing Company, 1950.

[17]II 3 (b). This book was unavailable.

der Logik (1807) he substituted triangles for Euler's circles.[18] This might be considered important as it demonstrates that the shape of the geometric patterns used is irrelevant to the validity of the diagram. Maass' system is an interesting early variation which has particular significance in the light of Hamilton's and Venn's comments.

3. Its Adolescence

George Boole was ignored in his own time. Yet his Laws of Thought (1854) changed the direction of logic after Jevons. It was an attempt at a coherent and comprehensive mathematical notation for logic. Boole used no diagrams but his influence on those who did does not allow us to ignore him. He broke radically with Aristotelian logic and paved the way for modern mathematical and symbolic logic. All of the developments in the logic diagram after Hamilton were instigated by a concern to apply the diagrams to the "Boole-Schroeder" algebra.[19]

Although Boole's major work was published sixteen years before Hamilton's, Boole was virtually unrecognized while Hamilton attained a powerful reputation based on his lectures and papers. Long before the publication of Lectures on Metaphysics and Logic (1860) Hamilton was widely accepted as the outstanding logician of the English-speaking world. It has been said that both Boole and de Morgan were deeply influenced by

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[18] J. G. E. Maass, Grundriss der Logik, Leipzig: Eduard Meissner, 1836. See II 2 (b) where the basic differences between Euler and Maass will be discussed.

[19] It is not within the scope of this work to give a description of Boolean algebra although some acquaintance with it is presupposed on the reader's part. A good introduction is Lewis's Survey.

Hamilton.[20] Their reaction against Hamilton's system laid the basis for a truly mathematical logic.

Sir William Hamilton rejected traditional Aristotelian logic on the grounds that it was too narrow. The solution to this narrowness, he felt, was not to be found in a new system but in an expansion of the old. This expansion was to be accomplished by the quantification of the predicate.[21] Such a move naturally made Euler's and Lambert's diagrammatic schemes obsolete in their original forms. In his lectures Hamilton used revised versions of both of these schemes but he also developed his own "geometric"[22] system. This is not a geometric system in our sense of the word "geometric". It consists of 1) a chart which illustrates Hamilton's concept of breadth and depth in reasoning, 2) a diagram, consisting of four concentric triangles, offering a condensed view of Hamilton's scheme of syllogistic notation and 3) a table of syllogistic moods illustrating Hamilton's wedges.[23] Thus Hamilton makes use of three systems of logic diagrams: 1) circular, adapted from Euler,[24] 2) linear, adapted from Lambert[25] and 3) wedges, original.[26] Hamilton mistook

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[20] Lewis, op. cit., p. 37.

[21] II 2 (a) and (c), II 5 (a) and (b), especially II 5 (a).

[22] Quoted by Venn, op. cit., p. 521 but I have not been able to find it in Hamilton.

[23] II 5 (a).

[24] II 2 (a).

[25] II 3 (b)

[26] See note 23 above.

Maass's triangular system, apparently because he judged the diagrams without reading the text, for an attempt at an angular system and dismissed it without further consideration.[27]

Augustus de Morgan fought a continuing battle with Hamilton for credit as discoverer of the quantification of the predicate. His logic, because he was a mathematician, took a mathematical point of view. De Morgan read Boole, apparently with little enthusiasm, although they have much in common.[28] De Morgan was not, however, the system-builder that Boole was so that much of his work concerned fine points. He also wrote many articles and wasted much time in his feud with Hamilton. De Morgan continued to write such articles long after Hamilton's death.[29] De Morgan's Syllabus for a Proposed System of Logic was published the same year as Hamilton's Lectures. In the Syllabus, de Morgan offers charts which are similar to those of Hamilton.[30] De Morgan makes no claims about logic diagrams. He does not seem to think of his chart as such but since it is necessary to examine Hamilton's charts it is valuable to look at de Morgan's as well.

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[27]Venn, op. cit., p. 516 and Sir William Hamilton, Lectures on Logic, ed. Rev. Henry L. Mansel and John Veitch, New York: Sheldon and Company, 1876, pp. 669 - 670.

[28]A basically mathematical approach was their greatest common ground.

[29]This is important because it forced de Morgan to develop a mathematical system. See Lewis, op. cit., pp. 37ff. for a thorough description of the relationship between de Morgan and Hamilton particularly concerning de Morgan's attempt to mathematecize logic.

[30]II 5 (b)

4. Its Maturity (1870 - 1882)

The contributions to the more esoteric branches of the literature of logic by W. Stanley Jevons included a paper "On the Mechanical Performance of Logical Inferences" (1870) and Studies in Deductive Logic (1880). Jevons seems to have been the first major logician to have realized the importance of Boole's discoveries and made a strong case for Boole as the discoverer of the quantification of the predicate.[31] He did not, despite a healthy respect for Hamilton's reputation, accept Hamilton's complex diagrammatic system, but suggested that we can get along quite well with Euler's diagrams.[32]

Jevons' importance for us rests in two instruments that he developed. In the 1870 article he describes a machine, played rather like a piano, which solves problems in logic.[33] In his Studies he describes a "slate" which operates on the same principles. The logical structure of these actually prefigures the diagrams of Venn. In some manner all possible combinations of the positive terms and their negatives in a syllogism are represented. Through mechanical means those which are inapplicable, because of the premises, are removed. From what remains we read off all possible conclusions.[34] All of the mechanics which appeared to be so original in Venn are represented in Jevons' machines. What is

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[31]Even Jevons did not fully appreciate Boole's significance as he applied Boole's system only to Aristotelian syllogisms.

[32]For Jevons' interpretation of Euler see II 2 (a).

[33]See also Gardner, op. cit., pp. 91ff.

[34]II 2 (c).

original in Venn is the depth of interpretation of these mechanics.

Jevons' own interpretation included the idea that every positive term must be represented in what remained on the machine. That is, he believed that we could not reach a negative existential conclusion.[35] We will see in Venn that Jevons was wrong yet we must face this serious problem of the import of existential conclusions later.[36]

Meanwhile on the continent the network diagram was invented and carried to virtual perfection by Gottlob Frege. His symbol of closure[37] is still in use. This symbol was part of a very complex notation fully described in Begriffsschrift, eine der arithmetischen nachgebildte Formelsprache des reinen Denken (1879). This notation is criticized as unwieldy by Venn who thought of it only as a notational system. Frege, of course, unlike Venn, Euler, etc., did not have a notational system apart from his diagrammatic system so that the criticism is partly valid but Frege's notation is no more unwieldy than Venn's diagrams.[38] Although more difficult for the beginner to master than most, Frege's system is comprehensive and consistent. It is a great improvement over Hamilton's and de Morgan's systems with regard to simplicity and over Euler's and Lambert's with regard to universality. We will find that network diagrams

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[35] For any A such that a is its negative at least one A must exist. Thus given AB=0 we must conclude that Ab≠0 and aB≠0. This is in direct opposition to Venn's position. See II 2 (c).

[36] III 4 (b)

[37] $\vdash$

[38] The whole question of the relationship between diagrams and notation is taken up by Peirce. See also Gardner's discussion of Marquand in Gardner, op. cit., p. 43 and III 1.

might well be more valuable for recording switching circuits[39] than are Venn's and that when Martin Gardner seeks a diagram which will be useful for teaching elementary logic he devises a network diagram.[40] We should, then, give serious attention to this branch of logic diagrams despite the fact that they are far less common than geometric diagrams.[41]

When logic diagrams are mentioned the Venn diagrams[42] immediately come to mind. They are easy to use and for the Boole-Schroeder algebra, at least, comprehensive. John Venn introduced these in 1880 in an article "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings", and further developed his system and examined other systems of diagrams in Symbolic Logic (1894). The latter work has much invaluable material including a thorough (though occasionally inaccurate) bibliography. Venn examines the diagrams of Euler and Hamilton and discovers in them two systems of logic (the "predication" and the "class inclusion and exclusion" views).[43] He accepts a third system which combines a "compartmental" view and an "existential" view.[44] This is derived from Boole but Venn uses inclusive disjunction prior even to Schroeder's use of it. Venn also examines the nature of the logic diagram and tries to

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[39]II 4 (c) and III 2 (c).

[40]II 4 (e).

[41]Included in network diagrams are the square of opposition, II 4 (a), and the diagrams of Lull, II 4 (b), as well as the more modern systems.

[42]II 2 (c).

[43]II 2 (a).

[44]II 2 (c).

show what is essential to these diagrams. Hamilton's and Frege's diagrammatic systems he examines as types of notation.[45] Frege's he finds awkward and Hamilton's incomprehensible. In Symbolic Logic he even introduced Marquand's diagram which he accepted for most purposes.[46] Of Venn's own system we shall see much. It is geometric, usually circular for three terms with ramifications as more terms are added. It gives one compartment to every possible combination of positive terms and their negatives and works through empty compartments being shaded and occupied compartments being marked in some way. For most purposes where a network diagram is not required we will find that Venn's diagrams, augmented by Marquand's or Carroll's for a large number of terms, are as adequate and practical as any we have.[47] Two further points concerning Venn should be noted. First, he rejected the complicated machines invented by Jevons. The feud between Jevons and Venn was second only to that between de Morgan and Hamilton. More important, although he explicitly introduced the universe of discourse[48] and accepted its importance he did not indicate this universe in any way in his diagrams.[49] Carroll makes much of this in his own system.[50]

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[45] See note 38 above for references concerning the relationship between diagrams and notation.

[46] Venn, op. cit., pp. 139-140, also II 2 (d) and I 5.

[47] E.g. teaching elementary logic, set theory, etc. (see III).

[48] Everything under discussion is designated the "universe of discourse".

[49] The entire page outside the diagram stands for the sub-class $\bar{a}\ \bar{b}\ \bar{c} \ldots \bar{n}$.

[50] II 2 (c) and (f).

Charles Sanders Peirce's diagrammatic systems span the whole period from Venn's first article to the publication of the Principia Mathematica. He began exploring graphs in 1882.[51] His systems are of three types: 1) the first system, 2) entitative graphs and 3) existential graphs.[52] These are all network systems but the lines of the network indicate objects and the variables indicate relations. The first system is little more than a convenient, rather idiosyncratic notation. The entitative graphs allow reasoning of a sort. We may reach positive conclusions but negation is more complicated. The existential graphs are much more subtle allowing diagrams about logic as well as of it. Perhaps the most interesting facet of Peirce's work is the fact that working independently with very different presuppositions Peirce arrived at a system which fits so well into the tradition of linear and network diagrams which includes Lambert and Frege.

5. Its Senescence

Venn's diagrams popularized a subject which had formerly been ignored by most non-logicians. In the three decades following the publication of Venn's article not only philosophers but also physicists and even art historians tried their hands at developing better diagrams for more terms. The logic diagram was stretched, reshaped and twisted until at one time it resembled corrals,[53] at another a patch from a

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[51] Letter to O.H. Mitchell, December 21, 1882. Unpublished, mentioned by Roberts in a lecture.

[52] II 4 (d).

[53] In Allan Marquand, "On a Logical Diagram for n Terms", Philosophical Magazine, XII (1881), 266-270.

quilt.[54] The period was notable more for its thoroughness than for its originality.[55]

Allan Marquand was an art historian who developed an interest in logic machines and diagrams and in 1883 published an article "On Logical Diagrams for n Terms" under the influence of Venn's system. Marquand's diagram is geometric:[56] a square divided and subdivided according to simple rules. This type of representation is suitable for arguments employing a large number of terms. It is important that Marquand closed his universe of discourse but that his various terms after two were broken into non-contiguous parts.[57]

A professor of physics, Alexander Macfarlane, stretched Marquand's grid into one long thin strip of rectangles which he called a "logical spectrum".[58] This is described in two cryptic articles, "The Logical Spectrum" (1885) and "Application of the Logical Spectrum to Boole's Problem" (1890). His method is noteworthy for its exact conjunction with Boolean symbolism.

Perhaps the most fascinating and frustrating figure in the history

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[54]In William Ernest Hocking, "Two Extensions of the Use of Graphs in Elementary Logic", University of California Publications, II (1909), 31 - 44.

[55]With the notable exception of Peirce.

[56]II 2 (d).

[57]Compare this to Hocking on the one extreme whose terms all occupy unbroken topological areas and to Macfarlane on the other who breaks every term from two upward into two or more individual areas.

[58]The term "spectrum" is to be taken analogously as we have a line of distinct squares not blending into one another. II 2 (e).

of logic is the Reverend Charles Lutwidge Dodgson,[59] known to the world as Lewis Carroll. There is something heroic in Carroll's struggle against "the Establishment" and something tragic in his failure. The reason for this failure was not the strength of dying Victorianism but that Carroll, the rebel, carried within himself the very seed of Victorianism. An anti-Romantic he was totally committed to Romanticism.[60] In logic he was anti-tradition but totally submerged in Aristotelianism. We have only two of his books on logic, The Game of Logic (1887) and Symbolic Logic (1896). The first of these uses two diagrams and coloured markers to solve problems in Aristotelian logic. The second is the first volume of a projected three volume survey of all logic. Carroll stated that he had a quantity of manuscript for the second and third volumes; this was apparently thrown out at his death[61] so that it is very difficult for us to make an accurate assessment of Carroll's position. Judging from what we have, he was superficial. Although he had read Venn he did not seem to have comprehended Boole. Like Marquand he closed his universe of discourse but he made a production of this, ignoring Venn's acknowledgement of such closure. In 1906 in "A New Logic Diagram" W.J. Newlin presented yet another geometric system[62] similar to those of Marquand and Carroll.

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[59] Lewis, op. cit., p. 312, lists him as "S.G. Hodgson".

[60] Carroll's failure as a logician is best understood in the context of his failures in other fields.

[61] See the introduction to The Diaries of Lewis Carroll, ed. Roger Lancelyn Green, London: Cassell, I, 1953.

[62] II 2 (g).

W. E. Hocking, in the first half of "Two Extensions of the Use of Graphs in Elementary Logic" (1909) carried this type of graph to its logical conclusion.[63] Every term was represented by a contiguous geometric area bounded by one line. Although this was the ideal toward which the diagram had been moving it was so confusing as to be almost unreadable.[64] The last part of the Hocking article presented a diagram[65] to aid in the immediate inferences of categorical propositions and is unlike anything else with which we will be involved.[66] To save confusion we will examine both of Hocking's diagrammatic systems together despite their different purposes.

6. Its Death (1910)

We can imagine what should have happened to the logic diagram. Further experiments and ramifications would have resulted in the acceptance of some one system, probably Marquand's or Macfarlane's. We would then have continued using this to aid our thought. This is what might have happened but it did not. It did not because in 1910 the logic diagram was murdered[67] by the publication of the Principia Mathematica, written

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[63] II 2 (h).

[64] Hocking realized this and was attempting only to show the theoretical infinite extensibility of diagrams.

[65] Also examined in II 2 (h).

[66] Though similar to Lewis's diagrams for the same purpose, II 2 (i).

[67] It might be questioned whether the logic diagram was really dead. There would seem to be some evidence, particularly in the cases of Roberts' work of expanding Peirce's system to include the functional calculus, and Gardner's network system, that such diagrams are very much

by Bertrand Russell and A. N. Whitehead. Because of the complexity of this work, diagrams, even if there were a way of adapting them to represent this system, would lose their value as illustrations of arguments. We would find it harder to follow a diagram of the developments of the Principia than we do to follow them in their symbolic form. Thus the diagram, except as a tool for the teaching of elementary logic, passed out of the field of logic.[68]

7. Post Mortem (1918 - 1958)

There have been brief revivals of interest in the logic diagram. In 1918 in A Survey of Symbolic Logic, C.I. Lewis discussed geometric diagrams as examples of the application of a logical system and presented his own diagrams for immediate inference.[69]

In 1937 in Qu'est-ce Que La Logique, F. Gonseth developed an interesting though seriously inadequate system of geometric diagrams.[70]

In 1958 Martin Gardner investigated them in Logic Machines and Diagrams. Unfortunately he thoroughly investigates only Venn's system.[71] In some cases he does not give examples, and in others he merely mentions

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alive, and that they had merely suffered a temporary setback. Further, the use of such diagrams in so many fields outside logic (see III) would indicate that they are, at least, as alive today as they were in 1910.

[68]With exceptions to be noted in I 7.

[69]II 2 (i).

[70]II 2 (j).

[71]Gardner is not, of course, claiming to do any more than he actually does nor can he be expected to within the context in which he is working.

systems without describing them.[72] He also develops his own network system.[73]

This historical résumé is necessarily brief. The many logicians who have used the diagrams without changing them have been omitted.[74] We have mentioned only those who have contributed something of lasting interest to the logic diagram or those who have influenced such contributors.[75]

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[72] This is particularly unfair to Hocking and to Peirce. It is also to be noted that Hocking's and Lewis's diagrams for immediate inferences go unmentioned.

[73] II 4 (e).

[74] Special note should be taken of Keynes whose improvements on Lambert will be noted in II 3 (b) and of Copi in whom I first found the logic diagram.

[75] The influences on Peirce, since they come from chemistry, not logic, must be discussed separately in II 4 (d). We might add that Peirce would seem to have developed a system which is adequate even for contemporary logic although his system is the only one that is.

II

THE CLASSIFICATION AND DESCRIPTION OF VARIOUS SYSTEMS OF LOGIC DIAGRAMS

1. The Scheme of Classification

The scheme of classification used in this chapter is not intended to be either the only or the best system. Its sole purpose is to arrange logic diagrams in a convenient form for description, analysis and comparison and someone with other aims might wish to classify the diagrams differently.[1] We will place each system of diagrams in one of four classes: geometric diagrams, linear diagrams, network diagrams, and unclassifiable diagrams.[2]

Geometric diagrams[3] we will define as all logic diagrams which use a closed curve to enclose a topologically distinct area for each term and which are used as an aid in the logical analysis of arguments.

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[1] One might, for example, wish to classify diagrams according to adequacy, use, or some other criterion. Although the adequacy and uses of these diagrams are relevant to this chapter they are not, since we are primarily concerned with description, the best criteria for our classification.

[2] These diagrams are, of course, classifiable and, in fact, are classified but this term is used to indicate that the charts of de Morgan and Hamilton are radically outside the geometric framework within which we are working. We might have used some such class as "other" but this does not sufficiently indicate how radically these charts fall outside our scheme.

[3] II 2.

Linear diagrams[4] will be diagrams which employ lines which do not enclose topologically distinct areas for the representation of terms.

Network diagrams[5] will be diagrams in which the argument is traced out on a topological network.

Unclassifiable diagrams[6] will be all logic diagrams which are not geometric, linear or network diagrams.

2. Geometric Diagrams

(a) Euler

Euler's system[7] is based on Aristotelian logic. In every proposition there are two terms A and B. A affirms or denies a subject; B is an attribute.[8] A general "notion",[9] either subject or attribute, contains an "infinite" number of individual objects.[10] Euler seems to mean "undefined" rather than "infinite". Otherwise he could not define

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[4] II 3.

[5] II 4.

[6] II 5.

[7] Euler seems to think of his system of logic as a science. It is introduced within the context of his psychology.

[8] Leonard Euler, Letters to a German Princess, trans. Henry Hunter, London: H. Murray, 1795, I, 452.

[9] This term is introduced in Letter C, p. 440. Euler defines it on p. 442 as "an idea formed by abstraction".

[10] Euler, op. cit., p. 453.

individual propositions as universal,[11] since an individual cannot contain an "infinite" number of individual objects. We may then consider the general notion as a space (i.e. two dimensional surface with only one boundary) in which all of these objects are contained. The purpose of such a diagrammatic method is to facilitate a more distinct comprehension.[12]

Euler has seven basic diagrams[13] (Diagram II).[14] The first (II a)[15] represents A or the subject of the proposition; the second (II b)[16]

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[11] Euler, op. cit., p. 480. "The same rules which take place in universal propositions apply, likewise, to singular propositions."

[12] Euler, op. cit., p. 454. "These circles, or rather these spaces, for it is of no importance what figure they are of, are extremely commodious for facilitating our reflections on this subject, and for unfolding all the boasted mysteries of logic, which that art finds it so difficult to explain; whereas, by means of these signs, the whole is rendered sensible to the eye."

[13] Hunter appends a note to the effect that Euler originally presented four basic diagrams which were eliminated from the Paris edition. These are repetitions of II c), d) (without bracket), e) and f) and would seem to be superfluous. It is worth noting that these four diagrams in Hunter's note (Euler, op. cit., p. 455) are drawn, for no apparent reason, with dotted rather than solid boundaries.

[14] Every effort has been made to retain the original peculiarities of drafting in all the diagrams in this work. Where we have been forced to depart from the original, note will be made of the fact. Two exceptions to this statement are size and position. Unless the size of a diagram is relevant to its logical import it has been made whatever size is most convenient; unless a diagram's position on the page is of logical significance it has been positioned wherever is most convenient. Hereafter, the word "diagram" will be omitted and diagrams will be referred to by number and letter. For example, (IIa) will refer to diagram II, subsection a).

[15] Euler, op. cit., Plate I, Second Series, facing page 460, Fig. 1. Other diagrams from this page will hereafter by referred to as Euler, I, Fig. n, where n is the number of the figure.

[16] Euler, I, Fig. 2.

represents B or the attribute of the subject. The universal affirmative proposition is represented by two circles (II c),[17] that representing B enclosing within its boundary that representing A ("All A is B"). The universal negative is represented by two mutually exclusive circles joined by a bracket (II d)[18] ("No A is B"). The particular affirmative is represented by two intersecting circles (II e)[19] with the letter A[20] in the common portion and B in the non-A part of circle B.[21] ("Some A is B"; "Some B is A"; "Some B is not A"; "Some A is not B")[22]. The particular negative is represented by similar circles (II f)[23] with the letter A moved to the non-B portion of circle A ("Some A is not B"). There is obviously some confusion here with two diagrams representing the same type of proposition. We may improve Euler's position[24] by positing that only that portion of the diagram for a particular proposition containing the letter A is claimed to have members. This would make II e represent

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[17] Euler, I, Fig. 3.

[18] Euler, I, Fig. 4.

[19] Euler, I, Fig. 5.

[20] Euler does not point out the position of the letter A thus leaving the door open for the confusion which arises between II e and II f.

[21] This seems to indicate that there is some B apart from the A portion if we are indicating the particular proposition by the letter A. See the following discussion.

[22] If we ignore letter placement as an indication of the existence of objects in a compartment we might read it any of these ways.

[23] Euler, I, Fig. 6.

[24] We are here making explicit what Euler was actually doing but he never states that he is using letter placement in this way.

"Some A is B" and "Some B is A". The seventh diagram (II g)[25] is a special case of II e in which there are no A which are not also B ("All A are B"; "Some B is A"; "Some B is not A"). Our interpretation would reduce the meanings of this diagram to "All A are B" and "Some B are A".

Notice should be taken of Euler's use, even in his basic diagrams, of parenthesis to link circles which are terms of the same proposition but are not otherwise linked in the diagram (II d).

Euler then goes on to attempt to give diagrams, by means of various combinations of his basic diagrams, for all possible combinations of two propositions to form syllogisms. In the first proposition in any of these syllogisms A is always the subject and B the predicate. In the second C always appears. The conclusion relates the term from the first proposition which does not appear in the second to C. An example will illustrate this:

Every A is B (II c)
No C is B or No B is C (II d)
∴ No C is A (III a)[26]

Sometimes it takes several diagrams viewed together to arrive at a conclusion. This happens when the diagrams for the two premises may be combined in more than one way:

No A is B (II d)
Some C is B or Some B is C (II e)
∴ Some C is not A (III b c d)[27]

We may also show with these diagrams if no conclusion follows from the

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[25] Euler, I, Fig. 7.

[26] Euler, I, Fig. 11.

[27] Euler, I, Fig. 21-23.

given premises:

All A is B (II c)
No C is A (II d)
∴ No conclusion[28] (III e.f.g)[29]

A slight discrepancy, which does not affect the result, may be noticed in the above situation. No diagram is given for the case in which C is totally excluded from B. One case is not examined. We will examine more serious difficulties in the examination of all cases along with discrepancies in the placing of letters later in this section.

Before we turn to the other logicians' assessment of Euler we must look briefly at his use of the asterisk.[30] He does not introduce this until he begins to apply his diagrams to actual arguments. A sample syllogism will illustrate the use of the asterisk.

No A is B (II.d)
Some B (the* portion) is C (IV.a)[31]
∴ No conclusion[32] (IV b.c d)[33]

We should note that IV.c meant to represent the situation in which all

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[28] Euler uses "no conclusion" to mean no valid conclusion. Euler, op. cit., pp. 458 ff.

[29] Euler, I, Fig. 12-14.

[30] Euler does not discuss or explain the asterisk. He simply introduces it (p. 446) in a problem and uses it. He only uses it in cases of actual arguments using words. We have reduced these arguments to syllogisms with variables by replacing the words with symbols.

[31] Euler, op. cit., Plate II, Second Series, facing page 468 (actually facing page 465 although 468 is printed at the top of Plate II - perhaps an error in binding), Fig. 15. Other diagrams from this page will hereafter be referred to as Euler, II, Fig. n, where n is the number of the figure.

[32] Euler, II, Fig. 16-18.

[33] Actually we may reach the conclusion "Some C is not B" but Euler does not mention this.

C which is not B is included in A not only fails to do this but also represents an impossible state of affairs according to the premises. The asterisk method may be used to show that a syllogism is invalid:

Some A (the * portion) is B (IV a)
No B is C (II d)
∴ Some C are not A (IV e f g)[34]

This is obviously invalid in IV f.

There can be no question about the impact of Euler's system of diagrams on logic. Venn found that the majority of logicians in the early nineteenth century used some type of diagrams to illustrate reasonings;[35] most of these simply used Euler's diagrams, as they were, to apply to Aristotelian logic.

It was not until the Lectures of Sir William Hamilton that Euler's system found a "broader"[36] application. Hamilton interpreted Euler as having four basic diagrams representing the four types of propositions in Aristotelian logic. Modifications in the drafting[37] may be noticed in the A proposition ("All A is B") in which the inner circle is not concentric to the outer (V a).[38] This has no logical implications. More important

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[34]Euler, II, Fig. 20-22.

[35]Venn, op. cit., p. 110 footnote.

[36]Broader for Hamilton in the sense that he believed that his system was broader. The claim that it is actually broader would seem to be dubious.

[37]Non-essential modifications will be pointed out only in the case of Euler. The reader will then be left to discover these for himself in other logicians, but it is necessary to present some so that the reader may know and form an opinion about those aspects of the diagram which this writer feels to be unimportant.

[38]Hamilton, op. cit., diagram V a-d, found at p. 180.

is the fact that Hamilton ignores Euler's letter placement and seems to substitute the placement of the circles themselves to indicate which proposition a particular diagram indicates.[39] The I proposition ("Some A is B") is illustrated by two horizontally linked circles (V c) while the O ("Some A is not B") is illustrated by two vertically linked circles (V d). Hamilton makes no mention of the asterisk. It would seem that Euler's letter placement or his asterisks is a simpler method of distinguishing the I and O propositions than Hamilton's differences in linkage. Only the E proposition ("No A is B") remains the same in Hamilton's interpretation of Euler's system as in the original system (V b).[40]

When we look at Hamilton's application of Euler's system we find that he did not, in fact, consistently maintain the two types of linking as a method of distinguishing the I and O propositions. The I proposition is represented four times by horizontal linking (VI a e g h), three times by vertical linking (VI b d n), and six times by diagonal linking (VI c i k l m o). The O proposition is represented by two different types of linking: once by horizontal linking (VI f) and once by diagonal linking (VI j). It is never, except in the introductory diagrams, represented by vertical linking. An examination of the diagrams used to solve syllogisms containing particular propositions (VI, e.g. g and j)[41] shows

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[39]"Seems" because this is never made explicit in Hamilton.

[40]Except that the bracket is not used by Hamilton but neither is it consistently used by Euler.

[41]Diagrams in VI are found in Hamilton, op. cit., pp. 290-301. We have repeated all the relevant diagrams although some redundancy is involved.

that Hamilton did not diagrammatically distinguish between the I and O propositions which leads to the conclusion that his interpretation of Euler is inadequate.

Hamilton expanded Euler's diagrams to include sorites.[42] Hamilton recognized an amazing variety of structure within sorites, compared to his predecessors, but all this could be reduced to three diagrammatic representations. First there is the affirmative sorites in which the concepts are coëxtensive (VII a):

A is B
B is C
C is D
D is E
∴ A is E

Secondly there is the affirmative progressive or regressive sorites (VII b):

All E is D
All D is C
All C is B
All B is A
∴ All E is A

and

All B is A
All C is B
All D is C
All E is D
∴ All E is A

And finally there is the negative sorites (VII c):

A is B
B is C
C is D
D is E
No A is P
∴ No E is P

Hamilton believed that he had extended Euler's diagrams to cover

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[42] Hamilton, op. cit., p. 261.

more relations of concepts. Hamilton listed five such possible relations.[43] Exclusion was illustrated exactly like an E proposition (VIII a). Coëxtension used one circle with one side slightly thickened and two letters placed in it (VIII b) to indicate that one concept was the same as the other. Hamilton's illustration of subordination is exactly like Euler's diagram for the argument <u>Barbara</u> <u>sans</u> letters and is self-explanatory (VIII c). The fourth of these diagrams of the relations of concepts is the most troubling. It has two circles joined by a curved line. One circle is divided in half and the other has three independent circles within it (VIII d). Hamilton defines coördination, which this illustrates, as follows:

> Two or more concepts are coördinated, when each excludes the other from its sphere, but when both go immediately to make up the extension of a third concept, to which both are cosubordinate.
> (<u>Lectures</u>, p. 134)

From this it is obvious that the curved line joining the large circles is meant to show that they are two distinct diagrams meant to illustrate the same type of relation. The first diagram fits the definition well but the second does not fit it at all. The three small circles do not go to make up the large one; Hamilton ought to have cut the large circle, like a pie, into three wedges. The final diagram (VIII e), when the circles are reduced to two which intersect, is simply Euler's diagram for the I proposition and this seems to be what Hamilton intends partial coïnclusion and partial coëxclusion to be.

Hamilton extended Euler's diagrams, as we have seen, in many ways

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[43] Hamilton, <u>op. cit</u>., p. 133.

but the most important, Hamilton felt, was the extension to cover the quantification of the predicate. He believed that traditional Aristotelian logic was too narrow to be of much use but that if the predicates of propositions were modified by "all" and "some" we would be able to construct many more syllogisms. This modification gives us eight basic propositions which may be illustrated by means of four diagrams.[44] The first proposition ("All C is all Γ") is really coëxtension (IX a). The choice of letters here was dictated by their position in their respective alphabets. The next two ("All C is some A" and "Some A is all C") may be illustrated by a diagram similar to Euler's for the A proposition (IX b). Hamilton should have added, for completeness, that "Some A is not all C" and "Some A is not some C" but he wanted to reserve representation of these propositions for the fourth diagram.[45] The diagram for the fourth proposition ("Any C is not any D") (IX c) might also be said to include "Any C is not some D"; "Some C is not any D" and "Some D is not any C", but again Hamilton reserves these propositions for the final diagram. The final diagram (IX d) supposedly represents four propositions ("Some C is some B"; "Any C is not some B"; "Some B is not any C" and "Some C is not some B"). That one diagram can represent so much is confusing; that individual propositions can, contrary to Hamilton's belief, be represented in so many ways is even more confusing. We will reserve further criticism of

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[44]Hamilton, op. cit., p. 529.

[45]Venn points out these and the following weaknesses. Venn, op. cit., p. 11. They are similar to those of which Carroll accuses Euler. Lewis Carroll, Symbolic Logic, New York: Dover Publications, 1958, pp. 173-174. See particularly II 5 (a).

the doctrine of the quantification of the predicate until such time as we have Hamilton's total position before us.[46]

Hamilton stretched Euler's system to include sorites, the relation of concepts and the quantification of the predicate. Most other philosophers adhered fairly closely to Euler's original diagrams. Jevons felt that Euler's diagrams were about as thorough as any could be. He did find it necessary to improve the representation of the I and O propositions. Jevons noticed and accepted Euler's letter placement. His criticism is well founded. Suppose "Some A is not B" (X a). It may or may not be the case that "No A is B" but the diagram prejudices this. Jevons uses a dotted line[47] to indicate the possibility that "No A is B" (X b). Thus a compartment bounded by a dotted line and having no letter in it may or may not exist. A similar problem arises with the proposition "Some A is B" (X c). Removing that portion of A which is excluded from B (X d) or bounding it with a dotted line (X e) will prevent us from overlooking the possibility that "All A is B".[48] The dotted line would seem to be a better method than the erasure of compartments as such erasure (e.g. X d) prejudices the case in exactly the opposite way making it difficult to

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[46]II 5 (a) and III 4 (a).

[47]William Thomson, Laws of Thought, London: Longmans, Green, 1869, p. 190 also makes use of dotted boundaries but he does not seem to realize that he has changed Euler's system. See p. 189 particularly. The first conscious and consistent use of such boundaries may be credited to Jevons.

[48]W. Stanley Jevons, Studies in Deductive Logic, London: Macmillan, 1880. For a criticism of Jevons' dotted boundaries see Venn, op. cit., p. 13. His criticism seems to presuppose that one is going to shade the diagrams to indicate that their compartments are empty but I am not entirely sure what Venn is saying. Note that (X d) also prejudices the case.

realize that there may be some A which is not B. Even the dotted line is difficult to work with and was superseded by Venn's method of shading compartments to show that they are empty and marking them in some other manner to show that they have contents.

Venn suggests that there are four possible forms of logical propositions[49] which are not always compatible.[50] At this point we must examine the first two. The first is the predication[51] view. It is essential to this view that subject and predicate be distinguished in any proposition. The predication view asserts that a subject possesses or does not possess a certain attribute. The predicate is not quantified except in convertible propositions. The predication view yields four possible propositions:

Universal affirmative:	"All A is B"
Universal negative:	"No A is B"
Particular affirmative:	"Some A is B"
Particular negative:	"Some A is not B"

There are no diagrams adequate to represent the predication or Aristotelian view.[52]

The class inclusion and exclusion view[53] may be represented by

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[49]See II 2 (c) for the two other views, the compartmental and existential views.

[50]See Venn, op. cit., chapter I.

[51]i.e. Aristotelian.

[52]If Venn is correct in this it would seem that translation from Aristotelian logic to the propositional calculus would be impossible as diagrams can be drawn for the propositional calculus (e.g. Gardner). Since such translation is possible it would seem that Venn is, to some degree, in error.

[53]i.e. Hamiltonian.

five diagrams[54] which are a modification of Euler's. The first of these diagrams (XI a) illustrates the case in which one class is totally included in the other and also wholly includes it ("All A is all B"). The second (XI b) and third (XI c) illustrate the cases in which one class is totally included within but does not include the other class completely ("All A is some B" (XI b), and "Some A is all B" (XI c)). The next diagram (XI d) illustrates the case in which a portion of each class is included within the other (i.e. the classes have a common portion) ("Some A is some B"). The final diagram illustrates the case in which the classes are mutually exclusive (XI e) ("Any A is not any B"). In all these propositions "some" signifies "some not all". It is doubtful whether Hamilton would have admitted that some means some not all since he permits individual indefinite propositions in which some <u>must</u> be equivalent to all (e.g. An Englishman generalized the law of gravitation). In the class inclusion and exclusion view subject and predicate are accidental; the terms may be taken in either order.

Venn then presents his interpretation of Hamilton's eight propositions:

"All A is all B"
"All A is some B"
"Some A is all B"
"Some A is some B"
"Any A is not any B"
"Any A is not some B"
"Some A is not any B"
"Some A is not some B"

The first five of these are the propositions which the above restatement

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[54] Venn, <u>op. cit.</u>, p. 7 and p. 31.

of Euler's logic diagrams illustrate. The last three are equivalent to one or more of the first five and on this ground are rejected by Venn.[55]

Finally, before he proceeds to the other two views, Venn compares the merits of the predication and class inclusion and exclusion views of logic. The former is more capable than the latter of expressing common language but the second has the advantage of being diagrammatically illustrable.[56]

Although negative terms had been introduced to logic through Boolean algebra long before this time, and although Venn and Jevons used such terms, they made no attempt to apply Euler's diagrams to them.[57] This was done by Lewis Carroll[58] with incredible results. Carroll seemed to believe that the diagrams that he was using were Euler's original basic diagrams. They have, in fact, been radically interpreted. The diagram in which y is totally included in x is found in Venn but is in neither Euler nor Hamilton's interpretation of him. The diagram illustrating the O proposition is dropped on the grounds, apparently, that it is the same as that for the I proposition. No account is taken of the placement of letters or of the use of the asterisk. With the introduction of negative terms Carroll's interpretation of the four diagrams is as follows:

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[55] Venn, op. cit., pp. 9 ff. See also II 5 (a).

[56] Venn, op. cit., pp. 16 ff.

[57] Unless, of course, one thinks of the Venn diagrams, as Peirce does, merely as an expansion of Euler's.

[58] Carroll, op. cit., pp. 173-174.

(XII a) "All x is y"; "No x is not-y"; "Some x are y"; "Some y are not-x"; "Some not-y are not-x"; "No not-y are x"; "Some y are x"; "Some not-x are y" and "Some not-x are not-y"

(XII b) "All y are x"; "No y are not-x"; "Some y are x"; "Some x are not-y"; "Some not-x are not-y"; "No not-x are y"; "Some x are y"; "Some not-y are x" and "Some not-y are not-x"

(XII c) "All x are not-y"; "All y are not-x"; "No x are y"; "Some x are not-y"; "Some y are not-x"; "Some not-x are not-y"; "No y are x"; "Some not-y are x"; "Some not-x are y"; "Some not-y are not-x"

(XII d) "Some x are y"; "Some x are not-y"; "Some not-x are y"; "Some not-x are not-y"; "Some y are x"; "Some not-y are x"; "Some y are not-x" and "Some not-y are not-x"

This system, according to Carroll, works out very well for universal propositions but for any particular proposition at least three diagrams are required to cover all cases. Even worse, "Some not-x are not-y" is invariably true. "Apparently," says Carroll, "it never occurred to him [Euler] that it might sometimes fail to be true!"[59] As a matter of fact, if Carroll had examined Euler's original work he would have realized that, since negative terms were not in use in Euler's time, his criticism is pointless. Euler's diagrams were designed expressly for Aristotelian logic which used no negative terms and if we eliminate the propositions containing negative terms from Carroll's analysis of the diagrams we find ourselves back with a set of propositions basically the same as Euler's.

Carroll further illustrates Euler's diagrams by applying them to a syllogism. Since the syllogism contains negative terms we may safely

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[59]Carroll, op. cit., p. 174.

ignore it.[60] One can charge Carroll, like Hamilton, with careless scholarship.[61]

Peirce uses the term "Euler diagrams" to apply to all geometric logic diagrams and does not feel that Euler's diagrams are different in kind from Venn's as Venn claims.[62] This makes it rather difficult to separate what he says about Venn's and Euler's diagrams. The two principles[63] which he discovers apply not to Euler but to Venn.

Peirce, as an introduction to his existential graphs, gives a list of all possible Euler diagrams with their meanings.[64] It will be

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[60]Carroll, op. cit., 180-182. The argument requires eighteen diagrams. It is:

No x are m
Some m are y'
∴ Some y' are x'

Carroll says (p. 182) ". . . he [Euler] seems to have assumed that a Proposition of this form [Some y' are x'] is always true." Since there is no mention of negative terms in Euler and since only the areas inside the circles are relevant in Euler, the whole example becomes, at best, irrelevant. Euler could not have considered either the second premise or the conclusion in his system, and Carroll's extension of the system to include these is clumsy, especially since he does not seem to feel that he has gone beyond Euler. x' is the negation of x.

[61]This charge in regard to Euler and Venn (See II 2 (c)) does not detract from Carroll's significance in other ways. His own diagrammatic system (II 2 (f)) is particularly valuable.

[62]Collected Papers of Charles Sanders Peirce, ed. Charles Hartshorne and Paul Weiss, Cambridge: Harvard University Press, IV, 350 ff. The final number in a reference to Peirce refers to a paragraph rather than to a page. Hereafter such references will be made according to the convention adopted by Peirce scholars (e.g. Roberts, op. cit., p. 6) as Peirce 4.350. The number before the decimal refers to the volume number and that following, to the paragraph number. The number above refers to Volume IV of Peirce's collected papers, and to paragraph 350 in that volume.

[63]Peirce 4.351.

[64]We have reserved Peirce's treatment of particular propositions until II 2 (c) as they really are treated within the framework of Venn's rather than Euler's system.

seen that this list[65] goes beyond Aristotelian logic:

(XIII a) Entire ignorance
(XIII b) Any P is S
(XIII c) No S is P
(XIII d) There is no P
(XIII e) Any S is P
(XIII f) S and P are identical
(XIII g) There is no S
(XIII h) There is neither S nor P

Note that Peirce does not use letter placement to indicate class membership. Similar possibilities which cannot be represented[66] include:

Everything is either S or P
Everything is P
No S is P but everything but S is P
Everything is S and nothing is P
Everything is S
Everything is both S and P
Nothing is S but everything is P
The universe is absurd and impossible

The failure of Euler's diagrams to represent such propositions is their major limitation. Other weaknesses include the inability of the system to represent existence, to present alternative states, to express quantitative notions and to exhibit relational reasonings.[67] Both Venn's and Peirce's systems are attempts to overcome these limitations. It is obvious that Peirce is not simply criticising Euler's diagrams as historically developed but that he has attempted to push them to the limit of their representational ability for universal propositions. If we accept this, we must point out that he might have overcome many of these weaknesses

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[65] Peirce 4.356.

[66] Peirce is able to represent these by means of his existential graphs (II 4 (d)). This list is also given at 4.356.

[67] Peirce 4.356.

by proper letter placement or asterisks. To Peirce Euler's diagrams were really nothing more than a prelude to his "chef d'oeuvre"[68] the existential graphs.[69]

C. I. Lewis presents Euler's four diagrams[70] for the A (XIV a), the E (XIV b), the I (XIV c), and the O (XIV d) propositions much as Euler himself had except for the placing of the small circle in the first diagram and the use and placement of lower case letters.[71] Instead of letter placement Lewis uses the asterisk in particular propositions.

Lewis points out that the representation in the diagrams goes beyond the relation of classes indicated by the proposition. For example, from the illustration of the A proposition (XIV a) we would draw the invalid conclusion that "some B are not A". This and other similar ambiguities result from the fact that there is no way of rendering any compartment empty and the ensuing general assumption that no compartment is null.[72] Venn's system is an attempt to rectify this.

Gardner mentions Euler's system[73] but dismisses it without examination to turn to Venn's more efficient method. We should realize before dismissing Euler so quickly that his method was developed expressly for

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[68]Peirce, title page between 4.346 and 4.347.

[69]Note the position of the section on Euler diagrams immediately prior to his exposition of his own existential graphs.

[70]Lewis, op. cit., p. 176, Fig. 1.

[71]These points are not logically relevant.

[72]Lewis, op. cit., p. 176.

[73]Gardner, op. cit., p. 31.

Aristotelian logic while Venn's was developed to apply to Boolean algebra, which includes a much wider variety of relations and allows negative terms, as well.[74] Even if we find that Euler's diagrams are insufficient for Aristotelian logic[75] they retain their historical significance, and their influence on later logicians should not be underestimated.

We should now be able to pass judgement on Euler's diagrams. Reference has already been made to their weaknesses when we try to go beyond Aristotelian logic so we will concentrate on those weaknesses which make it difficult to apply the diagrams even as they were intended.

Euler was generally very careful about the placement of his letters but he believed that the I proposition was convertible. For example he introduced the following valid syllogisms:[76]

No A is B
Some C is A or Some A is C
∴ Some C is not B

and

No A is B
Some C is B or Some B is C
∴ Some C is not B

But Euler's placement of letters does not allow "Some A is C" (XV a) to equal "Some C is A" (XV b) diagrammatically. Since the conversion of the I proposition can be established by any of several methods we would suspect that the placement of letters is incapable of dealing with I propositions.[77]

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[74] See II 2 (c).

[75] A discussion of this follows.

[76] Euler, op. cit., Letter CIII.

[77] Euler might have overcome this difficulty by the use of the asterisk.

Finally we must ask whether the diagrams work for Aristotelian syllogisms. The critical case would be the situation in which two particular propositions are premises. Any conclusion based on these will be invalid. Can this be shown diagrammatically? Suppose that "Some A is B" and "Some B is not C". When we diagram this we arrive at seven distinct diagrams (XV c d e f g h i). From this confusing collection it is hard to see whether there is a conclusion or not. Thus even in Aristotelian logic, Euler's diagrams fail to fulfill their purpose.[78] Acknowledging Euler's importance as discoverer of the modern logic diagram and his influence on the history of logic, we must, when we wish a diagram for practical purposes, turn to some other system. Such a comment as this would seem to be unnecessary in the face of all that other logicians have raised against Euler's system yet his diagrams continue to be used in elementary logic texts despite the advent of easier and more adequate systems of diagrams. Such continued use can only be a puzzle for those who study the history of the logic diagram.

(b) Maass

In Maass' system triangles are substituted for Euler's circles. The perimeter of a triangle with the letter "a" in one angle represents the boundary of the concept a[79] (XVI a). The area of the triangle thus represents the extent of the concept. If two or more terms are placed

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[78]See Venn's criticism on other grounds. Venn, op. cit., pp. 16ff.

[79]Maass, op. cit., facing page 290, Fig. I. Hereafter references to the diagrams in Maass facing page 290 will be abbreviated to Maass, Fig. n, where n is the number of the figure in Maass. Thus the above would be written, Maass, Fig. I.

in two or more angles of the triangle the equivalence of the concepts is represented (XVI b).[80] Maass introduces the dotted line to represent possibility. Thus XVI c[81] says that there is a concept a which includes everything bounded by the triangle with side k but may also include everything bounded by the triangle with side μ. This allows the possibility that a concept includes more than is stated in a proposition about that concept. A diagram (XVI d)[82] may be drawn in the same manner but which allows the narrowing of the concept rather than its broadening. A final diagram (XVI e) illustrates the point that everything outside the triangle is a negative of the concept represented by the triangle. With these basic diagrams in mind we may now draw diagrams for the four basic Aristotelian propositions and for a fifth proposition introduced by Maass. "All a is b" (XVII a)[83] shows that a is included within b but that b may or may not be broader than a. This would seem to be an improvement over Euler as in it Maass consciously attempts to avoid prejudicing the case as to whether the compartment which is b but not a has contents. To represent "No a is b" we put a and b in the acute angles of a rhombus and join the oblique angles with a straight line (XVII b).[84] This gives

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[80]Maass, Fig. IV.

[81]Maass, Fig. II.

[82]Maass, Fig. III.

[83]Maass, Fig. XII. We have made the lines of uniform thickness and eliminated two letters which serve a purpose in Maass' description but tend to lead to confusion in this context. In all of the following diagrams we will make such changes when there is danger that the essential point of the diagram will be lost if such changes are not made.

[84]Maass, Fig. VII.

us two triangles falling entirely outside each other, one representing a, the other b. The representation of "Some a is b" is simply two triangles overlapping to form a common triangle with the sides falling outside the common triangle represented by dotted lines and the angles opposite the dotted lines labeled a and b (XVII c).[85] The particular negative proposition is represented as a universal negative with dotted lines to indicate the possibility that the predicate may be partially or completely included within the subject taken in total (XVII d).[86] Maass also allows the relation of subsumption in which one concept is actually broader than another. A diagram for "a is subsumed under b" is the same as that for an A proposition except that the dotted line is drawn as solid (XVII e).[87] Maass generally uses his diagrams in two ways. The first is to show the relationship of two concepts given their relationships to another concept. For example we are given that a is subsumed under c, and b is subsumed under c. We may see from the diagram (XVIII a)[88] that there are three possible relationships between a and b: "a is equivalent to b"; "a and b are mutually exclusive" and "All a is b but all b is not a or vice versa". Maass' diagrams are more adequate than Euler's for such situations but they are still very complicated to read compared to Venn's. The second use of Maass' diagrams, which is really a subclass of the first, is as a method of illustration of the laws of logic. We might, for example, wish

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[85] Maass, Fig. XIII.

[86] Maass, Fig. XIV.

[87] Maass, Fig. V.

[88] Maass, Fig. X.

to show that two universal negative propositions cannot give any conclusion. Maass points out that if we have two universal negative premises we may draw either of two (at least) diagrams (XVIII b and c).[89] In the first, all a is c and in the second, no a is c. Thus no conclusion may be drawn as to the relationship of a and c.

Hamilton[90] believed that Maass' system was angular (i.e. that the angles represented the scope of the concepts) and rejected it outright as impossible. He also criticized Maass for not making his lines uniform and for using letters from more than one alphabet. These criticisms arise because when Hamilton used these devices[91] they had logical significance but when Maass used them, with the exception of the dotted line, they had only the psychological significance of making the diagrams more easily read. But Hamilton thought that these various lines and letters were employed for some logical reason and found himself unable to read them.

Venn clarified the nature of Maass' diagrams by pointing out that one could change the area of a concept by moving the line opposite the angle marked for that concept.[92] This is another way of describing the use of the dotted line although Venn does not mention that particular device. Venn does not, however, describe Maass' system with any degree of thoroughness.

Although Maass' diagrams are an interesting variation they are

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[89] Maass, Fig. XVII and XVIII.

[90] Hamilton, op. cit., pp. 669-670.

[91] E.g. his use of the comma, colon and lines of varying thickness.

[92] Venn, op. cit., pp. 515-516.

not of great significance as they do not have the intuitive clarity of Euler's and they are much less adequate than Venn's.

(c) Venn

John Venn undoubtedly would have been shocked to find an exposition of his system prefaced by remarks on his arch-rival, Jevons;[93] yet such remarks are necessary. The method that Jevons applied to Boolean algebra was to become the basis of the Venn diagrams. Let us take a simple problem. We are given the following syllogism[94] and asked to verify it:

A=Ab (1)
C=aC (2)
∴ C=BC

We write down all possible combinations of A, B, C, a, b, c[95] and strike out those which the premises make impossible (XIX a). "ABC" and "ABc" are eliminated by premise 1, "A=Ab". "ABC" and "AbC" are eliminated by premise 2, "C=aC". This leaves two combinations containing C, "aBC" and "abC". Thus "C=BC+bC"[96] and the conclusion is false.

Each of the positive terms, according to Jevons, must have members or exist within our universe of discourse.[97] Thus the elimination of one positive term means that we have a contradiction among the premises.

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[93]See Gardner, op. cit., pp. 104 ff. concerning this rivalry.

[94]Jevons, op. cit., p. 198.

[95]"a" represents the negation of A.

[96]"+" represents "either . . . or . . .".

[97]This is not actually stated in words by Jevons but since he practises it in his diagrams we may deduce that this is his position.

A=AC (1)
B=Bc (2)
B=AB (3)

When we diagram these premises[98] (XIX b) we see that B has disappeared. Therefore the premises are contradictory. We may also show how a conclusion may be drawn as to existence within this system.

A=AB (1)
B=BC (2)

From the chart[99] (XIX c) it will be seen that if "A" exists "ABC" must also exist. But for Jevons every term must have existential import within our universe of discourse. Therefore, given these premises "ABC" is a valid conclusion.

Jevons' charts are not really diagrams[100] but the method employed is exactly the same as that in Venn's diagrams. Both attempt to represent every possible "subdivision" of the classes or "compartments"[101] involved. For example, there are four possible subdivisions of the classes x and y: xy, x$\bar{y}$, $\bar{x}$y, $\overline{xy}$.[102] The general statement of the number of subdivisions is "'n' terms gives 2^n subdivisions." Thus for a diagram of an argument

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[98] Jevons, op. cit., p. 217.

[99] Jevons, op. cit., p. 216.

[100] This statement is true only if one makes the distinction between calculus and diagrams. Peirce points out that such a distinction is, at best, artificial and that all language is, in a sense, diagrammatic. See III 1. Gardner, however, makes this distinction and it would seem to be convenient if not convincing.

[101] Venn writes of the "compartmental" account of the import of propositions.

[102] "a" is a positive term; "$\bar{a}$" is the negation of "a"; "ab" represents "the conjunction of a and b."

involving 'n' terms there must be 2^n compartments. For each term Venn constructs a closed geometric figure which intersects each of the compartments already produced doubling their number.[103] Venn uses circles for two and three terms giving the familiar Venn diagrams[104] (XX a and b). For four terms he must abandon circles and he turns to elipses[105] (XX c). For five terms Venn is forced to abandon his geometrical plan and resort to a doughnut-shaped figure[106] (XX d). He feels that for more than five terms diagrams are of little value[107] but proposes that for six terms we might use two five term diagrams, one for the positive and one for the negative aspect of the sixth term[108] (XX e). These five diagrams are basic and what follows is a commentary and analysis of these.

Venn makes a general statement of the method of drawing diagrams for "n"[109] terms without resort to figures of more than one topological

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[103]One of the 2^n subdivisions lies outside all the circles in each of the following figures.

[104]Venn, op. cit., p. 114 and 115 respectively.

[105]Venn, op.cit., p. 116.

[106]Venn, op. cit., p. 117. The hole in the doughnut is required to bifurcate the compartment which is both y and w. Otherwise Venn would have to use a horse-shoe shaped figure which would complicate the diagram unnecessarily.

[107]Venn, op. cit., p. 117.

[108]This diagram is not drawn by Venn but it is described. Venn, op. cit., p. 117, footnote.

[109]Henceforth the symbol "n" or "n" will be used to represent any positive integer whatsoever.

class.[110] If we have used circles to create a three term diagram we may divide the compartments in such a fashion as to give a two-pronged curvilinear figure (XXI a). Dividing again in a similar manner gives us a horseshoe-shaped figure (XXI b). This may be continued ad infinitum[111] with the addition of ever more complex figures of this sort, the problem being, of course, that these figures will not be as clear as the ones Venn uses.[112]

Two general "deductions"[113] concerning the diagrams may be drawn:

> Any two compartments which are adjacent differ by the affirmation or denial of one symbol. When added we drop the symbol.

That is to say, $(abc)+(ab\bar{c})=(ab)$.[114]

> Any two compartments with two boundaries between must differ in two such terms. The adding of four such compartments allows the dropping of four terms.

That is $(abcd)+(abc\bar{d})+(ab\bar{c}d)+(ab\bar{c}\bar{d})=(ab)$. In both these deductions the crossing of the same line twice is equivalent to not crossing it at all.[115]

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[110] Venn does not give the diagrams in Symbolic Logic but he works out the proof. Venn, op. cit., p. 118, footnote. The diagrams given here are taken from an article by Venn "On the Diagrammatic and Mechanical Representation of Propositions and Reasonings", Philosophical Magazine, Series 5, X, July 1880. Further references to Venn will continue to refer to Symbolic Logic despite the introduction of this article.

[111] The $(4+x)^{th}$ figure introduced into such a diagram has 2^x prongs. Venn, p. 119, footnote.

[112] The lack of clarity is attributable to the lack of regularity in the diagrams.

[113] Venn, p. 119. He is not using the word in a rigorous sense but seems to mean deductions based solely on the diagrams.

[114] "a+b" represents "the alternation of a and b." The Boolean formula, by itself, can say nothing about adjacent areas.

[115] Venn, p. 119.

Venn's use of the inclusive "+" should be noted. In Boole $(a+b=1)=(\bar{a}\bar{b}+ab=0)$ but in Venn $(a+b=1)=(\bar{a}\bar{b}=0)$, that is $(a+b)=(ab+a\bar{b}+\bar{a}b)$.[116]

To diagram any universal proposition "p" it is only necessary to reduce it to the form $f(n)=0$[117] (where "n" designates a combination of classes derivable from "p"), and shade out all those compartments indicated by f(n) to show that they are empty. "All X is Y" is translated into Boolean algebra as "$x\bar{y}=0$." Thus we diagram the universal affirmative by shading out that compartment containing both x and $\bar{y}$ (XXII a).[118] Other examples of the diagramming of propositions in various systems are given by Venn:

$x=y$	(XXII b)[119]
$x=y+z$	(XXII c)[120]
$x(y+z)=1$	(XXII d)[121]

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[116] A third possibility was introduced by Jevons. Venn symbolizes these possibilities as $a(1-b)+b(1-a)$, $a+b(1-a)$ and $a+b$. ["1-a" is equivalent in meaning to $\bar{a}$.] The first is Boole's, the second Venn's and the third Jevons'. Jevons' expands to $ab+a(1-b)+b(1-a)+ab$ but since $a+a=a$ there would seem to be no essential difference between Jevons' version of alternation and Venn's except that Venn's is more easily used. There is, however, an essential difference between Boole's version and Venn's in that Venn's includes ab while Boole's does not.

[117] "f(x)" is "a perfectly general symbol for any class, group or arrangement of classes that includes x in it." Venn, p. 263. "0" is used to mean that a class is empty. Thus "x=0" would be read "there are no x".

[118] Venn, p. 122.

[119] Venn, p. 122.

[120] Venn, p. 124.

[121] Venn, p. 124. Venn ought to have shaded the $\overline{xyz}$ compartment but he does not mention this fact although he must have been aware of the problem as he points it out later in a similar case (Venn, p. 342). "1" represents everything in the universe of discourse.

Venn contrasts his system with Euler's by comparative diagrams. The first in each case is Euler's:

No Y is Z
All X is Y
∴No X is Z (XXIII a)[122]

translates to:

$yz=0$
$x\bar{y}=0$
$\therefore xz=0$ (XXIII b)[123]

and: All x is either y and z or not y
If any xy is z then it is w
No wx are yz

This may be represented by XXIII c[124] but this is not obvious. Translated to Venn's system it is easily diagrammed by XXIII d:[125]

$x[\overline{(yz)+y}]=xy\bar{z}=0$
$xyz\bar{w}=0$
$wxyz=0$

The diagramming is obvious and the conclusion, xy=0, is clear from the diagram. Euler would not have used four circles so that he probably would not have known what Venn was saying. It still holds that Venn's diagrams are more capable than Euler's of handling complex problems.

Venn suggests that the diagrams' main function is visual aid. In two problems he points this out. Given:

$x(\overline{y+z})=0$
$y(\overline{z+w})=0$
$z(\overline{w+y})=0$
$w(\overline{x+y})=0$

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[122] Venn, p. 125.

[123] Venn, p. 126.

[124] Venn, p. 127.

[125] Venn, p. 127.

What further condition is necessary to insure that $xy\bar{w}=0$? The premises may be reduced to:

$$x\overline{yz}=0$$
$$y\overline{zw}=0$$
$$z\overline{wy}=0$$
$$w\overline{xy}=0$$

Which gives us diagram XXIV a.[126] Of the surviving portion of xy only one compartment is $\bar{w}$ (i.e. $xyz\bar{w}$). We may destroy this by making $xyz\bar{\bar{w}}=0$. The condition is, therefore, that $xyz\bar{w}=0$. Another example shows how quickly we are able to see by this method that one class is null:

$$y[\overline{(x\bar{z})+(z\bar{x})}]=0$$
$$wx[\overline{(xz)+(\overline{xz})}]=0$$
$$xy(\overline{w+z})=0$$
$$yz(\overline{x+w})=0$$

When we diagram this it is obvious that $y=0$ (XXIV b).[127] These examples certainly do demonstrate the greatest strength of the Venn diagrams.

In his early work Venn did not even mention particular propositions but in Symbolic Logic he was forced to take them into account although he still did not seem to think them particularly important.[128] If we include particular propositions in our system we must be able to indicate unconditional preservation of compartments (by some mark such as an x)[129] as well as unconditional destruction (shading) and uncertainty. Particular propositions are of the form $ab\neq 0$. Thus there must be something which is both a and b. To indicate this we place a number in all compartments

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[126] Venn, p. 129.

[127] Venn, p. 129.

[128] At least he devotes a relatively small amount of space to particular propositions.

[129] Venn uses Arabic numerals rather than mere marks.

containing both a and b, one number for each proposition. If any particular number appears only once we know that the compartment containing it is occupied. Venn gives the following argument as an example (XXV a):[130]

$G+L=1$
$(G+L)(\overline{EF})=0$
$(GL)(\overline{E+F})=0$
$GLE\neq 0$ (1)
$GLF\neq 0$ (2)
$GEF\neq 0$ (3)
$LEF\neq 0$ (4)

Must $GLEF\neq 0$? From the diagram it is seen that it is not necessary that $GLEF\neq 0$ because (1), (2), (3) and (4) may occupy the four compartments with single numerals in them while GLEF is shaded.

From the above considerations we can see what Venn means when he says that his system combines a compartmental with an existential view of logic.[131] Venn's diagrams are still the most commonly used and most influential of all diagrammatic systems in the history of logic. No geometric system has escaped the tyranny of Venn's diagrams.[132] It is in terms of the scope and adequacy and clarity of Venn's system that all that followed must be judged.

Marquand[133] criticized Venn on three counts. First, he said that Venn's diagrams, because of the variety of figures employed, became

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[130] Venn, p. 132.

[131] Venn, p. 2.

[132] Even Peirce based his system on Venn's to a large degree although he changed the terminology and expanded the system to include much more.

[133] Marquand, op. cit., p. 226.

unintelligible too quickly.[134] Marquand's own system is an attempt to overcome this limitation. Next he suggests that Venn's diagrams are not infinitely extensible. Venn has, of course, shown that they are in theory but not in practice.[135] Marquand is concerned to show that they are in theory and in practice. Finally he points out that there is no compartment indicating $\bar{a}$, $\bar{b}$, $\bar{c}$, . . . $\bar{n}$. Venn was aware of this and carried the shading of the entire page outside the circles, in his head when necessary.[136] When constructing his "logic diagram machine"[137] Venn allowed a portion to represent the case in which all terms are negative. Thus the edge is taken from Marquand's criticism.

Macfarlane looks on Venn's diagrams as a modified use of Euler's circles.[138] Circles are only capable of making general diagrams for three terms. Although he does not specifically mention Venn it would seem that this was an indirect criticism of him and a reason for developing his own more adequate "logical spectrum".

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[134] This common criticism has a large degree of truth in it. Certainly Venn's diagrams would be of little use for the complex problems that Marquand's and Macfarlane's diagrams have been used for.

[135] Venn does give a rule for extending such diagrams in the above cited article, p. 8. ". . . for merely theoretical purposes the rule of formation would be very simple. It would merely be to begin by drawing any closed figure, and then proceed to draw others, subject to the one condition that each is to intersect once and once only all the existing subdivisions produced by those which had gone before." This is an adequate statement of a factual possibility. Hocking produced a rigorous proof that Venn's method would work. See II 2 (h).

[136] Venn, p. 342.

[137] Venn, p. 136.

[138] Alexander Macfarlane, "The Logical Spectrum", Philosophical Magazine, XIX, 1885, p. 286-287.

Carroll misinterprets Venn almost as badly as he did Euler.[139] He applies Venn's diagrams only to Aristotelian propositions.[140] Even then, since he has introduced negative terms, he rejects the O proposition entirely. This leaves only the A (XXVI c), the E (XXVI b), and the I (XXVI a) propositions.[141] Venn interprets the A proposition as $a(1-b)=0$.[142] This means that the proposition has no positive existential import. Carroll, in his interpretation of Venn attributes to him the interpretation of the A proposition, $a(1-b)=0$ and $ab\neq 0$, (using Venn's language).[143] The existence which is marked in XXVI c Venn would have rejected.

Carroll criticizes Venn because his diagrams fail to correspond to the universe of discourse[144] and are not extended beyond six terms.[145] Both of these criticisms have been dealt with under Marquand.

Carroll's example of Venn's diagrams in use[146] is more informative with regard to Carroll's prejudices than with regard to Venn's

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[139] Carroll, <u>op. cit.</u>, pp. 174-176 and p. 182.

[140] E.g. See the bottom of p. 174, Carroll, <u>op. cit</u>.

[141] Carroll, <u>op. cit.</u>, p. 174, gives his own diagrams.

[142] Venn, pp. 164 ff.

[143] Carroll, <u>op. cit.</u>, p. 174, third diagram.

[144] Carroll, <u>op. cit.</u>, pp. 174-175. "The class [a†b'] which, under Mr. Venn's liberal sway, has been ranging at will through Infinite Space, is suddenly dismayed to find itself 'cabin'd, cribb'd, confined', in a limited cell like any other class." (p. 176). "a'" represents the negation of <u>a</u>. "†" represents "both . . . and . ."

[145] If "beyond six letters Mr. Venn does not go," can be construed as a criticism. Carroll, <u>op. cit.</u>, p. 174.

[146] Carroll, <u>op. cit.</u>, p. 182.

system. The diagram handles the problem in Aristotelian logic well but Venn's diagrams are at their most effective grappling with the complexities of Boolean algebra and Carroll seemed incapable of even mentioning that field.[147]

Peirce performed a thorough and complex analysis of the use of Venn's diagrams.[148] He uses o instead of shading to indicate empty compartments (XXVIIa b c d).[149] x may be used to mark the existence of at least one occupant in a compartment (XXVIII a b c)[150] and the precise denial of a proposition diagrammed with an x is produced by substituting o for the x (XXVIII d e f).[151] Two contradictory signs in one compartment are absurd and render the premises impossible but if they are connected by being produced by the same premise they cancel each other out.[152] A cross on a boundary is equivalent to crosses in the compartments so bounded joined by a line (a real improvement over Venn's figures).[153] Finally Peirce considers the relations of the signs in the various compartments. Disconnected signs are to be taken conjunctively and connected signs disjunctively.[154] All of the above crystallizes into a set of rules

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[147]We may well be doing Carroll a serious injustice. See II 2 (f).

[148]Peirce, 4.357 to 4.371.

[149]Peirce, 4.357.

[150]Peirce, 4.359.

[151]Peirce, 4.359. This rule receives further modifications when more than two terms are present.

[152]Peirce, 4.359.

[153]Peirce, 4.359. See diagrams XXIX to XXXI for examples.

[154]Peirce, 4.360.

to be used in the manipulation of Venn diagrams.

Rule 1. Any entire assertion . . . can be erased.[155]
Rule 2. Any sign of assertion can receive any accretion [XXIX a and b].[156]
Rule 3. Any assertion which could permissively be made if there were no other assertion can be written at any time detachedly.[157]
Rule 4. In the same compartment repetitions of the same sign, whether mutually attached or detached, are equivalent to one writing of it. Two different signs in the same compartment, if attached to one another are equivalent to no sign at all, and may be erased or inserted. But if they are detached from one another they constitute an absurdity. All the foregoing supposes the signs to be unconnected with any other compartments. If two contrary signs are written in the same compartment, the one being attached to certain others, P, and the other to certain others, Q, it is permitted to attach P and Q and to erase the two contrary signs [XXIX c and d].
Rule 5. Any area-boundary, representing a term, can be erased, provided that, if, in so doing, two compartments are thrown together containing independent zeros, these zeros be connected, while if there be a zero on one side of the boundary to be erased which is thrown into a compartment containing no independent zero, the zero and its whole connex be erased [XXIX e and f].[158]
Rule 6. Any new term-boundary can be inserted; and if it cuts every compartment already present, any interpretation desired may be assigned to it. Only, where the new boundary passes through a compartment containing a cross, the new boundary must pass through the cross, or what is the same thing, a second cross connected with that already there must be drawn and the new boundary must pass between them, regardless of what else is connected to the cross. If the new boundary passes through a compartment containing a zero, it will be permissible to insert a detached duplicate of the whole connex of that zero so that one zero shall be on one side and the other on the other side of the new boundary [XXIX g and h].[159]

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[155] No example is given of this rule by Peirce. It simply means that any unconnected cross, or zero, or entire connex of crosses and/or zeros may be erased.

[156] Accretion refers to disjunctive connection only.

[157] No example is given by Peirce.

[158] Peirce's example (XXIX e and f), unfortunately, only illustrates part of this rule. Two compartments containing independent zeros are not thrown together in the example.

[159] These rules have been quoted verbatum from Peirce, 4.362. The

It would, perhaps, be suggestive of the depth of Peirce's insight into the workings of the logic diagram to examine just one example of the solution of a problem using Peirce's interpretation of Venn's diagrams:[160]

Given: Some M is P (XXX a)
No S is M (XXX b)

We can deduce:

XXX c (by rule 6 from XXX a)
XXX d (by rule 6 from XXX b)
XXX e (by rule 3 and 4 from XXX c and d)
XXX f (by rule 5 from XXX e)
∴ Some P is not S

This is the introduction to a discovery of Peirce's concerning particular premises which is seen very clearly in the following diagrams:

Suppose: Some M is P (XXX a)
and Some S is not M (XXX g)
XXX c (by rule 6 from XXX a)
XXX h (by rule 6 from XXX g)
XXX i (by rule 7 from XXX c and h)
XXX j (by rule 6 using two undescribed terms from XXX i)
XXX k (by rule 5 from XXX j)

If it be objected that the step leading to XXX j is illegitimate[161] we may put the x's in XXVIII i on the boundary and work as follows:

XXX l (by putting x's on boundary from XXX i)[162]
XXX m (by rule 5 from XXX l)
∴ Some S is not some P

This attempt to derive a valid syllogism with two particular premises is significant for us only as a demonstration of Peirce's use of Venn's

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diagrams are given by him there. The logical interpretations of these diagrams given below them at XXIX is that supplied by Peirce's editors.

[160] Peirce, 4.363.

[161] There is no rule in Peirce that would allow this step.

[162] Since a cross on a boundary is the same as a pair of crosses joined by a line which is divided by that boundary.

diagrams in a practical situation.

One of the major problems for any diagrammatic scheme is the representation of disjunctions of conjunctions.[163] Peirce believes that he has overcome this problem and gives two possible diagrams for "Either some A is B while everything is either A or B, or else all A is B while some B is not A" (XXXI a and b).[164] The outer circles in the second diagram represent the "Universe of Hypothesis" of the proposition; the disjunction is represented by the rectangular compartments.

Venn's diagrams with Peirce's interpretation are capable of illuminating at least some quantitative notions. Peirce gives, as an example, the method of illustrating minimal multitudes (XXXII a) and their precise denial (XXXII b).[165]

Venn's diagrams, Peirce feels, are no more capable than Euler's of illuminating abstract or relational reasoning.[166]

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[163] Gardner suggests that this may be done by drawing Venn diagrams of Venn diagrams. Gardner, op. cit., pp. 53-54. See Peirce, 4.365.

[164] The first of these diagrams is rather complex. It may be worked out as follows:

Let some A is B=W
everything is either A or B=X
all A is B=Y
some A is not B=Z

In modern symbolism our proposition would then be:

W.XvY.Z.

which may be transformed to:

(WvY).(WvZ).(XvY).(XvZ)

Substituting the short porpositions for W, X, Y and Z we may draw the diagram easily from the resulting proposition and we now have conjunctions of disjunctions rather than disjunctions of conjunctions.

[165] Peirce, 4.366. The small circles in these diagrams seem to be being used merely to set off or separate individuals and without introducing other terms as we might expect.

[166] Peirce, 4.367.

When Peirce sets up his own system of diagrams he incorporates the insights and sometimes the structures of Venn's system into it. Venn's system, according to Peirce, is the best method of demonstrating simple two value logic. It is, however, more iconic for basic operations and is therefore of more interest to the philosophically oriented logician than to the mathematically oriented logician.[167]

Newlin, who had recognized the improvement of Venn's diagrams over Euler's, nevertheless found Venn's to be confusing.[168]

Hocking praised Venn's ingenuity but failed to realize that he had produced a statement of the theoretical infinite extensibility of graphs using only geometric figures of one boundary. The purpose of Hocking's work was to rectify this alleged theoretical failure but, as we have shown, Venn had already done this. Hocking believed that Venn's diagrams were adequate in practice, since diagrams going beyond five or six terms are not much use.[169]

Lewis describes the Venn diagrams at length.[170] The basic diagram for <u>n</u> terms may be briefly but completely defined by the following equation:

$$1=(a+\bar{a})(b+\bar{b}) \; . \; . \; . \; (n+\bar{n})$$[171]

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[167] Peirce, 4.368, ff. If a diagram is iconic the elements of the diagram stand in a one to one correspondence to the elements that it represents.

[168] William J. Newlin, "A New Logic Diagram", <u>Journal of Philosophy, Psychology, and Scientific Methods</u>, III, 1906, p. 539.

[169] Hocking, <u>op. cit</u>., p. 31; II 2 (h).

[170] Lewis, <u>op. cit</u>., p. 176 ff.

[171] Lewis does not generalize but we have felt that from his particular examples such a generalization is valid. We have substituted "a" for Lewis' "-a" since "+-" is rather awkward to read.

The diagrams for two to four terms are exactly like those of Venn (XXXIII b,c and d).[172] Lewis adds a diagram for one term (XXXIII a)[173] and introduces a perforated square rather than a doughnut to bring in the fifth term (XXXIII e).[174] Lewis closes the universe of discourse in the earlier of these diagrams but soon accepts the convention of leaving it open. As long as we remember that the $\overline{ab}$ area is there we may represent it by the area around the diagram.[175]

To illustrate particular propositions Lewis suggests the use of asterisks joined disjunctively by a broken line (XXXIV a and b).[176] Shading, as in Venn, represents all other propositions. Lewis proceeds to give several examples of the use of Venn diagrams in solving problems in Boolean algebra.[177] These may be passed over here as they do not contribute significantly to the history of the logic diagram.

Gardner's introduction to the Venn diagrams gives a brief account of exactly what we have described earlier including some of

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172 Lewis, op. cit., pp. 177-178.

173 Lewis, op. cit., p. 177.

174 Lewis, op. cit., p. 179.

175 He leaves it open for the first time in the diagram for four terms, p. 178. He establishes the convention p. 177-178. "It is not really necessary to draw the square, 1, since the area given to the figure, or the whole page, may as well be taken to represent the universe. But when the square is omitted, it must be remembered that the unenclosed area outside all the lines of the figure is a subdivision of the universe — the entity -a, or -a-b, or -a-b-c, etc., according to the number of elements involved."

176 Lewis, op. cit., p. 183.

177 Lewis, op. cit., pp. 184, 201-207 and 211-216.

Peirce's comments.[178] He points out the possibility of using these diagrams to represent disjunctive relations such as "All X are either Y or Z" taking "or" in the inclusive sense (XXXV a)[179] and in the exclusive sense (XXXV b).[180]

Gardener also adapts the Venn diagrams, by introducing rectangles, to handle syllogisms in which terms are quantified by "most" or a number.

There are ten A's of which four are B's
Eight A's are C's
∴ At least two B's are C's (XXXVI a)[181]

Perhaps the most significant contribution of Gardner to the examination of the nature of the logic diagram is his discussion of the relationship between a class calculus and the propositional calculus from a diagrammatic point of view.[182] Any formula in the one calculus may be restated in terms of the other but such restatement is not necessary in the application of the diagrams. For the class calculus, as we have seen, we shade areas of the diagram to show that a class is empty. For the propositional calculus such shading is reinterpreted as an indication of the falsity of a proposition or of a particular combination of propositions. Gardner next proceeds to give diagrams for the simplest formulae containing each of the logical constants of the propositional calculus.

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[178] Gardner, op. cit., pp. 39 ff.

[179] Gardner, op. cit., p. 41, Figure 26.

[180] Gardner, op. cit., p. 41, Figure 27.

[181] Gardner, op. cit., p. 42, Figure 29. This, to some degree, overcomes Peirce's objection that Venn diagrams are not capable of being used for quantitative notions except in the simplest cases.

[182] Gardner, op. cit., p. 49.

He uses a small circle to the lower right of the diagram to represent the case in which all the terms are negated. The first diagram illustrates simple assertion and the second simple negation.

A (i.e. A is true) (XXXVII a)[183]
$\sim A$ (i.e. A is false) (XXXVII b)[184]

The remaining diagrams[185] in XXXVII and XXXVIII illustrate the binary relations and their exact negations. Those diagrams in the left columns indicate the binary relations:

$A \supset B$ (XXXVII c)
$B \supset A$ (XXXVII e)
$A \vee B$ (XXXVII g)
$A \not\equiv B$ (XXXVIII a)
$A | B$ (XXXVIII c)
$A \equiv B$ (XXXVIII e)
$A . B$ (XXXVIII g)

Those in the right column represent the exact negation of the diagram directly to their left:

$\sim(A \supset B) \equiv (A . \sim B)$	(XXXVII d)
$\sim(B \supset A) \equiv (B . \sim A)$	(XXXVII f)
$\sim(A \vee B) \equiv (\sim A . \sim B)$	(XXXVII h)
$\sim(A \not\equiv B) \equiv (A \equiv B)$	(XXXVIII b)
$\sim(A | B) \equiv (A . B)$	(XXXVIII d)
$\sim(A \equiv B) \equiv (A \not\equiv B)$	(XXXVIII f)
$\sim(A . B) \equiv (A | B)$	(XXXVIII h)

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[183] Gardner, <u>op. cit.</u>, p. 50, Figure 38.

[184] Gardner, <u>op. cit.</u>, p. 50, Figure 39.

[185] Gardner, <u>op. cit.</u>, p. 52, Figure 42. There is some unnecessary repetition in this list. Gardner could have managed without "≡" and " " but he wished to show how the commonly accepted binary operators were used. "$A \supset B$" means "if A then B." "$A \vee B$" means "A or B." "$A \not\equiv B$" means that "if A then not B and if B then not A." "$A | B$" means "not both A and B." "$A \equiv B$" means "if A then B and if B then A." "$A . B$" means "both A and B."

The Venn diagrams render "tautologous or equivalent statements"[186] obvious. For example, independent diagrams for A∨~B (XXXIX a) and B⊃A (XXXIX b) prove to be identical.[187]

Gardner gives a sample problem to demonstrate the use of Venn diagrams in the solution of problems in the propositional calculus:

A⊃B
B≢C
A∨C
C⊃A
∴A.B.~C (XXXIX c)[188]

We may solve more complex problems, for example those involving compound statements, by drawing Venn diagrams in which the circles represent simpler Venn diagrams. For example, if we are given (A∨B)⊃(B∨C) we draw a diagram for (A∨B), another for (B∨C), and a third in which one term is (A∨B) and another (B∨C) (XL a).[189]

Gardner has shown how these diagrams may be adapted to illustrate in principle any type of logical statement, including statements which combine class and propositional assertions. The only problem is the increasing complexity of the diagram with the increase in the number of propositions and this would appear to be a problem for any geometric system.

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[186] Gardner, op. cit., p. 51.

[187] Gardner does not give these diagrams but he does suggest them (p. 51).

[188] Gardner, op. cit., p. 53.

[189] Gardner, op. cit., p. 54. This is rather awkward although workable with practise. Gardner's method of dealing with complex expressions (II 4 (e)) and Peirce's system (II 4 (d)) are both better equipped to handle such expressions.

Thus we have discovered in Venn's system a geometric diagram which is suitable for any type of argument in any type of logic which had been developed up to that time. Most of the attempts of the subsequent twenty years to clarify and simplify what Venn had accomplished were, as we shall see, either ill-founded or trivial, although almost all of them added something of value to our understanding of these diagrams.

The major question we must ask about Venn diagrams is whether they can, in fact, represent the Boole-Schroeder algebra adequately; their further application rests on this primary one. Let us then look at the diagrams for basic formulae using the various symbols of operation. We have already looked at some of these but it would be well to review them. There are, of course, four operations which must be diagrammed: grouping (+), exclusion (-), selection (X), and restriction (÷). For the sake of convenience we will allow these operators to form binary relations giving an empty class. The diagramming of the first three is no problem; an example of each will suffice:

$A+B=0$ (XLI a)
$A-B=0$ (XLI b)
$AB=0$ (XLI c)

Restriction cannot be shown on Venn diagrams quite so directly.[190] It is first necessary to convert the restriction to selection and equivalence.

$$\frac{A}{B} = 1$$

$\therefore A=B$ (XLI d)

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[190]Venn seems to arrive at restriction because of his use of mathematical operators. Restriction cannot be rendered in ordinary language as the other operators can. It is to be understood as a secondary operation which is derived from selection.

$\frac{AB}{C} = 1$

$\therefore AB=C$ (XLI e)

The critical case is the type of restriction that occurs when a term is multiplied by $\frac{0}{0}$.[191] $\frac{0}{0}$ A=n means that, whatever n is, the truth functional value or the class membership of A is absolutely indeterminate. Such an equation cannot be shown by any of the normal techniques on Venn diagrams. One possible solution would be to introduce some mark, say "I", into the diagram to indicate that a class is indeterminate. Thus if n=f[A +(B+C)][192] diagram XXXIX f illustrates $\frac{0}{0}$A=n.

There is one type of formula within Venn's version of Boolean algebra that Venn's diagrams cannot represent. Any formula of the type A=f(B) expresses such a degree of uncertainty about the nature of the relationship between A and B that it cannot be expressed by the techniques used with compartmental geometric diagrams. It is possible to express functions diagrammatically by lines linking separate areas or terms[193] (XLII a) but this carries us well beyond Venn's diagrams. The inability of Venn's diagrams to express functions would not seem to be a weakness in their use because of the indeterminacy involved which could not be illustrated on a geometric diagram without being made at least somewhat more determinate and thus deceptive.

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[191] $\frac{0}{0}$ is, as in mathematics, absolutely indeterminate. It may be 0 or 1 or anything between.

[192] For Venn f(x) always refers to a class determined by x. Thus f(A+B) is a class determined by A or by B or by both.

[193] This diagram is adapted from Seymour Lipschutz, Set Theory and Related Topics, New York: Schaum Publishing Co., 1964, p. 77.

It would seem, then, that the Venn diagrams are absolutely iconic and that, for two to four or five possible terms, they are as capable as any of their progeny of giving geometric demonstration of both two value truth functional logic and class logic.

(d) Marquand

Marquand's diagrams work on exactly the same principle as Venn's. He begins with a square which is to represent the universe of discourse. He then drops a perpendicular from the center of the top line to the bottom line, bisecting the square. The left compartment represents all *A* and the right all *a* (XLII a).[195] To introduce a second term he bisects the center line at right angles. The upper half of the diagram represents *B*, the lower half *b* (XLII b).[196] A four term diagram requires two more horizontal and two more vertical dividing lines (XLII c).[197] It is noticeable that Marquand is more concerned with regularity than with retaining singly bounded geometrical areas for his terms. Thus *C* and *D* are both divided into two areas.

Marquand gives a general formula for the number of dividing lines required. If *n* is even and $n>2$ there will be $2+2^2+2^3. . . . 2^{\frac{n}{2}}$ lines. When n is odd diagrams for n-1 terms require $2+2^2+2^3. . . . 2^{\frac{n}{2}}-2^{\frac{n}{2}-1}$ such

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[195]For Marquand *a* is the negation of *A*. The diagram is found in Marquand, *op. cit.*, p. 267.

[196]Marquand, *op. cit.*, p. 267.

[197]Marquand, *op. cit.*, p. 267.

lines.[198]

Marquand simplifies the labour of writing out the letters by joining various squares with brackets (XLIV). The horizontal bracket immediately to the left of a letter indicates that the argument[199] represented by that letter is true, as does the vertical bracket immediately above a letter. Otherwise the argument is false.[200] The star in XLIV indicates the compartment adcdefgh, which, as Marquand points out, does not exist in Venn's diagrams.[201]

We will now look at Marquand's only example of his diagrams in use:

> The[re] are eight arguments, A, B, C, D, E, F, G, H, thus related to each other: — When E is true, F is true; and when F is true, either E is true or B and C are both false. When either G is true or E and F are both false, D is true. If B is false when either F or G (but not both) are true, then H is true and either C is false or D is true. It [D] is true only when an even number of

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[198]These formulae are given by Marquand, p. 268, without further comment. The formulae are valid, the first for an even number of terms, the second for an odd number of terms. The reason for this validity is a topological question which will not be examined here. Much more thorough work on the structure of the logic diagram is to be found in Hocking, II 2 (h).

[199]Marquand uses the various terms to represent "arguments". It is difficult to see exactly what Marquand means by "argument". He would seem to mean something like premises or statements which are interdependent with regard to their validity.

[200]Marquand says only that the work of writing out the letters may be decreased by the use of brackets. Our description is abstracted from his diagram on p. 269.

[201]We have already discussed this problem and shown that Venn was often misunderstood regarding his intentions in his diagrams. The whole paper outside the diagram was, in Venn, meant to represent the compartment in which all the terms were negated. See II 2 (c).

> the remaining arguments are true; it [D] is false only when an odd number of the remaining arguments are false.
> Supposing any combination not inconsistent with the premises to exist, (1) What follows from A being true either when B is true and D false or C false and F true? and (2) From what combination of arguments may we conclude that A and H are both true when E and G are false?[202]

The answer to (1) is represented by the combination of squares marked 1 - 8 in XLII a. The answer to (2) is represented by 8 - 10 in the same diagram.[203]

Venn, in the second edition of Symbolic Logic, accepts Marquand's diagrams for problems having a large number of terms.[204] Gardner describes Marquand's system and points out that it is less iconic than Venn's graphs but it is more iconic than algebraic notation.[205] Thus it seems to occupy a medial position between notation and graphs.

The only major problem with Marquand's system is apparent in his example above. The geometrical areas representing each letter are broken up and there is an increase in confusion which is not proportionate to the increase in the number of terms.

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[202]Marquand, op. cit., p. 268. Note that, since we are interested only in those cases in which A is true we may ignore that part of the diagram in which A is false. The exact method by which these premises are diagrammed will not be examined as it is the general principle in which we are interested.

[203]Marquand, op. cit., p. 269. These answers are simply given, presumably read from the diagram.

[204]Venn, p. 140.

[205]Gardner, op. cit., pp. 43-44. The diagram that Gardner gives is taken from Venn with some changes in the lettering. It is especially noticable because of the position of the letter "x" in Gardner's diagram which is exactly the same as that in Venn's.

(e) Macfarlane

Whereas everyone else used diagrams as an aid in the solution of problems, Macfarlane used them only as a method of verification.[206] The solution was accomplished by algebraic methods. Macfarlane, in "The Logical Spectrum", presents and solves a problem, then verifies his solution. We are given U ax+by=c [207] and U dx-ey=f and asked to solve for x and y. By the process of solution of simultaneous equations:[208]

$$x=\frac{ce+bf}{ae+bd} \text{ and } y=\frac{cd-af}{ae+bd}$$

$$\frac{ce+bf}{ae+bd}=A_1abcdef+A_2abcdef'+A_3abcde'f+A_4abcde'f' \ldots +A_{61}a'b'c'd'ef$$
$$+A_{62}a'b'c'd'ef'+A_{63}a'b'c'd'e'f+A_{64}a'b'c'd'e'f'$$ [209]

where the coefficients of $A_1, A_2, A_3, \ldots A_{64}$ are numerical. The coefficient for any term is found by substituting 1 or 0 for each term (depending on its assertion or negation) within U. The final numerical

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[206] Macfarlane is careful to make this point in both the articles. Macfarlane, op. cit., p. 287. "I shall apply this method to verify the logical equations . . . " (Italics mine). Further evidence of this is the fact that an algebraic solution is given before the diagrams are applied.

[207] "U" is the symbol for everything considered; it corresponds to the strip of paper on which the spectrum is drawn.

[208] e.g. ax+by=c and dx-ey=f

$$x+\frac{by}{a}=\frac{c}{a} \text{ and } x-\frac{ey}{d}=\frac{f}{d}$$

$$\frac{by}{a}+\frac{ey}{d}=\frac{c}{a}-\frac{f}{d}$$

$$bdy+aey=cd-af$$
$$y(bd+ae)=cd-af$$

$$y=\frac{cd-af}{ae+bd}$$

[209] "x'" is used to mean the negation of "x". "A_n" means the numerical value of the particular combination of terms that occurs in the nth case, when numerical values are substituted for each of the terms.

solution can be interpreted in only four ways:[210] 1 means all, 0 means none, $\frac{0}{0}$ means none or a portion or all,[211] and every other coefficient shows that the term is impossible.[212]

Macfarlane managed to cut out almost all of the above steps which are found in Boole. He simply substituted the special values (i.e. 1 or 0) for a, b, c, d, e, and f in the original equations and solved for x and y. For example let Uabcdef=U:[213]

$$x+y=1$$
$$x-y=1$$
$$\therefore 2x=2$$
$$\therefore x=1$$ [214]

$$1+y=1$$
$$\therefore y=0$$ [215]

Macfarlane was able to arrive at conclusions by this method that Boole missed. For example let Uabcd'e'f'=U. Substituting we get x+y=1 and 0=0. Thus x and y are indeterminate but complementary.[216]

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[210] Macfarlane attributes this to Boole but does not give a reference. Macfarlane, op. cit., p. 288.

[211] This is exactly the use Venn makes of $\frac{0}{0}$. See II 2 (c).

[212] E.g. solving for x in the above problem we would get:

$$A_1=\frac{1+1}{1+1}=1$$
$$A_2=\frac{1+0}{1+1}=\frac{1}{2}$$
etc.

[213] This, simply translated, means "let the universe consist of the one case in which abcdef=U".

[214] In this case everything is x.

[215] In this case nothing is y.

[216] From x+y=1 and 0=0 we cannot derive anything but we still know that whatever x and y are together they make up everything.

Macfarlane verifies his solutions by drawing a "logical spectrum". A logical spectrum is a rectangular strip divided into two subdivisions for every "mark" (term) employed.[217] It is simply a Marquand square stretched into one horizontal line. Macfarlane draws examples for one, two, three, and four terms (XLV a,b,c and d).[218] In the logical spectrum null terms are shaded;[219] wholly included terms are white;[220] totally excluded terms are black;[221] and indeterminate terms are half black and half white. Complementary indeterminates have complementary parts white;[222] identical indeterminates have identical parts white.[223]

Macfarlane diagrams his solutions for x and y in the above problem on a pair of corresponding logical spectrums (XLVI).[224] To verify the

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[217]Actually each subdivision is so subdivided when a new term is introduced.

[218]Macfarlane, op. cit., p. 287.

[219]E.g. abcdef' in Ux (XLVI).

[220]E.g. abcdef in Ux (XLVI).

[221]E.g. abcde'f' in Ux (XLVI).

[222]E.g. abcd'e'f' in Ux and in Uy (XLVI).

[223]E.g. a'b'c'def' in Ux and in Uy (XLVI).

[224]Macfarlane, op. cit., p. 287. We might wish to illustrate the principles by which it was drawn.

Given U ax+by=c and U dx-ey=f

Cases 1 and 2: Let Uabcdef=U

x+y=1 and x-y=1

2x=2

∴x=1

Therefore the area representing abcdef in Ux will be left white.

and: y=0

Therefore the area representing abcdef in Uy will be black.

Case 3: Let Uabcdef'=U

solution we simply see if the ax together with the by (both discovered by inspection of the diagram) is identical with the c and whether the dx excepting the ey is identical with the f.[225]

Macfarlane's second article is simply another example of the application of the logical spectrum to a complex problem and does not warrant examination here as it adds nothing new to his diagrammatic system.[226]

Gardner describes the logical spectrum briefly but does not give

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$x+y=1$ and $x-y=0$
$2x=1$
$\therefore x=\frac{1}{2}$

Therefore the area representing abcdef' in Ux will be shaded.

Case 4: See note 216 above where we found that in Uabcd'e'f' x+y=1 (i.e. make up everything there is) but we know nothing else about them. Therefore abcd'e'f' in Ux is divided into two portions, one white, the other black, and abcd'e'f' in Uy is divided into two portions with the portion corresponding to the black in Ux left white and the portion corresponding to the white, black.

Case 5: Let Uab'cde'f=U
$\therefore x=1$ (from either equation)

We have no information about y, so y must be divided into two portions, one black and the other white but it does not matter which is which.

Case 6: Let Ua'b'c'def'=U
$x-y=0$
$\therefore x=y$

Therefore x and y are both indeterminate but are also equal. Thus the portion of x which is black must correspond to the portion of y which is black, and the white to the white.

[225] By "verify" Macfarlane means only that we may discover whether we have made a mistake by finding out by inspection whether the diagram corresponds to the premises. This is the same sort of verification that is found in arithmetic. For example to verify a+b=c we subtract b from c and if we get a we have verified the original answer.

[226] Alexander Macfarlane, "Application of the Method of the Logical Spectrum to Boole's Problem", (in Gardner cited as "Adaption of the method . . .") Proceedings of the American Association for the Advancement of Science, XXXIX, 1890, pp. 57-60.

an example of its operation.[227] Such an example is necessary to display its value as a method of verification. It follows an extremely regular pattern of transformation as we read from left to right. This makes it easier to read than Marquand's square when we are dealing with a large number of terms. It does, however, have two disadvantages. The various areas representing terms are broken up into a great many topologically distinct areas and this problem increases in geometric proportion to the increase in terms; and one requires an extremely long strip of paper if a large number of terms are used.

(f) Carroll

Carroll's "game of logic" is played on two diagrams (XLVII).[228] These consist of a small square bisected for two terms *à la* Marquand and a large square similarly bisected with a smaller square in its center to represent the middle term of our argument. Nine counters, five grey (shaded) and four red (black) are also required. At least one player is the final requirement.[229] All one need do to represent any Aristotelian syllogism, even those containing negative terms,[230] is place counters in the appropriate locations to show which compartments are occupied and

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[227]Gardner, *op. cit.*, pp. 44-45.

[228]Lewis Carroll, *Symbolic Logic and The Game of Logic*, (both books bound as one), *The Game of Logic*, (hereafter referred to as *G. L.* in notes), facing full title page and inside back cover.

[229]These requirements are listed by Carroll (*G. L.*) in the preface. The page is unnumbered.

[230]Aristotelian syllogisms containing negative terms are acceptable to Carroll.

which are empty. The premises are worked out on the large diagram and the conclusion transferred to the small one. An empty compartment receives a grey counter; an occupied one, a red. Carroll supplies a rhyme to help us remember this:

See, the Sun is overhead,
Shining on us FULL and
RED!

Now the Sun is gone away.
And the EMPTY sky is
GREY![231]

Carroll's game is an interesting method of teaching young children the basic principles of elementary Hamiltonian Logic but nothing more. We need not examine an example of this game in use as it corresponds exactly to the system, using numbers, described in Symbolic Logic.[232] Even in The Game of Logic Carroll uses "1" to represent the red markers and "O" to represent the grey on his diagrams.[233]

Much of Symbolic Logic is taken up with a polemic against "the Logicians".[234] Carroll would seem to have been applying this "inoffensive"[235] title to those who still pursued Aristotelian logic since he never considers Boolean algebra. Carroll was familiar with Venn[236] whose work was based on Boole. Why, then, did he never acknowledge any

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[231] Carroll, G. L., facing full title page.

[232] Also in Carroll, Symbolic Logic and The Game of Logic. Symbolic Logic will hereafter be referred to as S. L.

[233] Carroll, G. L., pp. 5 ff.

[234] Carroll, S. L., pp. 165 ff.

[235] Carroll, S. L., p. 165.

[236] Carroll, S. L., pp. 174 ff.

more complex logic than that discussed by Hamilton? We may be doing Carroll an injustice in criticizing him on these grounds as we have only the first part of his projected system.[237] but, as it stands, Carroll was fifty years out of date even in his own time. This makes it even more amazing that he developed the most adequate set of diagrams for up to four terms to be found prior to the turn of the century.[238]

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[237]The brief account that Carroll has left, S. L., p. 185 does not indicate that these parts would have included Boolean algebra but we cannot be sure. We quote the passage in full so that the reader may exercise his own judgement:

> In Part II. will be found some of the matters mentioned in this Appendix, viz., the "Existential Import" of Propositions, the use of the negative Copula, and the theory that "two negative Premises prove nothing." I shall also extend the range of Syllogisms, by introducing Propositions containing alternatives (such as "Not-all x are y"), Propositions containing 3 or more Terms (such as "All ab are c", which taken along with "Some bc' are d", would prove "Some d are a'"), &c. I shall also discuss Sorites containing Entities, and the very puzzling subjects of Hypotheticals and Dilemmas. I hope, in the course of Part II., to go over all the ground usually traversed in the text-books used in our Schools and Universities, and to enable my Readers to solve Problems of the same kind as, and far harder than, those that are at present set in their Examinations.
>
> In Part III. I hope to deal with many curious and out-of-the-way subjects, some of which are not even alluded to in any of the treatises I have met with. In this Part will be found matters as the Analysis of Propositions into their Elements (let the Reader, who has never gone into this branch of the subject, try to make out for himself what additional Proposition would be needed to convert "Some a are b" into "Some a are bc"), the treatment of Numerical and Geometrical Problems, the construction of Problems, and the solution of Syllogisms and Sorites containing Propositions more complex than any that I have used in Part II.

[238]This is Lewis' opinion, Lewis, op. cit., p. 180, and would seem to be true.

Carroll allows only three figures of syllogisms, since he used negative terms, and we may show his application of his diagrams by his use of them for these figures. Given $xm_0 \dagger ym'_0$[239] we may put "O" in compartments xym and xy'm (from xm_0) and xym' and x'ym' (from ym'_0). From the diagram (XLVIII a) we see that both xym' and xym are empty. Therefore we may put a "O" in xy in the smaller diagram in XLVIII a. That is:

Fig. I. $xm_0 \dagger ym'_0 \,|\!\!P\, xy_0$[240]

There are several variations of this figure possible by substituting m_1, x_1, etc. for m, x, etc. in the first term of each premise but the final term in each proposition must be quantified by "O".[241] These variations are easily worked out and need not concern us here. Figures two and three are similarly diagrammed:

Fig. II. $xm_0 \dagger ym_1 \,|\!\!P\, x'y_1$ (XLVIII b)[242]

Fig. III. $xm_0 \dagger ym_0 \dagger m_1 \,|\!\!P\, x'y'_1$ (XLVIII c)[243]

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[239]"$P_1 \dagger P_2$" means "the premises P_1 and P_2 taken together" where P_1 and P_2 are two premises. "ab_0" means "there are no <u>a</u>'s which are also <u>b</u>'s." "a'" is the negation of "a". "a_1" means "there are <u>a</u>'s".

[240]"$P_n \,|\!\!P\, C_m$" means "C_m is the conclusion of the premise(s) P_n" where P_n is a set of premises and C_m is a conclusion validly drawn from them.

[241]E.g. $m_1 x_0 \dagger ym'_0 \,|\!\!P\, xy_0$
$xm'_0 \dagger m_1 y_0 \,|\!\!P\, xy_0$
$m'_1 x'_0 \dagger m'_1 y'_0 \,|\!\!P\, x'y'_0$
etc. Carroll, <u>S. L.</u>, p. 75.

[242]Carroll, <u>S. L.</u>, p. 76.

[243]Carroll, <u>S. L.</u>, p. 77.

When Carroll is uncertain which of two compartments should contain a "1" he simply places it on the line between the compartments in question.[244]

$xm'_0 \dagger m'y'_0 \dagger xm_1 \dagger ym'_1 \P x'y_1$ (XLVIII d)[245]

In case the reader has not noticed, it may be pointed out that Carroll is rather careless in his interpretation of the A proposition. In some cases he interprets it as xm'_0[246] in others as $xm'_0 \dagger x_1$[247] and in yet others as $xm'_0 \dagger xm_1$.[248] It is true that the latter two are logically equivalent but they make it necessary to accept the existential import of the A proposition. Carroll seems to believe[249] that the existential import of the E and I propositions is a matter of convention. It is logically possible to say that the A and E propositions assert existence while the I does not. This is, however, in opposition to common usage.[250] It is logically impossible to hold any other view of the existential import of propositions according to Carroll.[251]

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[244]Carroll, S. L., p. 43.

[245]Carroll, S. L., p. 141.

[246]E.g. Fig. I above.

[247]This might also be written as $x_1 m'_0$. An example of the version in the text is Fig. III above.

[248]E.g. final example above (i.e. $xm'_0 \dagger m'y'_0 \dagger xm_1 \dagger ym'_1 \P x'y'_1$).

[249]Carroll, S. L., pp. 165 ff.

[250]We do not normally think that "no a is b" asserts that there are a's while "some a is b" does not. There is, according to Carroll, no logical reason why this should not be the case.

[251]There are eight possible views of existential import as far as Carroll is concerned:
(1) the A asserts, the E asserts and the I asserts,
(2) the A asserts, the E asserts and the I does not assert,
(3) the A asserts, the E does not assert and the I asserts,

Carroll's diagrams have, thus far, been limited to three terms but he gives further diagrams in the appendix which he plans to discuss in the second volume. For four terms we replace the small center square with two intersecting rectangles (XLIX a);[252] for five terms we divide each area diagonally (XLIX b);[253] for six terms we replace the diagonal stroke with a cross (XLIX c);[254] for seven terms we place a three term diagram in each of the compartments of a four term diagram (XLIX d);[255] and for an eight term diagram we place a four term diagram in each of the compartments of another, larger, four term diagram (XLIX e).[256] For nine terms we place two eight term diagrams side by side and for ten terms we place two more eight term diagrams below.[257]

C. I. Lewis finds Carroll's modifications of the square diagram to be the "most convenient".[258] He uses a small key diagram to one side to aid in the interpretation of the larger diagram much as we shall find

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(4) the A asserts, the E does not assert and the I does not assert,
(5) the A does not assert, the E asserts and the I asserts,
(6) the A does not assert, the E asserts and the I does not assert,
(7) the A does not assert, the E does not assert and the I asserts,
(8) the A does not assert, the E does not assert, and the I does not assert.
Only (2) and (3) are logically possible according to Carroll.

[252] Carroll, S. L., p. 177.

[253] Carroll, S. L., p. 177.

[254] Carroll, S. L., p. 177.

[255] Carroll, S. L., p. 178.

[256] Carroll, S. L., p. 179.

[257] We do not include examples of these last two cases nor does Carroll although he mentions them (S. L., pp. 178-179).

[258] Lewis, op. cit., p. 180.

in Newlin.[259] Gardner shows Carroll diagrams for three terms but does mention that they are infinitely extensible.[260]

As we have said before, one of Carroll's major claims is to the closure of the diagrammatic universe of discourse.[261] That he did do this is true, but he ignored Venn's acknowledgement that we are, indeed, working within such a universe and that the paper outside the diagram must be taken as representing $\bar{a}$, $\bar{b}$, $\bar{c}$, . . . $\bar{n}$. Carroll does not seem to have read Marquand or Macfarlane both of whom had created systems with closed universes of discourse prior to him.

How then are we to evaluate Carroll's diagrams? For up to four terms they are able to keep topologically distinct and undivided areas and, since they illustrate the closed universe of discourse, Carroll's system would seem to be the most adequate of the geometric systems yet examined.[262] For arguments involving four or more terms it loses the simplicity of Macfarlane's and Marquand's systems.

One further problem is the existential import of the A proposition. Carroll represents the A proposition as positing existence but Venn, Marquand, etc. do not. Since the universal affirmative proposition can

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[259] Lewis, op. cit., pp. 180-181.

[260] Gardner, op. cit., pp. 45 ff.

[261] See above, p. 54, fn. 144 for Carroll's statement of this.

[262] This is a personal evaluation which may be questioned. It is also necessary to take into consideration the purposes for which the diagrams are being used.

usually be translated into a hypothetical proposition[263] it would seem that Carroll was wrong.

(g) Newlin

Newlin points out that the logic diagram is not essential to logic;[264] its main purpose is illustrative and educational. This should be kept in mind when choosing a diagrammatic system. The above qualification leads to three requirements of a good logic diagram.[265] First, there must be one to one correspondence between the elements of the diagram and the elements of the logical universe of discourse. Second, the diagram must be simple in construction and appearance, free from sources of confusion, quick, reliable, and modifiable. Finally, any system of logic diagrams which is definitive must be "simply-extensible".[266] Newlin coins this word to apply to diagrams which may be extended to be applicable in cases of any number of terms and in which this extension

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[263] This is a matter of controversy. If the universal affirmative is equivalent to a hypothetical, Carroll was most certainly wrong. This problem is taken up from a different angle in the third chapter. In that chapter the meaning of the term "exist" is considered as that term is used in logic and the conclusions reached indicate that neither Carroll's position nor that which opposed it is entirely correct. It should be pointed out that Venn, although he did not accept the existential import of the A proposition, did believe that <u>something</u> must exist in any universe of discourse. This is approximately the position that will be adopted in the third chapter.

[264] Newlin, <u>op. cit.</u>, p. 535.

[265] These "few but vital" demands of a satisfactory diagram are given at length by Newlin, <u>op. cit.</u>, p. 540.

[266] Newlin, <u>op. cit.</u>, p. 540.

is carried on by simple and invariable rules. We may see the faults of other diagrammatic systems in the light of these rules. Euler's diagrams are limited to the representation of three classes; Venn's for four and five classes are confusing; Marquand's are simply-extensible but even in simple cases elements of single classes are discrete.[267]

With these requirements in mind Newlin proposes his diagrammatic system. We begin with a primary square which is our universe of discourse. This square is divided into sixteen smaller squares by means of three perpendiculars dropped from the top to the base and three perpendiculars running from one side to the other. The diagram may be thought of as having four horizontal and four vertical strips. The first and second vertical strips at the left will represent *a*; the second and third, *b*. The first and second horizontal strips beginning at the top will represent *c*; the second and third, *d* (L a).[268] The square $\bar{a}b\bar{c}d$, marked in the diagram, is located by finding the square where the $\bar{a}b$ and $\bar{c}d$ strips meet.[269]

To construct a diagram for three terms we make c=0 by erasing the top two rows of squares; for two terms we make d=0 by erasing the third row as well.

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[267] Newlin considers only the three systems mentioned. Newlin, *op. cit.*, p. 540.

[268] Newlin, *op. cit.*, p. 541.

[269] Although Newlin's diagram for four terms looks like Marquand's it has the advantage of representing each of these terms by a distinct singly bounded area. Note also that if the rectangles in the Carroll diagram for four terms are extended to the border of the universe we have a Newlin diagram for four terms.

Now we must extend the diagram. First we draw in all the diagonals of the secondary squares, in a four class diagram, which run in one direction. If we think of these diagonals as borders of diagonal strips we may use alternate diagonal strips to represent our fifth term *e*. Similarly, inserting the diagonals in the other direction, we are able to represent a sixth term *f*. Finally we divide the secondary squares vertically and horizontally through the point where the diagonals meet making eight triangles in each secondary square (L b).[270] In the diagram the triangle marked *x* represents $\bar{a}b\bar{c}de\bar{f}g$ and is located by first locating $\bar{a}b$, and within this $\bar{c}d$, then *e*, then $\bar{f}$, and finally *g*. Upper or left triangles represent the seventh term.

To go beyond seven classes we substitute a four class diagram in the secondary squares of a larger four class diagram (LI a).[271] The small squares resulting are called tertiary squares. The class marked *x* is defined by eight terms $a\bar{b}c\bar{d}\bar{e}f\overline{gh}$ and may be located in four steps. First we locate $a\bar{b}$ and then within it $c\bar{d}$, then $\bar{e}f$ and finally $\overline{gh}$. Diagrams for nine, ten and eleven terms may be drawn by treating the secondary square as a primary square (LI a). For example the class *y* is defined by ten terms $\overline{ab}cd\bar{e}f\bar{g}hi\bar{j}$. To go beyond eleven terms we simply substitute another four class diagram in the tertiary squares.

The use of a key (LI b)[272] will prevent the confusion arising

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[270] Newlin, *op. cit.*, p. 542.

[271] Newlin, *op. cit.*, p. 544.

[272] Newlin, *op. cit.*, p. 544.

when all the letters needed are put on one diagram and will allow us to carry out the more complex construction only on those portions of the diagram with which we are working. The key is simply a diagram which shows how the various classes beyond the first four are distributed in the diagram which has been substituted in the secondary square.

Newlin felt that his system had two advantages over the others which had been developed: there is a "perfect correspondence for any number of classes"[273] and it never becomes more complex than the seven class diagram.[274]

The choice between Newlin's diagrams and those of Marquand, Macfarlane and Carroll would seem to be a matter of personal preference. After four terms Newlin is unable to keep the various classes within singly bounded topological areas so his advantage extends only two terms beyond Marquand's diagrams and, except for simplicity of construction, there is no advantage over Carroll's four term diagram. Further, the use of diagonal strips and discrete triangles is quite confusing when compared to the regularity of Macfarlane's spectrum. Newlin's major contribution to the logic diagram would seem to rest not so much in his diagrammatic system as in his methodical description of such diagrams and his ennumeration of their requirements.[275]

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[273]Newlin, op. cit., p. 545.

[274]Newlin, op. cit., p. 546.

[275]Even in this area it should be noted that Hocking is much more thorough. II 2 (h).

(h) Hocking

Hocking's analysis of the logic diagram is even more profound than Newlin's and his final system is even more obscure, but he himself realizes this. He is not attempting to develop a workable diagram but to extend the diagram to cover the theoretical necessity of drawing diagrams for n terms.[276]

The logic diagram represents the class calculus. The major considerations of a good representation is that it be "rich" enough to imply all the relationships which are to be found in that which is represented, while it remains "poor" enough that it does not imply any factors that are not present in the original. That the spatial figure of the geometric diagram is this rich cannot be doubted. It does, however, convey the impression that any class is bounded by logically "next" neighbours and that a universe of discourse may be "carved out" around a class without regard to those classes which determine such a universe. These "superstitions" are, however, due more to our ignorance of class logic than to the suggestiveness of the graph.[277]

As well as the necessary correspondence of the parts in the original

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[276]". . . and indeed there is little practical need to devise graphs for more than five classes at a time. But it would be a serious failure in the principle of the class-graph if there were a theoretical limit to the number of classes which can be drawn within a given universe; . . . I wish to propose here a simple generalization of the graphic process which is demonstrably extensible to n classes without sacrifice of unity or continuity in the class boundary." Hocking, op. cit., p. 31.

[277]This paragraph is a paraphrase of Hocking, op. cit., p. 32. It is meant to be a description of an adequate diagram with its limitations indicated.

and in the diagram it is also possible to introduce "proportion". That is, it is desirable that the relations of these parts and the relations of these relations, etc., should also correspond.[278]

There are particulars in which spatial figures must fail to correspond exactly to the logical relations. For example, the various classes introduced are coördinate, equivalent in logical denotation, etc., but it is only possible to maintain the geometric symmetry and indifference necessary to connote this in graphs of three or four terms. Geometric impartiality is virtually impossible for the various sub-classes even at three or four terms.[279]

Hocking feels that any tabular arrangement is a primitive diagram. The first stage in such a diagram would be to order sub-classes according to their "connotative rank". This might be defined as the breadth of connotation of the sub-class. If we adopt the convention of using the number "1" when the universe is connoted, we may list the sub-classes according to rank from 0 upward with 0 representing the broadest connotation. A table for two classes would be as follows:[280]

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[278] Hocking, op. cit., pp. 32-33. The remainder of the first part of Hocking's article is an attempt to show to what degree this may be done in an ideal geometric diagram.

[279] For example the sizes of the various components of the Venn diagram vary with regard to size whereas there is no logical reason for this variation.

[280] Hocking does not give this table. He does, however, give a similar table from which this was drawn but he does not include rank. The table, Hocking, op. cit., p. 33 is:

Rank	Sub-classes	Connotations
0	$\overline{ab}$	1
1	$a\bar{b}$, $\bar{a}b$	a, b
2	ab	ab

Similar tables for three and four classes may aid the reader's understanding of connotative rank:

Rank	Sub-classes	Connotations
0	$\overline{abc}$	1
1	$a\overline{bc}$, $\bar{a}b\bar{c}$, $\overline{ab}c$	a, b, c
2	$ab\bar{c}$, $a\bar{b}c$, $\bar{a}bc$	ab, ac, bc
3	abc	abc
0	$\overline{abcd}$	1
1	$a\overline{bcd}$, $\bar{a}b\overline{cd}$, $\overline{ab}c\bar{d}$, $\overline{abc}d$	a, b, c, d
2	$ab\overline{cd}$, $a\bar{b}c\bar{d}$, $a\overline{bc}d$, $\bar{a}bc\bar{d}$, $\bar{a}b\bar{c}d$, $\overline{ab}cd$	ab, ac, ad, bc, bd, cd
3	$abc\bar{d}$, $ab\bar{c}d$, $a\bar{b}cd$, $\bar{a}bcd$	abc, abd, acd, bcd
4	abcd	abcd

It will be seen that in a universe of n classes there will be one sub-class of rank n and one sub-class of rank 0, n sub-classes of rank 1 and n sub-classes of rank n-1, and the distribution of the remainder of the sub-classes among the ranks will be according to the law of Pascal's triangle.[281]

Sub-classes	Connotations
ab	ab
$a\bar{b}$	a
$\bar{a}b$	b
$\overline{ab}$	1

Note that Hocking does not include the possibility of negative connotations.

[281]"This is a triangle of numbers which is formed in such a way that the numbers in any row, after the first two, are obtained from those in the preceeding row by copying down the terminal 1's, and adding together the successive pairs of numbers from left to right to give a new row. Thus, any number is the sum of those two numbers immediately above it." C. C. T. Baker, Dictionary of Mathematics, London: George Newnes Limited, 1961, p. 228. The particular Pascal triangle about which Hocking is writing is as follows:

Any satisfactory graph must so group the ranks that those sub-classes having the same rank will be at the same approximate distance from the center and periphery and that the ranks are ordered either inward or outward (i.e. 0 and *n* will be at the two extremes in the ordering of the ranks).[282]

The second method of classification of the sub-classes is by connotative kinship.[283] Kinship is measured by the simplicity of transition from one sub-class to another. For example, 1 may pass into *a* or *b* by a single change; *a* may also pass into *ab* or *ac* by a single change but it takes two changes for it to pass into *b* or *c*. In general, it may be stated that the sub-classes of a given rank are related to each other more remotely than to the sub-classes of the adjacent ranks. We may, in our diagrams, allow the number of boundaries which must be crossed in moving from one sub-class to another to represent the degree of kinship with one boundary representing one change, two boundaries two, etc.

In order to list sub-classes according to both rank and kinship it is necessary to abandon the linear form and substitute a cyclical order. The diagram for three classes is the Venn diagram (LII a)[284]

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
. . . .

[282] Hocking, *op. cit.*, p. 34. Further reference will be made to this rule in III 3 (c) as well as to kinship.

[283] Hocking discusses kinship *op. cit.*, p. 34-36.

[284] Hocking, *op. cit.*, p. 36, Fig. 2.

which perfectly orders the sub-classes according to both rank and kinship.

For four classes it would require three dimensions to make a representation which would perfectly accommodate both rank and kinship. In general, a representation of n classes would require a space of n-1 dimensions.[285] Thus we must modify our origianl plan. We will hereafter require only that any set of four sub-classes about an intersection shall be related with regard to kinship as in the above scheme and that all sub-classes containing, in common, any connotative factor shall be placed together. A table for four terms which complies with these regulations may be drawn (LII b).[286] When the redundancies are removed we are left with diagrams similar to Marquand[287] (LII c),[288] Carroll (LII d),[289] and Venn and Newlin (LII e).[290] Thus we see why all of these systems have proven relatively adequate. The choice among these diagrams will be determined by such factors as "the simple and fluent outlines of the classes, their equivalence in area, approximate equivalence in area of the sub-classes; symmetry and openness of the whole graph, etc."[291]

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[285] Hocking simply states this without making any attempt to prove it.

[286] Hocking, op. cit., p. 37, Fig. 3.

[287] Hocking does not point out the equivalence of this and the following diagrams to earlier ones but the relationships are so obvious that it seems clear that he had these other systems in mind.

[288] Hocking, op. cit., p. 37, Fig. 4.

[289] Hocking, op. cit., p. 37, Fig. 5.

[290] Hocking, op. cit., p. 37, Fig. 6.

[291] Hocking, op. cit., p. 38.

Hocking now attempts to prove that it is possible to draw a graph for n terms that will satisfy all three primary conditions: "(1) that each class be represented as a closed figure within a single boundary; and (2) that the sub-classes be arranged in order of connotative rank, and (3) of connotative kinship within the limits above defined."[292]

If, in a universe of n-1 classes, a closed figure can be drawn that passes once and only once through each compartment we have the requisite for a graph of n classes.[293] As the new boundary passes through any compartment it will divide the compartment into two parts: one, destined to be within the new figure, is raised one rank; the other, remaining without, retains its previous rank. If rank and kinship are observed in the original diagram they will be observed in the new one. This is evident in the transition of any typical part of the graph as in the transition of LIII a[294] to LIII b.[295] Now we can be certain that if such a closed curve can be drawn we will have fulfilled the other requirements. We can prove as follows that if such outlines can be drawn for the (n-1)th class they can be drawn for n classes. By hypothesis the outline of the (n-1)th class passes through all previous sub-classes arranging all of the sub-classes along its boundary. Now all the sub-classes are related to that boundary as m and n are related to the line

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[292]Hocking, op. cit., p. 38.

[293]The following proof is carried out by Hocking, op. cit., pp. 38-39.

[294]Hocking, op. cit., p. 38, Fig. 7.

[295]Hocking, op. cit., p. 38, Fig. 8.

xy in LIII c.[296] Therefore a new boundary can follow pq and, since there will always be an even number of turns, the end of the line will be able to join the beginning. Therefore, if we can draw such a diagram for n-1 classes we can draw such a diagram for n classes. But we can draw such figures for a number of classes.[297] Therefore, we must be able to draw them for n classes.

Finally Hocking draws a set of figures up to and including seven classes (LIV a-f).[298] Hocking did not mean these figures to be of practical use in logic but of value only in demonstrating the theoretical extensibility of the diagram.

In the second part of his article Hocking develops a graphic method of dealing with immediate inferences. He first attempts to show that the eight variants of immediate inference are functions of two fundamental processes: conversion and obversion. Let the direct inference be represented by 1 and let C. O. be read "the converse of the obverse" etc.

Direct =	1	=SP=	O. C. O. C. O. C. O. C.
Obverse =	O.	=$S\bar{P}$=	C. O. C. O. C. O. C.
Contraverse =	C. O.	=$\bar{P}S$=	O. C. O. C. O. C.
Contra-positive =	O. C. O.	=$\bar{P}\bar{S}$=	C. O. C. O. C.
Oppositive =	C. O. C. O.	=$\bar{S}\bar{P}$=	O. C. O. C.
Inverse =	O. C. O. C. O.	=$\bar{S}P$=	C. O. C.
Contra-inverse =	C. O. C. O. C. O.	=$P\bar{S}$=	O. C.
Converse =	O. C. O. C. O. C. O.	=PS=	C.
Direct =	C. O. C. O. C. O. C. O.	=SP=	1

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[296] Hocking, op. cit., p. 39, Fig. 9.

[297] We can, for example, draw such a diagram for 1, 2, 3, and 4 classes as was pointed out earlier (e.g. LII a, etc.).

[298] Hocking, op. cit., p. 39, Fig. 10 and 11; p. 40, Fig. 12, 14, 16 and 17. Note that Hocking does not give a diagram for three terms but such a diagram could easily be drawn by eliminating one of the ovals in the four term diagram although Hocking would have been satisfied with the Venn circle diagram for three terms.

This may be demonstrated by a graphic method which Hocking also believes to be of assistance in solving any logical problem involving immediate inference. We draw a universe of discourse containing S, P and their negatives (LV a).[299] We will adopt the convention that the letter in the lower left corner will always be the subject of a proposition[300] and that in the lower right corner will be the predicate.

The graph will be thought of as having two axes: the axis of conversion is a line perpendicular to the base through the meeting-point of the diagonals (LV b)[301] and the axis of obversion is the diagonal running from the lower left to the upper right corner of the graph (LV d).[302] To convert a proposition we simply rotate the graph on the axis of conversion 180 degrees and read off the resulting proposition (LV c);[303] to obvert a proposition we do the same using the axis of obversion (LV e).[304]

We may, if we wish, go further and prove all of the immediate inferences to be functions of conversion and obversion simply by following the procedures listed in each case.

Hocking gives us an example of this graph in use. Let us assume that every categorical proposition implies the existence of both P and S

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[299]Hocking, op. cit., p. 42, Fig. 18.

[300]What proposition will depend on the markings on the particular card (See LVI).

[301]Hocking, op. cit., p. 43, Fig. 19.

[302]Hocking, op. cit., p. 43, Fig. 19.

[303]Hocking, op. cit., p. 43, Fig. 19.

[304]Hocking, op. cit., p. 43, Fig. 19.

and that an affirmative proposition implies the existence of members in the class $\overline{S}P$. We indicate existence with a checkmark and non-existence with shading. The A, E, I and O propositions give us four distinct diagrams (LVI a-d).[305]

It is easily seen, using these diagrams and applying the above methods, that the obverse of an A proposition is an E proposition, that the converse of both the E and I propositions remain E and I propositions because they are symmetrical about the vertical axis, that the O proposition cannot be converted to any of the forms we have mentioned and that the converted A proposition will be partly covered (i.e. by limitation) by the I proposition.

All of this is very interesting but one is tempted to ask, "So what?" Immediate inferences are reasonably simple and require the memorizing of only eight forms. Hocking, himself, has shown the dependence of all immediate inferences on conversion and obversion. Why then do we need the diagrams? One should add, in fairness, that this subject had been very complicated prior to Hocking's time. It may be that it only appears trivial to us in the light of C. I. Lewis' brilliant analysis,[306] and perhaps the highest tribute we could pay to Hocking is to point out that his analysis, though less profound, parallels Lewis' almost point for point.

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[305]Hocking, op. cit., p. 44, Fig. 20. Note that Hocking makes these existential assumptions arbitrarily and would have to be willing to allow other assumptions. Thus his examples are only examples. They might be quite different in their conclusions given different assumptions.

[306]See II 2 (i).

(i) Lewis

C. I. Lewis has also produced a diagrammatic method of discovering immediate inferences.[307] Since we have already discussed Hocking's[308] method, and since Lewis makes use of Venn circles,[309] we shall consider this method here although the diagrams are not strictly geometric in the sense in which we have defined that term. Lewis' aim is to show how Boolean algebra has "done a real service" in "the clearing of certain difficulties" concerning immediate inferences. He gives the following series of inferences which would be accepted by "some" logicians:[310]

"No a is b"	gives	"No b is a"
"No b is a"	gives	"All b is not-a"
"All b is not-a"	gives	"Some b is not-a"
"Some b is not-a"	gives	"Some not-a is b"

This series of inferences would lead to such conclusions as: if no mathematicians have squared the circle, some non-mathematicians have squared the circle.[311] Something is obviously wrong. The problem lies in the inference of the particular premise from the general. Given $ba=0$ we must also have $b\neq 0$ before we can infer $b-a\neq 0$.

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[307] Lewis, op. cit., pp. 191-195.

[308] II 2 (h).

[309] Lewis uses only two such circles but each stands for a term. He does, however, use other, non-geometric symbols (viz. arrows).

[310] Lewis, op. cit., p. 190.

[311] One of Lewis' examples. The other is: "'No cows are inflexed gastropods' implies 'Some non-cows are inflexed gastropods.'" We have eliminated the letters "a" and "b" from these examples. See Lewis, op. cit., p. 190.

$b=b[a+(-a)]$
$=ab+(-a)b$
if $b=0$
$ab+(-a)b=0$
the inference $(-a)b\neq 0$ is not possible.
but if $b\neq 0$
and $ab=0$
$\therefore (-a)b\neq 0$

That is to say, there is a suppressed premise in the conversion of an A proposition to an O proposition: that one of the classes involved has members.[312]

The inferences that will be possible are eight in number and are indicated on our diagrams by the arrows (LVII a).[313] One simply follows the arrows to read off the subject and predicate in their correct order. The actual inferences are dependent on our given information. Null classes are shaded on the diagram; classes known to have members are marked with an asterisk. Lewis gives two typical examples: given $SP=0$, $S\neq 0$, and $P\neq 0$ (LVII b)[314] we may read along the arrows to get:

1) No S is P and some S is not-P
2) All S is not-P and some S is not-P
3) All P is not-S and some P is not-S
4) No p is S and some P is not-S
5) Wanting
6) Some not-S is P
7) Some not-P is S
8) Wanting

given $S-P=0$, $S\neq 0$ and $P\neq 0$ (LVII c)[315] we may read along the arrows to get:

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[312] Lewis treats these matters op. cit., pp. 190-191.

[313] Lewis, op. cit., p. 191, Fig. 13. This is the diagram when we are in entire ignorance as to the existence of S and P.

[314] Lewis, op. cit., p. 192, Fig. 14.

[315] Lewis, op. cit., p. 193, Fig. 15.

1) All S is P and some S is P
2) No S is not-P
3) Wanting
4) Some P is S
5) Some not-S is not-P
6) Wanting
7) No not-P is S
8) All not-P is not-S and some not-P is not-S

The great value of Lewis' analysis of this subject is that it shows in final and simple diagrammatic form the absolute limits of immediate inference. Boolean algebra showed its power in defining this field that had plagued logicians using Aristotelian methods. It is significant that both Hocking and Lewis found it expedient to use diagrams when working in this field and their ultimate conclusions show that diagrammatic representation can be a significant tool on the path toward the drawing of conclusions from symbolic representation.

(j) Gonseth

Gonseth's diagrams[316] were given only for two terms and were exactly like those of Marquand except that instead of shading out the appropriate areas he eliminated them entirely from the diagram. Thus one may represent AvB as in Diagram LVIII a,[317] A&B as in Diagram LVIII b,[318] A→B as in Diagram LVIII c,[319] etc.

Gonseth's departure from the methods of his predecessors is so

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[316] F. Gonseth, Qu'est-ce Que La Logique, Paris: Hermaan & C^{ie}, 1937, section 53, pp. 76-78.

[317] Gonseth, op. cit., p. 77

[318] Gonseth, op. cit., p. 77. "A&B" means "A.B".

[319] Gonseth, op. cit., p. 77. "A→B" means "A⊃B".

insignificant that it would not merit mention except for Bocheński's interpretation of it.[320] Bocheński also begins with a two term primary square but he shades the areas which the proposition does not negate. For example pvq demands the shading of the p area and of the q area (LVIII d).[321] Similarly p.q demands the shading of that area which is both p and q (LVIII e) and p⊃q, the shading of all but the area which is p and not-q (LVIII f).

There are several problems with Bocheński's interpretation of Gonseth's system. When, in Marquand's system,[322] an area proves to be empty it is shaded out and as more propositions are added more areas are shaded. In Bocheński's diagrams all but the areas that prove to be empty are shaded and the addition of more propositions would necessitate the erasure of shading. This is impractical. It is also difficult and confusing to mark a shaded compartment for existential propositions. Since this final geometric system is riddled with difficulties we shall leave it and turn to the linear systems.

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[320] J. M. Bocheński, A Precis of Symbolic Logic, Translated by Otto Bird, Dordrecht: D. Reidel Publishing Company, 1959, pp. 13-14.

[321] All of these diagrams are found in Bocheński, op. cit., p. 14.

[322] II 2 (d) above.

3. Linear Diagrams

(a) Leibniz

In any English work on the logic diagram Leibniz is certain to receive unfair treatment. His papers in this field have not been published in German let alone translated. That he did use both geometric and network diagrams is evident from the page of manuscript reproduced by Bochénski[323] but no material is available on this except for that one nearly illegible page. Thus we are left with the brief fragments translated by Lewis.[324]

One example of Leibniz's diagrams will show us how primitive, and yet how effective, they were. In the second fragment, proposition 9, Leibniz wishes to prove that if A=B, then A⊕C=B⊕C.[325] As part of the proof he draws a line of which the various segments represent the various parts of the proposition (LIX).[326]

Let RS represent A
    A=B
    ∴ RS may also represent B
Let SX represent C
    RS⊕SX=RX
    ∴ RX represents A⊕C
but A=B
    ∴ RX represents B⊕C

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[323] I. M. Bochénski, A History of Formal Logic, trans. and ed. Ivo Thomas, Notre Dame: University Press, 1961, facing page 260. Bochénski also gives examples of Venn diagrams, p. 261.

[324] G. W. Leibniz, Die Philosophischen Schriften von G. W. Leibniz, Band VII, 'Scientia Generalis. Characteristica,' XIX and XX, published as "Two Fragments from Leibniz", in Lewis, op. cit., pp. 291-305.

[325] "A ⊕ B", though not defined by Leibniz in these fragments, seems to mean A and B taken together or the sum of A and B.

[326] Leibniz, op. cit., p. 298.

The further extension of this system will be obvious to the reader without further examples.

One might represent almost any proposition by means of such a line; the problem with Leibniz's system is that such diagrams would soon become so confusing as to be of no aid at all in reasoning. It should, nonetheless, be pointed out that Leibniz's diagrams, the first of the linear systems, were much more effective than those that were produced later.[327]

(b) Lambert

Accurate information on Lambert's system of linear diagrams is difficult to find. Because of its weaknesses it has generally been ignored. However, piecing together the information found in Venn[328] and Hamilton,[329] we are able to get an approximate picture of Lambert's system. Each term is represented by one horizontal line. The relation of these lines to one another represents the relationship of the terms. If the line A is shorter than the line B and would be between perpendiculars dropped from the ends of B we read "all A is B" (LX a). Similar diagrams for the other Aristotelian propositions are obvious (LX b-d). If we interpret "some" as "some or all" we may use a dotted line to indicate that portion concerning which we are in ignorance (LX e and f). The system has obvious weaknesses. It cannot represent disjunctions. Euler's system is much more flexible and Venn's even more so.[330] It is

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[327]With the possible exception of Keynes.

[328]Venn, op. cit., pp. 517-520.

[329]Hamilton, op. cit., 180, 214-217, 133, 261, 584-586, 595-597, 642-645, and especially 667-669.

[330]This is the obvious criticism. Venn goes further to criticize the system as actually employed by Lambert. Venn, op. cit., pp. 518-519.

not surprising, then, that Lambert's diagrams have virtually fallen out of use.

Hamilton disagrees with almost everything that Lambert does.[331] He calls the dotted line, Lambert's finest insight, "a line different by an accidental quality, not by an essential relation".[332] It is clear that Hamilton did not grasp the significance of the dotted line. There would be little point in itemizing Hamilton's scathing attack on Lambert. Much of the criticism is directed against Lambert's inability to cope with forms that were not even discovered in his time. It will be more useful to turn to Hamilton's attempt to improve Lambert's diagrams. We will then see that Lambert's diagrams have a natural simplicity, an obviousness, that makes them preferable to Hamilton's.

We can do no better in describing Hamilton's system of linear diagrams than to quote directly from his own description:

> Herein, four common lines are all the requisites: three (horizontal) to denote the terms; one (two?-perpendicular), or the want of it, at the commencement of comparison, to express the quality of affirmation or of negation; whilst quantity is marked by the relative length of a terminal line within, and its indefinite excurrence before, the limit of comparison. This notation can represent equally total and ultra-total distribution, in simple Syllogisms and in Sorites; it shows at a glance the competence or incompetence of any conclusions; and every one may easily evolve it.
>
> Of these, the former, [LXI a] with its converse, includes Darii, Dabitis, Datisi, Disamis, Dimaris, etc.;

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[331] ". . . and although I think linear diagrams do afford the best geometrical illustration of logical forms, I have found it necessary to adopt a method opposite to Lambert's, in all that is peculiar to him. I have been unable to adopt, unable to improve, anything." Hamilton, op. cit., p. 667.

[332] Hamilton, op. cit., p. 668.

> whilst the latter, [LXI b] with its converse, includes Celarent, Cesare, Celanes, Camestres, Cameles, etc. But of these, those which are represented by the same diagram are, though in different figures, formally the same mood. For in this scheme, moods of the thirty-six each has its peculiar diagram; whereas, in all the other geometrical schemes hitherto proposed (whether by lines, angles, triangles, squares, parallelograms, or circles), the same (complex) diagram is necessarily employed to represent an indefinite plurality of moods. These schemes thus tend rather to complicate than to explicate, - rather to darken than to clear up.[333]

From this we are able to see the weakness of Hamilton's system clearly. Euler, Lambert, Venn, and in fact everyone involved in the development of logic diagrams, was seeking a diagram to which various propositions might be applied and in which the logical conclusions of such propositions would be made obvious. In other words, most logicians were seeking a diagram which would be universally valid. Hamilton, on the other hand, was seeking particular diagrams for particular situations. This would give us a large number of distinct diagrams each representing a certain situation. Our basic aim in logic is the solution of problems. Hamilton is concerned with illustrating particular arguments; everyone else is concerned with solving them. For such solutions a universal diagram is required. In any practical problem Hamilton's linear diagrams would be of little use.[334]

Hamilton applied linear as well as circular diagrams to the relations of concepts. The faults which we have already pointed out

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[333]Hamilton, <u>op. cit</u>., 670-671.

[334]Hamilton, himself, describes his linear system as "easy, — simple, — compendious, — all-sufficient, — consistent, — manifest, — precise, — complete." Hamilton, <u>op. cit</u>., p. 672.

in his circular diagrams are aggravated by the use of linear ones (LXI c-g).[335]

There is nothing surprising in Hamilton's application of linear diagrams to sorites (LXII a and b) which is exactly analogous to his use of circular diagrams.[336]

When we turn to Hamilton's application of his diagrams in traditional logic it is with a shock that we recognize that he has abandoned his own diagrams in favour of Lambert's. The first of these diagrams (all without any perpendicular lines) shows extensive affirmation (LXIII a),[337] the second, (LXIII b)[338] intensive affirmation. The third diagram (LXIII c)[339] applies to either; we read the right side for extensive and the left side for intensive affirmation. The fourth (LXIII d)[340] is capable of a similar dual reading for extensive and intensive negation. Hamilton's linear system is at best confusing and not as adequate as Lambert's.

Keynes,[341] on the other hand, made a serious attempt to develop a more adequate system which would remain true to the principles of Lambert. There are only seven relationships which a predicate may bear

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[335] Hamilton, op. cit., p. 133.

[336] Hamilton, op. cit., p. 261.

[337] Hamilton, op. cit., p. 214.

[338] Hamilton, op. cit., p. 214.

[339] Hamilton, op. cit., p. 214.

[340] Hamilton, op. cit., p. 215.

[341] John Neville Keynes, Studies and Exercises in Formal Logic, London: Macmillan, 1928. Although earlier editions of this book discussed Lambert's system it was in this edition that Keynes introduced his own. A similar system was developed by J. Welton in A Manual of Logic, Volume I, London: University Tutorial Press, 1922, pp. 223-224.

to a subject. These relationships are all represented in a condensed diagram (LXIV a). Each line represents the entire universe of discourse. The first line bisects the universe between S and S'. Each of the other seven lines represents one of the possible ways in which that same universe may be divided between P and P'. This is the most thorough example of linear diagrams that we have.[342]

Keynes goes on to use these diagrams to represent propositions (LXIV b).[343] Dotted lines represent areas the constitution of which is uncertain. Because of this uncertainty Keynes points out that we must be careful not to represent any of the middle term in a syllogism by dotted lines as premises so represented cannot lead to a valid conclusion.[344]

The diagrams of Keynes are particularly appealing because of their value in reading off immediate inferences. Given SaP it is at once apparent from the diagram PiS, P'aS' and S'iP'.[345]

Keynes would seem to have pushed the linear diagram to the limit of its representational ability. He has made it adequate for Aristotelian logic but its weakness in more modern logic remains.

This weakness of all linear systems is obvious. Any system, such as the Lambert-Hamilton system, simple enough to be meaningful, is not capable of application to complex relations; any system capable of covering the complex subtleties found in the geometric systems would be so complicated as to be of no help.

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[342] Keynes, op. cit., pp. 174-176.

[343] Keynes, op. cit., p. 176.

[344] Keynes, op. cit., p. 344.

[345] Keynes, op. cit., p. 176.

4. Network Diagrams

(a) Squares of Opposition

The square of opposition first made its appearance in Apuleius (LXV a)[346] and a somewhat simplified form (LXV b)[347] has been in common use for centuries. The square of opposition is a network made up of the four sides and two diagonals of a square (or rectangle). The corners represent the A, E, I and O propositions. The lines represent the relationships between them. There are, in all, four relations: contrariety, subcontrariety, contradiction and subalternation. The A and E are contraries: they may both be false but they cannot both be true. The I and O may both be true but cannot both be false and are, therefore, called subcontraries. The A and O are called contradictories because either the one or the other, but not both, must be true. The E and the I are a second pair of contradictories. If the A is true the I is true but if the I is true the A may or may not be true; this relationship is called subalternation, the I is called the subaltern of the A and the A is called the superaltern of the I. Similarily the O is the subaltern of the E and the E is the superaltern of the O. It is to be noted that all of these relationships presuppose that the subject of the propositions has existential import.

Boolean algebra allows the possibility that S=0 and P=0. This introduces serious problems into the square of opposition (LXVI a).

Let $S=0$
$\therefore S\bar{P}=0$
and $SP=0$
$\therefore SP\neq 0$ is not the subaltern of $S\bar{P}=0$
and $SP=0$ is not the contrary of $S\bar{P}=0$
and $S\bar{P}=0$ is not the subaltern of $SP=0$

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[346] I. M. Bocheński, A History of Formal Logic, p. 141.

[347] Irving Copi, Introduction to Logic, New York: MacMillan, 1961, pp. 142-149 and p. 161.

and SP≠0 and $S\bar{P}$≠0 cannot be subcontraries.
∴ all the relationships on the sides of the square break down when S=0

The Boolean square of opposition is left with a cross made up of the diagonals.

A similar square of opposition (LXVI b)[348] may be produced for a propositional calculus using quantifiers. If there is at least one x the various relationships between $(x)\phi x$, $(x){\sim}\phi x$, $(\exists x)\phi x$ and $(\exists x){\sim}\phi x$ are the same as those in the traditional square of opposition. In other cases[349] the relationships are the same as those in the Boolean square.

Finally we turn to the square of opposition for particular formulae in the propositional calculus. This square shows the relationship of four basic propositions: (p.q), (~p.~q), (pvq) and (~pv~q). For convenience in diagramming we may translate to the Lukasiewicz's notation. If EJpqEpNq and EDpqNKpq[350] we may relate the four propositions within six tautologies and diagram the entire system with the vertices representing the propositions and the lines representing their tautologous relationships (LXVI c).[351] The tautologies are:

DKpqKNpNq

AApqANpNq

CKpqApq

CKNpNqANpNq

JKpqANpNq

JKNpNqApq

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[348] Copi, op. cit., p. 311.

[349] ie. cases where there is not one x.

[350] Definitions of D and J

[351] Bocheński, A Precis of Mathematical Logic, p. 14.

The square of opposition has proven itself to be a convenient device for showing relationships. It is not a true logic diagram in the sense that it is not generally capable of solving problems. It is, nonetheless, a very convenient instrument when carrying out some of the basic mechanical steps necessary to logical solutions.

(b) The Pons Assinorum and the Ars Magna

In the Mediaeval pons assinorum there was a serious attempt to show all the possible relationships between the terms in the various categorical propositions and especially to reduce to a simple formula the position and relationship of the middle term of a syllogism. Two such diagrams are reproduced here (LXVII a and b).[352] It will be seen that they conveniently give all possible combinations of the premises but that they are no aid in reaching the conclusion. The following chart of definitions will assist in reading our examples of the pons assinorum:

A - the proposition contains P
B - the middle term follows the predicate
C - the middle term is antecedent to the predicate
D - the position of the middle term is extraneous
E - the proposition contains S
F - the middle term follows the subject
G - the middle term is antecedent to the subject
H - the position of the middle term is extraneous.

Lull also produced diagrams in his ars magna meant to aid in combinations but he was interested in combining terms, not premises. Our brief account of one such diagram follows Gardner who devotes an entire chapter to Lull.[353] The diagram we are using for illustration

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[352] Bocheński, A History of Formal Logic, pp. 143-144 and especially 219-221 and diagram facing p. 220.

[353] Gardner, op. cit., pp. 1-25. Bocheński also discusses Lull in A History of Formal Logic, pp. 272-273 and gives one diagram facing p. 274.

(LXVIII)[354] contains, around its border, a list of sixteen attributes of God. Each attribute is represented by a letter in the second circle. We know these sixteen facts about God. To gain further knowledge we simply choose one attribute and follow one of the lines leading from it. It may pass through A, which represents God;[355] we continue on the other side until we arrive at another attribute. Thus we may, for example, reach the conclusion BC ("God's goodness is great"), BD ("God's goodness is eternal") and CD ("God's greatness is eternal"), etc. To simplify matters we may rotate the inner circle on the outer bringing together the various attributes in the respective circles. This frees us from the maze of lines but accomplishes exactly the same task.[356] Such a device is able to give the operator access to all possible combinations of terms. It is not actually able to assist in finding the solution of problems. Thus we are justified in turning to the more sophisticated network systems which can, indeed, make such solutions more apparent.

(c) Frege

It is necessary to defend the examination of Frege's notation. It is, after all, simply another form of notation for mathematical logic somewhat different from Russell's. Why then should we be studying it as a type of logic diagram? Although a full defense must wait until we have examined his notation in some detail, it may be said here that the structure of Frege's system seems to have diagrammatic as well as algebraic

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[354]Gardner, op. cit., figure 4, p. 11, described pp. 10 and 12.

[355]If it does not pass through A we do exactly the same thing. The introduction of the "A" in the diagram has no effect on its use.

[356]It is obvious from this that Lull's diagrams are on the borderline between diagrams and machines.

import. We shall conclude this section with a comparison of Frege's notation and the diagrams used for switching circuits to prove this point.

The basic unit of Frege's system is the proposition. Simple propositions are of two sorts: Judgments and thoughts. A thought is represented by a horizontal stroke (in the Grundgesetze) or a "content" stroke (in the Begriffsschrift) followed by a name (LXIX a).[357] A judgment is an asserted thought. The vertical judgment-stroke to the left asserts that the content of the proposition is true (LXIX b).[358] Judgments always assert. It is necessary to indicate negation by means of a short vertical stroke below the horizontal (LXIX c).[359] If we are given two or more horizontals, one or less of which has a negation stroke, they may be amalgamated into one horizontal (LXIX d-g) and the process may be reversed (LXIX h-k).[360]

Now we must choose a basic unit of combination, a binary relation in terms of which our system may be developed. Frege, although conscious

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[357] Gottlob Frege, Begriffsschrift und Andere Aufsätze, Hildesheim: Georg Olms Verlagsbuchlandlung, 1964, p. 1 (Hereafter called Begriffsschrift). Also Gottlob Frege, The Basic Laws of Arithmetic. Trans. and ed. Montgomery Furth, Berkeley and Los Angeles: University of California Press, 1964, p. 38 (Hereafter called Grundgesetze).

[358] Frege, Begriffsschrift, p. 2 and Grundgesetze, p. 38.

[359] The negation is then a part of the thought and not a part of the judgment. Frege, Begriffsschrift, p. 10 and Grundgesetze, p. 39.

[360] Frege, Grundgesetze, pp. 39-40.

that he might have chosen differently, chooses that state of affairs in which Δ cannot be true while Γ is false. This state of affairs is indicated by inserting the conditional stroke, a vertical line perpendicular to the horizontal of Γ and intercepting the left tip of the horizontal of Δ (LXX a).[361] The negation of this entire statement requires a negation stroke immediately before the conditional stroke (LXX b).[362] A negation stroke after the conditional stroke negates only the particular proposition involved (LXX c-e).[363] This may be further expanded by the addition of another term. A new term implying the first proposition is simply attached to the horizontal by means of a new conditional stroke (LXX f)[364] and one implying only one of the secondary terms is attached to the horizontal of that term in the same manner (LXX g).[365]

Frege introduces Gothic letters to limit the scope of arguments. A generality[366] is preceded by a Gothic letter which appears in the argument. This letter is put over an identation in the horizontal

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[361] Frege, Begriffsschrift, p. 5 and Grundgesetze, p. 51.

[362] Frege, Grundgesetze, p. 51.

[363] Frege, Begriffsschrift, p. 11. Diagram LXX e is not actually given there.

[364] Frege, Begriffsschrift, p. 6 and Grundgesetze, p. 52.

[365] Frege, Begriffsschrift, p. 7.

[366] Perhaps "universal generalization" would be better than Furth's word "generality" but we have retained it for simplicity of reference. The term is introduced on p. 40 of Grundgesetze.

(LXXI a).[367] The Gothic letter is placed before the negation stroke to express the generality of a negation (LXXI b)[368] and after to express the negation of a generality (LXXI c).[369] Particularity may be thus expressed by negating the generality of a negation. Such an expression (LXXI d)[370] is read "There is at least one . . ." If the Gothic letter appears to the right of the conditional stroke (LXXI e)[371] it applies only to that argument or those arguments traced out to its right. If it appears to the left of all the conditional strokes it applies to all the arguments in the proposition (LXXI f).[372]

Frege recognizes two methods of interchangeability. Two subcomponents of a proposition may be interchanged. Diagram LXXII a can become diagram LXXII b.[373] The second method of interchangeability is called contraposition and is thus described by Frege:

> A subcomponent may be interchanged with the main component if the truth-value of each is simultaneously reversed [LXXII c-f].[374]

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[367]Frege, Begriffsschrift, p. 19 and Grundgesetze, p. 41.

[368]Frege, Begriffsschrift, p. 23 and Grundgesetze, p. 41.

[369]Frege, Begriffsschrift, p. 22 and Grundgesetze, p. 41.

[370]Frege, Begriffsschrift, p. 23 and Grundgesetze, p. 42.

[371]Frege, Begriffsschrift, p. 21.

[372]Frege, Begriffsschrift, p. 24 and Grundgesetze, p. 55.

[373]Frege, Grundgesetze, p. 53. His treatment of interchangeability in the Begriffsschrift deals with specific cases.

[374]Frege, Grundgesetze, p. 60, italicized in the original. In the Begriffsschrift he treats contraposition as he did interchangeability proving its validity in each particular case.

Frege introduces three methods of inference. The first is stated as follows:

> If a subcomponent of a proposition differs from a second proposition only in lacking the judgment-stroke, then a proposition may be inferred that results from the first proposition by suppressing that subcomponent.[375]

To simplify matters each of the original propositions is given an "index" by means of the Greek letters "α", "β", etc.[376] If the proposition with the lower index is written in full in the argument the second proposition is indicated by means of its index and a double colon before a single long solid stroke between the initial proposition and the conclusion (LXXIII a). If the second proposition is used a single colon is placed after the index of the first proposition but the remainder of the solution is the same (LXXIII b). The long single line before the conclusion is to be taken as the mark of the first method of inference. The number of times this line appears indicates the number of times the method must be applied to reach the conclusion.

The second method of inference is marked by a broken instead of a solid line and may be described as follows:

> If the same combination of signs occurs in one proposition as main component and in another as subcomponent, a proposition may be inferred in which the main component of the second is main component, and all subcomponents of either, save the one mentioned, are subcomponents. But subcomponents occurring in both need be written only once [LXXIV a and b].[377]

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[375]Frege, Grundgesetze, p. 58, italicized in the original. Diagrams were on p. 57. This is the sole method of inference in the Begriffsschrift p. 7 ff.

[376]Frege, Grundgesetze, p. 57.

[377]Frege, Grundgesetze, p. 63, italicized in the original. The diagrams are on p. 59.

The third method of inference differs in symbolism in its use of a line broken into dots and dashes. Frege states the rule for the third method thus:

> If two propositions agree in their main components, while a subcomponent of one differs from a subcomponent of the other only in a negation-stroke's being prefixed, then a proposition may be inferred in which the common main component is main component, and all subcomponents of either, save the two mentioned, are subcomponents.[378]

Frege's example (LXXV a) is not the simplest case and we have added a diagram for that case in which only one subcomponent is present in each proposition (LXXV b)[379] to cover the simplest possible situation.

The following comparative table, using Copi,[380] shows how Frege's methods of inference are related to those in a standard textbook of contemporary symbolic logic:

Frege		Copi
1. First type of interchangeability	=	commutation of conjunction plus material equivalents (LXXII a and b)
2. Second type of interchangeability	=	a) transposition (LXXII c and d) or b) commutation of disjunction (LXXII e) or c) commutation of negated conjunction (LXXII f)
3. First method of inference	=	Modus Ponens (LXXIII)
4. Second method of inference	=	Hypothetical Syllogism (LXXIV)
5. Third method of inference	=	elimination of tautologous alternatives (not in Copi) (LXXV)

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[378] Frege, Grundgesetze, p. 65, italicized in the original. Diagram LXXV a is on pp. 64-65.

[379] This diagram does not appear in Frege.

[380] Copi, op. cit., These rules are given on pp. 277 and 283.

We have generally followed the symbolism of the Grundgesetze which is much more condensed than that in the Begriffsschrift but the basic principles of symbolism are similar in both.

There are, of course, weaknesses in Frege's system examined as either symbolism or diagrams. This symbolism is cumbersome and at times difficult to read. Nonetheless, it is significant for its attempt at universality and especially for the introduction of Gothic letters to indicate quantification for the first time in the modern sense.

As diagrams Frege's branching figures have several faults. The most important is their failure to universalize. It is possible to draw almost any proposition or set of propositions using them but there is no general diagram such as we find in Venn, Marquand, etc. This means that we must begin anew to draw the diagram for each new proposition. It is true that there is a general structure of the diagrams but there is no single universal diagram which incorporates all the alternatives.

We must, finally, ask whether we should have examined Frege's branching structures as diagrams or whether we should have relegated them to the realms of symbolism and ignored them. In order to defend our examination of Frege it is necessary to look briefly at the diagrams used in electrical engineering to map switching circuits in computers. There are three basic switching circuits: the AND circuit (LXXVI a), the OR circuit (LXXVI b) and the inverter circuit (LXXVI c).[381] Usually Venn or Marquand diagrams are used to illustrate these if diagrams are

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[381]Allan Lytel, abc's of Boolean Algebra, New York: The Bobbs-Merrill Company, 1965, p. 15. Montgomery Philster Jr., Logical Design of Digital Computers, New York: John Wiley, 1963, pp. 30 ff.

used at all (e.g. LXXVI d[382] and e).[383] It is easily seen (LXXVI f and g) that Frege's diagrams do not fit the switching circuits well but, with a small adaptation we can make them far more adequate than the Venn diagrams. We simply let the basic Frege structure represent A . B instead of B ⊃ A. Now it will be seen that Frege diagrams represent switching circuits with a one to one correspondence (E.g. LXXVI h, i and j) while the Venn diagrams, far from clarifying the situation, add considerable confusion by attempting to illustrate a network by means of a geometric area. Thus Frege's diagrams, despite their weakness as logic diagrams, would seem to be very valuable for illustrating the logical structure of networks. In fact, it is the fact that they strip away that part of the logical universe which is irrelevant to the proposition that makes them useful in illustrating networks and deceptive in illustrating logic. Thus whether they were originally meant to be diagrams or not there are cases in which they function extremely well as logic diagrams.

(d) Peirce[384]

When we come to Peirce's diagrams we find ourselves with an embarrassment of riches. Peirce's final system is the most complete

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[382] Lytel, op. cit., p. 26. See also Philster, op. cit., pp. 34-35.

[383] Lytel's squared diagram given here is a negative of a Marquand diagram for 3 terms. Lytel, op. cit., p. 81. See also Philster, op. cit., pp. 48-49.

[384] The pivotal works by Peirce on diagrams are Peirce 4.347-4.584 but other significant insights are scattered throughout Peirce's work. In an unpublished lecture of D. D. Roberts, Toronto, Dec. 1965 it was pointed out that many of the significant papers on diagrams by Peirce have not been published.

system of logic diagrams yet developed. In addition to this, Roberts[385] has added to this system to make it adequate for almost any purpose. To examine Peirce's diagrams thoroughly would require much more time and space than are available here; yet we profess to examine all of the logical systems. What then are we to do? Our plan for this section will be as follows: (1) we shall describe the essential symbols of Peirce's various systems; we shall not go on to explain the rules of operation etc., but anyone wishing a thorough analysis will be directed to the relevant portions of Roberts' thesis, (2) we shall describe briefly Roberts' interpretation of Peirce's system with emphasis on his improvements of that system rendering it operative for functional calculus, and (3) we shall discuss evaluations, especially Peirce's own evaluation, Gardner's unfavourable evaluation and Roberts' favourable evaluation.[386]

Before we present the systems it might be well to mention the influences which were most important in their development. During his period of teaching at Johns Hopkins University, Peirce associated with, and was influenced by William Clifford. Clifford, with James J.

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[385]Roberts, The Existential Graphs of C. S. Peirce.

[386]It is, of course, recognized that this gives us a very superficial picture of Peirce's system. The justification for this superficiality rests in the fact that we have Roberts' analysis of Peirce while we have little on the other systems discussed.

Sylvester, developed a method of writing algebraic formulae using chemical diagrams.[387] These diagrams suggested to Peirce that logical variables might be thought of as having valences and might be represented in diagrams similar in structure to chemical diagrams. A. B. Kempe's "A Memoir on the Theory of Mathematical Form",[388] in which valence diagrams of unordered pairs were used as a basis for mathematics, further suggested that there was something basic about such diagrams, that they well might be the basis for all human thought. This concept of valency will appear especially in the gamma part of existential graphs but appears, at least to some degree, even in Peirce's very early diagrams which were developed prior to the Kempe article.[389]

Peirce's first system of diagrams appeared in a letter to his student O. H. Mitchell.[390] In these diagrams, as in all Peirce diagrams, the lines represent individuals and the variables relationships.[391] Each

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[387]J. J. Sylvester, "On an Application of the New Atomic Theory to the Graphical Representation of the Invariants and Covariants of Binary Quantics, — With Three Appendices", American Journal of Mathematics Pure and Applied, Vol. I, 1878, pp. 64-125. W. K. Clifford, "Remarks on the Chemico- Algebraical Theory", (Extract from a letter to Mr. Sylvester from Prof. Clifford of University College, London), American Journal of Mathematics Pure and Applied, Vol. I, 1878, pp. 126-128. Peirce and Clifford were both regular contributors to this Journal of which Sylvester was editor-in-chief.

[388]A. B. Kempe, "A Memoir on the Theory of Mathematical Form", Philosophical Transactions of the Royal Society of London, Vol. 177, 1886, pp. 1-70.

[389]It was prior also to the Clifford and Sylvester articles but not prior to Peirce's association with Clifford.

[390]C. S. Peirce, "Letter to O. H. Mitchell", Dec. 21, 1882, Unpublished. This system was described by Roberts in his lecture cited above. See fn. 383. See also Roberts, The Existential Graphs, pp. 34-38.

[391]This dramatic difference in representation would seem to stem from Peirce's metaphysics. For Peirce relationships would seem to be the basic category and individuals are functions of relationships.

line can, then, be read as "something" or "there exists an". Thus Diagram LXXVII a[392] may be read $\Sigma x \Sigma y bxy\ Lxy > 0$,[393] or "there exists an individual x and there exists an individual y, such that x is a b of y and x is an L of y." One may express a thing's relationship to itself as in Diagram LXXVII b, $\Sigma x\ Lxx > 0$.

To express "everything" one draws a perpendicular line through the line representing the individual as in Diagram LXXVII c, $\Pi x\ Lxx > 0$ and Diagram LXXVII d, $\Pi x\ \Pi y(Lxy\ \ bxy) > 0$.

Peirce attempted to distinguish between alternation and conjunction by using straighter and shorter lines to represent bonds which are attached later. Thus Diagram LXXVII e represents $\Sigma y \Pi x(Lxy \psi bxy) > 0$ [394] while Diagram LXXVII f represents $\Pi x \Sigma y(Lxybxy) > 0$.

This primitive system contains much that will appear in the later systems but lacks a notation for negation and Peirce's own estimate of it was much lower than that of his later work.

Peirce's second system[395] makes more use still of valency[396]

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[392] All of the diagrams in LXXVII are from Roberts' lecture.

[393] Peirce has a great many symbolic systems. In this one "Σ" is the existential quantifier. x and y are objects and b and L are relations between objects. The "$>$" is read "is greater than" and is equivalent to "$\neq$" in Boolean algebra.

[394] "ψ" is the symbol for alternation.

[395] "Entitative graphs". Most of Peirce's work in this area is found in 3.456-3.552.

[396] Of course any network system of diagrams will entail valency as far as variables are related to a finite number of other variables. What makes Peirce so significant is his awareness of this and his deliberate attempt to construct his system with it in mind.

Compare Peirce's diagram of "John gives John to John" (LXXVII a) to the chemical formula diagram for ammonia (LXXVII b).[397] As we are about to see, however, this particular type of diagram is in contradiction to the conventions of entitative graphs and particularly to the use of the "cut" for negation. Peirce's aims in using such diagrams are, nevertheless, obvious from the above example and improvements on the valency diagram will be seen in the gamma part of existential graphs.

The conventions of entitative graphs are as follows:[398]

(1) To write a proposition is to assert it. Thus Diagram LXXVII c asserts "P".[399]

(2) To write two propositions is to assert their alternation. Thus Diagram LXXVII d asserts "P or Q".[400]

(3) To encircle a proposition is to negate it. Thus Diagram LXXVIII e asserts "not P".[401]

(4) To write a conditional proposition we encircle the antecedent. Thus Diagram LXXVIII f asserts "if P then Q".[402]

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[397] This comparison is made by Peirce, 3.469.

[398] We are following Roberts' description of "Entitative Graphs". To do a thorough analysis of Peirce's paper would entail another paper as long as the present. It should be pointed out that Roberts' description is a condensed view abstracted from Peirce who deals with particular cases. It is not immediately evident that they are actually doing the same thing but it will be revealed by a careful comparison.

[399] Roberts, op. cit., p. 46, Fig. 7. We have used variables in place of the propositions used by Roberts in all of these diagrams. Our reasons will become evident below.

[400] Roberts, op. cit., p. 46, Fig. 8.

[401] Roberts, op. cit., p. 46, Fig. 9.

[402] Roberts, op. cit., p. 47, Fig. 10. This could be derived from Diagram LXXVIII d and e.

(5) To assert a conjunction we negate both propositions and then encircle the entire graph. Thus Diagram LXXVIII g asserts "both P and Q".[403]

(6) A line or dash represents an individual object.[404] Two corollaries follow from (6).[405]

(6a) A line whose least enclosed extremity is unencircled or is encircled an even number of times is read "all" or "every".[406]

(6b) A line whose outermost extremity is encircled an odd number of times is read "some".

Thus "everything P is Q" would be represented by Diagram LXXVIII h[407] and "Something P is not Q" by Diagram LXXVIII i[408] etc.

The weaknesses of this system are obvious. Why is it to assert "P" to write it, yet, to assert "P" or "Q" to write both P and Q? Such a decision seems purely arbitrary and makes the system unnecessarily complex. Peirce rectified this in the alpha part of existential graphs.

The conventions of the alpha part of existential graphs are few and simple.[409] For the alpha part we may ignore valency and individuals

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[403] Roberts, op. cit., p. 47, Fig. 11. This could be derived from Diagram LXXVIII d and e.

[404] Roberts, op. cit., p. 47. Thus P and Q are not really propositions. They might be said to be propositions about particular individuals but what they represent is the various situations or relationships into which individuals may enter.

[405] These really follow, of course, from (6) in conjunction with the foregoing conventions. These two rules are given at Peirce, 3.479.

[406] We always begin reading with the least enclosed extremity. Peirce, 3.479.

[407] Roberts, op. cit., p. 48, Fig. 12.

[408] Roberts, op. cit., p. 48, Fig. 13.

[409] Peirce, 4.394-4.402, 4.414-4.415, 4.424-4.437, 4.485-4.498. Roberts, op. cit., pp. 60-84.

and limit ourselves to two symbols: the variable and the cut. To write or "scribe" a proposition is to assert it.[410] Thus Diagram LXXIX a asserts "P",[411] and Diagram LXXIX b asserts "P and Q".[412] To negate a proposition we enclose it within a cut because any proposition written on the sheet of assertion is asserted. The cut is, then, an area separated off as apart from the sheet.[413] Thus Diagram LXXIX c represents "not P".[414] We express a conditional by enclosing the

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[410]Peirce, 4.397. Roberts, op. cit., p. 61. It may be noted here that Peirce's use of technical terms makes his work difficult but he believed that they were essential to his intention and we will find ourselves resorting to them. In another context Peirce defended the use of such terms:

> For philosophical conceptions which vary by a hair's breadth from those for which suitable terms exist, to invent terms with a due regard for the usages of philosophical terminology and those of the English language but yet with a distinctly technical appearance. Before proposing a term, notation, or other symbol, to consider maturely whether it perfectly suits the conception and will lend itself to every occasion, whether it interferes with any existing term, and whether it may not create an inconvenience by interfering with the expression of some conception that may hereafter be introduced into philosophy. Having once introduced a symbol, to consider myself almost as much bound by it as if it had been introduced by somebody else; and after others have accepted it, to consider myself more bound to it than anybody else. (Peirce, 2.226)

Peirce seems to have followed this program rigorously with regard to his existential graphs.

[411]Roberts, op. cit., p. 61. We shall use Roberts' diagrams since they are given systematically. The same diagrams will be found scattered through Peirce. Again we have substituted variables for propositions where necessary. Note: Roberts, pp. 60-79, diagrams are unnumbered.

[412]Roberts, op. cit., p. 63.

[413]Peirce, 4.414. "A cut drawn upon the sheet of assertion severs the surface it encloses, called the area of the cut, from the sheet of assertion; so that the area of a cut is no part of the sheet of assertion."

[414] Roberts, op. cit., p. 71, Fig. 3.

antecedent in the "outer close" and the consequent in the "inner close" of a "scroll". A scroll is simply a double cut or a cut within a cut (LXXIX d).[415] Finally the empty cut (LXXIX e) represents or expresses any absurdity and is called by Peirce the "pseudograph".[416]

The beta part[417] of existential graphs includes everything from the alpha part and introduces the dot (LXXX a)[418] or the dash (LXXX b),[419] called the line of identity, to represent the individual. Either Diagram LXXX a or LXXX b would, thus, be read "something exists". The second beta convention allows such a line to join two propositions or variables. When this is done the resulting proposition is read as "something is both P and Q" (LXXX c).[420] Peirce applies the term ligature to the point where a line of identity branches. A branching line of identity expresses the identity of the n individuals at its n extremities. Thus Diagram LXXX d is read "something is P and Q and R".[421] When a line of identity is entirely enclosed by a cut (LXXX e)[422] the resulting graph is read "it is

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[415] Roberts, op. cit., p. 66

[416] Roberts, op. cit., p. 69.

[417] Peirce, 4.403-4.408, 4.416-4.417, 4.438-4.462, 4.475-4.484, 4.499-4.509; Roberts, op. cit., pp. 87-121.

[418] Roberts, op. cit., p. 88. Roberts' diagrams are unnumbered pp. 87-91.

[419] Roberts, op. cit., p. 88.

[420] Roberts, op. cit., p. 89.

[421] Roberts, op. cit., p. 91.

[422] Roberts, op. cit., p. 92, Fig. 2.

false that something is P" or "nothing is P". If a line crosses a cut (LXXX f)[423] it may more easily be read if it is broken at the ligature (i.e. where it crosses the cut). We then have examples of Diagrams LXXX b and e. Thus Diagram LXXX f is read "something exists and it is false that this is P" or more simply "something is not P". If the line of identity passes through an empty cut it asserts the existence of the two individuals at its extremities and renders their identity absurd. Thus Diagram LXXX g[424] is read "P and Q are not the same individual". As we have seen, a ligature (e.g. LXXX h)[425] asserts the identity of all individuals with lines of identity opening on the ligature. Peirce introduces the "bridge" which allows lines of identity to cross without forming a ligature (LXXX i).[426] This is merely a convenient and sometimes necessary notation.

The gamma part of existential graphs may be divided into three parts, the first dealing with metagraphs,[427] the second with abstraction[428]

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[423] Roberts, op. cit., p. 93, Fig. 3.

[424] Roberts, op. cit., p. 99, Fig. 15.

[425] Roberts, op. cit., p. 101, Fig. 2.

[426] Roberts, op. cit., p. 101, Fig. 4.

[427] The metagraph is a graph of a graph. This is one form of "abstraction" in Peirce's sense of the word. Abstraction is to treat a symbol as an ens rationis. On the other hand there seems to be a distinct difference in dealing with graphs in this manner (what we have termed "metagraphing") and in doing the same thing in treating qualities, relations and particular objects (for which we have retained the term "abstraction"). Peirce, 4.409-4.413, 4.528-4.529, Roberts, op. cit., pp. 124-130. "Metagraphing" is really a particular application of "abstraction".

[428] Peirce, 4.409-4.413, 4.463-4.474, 4.524-4.527. Roberts, op. cit., 123-124, 130-136.

and the third with modality.[429] When Peirce develops the system of graphs to be used in diagramming his graphs he introduces a great many symbols all of which are structured according to the same principle: a variable is identified with some aspect of the graph to be represented. We will give five of these and use them in examples so that the reader may see Peirce's aims in this section. Anyone wishing a more detailed analysis or a more complete list of symbols is referred to Roberts or to Peirce's original work.[430] Let us then draw diagrams to express "x is the sheet of assertion" (LXXXI a), [431] "x is a graph precisely expressing 'P'" (LXXXI b), "x is scribed on y" (LXXXI c), "x is the area of y" (LXXXI d) and "x is a cut" (LXXXI e). Any graph may be precisely defined in graphical terms. For example the graph for "P" (LXXXI f) may be graphically defined by (LXXXI g)[432] which is read "a graph precisely expressing 'P' is scribed on the sheet of assertion". A more complex example is the graphical statement of the graph for "not P" (LXXXI h). This graph may be precisely described by Diagram LXXXI i[433] which is read "a graph precisely expressing 'P' is scribed

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[429] Peirce, 4.510-4.523. Roberts, op. cit., pp. 136-140.

[430] See fn. 426 above.

[431] Diagrams LXXXI a-e are given by Roberts, op. cit., p. 125 and by Peirce, 4.528-4.529.

[432] Roberts, op. cit., p. 127, Fig. 4.

[433] Roberts, op. cit., p. 128, Fig. 6.

on the area of a cut which is itself scribed on the sheet of assertion". These examples should be sufficient to give the reader the general idea.

A second similar set of symbols represent abstraction.[434] In this section Peirce distinguishes between monadic, dyadic and triadic relations. All greater relations may be worked out from these three. A dyadic relation may be expressed, for example, as in Diagram LXXXII a which is read "y is in relation x to z".[435] An example of such relations in use is Diagram LXXXII b[436] which is read "Cyrano loves Roxanne but Roxanne does not love Cyrano" or in its expanded form "there are two individuals — Cyrano and Roxanne — and a dyadic relation 'loves' such that Cyrano loves Roxanne but Roxanne does not love Cyrano". Now the line of identity indicating the relation "loves" is somewhat suspect for such lines are supposed to indicate individuals. Peirce therefore proposed that we enclose such a line with dots (LXXXII c)[437] or replace it with "R" at its termini (LXXXII d)[438] to indicate that a relation is represented and not an individual. Again the reader is referred to Roberts for a more detailed exposition than can be given here.

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[434]Since these work on the same basic principle as those used for metagraphing we have not given them. The interested reader will find them in Peirce, 4.524 or in Roberts, op. cit., p. 130.

[435]Roberts, op. cit., p. 130.

[436]Roberts, op. cit., p. 131, Fig. 13.

[437]Roberts, op. cit., p. 132, Fig. 14.

[438]Roberts, op. cit., p. 132, Fig. 15.

Finally we mention Peirce's attempt to deal with modality (i.e. to express possibility and impossibility, necessity and contingency). To do this we introduce the broken cut. The broken cut expresses the fact that the entire graph on its area is contingent. Thus Diagram LXXXIII a means "it is possible that 'P' is false".[439] To express the fact that P cannot be false we need only enclose Diagram LXXXIII a in a closed cut (LXXXIII b) and this may be read "it is false that 'P' is possibly false" or "'P' is necessarily true".[440] To express the possibility that P is true we need only enclose not P in a broken cut. Thus Diagram LXXXIII c reads "it is possible that it is false that 'P' is false".[441] If we wish to express the falsity of this we simply enclose Diagram LXXXIII c in a solid cut (LXXXIII d) and read the result as "it is not true that it is possible that 'P' is true" or simply "'P' is impossible".[442] Concentric broken cuts (LXXXIII e) may be read "it is possible that 'P' is necessary".[443] This system is much more adequate than one would expect of an attempt to produce diagrams for modality, but Peirce wanted to improve on it in situations where various cases of contingency were encountered. For this purpose he introduced the tinctures.[444] One imagines cuts actually cutting through the surface

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[439] Roberts, op. cit., p. 138, Fig. 4.

[440] Roberts, op. cit., p. 138, Fig. 5.

[441] Roberts, op. cit., p. 138, Fig. 8.

[442] Roberts, op. cit., p. 139, Fig. 9.

[443] Roberts, op. cit., p. 139, Fig. 12.

[444] Peirce, 4.552-4.572. Much of this is a repetition of the alpha and beta conventions but 4.553 ff. introduce the tinctures. See also Roberts, op. cit., pp. 140-142. A similar attempt was made by Peirce using the "verso" of the sheet of assertion (4.573-4.584) but this is equally obscure.

of the sheet of assertion and exposing various layers which are differentiated by having various emblems representing twelve tinctures which are grouped in three groups of four according to modality. Peirce himself soon realized that this system was unworkable[445] but the remainder of his work was just as relevant when the tinctures were discarded.

Roberts, who is very sympathetic toward Peirce's system, made an attempt to improve the beta part of existential graphs in such a way that these graphs would be an adequate substitute for the functional calculus. The beta graphs lack a symbol for quantification. Thus Diagram LXXXIV a expresses $Fx \supset Gx$. What is needed is the possibility of expressing $(\forall x)(Fx \supset Gx)$.[446] Roberts proposes that the quantifier be placed next to the line of identity. Since the existential quantifier may be derived from the universal quantifier we need only consider the latter.[447] Thus $(\forall x)(Fx \supset Gx)$ may be expressed as in Diagram LXXXIV b.[448] This forces Roberts to add a graphical equivalent to the rule of universal generalization. It is simply that given A (LXXXIV c) we may always infer that it is not true that for any x whatsoever not A (LXXXIV d).[449] These are the major changes that Roberts made in Peirce's system.

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[445] See Roberts, op. cit., p. 142 for an account of this.

[446] The problem, of course, is not so much the expression of $(\forall x)(Fx \supset Gx)$: Diagram LXXXIV a could be taken by convention to express this. Rather it is the need to express $(\forall x)(Fx \supset Gx)$ in such a way that it may be distinguished from $(\forall y)(Fy \supset Gy)$ or any other expression of the same form.

[447] $((\exists x)(Ax)) \equiv (\sim(\forall x)\sim(Ax))$.

[448] Roberts, op. cit., p. 212.

[449] Roberts, op. cit., p. 212.

With the amount of information given here we could not reach an adequate evaluation of Peirce's system; we therefore presuppose Roberts' more thorough analysis in making our comments.

The consensus of logicians has been against Peirce's diagrams. Most logicians simply ignored them and turned to what they considered to be Peirce's significant contributions to symbolic logic. Those who treated them at all were generally unsympathetic. Gardner, whose work on Venn and Carroll was very astute and whose own system is exciting, was unimpressed by Peirce's diagrams. Since we will be arguing against the critics and on the side of Roberts, we had, perhaps, best let Gardner speak for himself so that there will be less danger of our distorting his position.

> His [Peirce's] several papers on the topic (reprinted in Vol. 4 of his Collected Papers) are written in such an elliptic, involuted style that one is led to wonder if Peirce harbored unconscious compulsions toward cloudy writing that would enable him to complain later of his critics' inability to understand him. Add to this opaque style his use of scores of strange terms invented by himself and altered from time to time, and the lack of sufficient drawings to illustrate the meaning of these terms, and the task of comprehending his system becomes formidable indeed.[450]
>
> These noniconic aspects of Peirce's system give it an air of arbitrariness and disjointedness. The parts do not seem to hang together. One has the feeling that, if twelve competent modern logicians were to set themselves the task of constructing similar graphs that would encompass the whole of logic, each would come up with a different system, and each as good if not better than Peirce's. At any rate, there is no question that Peirce, like Ramon Lull (whom Peirce in an unguarded moment called an "acute logician"), held a greatly exaggerated notion of the value of his diagrams.[451]

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[450] Gardner, op. cit., pp. 55-56.

[451] Gardner, op. cit., p. 58.

This is the case for the opposition. It ought to be pointed out here that Gardner praises Peirce for the attempt and says that his work may suggest future lines of thought.[452] On the whole, however, there was no sympathetic examination of Peirce's diagrams until Roberts'.

We will confine our evaluation of Peirce's system to the existential graphs since the early graphs are very similar to the beta part of existential graphs and the alpha part is entitative graphs "turned inside out".[453] This evaluation will consider three functions of the graphs: (1) the value of the graphs for logical experimentation, (2) the value of the graphs as a calculus, and (3) the value of the graphs for logical analysis. Again our comments are necessarily brief and the interested reader is referred to Roberts.

That the diagrams do, in fact, encourage experimentation[454] there can be no doubt. With the logical relations spread over two dimensions, as opposed to one in algebraic notation, one is, so to speak, invited to insert double cuts, iterate and deiterate[455] and carry out the various other possible transformations which become more clear and obvious for being vizualized. This experimentation may, and according to Peirce and Roberts does, lead to the discovery of logical truths which may not be obvious or may, in fact, be very obscure in algebraic

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[452]Gardner, op. cit., p. 58. Even in this passage, however, Gardner mentions Peirce's "eccentricity" as well as his industry and brilliance.

[453]Roberts, op. cit., p. 49.

[454]Roberts, op. cit., p. 195-197.

[455]Scribe and erase - these represent the basic methods by which thought is carried on. See III 1 (a) and Roberts, op. cit., p. 268.

formulation.[456] This same sort of experimentation also makes it much easier to discover the relationship between various propositions, thus making complex problem solving a matter of insight and controlled experiment rather than chance.

Roberts has shown that the alpha part of existential graphs as a calculus is adequate to carry out all the operations of a propositional calculus and the steps in developing the graphical calculus are easier, clearer and fewer than those in developing a propositional calculus.[457] The beta part, with Roberts' improvements, is capable of substituting for the functional calculus and it too requires generally fewer steps, and those clearer, than the corresponding algebra.[458] Further there would seem to be no reason why a graphical calculus should not be extended to include abstraction, metagraphs, and perhaps modality, since nothing new is introduced in the graphs of these, at least of the first two, except a shorthand which represents graphs. Thus Peirce's system with few changes would seem not only an adequate but also a superior calculus. This is particularly interesting since Peirce, himself, thought of his system, not as a calculus, but as an instrument of logical analysis.

The value of Peirce's diagrams as an instrument of logical analysis rests in Peirce's concept of the nature of reasoning.[459] Peirce believed that all reasoning proceeded, at its most basic level, <u>via</u> erasure and insertion. That is, when <u>A</u> and <u>B</u> are related and <u>B</u> and <u>C</u> are related

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[456] Compare one diagram (Roberts, <u>op. cit.</u>, 291) with its algebraic formula (p. 293) for a clear example of the relative simplicity of the diagrams in certain cases.

[457] Roberts, <u>op. cit.</u>, pp. 150-200.

[458] Roberts, <u>op. cit.</u>, pp. 207-254.

[459] Roberts, <u>op. cit.</u>, pp. 258-300. See also III 1 (a).

etc., I see A and I think of AB, then ABC, then BC, then C, etc. Further he believed that all reasoning was essentially diagrammatic.[460] Thus diagrams closely represent thought, and the operations on the diagrams, all of which operations are reducible to erasure and insertion, closely represent reasoning. It is much easier to see this nature of reasoning, to reduce complex propositions to their simple elements, and to grasp the essential structure of thought in diagrams than in any other formulation, and especially are diagrams superior to algebraic formulae. Such was Peirce's estimate of the real importance of the diagrams. Although we may wish to disagree with Peirce with regard to the nature of thought and the operations of reasoning, or perhaps we merely wish to remain agnostic on these subjects, there can be no doubt that the diagrams can be used to reduce logical arguments into a much simpler series of steps than algebra can without achieving such complexity as to be useless and

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[460] A passage cited by Roberts (p. 268) in this regard is worth quoting in full.

> Looking out of the window, I see the cow whose milk we generally drink. There are certain difficulties which have occasioned a good deal of thought, so that I imagine I see a boy sitting by the cow milking her. The boy, and the stool, and the pail are added to my idea. Thence, I imagine the boy carrying the pail to the house. The cow and the stool have dropped out. The straining of the milk presents itself to my imagination. A bowl is there and the milk. The boy is standing by; but I lose sight of him. I am following along the train diagrammatically, that is, following the interesting history.
>
> As I followed that series of events in my mind (as I did; for I do not take make-believe observations), there was always something identical being carried along. The boy going up to the house with the pail, was thought as that same boy, the pail that same pail, and the occasion that same occasion that I had just before been thinking of. The new ideas must, therefore, have always been taken in before the old ones were allowed to drop. By the time the milk was

that the diagrams can render propositions more obviously and reduce them to simple elements more clearly and easily than can algebra.[461] Examples of these various uses of the diagrams would take more space than we have but the reader is again referred to Roberts.[462]

Thus Peirce's diagrams would seem to be valuable as experimental models, as a logical calculus, and as instruments of logical analysis.

(e) Gardner

Martin Gardner was dissatisfied with the diagrams using geometric figures because, when applied to the propositional calculus, they seemed to lack the iconicity that they displayed when applied to class logic.[463] In 1951 he developed a network system which would give a more exact correspondence.[464] To achieve such correspondence we begin by allowing two parallel vertical lines to represent the truth and falsity of each proposition (LXXXV a).[465] By convention the left line represents truth. Horizontal shuttles between these lines will represent the truth value of the terms. There are four such shuttles for two terms

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> straining in my imagination I had already begun to think that it would be good for my wife, who is threatened with nervous prostration.
>
> To one skeleton-set another is added to form a compound set. Then, the first, perhaps, is dropped and the ideas which remain are viewed in a new light. (Pierce 7.428-7.430)

[461] Again the reader is referred to the diagram and formula cited by Roberts. See fn. 455.

[462] Particularly to his last chapter. Roberts, op. cit., pp. 258-300.

[463] Gardner, op. cit., p. 60. Gardner also criticizes the difficulty of separating the various premises in one diagram (p. 61).

[464] Gardner, op. cit., pp. 60-79.

[465] Gardner, op. cit., p. 62, Fig. 45.

(LXXXV b).[466] If any proposition P_1 allows one and only one shuttle to run from a particular line L_1 to some other line L_2 we may move[467] from L_1 along the shuttle to its termination on L_2. If there is more than one shuttle from L_1 in some proposition no move may be made from that line on the shuttles representing that proposition.[468]

The basic proposition A is marked by placing a cross on A's line representing the truth of A (LXXXVI a) and falsity is represented in an analogous manner (LXXXVI b).[469] To understand the Gardner diagrams for binary propositions it is necessary that we look briefly at truth tables.[470] If A is true and B is true we may say that A.B is true. If A is false or B is false or both we may say that A.B is false. This may be illustrated by a truth table which shows all possible combinations of the truth and falsity of A and B in relation to some binary operator. Such a table for A.B would be:

A	B	A.B
T	T	T
T	F	F
F	T	F
F	F	F

Now to apply Gardner's diagrams all that one need do is draw shuttles to represent all the lines in the truth table in which the proposition

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[466] Gardner, op. cit., p. 62, Fig. 47.

[467] Gardner uses the term "ride", op. cit., p. 63.

[468] Gardner does not give this rule; it is, however, implied in his use of the system.

[469] These two diagrams are given by Gardner, op. cit., p. 62, Fig. 46, as one.

[470] Any standard text will discuss truth tables. We have used Copi, op. cit., pp. 237-268. Since there is almost universal agreement on this subject, at least on the elements of it presented here, we have not given specific references.

formed by the terms joined by the binary operator is true. Thus to represent A.B we draw one line joining the truth line of A and the truth line of B (LXXXVI c).[471] In a similar manner we may diagram any case of conjunction (LXXXVI d[472] and e).[473] Further truth tables will illustrate this:

A	B	A≡B	A≢B	AvB	A|B	A⊃B	B⊃A
T	T	T	F	T	F	T	T
T	F	F	T	T	T	F	T
F	T	F	T	T	T	T	F
F	F	T	F	F	T	T	T

These cases are shown in (LXXXVI f-k).[474] If we are given that such a proposition is true we may place crosses on its termini, otherwise not (LXXXVI e).

The application of these diagrams to the propositional calculus is extremely simple, as will be seen in the following examples:

Given: A⊃B
B≢C
AvC
B

We may diagram these premises[475] as in Diagram LXXXVII a.[476] We may start on line B since there is a cross on it and we may move along that line seeking a shuttle terminating on it. Two such shuttles occur in the first proposition eliminating the possibility of their use. Only one such shuttle, however, occurs in the second. We may place a cross

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[471]Gardner, op. cit., p. 62, Fig. 48, top half.

[472]Gardner, op. cit., p. 62, Fig. 48, bottom half.

[473]Gardner, op. cit., p. 62, Fig. 49.

[474]Gardner, op. cit., pp. 63-64, Fig. 50 - 54 taken in order.

[475]The problem appears in Gardner, op. cit., p. 66.

[476]Gardner, op. cit., p. 67, Fig. 55.

at the point where the shuttle meets the line since we know that B is true. Henceforth we will be able to place a cross anywhere on our path since if B is true we will be led only to true statements. Should crosses appear on both the true and false lines of a term we should be forced to the realization that our premises were self-contradictory. We move, then, across the shuttle to find ourselves on C's false line where we place another cross. Moving down C's false line we come to a single shuttle in the third proposition which carries us across to A's true line. We again place crosses at both ends of the shuttle. Finally we cross and mark the single shuttle leading from A's true line to B's true line in the first proposition. We have now marked both A and ~C as true. Thus we may arrive at the conclusion A.B.~C or:

A⊃B
B≢C
AvC
B
∴ A.B.~C

The finished diagram will be LXXXVII b.[477]

Sometimes we lack an existential or individual proposition,[478] which lack complicates our work:

Given: A≢C
B|C
AvB

We may draw Diagram LXXXVII c.[479] We may begin anywhere as we have no crosses. Let us begin by supposing A to be true. We may cross via the

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[477]Gardner, op. cit., p. 67, Fig. 56.

[478]This problem appears in Gardner, op. cit., p. 67.

[479]Gardner, op. cit., p. 67, Fig. 57.

top shuttle to ~C but we can get no further. Therefore we know that $A \supset \sim C$. Now let us begin at ~A. We may move *via* the second shuttle in the top proposition to *C* and thence by the second shuttle in the second proposition to ~B and finally across the second shuttle in the final proposition to A. Thus we have moved from ~A to *C* to ~B to *A*. We have proven that $\sim A \supset A$ and thus $A . \sim A$.[480] Since this is a contradiction we must discard the possibility that ~A is true. Thus A is true. But if *A* is true ~C is also true. We next test B and ~B for such contradictions and find that either *B* or ~B is possible. Thus our conclusion is $A . \sim C . (B \vee \sim B)$ or simply:

$A \not\equiv C$
$B | C$
$A \vee B$
$\therefore A . \sim C$

Thus far we have considered only compound propositions with one connective. Gardner provides two methods of dealing with compound statements "involving parentheses": horizontal truth-value lines[481] and "chains".[482]

If we are given a compound statement, for example $(A \vee B) \supset (C \vee D)$, we may diagram it by drawing dotted or broken shuttles for the subordinate parts of the proposition $(A \vee B)$ and $(C \vee D)$. Each of these subordinate parts

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[480] A symbolic proof of this is very simple:

$\sim A$
$\sim A \supset A$ (derived from first 3 premises + ~A)
$\therefore A$
$\sim A \supset A$ (*reductio ad absurdum* proof)
$\therefore A . \sim A$

[481] Considered by Gardner, *op. cit.*, pp. 69 and 72.

[482] Gardner, *op. cit.*, pp. 69-71.

is then represented by a pair of horizontal truth-value lines and the relationship between these parts is represented by vertical shuttles. By convention in drawing these truth-value lines we give the page a quarter turn clockwise. Since these final relations are not tentative they are represented by solid shuttles. By convention we make the lower line of a pair of horizontal lines representing the truth-value of a term or complex of terms true. Thus (AvB)⊃(CvD) may be represented by Diagram LXXXVIII a.[483] In some cases it is necessary to allow one pair of horizontal truth-value lines to represent a single term. Thus in Diagram LXXXVIII b the lower pair of lines is used for A.[484]

Suppose that we have a proposition of the form A.B.C. This may be represented most simply as a shuttle intersecting more than two truth-value lines. We know, of course, that the two ends of a shuttle intersect truth-value lines, but Gardner allows the shuttle to intersect more than two lines by marking such intersections with small circles. Thus A.~B.D may be represented by Diagram LXXXIX a.[485] Other chains may be represented in a similar manner after the proposition is reduced to disjunctive normal form. For example a chain of equivalences A≡B≡C may be reduced to (A.B.C)v (~A.~B.~C) and may be diagrammed as in LXXXIX b.[486] Some complex statements are capable of similar representation. A⊃(B.C) may be reduced to

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[483] Gardner, op. cit., pp. 69-71.

[484] Gardner, op. cit., p. 70, Fig. 59.

[485] Gardner, op. cit., p. 70, Fig. 60.

[486] Gardner, op. cit., p. 71, Fig. 61.

(A.B.C) v (~A.B.C) v (~A.B.~C) v (~A.~B.C) v (~A.~B.~C) and may be represented as in Diagram LXXXIX c.[487]

Gardner presents one example of the reduction of a proposition to its simplest form by the use of diagrams. We are given (A.~B)v(~A.~B) and asked to represent it. It may be drawn using horizontal truth-value lines as above but since we are dealing with only two terms and since shuttles belonging to the same binary operation represent disjunctive possibilities it is possible to represent it by two horizontal shuttles as in Diagram XC a.[488] This is still not the simplest diagram as it is apparent from Diagram XC a that the truth-value of A is irrelevant to the truth-value of B. ~B is always true. Thus a cross on B's false line (XC b)[489] represents this proposition precisely. Gardner feels that the network diagram is a visual aid in such reductions which are an important aspect of propositional calculus.

This system allows the representation of propositions of any complexity by the use of alternating vertical and horizontal truth-value lines (XC c).[490] All of the shuttles, of course, must be dotted except those between the last set of truth-value lines which are always solid. Gardner does not attempt to give a complete description of the rules for solving complex problems but he does give the four most important rules and solves a problem by way of illustration. The rules are as follows:

> 1. If the truth values of all individual terms within a parenthetical statement are known, and they conform to one of the dotted shuttles for that statement, then the entire statement is known to be true.[491]

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[487]Gardner, op. cit., p. 71, Fig. 62.

[488]Gardner, op. cit., p. 72, Fig. 63.

[489]Gardner, op. cit., p. 72, Fig. 64.

[490]Gardner, op. cit., p. 72, Fig. 65.

[491]Gardner, op. cit., p. 72.

2. If the terms are known to have a combination of truth values <u>not</u> indicated by a shuttle, the entire relation is known to be false.[492]

3. Whenever a parenthetical statement is known to be true, either because of knowledge of its terms or because it is found to be true in the process of exploring the entire structure, its shuttles are changed to solid lines or its half crosses [introduced to allow for the representation of possible existence (e.g. in $(AvB)\supset C$, C will be marked with a half cross until we know the value of AvB)][493] to crosses. The truth of the entire statement is then indicated by a cross mark on the <u>T</u> line in the pair of truth-value lines (to the right or below) that correspond to the statement.[494]

4. Whenever a parenthetical statement is known to be false, in either of the two ways mentioned above, we add the missing shuttle or shuttles [i.e. those <u>not</u> represented in dotted lines] in solid lines. The falsity of the entire relation is then indicated by a cross mark on the <u>F</u> line in the pair of truth-value lines that correspond to the statement.[495]

To illustrate this Gardner proves that $(A\supset B) \supset (B\supset A)$ is not a valid theorem.[496] If it were a valid theorem it must be true for all values of <u>A</u> and <u>B</u>. We draw the complex diagram XCI a[497] for $(A\supset B) \supset (B\supset A)$. Our testing procedure would show that if <u>A</u> is true or if <u>B</u> is false or both, the proposition is valid. The critical case is that in which <u>A</u> is false and <u>B</u> is true. Therefore we make a cross on A's false line and another on B's true line. Since this combination is represented by a shuttle in the lower part of the diagram we know that the lower

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[492] Gardner, <u>op. cit.</u>, p. 73.

[493] Gardner does not give an account or an example of the use of the half cross and it would not seem to be essential to the operation of the system.

[494] Gardner, <u>op. cit.</u>, p. 73.

[495] Gardner, <u>op. cit.</u>, p. 73.

[496] Gardner, <u>op. cit.</u>, pp. 73-74.

[497] Gardner, <u>op. cit.</u>, p. 74, Fig. 66.

proposition is true. We may then draw the dotted shuttles as solid lines and place a cross on the true line of A⊃B. Since there is only one shuttle attached to this line we may move up that shuttle to the true line of B⊃A. Thus B⊃A is also true and we may place a cross on that line and change its dotted shuttles to solid ones. The graph at this point is represented by Diagram XCI b.[498] We are now caught in a contradiction. If we move up A's false line from our cross we encounter a shuttle leading to B's false line. Thus, since A's false line is affirmed, B's false line must also be affirmed which means that B is both true and false. Thus (A⊃B) ⊃ (B⊃A) cannot be a logical theorem.

Gardner suggests that these diagrams, combined with Venn diagrams, might be capable of extension to include systems combining class-inclusion and truth-value statements but he does not give any examples of this.[499]

Gardner does, however, attempt to apply network diagrams to three-value logic.[500] Let us call the third value "?". We must now include three truth-value lines for each term. So far there have been only two kinds of shuttles, those that were drawn and those that were not. Since we have introduced a third value besides truth and falsity it becomes necessary to introduce a third type of shuttle to represent the ? relationship between the truth-value lines of any pair of terms. Gardner chooses to use a wavy line for this purpose. In a three-value logic there are many possible interpretations of any particular binary relation.

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[498] Gardner, op. cit., p. 74, Fig. 67.

[499] Gardner, op. cit., p. 75.

[500] Gardner, op. cit., pp. 75-78. An excellent brief introduction to this subject is J. Barkley Rosser, "On the Many-Valued Logics", American Journal of Physics, Vol. 9, Aug. 1941, pp. 207 ff.

Clearly A.B requires that a shuttle be drawn from A's true line to B's true line, but what shuttles should be drawn which will terminate on A's ? line and B's ? line? Gardner draws two of the many possible A.B's (XCII a[501] and b).[502] The first is that preferred by Lukasiewicz, Post and Rosser; the second by Bochvar.[503] He further illustrates the use of these diagrams for three-value logic by drawing the diagram for Lukasiewicz's and Tarski's three-value interpretation of A⊃B (XCII c).[504]

Finally Gardner suggests that Carroll's method of placing counters on a diagram may be adapted to a network system. He illustrates this with a diagram for (A⊃B) v (B≡C) (XCIII a).[505] The major value of such a method would be the elimination of the need for erasing and drawing. White counters indicate uncertain or parenthetical relations while coloured indicate certain relations. Since the method works exactly like Gardner's network diagrams a detailed discussion will not be necessary.

In his footnotes Gardner suggests that we might be able to replace the shuttles with vectors and gives a vector diagram (XCIII b)[506] for A⊃B. The point of this diagram is that there is no necessity for the rule that if two shuttles terminate on the same truth-value line in the same proposition one may not move across those shuttles from that line. Instead

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[501]Gardner, op. cit., p. 76, Fig. 69.

[502]Gardner, op. cit., p. 77, Fig. 70.

[503]Gardner does not give references for his sources for particular logicians although he mentions general works on three-value logic, op. cit., p. 79, fn. 2.

[504]Gardner, op. cit., p. 77, Fig. 71.

[505]Gardner, op. cit., p. 78.

[506]Gardner, op. cit., p. 79, fn. 1, unnumbered diagram.

the rule is substituted that one may move only in the direction of the vector. Experiments[507] with vectors in simple logic have produced several advantages that Gardner did not point out. The most significant of these is the possibility of eliminating truth-value lines in favour of truth-value points. Diagrams using points eliminate the pause in movement from one truth-value to another making operation quicker and more certain and also representing in a very iconic manner alternation, equivalence and implication. As it is not our aim to produce a new system we will not go into detail here regarding this use of vectors but we should suggest that such diagrams would be very useful in representing such things as current flow and programming which may be diagrammed according to logic. Further, the rules, because motion is possible only with the vectors, are very much simplified over any other thorough system of logic diagrams.

Gardner's system would seem to be the most thorough, except for Peirce's, of any that we have examined. It is simple to operate and certain. We should, however, point out that its greatest strength lies in its possibilities for expansion beyond the propositional calculus into three-value logic, etc. as other systems (e.g. Venn, Marquand, etc.) are capable of doing almost anything that Gardner's can do in the propositional calculus and are somewhat more familiar and thus easier to use. It is the great versatility of Gardner's diagrams that gives them their value as much as their iconicity.

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[507] By the author of this paper who, at the time, had intended to develop his own system as part of this paper.

5. Unclassified Diagrams

(a) Hamilton

In this section of Chapter II we will discuss the systems of Hamilton and de Morgan. The shift from the frame of mind of Frege, Peirce and Gardner to that of Hamilton is not an easy one and the systems which Hamilton and de Morgan developed now seem trivial to us. It should, however, be remembered that these were the first serious attempts to break away from the narrow bounds of Aristotelian logic by means of symbolism. It is in this light that they should be read.

Although his work was not published until after his death, Hamilton's system was developed prior to de Morgan's and we will, therefore, treat it first. It will not, indeed could not, be our aim to present Hamilton's entire system. Nor will we recapitulate Hamilton's reinterpretation of Euler[508] and Lambert.[509] We will simply examine the finished system as presented at the end of his Lectures.[510]

In order to understand anything of what Hamilton is trying to do we must keep certain definitions in mind. The definitions of "quantity", "internal" and "external" (or "intensive" and "extensive") are now given:

> In relation to their objects, [things] — they [concepts]are considered as inclusive of a greater or smaller number of attributes, that is, as applicable to a greater or smaller number of objects; this is technically styled their Quantity.[511]

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[508] II 2 (a).

[509] III 3 (b).

[510] The scheme of the two quantities is given on p. 108. Both of the other diagrams are at the end of the appendix, p. 674 and p. 678-679. Hamilton, op. cit.

[511] Hamilton, op. cit., p. 100.

> This quantity is thus of two kinds; as it is either an Intensive or an Extensive. The Internal or Intensive Quantity of a concept is determined by the greater or smaller number of constituent characters contained in it. The External or Extensive Quantity of a concept is determined by the greater or smaller number of classified concepts or realities contained under it.[512]

The intensive quantity is also called depth; the extensive, breadth.[513] Now we may see what Diagram XCIV attempts to illustrate.[514] In the diagram vowels are reserved for classes, consonants for individuals. The earlier a vowel comes in the alphabet the broader the concept it represents. Every higher class is divided by a lower class and its (the lower class's) contradictory, into two parts.[515] A vertical stroke (|) followed by an italicized capital represents the first term in the negative series. The figures to the left of the chart represent the position of the concept directly in a horizontal line with them in breadth and depth respectively. The arrow of affirmation indicates that affirmation moves from the particular to the general. The object z is affirmed to be classified under the concept Y; the concept Y is affirmed to be classified under the concept U, etc. Exactly what the second arrow means I am not sure. It cannot mean negation in the traditional sense as "no A is B" is supposed to be equivalent to "no B is A".

We are now ready to reason either in breadth or in depth. In the first case we begin with the concept of greatest breadth and work downward. "Some A is all E, some E is all I, some I is all O, some O

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[512] Hamilton, op. cit., p. 100.

[513] Hamilton, op. cit., p. 100.

[514] Hamilton, op. cit., p. 108.

[515] This is the import of the dark vertical lines though Hamilton does not mention this fact.

is all U, some U is all Y, some Y is z. Therefore some A is z." If we reason in depth we begin with the individual and work from concepts of greater depth to those of lesser. "z is some Y, all Y is some U, all U is some O, all O is some I, all I is some E, all E is some A. Therefore z is some A."[516] All concepts are ideal; only individuals are real.[517] The ground of reality is, for this reason, at the bottom of the chart with particular objects.

With these distinctions in mind we may turn to Hamilton's most diagrammatic[518] diagram (XCV).[519] A few definitions are again needed to make the diagram readable. An analytic syllogism is a syllogism beginning with a conclusion and deducing premises; a synthetic syllogism is one deducing a conclusion from premises.[520] All the concentric triangular figures are presumed to have Ⓜ at their upper vertices, Ⓒ at their lower left vertices and (Γ) at their lower right vertices. The lines ⒸⓂ and (Γ)Ⓜ represent the premises and the line Ⓒ(Γ) represents

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[516] At this stage Hamilton has not allowed for propositions stating equivalence. This possibility was, however, introduced in his Euler diagrams, II 2 (a).

[517] Hamilton, op. cit., p. 110. This is a metaphysical, not a logical, statement.

[518] The others are more charts than diagrams.

[519] Hamilton, op. cit., p. 674, description pp. 673-676.

[520] Hamilton, op. cit., p. 673. Just how the analytic syllogism is supposed to work I am uncertain. It would seem to be impossible to deduce the premises from the conclusion as Hamilton suggests.

the conclusion. The triangles represent, moving inward, the unfigured syllogism, the first figure, the second figure and, finally, the third figure. The direction of the wedge indicates that we are moving from one concept to another in order of depth. The order of breadth is the opposite to the direction of the wedge. If the line is not a wedge we may assume that we may consider it either in order of breadth or in order of depth or in neither, whichever is convenient. A broken or dotted line or wedge indicates a weakened conclusion.

This condensed view may be expanded into the complete table (XCVI)[521] which represents Hamilton's final scheme of notation. Although Hamilton did leave a record of this scheme of notation he did not leave directions for reading it. Thus all of the information for reading the symbols is given to us by Hamilton's editors.[522] All of the above definitions from Diagram XCV apply equally to Diagram XCVI but certain new ones need to be introduced. The quantification of the predicate was felt by Hamilton to be his most significant contribution to logic. This was accomplished by modifying the predicate as well as the subject of a proposition by "any" or "all" for definite quantity and by "some" for indefinite quantity. Definite quantity is indicated by a colon (:) before or after the appropriate term depending on its place in the proposition; indefinite quantity is represented by a comma or reversed comma (in this diagram we have used only the comma (,) for typographical reasons) placed in the same manner. An affirmative proposition is represented by two

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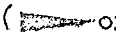
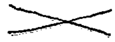
[521] Hamilton, <u>op. cit</u>., pp. 678-679.

[522] Hamilton, <u>op. cit</u>., pp. 676-677.

terms quantified, and joined by a tapering horizontal line ( or ). To negate a proposition one simply draws a vertical stroke through this line (). The symbol  shows that when the premises are converted the syllogism remains the same. The symbol  shows that the two moods between which it stands are convertible into each other by the conversion of their premises.[523] The moods are either "A" balanced or "B" unbalanced. For a mood to be balanced both the terms and the propositions must be balanced (i.e. the major and minor terms must be quantified in the same way and the middle term must be quantified by the same quantifiers in the same positions). If these two statements are not both true the syllogism is unbalanced. There are only two cases of balanced syllogisms. In the others either the terms are unbalanced as in iii and iv (i.e. the major and minor terms are quantified differently or the middle term is quantified differently in the two premises or both) or both the terms and the propositions are unbalanced as in v to xii (the propositions also contain at least one case in which the quantifier is different in the same position in one premise from that in the other). With these definitions Diagram XCVI should be clear.

We shall now attempt to translate this symbolism into ordinary language. In chart XCVI we have seen that every term is modified by "," or ":" depending on its quantity. If we use <u>A</u> to indicate the subject of a proposition and <u>B</u> to indicate the predicate and Hamilton's

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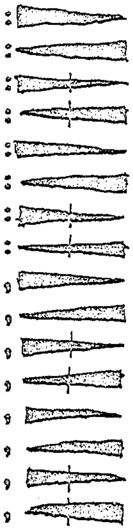
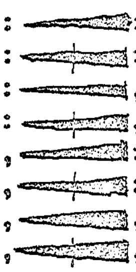
[523]Both symbols are defined on p. 676, Hamilton, <u>op. cit.</u>

wedges to represent the relationship between the subject and predicate we may write only the following sixteen propositions:



1. A: :B
2. A: :B
3. A: :B
4. A: :B
5. A: ,B
6. A: ,B
7. A: ,B
8. A: ,B
9. A, :B
10. A, :B
11. A, :B
12. A, :B
13. A, ,B
14. A, ,B
15. A, ,B
16. A, ,B

Now agreeing that A and B are variables and that in any of these propositions A may be substituted for B and _vice versa_, this list may be reduced to eight propositions (the so-called Hamiltonian system discussed by Venn).[524]



1. A: :B (from 1 and 2 above)
2. A: :B (from 3 and 4 above)
3. A: ,B (from 5 and 10 above)
4. A: ,B (from 7 and 12 above)
5. A, :B (from 6 and 9 above)
6. A, :B (from 8 and 11 above)
7. A, ,B (from 13 and 14 above)
8. A, ,B (from 15 and 16 above)

Hamilton uses the words "any" and "all" synonymously. Our practice will be to use only "all" because "any" sometimes leads to ambiguity. We shall translate the wedge as "included within" beginning to read at the narrower end. This, it is hoped, will eliminate the ambiguity of the word "is" as a translation of the wedge. When so read the above propositions render the following sentences:

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[524] Venn, _op. cit._, pp. 8-9. See also II 2 (a).

1. All A is included within all B.
2. All A is not included within all B.
3. All A is included within some B.
4. All A is not included within some B.
5. Some A is included within all B.
6. Some A is not included within all B.
7. Some A is included within some B.
8. Some A is not included within some B.

Finally we must evaluate Hamilton's diagrams as diagrams. The first chart (XCIV) does not warrant much comment. The ontological presuppositions might be pointed out, and the lack of recognition of the possibility of overlapping classes. The second diagram (XCV), after one's initial confusion, is recognizable as a condensed version of the third (XCVI). The third is a thorough rendering of all the three term syllogisms which may be written in Hamilton's symbolism. Thus the second diagram's value rests on the third. The only aspect of the third diagram which might be called diagrammatic is the use of "" to mean "included in", but this is, surely, a symbol like "$<$" in $1 < 2$ and not a diagram. Thus when we think of Hamilton's system as diagrammatic it fails to be so but falls back into symbolism. It would seem that on this point Venn's estimate was certainly correct.

> To my thinking it does not deserve to rank as a diagrammatic scheme at all, though he does class it with the others as "geometric"; but is purely symbolical.[525]

(b) De Morgan

Finally we come to the last system: that of de Morgan. It should be pointed out at the outset that de Morgan's system is no more diagrammatic than Hamilton's and that it is being examined here only because Hamilton's was. It is valuable in judging Hamilton's work to be able to compare it to de Morgan's.

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[525] Venn, op. cit., p. 521.

Again we should begin with some definitions. Any term "X" taken in total may be called "X)" or "(X" whichever is more convenient.[526] Any term taken in part is ")X" or "X(".[527] A proposition is negated by the placing of a dot between the pair of spiculate (parenthesis) modifying the terms (e.g. (.().[528] The affirmation of a proposition uses two dots or none (usually none) in a similar manner.[529] It may be added, although no use will be made of the fact in this paper, that de Morgan introduced negative terms using "x" to mean the negative of X.[530]

We are now ready to read charts XCVII and XCVIII. For any two terms X and Y there are eight propositions:

X))Y
X).(Y
X(.)Y
X((Y
X(.(Y
X()Y
X)(Y
X).)Y

These eight propositions are listed to the left in chart XCVII[531] and the similar propositions, substituting X and Y appropriately, which affirm them, contradict them, are consistent with them, etc. are arranged in a convenient chart form.

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[526]Augustus de Morgan, Syllabus of a Proposed System of Logic, London: Walton and Maberly, 1860, p. 14.

[527]De Morgan, op. cit., p. 14.

[528]De Morgan, op. cit., p. 14.

[529]De Morgan, op. cit., p. 14.

[530]De Morgan, op. cit., p. 13.

[531]De Morgan, op. cit., p. 16.

The next table (XCVIII)[532] goes on to show the syllogisms possible using these same formal methods. In the middle column stands the universal, horizontally between the two particulars into which it may be weakened by weakening one of the concluding terms.[533] Each strengthened particular stands vertically between the two particulars from which it may be formed by altering the quality of the middle term in the particular premise only.[534]

Another interesting arrangement of syllogisms is the "logical zodiac" (IC).[535] In this case the universal and particular syllogisms are grouped in threes, each of any three having the other two for its opponents. If A and B are two propositions leading to conclusion C then Ac gives b and Bc gives a. a and b are called the opponents of C.[536] Other interesting formations occur in the logical zodiac. For example the universal propositions at the four cardinal points are so placed

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[532] De Morgan, op. cit., p. 20.

[533] That is to say any of the three two term arguments in a horizontal line will lead to the same conclusions due to this relationship described. For example the first row reads from left to right: "Some A are some B, All B are some C," and "All A are some B, All B are some C," and "All A are Some B, All B are All C." All three lead to the conclusion "Some A are some C."

[534] The same sort of a relationship exists between the argument in the "strengthened particular" column. This seems very puzzling, especially since de Morgan introduces premises such as "(.)(.)" which cannot possibly lead to any conclusion. On the whole I am inclined to think that the diagram, as a whole, is indeciferable without more information than de Morgan gives.

[535] De Morgan, op. cit., p. 21.

[536] "Opponents" is a very difficult term. See de Morgan, op. cit., p. 20 for a more thorough definition.

that any two contiguous, whether read forwards or backwards, give the premises of a valid universal syllogism.[537]

We will cut our description of de Morgan's system short at this point since, as is obvious by now, these charts, interesting and suggestive as they are, are not diagrams.

Obviously de Morgan's system is superior to Hamilton's. The most important difference is the introduction of negative terms. On the other hand Hamilton's use of "◄" as an operator gives him a logically powerful system and some such operator would seem to be necessary for any adequate system of symbolism.

6. Conclusion

To summarize such a chapter as this is almost impossible. We might point out that there has been evolution within the field of logic diagrams, that some diagrammatic systems are more adequate than others, that some systems are more capable of performing some functions than others. All of this is obvious or where it is not it will be discussed in the last chapter. We are left with the simple descriptions; these, then, were the diagrammatic systems which contributed not only to the growth in adequacy of the logic diagram but also to the development of logic. A brief summary of the development of the logic diagram might be given by means of a graph (C). The horizontal axis represents the years in which the various contributions to the logic diagram were

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[537] It is difficult to see what de Morgan means by "universal proposition". "(.)" seems to be particular. Further "A(.)B" and "B).(C", if they can be said to lead to any conclusion lead to "B()C" which is a particular, not a universal proposition.

written; the vertical axis represents the value, in this author's estimate, of the contributions. A diagram which would be completely adequate for all logic would be classed at 100; one which would be totally inadequate at 0.[538]

From this chart certain conclusions may be drawn. (1) most of the activity in this field occurred between 1850 and 1925. (2) There was a sharp rise in the adequacy of the diagrams 1850-1880 followed by a more gradual rise in the value of the better systems. (3) Inadequate systems continued to be produced and inadequate interpretations of systems continued long after the more adequate systems were developed. (4) Two sections of the chart (really within one time period) show an exceptional amount of activity. These are the sections containing diagrams of value 20 to 35 between 1855 and 1890 and of value 55 to 90 between 1875 and 1920. These are reproduced in enlargment with a key to allow the reader to form an estimate of the author's judgment of the value of these systems and to give an overall view of the major period of development for the diagrams (CI) and (CII). The reader may disagree with the position of some of these developments with regard to the vertical axis but the main purpose of the chart is to indicate the sort of growth that occurred and to show periods of greatest activity. Even allowing for disagreement these charts should be adequate for this purpose.

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[538]A very subjective standard meant only to be taken as a guide. The reader is invited to disagree, forming his own evaluation.

III

THE LOGIC DIAGRAM TODAY

1. What is the Logic Diagram?

(a) Peirce's Concept of Language

Peirce, as we have noted, believed that all human thought proceeded diagrammatically.[1] This is not say that we proceed to form a visual picture of the objects of which we are thinking but rather that our thoughts are a simple pattern, a map, of the objects of our thoughts in their relationships. Peirce gives several examples of this process. The one cited below will demonstrate his position clearly:

> Consider any argument concerning the validity of which a person might conceivably entertain for a moment some doubt. For instance, let the premise be that from either of two provinces of a certain kingdom it is possible to proceed to any province by floating down the only river the kingdom contains, combined with a land-journey within the boundaries of one province; and let the conclusion be that the river, after touching every province in the kingdom, must again meet the one which it first left. Now, in order to show that this inference is (or that it is not) absolutely necessary, it is requisite to have something analogous to a diagram with different series of parts, the parts of each series being evidently related as those provinces are said to be, while in the different series something corresponding to the course of the river has all the essential variations possible; and this diagram must be so contrived that it is easy to examine it and find out whether the course of the river is in truth in every case such as is here proposed to be inferred.[2]

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[1] See II 4 (d).

[2] Peirce, 3.418.

This position would be very controversial if we were to take Peirce's word "diagram" too literally; but the passage continues in a way that prevents this. Peirce shows that he is concerned with the one to one correspondence of the "diagram" to the original, and not with the geometrical properties of it. Thus the concept of "diagram" is broadened to include all language and all types of symbolism insofar as these may be meaningful.

> Such a diagram has got to be either auditory or visual, the parts being separated in the one case in time, in the other in space. But in order completely to exhibit the analogue of the conditions of the argument under examination, it will be necessary to use signs or symbols repeated in different places and in different juxtapositions, these signs being subject to certain "rules", that is, certain general relations associated with them by the mind. Such a method of forming a diagram is called algebra. All speech is but such an algebra, the repeated signs being the words, which have relations by virtue of the meanings associated with them. What is commonly called logical algebra differs from other formal logic only in using the same formal method with greater freedom. I may mention that unpublished studies have shown me that a far more powerful method of diagrammatisation than algebra is possible, being an extension at once of algebra and of Clifford's method of graphs; but I am not in a situation to draw up a statement of my researches.[3]

A diagram, a sentence and a logical formula are all of the same character. Each is an iconic representation of the object of thought. The diagram differs from the other two in that its parts are spatially, rather than temporally, ordered. Next the question arises of what, exactly, the diagram, the language and the algebra represent. Peirce points out that no matter how complex our diagram becomes, no matter how carefully we expand it, the diagram can never show to what it is intended to be applied. Since a diagram cannot, algebra cannot for

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[3]Peirce, 3.418.

algebra is a sort of diagram; since algebra cannot, a language cannot for language is a sort of algebra. Thus the referent of a logic is an extralogical matter and the referent of a language is an extralinguistic matter. Reference must be given before the diagram, algebra or language can have meaning other than as a closed analytic system. In language this step may be taken in any of several ways. The most common is the use of demonstratives which are, so to speak, fixed points in our language relating it to the world or to whatever world we are discussing. Nouns (Peirce calls them "prodemonstratives") may then be substituted for, or attached to, demonstratives and our correspondence is set up. The situation with diagrams is exactly analogous.[4]

> If upon a diagram we mark two or more points to be identified at some future time with objects in nature, so as to give the diagram at that future time its meaning; . . . the professedly incomplete representation resulting may be termed a <u>relative rhema</u>.[5]

Thus a diagram, or for that matter a language, receives its meaning within a context. If there is no context, no world or worlds to which the diagram applies, there is no meaning. Further it is to be noted that this meaning can only be given by demonstratives. Thus, even though the relationships are drawn in the diagram, it receives meaning only when it is applied to something.

Peirce's position is, then, that a logic diagram is a language (or better that a language is a logic diagram) and it is, when demonstratives are substituted for its marks, a representation of reality

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[4]All of this is said by Peirce in more technical language, 3.419.

[5]Peirce, 3.420.

or whatever world we are discussing. This description gives the logic diagram a very high position of priority among the things philosophers ought to be studying in order to understand reality. We might wish to call this position the strong statement of the value of the logic diagram.

(b) The Weak Statement of the Value of the Logic Diagram

Many logicians would wish to reject the idea that we think diagrammatically and that language and calculus are types of diagrams, yet would wish to retain the logic diagram as a useful device. The starting point of such a position would be a statement of the relationship between the logical language or algebra and the diagrams. Hocking states this relationship in terms of representation.

> It is possible to represent anything by anything else, provided the system of such parts in the given object as are significant for the purpose in hand can find corresponding parts in the representative object.[6]

There is no reason why a diagram would be superior to any other type of representation.[7] The diagram is chosen for other reasons. There is no ontological or epistemological ground common to the diagram and the logic. Parts happen to correspond (or are so drawn that they correspond) so the diagram may be used in this way.

Gardner puts this in another way in his discussion of Venn diagrams.

> What we have been doing, in a sense, is to translate the verbal symbols of a syllogism into a problem of topology.

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[6] Hocking, *op. cit.*, pp. 31-32.

[7] Hocking speaks of "other possible ways of making sensible, these ideal relations" but does not tell us what ways he means. Hocking, *op. cit.*, p. 32.

> Each circle is a closed curve, and according to the "Jordan theorem" of topology a closed curve must divide all points on the plane into those which are inside and those which are outside the curve. The points inside each circle constitute a distinct "set" or "class" of points. We thus have a simple geometrical model by means of which we can show exactly which points lie within or without a given set. The question now arises, do the topological laws involved here underlie the logic of class inclusion, or do the laws of class inclusion underlie the topological laws? It is clearly a verbal question. Neither underlies the other. We have in the Venn circles and in the syntax of a syllogism two different ways of symbolizing the same structure — one grammatical, the other geometrical. Neither, as Peirce expresses it, is "the cause or principle of the other".[8]

The basic difference between Peirce and Gardner on this point is that for Peirce the underlying structure symbolized is, itself, "diagrammatic" in that it represents something. For Gardner and Hocking no such description is, or could be, made.

> Logic diagrams stand in the same relation to logical algebras as the graphs of curves stand in relation to their algebraic formulas; they are simply other ways of symbolizing the same basic structure.[9]

> Clearly, the parabola and its formula are simply two different ways of asserting the same thing. The parabola is a spatial way of representing an equation; the equation is an algebraic expression of a parabola.[10]

Clearly, for Gardner and most other modern logicians, diagrams and algebras are simply two languages expressing a basic structure. Neither comes nearer to expressing that structure; they are of equivalent value in such expression and are to be judged by other standards.

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[8]Gardner, _op. cit._, p. 41.

[9]Gardner, _op. cit._, p. 28.

[10]Gardner, _op. cit._, p. 28.

(c) Summary

A third case might be made: for the uselessness of diagrams in the representation of logic. A defense of such a position would attempt to point to structures within logic which are not, in fact, represented by the diagrams or, better yet, to show that some logical structures are, in principle, unrepresentable by any diagram.[11] We would reject such a position and point to the work of Venn, Peirce and Roberts as a sufficient refutation of it. We have attempted to show that Venn's diagrams are adequate for the Boole-Schroeder algebra[12] and Roberts has pointed out that Peirce is, or may be made, adequate for the proposition calculus and the functional calculus.[13] We have also mentioned attempts to develop diagrams for multi-valued logics,[14] for modality,[15] and for meta-logic.[16] Thus we would wish to hold that it is (at least on the present evidence), in principle, possible to develop adequate logic diagrams for any system of logic.

Since we accept this, there can be no doubt that we must hold, at least, to the weak statement of the value of the logic diagram. There is one to one correspondence between certain features of the structure of logic and the structure of the diagrams. Otherwise the diagrams would not be adequate to represent the logical system.

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[11]One might wish, for example, to single out some aspect of a logical system and to prove that there is no way in which diagrams could be used to represent that aspect.

[12]II 2 (c).

[13]II 4 (d).

[14]II 4 (e).

[15]II 4 (d).

[16]II 4 (d).

Finally we must ask about Peirce's strong statement. Is language an algebra and algebra a diagram and all thinking diagrammatic? A thorough examination of languages, algebras and diagrams in general, and of the relationships of these with universes, real or ideal, and of all this with experience would be necessary before this question could be answered.[17]

We are able, at least, to accept the weak statement of the value of the logic diagram. Gardner has stated this position thus:

> A logic diagram is a two-dimensional geometric figure with spatial relations that are isomorphic with the structure of a logical statement.[18]

2. The Uses of the Logic Diagram

(a) In Logic

The logic diagram is used within logic primarily for two related purposes: the teaching of elementary logic and the illustration of some aspects of logic.

At the present time the Venn diagrams are the most popular set of diagrams used in the teaching of logic. Copi, for example, uses them to great advantage. He introduces them to illustrate categorical propositions[19] but goes on to use them as a simple method of solving categorical syllogisms.[20]

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[17] Peirce saw the importance of such an investigation and began it 3.418-3.420, but little further work has been done in this area from this point of view.

[18] Gardner, op. cit., p. 28.

[19] Copi, op. cit., pp. 161-166.

[20] Copi, op. cit., pp. 176-186.

> The Venn Diagrams constitute an iconic representation of the standard-form categorical propositions, in which spatial inclusions and exclusions correspond to the nonspatial inclusions and exclusions of classes. They not only provide an exceptionally clear method of notation but also are the basis for the simplest and most direct method of testing the validity of categorical syllogisms, . . .[21]

We have already explained this use of the Venn diagram. Occasionally an author, such as Bittle, still uses the Euler diagrams for this purpose.[22]

Diagrams are also used to illustrate many different aspects of logic. For example, in the Precis, Bocheński makes use of the following diagrams:

(1) Gonseth's graphical representation is used to illustrate a truth functional calculus.[23]

(2) A square of opposition is used to illustrate the relationships between the operators of the same truth functional calculus.[24]

(3) A negative version of squared Venn diagrams is used to illustrate class relations.[25]

(4) A diagram of functions is used to illustrate individual, plural and bi-plural descriptions.[26]

(5) A directed network is used to illustrate isomorphic relations.[27]

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[21] Copi, op. cit., p. 166.

[22] Bittle, op. cit., pp. 187 ff.

[23] Bocheński, A Precis of Mathematical Logic, pp. 13-14.

[24] Bocheński, op. cit., p. 14.

[25] Bocheński, op. cit., pp. 56-57.

[26] Bocheński, op. cit., pp. 68-69.

[27] Bocheński, op. cit., p. 75.

The question that one must always raise with regard to the use of diagrams in logic, or anywhere else for that matter, is whether a better diagram may not be available. Gardner, for example, does not feel that the geometric areas of the Venn diagrams adequately represent the truth values of propositions, and therefore suggests a network system.[28] One might wish to question Bocheński's use of Gonseth diagrams on similar grounds. One might also question the value of his diagrams used for the representation of classes on the grounds that they reverse the accepted conventions of such diagrams.

If diagrams are being used to help students beginning to work in logic it would seem to be important that they should be useful in as many situations as possible, that they should be simple to use, and that they should distort the relations between terms as little as possible. This would make it important for the teacher to use the diagrams which he considered best after careful examination of various systems with the limitations of the students in mind. One can only suggest that caution should be exercised in the use of diagrams. An adequate diagram, well explained, can be an invaluable aid; a poor one, or one given without sufficient information, will simply add to the confusion.

Other possible uses of logic diagrams within logic have been mentioned from time to time. Note particularly Roberts' suggestion of the use of existential graphs as a calculus.[29] We will not discuss these uses in this chapter as we are concerned only with the ways in which the diagrams are actually used at the present time and not with

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[28]Gardner, op. cit., p. 60.

[29]Roberts, op. cit., pp. 150-256.

ways in which they might be used.

(b) In Mathematics

Diagrams or graphs with the characteristics of logic diagrams are found throughout mathematics. The two areas in which such diagrams are used most frequently are theory of graphs and set theory.

The first paper on theory of graphs was written by the famous Swiss mathematician Euler and appeared in 1836.[30] It is interesting that Euler, the midwife of the logic diagram, should crop up as well as the father of the mathematical graph. The linear graph has exactly the appearance of an irregular network logic diagram. Various points called vertices are joined by lines called edges. These may be used to diagram various situations. One may then develop an algebra for describing these graphs. The kinship of the graphs and the diagrams is particularly clear when we turn to directed graphs. Graphs are directed when we may move only in a prescribed direction along the edges either by the use of vectors or by some other rule. Despite superficial differences Gardner's diagrams are exactly defined as mixed (i.e. directed and non-directed) graphs.[31] Peirce's valency diagrams also behave exactly like linear non-directed graphs.[32] The field of graph theory then is going to be very significant for at least some systems of logic diagrams. We have already mentioned the Jordan theorem and its importance in understanding the basic nature of geometric diagrams. This theorem is a topological

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[30] This, of course, was the paper proving that the "seven bridges of Königsberg" puzzle was insoluble.

[31] II 4 (e).

[32] II 4 (d).

theorem and is developed *via* theory of graphs. Various other theorems may be developed concerning the nature of both network and geometric logic diagrams by means of theory of graphs. Thus theory of graphs is a valuable instrument for the exact understanding of logic diagrams and for systematizing them. From a mathematical point of view, at least, theory of graphs would appear to be preliminary to the study of the topological characteristics of logic diagrams and most sorts of logic diagrams would be thought of as examples or illustrations of the theory of graphs or objects for analysis of the theory of graphs.[33]

In set theory the Venn-Euler diagrams are a very valuable instrument.[34] If one is attempting to teach the operations involved in set theory one may begin with concrete geometric examples. Imagine that a set A is the class of all objects having characteristic A. This class is represented by a circle or other geometric figure which is thought of as containing all the objects having the characteristic. Now one may represent the various operations of set theory by means of the diagrams. Thus the diagram for $A \cup B$ is the same as that for $A+B=1$ in Boolean algebra;[35] that for $A \cap B$ the same as that for $AB=1$,[36] etc.

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[33]There are relatively few works in this field, especially for the general reader. Oystein Ore, *Graphs and Their Uses*, Toronto: Random House, 1963 is useful. In a more popular style is Stephen Barr, *Experiments in Topology*, New York: Thomas Y. Crowell, 1964 (especially the first chapter).

[34]Almost every standard work on set theory uses some diagrams. We have used Lipschutz, *op. cit.*, because of the author's limited knowledge of mathematics.

[35]Lipschutz, *op. cit.*, p. 17.

[36]Lipschutz, *op. cit.*, p. 18.

Since set theory is now being taught in public school, Venn diagrams have found their way into texts for the teaching of elementary arithmetic.[37] This leads us to emphasize again the importance of a thorough analysis of logic diagrams and of their use in extralogical fields in order that they may be used to the greatest advantage and with the utmost accuracy.

(c) In the Sciences

Logic diagrams are to be found scattered through many science books in widely diversified areas. We have chosen three fields to illustrate this: geography, psychology, and electrical engineering.

Peter Haggatt, in Locational Analysis of Human Geography, makes use of Venn diagrams to show the relationship between geography and other areas of study[38] and to illustrate regional geography.[39] These diagrams would seem to be ideally suited to the latter use since Haggatt is able to abstract the topological characteristics of regional geography and to represent them with the topological features of Venn diagrams.

Kurt Lewin attempted to make psychology an exact science and as part of this process he used topological models of human behaviour. These diagrams included Venn-Euler diagrams, sometimes combined with Peirce-like directed graphs, and various other topological structures resembling, in varying degrees, logic diagrams.[40] Many, if not all of

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[37] See for example E. P. Rosenbaum, "The Teaching of Elementary Mathematics". Scientific American, Vol. 198, Number 5, May 1958, pp. 62-73.

[38] Peter Haggatt, Locational Analysis in Human Geography, London: Edward Arnold Publishers Ltd., 1965, pp. 14-15.

[39] Peter Haggatt, op. cit., pp. 243-245.

[40] Kurt Lewin, A Dynamic Theory of Personality, trans. Donald K. Adams and Karl Zener, New York: McGraw Hill, 1935. Kurt Lewin, Field

them, might be represented by logical formulae although Lewin did not do this. The use of diagrams of this sort has spread well beyond Lewin's immediate followers and seems now to be generally accepted in psychology. In Psychology: An Introduction to a Behavioral Science,[41] for example, valency graphs like those of Peirce are used to represent the relationships of patients on wards and to illustrate the principles of group dynamics.[42]

In the practical sciences logic diagrams find many uses. For example, the printed electric circuit is patterned exactly like a network diagram and the logic diagrams are used to illustrate switching circuits in computers. It is significant that both Lytel[43] and Philster[44] use Venn and Marquand diagrams for this purpose. This may be evaluated in either of two ways depending on what we consider their purpose to be. A geometrical logical diagram may be thought of as being so remote topologically from an electrical network as to be of very little value as illustration; on the other hand, the geometric area may be a visualization

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Theory in Social Sciences, ed. Dorwin Cartwright, New York: Harper & Brothers, 1951. Kurt Lewin, Principles of Topological Psychology, trans. Frith and Grace M. Heider, New York: McGraw Hill, 1936. Kurt Lewin, Resolving Social Conflicts, ed. Gertrud Weiss Lewin, New York: Harper & Brothers, 1948. There is no point in citing page numbers as Lewin's diagrams are so integral a part of his system that they appear in every chapter and are implied in everything he says.

[41]A standard text selected only because it was close at hand.

[42]Henry Clay Lindreu, Donn Byrne and Lewis Petrinovich, Psychology: An Introduction to a Behavioral Science, New York: John Wiley & Sons, 1966, pp. 313, 319 and 413. Venn-like diagrams are presented on pp. 277 and 445.

[43]Allan Lytel, op. cit., pp. 24-27, 66-69 and 80-82, although he uses network diagrams as well, pp. 101-102.

[44]Philster, op. cit., pp. 48-49.

of something which in itself is difficult to understand. Thus, properly used, such diagrams may help elucidate the subject matter of electrical engineering, but badly used they will add confusion to it.

Again it should be pointed out that the use of graphs in science can be deceptive if they are not accompanied by careful commentary. It might also be suggested that scientists should be familiar with various sorts of graphs so that they may use those which are clearest, simplest, and most adequate for their purpose.

(d) In Business

Logic diagrams can be, and have been, used outside the academic world. One outstanding example of the use of the logic diagram in business occurs in an article by Edmund C. Berkeley called "Boolean Algebra (The Technique for Manipulating 'And', 'Or', 'Not', and Conditions) and Applications to Insurance".[45] Although Berkeley is using Venn diagrams he expands them to incorporate six terms all represented by topological areas bounded by only one line by using complicated horseshoe-shaped areas,[46] which tend at times to be very confusing. He also makes use of irregularily shaped areas to represent unknowns.[47] To make it clear just how powerful these diagrams are in practical situations within business, we will simply give, without working out, two of the problems to which Berkeley applies them:

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[45]Edmund C. Berkeley, "Boolean Algebra (The Technique for Manipulating 'And', 'Or', 'Not', and Conditions) And Applications to Insurance", The Record of the American Institute of Actuaries, Vol. 28, Number 3, October, 1937, pp. 373-414.

[46]Berkeley, op. cit., p. 399.

[47]Berkeley, op. cit., p. 400.

Problem 1: An employer has a contributory group insurance contract. On any given date, what are the possible statuses of those of his employees who are not insured with reference to: being eligible for insurance; having turned in an application for insurance; having the application for insurance approved; requiring a medical examination for insurance? Assume:

1. Any employee, to be insured, must be eligible for insurance, must make application for insurance, and must have such application for insurance approved.

2. Only eligible employees may apply for insurance.

3. The application of any person eligible without medical examination is automatically approved.

4. (Naturally) an application can only be approved if the application is made.

5. (Naturally) a medical examination will not be required from any person not eligible for insurance.[48]

Problem 3: (Joint Associateship Examination, 1935, Part 5, question 9b): Certain data obtained from a study of a group of 1,000 employees in a cotton mill as to their race, sex, and marital state were unofficially reported as follows: 525 colored lives; 312 male lives; 470 married lives; 42 colored males; 147 married colored; 86 married males; 25 married colored males. Test this classification to determine whether the numbers reported in the various groups are consistent.[49]

These are no remote arguments that the diagrams are being used to test but the sort of problems that an insurance company might meet in the field. This would suggest that the Venn diagram, and perhaps some of the other diagrams, might well be studied by business and industry as a problem solving method.

(e) Summary

Anything that may be treated by means of logical symbols is amenable to some type of logic diagram. This means that the diagrams will be useful in the examination of any type of argumentative or

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[48]Berkeley, op. cit., p. 403-404.

[49]Berkeley, op. cit., p. 409.

deductive thinking. The various diagrams are not of equal use in treating such deductions. Some arguments are more clearly understood with symbolic notation but the diagrams are very useful in a surprising number of cases. We have mentioned a broad cross-section of such cases in this section and suggested that there are many other areas in which the diagrams might find use. We add again, however, that the user of the diagrams must exercise caution. We now turn to methods of evaluating the diagrams, which methods will give the users of these diagrams instruments for checking the adequacy of their diagrams.

3. The Evaluation of the Logic Diagram

(a) Iconicity[50]

There must be a one to one correspondence of the structure to be represented and the structure representing. This is intuitively obvious but might easily be proven. Let us use a Venn[51] diagram to represent a number of classes and let the classes to be represented be *a*, *b*, *c*, . . . *n*, and the elements (e.g. the geometric areas) of the diagram which are to represent these classes *a'*, *b'*, *c'*, . . . *n'*-1. where *n* is the same numerically as *n'*. The number of elements in the diagram is clearly

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[50]"Icon" is Peirce's term and is defined at 2.247.

> An *Icon* is a sign which refers to the Object that it denotes merely by virtue of characters of its own, and which it possesses, just the same, whether any such object actually exists or not. It is true that unless there really is such an Object the Icon does not act as a sign; but this has nothing to do with its character as a sign. Anything whatever, be it quality, existent individual, or law, is an Icon of anything, in so far as it is like that thing and used as a sign of it.

[51]This section deals particularly with the Venn diagram. Similar proofs might be given for other systems. We are also assuming in this case that the terms stand for classes. If the terms stand for something other than classes similar proofs may be constructed.

less than the number of classes to be represented. Now we must be able to represent any subclass made up of any combination of classes i.e. $(abc \ldots n)$, $(abc \ldots \bar{n}) \ldots (a\bar{b}\bar{c} \ldots \bar{n})$, $(\bar{a}\bar{b}\bar{c} \ldots \bar{n})$ as well as subclasses composed of combinations excluding some of the possible classes between *a* and *n*. There are only three sorts of propositions that may be made about the subclasses containing *n* terms: 1) $(abc \ldots n)=0$, 2) $(abc \ldots n)=1$ and, 3) $(abc \ldots n)\neq 0$. In the first case we must shade out the compartment representing $(abc \ldots n)$; in the second we must shade out everything but this compartment; in the third we must make a mark of some sort in the compartment. Now we may pair off the elements of the diagram with the classes, *a'* with *a*, *b'* with *b*, *c'* with *c*, . . . *n'*-1 with *n*-1. We are left with the unrepresented class *n*. Thus there can be no compartment representing the subclass $(abc \ldots n)$. Therefore to be adequate the diagram must contain at least as many elements as there are classes or terms to be represented.

Further, the diagram must represent the classes. That is, it must be understood that the element *a'* represents the class *a*, the element *b'* represents the class *b*, etc. This qualification is trivial but it is also significant. It is trivial in that all it means is that a diagram, the elements of which do not represent anything, is useless. A diagram must be attached by means of what Peirce calls "fixed points" to the universe which the diagram represents.[52] Thus the various subdivisions of the diagram are iconic in that they stand for something extradiagrammatic.

Iconicity in this sense is the primary requirement of the logic

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[52] See particularly Peirce, 3.419 in this regard.

diagram. Without it the diagram is irrelevant; with it even an inadequate diagram may have some value. There must be a correspondence of the diagram with the elements to be represented and the representation must take place.

(b) Abstraction

What if there are a greater number of elements in the diagram than there are elements to be represented? Does this shatter the diagrams iconicity? It does not. Suppose that the elements *a*, *b*, *c*, . . . *n* are to be represented by a diagram having the elements *a*', *b*', *c*', . . . *n*', *n*'+1. We must be able to represent the subclasses derived in the last section when they are equal to 1 or to 0 or when they are not equal to 0. Let the subclass (abc . . . n)=0. This includes two compartments which must in conjunction equal 0: a', b', c', . . . n',(n'+1) and a', b', c', . . . n',$(\overline{n'+1})$. Thus:

$$(abc \ldots n\,[n+1]) + (abc \ldots n\,[\overline{n+1}]) = 0$$
$$([n+1] + [\overline{n+1}])\,(a\,b\,c \ldots n) = 0$$
$$\text{But } x+\bar{x} = 1$$
$$(abc \ldots n) = 0$$

Thus the introduction into the diagram of an element, n'+1, which is not needed to represent the classes does not affect the iconicity of the diagram, as its effect cancels itself out. It does however violate the principle of abstraction.

The principle of abstraction as given by Hocking attempts to include iconicity. It is evident, however, that Hocking's major concern is the reduction of the elements of the diagram to the bare essentials:

> ...if a representation is to be of any value, it must have the force of an abstraction. That is to say, while it must be rich enough in prominent features to correspond to the entire system which our purpose defines, it must be poor

> enough to distract us as little as possible with other features. The primary recommendation of the graph in logic, in contrast to other possible ways of making sensible these ideal relations, is that it is almost poor enough to tell nothing but the truth. . . . We do not need to exert a great additional heave of abstraction from the graph in order that it may aid in discriminating the logic of a situation from its psychology.[53]

Abstraction is the complement to iconicity. Iconicity gives the lower limit of the elements of the diagram; abstraction gives their upper limit. If a diagram violates the principle of iconicity it is inadequate because it cannot deal with all the elements that it must represent; if it violates the principle of abstraction it becomes unnecessarily confusing. Thus iconicity and abstraction together define the diagram with regard to its elements.

(c) Proportion

Hocking lists proportion as a secondary value of diagrammatic representation in logic.[54] Once the correspondence is established between the elements of the diagram and the elements represented, it may be desirable to establish a correspondence between the relationships of the elements of the diagram and the relationships of the elements represented, and between second degree relationships, etc. The two most important logical relations (i.e. relationships between terms or classes) that may be illustrated by means of diagrams are connotative rank and connotative kinship. These have already been discussed and the limits of the diagrams' representational ability established with regard to

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[53] Hocking, op. cit., p. 32.

[54] See II 2 (h). Hocking's major aim was to show to what degree proportion was possible in a geometric diagram.

rank and kinship for the geometric diagram. The more nearly that a diagram is able to approximate representation of connotative rank and kinship in the relationships of its parts while remaining iconic and abstract the more adequate the diagram will be.

(d) Simplicity

It is obvious that the rules of operation applying to the diagram should be as simple as possible and that the diagram itself should be uncluttered and should use as few basic devices as possible. A Venn diagram for three terms can give the solution of an Aristotelian syllogism much more easily than the clumsy devices of nineteenth-century logic.[55] A Peirce diagram, as Roberts has shown, is easily used as a calculus, in many cases more easily than the complicated apparatus of formal symbolic logic.[56] When diagrams lose simplicity, as Hocking's do[57] for example, they lose their value as logic diagrams for we then see things not about logic but about diagrams. It is to be noted that Peirce requires simple diagrams for logical analysis in order to arrive at the most basic relations.[58] Thus an adequate logic diagram (i.e. one which has iconicity, abstraction and proportion) which is simple is to be preferred to one that is complex whether the diagram is to be used as an example, a calculus or an instrument of logical analysis.

(e) Purpose

One very important element in the evaluation of logic diagrams that is easily overlooked is the purpose for which those diagrams were

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[55] II 2 (c).

[56] II 4 (d).

[57] Hocking was of course aware of this. He did not mean to have his diagrams used in practical problems, II 2 (h).

[58] II 4 (d).

intended. The simplest and most iconic diagram, if it serves the purpose for which it was intended, is to be more highly evaluated than one that does not serve its purpose. Thus an engineer attempting to approximate an electrical network on paper would be better advised to use a Gardner network than a Venn diagram. On the other hand, a teacher attempting to communicate the nature of the networks in computers might wish to give a two-dimensional model in order to give a fresh point of view of the relationships involved and would then be better off to use a Venn diagram than a Gardner network. A logician teaching elementary logic would find Venn's diagrams much more useful than Peirce's (at least in this author's opinion) but if the same logician were developing a diagrammatic calculus he would find Peirce convenient and Venn confusing. This standard of evaluation is, to some degree, subjective but it is important and valid.

(f) Summary

The methods of evaluating any particular system of logic diagrams may be reduced to a number of rules:

Rule 1: Any logic diagram must be iconic.

Rule 2: Any logic diagram must be useful for the purpose to which it is being turned.

These are the primary requirements of the logic diagram and if either or both of these rules is broken the diagram may be judged to be inadequate.

Rule 3: Any logic diagram must accommodate the least necessary number of terms.

This is the rule of abstraction.

Rule 4: Any logic diagram should display proportion whenever possible.

Rule 5: Any logic diagram should be as simple as possible in appearance and operation.

These rules form a hierarchy. The first two are absolutely essential for adequate diagrams. The third is logically possible whenever the first applies and should thus always be carried out. The last two are not essential to a diagram's being useful but the nearer that the diagram approaches the achievement of these ideals the more adequate it becomes. It may, at times, be necessary to sacrifice rule four for the sake of rule five or vice versa. These then are the five basic points on which the logic diagram ought to be evaluated and these should be taken into consideration whenever diagrams are to be used.

4. The Central Issues Raised by this Paper

(a) Quantification of the Predicate

In the time of Boole, de Morgan, and Hamilton the question of whether the predicates of propositions were to be quantified became a central issue for everyone interested in the use of logic diagrams. In Euler,[59] of course, predicates were not quantified as he used only the basic Aristotelian propositions in his syllogisms:

All A are B
No A are B
Some A are B
Some A are not B

Hamilton[60] attempted to extend the capacity of logic to represent arguments by developing these four basic propositions into eight:

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[59] II 2 (a).

[60] II 2 (a) and II 5 (a).

All A is all B
All A is some B
Some B is all A
Any A is not any B
Some A is some B
Any A is not some B
Some B is not any A
Some A is not some B

We have attempted to show that this group of propositions is neither as comprehensive nor as simple as possible from Venn's point of view.[61] One major reason for this problem is the failure of Venn to come to grips with the representation of particular propositions.[62] This makes it impossible to manipulate his diagrams in order to solve problems.

Even while Hamilton was attempting, unsuccessfully, to quantify the predicate, a new approach was being developed by Boole. Boole applied mathematical techniques to logic. Although Boole is beyond the scope of this paper de Morgan and Venn represent two distinct mathematical systems of logic based more or less on Boole and are within our jurisdiction.

De Morgan,[63] too, derives eight propositions from all possible combinations of the brackets and dots in his system:

X))Y
X).(Y
X(.)Y
X((Y
X(.(Y
X()Y
X)(Y
X).)Y

Obviously all "predicates" are quantified. In fact there is no subject

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[61] II 2 (a).

[62] II 2 (c). A more important reason is the weakness of Hamilton's position.

[63] II 5 (b).

or predicate but simply two terms which may be reversed: X))Y is equivalent to Y((X. Of these eight propositions X(.(Y has no equivalent in Hamilton's system and X).)Y is equivalent to both "All X is not some Y" and "Some Y is not any X". De Morgan's system seems to solve the problem of the quantification of the predicate by eliminating the predicate entirely. There are two problems with this view. De Morgan's dots and brackets constitute a private language understandable only after considerable study and practice - and Boole had already discovered a simpler calculus. Further, de Morgan's system is a closed system drastically limiting the number of propositions that can be expressed. In Boole's system the number of possible propositions expressible is unlimited.

Boole used the symbols of elementary algebra for his calculus and Venn followed his example. In Venn,[64] as in de Morgan, the problem of the quantification of the predicate is solved by the elimination of the predicate as an element of the logic system. For Venn, however, there are no general basic propositions. One could not say that x=1 was more basic than (x+y)=1. They are both simply propositions. Thus, in Boolean algebra, the problem vanishes. The problem is basically one which arises when one attempts to develop a "class inclusion and exclusion" system of logic but does so retaining the language of traditional "predicative" logic.[65] When one develops such a view from scratch, as in de Morgan, or when one begins with a "compartmental" or "existential" view,[66] the problem of whether or not to quantify the predicate and how to do so disappears.

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[64]II 2 (c).

[65]These are Venn's terms, discussed in II 2 (a).

[66]These are also Venn's terms and are discussed in II 2 (c).

(b) Existence

Several other related problems have been raised in this paper: the question of the existential import of the A proposition,[67] the question of the implications of specific propositions with regard to existence,[68] the question of the relationship of logic to reality,[69] etc. These problems are all bound up in one difficult question that transcends logic but is still a most important question for it: what exists? This problem might be briefly stated as follows: what sort of proposition must we be given before we can derive the existence of something from that proposition? We have seen a variety of views in this paper.

The first such view is that supported by the Euler diagrams.[70] If these diagrams are valid representations of Aristotelian logic the moment that we are given a particular proposition we know that the subject, at least, exists. This is true for any particular proposition. Thus "Some unicorns have one horn" means "there is at least one unicorn having one horn". This view is strengthened when x's are used as in Venn, to indicate O and I propositions. If this view is rigorously maintained logic is separated from reality and becomes a game played with concepts, or the term "reality" is broadened to allow talk about unicorns or anything else that we wish to say exists.

The second view concerning existence is that of Jevons.[71] For

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[67]Particularly in II 2 (a) and II 2 (f).

[68]II 2 (a), II 2 (c), II 2 (f), II 3 (b), II 5 (a).

[69]Although this was mentioned often specific attention was drawn to the problem in II 4 (d) and III 1 (a).

[70]II 2 (a).

[71]II 2 (c).

Jevons every term must have existential import. Thus A=Ab tells us not only that there are A's and that these A's are not B's but also that there are B's and, obviously, that these B's are not A's. This would seem to mean that the moment a term is introduced into a proposition we are committed to the existence of the things represented by that term. It is interesting that this is not so for the negative of such terms. Thus from A=AB, B=BC and C=Ca we may derive ABC but not abc. It makes no difference to Jevons what we are talking about: it exists.

A third view, in many ways the most moderate of those presented, is that of Venn.[72] For Venn the universe of discourse, everything there is, is represented by the number 1. In other words, $(x+\bar{x})(y+\bar{y}) \ldots (n+\bar{n})=1$. If this is so something must exist. Thus, although we are uncertain whether $x=0$ or $\bar{x}=0$ these cannot both be true. This is simply the law of excluded middle and it would seem to be an absolute for all geometric diagrammatic systems. If the law of excluded middle does not apply in any system of logic the diagrams of Euler, Venn, Marquand, Macfarlane, etc. are useless. If, then, we are given the proposition "All unicorns are white" there must be either unicorns or things which are not unicorns. For Venn particular propositions have the same implications that they did for Euler. This is, perhaps, why Venn tends to shun particular propositions and the problems they present.

Another complication is added to this problem by the view of Lewis Carroll.[73] For Carroll the A proposition implies the existence

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[72] Also II 2 (c).

[73] II 2 (f).

of its subject. This view is in direct opposition to both Aristotelian logic and Boolean algebra. If we accept Carroll, Aristotelians are mistaken with even such a simple argument as Barbara: As well as "all A are B and all B are C ∴ all A are C" we are able to write "all A are B and all B are C ∴ some A are C". This would put us in the position of affirming the existence of the subject of every proposition except the universal negative and thus of affirming the existence of some particular thing in every argument.

The one thing which is common to all of these views is the fact that something must exist. So far as Euler, Jevons, Venn and Carroll are concerned if nothing exists we are caught in a contradiction. For example ((AvB).(~A).(~B)) does not give the conclusion "there is nothing" but rather "there is a contradiction in the premises". At first this may not be apparent from the diagrams: given ((AvB).(~A).(~B)) we ought to be able to shade out first (~A.~B) then (A.~B).(A.B) and finally (~A.B). This gives us a diagram with no contents. But it is not so simple. A diagram shaded out is precisely equivalent, according to Peirce, to no diagram at all. This diagram, a blank page, represents Peirce's absurd universe and is read "nothing exists". But all absurd universes are equivalent; thus the introduction of terms is impossible.[74] This means that a diagram consisting of compartments completely shaded out cannot represent the absurd universe, but it cannot represent anything else. This leads to the inevitable conclusion that such a diagram does, in fact, mean that there is a contradiction in the premises and that either a correction of the premises is in order or we must erase the entire

[74] II 4 (d) and II 2 (a) and (c).

diagram. Thus we must say that something exists whenever we have at least one premise and no contradictions.

The disagreements among the various logicians discussed above are as to <u>what</u> exists and we have already mentioned the various alternatives suggested by them.

Logic applies, presumably, to all language, not just to language about the real world of existent things whatever that may be. But as we have said, for logicians using diagrams, something exists the moment a proposition is produced. All of this discussion of existence in logic serves the purpose of showing that those logicians who rely on diagrams seem to have a different meaning for the word "existence" than that which we normally use. For these logicians existence represents a certain logical state of affairs and not necessarily a fact of experience.

All of this presupposes the validity of the logic diagram, and even more specifically the geometric diagram, as an instrument for representing the propositions of logic. The position is not universally accepted but it is incumbent on its critics to show either the invalidity of the diagrams in logic or the invalidity of logic as an abstraction from language. Peirce believed, as we have shown, that neither of these could be demonstrated and that language was actually diagrammatic.[75] If he was correct I can see no conclusion other than that above distinguishing two distinct uses of the word "exist": one referring to logical "existence", one to factual existence.

5. Conclusion

There are many other problems in the field of the logic diagram:

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[75] III 1 (a).

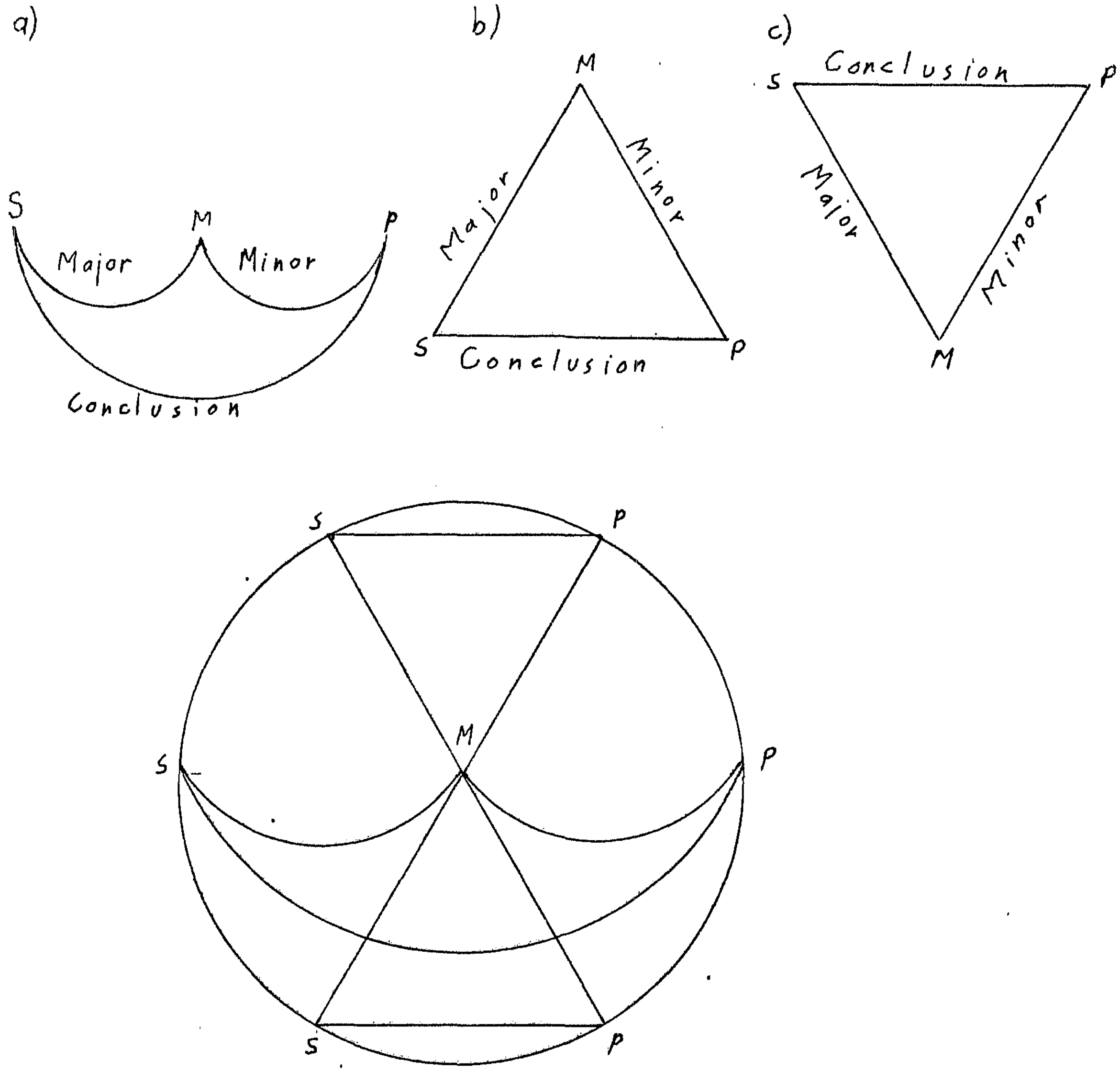
the relationship of geometric, linear and network diagrams topologically, the relationship of diagrams to contemporary logic, and especially the relationship between diagrams, logic and language. Although these questions are beyond the scope of this paper it is hoped that their significance was demonstrated.

It has been the author's hope at least to raise the issues surrounding the logic diagram and thus to open up a fascinating and much neglected field for the reader. It has, further, been his hope that in the logic diagram the reader will discover a powerful tool for manipulating ideas and discovering inconsistencies in arguments. These are, however, lofty aims and the writer will be satisfied if some future researcher finds it useful to have all of the various systems of logic diagrams collected and described in one paper.

APPENDIX I

DIAGRAMS

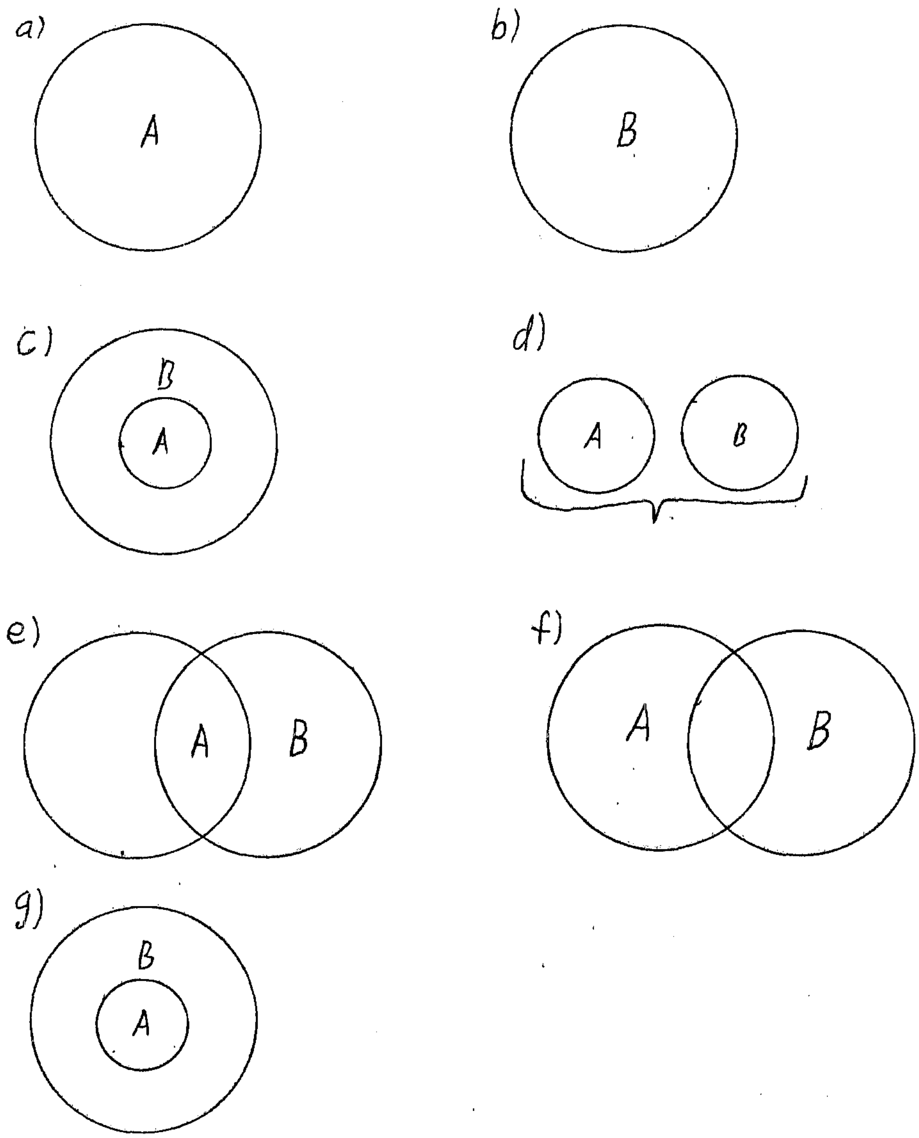
Diagram I   Pre-Eulerian Diagrams



a), b) and c) Mediaeval diagrams of categorical syllogisms

d) Bruno's diagram containing a), b) and c) above and encircling them.

Diagram II   Euler's System - Basic Diagrams



a) A

b) B

c) All A is B

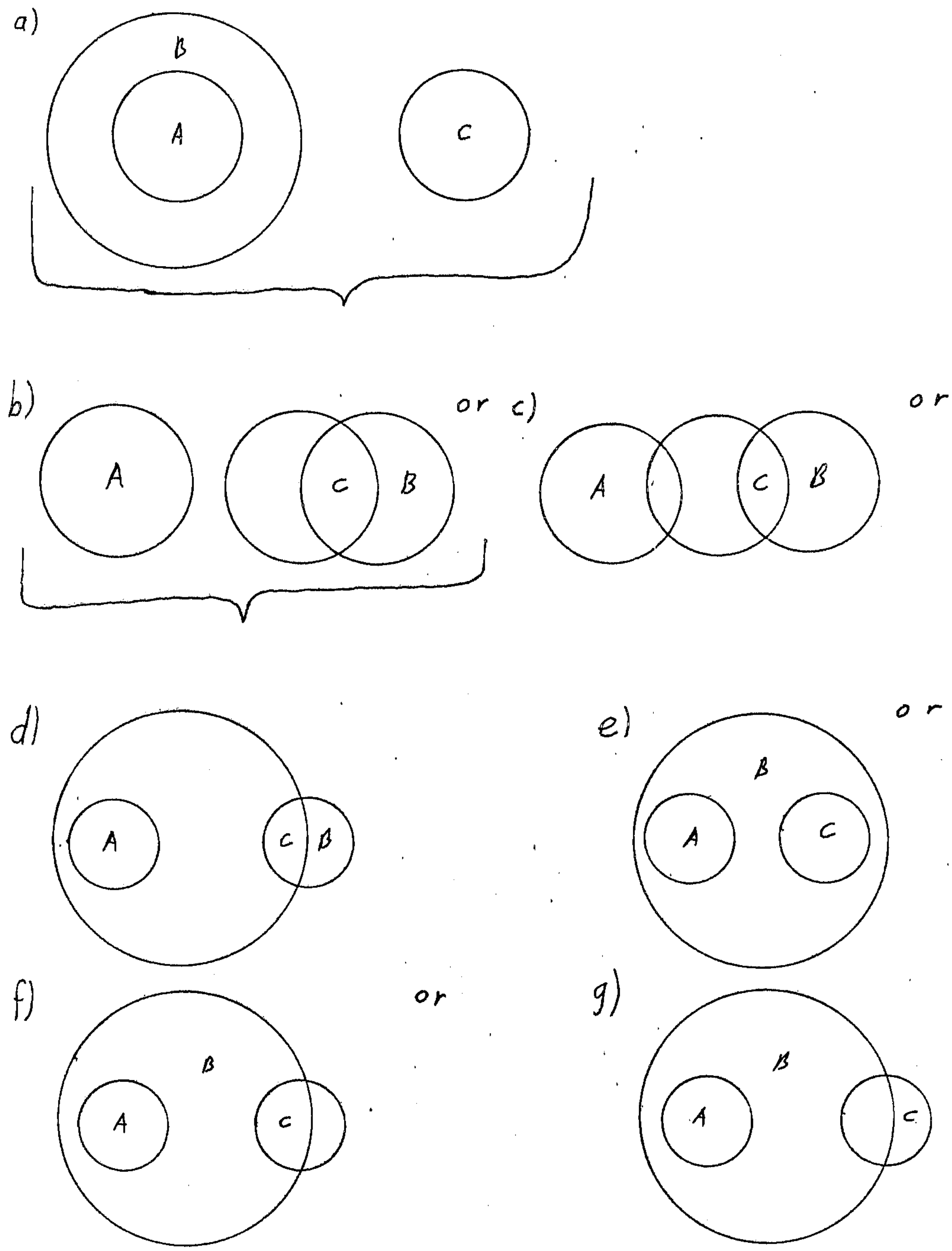
d) No A is B

e) Some A is B; Some B is A; Some B is not A; Some A is not B

f) Some A is not B

g) All A is B; Some B is A; Some B is not A

Diagram III   Euler's System - Use of Diagrams

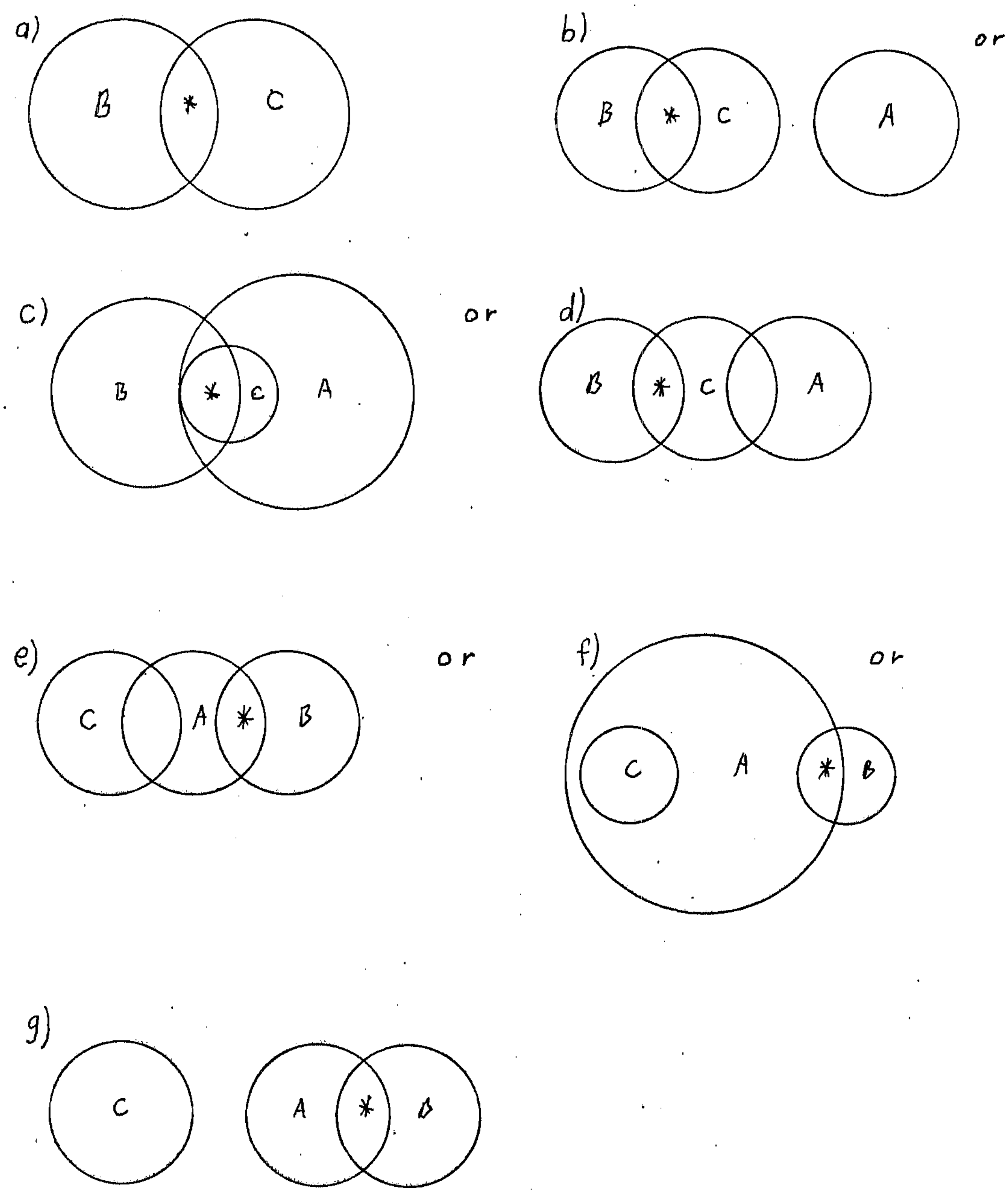


a) All A is B
No C is B
∴ No A is C

b), c) and d) No A is B
Some C is B
∴ Some C is not A

e), f) and g) All A is B
No C is A
∴ No conclusion

Diagram IV   Euler's System - The Use of the Asterisk

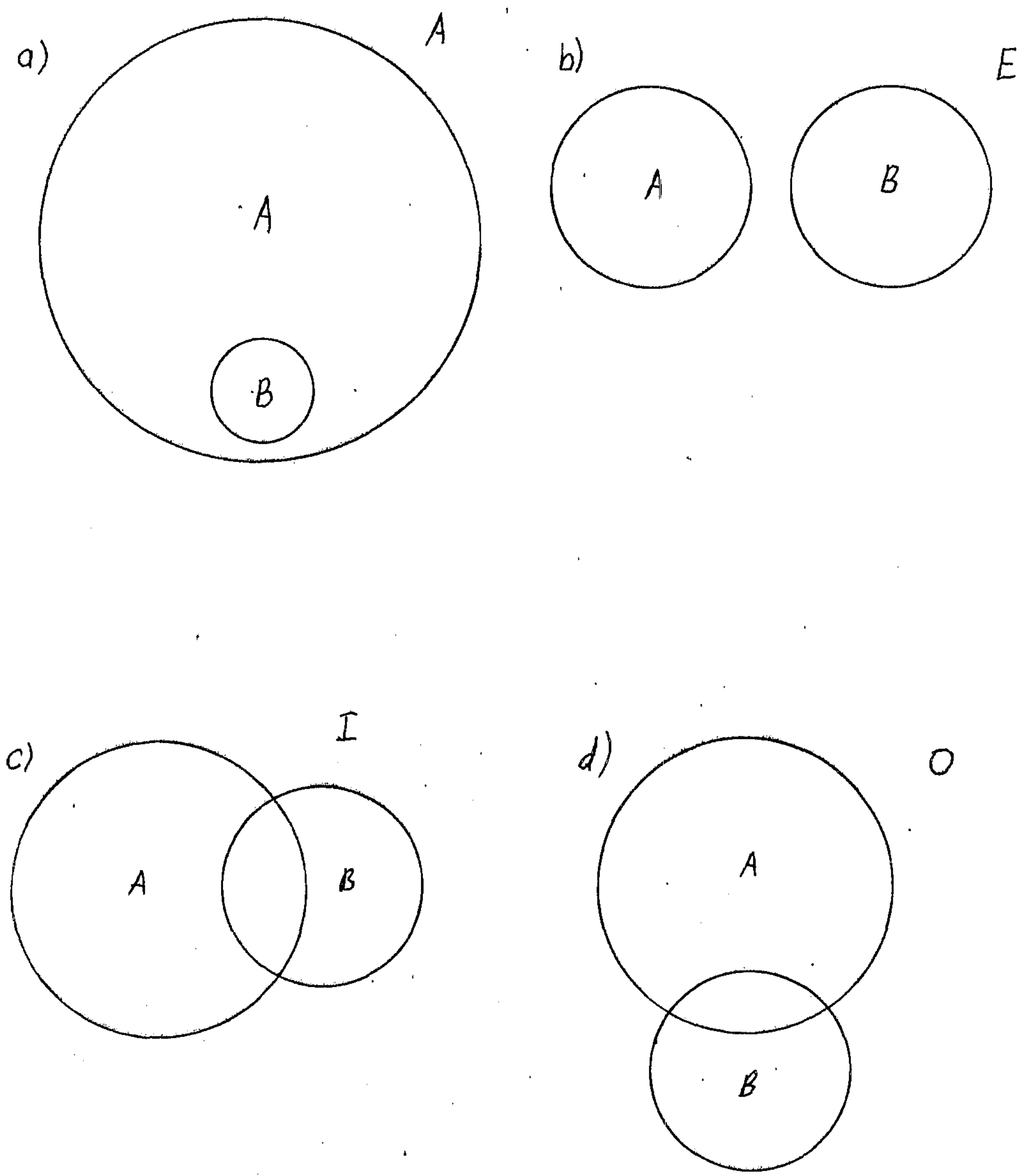


a) Some B is C

b), c) and d) No A is B
Some B are C
∴ No conclusion

e), f) and g) test of:
Some A is B
No B is C
∴ Some C are not A
Conclusion is invalid in f)

Diagram V   Hamilton's Interpretation of Euler's Basic Terms



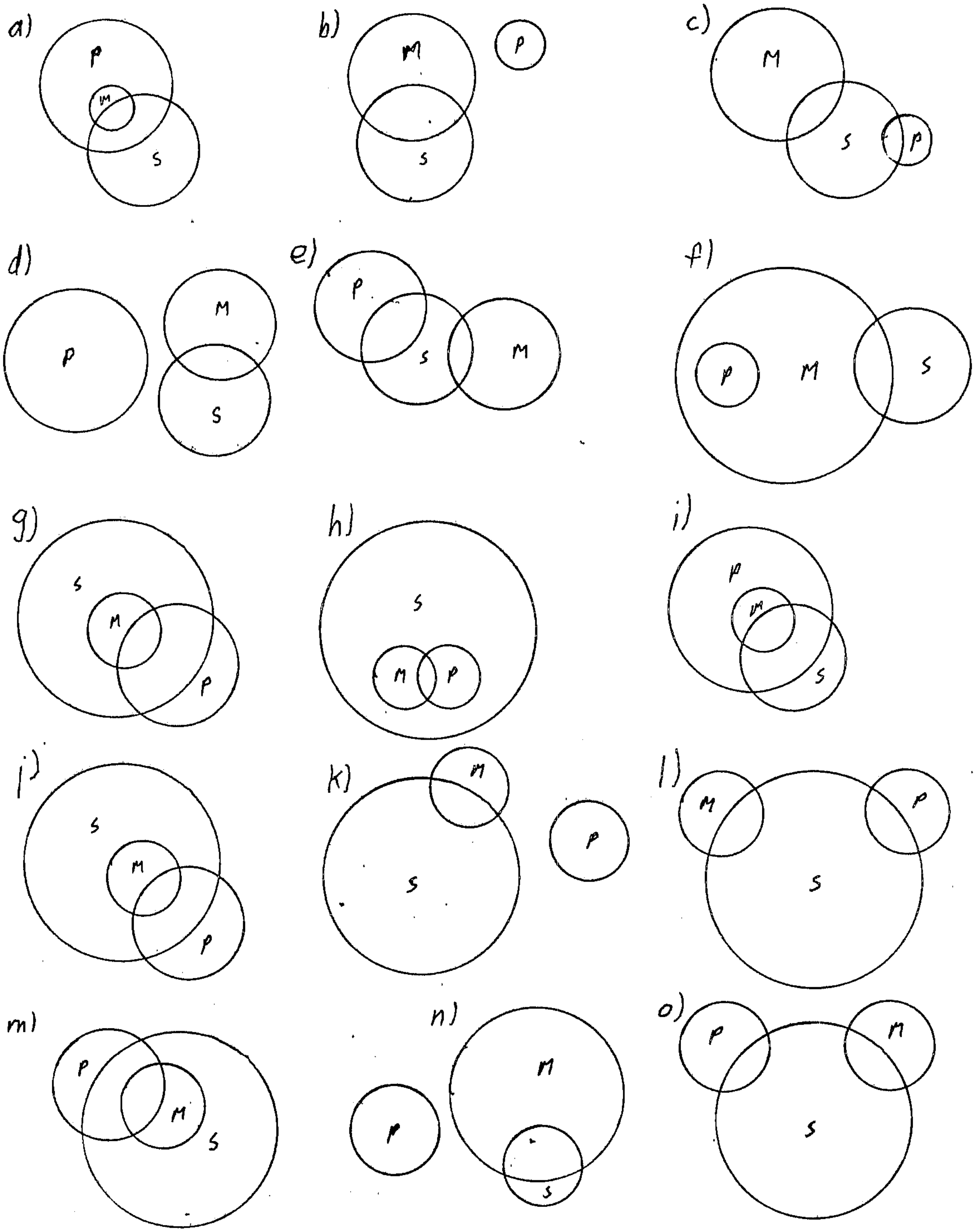
a) All B is A

b) No A is B

c) Some A is B

d) Some A is not B

Diagram VI Hamilton's Representation of Particular Propositions
Using Euler's Diagrams



a), b), c), d), e) Some S are M

f) Some S are not M

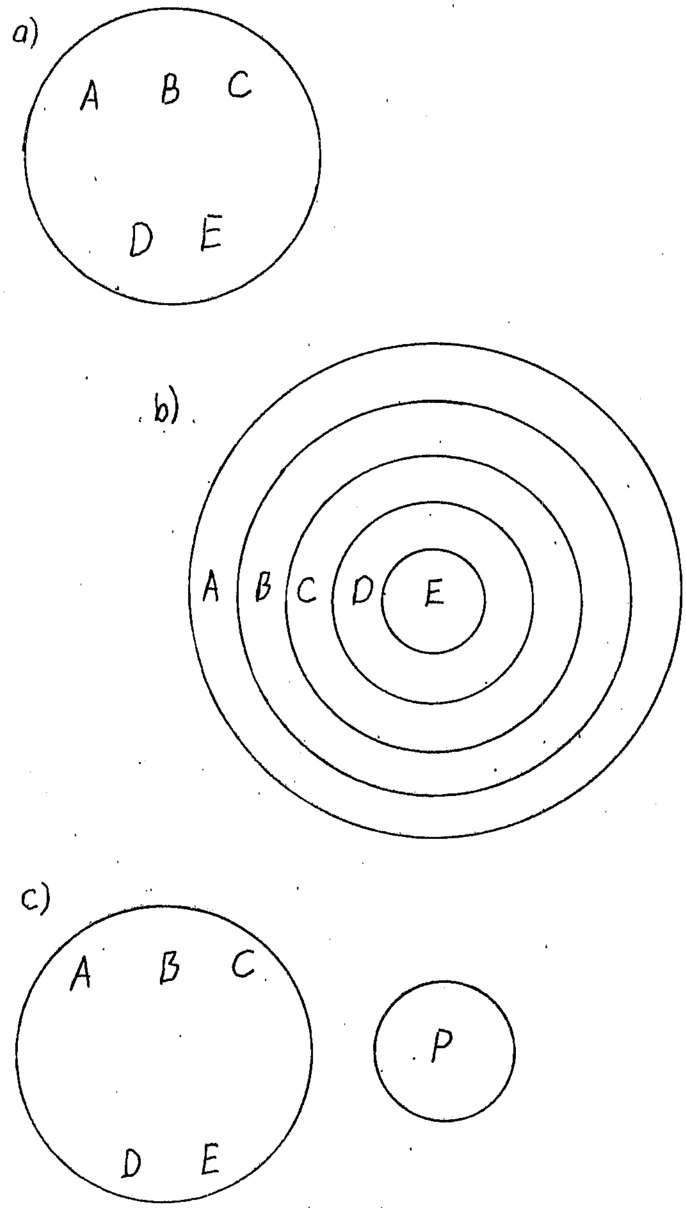
g), h) Some M are P

i), k), l), n), o) Some M are S

j) Some M are not P

m) Some P are M

Diagram VII Hamilton's Extension of Euler's Diagrams to Sorites

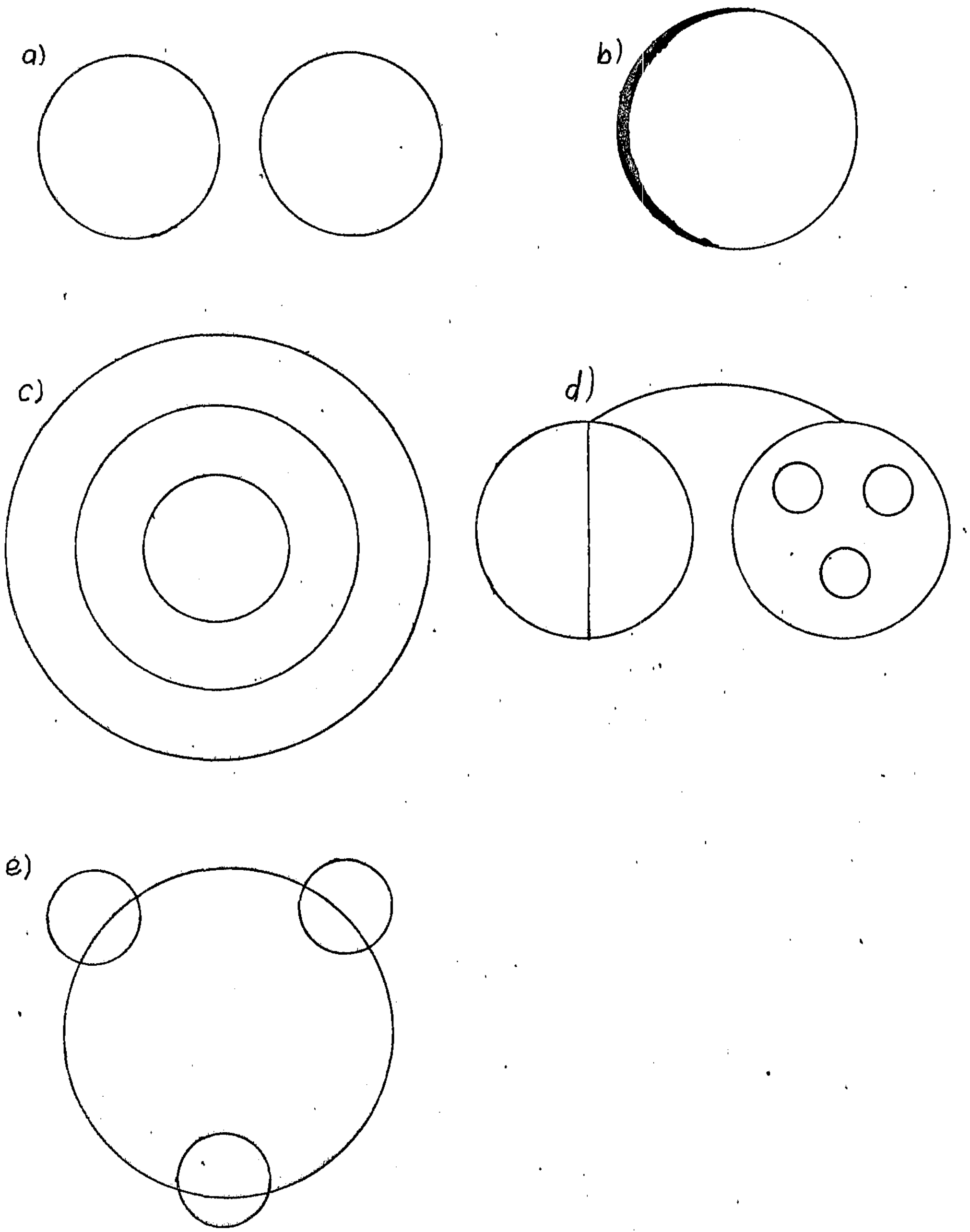


a) All A is the same as B
All B is the same as C
All C is the same as D
All D is the same as E
∴ All A is the same as E

b) All E is D
All D is C
All C is B
All B is A
∴ All E is A

c) E is the same as D
D is the same as C
C is the same as B
B is the same as A
No A is P
∴ No E is P

Diagram VIII Hamilton's Extension of Euler's Diagrams
to Include the Relations of Concepts

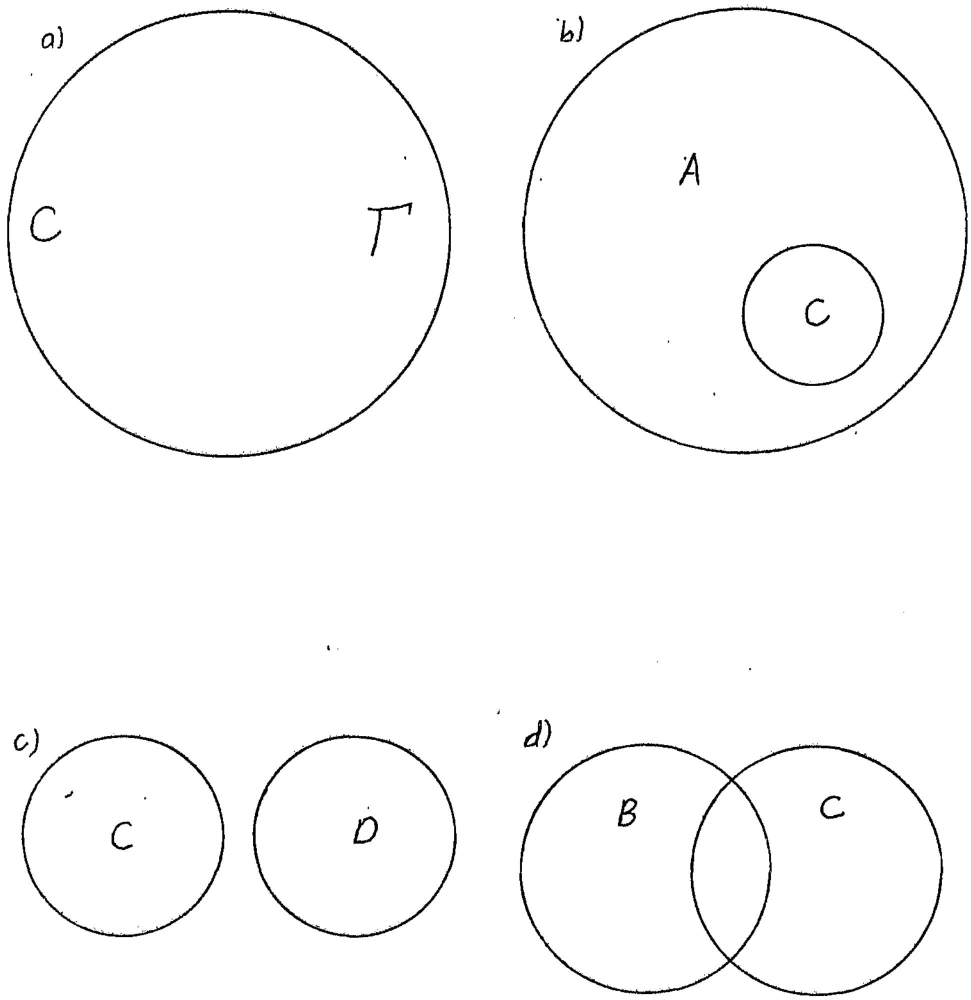


a) Exclusion b) Coëxtension

c) Subordination d) Coördination

e) Intersection, or partial coïnclusion and partial coëxclusion

Diagram IX   Hamilton's Extension of Euler's Diagrams
to Include the Quantification of the Predicate



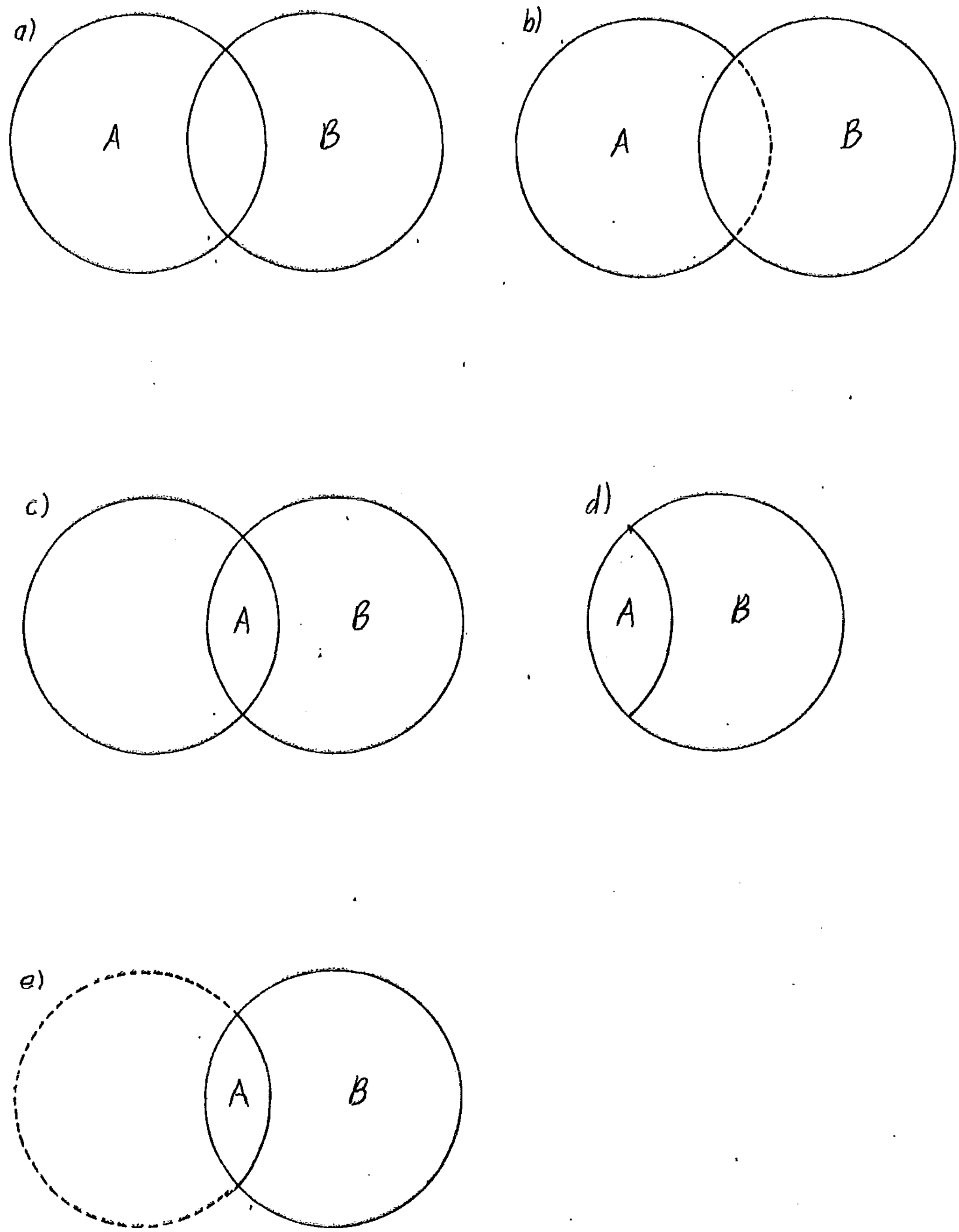
a) All C is all T

b) All C is some A
Some A is all C

c) Any C is not any D

d) Some C is some B
Any C is not Some B
Some B is not any C
Some C is not some B

Diagram X Jevons' Improvement of Euler's Diagrams for the I and O Propositions



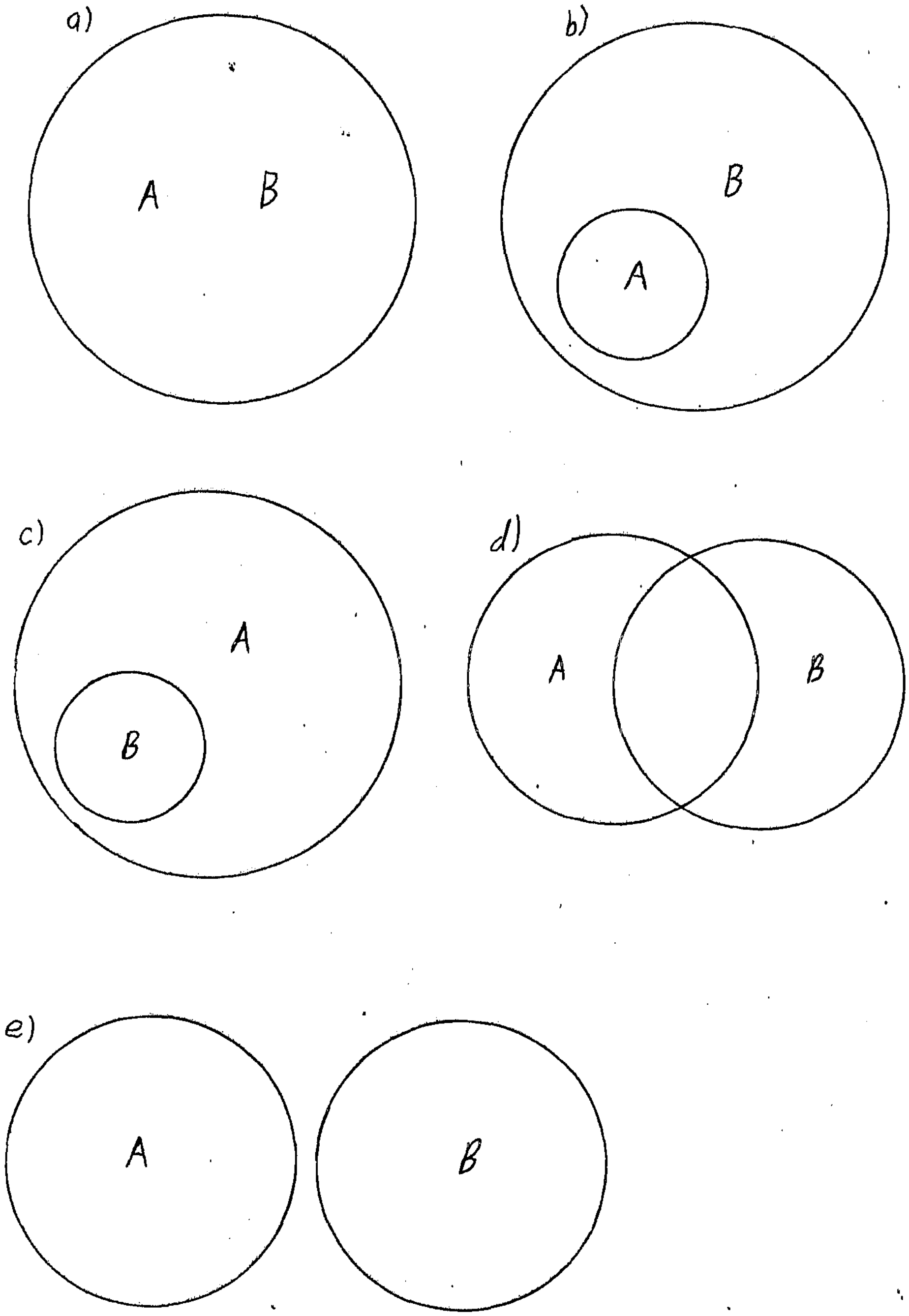
a) Some A are not B (Euler)

b) Some A are not B (Jevons)

c) Some A are B (Euler)

d) e) Some A are B (Jevons)

Diagram XI   Venn's Restatement of Euler's Basic Diagrams



a) All A is all B

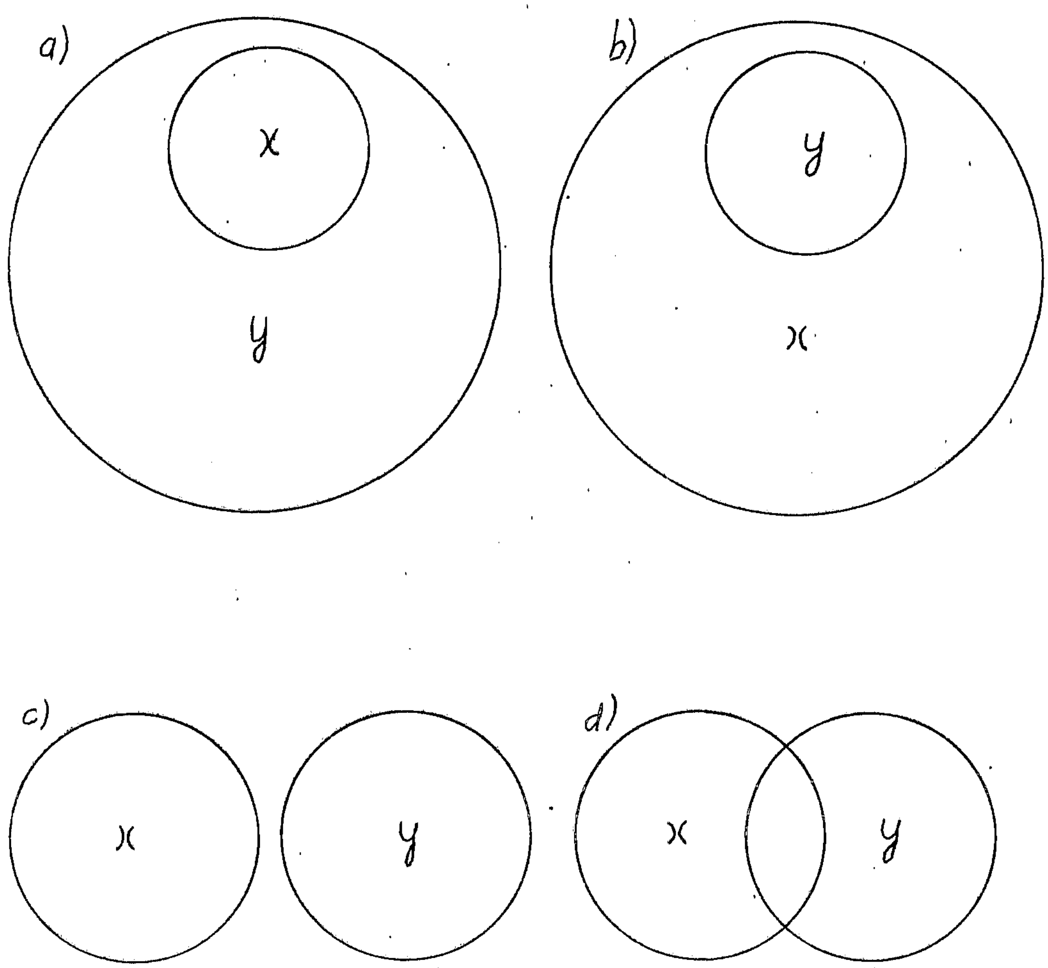
b) All A is some B

c) Some A is all B

d) Some A is some B

e) Any A is not any B

Diagram XII Carroll's Interpretation of Euler's Basic Diagrams



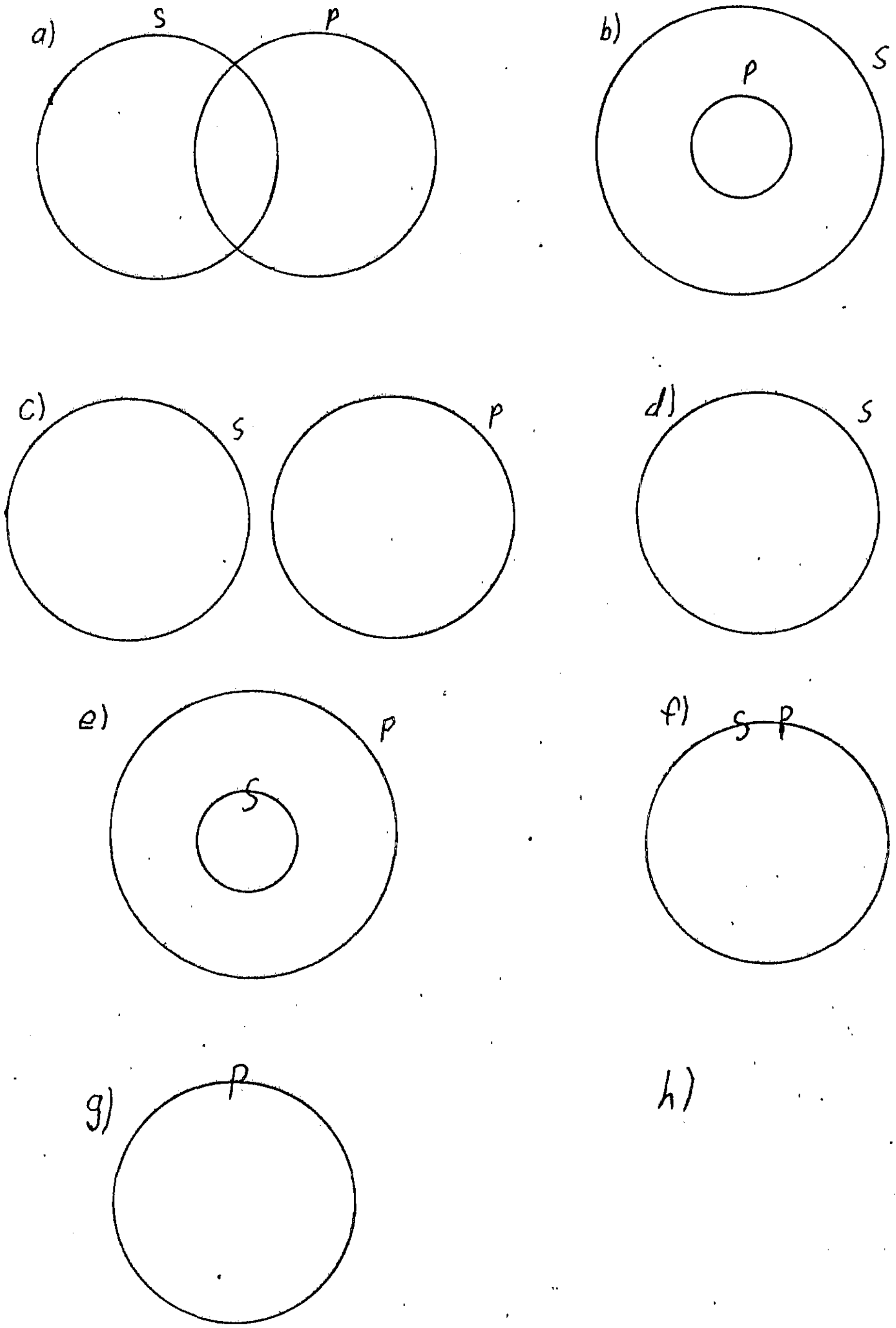
a) All x are y; No x are not-y; Some x are y; Some y are not-x; Some not-y are not-x; No not-y are x; Some y are x; Some not-x are y; Some not-x are not-y

b) All y are x; No y are not-x; Some y are x; Some x are not-y; Some not-x are not-y; No not-x are y; Some x are y; Some not-y are x; Some not-y are not-x

c) All x are not-y; No x are y; Some x are not-y; Some y are not-x; Some not-x are not-y; No y are x; All Y are not-x; Some not-y are x; Some not-y are not-x

d) Some x are y; Some x are not-y; Some not-x are y; Some not-x are not-y; Some y are x; Some not-y are x; Some y are not-x; Some not-y are not-x

Diagram XIII   Peirce's Interpretation of Euler's System



a) Entire ignorance

b) Any P is S

c) No S is P

d) There is no P

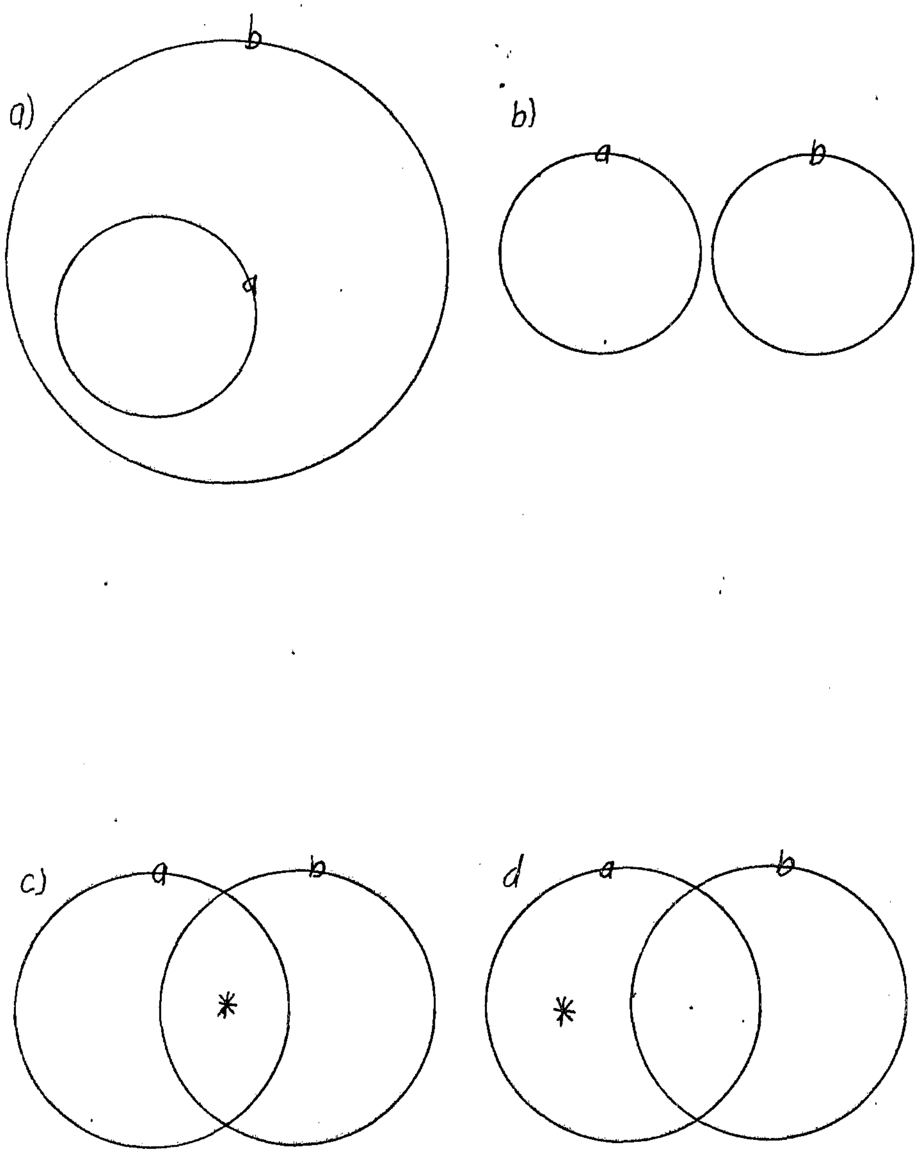
e) Any S is P

f) S and P are identical

g) There is no S

h) There is neither S nor P

Diagram XIV Lewis' Restatement of Euler's Basic Diagrams



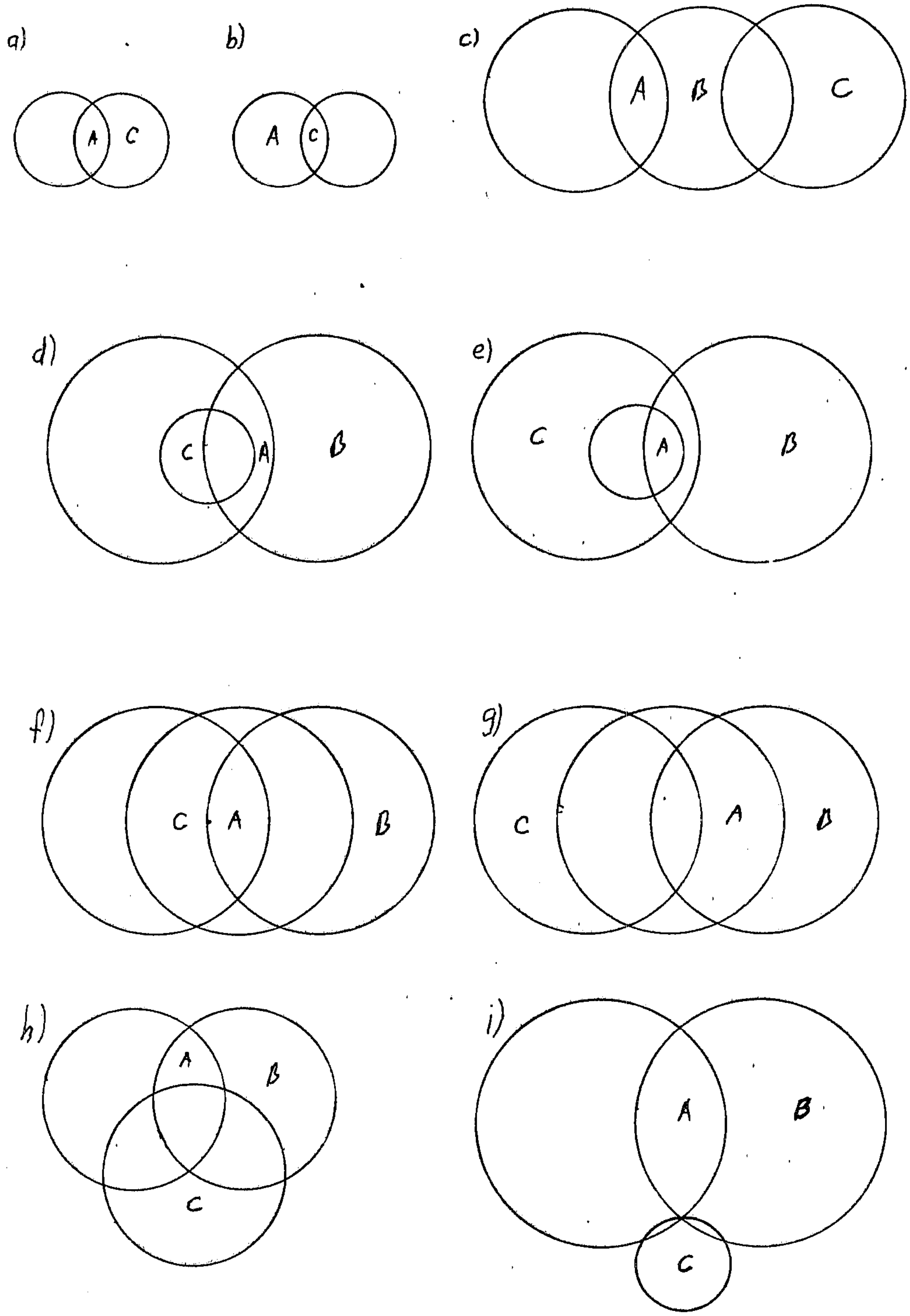
a) All a is b

b) No a is b

c) Some a is b

d) Some a is not b

Diagram XV   Euler's System - Inadequacies

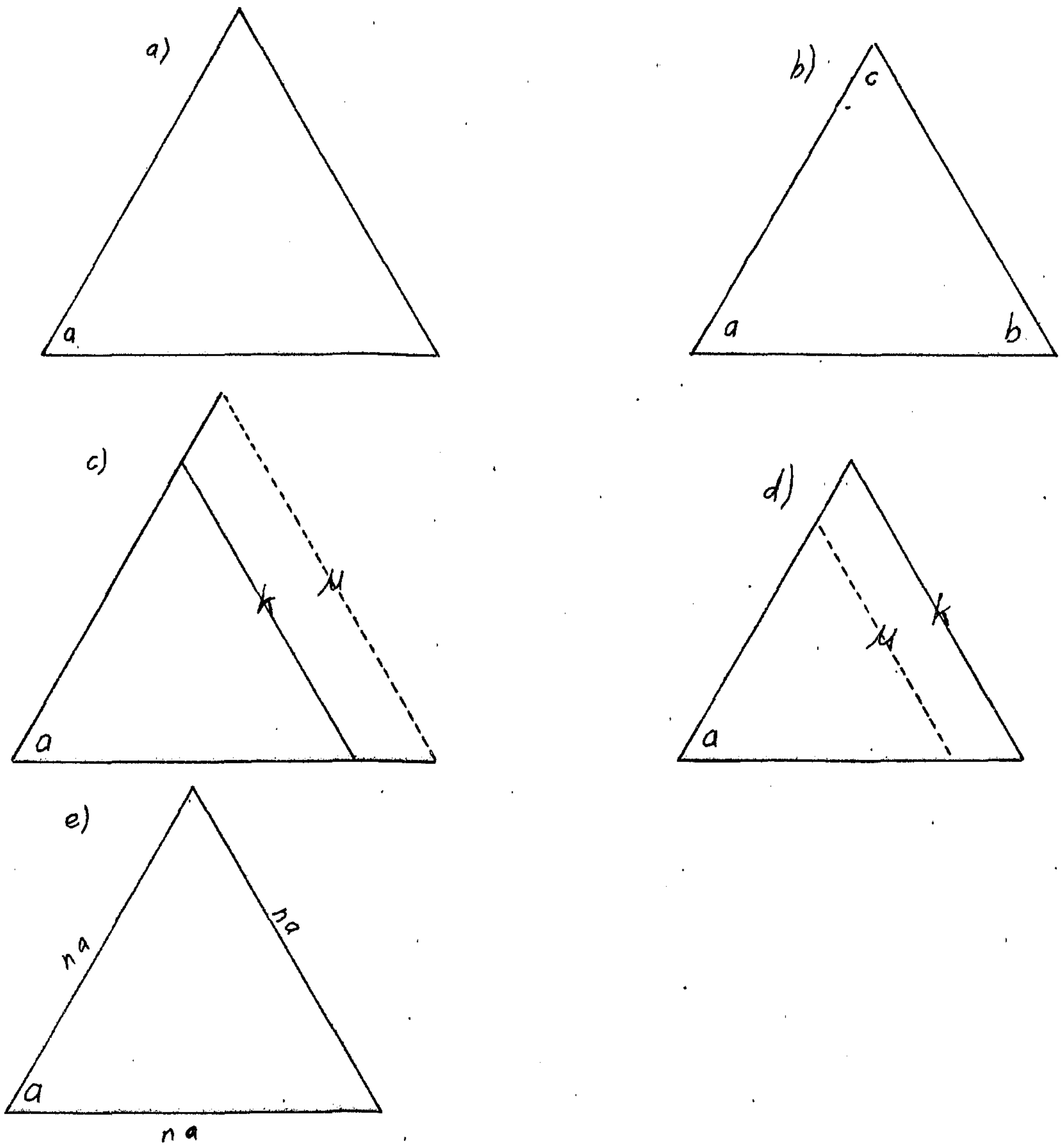


a) Some A is C

b) Some C is A

c) to i) Some A is B
Some B is not C
∴ No conclusion

Diagram XVI   Maass' Basic Diagrams



a) a

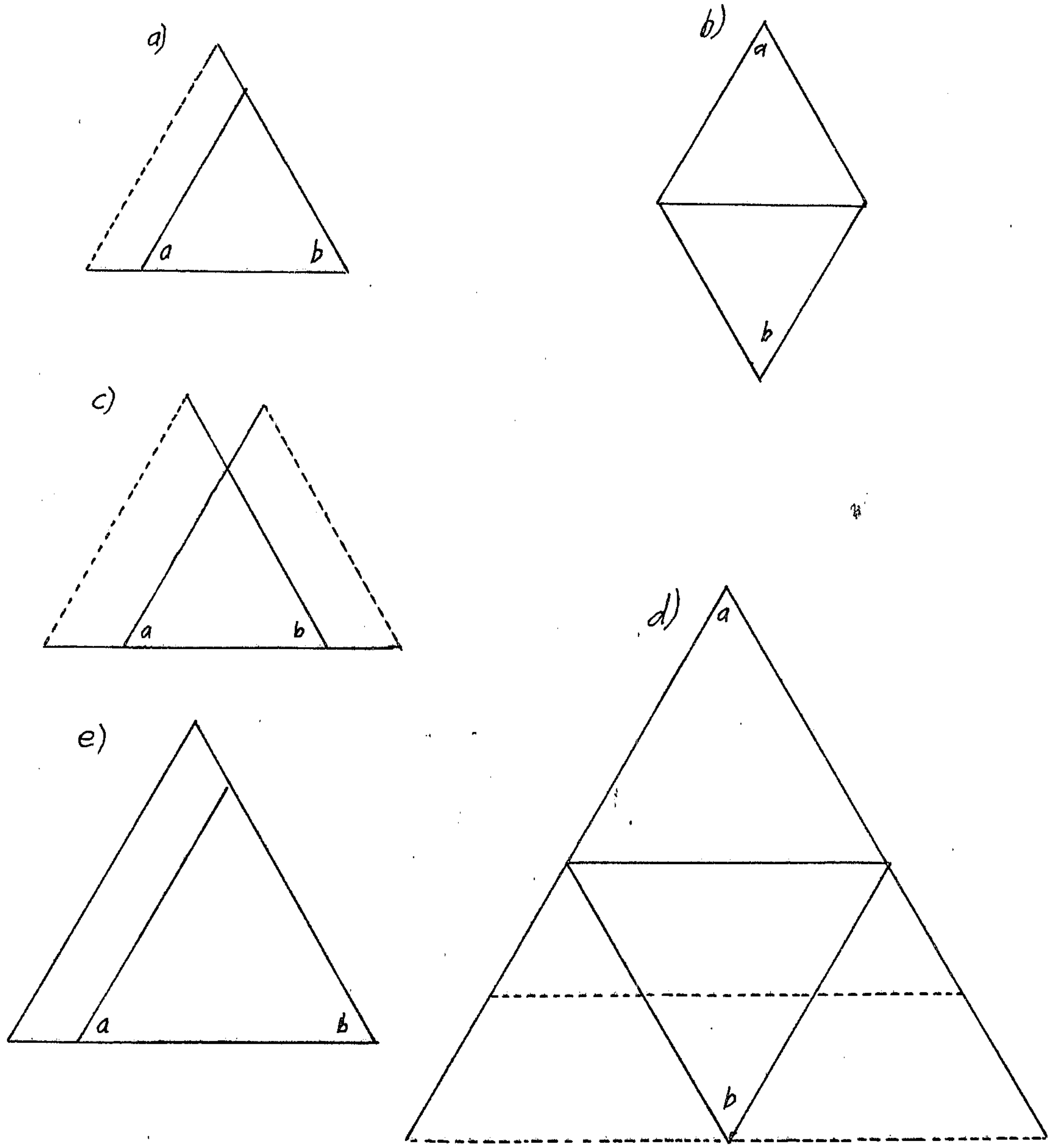
b) a=b=c

c) a includes k but may also include μ

d) a includes k but may only include μ

e) na represents not a

Diagram XVII   Propositions in Maass' System



a) All a is b

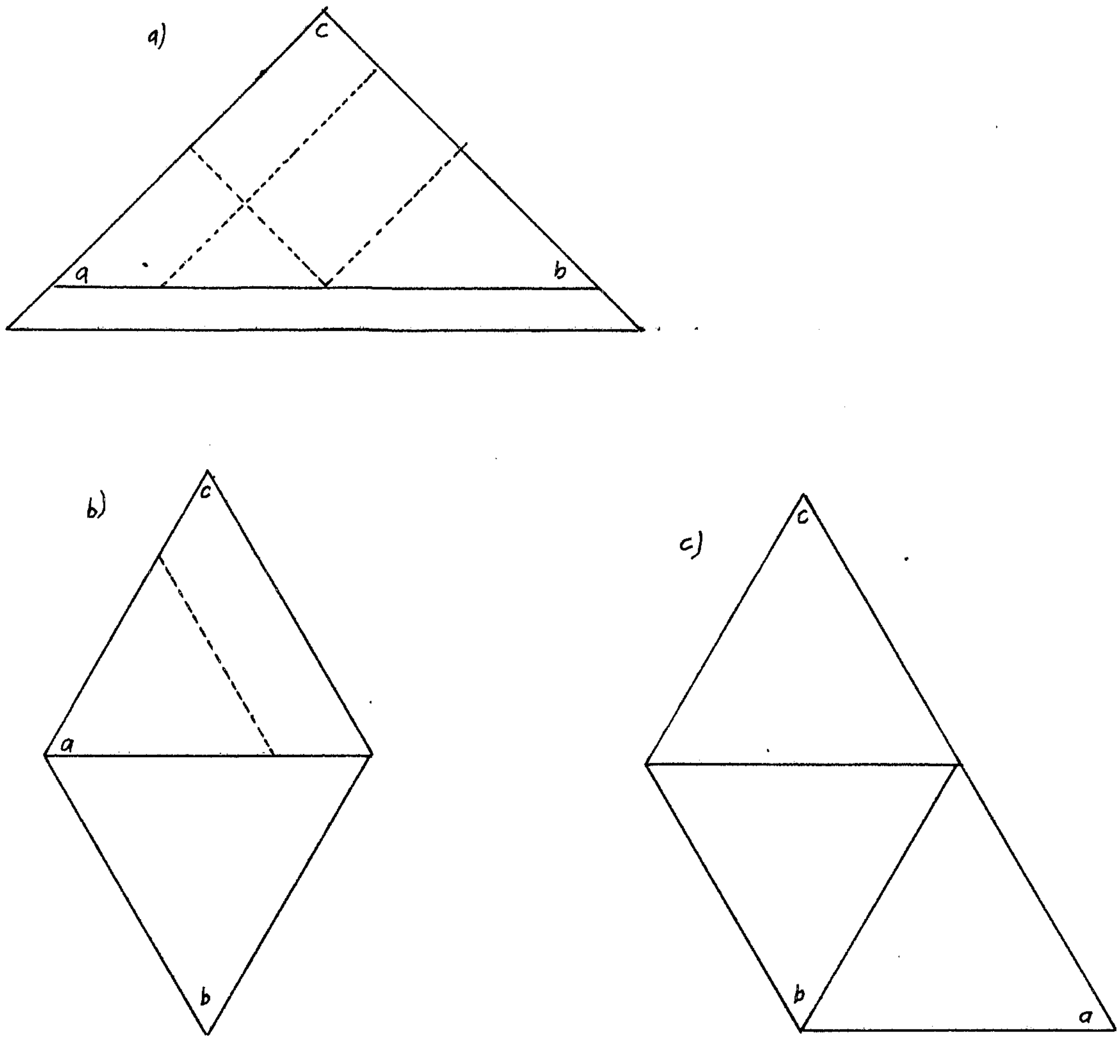
b) No a is b

c) Some a is b

d) Some a is not b

e) a is subsumed under b

Diagram XVIII   Maass' Diagrams in Use



a) a is subsumed under c
b is subsumed under c

b) No a is b
No b is c

c) Same as b) above

Diagram XIX   Jevons' Method of Verification

a)

(2) (1)

~~ABC~~
~~ABc~~
~~AbC~~
Abc
aBC
aBc
abC
abc

b)

(1) (2) (3)

~~ABC~~
~~ABc~~
AbC
~~Abc~~
~~aBC~~
~~aBc~~
abC
abc

c)

(1) (2)

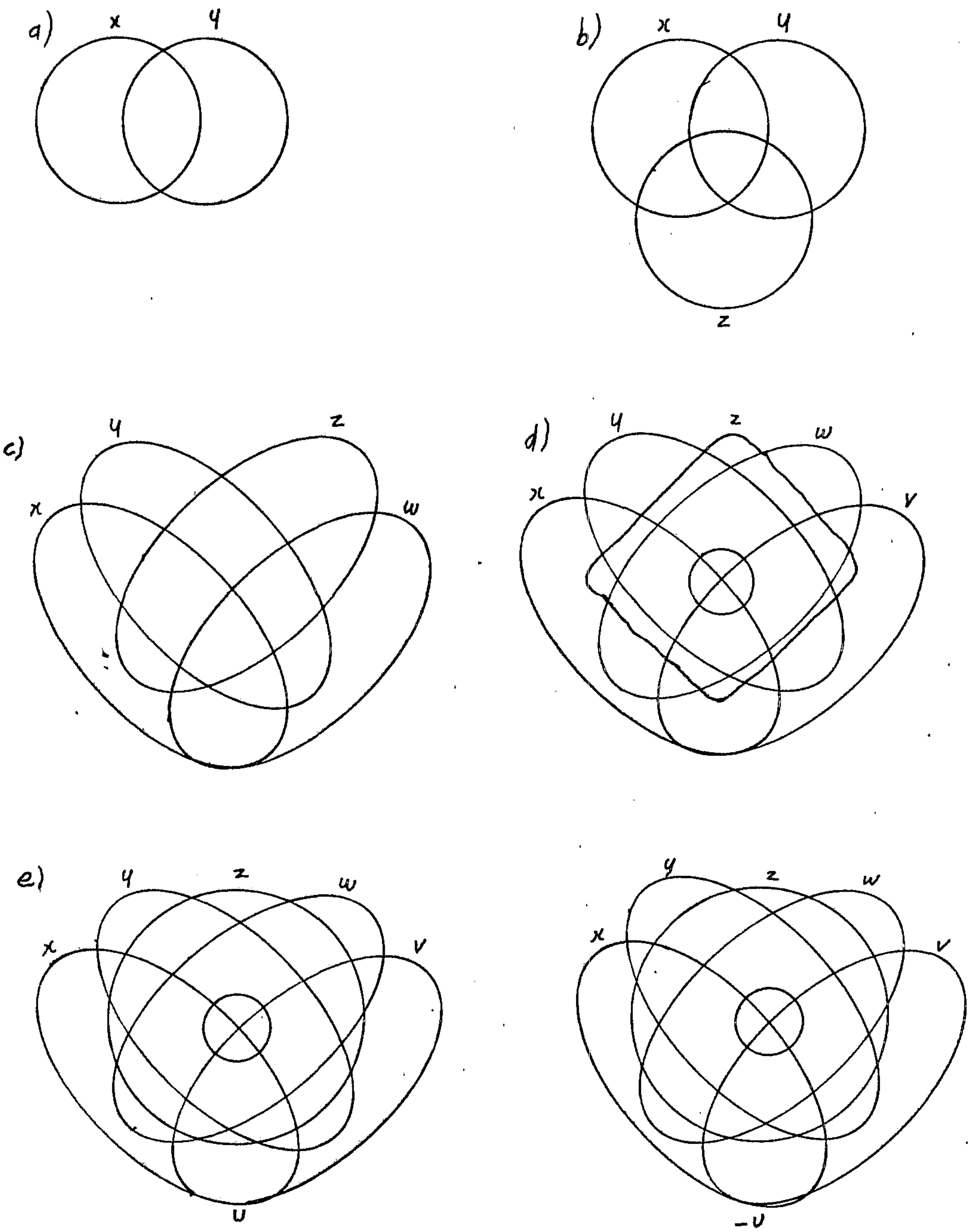
ABC
~~ABc~~
~~AbC~~
~~Abc~~
aBC
~~aBc~~
abC
abc

a) A=Ab (1)
C=aC (2)
∴ C=BC·|·bC

b) A=Ab (1)
B=Bc (2)
B=AB (3)
∴ B does not exist
∴ the premises are contradictory

c) A=AB (1)
B=BC (2)
∴ ABC exists

Diagram XX Venn's Basic Diagrams



a) two term diagram

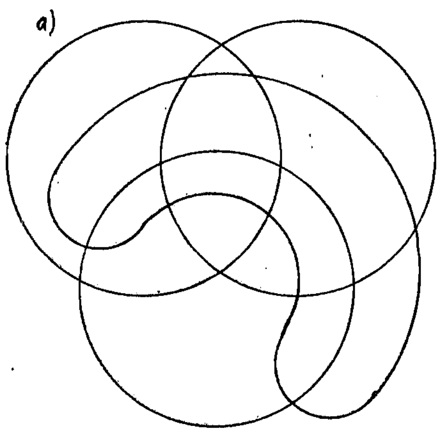
b) three term diagram

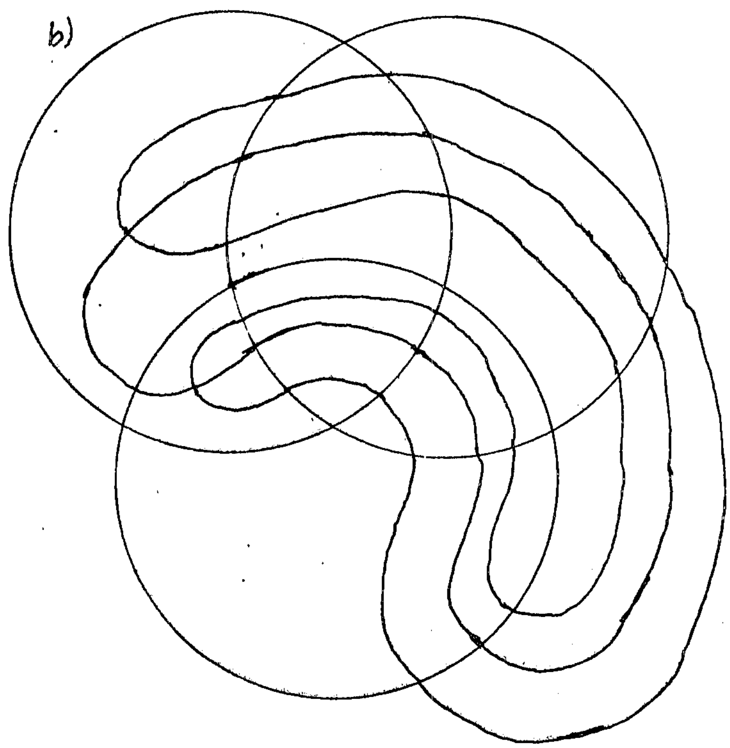
c) four term diagram

d) five term diagram

e) six term diagram

Diagram XXI   Venn's Proof that Logic Diagrams are Infinitely Extensible

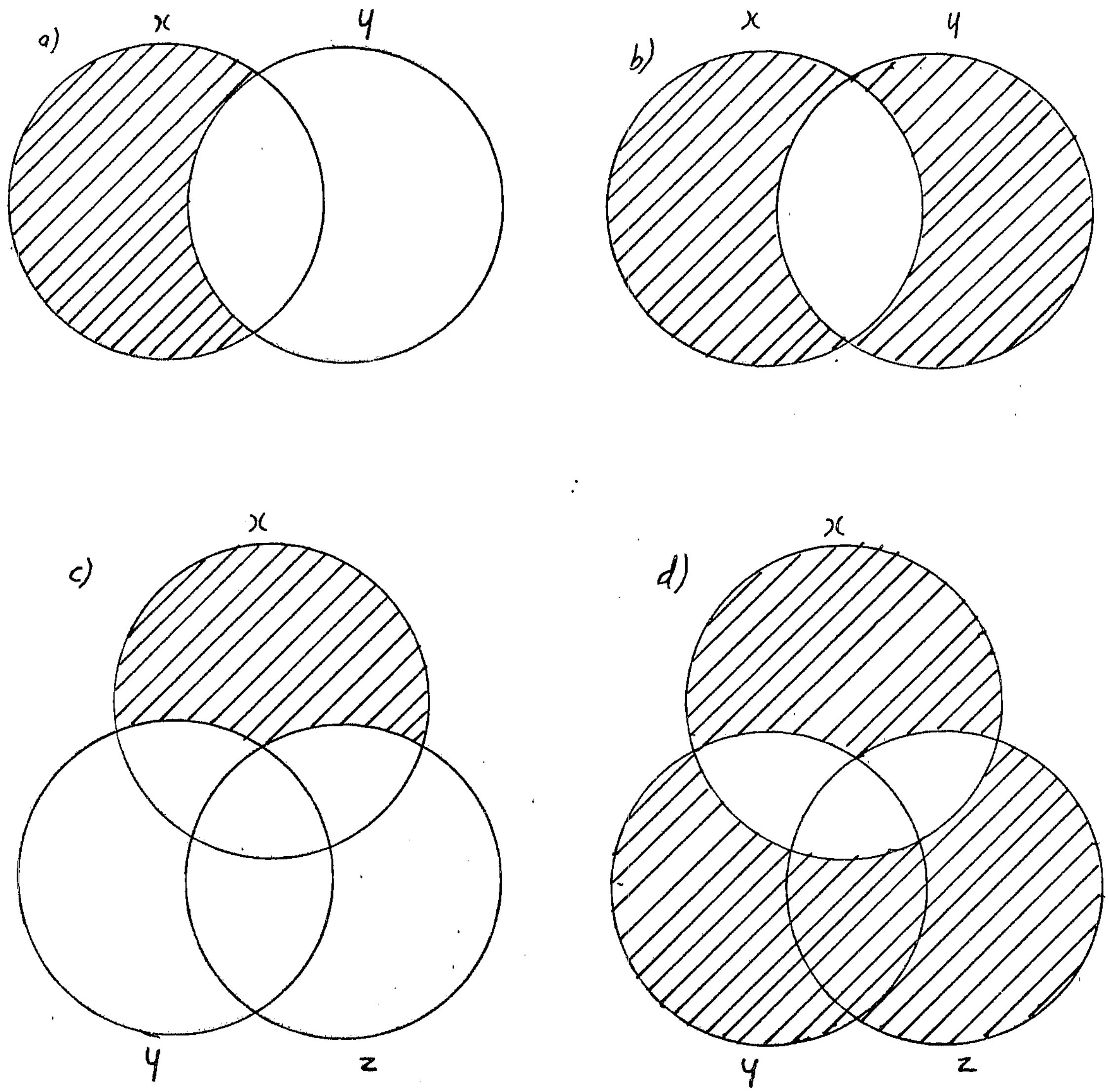




a) four terms

b) five terms

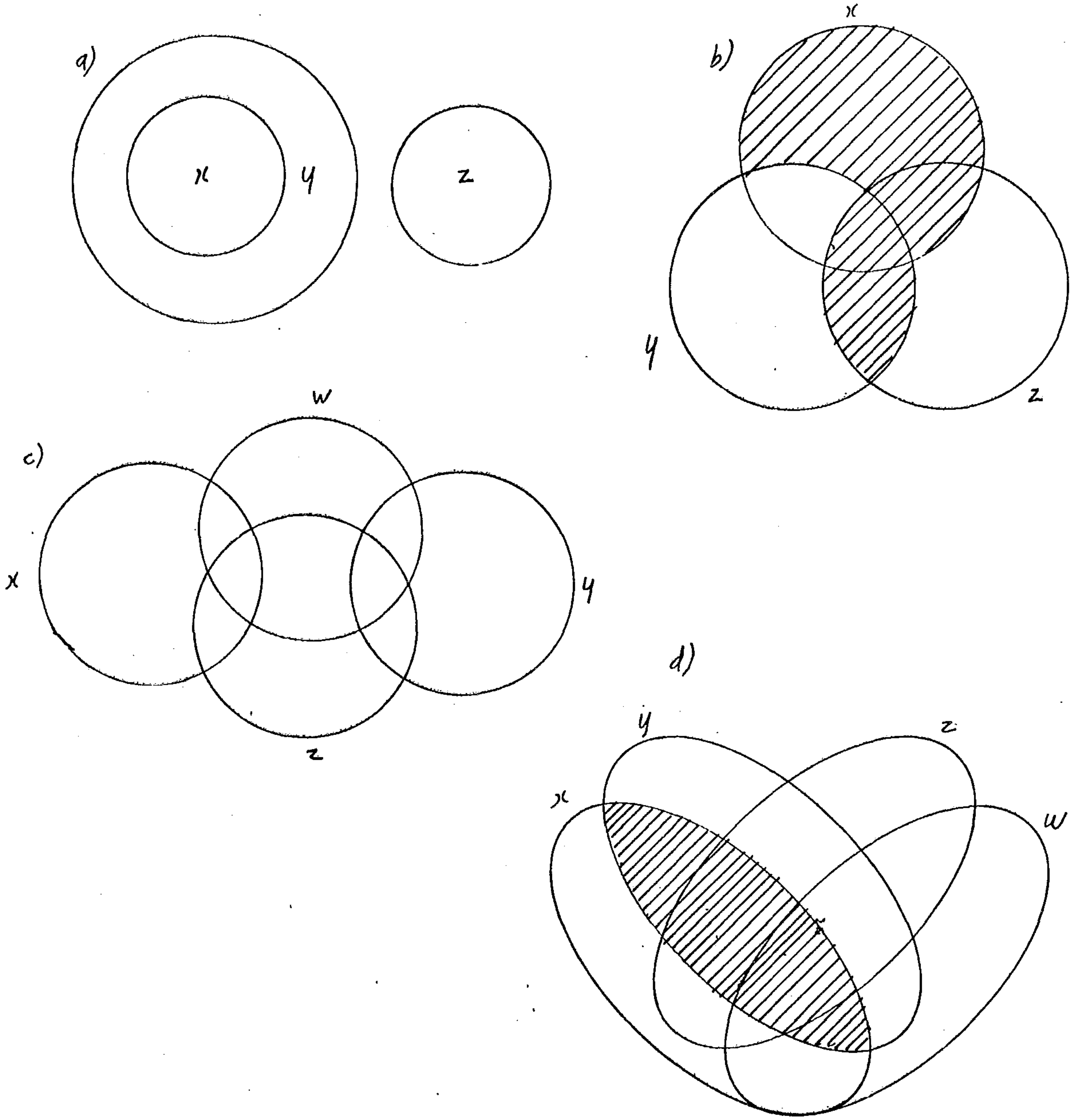
Diagram XXII   Propositions in Venn's System



a) $x\bar{y}=0$ b) $x=y$

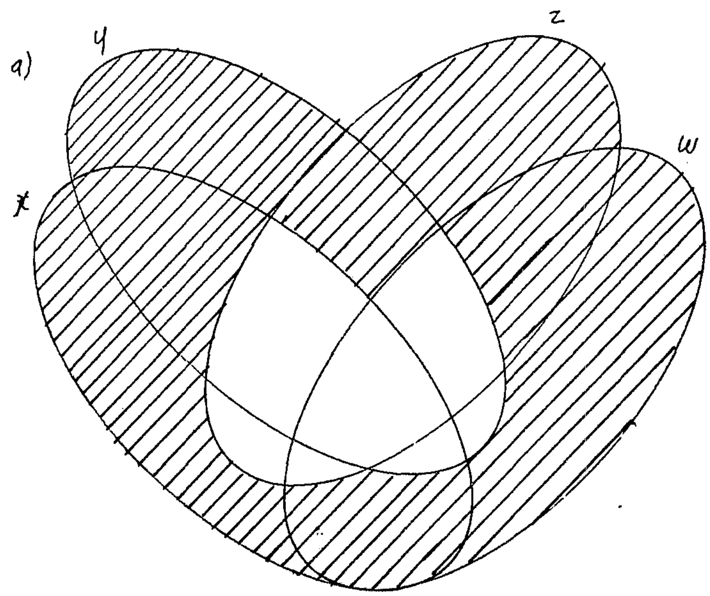
c) $x=y+z$ d) $x(y+z)=1$

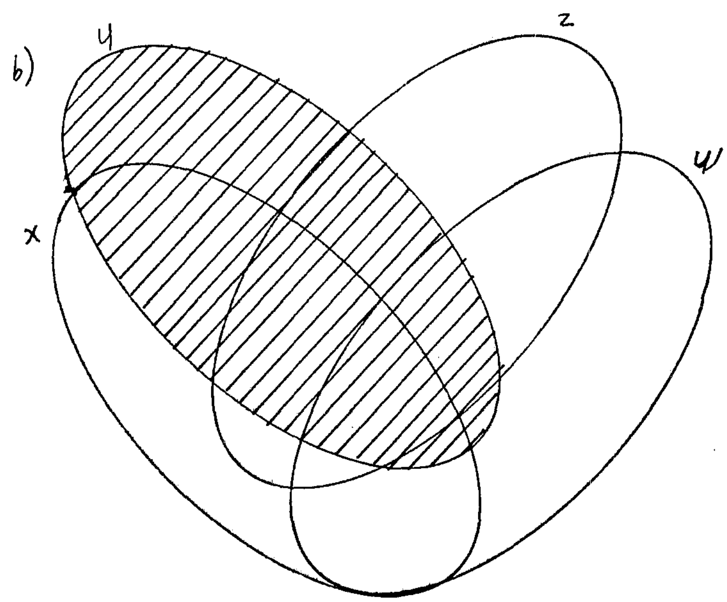
Diagram XXIII   Comparison of Euler's and Venn's Diagrams



a)	No y is z All x is y ∴ No x is z	b)	$yz=0$ $x\bar{y}=0$ ∴ $xz=0$
c)	All x is either y and z or not y If any xy is z then it is w No wx is yz ∴ No x is y	d)	$x[\overline{(yz)+y}]=0$ $xyz\bar{w}=0$ $wxyz=0$ ∴ $xy=0$

Diagram XXIV   Visual Aid as a Function of Venn Diagrams

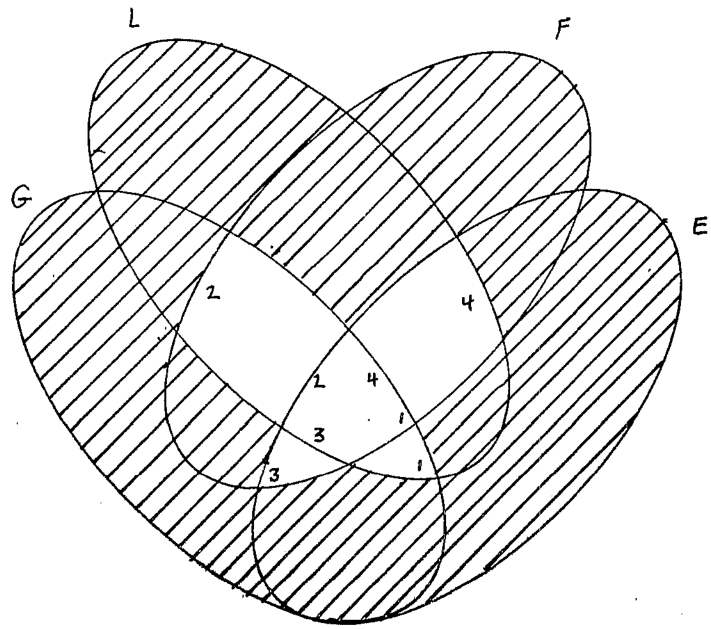




a) $x\overline{yz}=0$
$y\overline{zw}=0$
$z\overline{wy}=0$
$wxy=0$

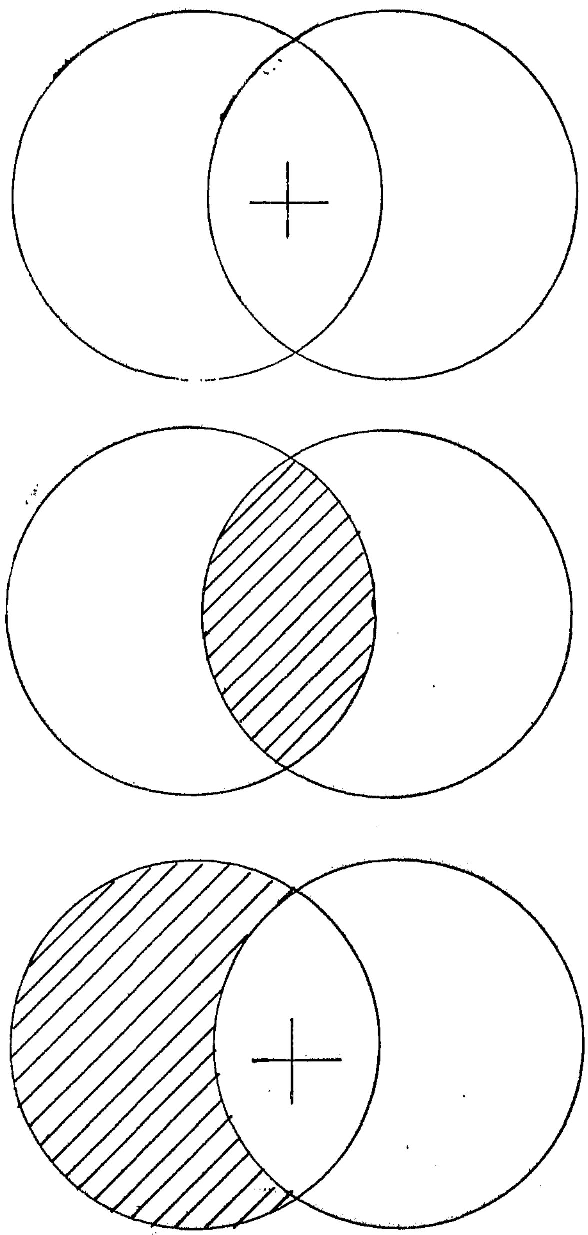
b) $y[\overline{(x\bar{z})+(z\bar{x})}]=0$
$wx[\overline{(xz)+(\overline{xz})}]=0$
$xy(\overline{w+z})=0$
$yz(\overline{x+w})=0$

Diagram XXV   Venn's Representation of Particular Propositions



$$G+L=1$$
$$(GL)(\overline{E+F})=0$$
$$(G+L)(\overline{EF})=0$$
$$GLE\neq 0 \quad (1)$$
$$GLF\neq 0 \quad (2)$$
$$GEF\neq 0 \quad (3)$$
$$LEF\neq 0 \quad (4)$$

Diagram XXVI Carroll's Interpretation of Venn's Basic Diagrams

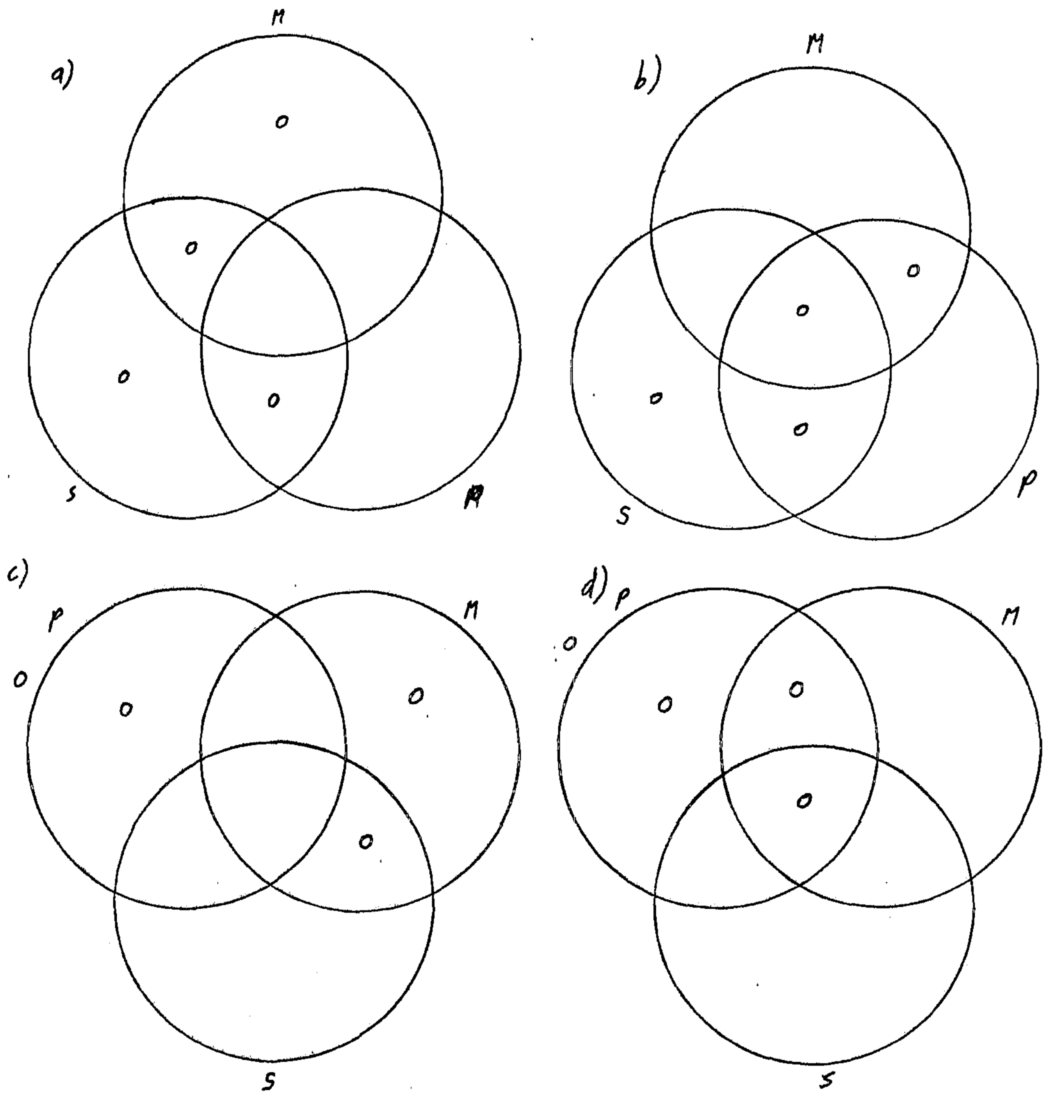


a) Some x is y

b) No x is y

c) All x is y

Diagram XXVII   Peirce's Modification of Venn's Diagrams - Non-existential Propositions



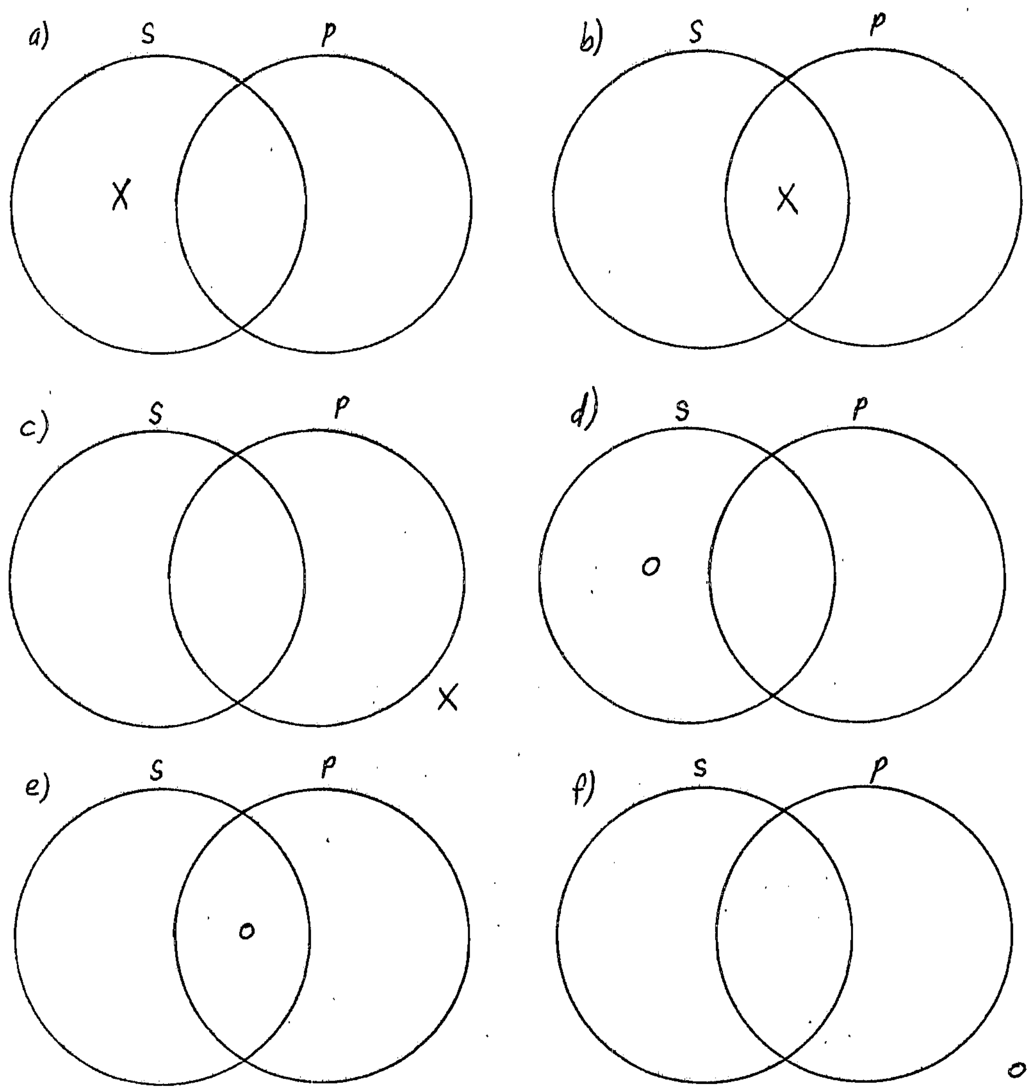
a) All M are P
All S are M

b) No M are P
All S are M

o) All M are P
All ~S are M

d) No M are P
All ~S are M

Diagram XXVIII Peirce's Modifications of Venn's Diagrams - Existential Propositions



a) Some S is not P

b) Some S is P

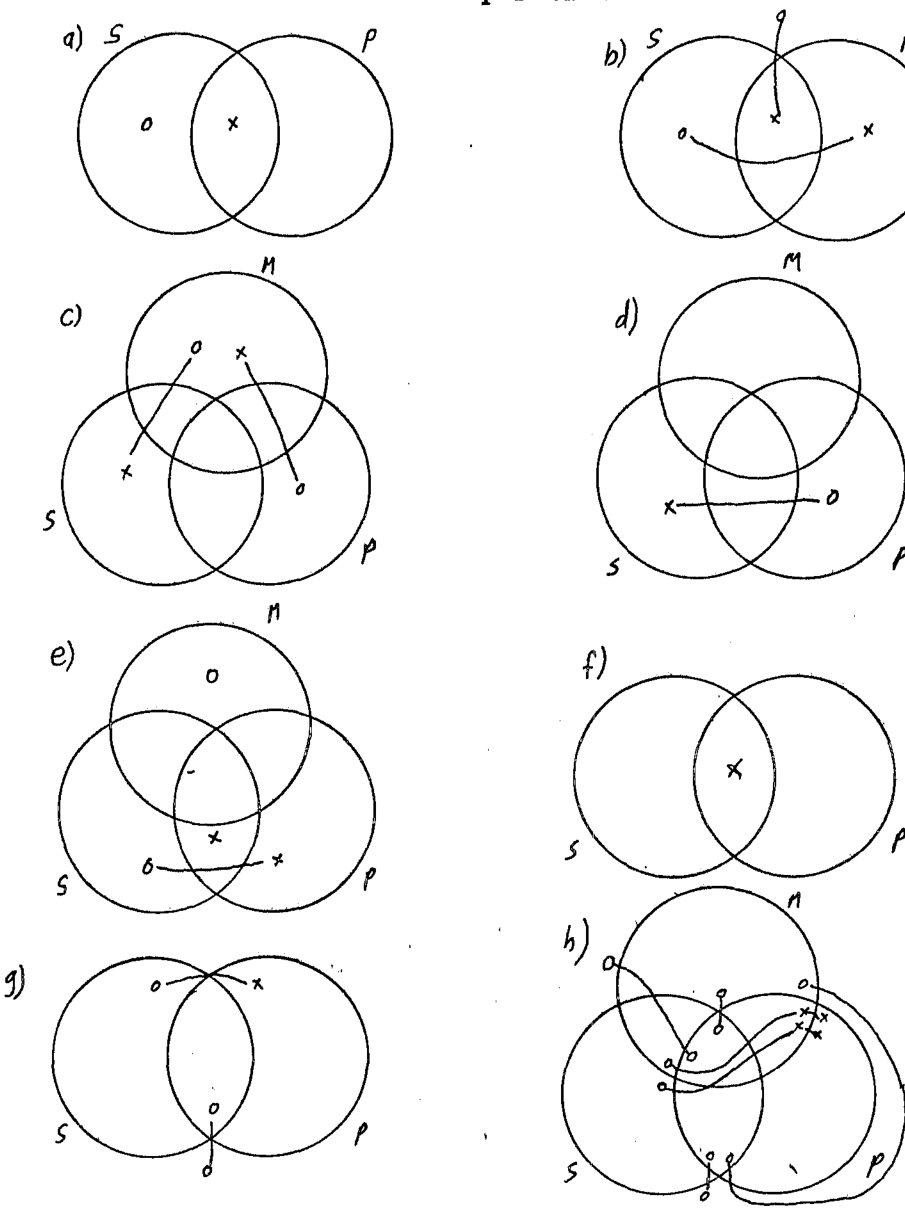
c) There is something besides S and P

d) No S is not P (the precise denial of a)

e) No S is P (the precise denial of b)

f) There is nothing besides S and P (the precise denial of c)

Diagram XXIX Peirce's Modification of Venn's Diagrams
- Rules of Operation



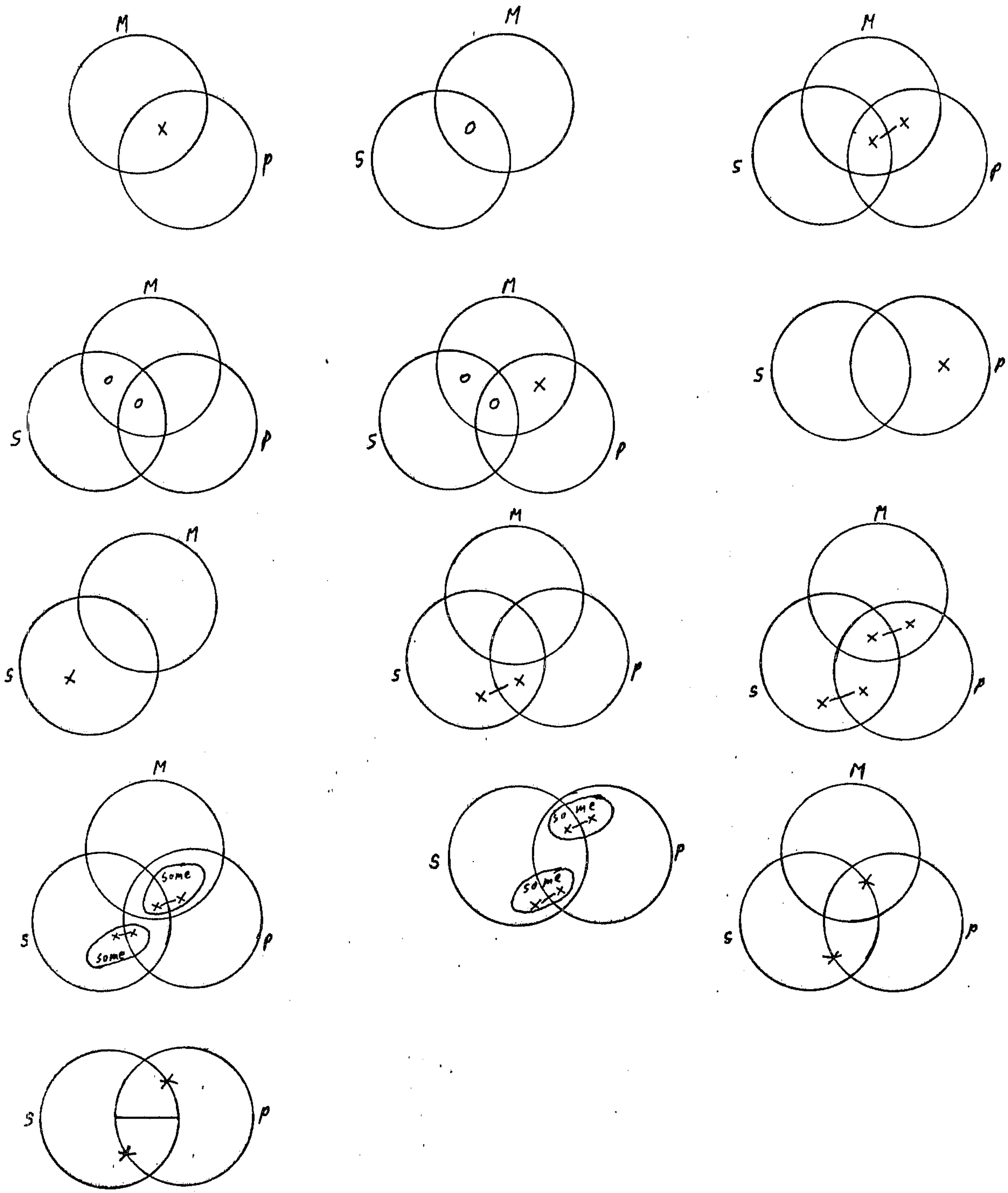
a) and b) "All S is P and Some P is S" can be transformed to "Either All S is P or Some $\bar{S}$ is P and Some S is P or All $\bar{S}$ is P"

c) and d) "Either Some $S\bar{P}$ is $\bar{M}$ or All M is SvP and Some $M\bar{P}$ is S or All P is MvS" is transformable into "Either Some $S\bar{P}$ is $\bar{M}$ or All P is MvS"

e) and f) "Either All S is PvM or Some $P\bar{M}$ is $\bar{S}$, and Some SM is P and All M is SvP" is transformable into "Some S is P"

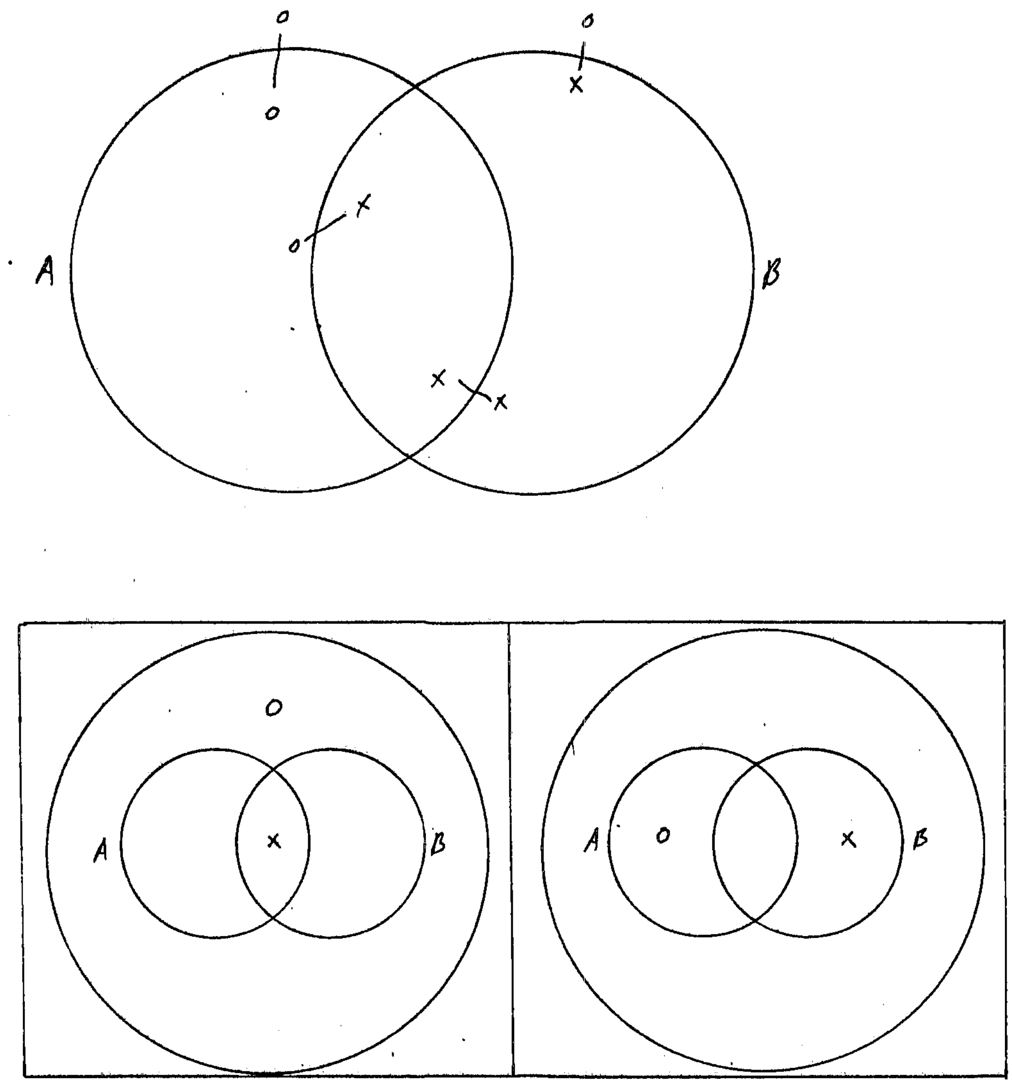
g) and h) "Either All S is P or Some P is $\bar{S}$ and either No S is P or No $\bar{S}$ is P" is transformable into "$\bar{M}\bar{S}\bar{P}$=0 or MSP=0; and MSP=0 or $M\bar{S}\bar{P}$=0; and $M\bar{S}\bar{P}$=0 or $\bar{M}SP$=0; and $\bar{M}SP$=0 or $\bar{M}\bar{S}\bar{P}$=0; and $MS\bar{P}$=0 or Some $\bar{S}M$ is P or Some $\bar{S}\bar{M}$ is P; and $\bar{M}S\bar{P}$=0, or Some $\bar{S}M$ is P or Some $\bar{S}\bar{M}$ is P"

Diagram XXX   Peirce's Modification of Venn's Diagrams - Use



a) to m) Peirce's proof that a conclusion may be reached with two particular premises.

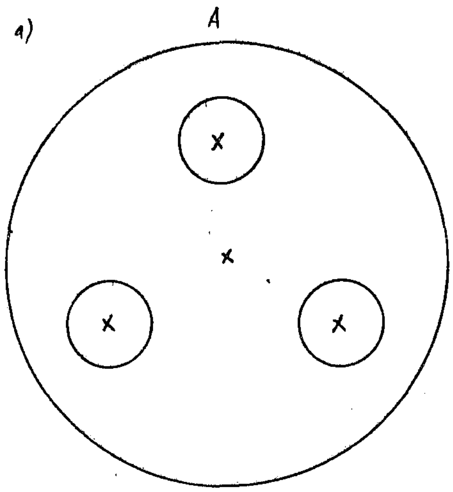
Diagram XXXI Peirce's Modification of Venn's Diagrams - Disjunction of Conjunctions

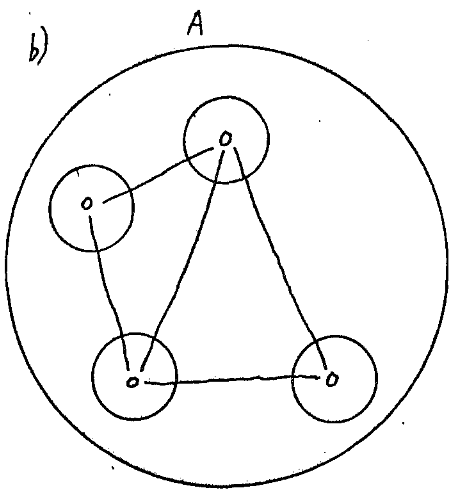


a) Either some A is B while everything is either A or B, or else all A is B while some B is not A.

b) Same as a) with the inclusion of a "Universe of Hypothesis" in the diagram.

Diagram XXXII   Peirce's Modification of Venn's Diagrams - Minimal Multitudes

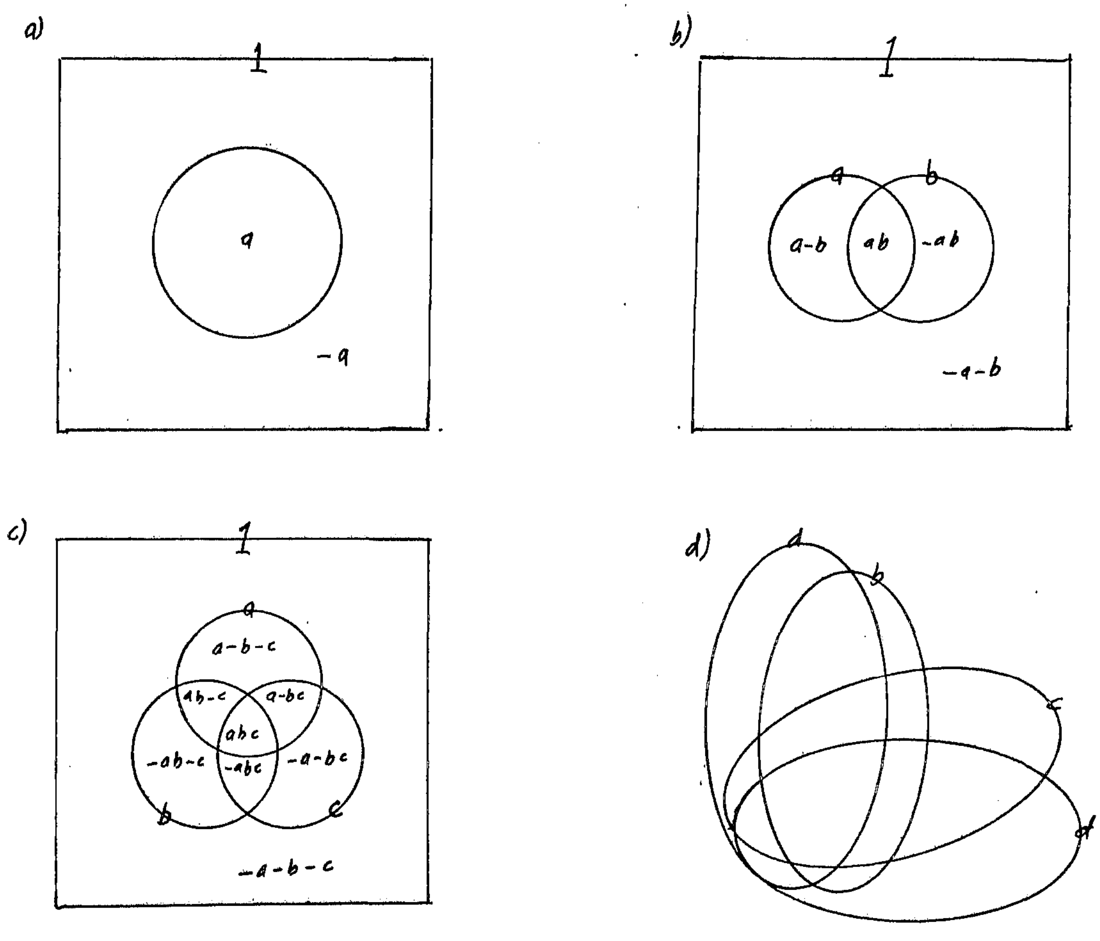


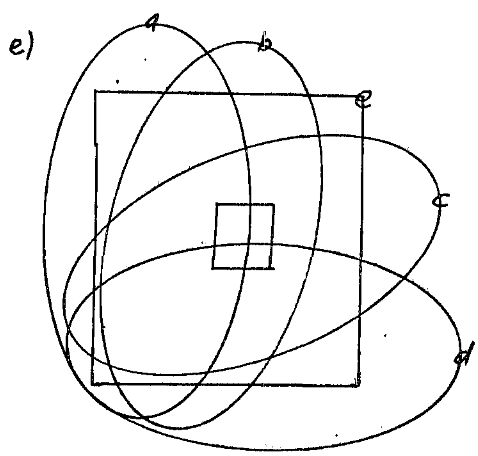


a) There are at least four A's.

b) There are not as many as four A'x.

Diagram XXXIII   Lewis' Reiteration of Venn's Diagrams





a) One term

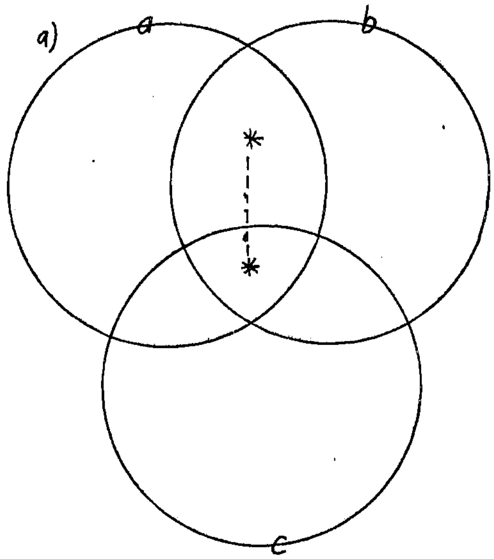
b) Two terms

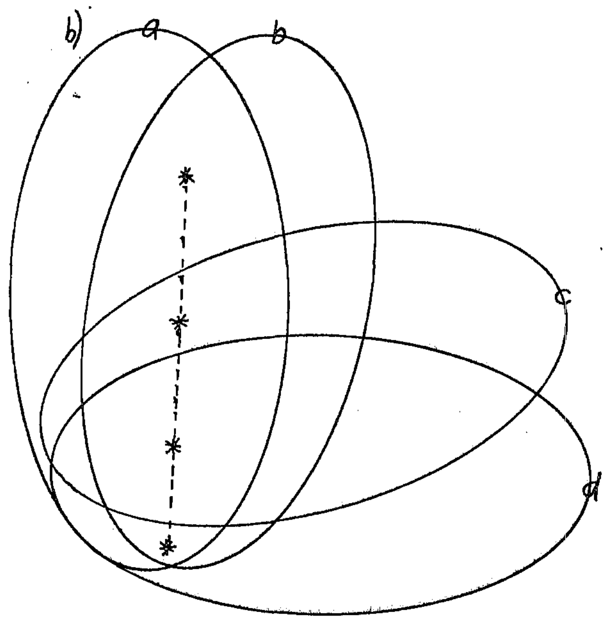
c) Three terms

d) Four terms

e) Five terms

Diagram XXXIV   Lewis' Method of Diagramming Particular Propositions

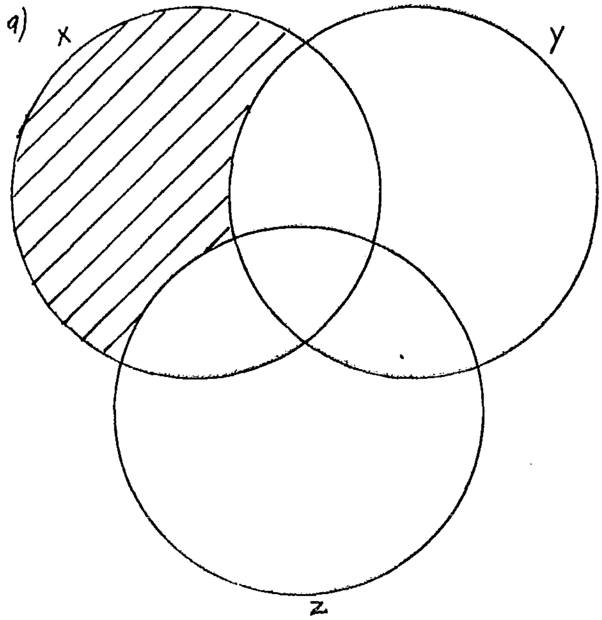


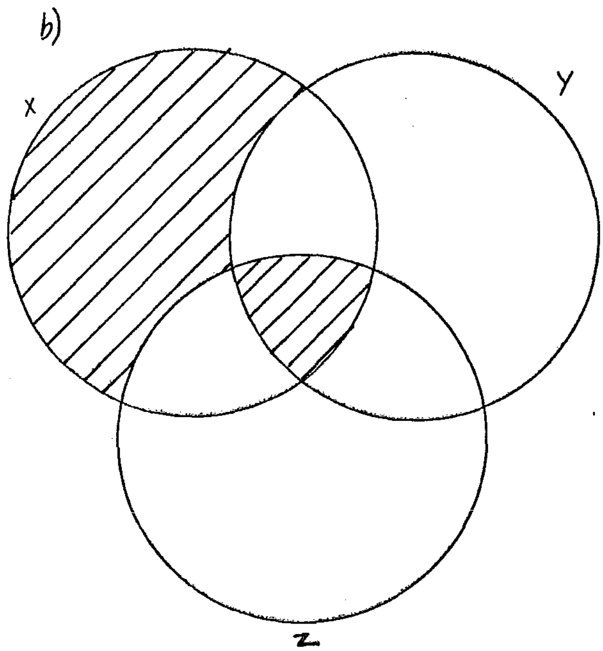


a) ab≠0

b) ab≠0

Diagram XXXV Gardner's Application of Venn's Circles to Disjunction

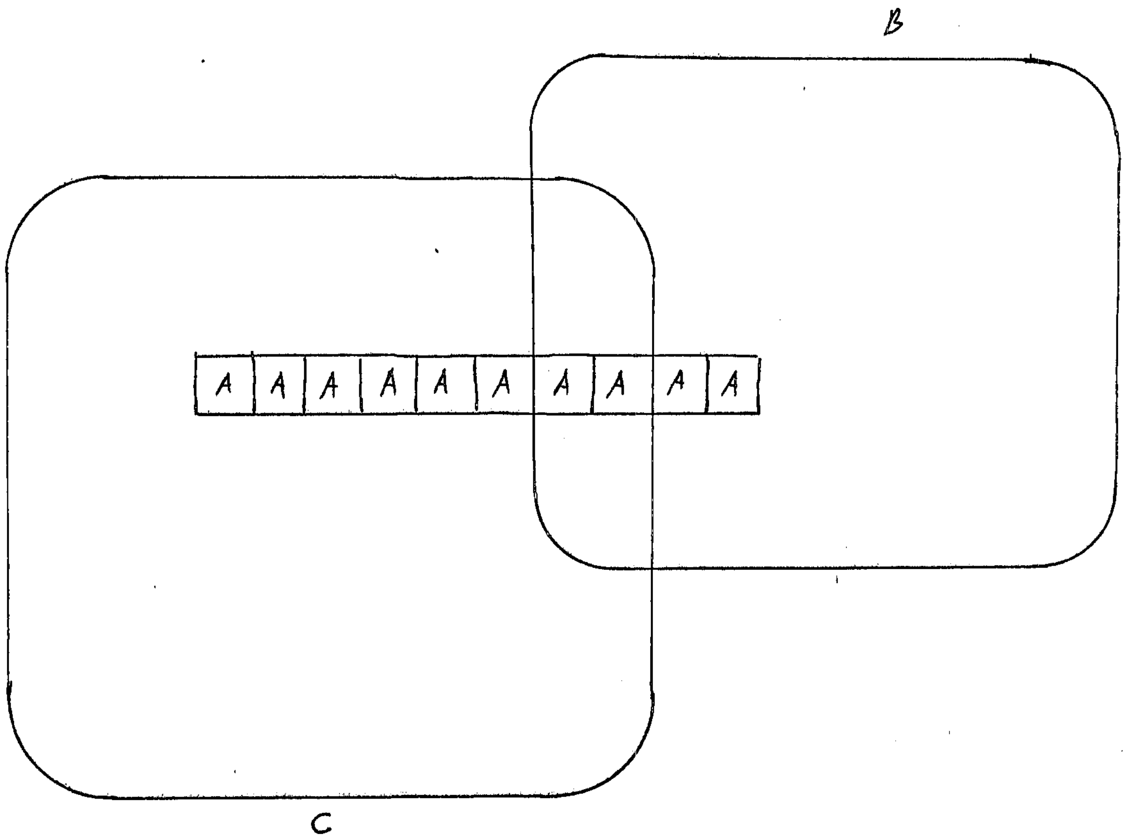




a) All X are either Y or Z (inclusive "or")

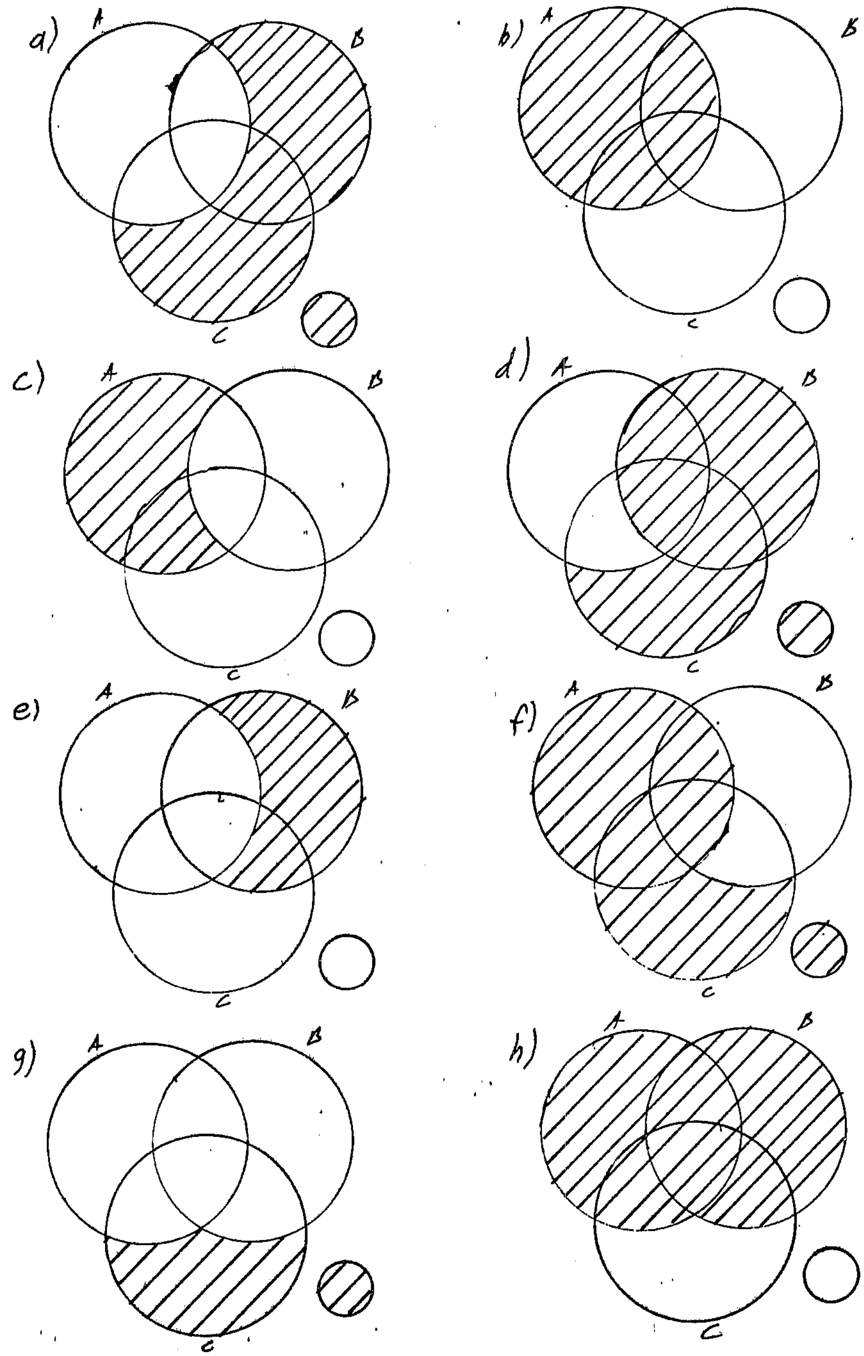
b) All X are either Y or Z (exclusive "or")

Diagram XXXVI Gardner's Adaption of Venn's Diagrams to Numerical Syllogisms



There are at least ten A's of which four are B's
Eight A's are C's
∴ At least two B's are C's

Diagram XXXVII Gardner's Adaptation of Venn's Circles to the Propositional Calculus I



a) A

b) ~A

c) A⊃B

d) A·~B

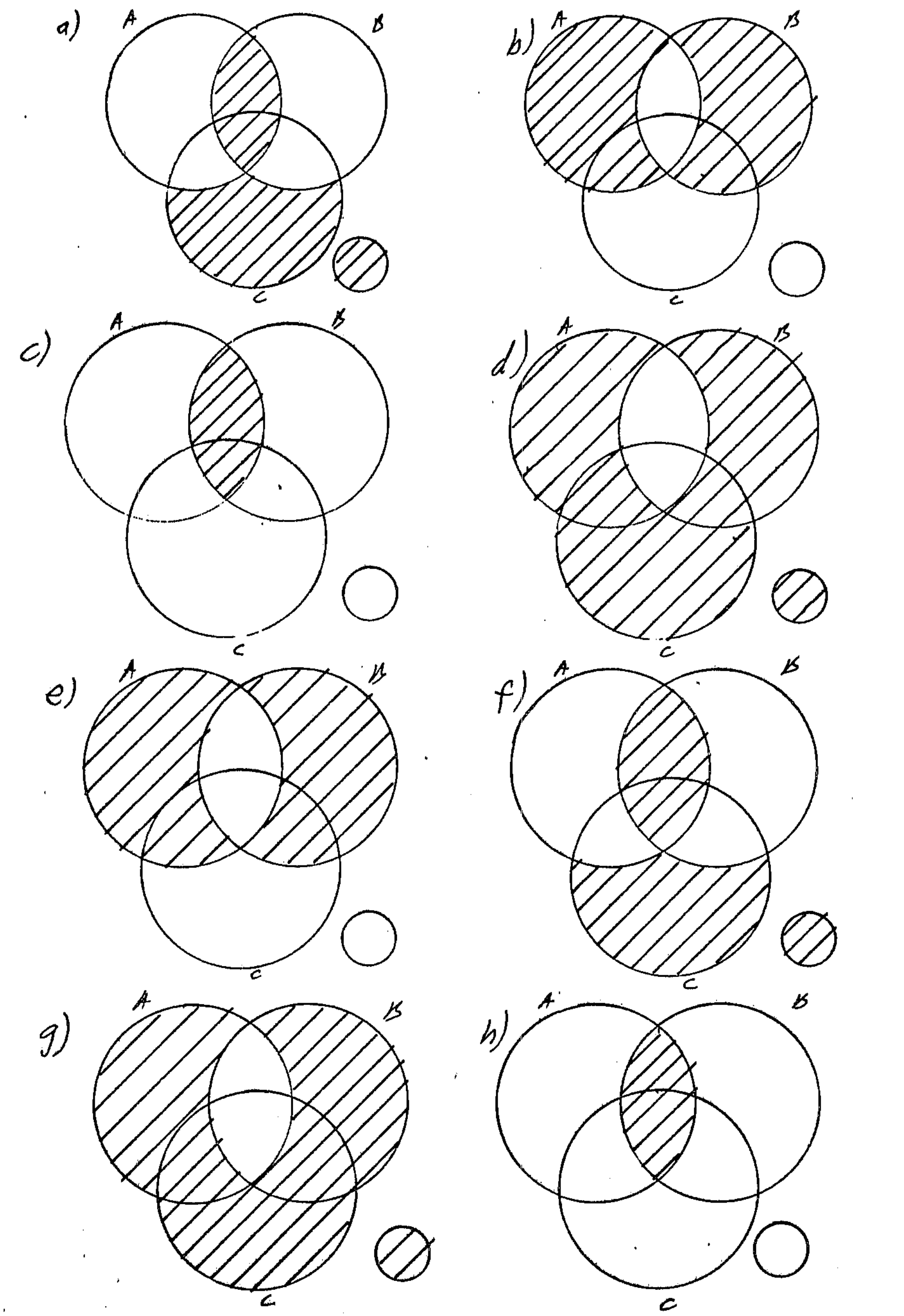
e) B⊃A

f) B·~A

g) AvB

h) ~A·~B

Diagram XXXVIII Gardner's Adaptation of Venn's Circles to the Propositional Calculus II



a) $A \not\equiv B$

b) $A \equiv B$

c) $A \mid B$

d) $A \cdot B$

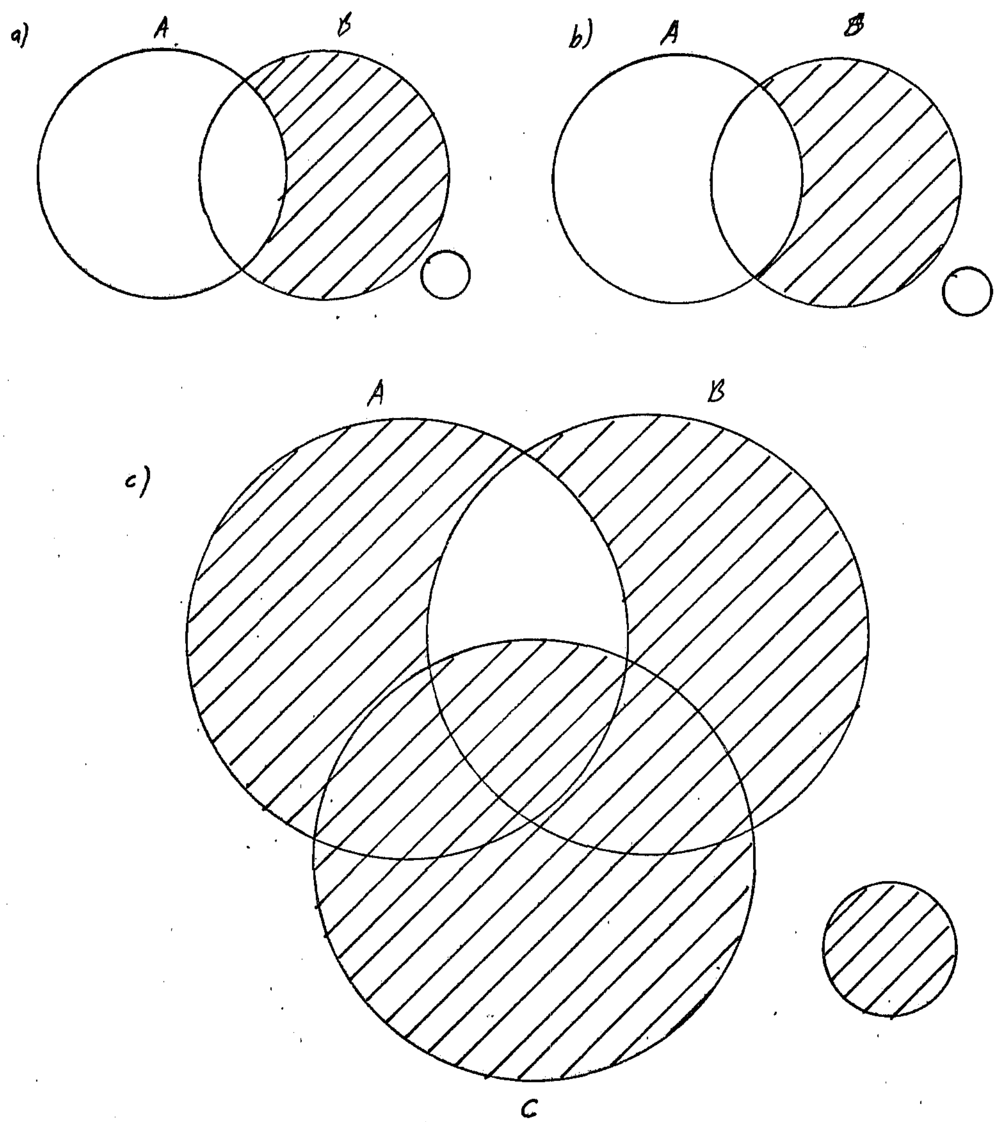
e) $A \equiv B$

f) $A \not\equiv B$

g) $A \cdot B$

h) $A \mid B$

Diagram XXXIX   Gardner's Use of Venn Diagrams for the Solution of Problems in the Propositional Calculus

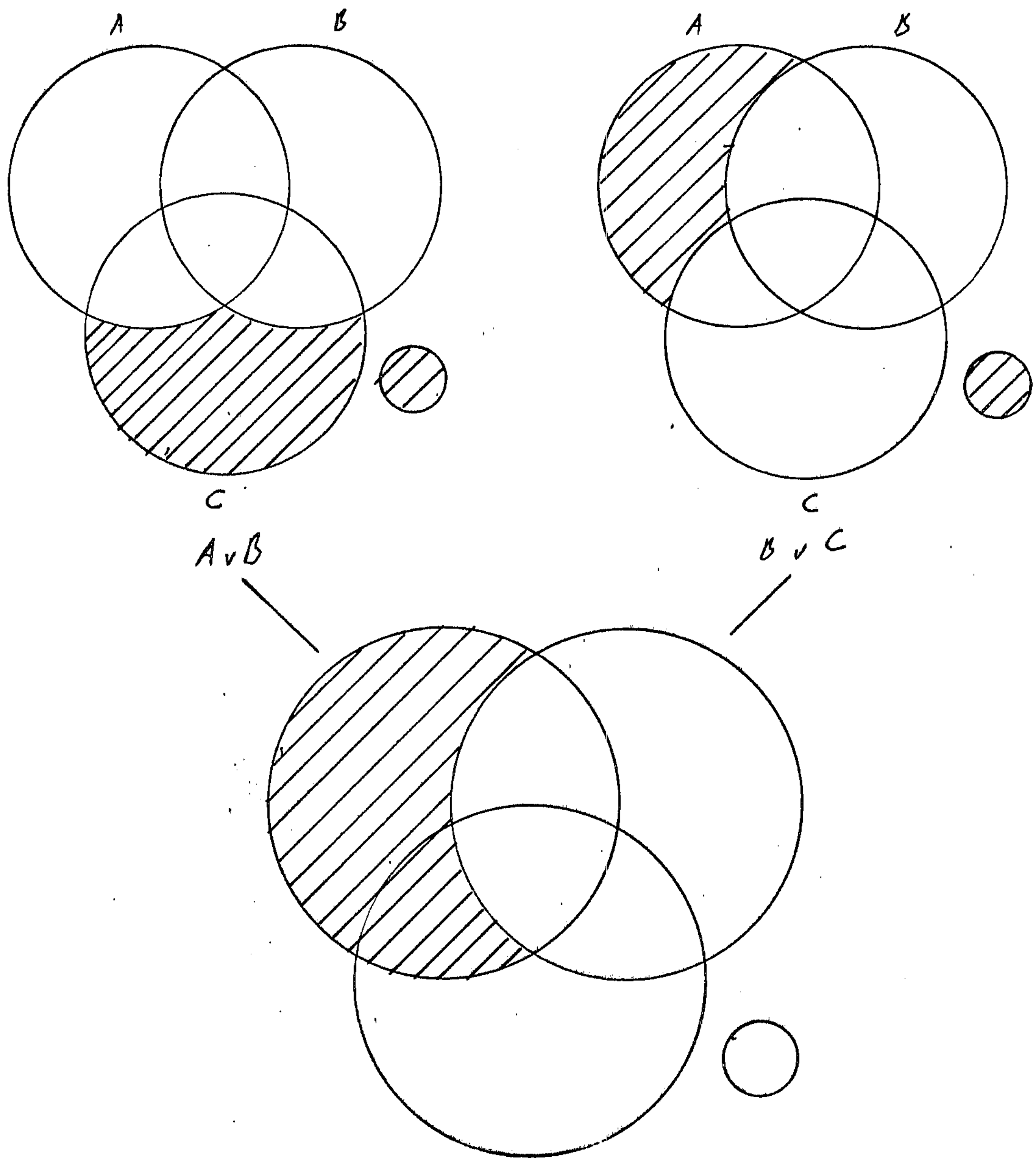


a) $A \vee \sim B$

b) $B \supset A$

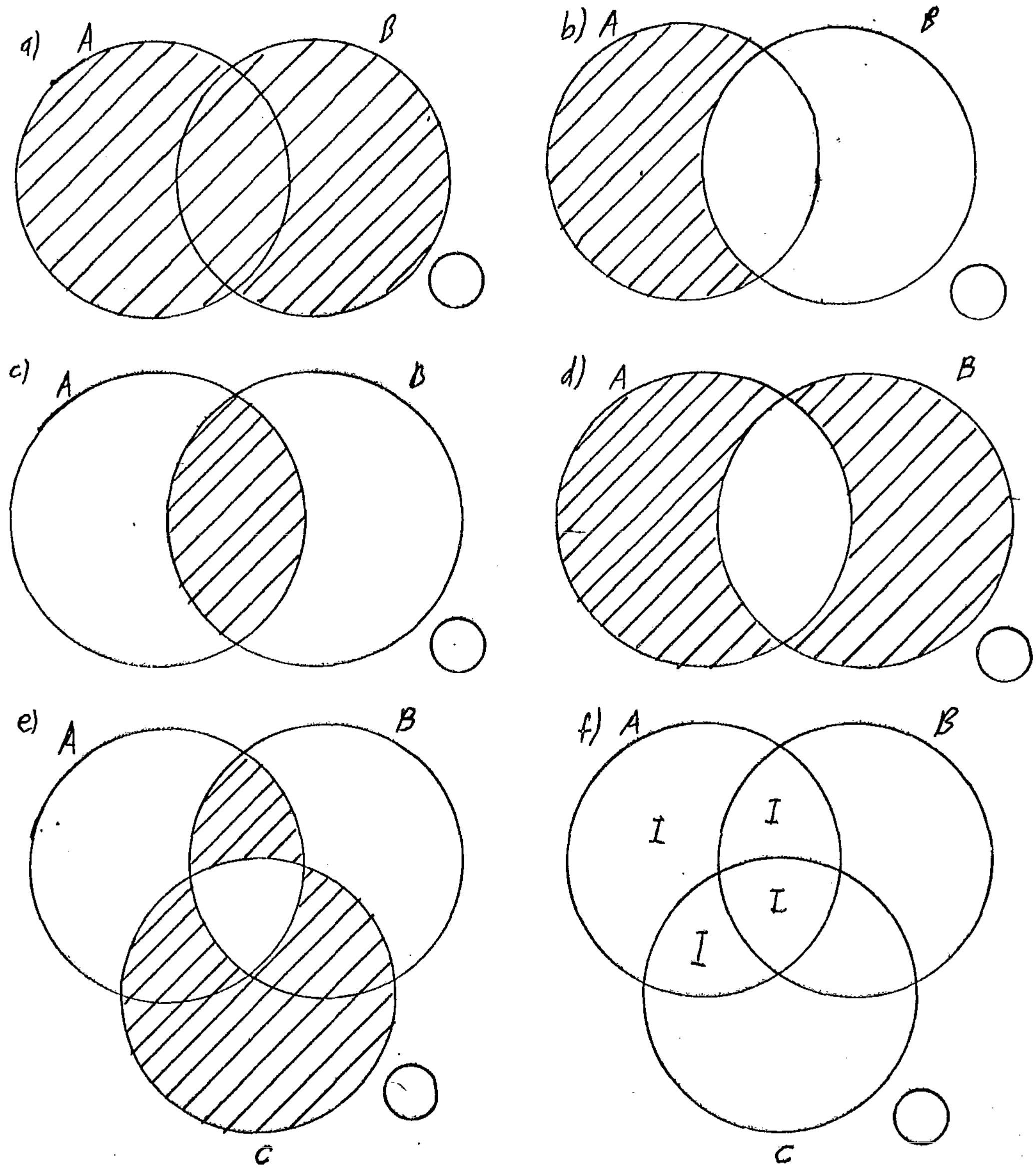
c) $A \supset B$
$B \not\equiv C$
$A \vee C$
$C \supset A$
$\therefore A . B . \sim C$

Diagram XL   Gardner's Method of Diagramming Compound Statements with Venn's Diagrams



(AvB)⊃(BvC)

Diagram XLI Application of Venn Diagrams to Boolean Propositions



a) $(A+B)=0$

b) $(A-B)=A(1-B)=0$

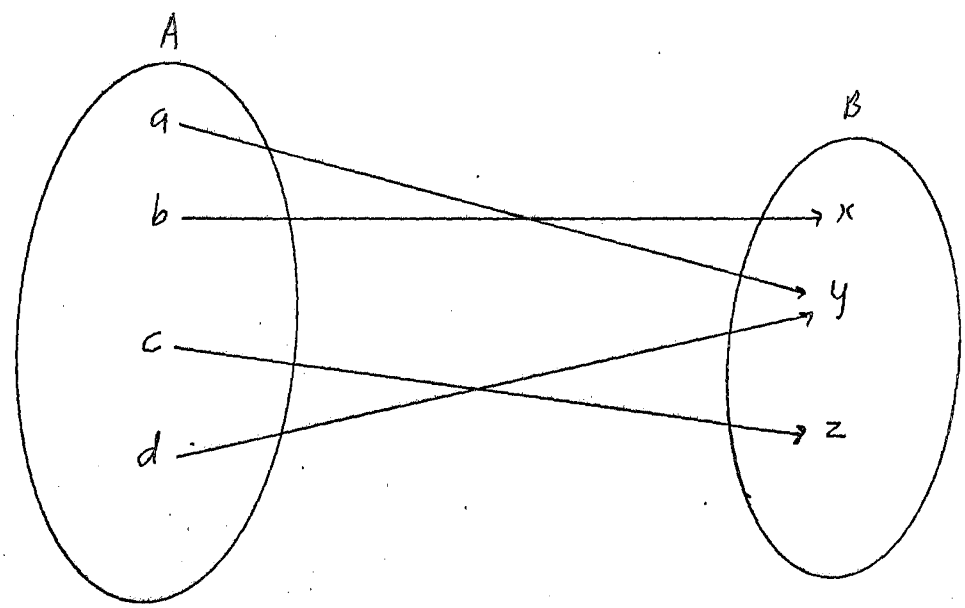
c) $AB=0$

d) $\frac{A}{B}=1$
$\therefore A=B$

e) $\frac{AB}{C}=1$
$\therefore AB=C$

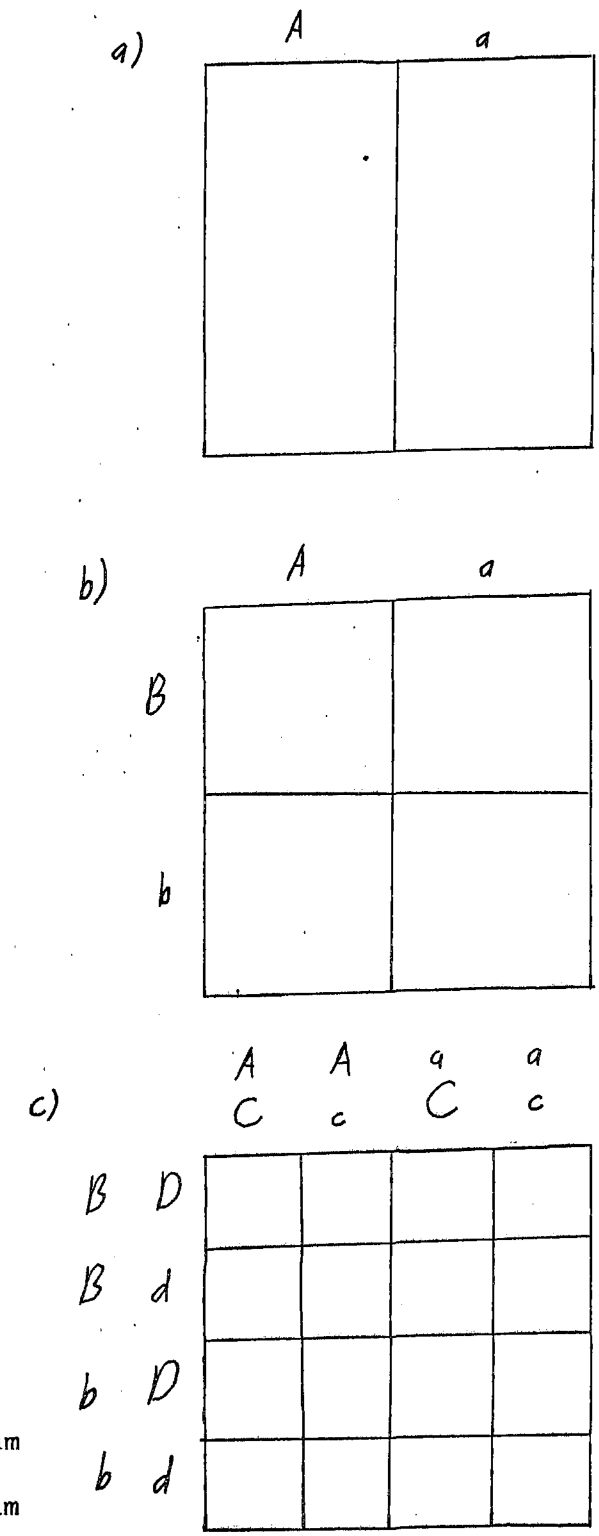
f) $\frac{0}{0}A=n$ (where $n=f[A(B+C)]$)

Diagram XLII Diagram Representing Functions



f(A)=B [where A=(a,b,c,d) and B=(x,y,z)]

Diagram XLIII   Marquand's Basic Diagrams

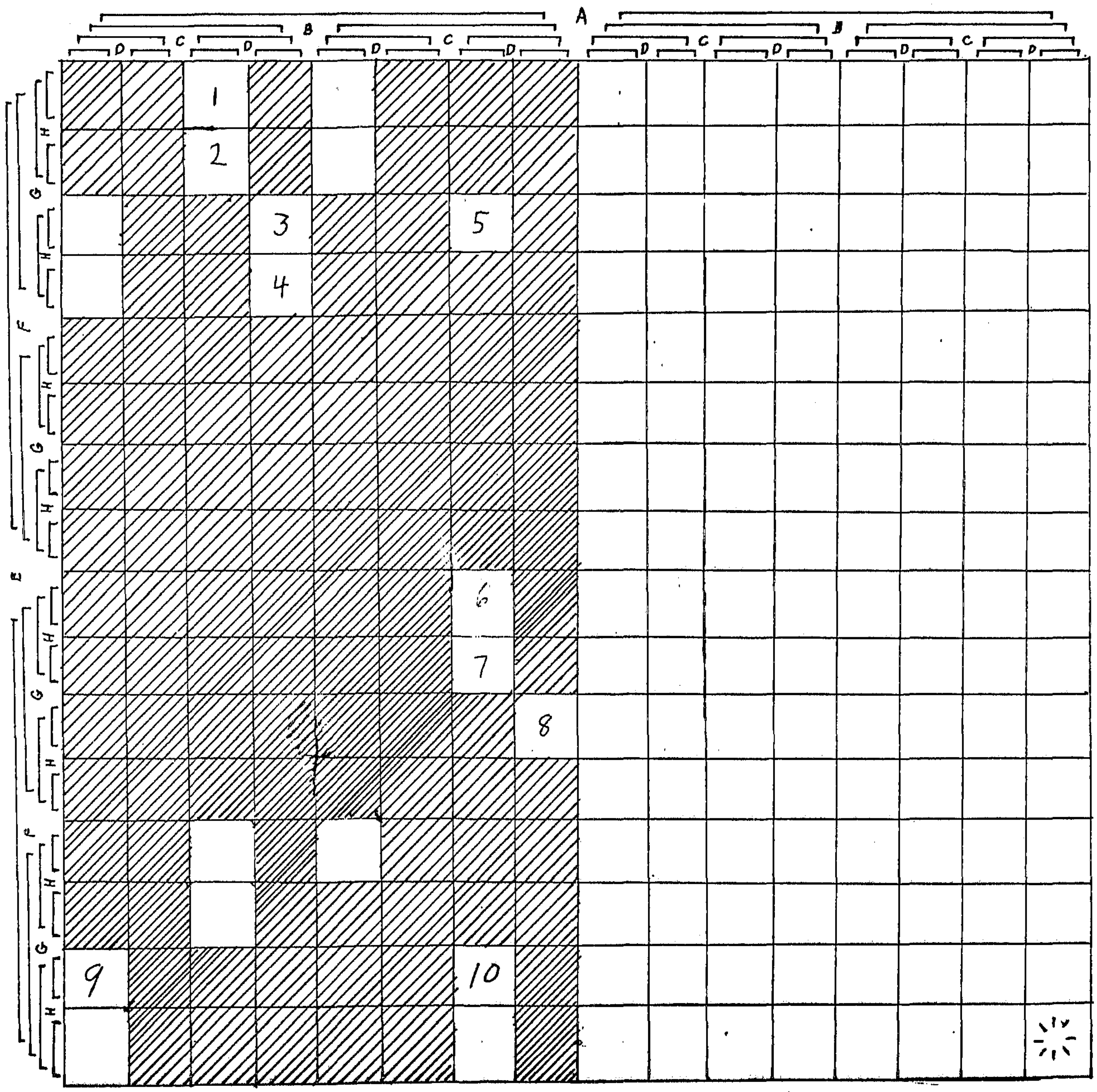


a) One term diagram

b) Two term diagram

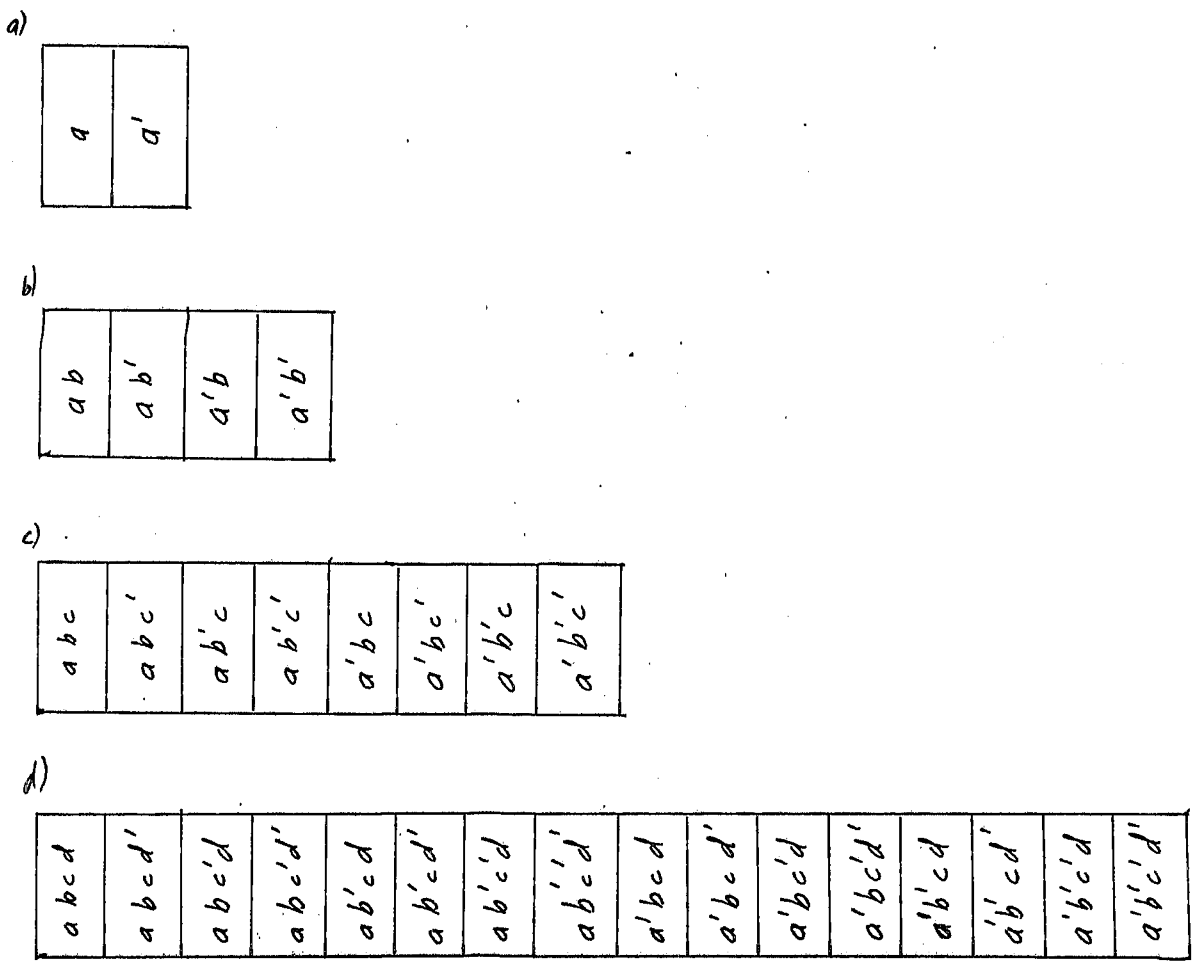
c) Four term diagram

Diagram XLIV Marquand's Example



Example of Marquand's diagrams in use

Diagram XLV Macfarlane's Logical Spectrum



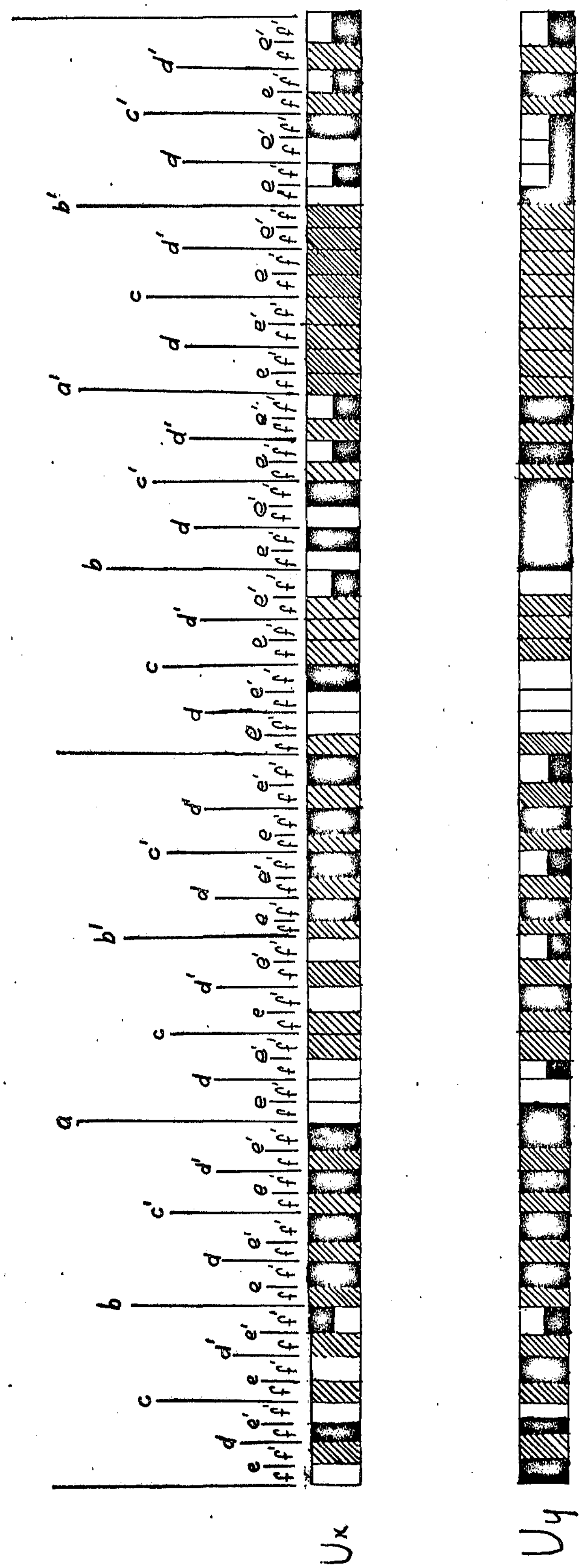
a) One term

b) Two terms

c) Three terms

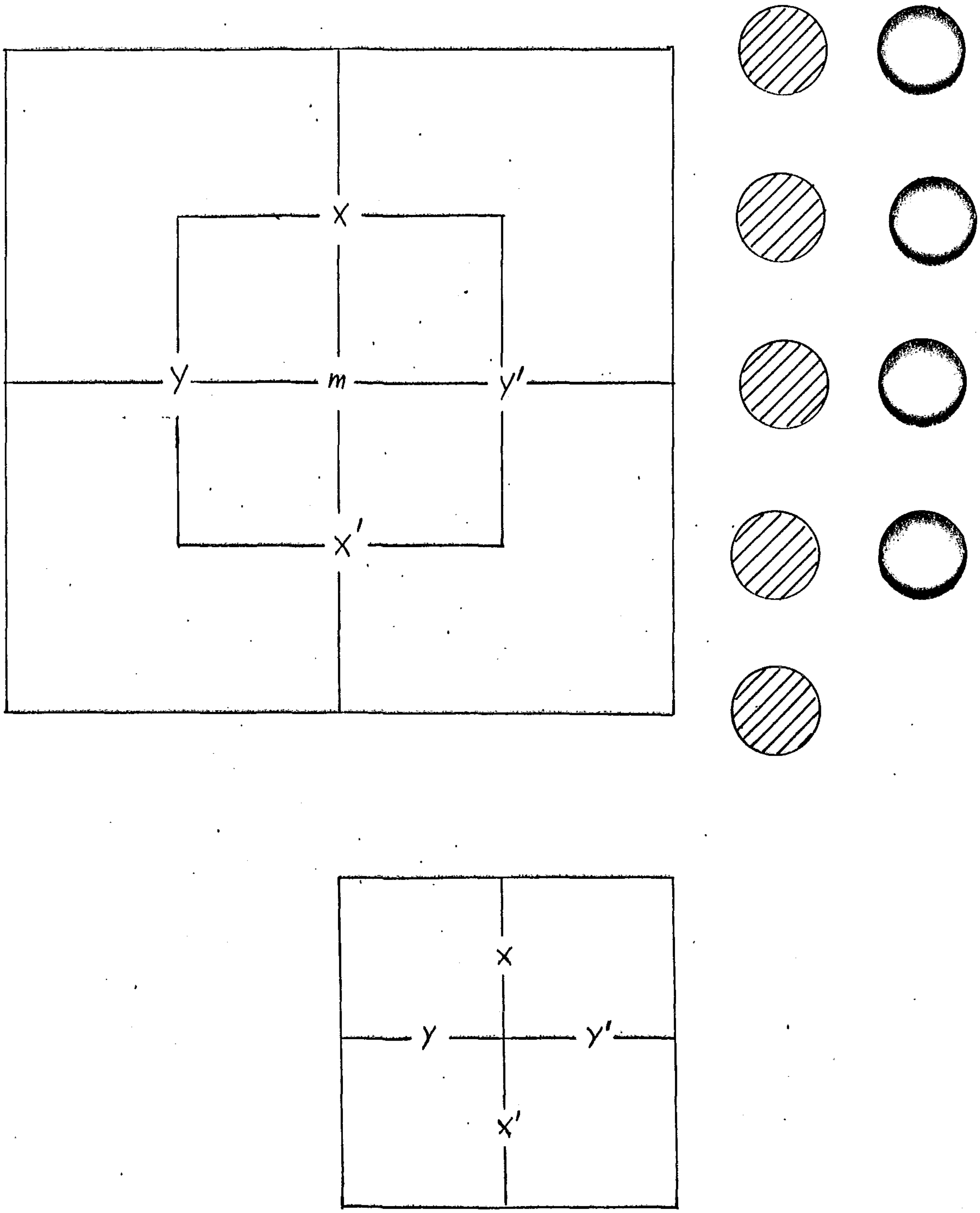
d) Four terms

Diagram XLVI Macfarlane's Use of Diagrams



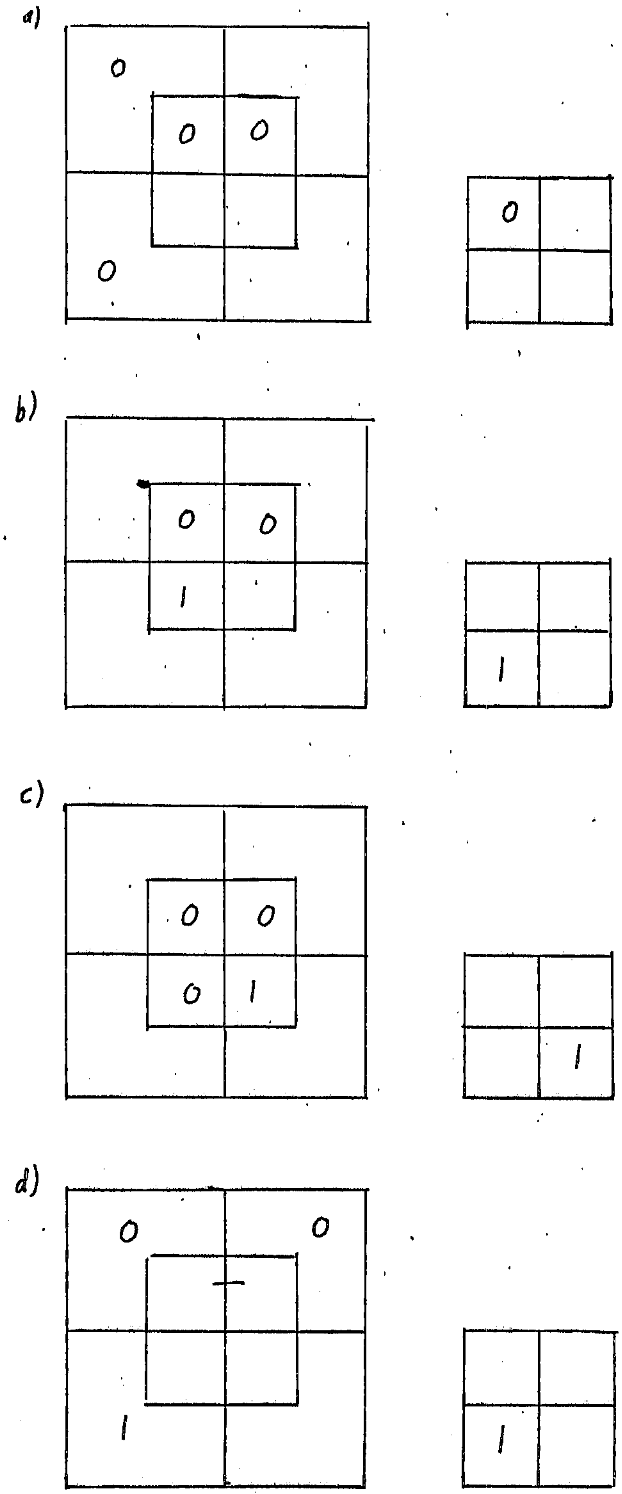
U $ax + by = c$
U $dx - ey = f$
solving for <u>x</u> and <u>y</u>

Diagram XLVII   Carroll's Game of Logic



Diagrams and counters for the game of logic

Diagram XLVIII Carroll's Diagrams in Use



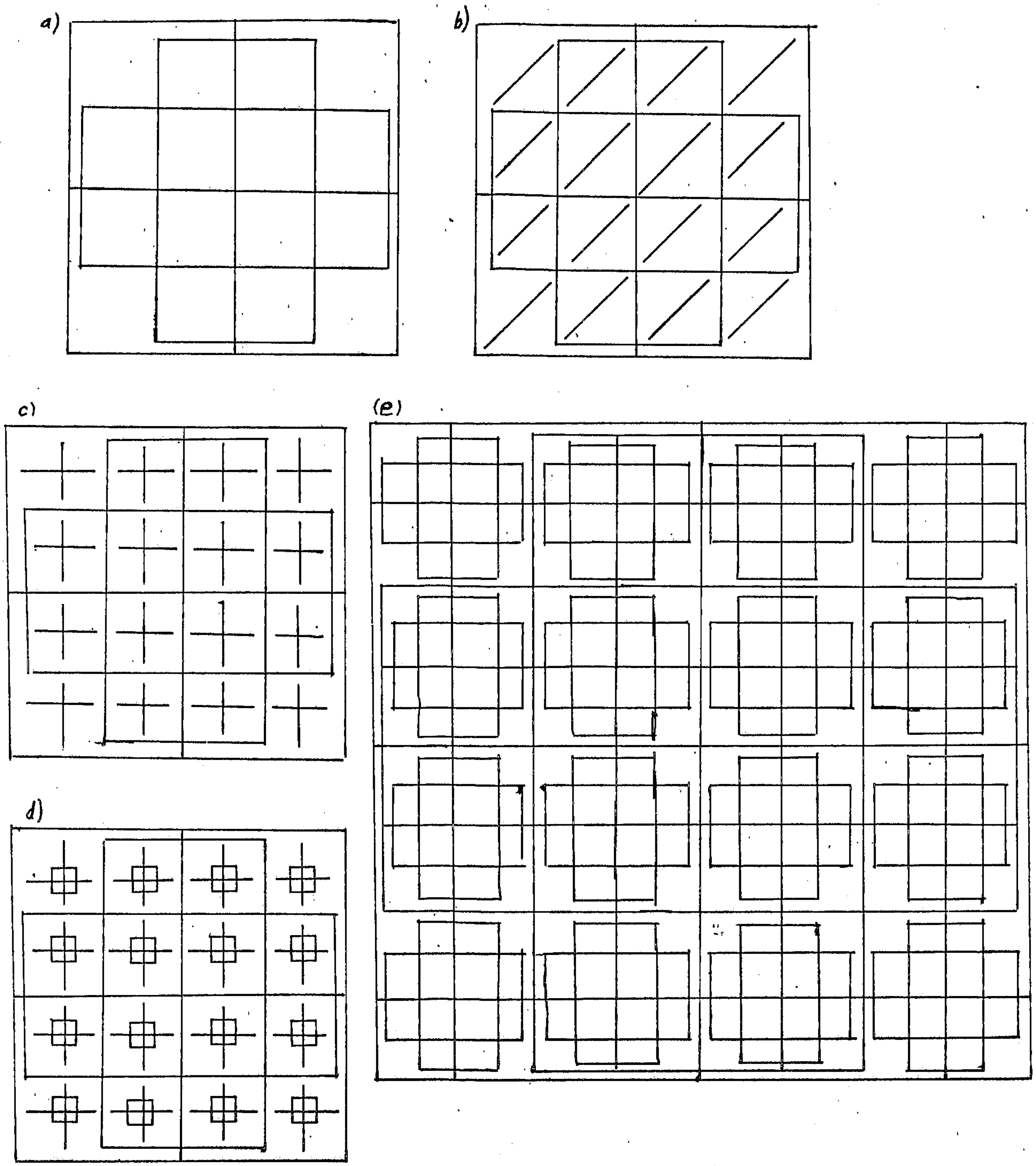
a) $xm_o \dagger ym'_o$ ¶ xy_o

b) $xm_o \dagger ym_1$ ¶ $x'y_1$

c) $xm_o \dagger ym_o \dagger m_1$ ¶ $x'y'_1$

d) $xm'_o \dagger m'y'_o \dagger xm_1 \dagger m'_1$ ¶ $x'y_1$

Diagram XLIX   Carroll's Basic Diagrams



a) Four terms

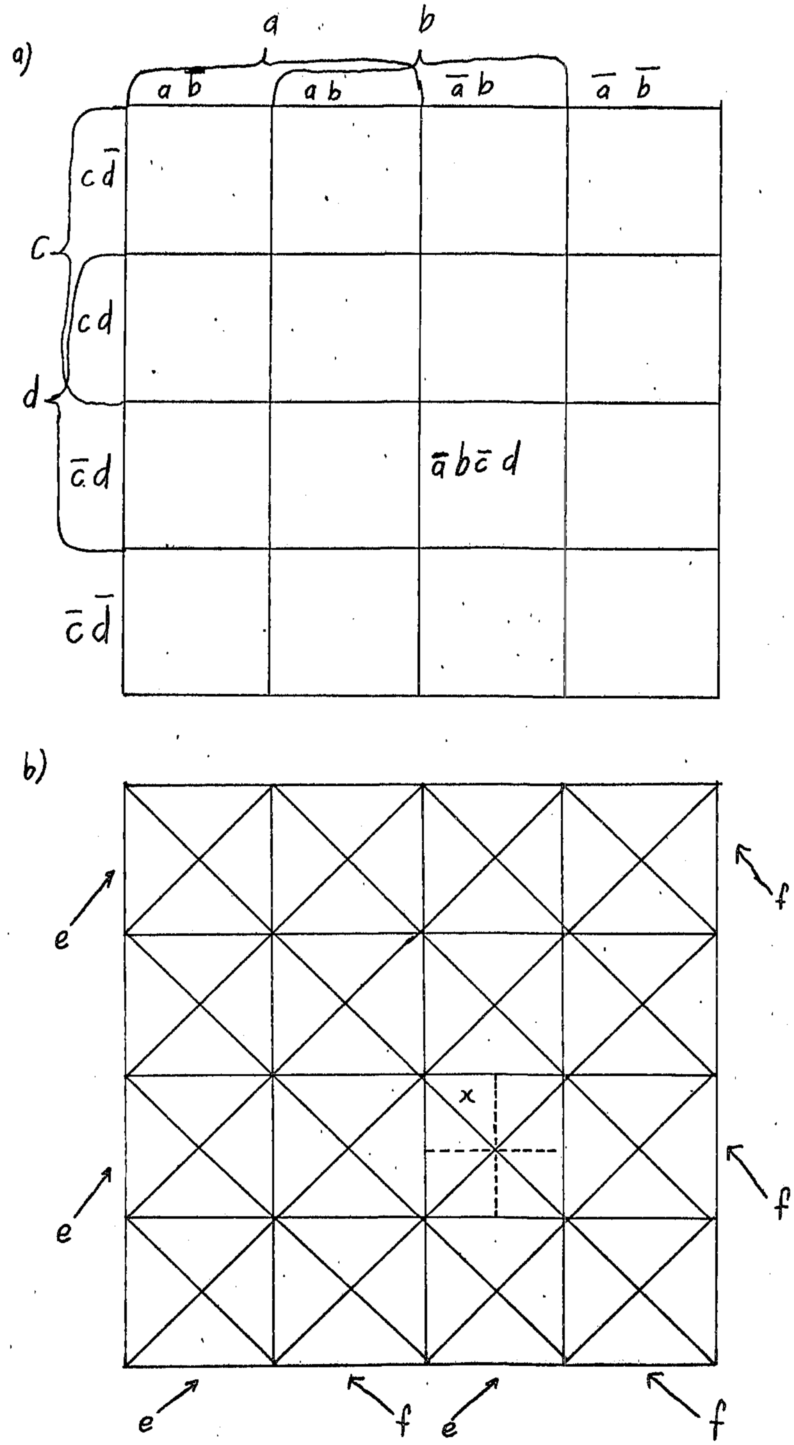
b) Five terms

c) Six terms

d) Seven terms

e) Eight terms

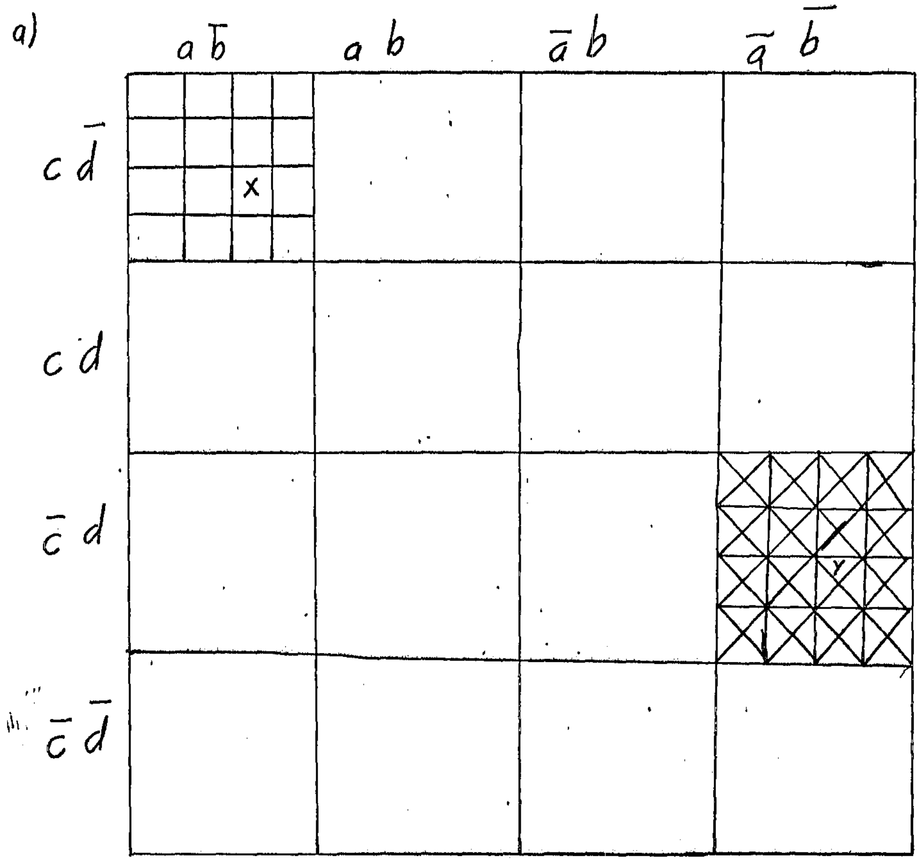
Diagram L   Newlin's Primary Square Subdivided
for Four and Seven Classes

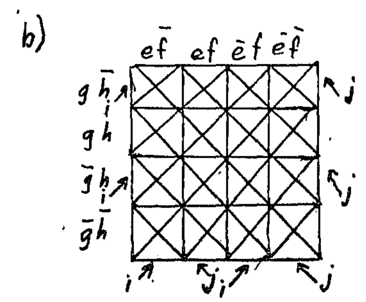


a) Four class diagram

b) Six class diagram subdivided at abcd to allow for a seventh class

Diagram LI   Newlin's Extended Diagram with its Key

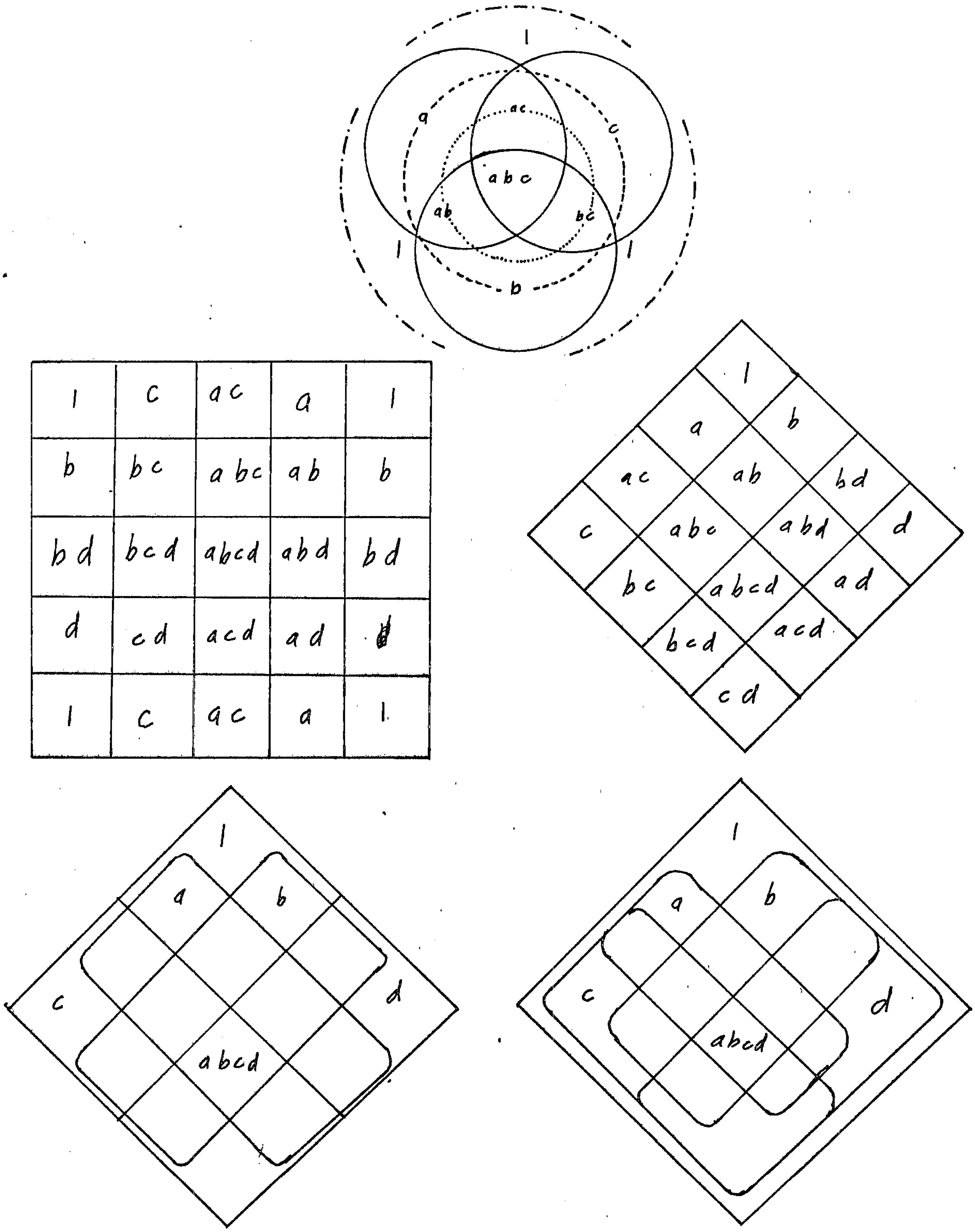




a) Newlin diagram extended to eight classes at $a\bar{b}c\bar{d}$ and to ten classes at $\overline{ab}cd$

b) Key for secondary squares of ten term diagram

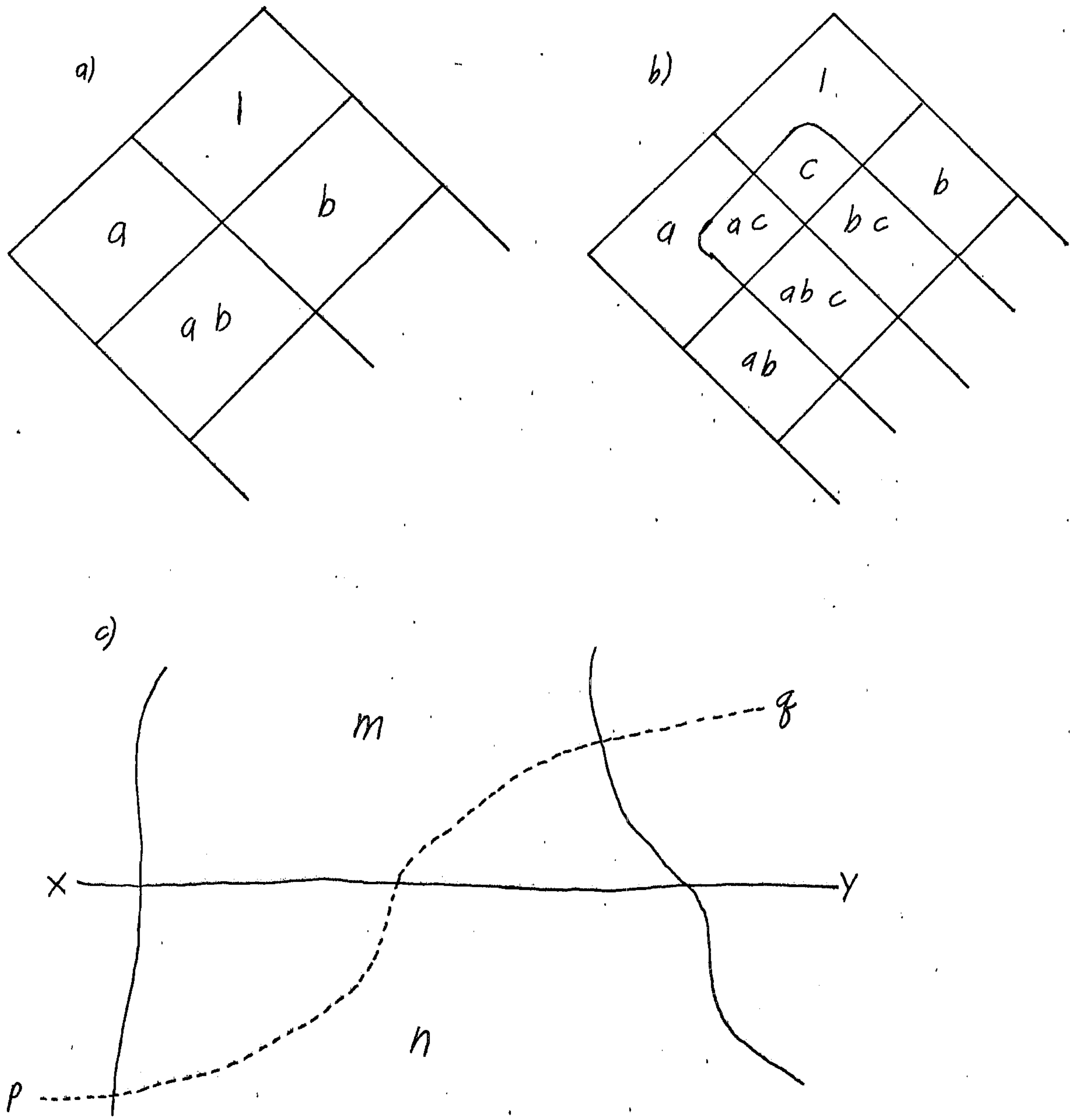
Diagram LII Hocking's Theory of Graphs



a) Three class diagram   b) Four class diagram

c) to e) Four class diagram modified to remove redundancies

Diagram LIII   Hocking's Proof of the Infinite Extensibility of the Logic Diagram

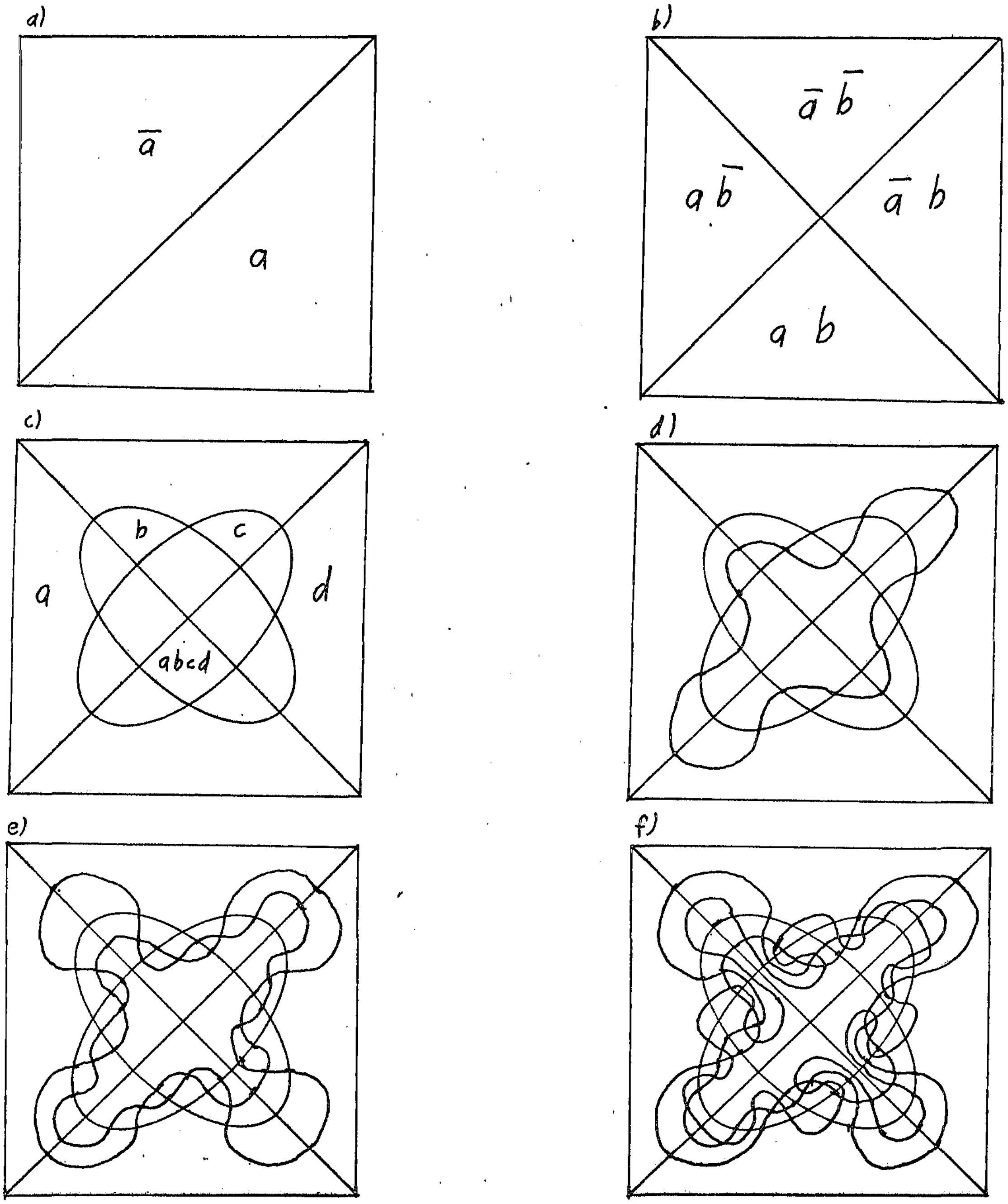


a) Portion of graph

b) Same portion of graph after introduction of another term

c) Proof of infinite extensibility of the logic diagram

Diagram LIV   Hocking's Extended Diagrams



a) One term

b) Two terms

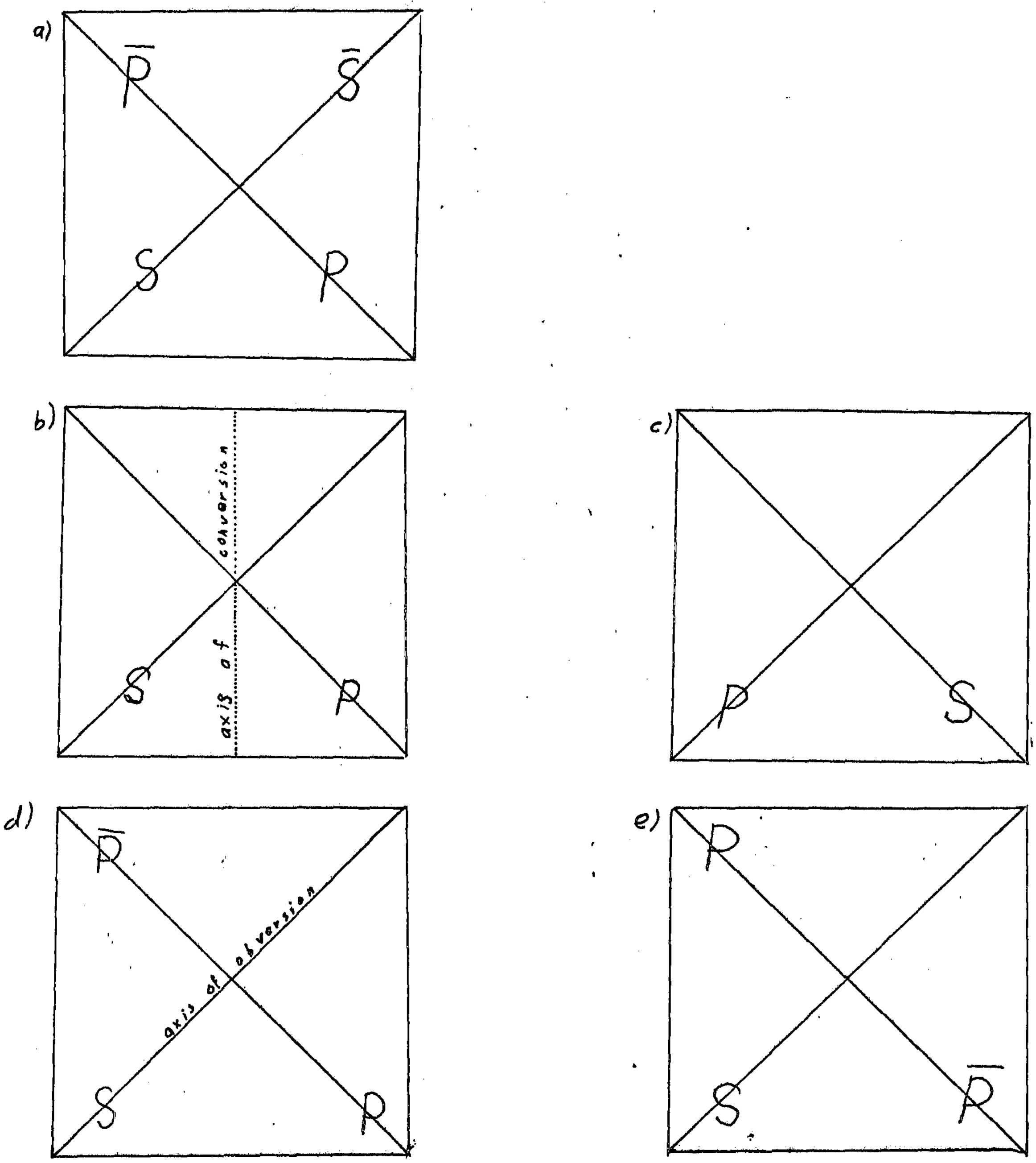
c) Four terms

d) Five terms

e) Six terms

f) Seven terms

Diagram LV   Hocking's Method of Immediate Inference



a) Basic diagram for immediate inference

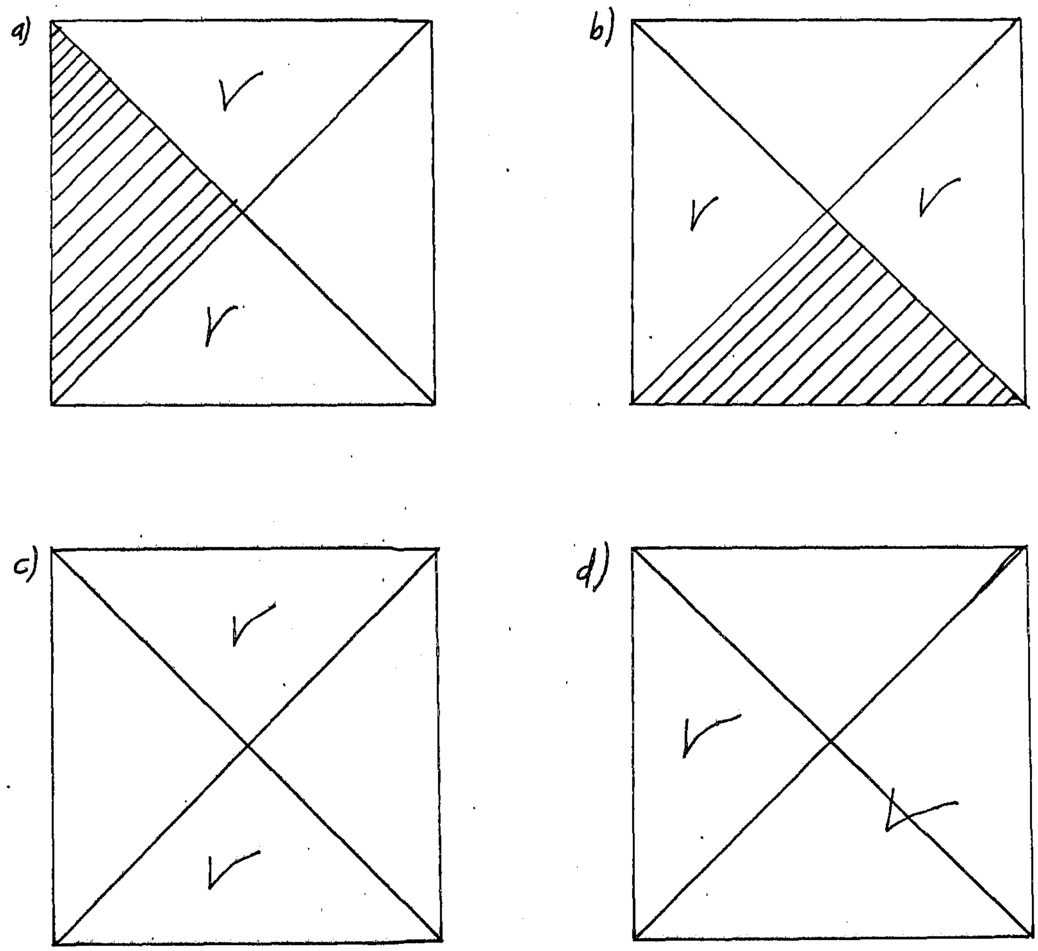
b) Axis of Conversion

c) PS (converse of SP)

d) Axis of Obversion

e) $S\bar{P}$ (obverse of SP)

Diagram LVI Hocking's System of Immediate Inference in Use



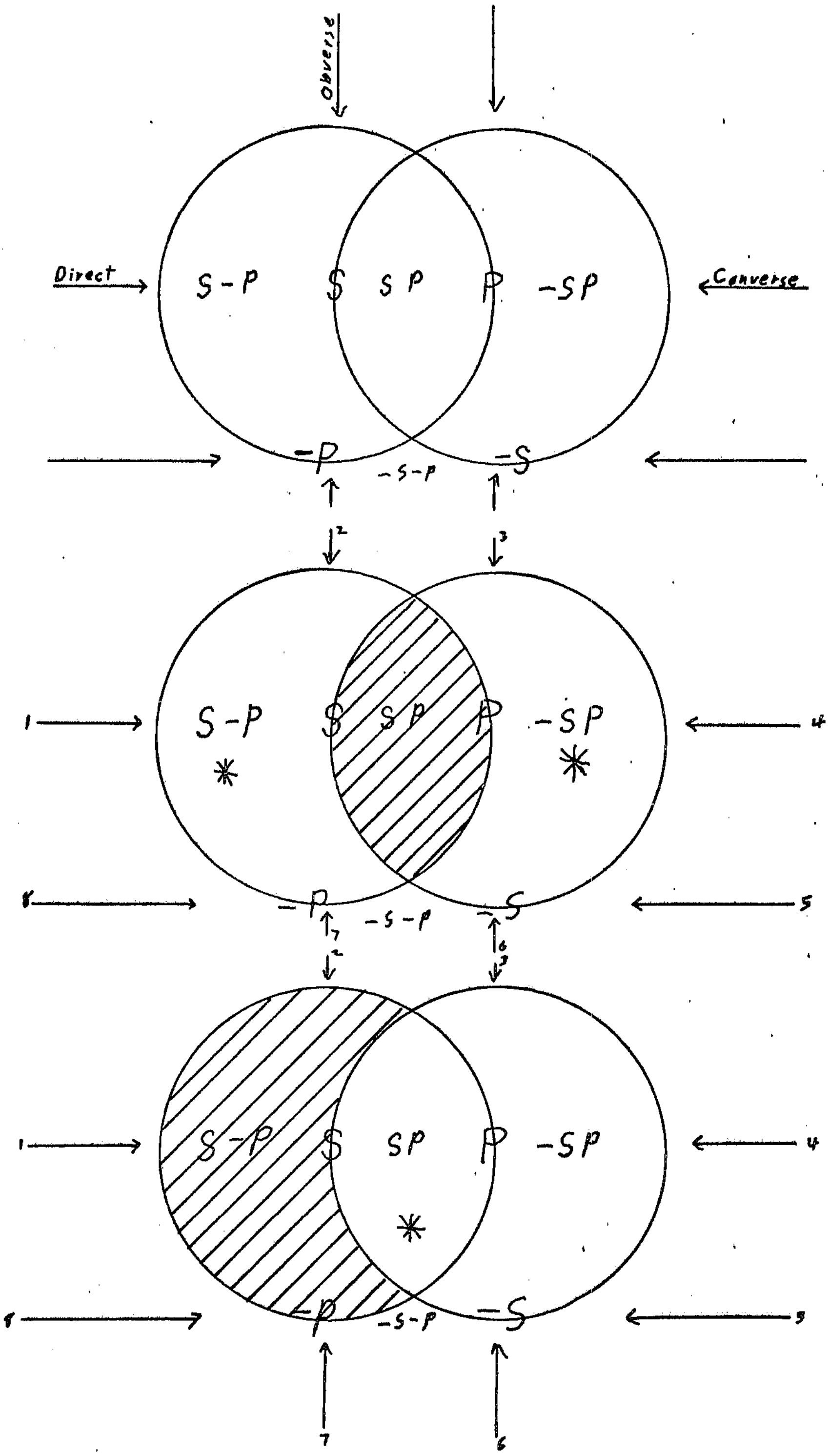
a) The A proposition (All S is P)

b) The E proposition (No S is P)

c) The I proposition (Some S is P)

d) The O proposition (Some S is not P)

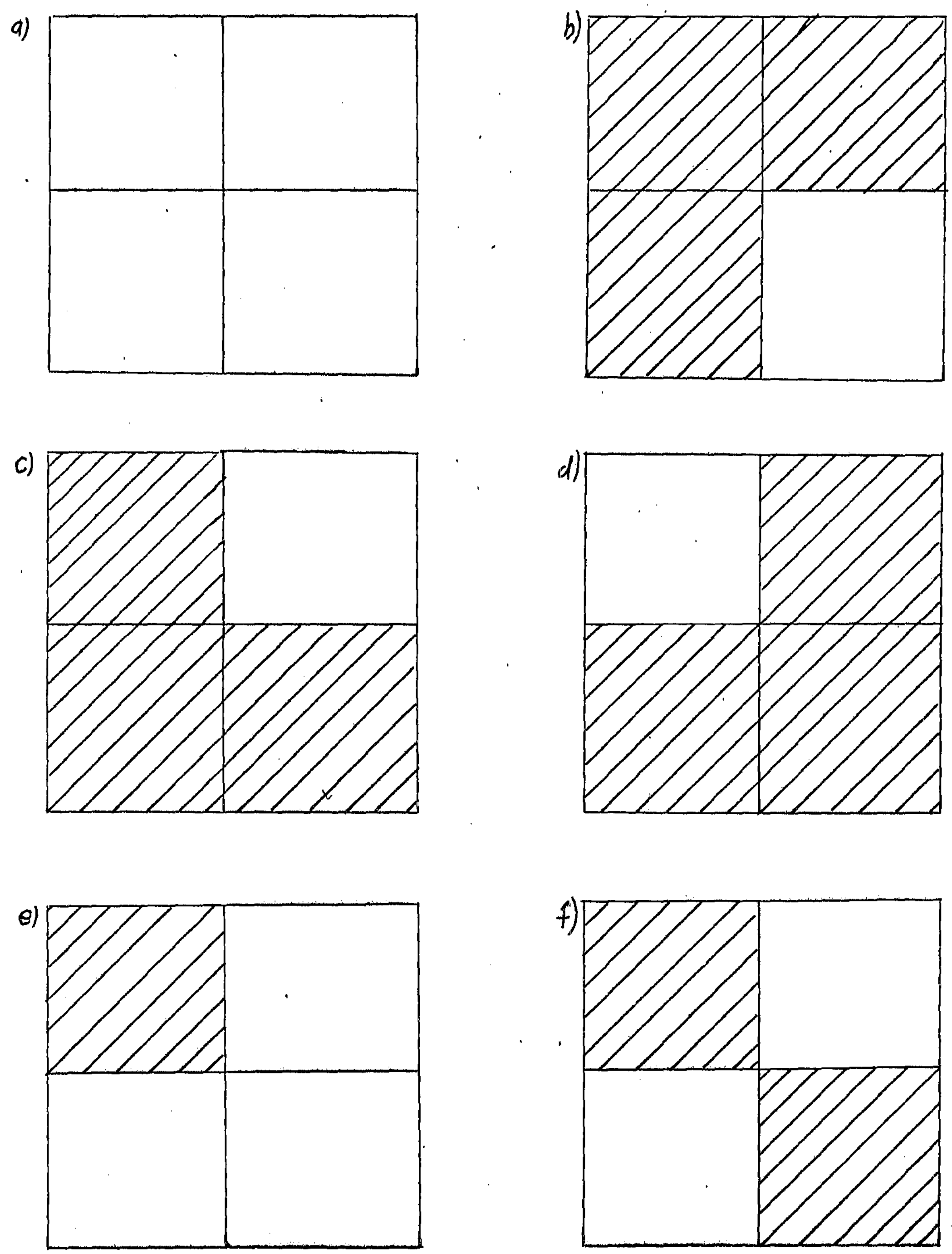
Diagram LVII   Lewis' Diagrams for Immediate Inferences



a) Basic diagram for immediate inferences

b) SP = 0 , S ≠ 0 , P ≠ 0   c) S - P = 0 , S ≠ 0 , P ≠ 0

Diagram LVIII Gonseth's Diagrams



a) Basic Diagram

b) $p \vee q$

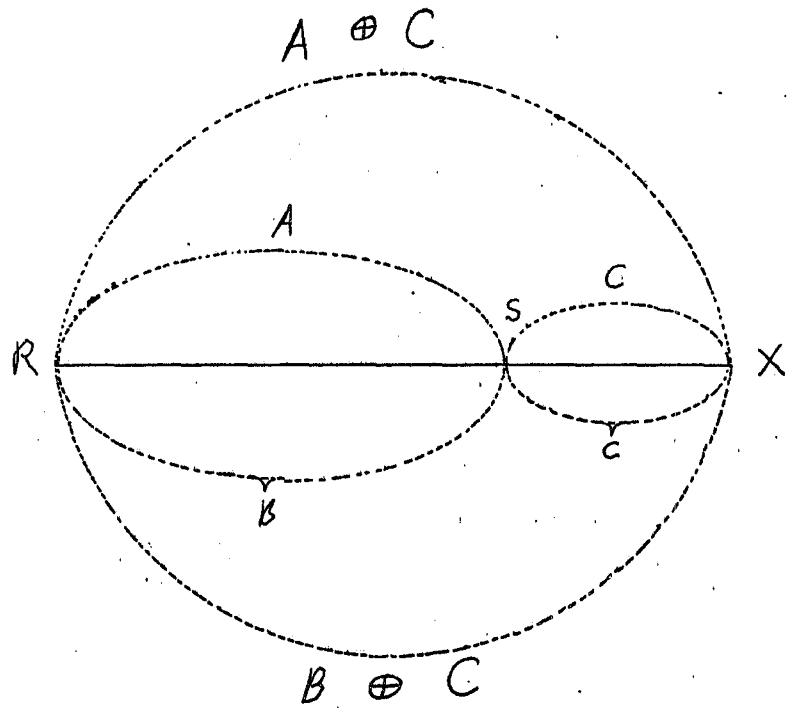
c) $p \supset q$

d) $p | q$

e) $p \cdot q$

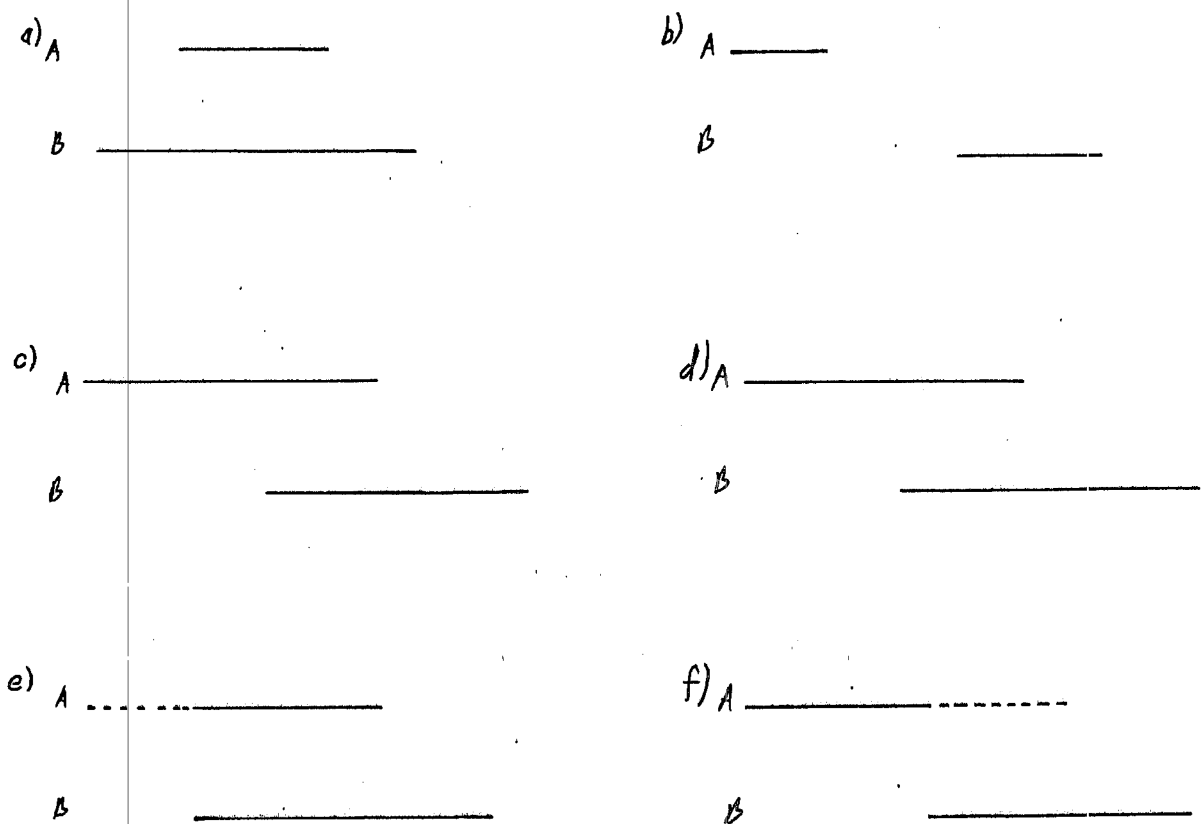
f) $p \equiv q$

Diagram LIX   Example of Leibniz's System



If A=B
then $A \oplus C = B \oplus C$

Diagram LX   Lambert's Linear Notation



a) All A is B

b) No A is B

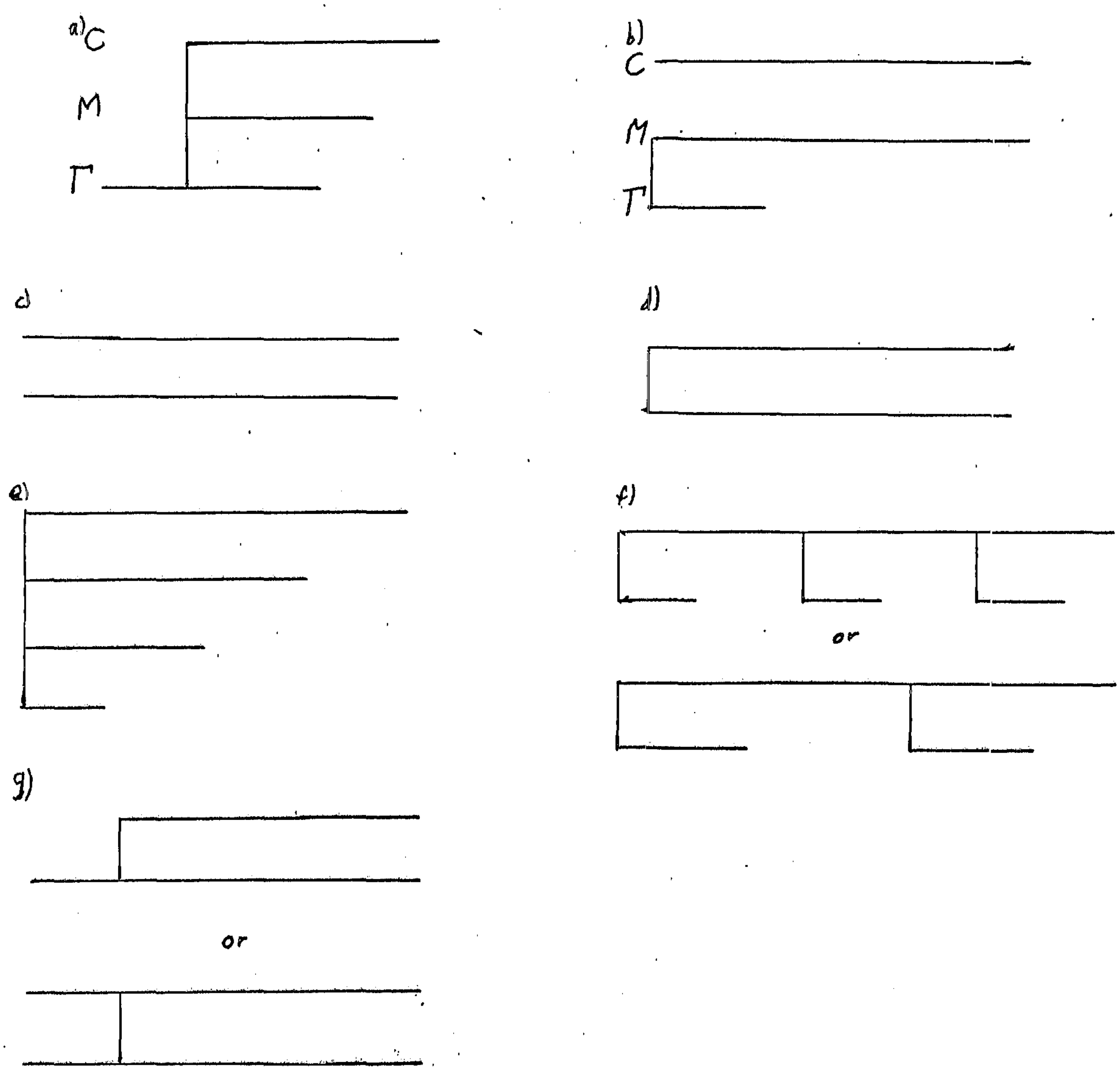
c) Some (not all) A is B

d) Some (not all) A is not B

e) Some (all or some) A is B

f) Some (all or some) A is not B

Diagram LXI   Hamilton's Improved Linear Notation



a) and b) Basic diagrams (actually not describable in logical symbolism)

c) Exclusion

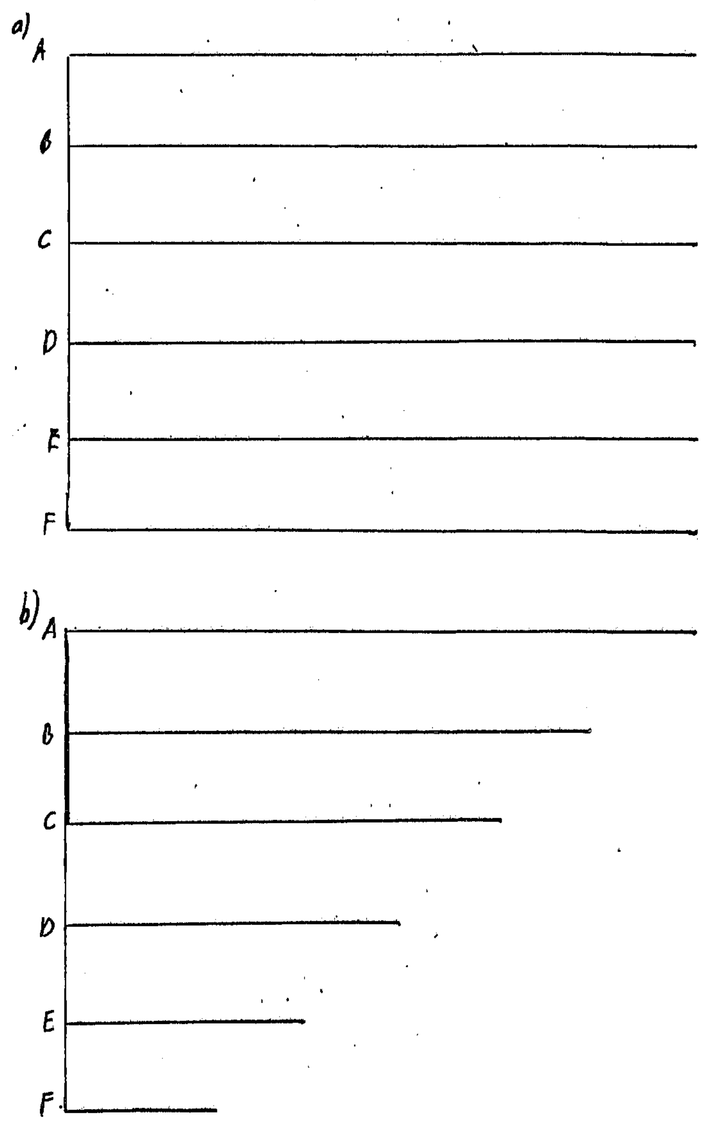
d) Coëxtension

e) Subordination

f) Coördination

g) Intersection or partial coïnclusion and partial coëxclusion

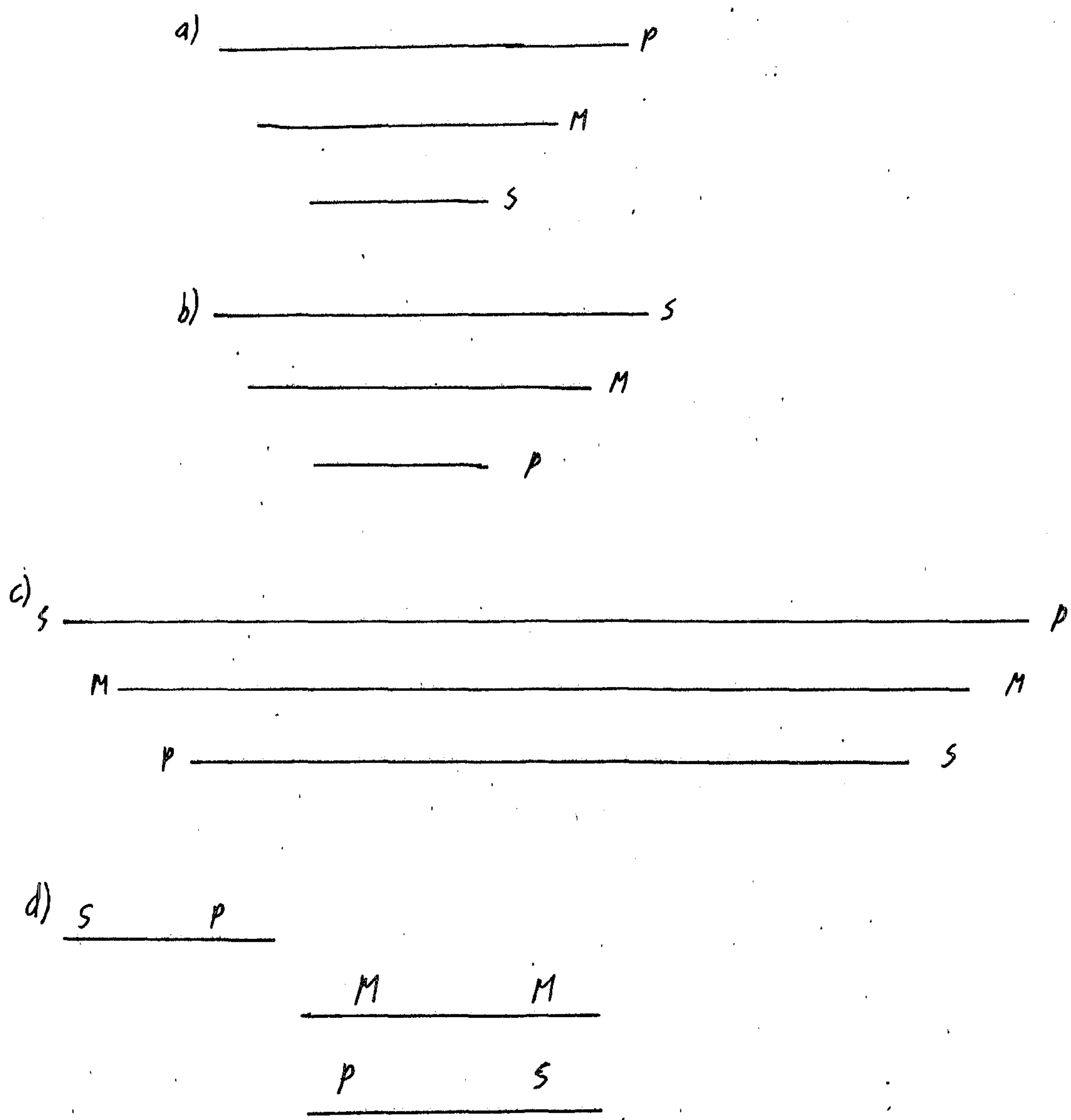
Diagram LXII   Hamilton's Linear Diagrams Applied to Sorites



a) A=B
B=C
C=D
D=E
E=F
∴ A=F

b) All B is A
All C is B
All D is C
All E is D
All F is E
∴ All F is A

Diagram LXIII   Hamilton's Use of Linear Diagrams



a) All M are P
All S are M

b) All M are S
All P are M

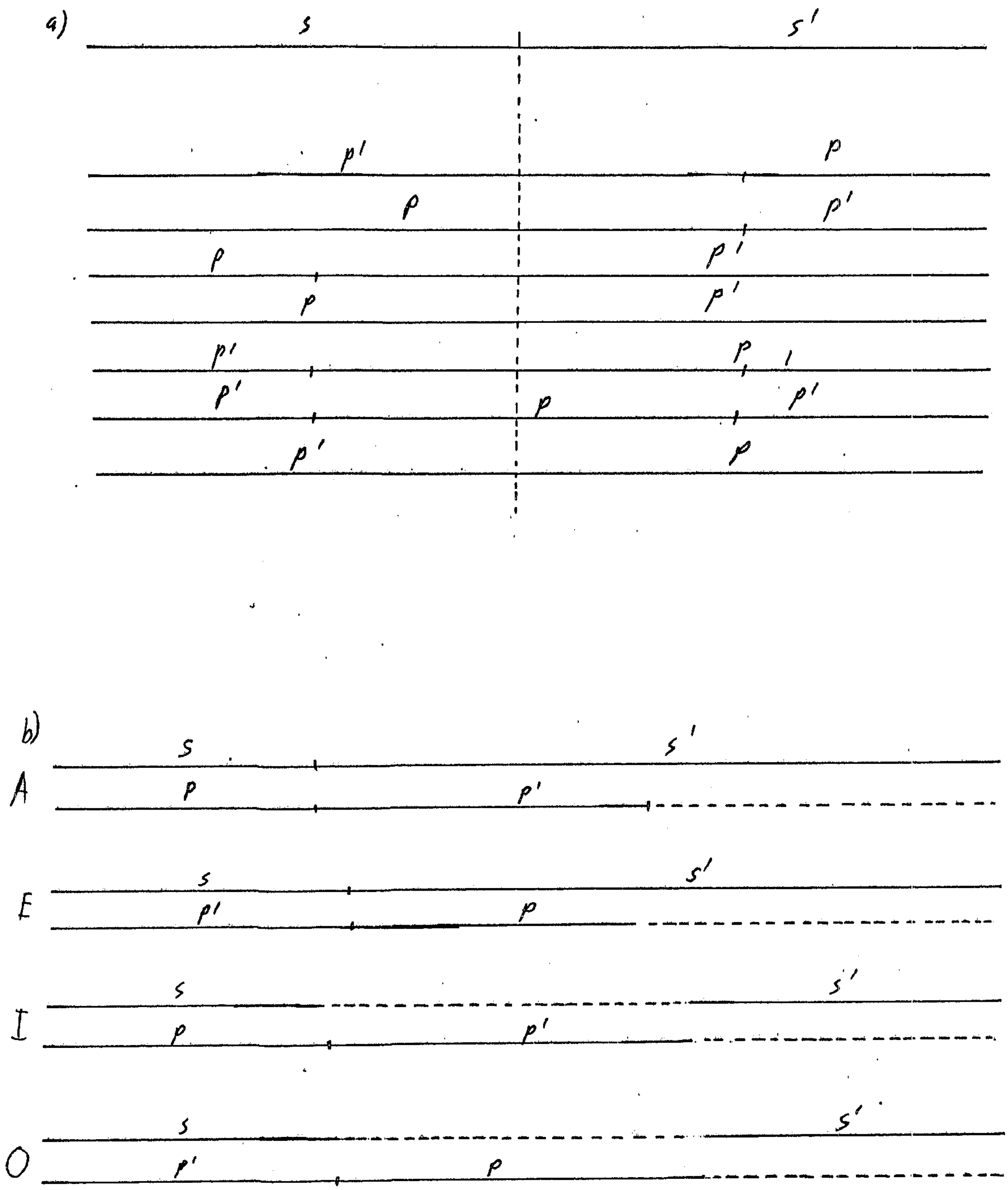
c) left side
All M are S
All P are M

right side
All M are P
All S are M

d) left side
No S are M
All P are M

right side
No P are M
All S are M

Diagram LXIV Keynes' Improvement on Lambert's Linear System

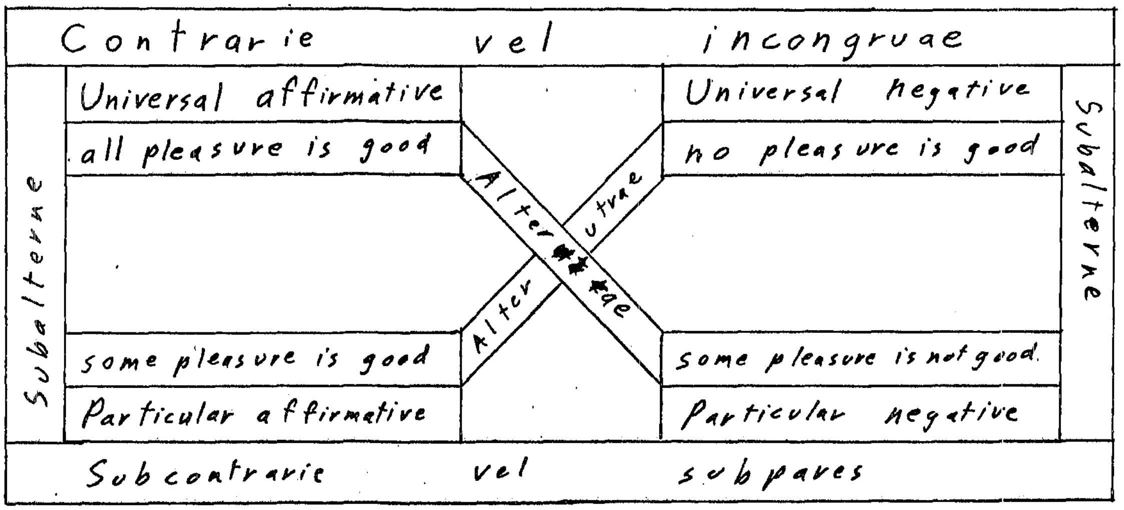


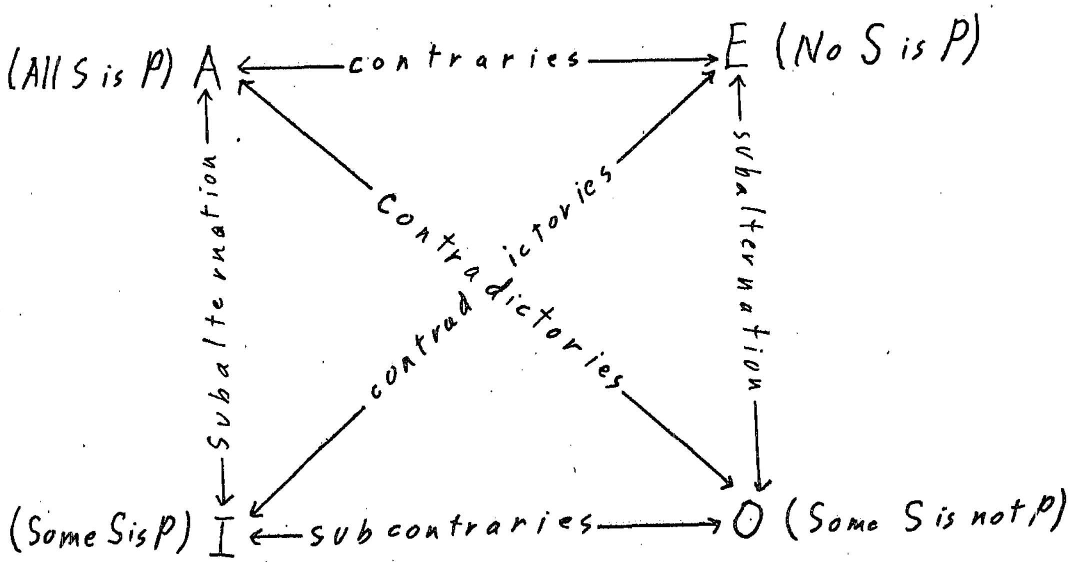
a) Diagram of all possible relations of S and P

b) Diagrams for the four Aristotelian propositions

Diagram LXV Traditional Squares of Opposition

a)

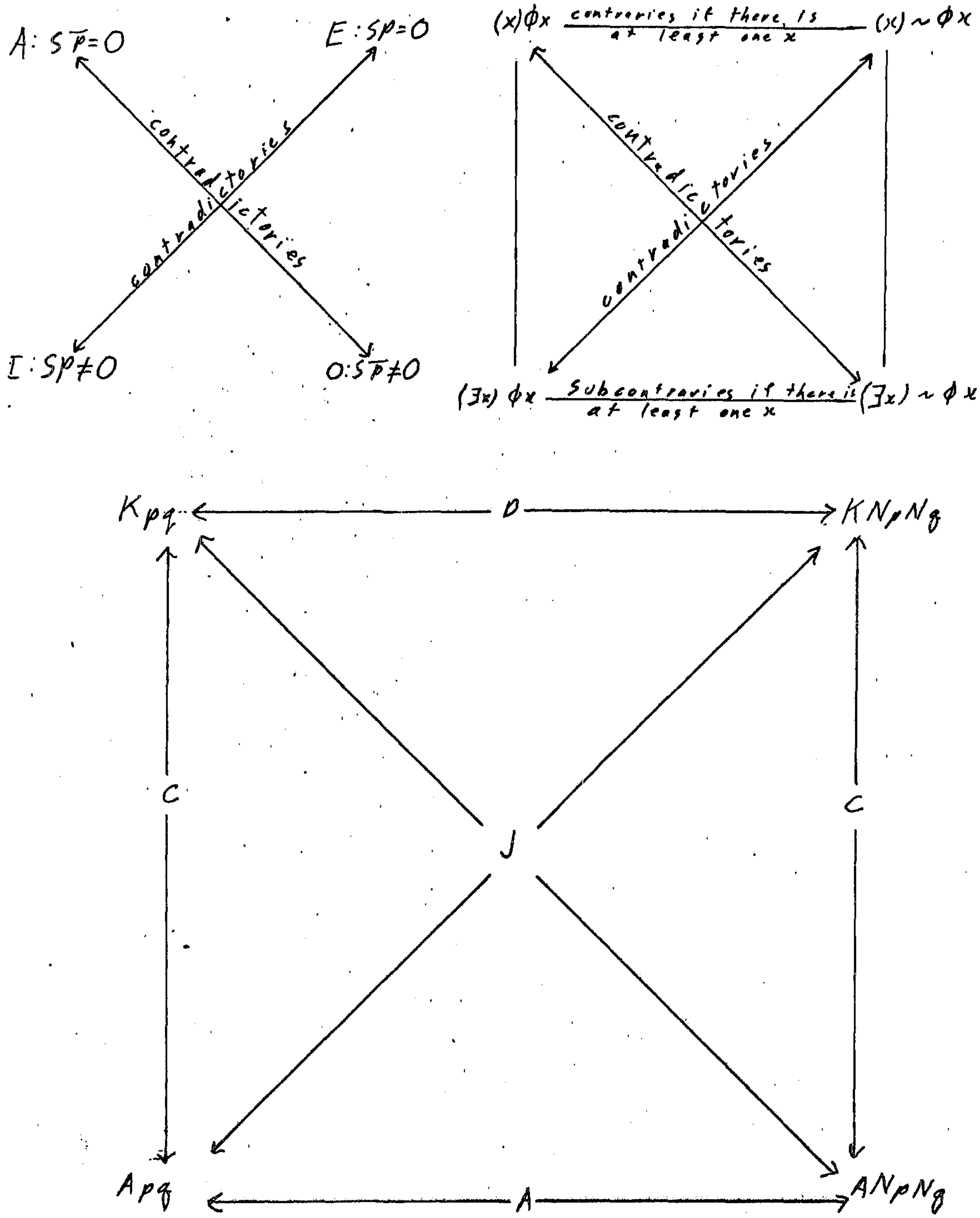




a) After Apuleius

b) Traditional square of opposition

Diagram LXVI Modern Squares of Opposition

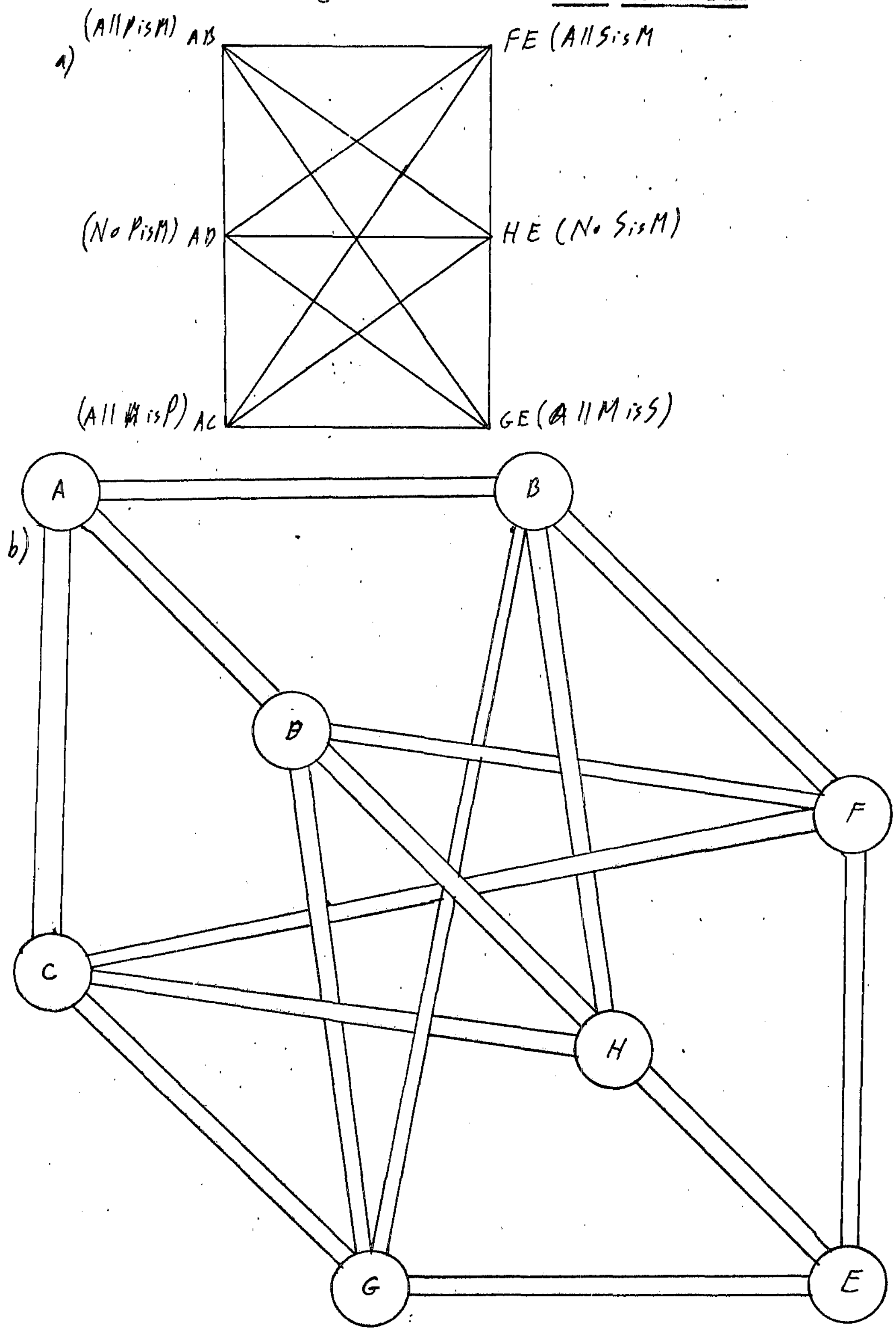


a) Boolean square of opposition

b) Square of opposition with quantifiers

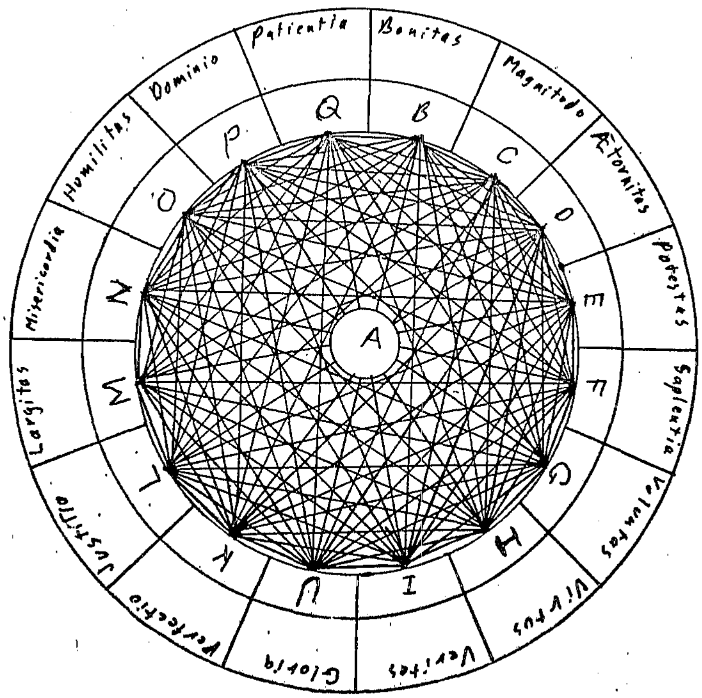
c) Square of opposition for propositional calculus

Diagram LXVII The Pons Assinorum



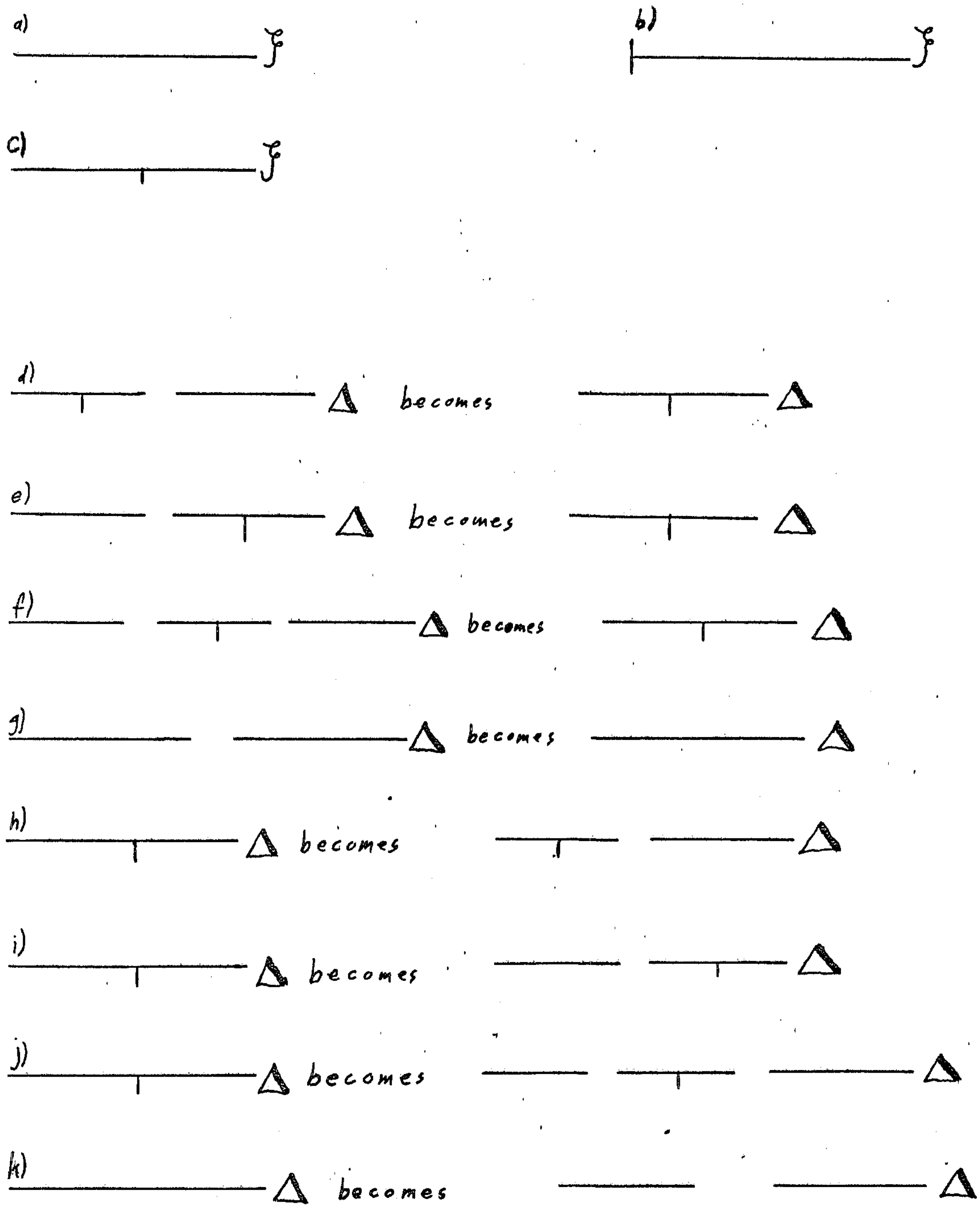
a) After Albert the Great b) After Peter Tarteret

Diagram LXVIII The Ars Magna



After Lull's circle for discovering new truths about the attributes of God

Diagram LXIX   Frege's Basic Diagrams

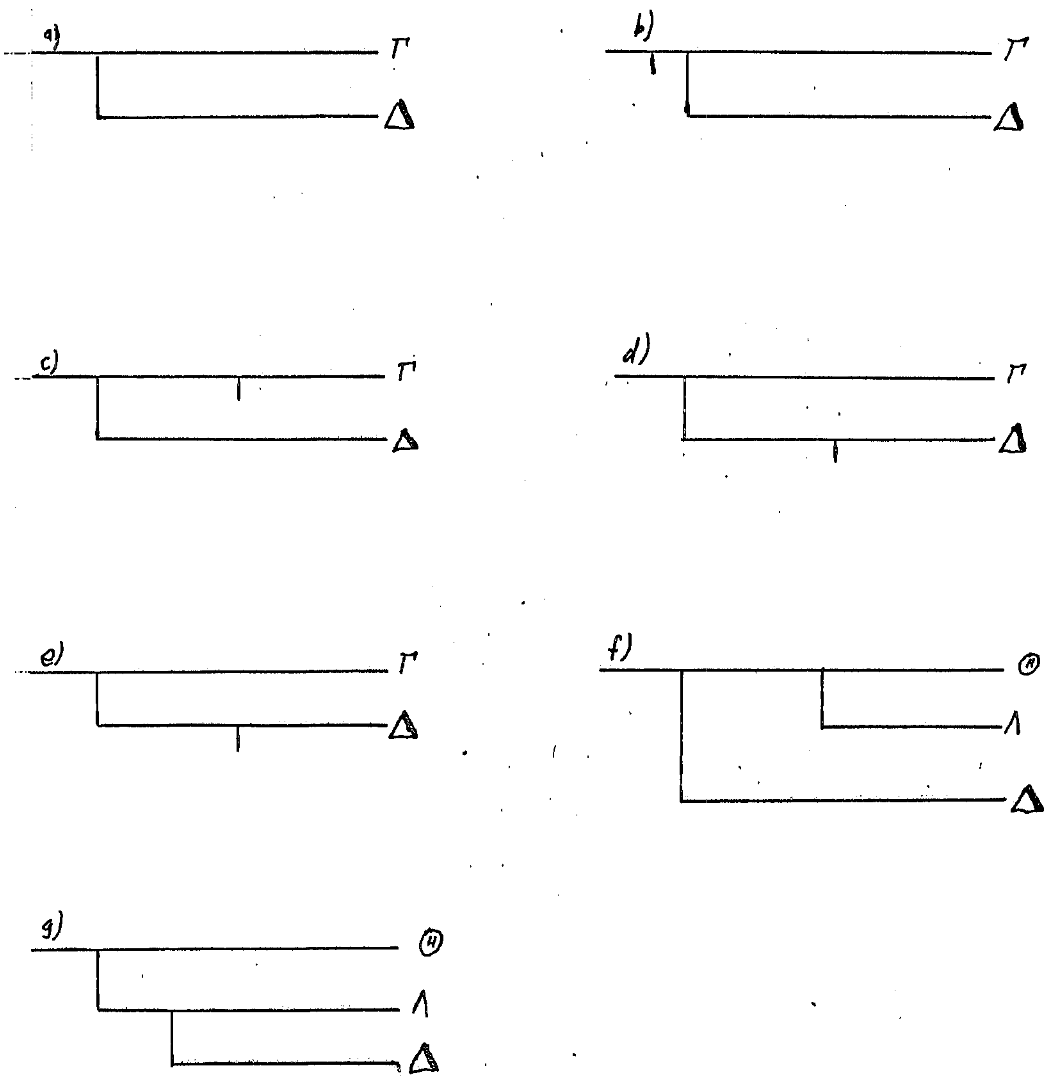


a) The thought ℑ   b) ℑ

c) The thought ~ℑ   d) to g) Amalgamation of horizontals

h) to k) Inverse of d) to g)

Diagram LXX. Frege's Binary Relations



a) $\Delta \supset \Gamma$

b) $\Delta \,.\!\sim \Gamma$

c) $\sim(\Delta . \Gamma)$

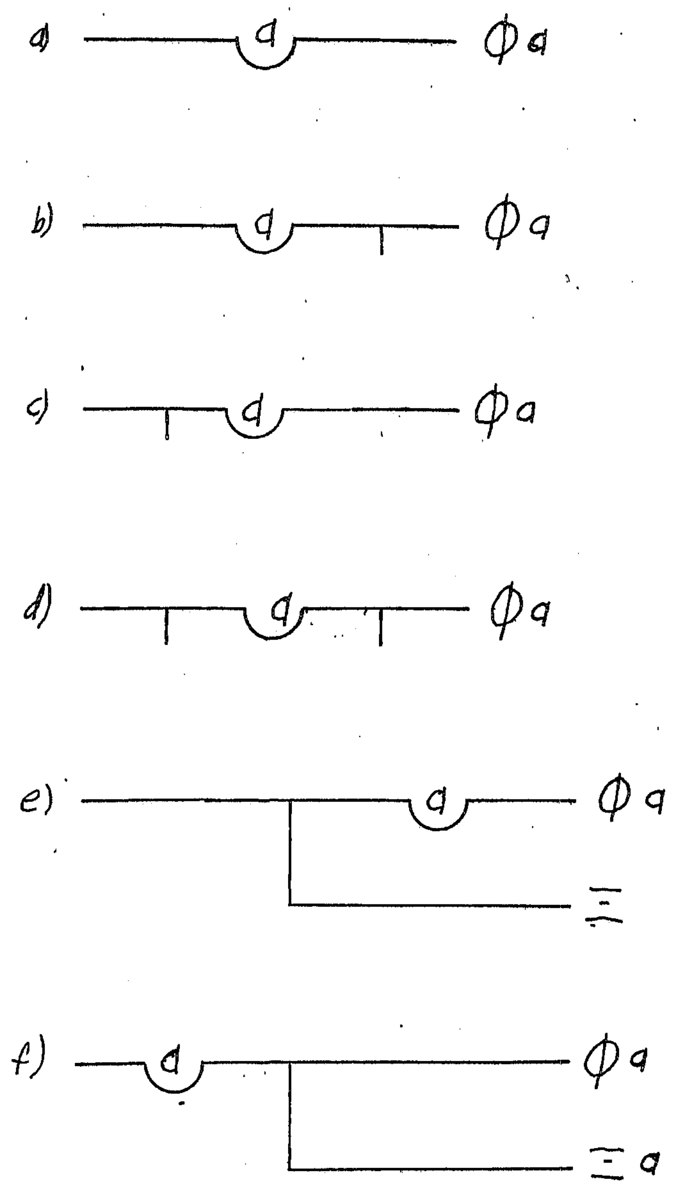
d) $\Delta \vee \Gamma$

e) $\Gamma \supset \Delta$

f) $\Delta \supset (\Lambda \supset \Theta)$

g) $(\Delta \supset \Lambda) \supset \Theta$

Diagram LXXI   Frege's Expression of Generality



a) $(\alpha)\ (\phi\alpha)$

b) $(\alpha){\sim}(\phi\alpha)$

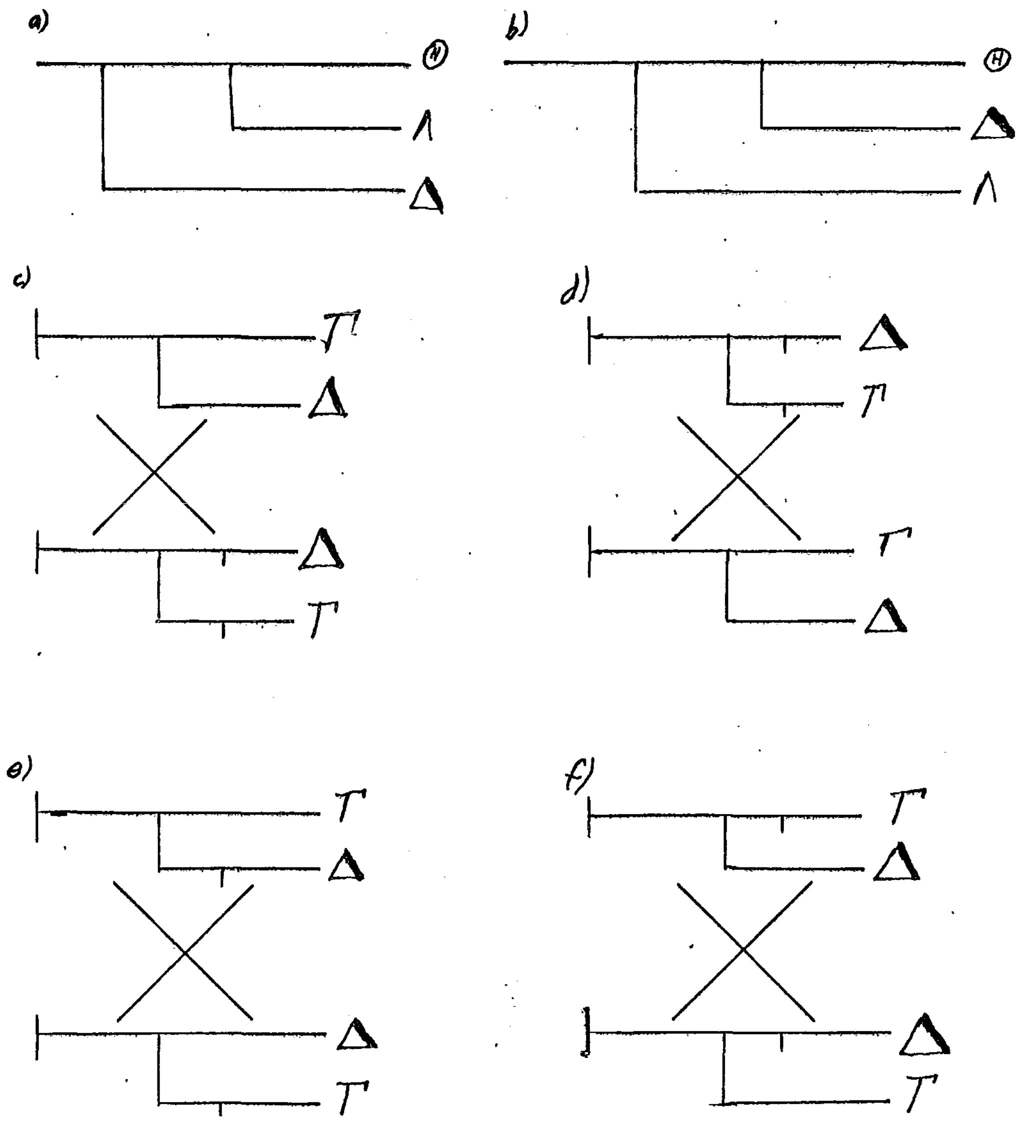
c) ${\sim}(\alpha)\ (\phi\alpha)$

d) $(\exists\alpha)\ (\phi\alpha)$

e) $(\Xi)\supset(\alpha)\ (\phi\alpha)$

f) $(\alpha)\ ([\Xi\alpha]\supset[\phi\alpha])$

Diagram LXXII   Frege's Interchangeability and Contraposition



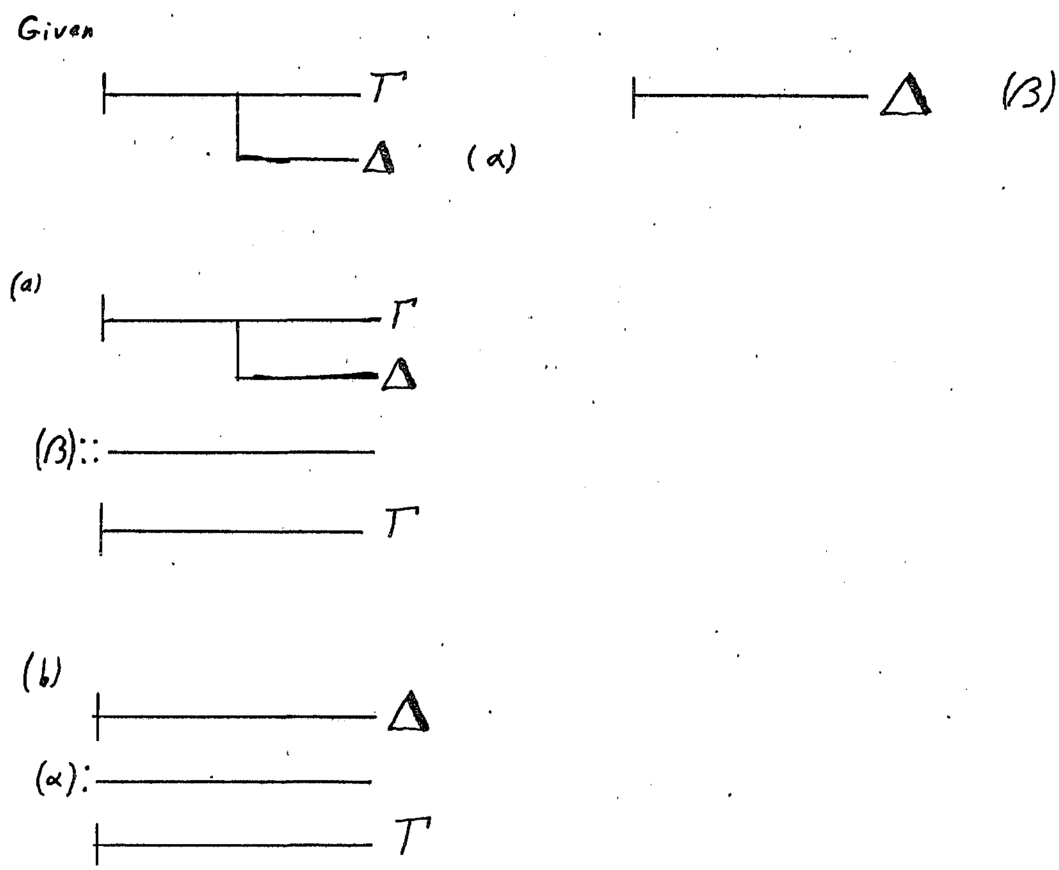
a) and b) Interchangeability   c) to f) Contraposition

a) and b) $(\Delta \supset [\Lambda \supset \Theta]) \equiv (\Lambda \supset [\Delta \supset \Theta])$

c) $(\Delta \supset \Gamma) \equiv (\sim\Gamma \supset \sim\Delta)$ d) $(\sim\Gamma \supset \sim\Delta) \equiv (\Delta \supset \Gamma)$

e) $(\Delta \vee \Gamma) \equiv (\Gamma \vee \Delta)$ f) $(\sim[\Delta . \Gamma]) \equiv (\sim[\Gamma . \Delta])$

Diagram LXXIII Frege's First Method of Inference

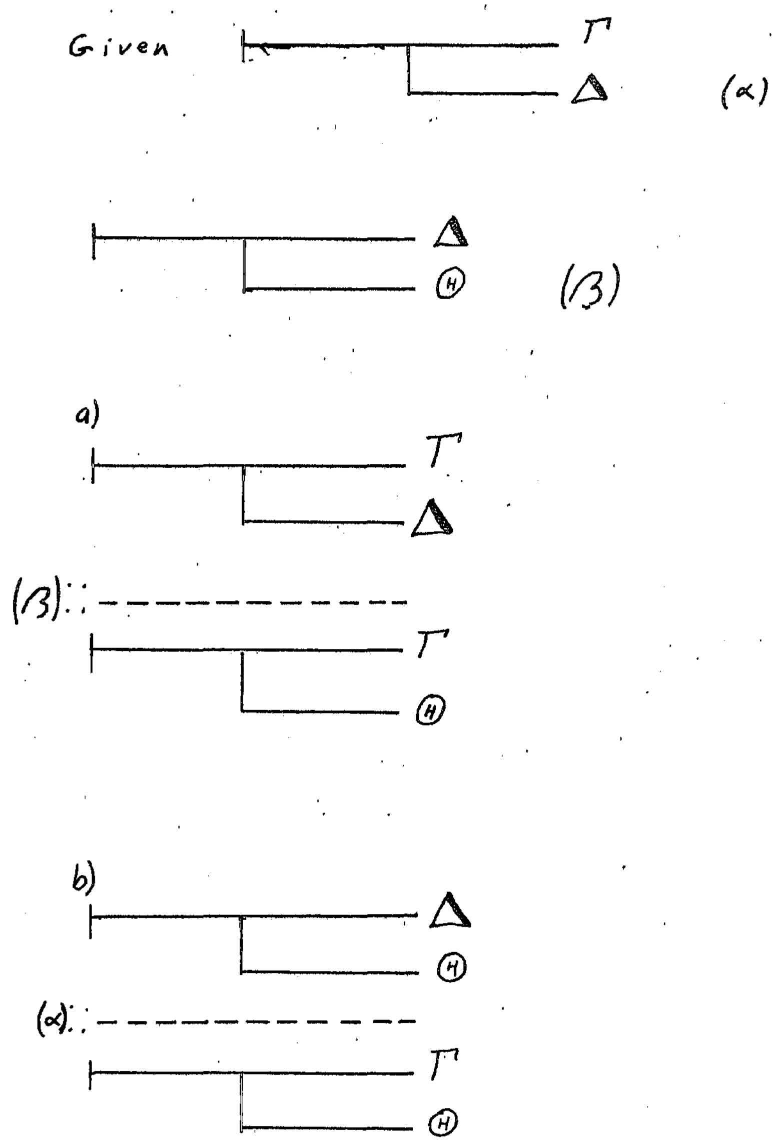


Given: $\Delta \supset \Gamma$ and Δ

a) $\Delta \supset \Gamma$
Δ
$\therefore \Gamma$

b) Δ
$\Delta \supset \Gamma$
$\therefore \Gamma$

Diagram LXXIV   Frege's Second Method of Inference



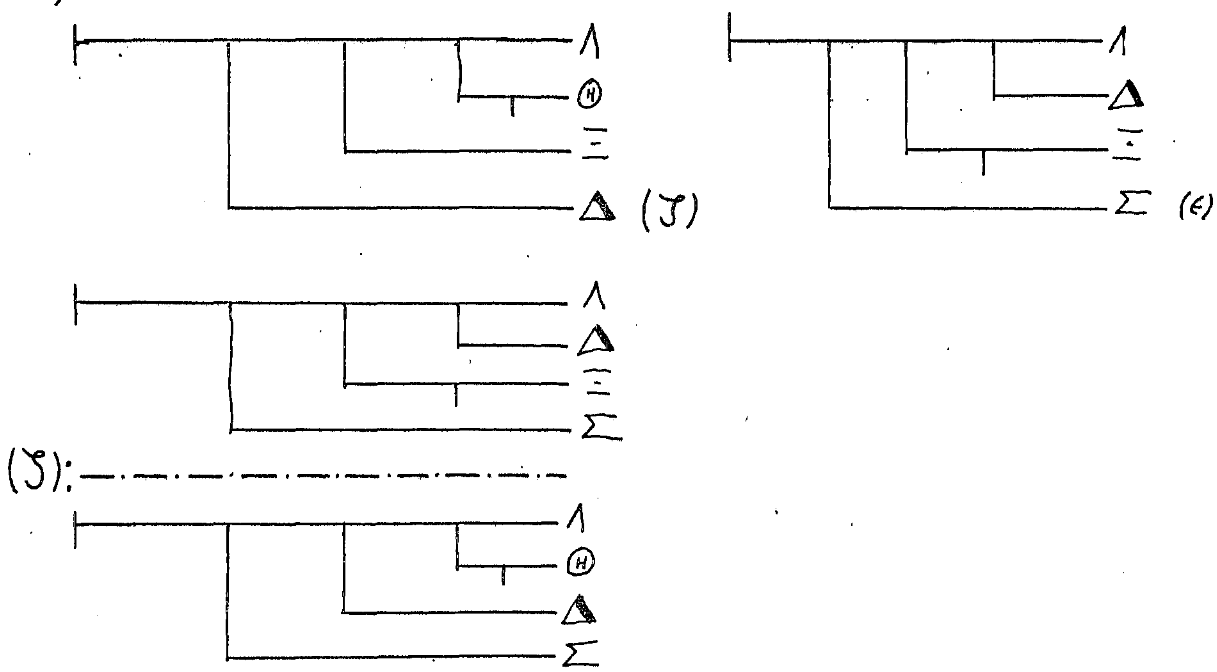
Given: $\Delta \supset \Gamma$ and $\Theta \supset \Delta$

a) $\Delta \supset \Gamma$
$\Theta \supset \Delta$
$\therefore \Theta \cdot \Gamma$

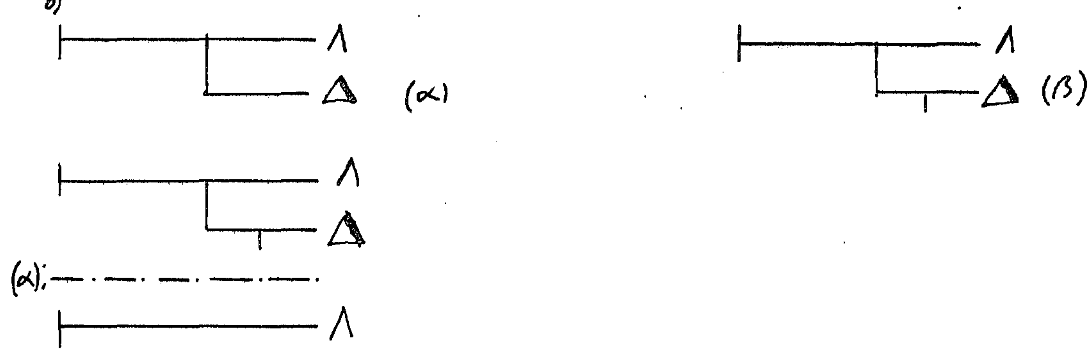
b) $\Theta \supset \Delta$
$\Delta \supset \Gamma$
$\therefore \Theta \supset \Gamma$

Diagram LXXV Frege's Third Method of Inference

a)



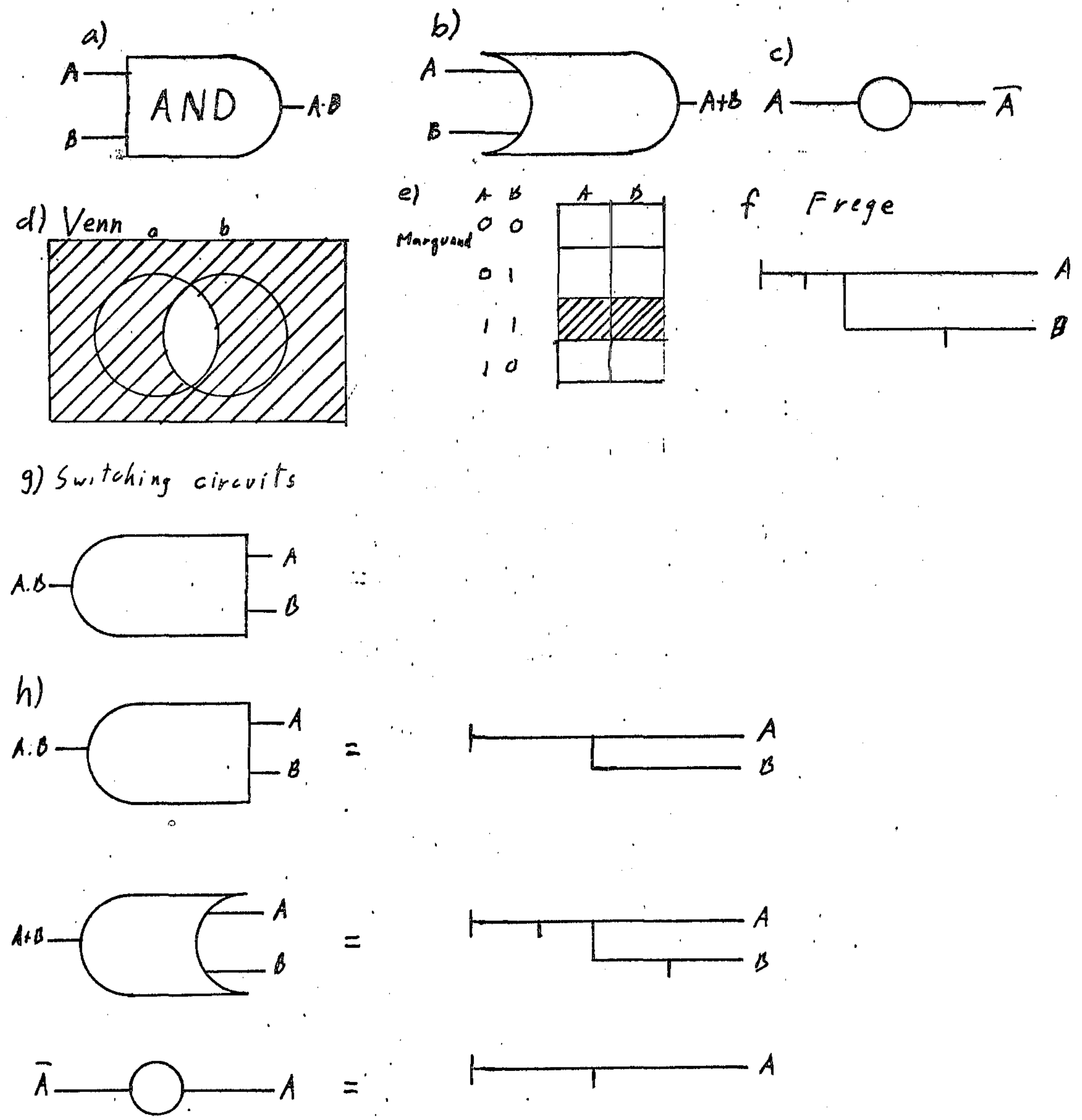
b)



a) $\Sigma \supset (\sim \Xi \supset [\Delta \supset \Lambda])$
$\Delta \supset (\Xi \supset [\sim \Theta \supset \Lambda])$
$\therefore \Sigma \supset (\Delta \supset [\sim \Theta \supset \Lambda])$

b) $\sim \Delta \supset \Lambda$
$\Delta \supset \Lambda$
$\therefore \Lambda$

Diagram LXXVI   Application of Frege's Diagrams to Switching Circuits



a) AND circuit

b) OR circuit

c) Inverter

d) A.B in Venn

e) A.B in Marquand

f) A.B in Frege

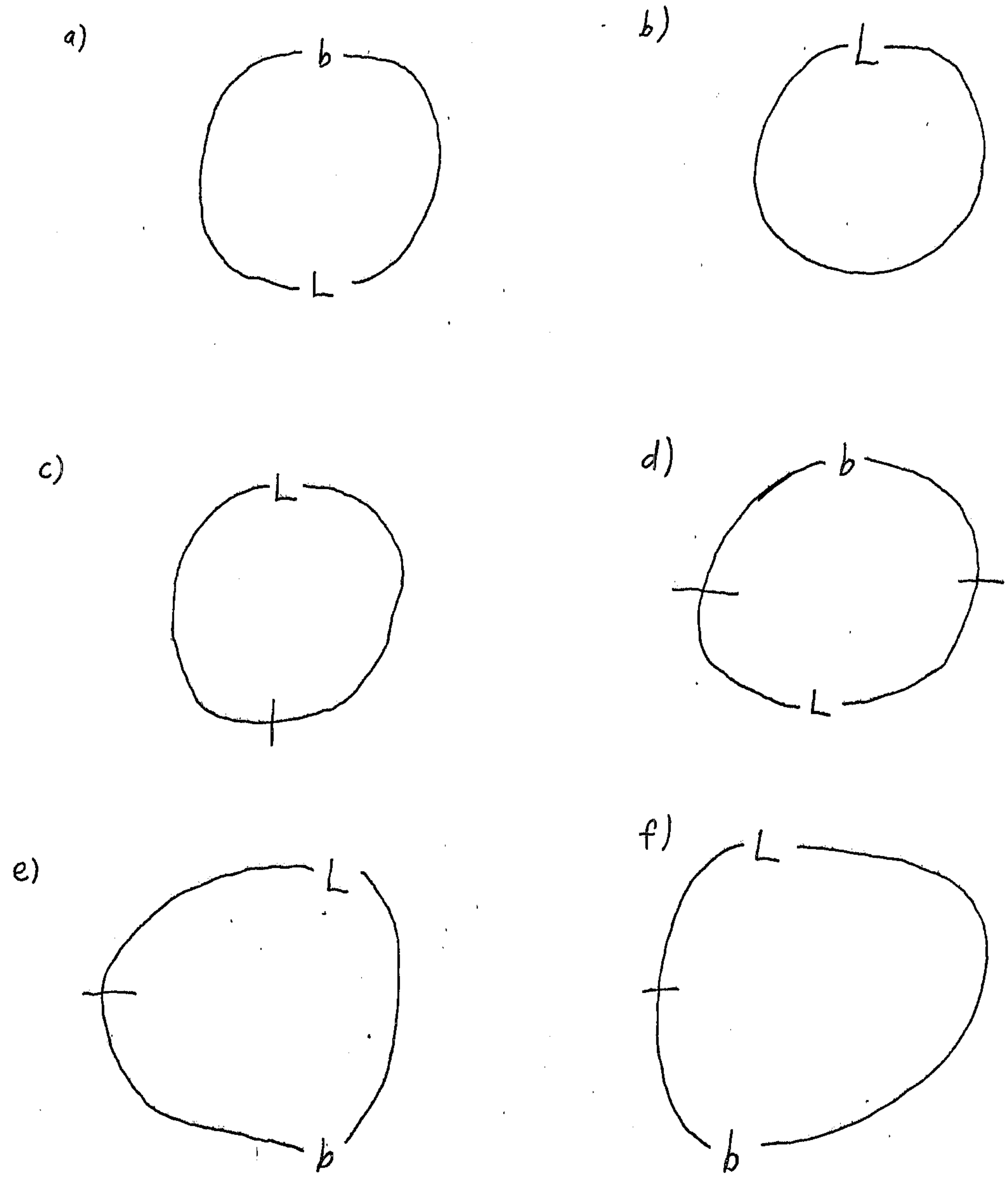
g) A.B in switching circuits

h) AND circuit represented in adapted Frege diagrams

i) OR circuit similarly represented

j) Inversion similarly represented

Diagram LXXVII Peirce's First Diagrammatic System



a) $\Sigma x \Sigma y \quad bxyLxy > 0$

b) $\Sigma x \ Lxx > 0$

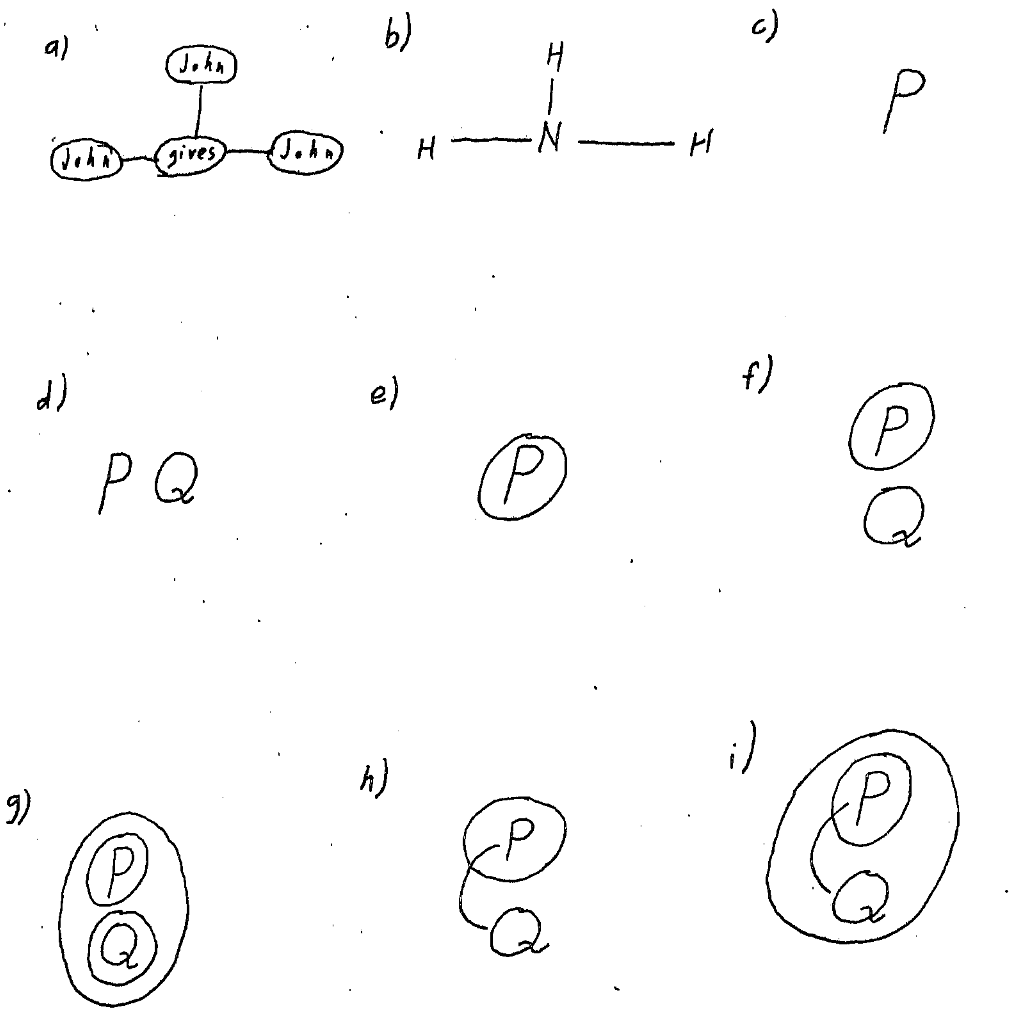
c) $\Pi x \ Lxx > 0$

d) $\Pi x \Pi y \ (Lxy \Psi bxy) > 0$

e) $\Sigma y \Pi x \ (Lxy \Psi bxy) > 0$

f) $\Pi x \Sigma y \ (Lxy \ bxy) \quad 0$

Diagram LXXVIII Peirce's Entitative Graphs



a) John gives John to John

b) Chemical graph of ammonia

c) P

d) P or Q

e) not P

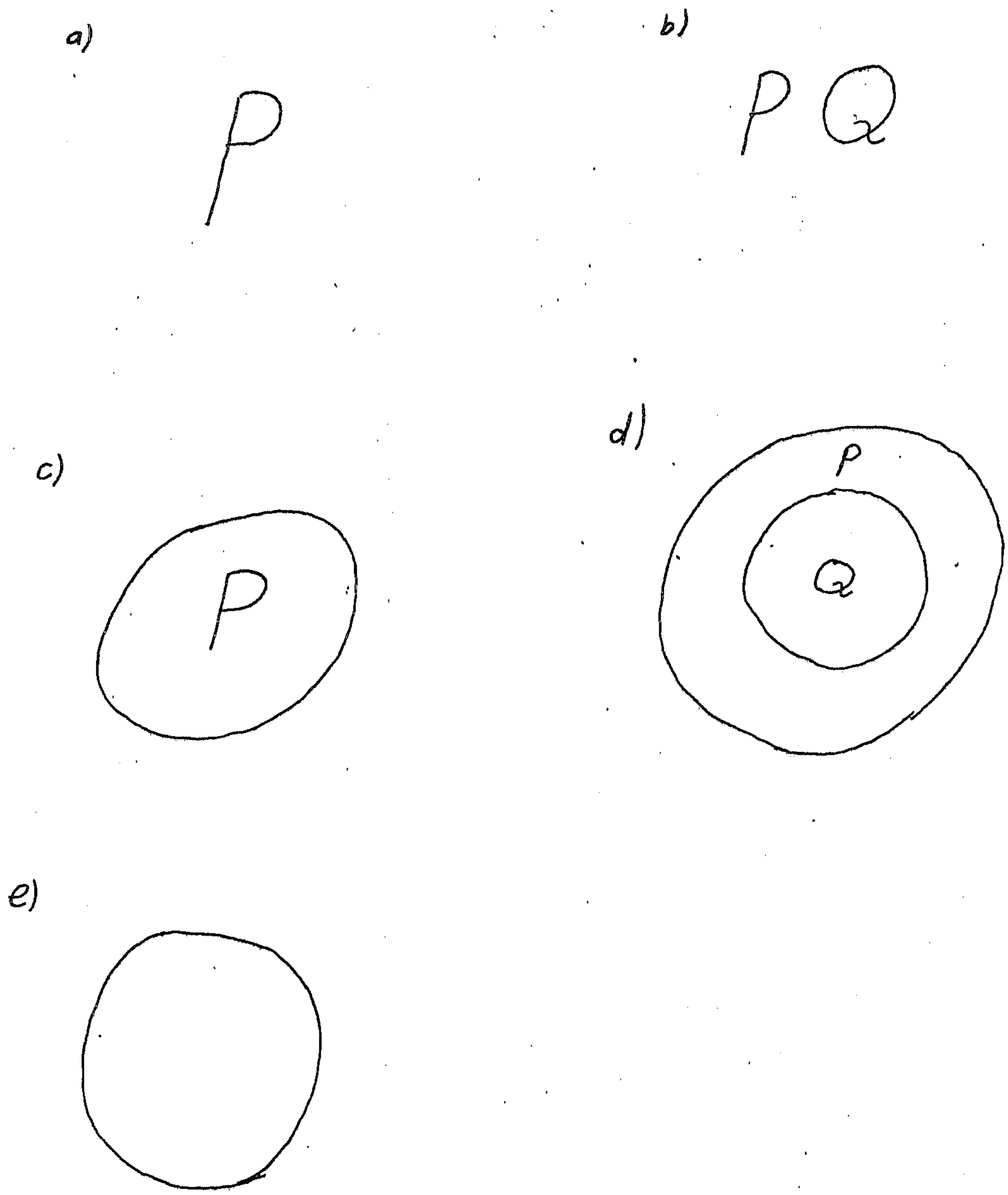
f) If P then Q

g) P and Q

h) Everything that is P is Q

i) Something is P and not Q

Diagram LXXIX The Alpha Part of Existential Graphs



a) P

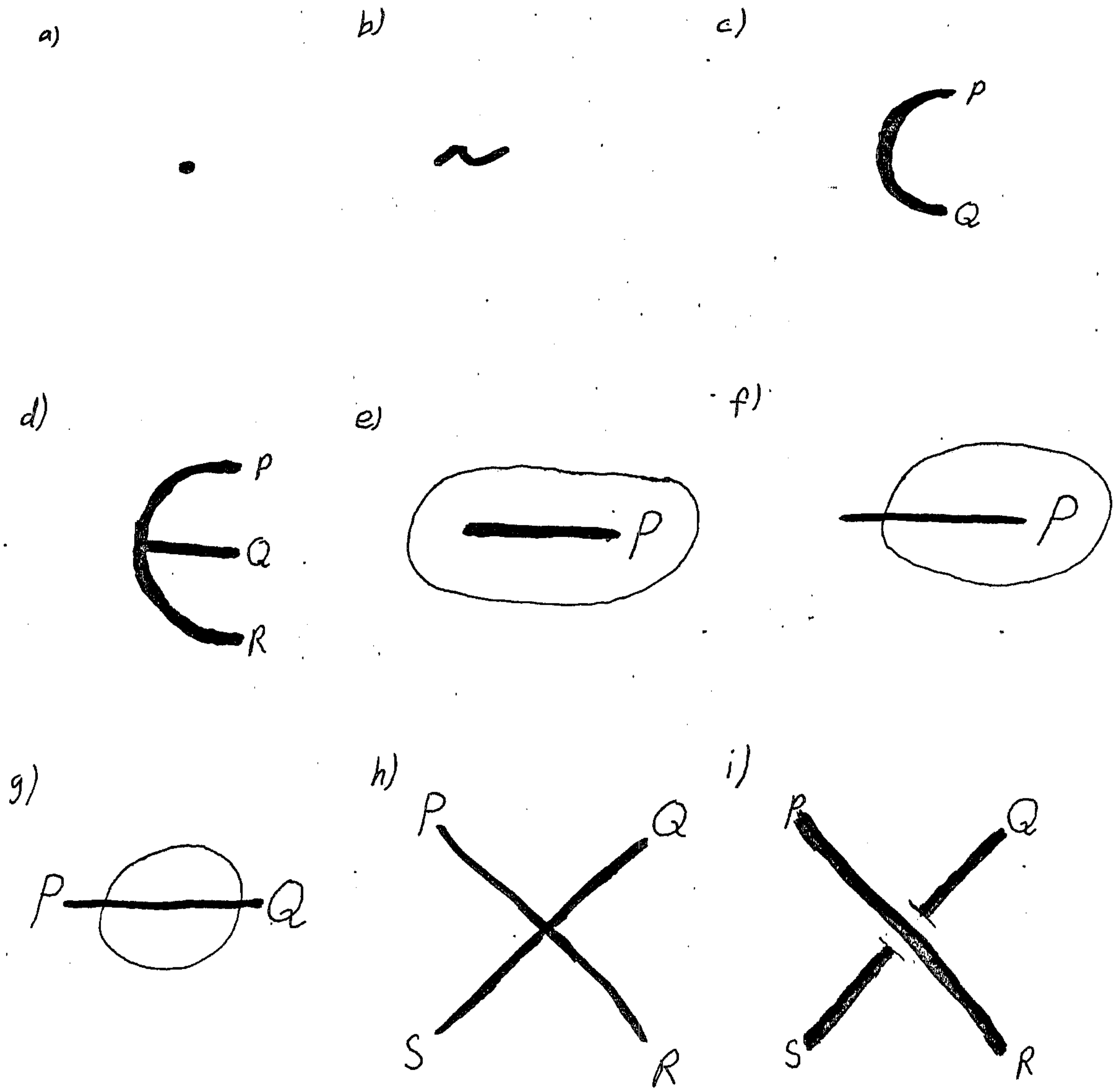
b) P and Q

c) not P

d) If P then Q

e) The empty cut

Diagram LXXX The Beta Part of Existential Graphs



a) and b) Something exists

c) Something is both P and Q

d) Something is P, Q and R

e) Nothing is P

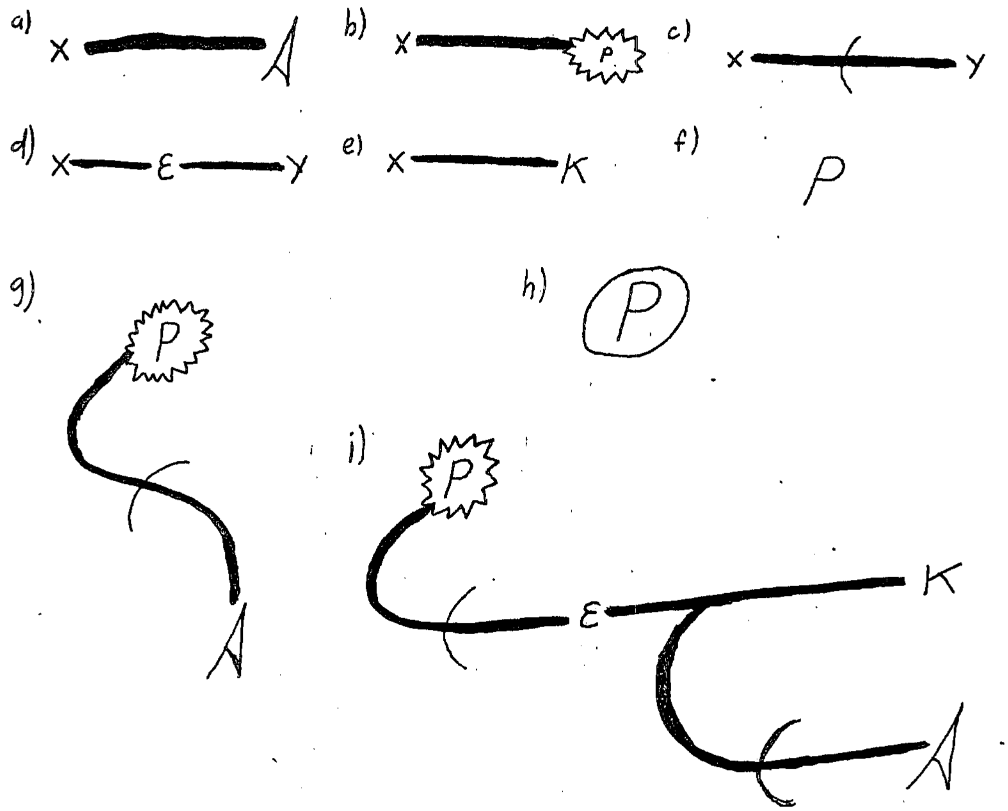
f) Something is not P

g) P and Q are not the same individual

h) Something is P and Q and R and S

i) Something is P and R and something is Q and S

Diagram LXXXI The Gamma Part of Existential Graphs - Metagraphs



a) x is the sheet of assertion

b) x is a graph precisely expressing "P"

c) x is scribed on y

d) x is the area of y

e) x is a cut

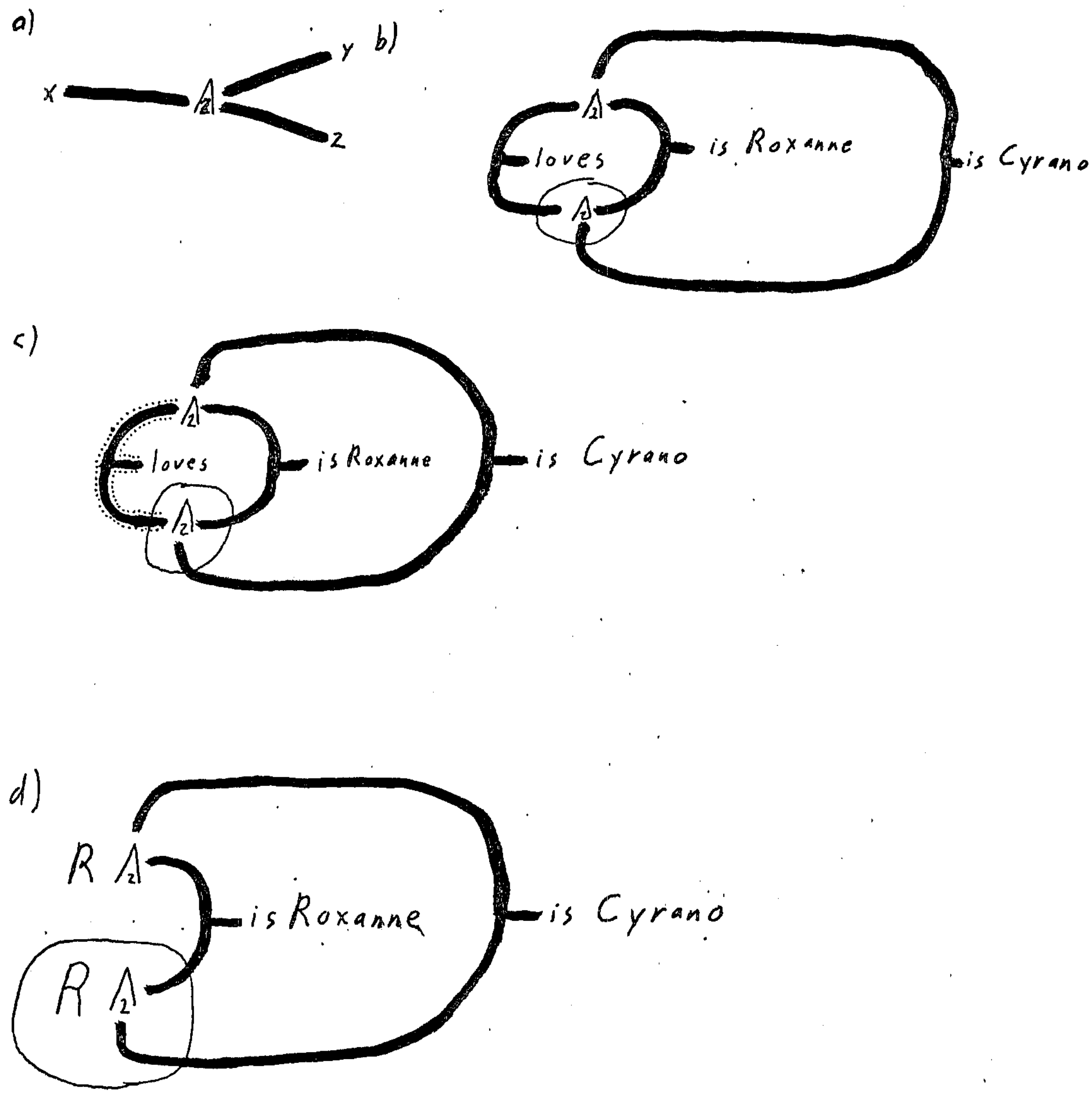
f) P

g) A graph precisely expressing "P" is scribed on the sheet of assertion

h) Not P

i) A graph precisely expressing "P" is scribed on the area of a cut which is itself scribed on the sheet of assertion

Diagram LXXXII The Gamma Part of Existential Graphs - Abstraction



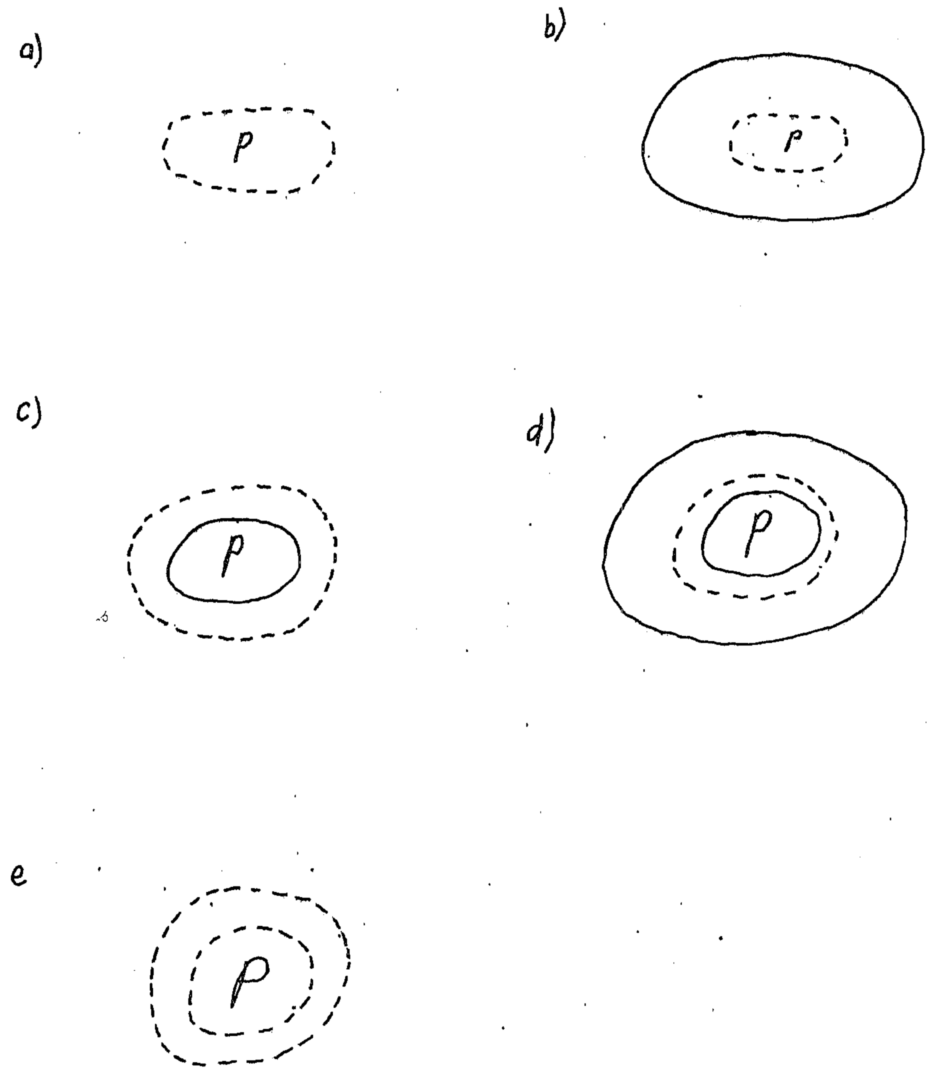
a) y is in relation x to z

b) Cyrano loves Roxanne but Roxanne does not love Cyrano

c) Same as b)

d) Same as b) and c)

Diagram LXXXIII   The Gamma Part of Existential Graphs - Modality

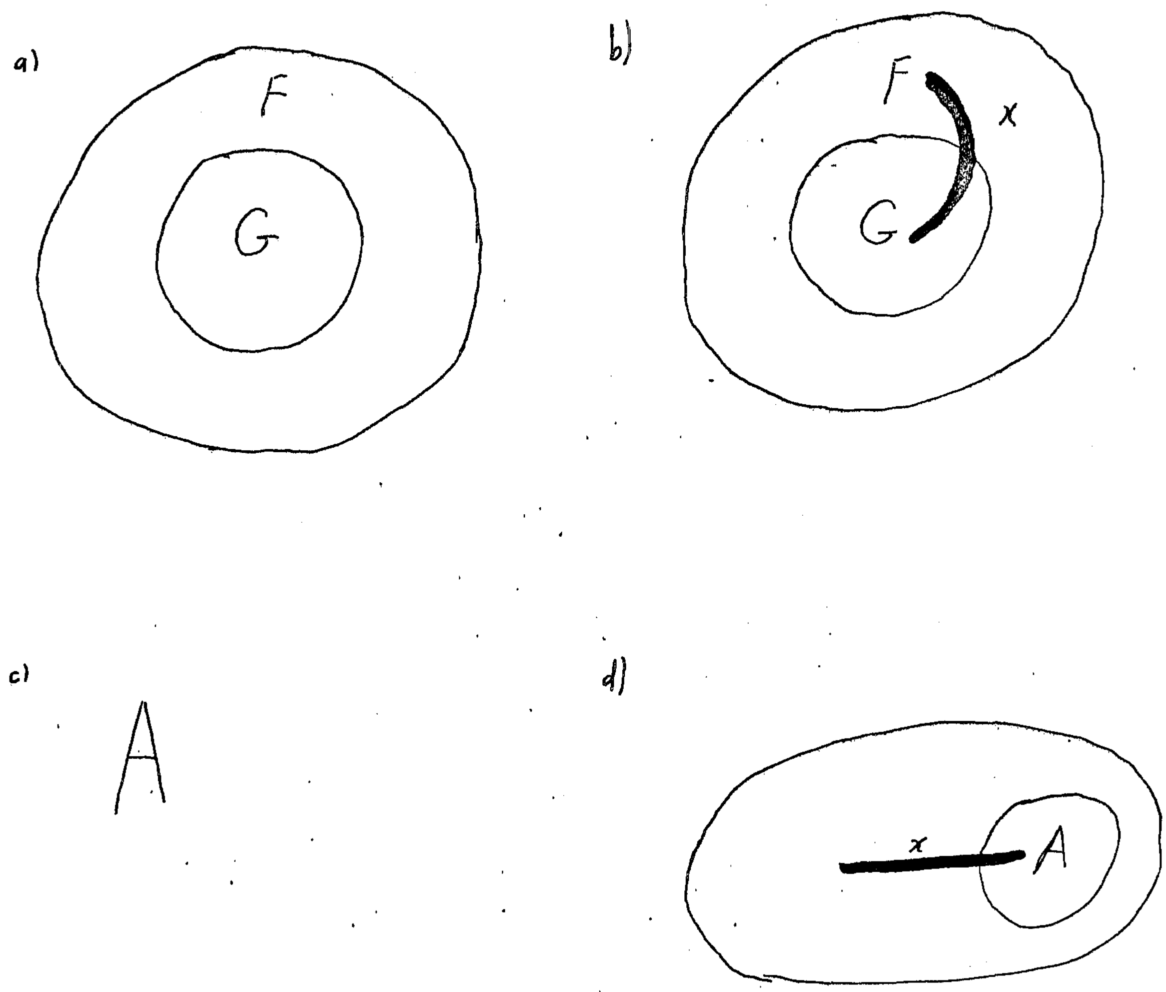


a) It is possible that P is false   b) P is necessarily true

c) It is possible that P is true   d) P is necessarily false

e) It is possible that P is necessary

Diagram LXXXIV Robert's Reinterpretation of the Beta Part of Existential Graphs

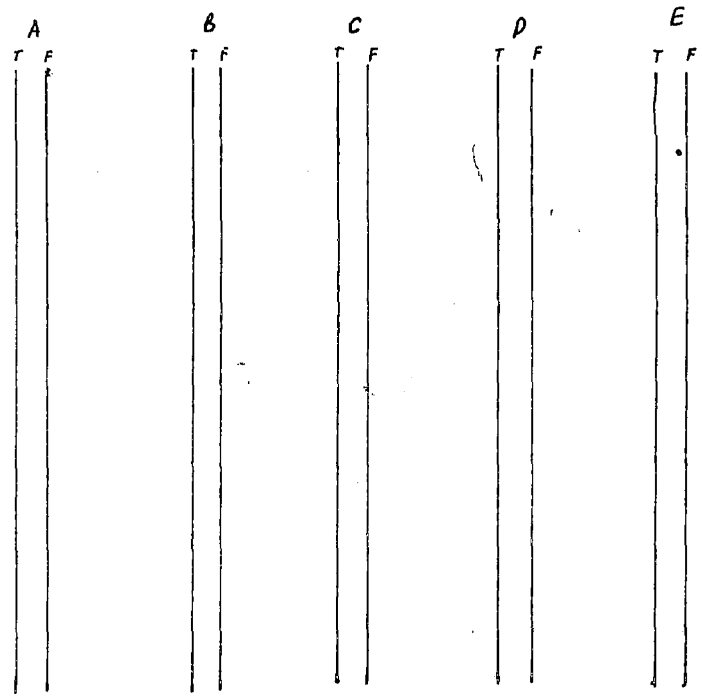


a) $Fx \supset Gx$ b) $(\forall x)(Fx \supset Gx)$

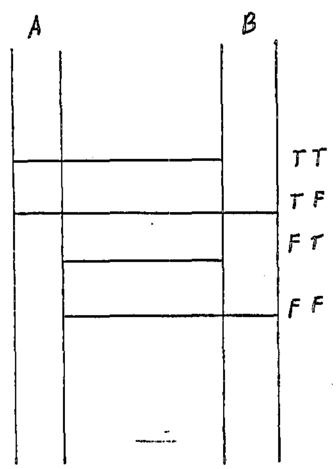
c) A d) $\sim(\forall x)\sim(Ax)$

Diagram LXXXV   Gardner's Basic Diagrams

a)



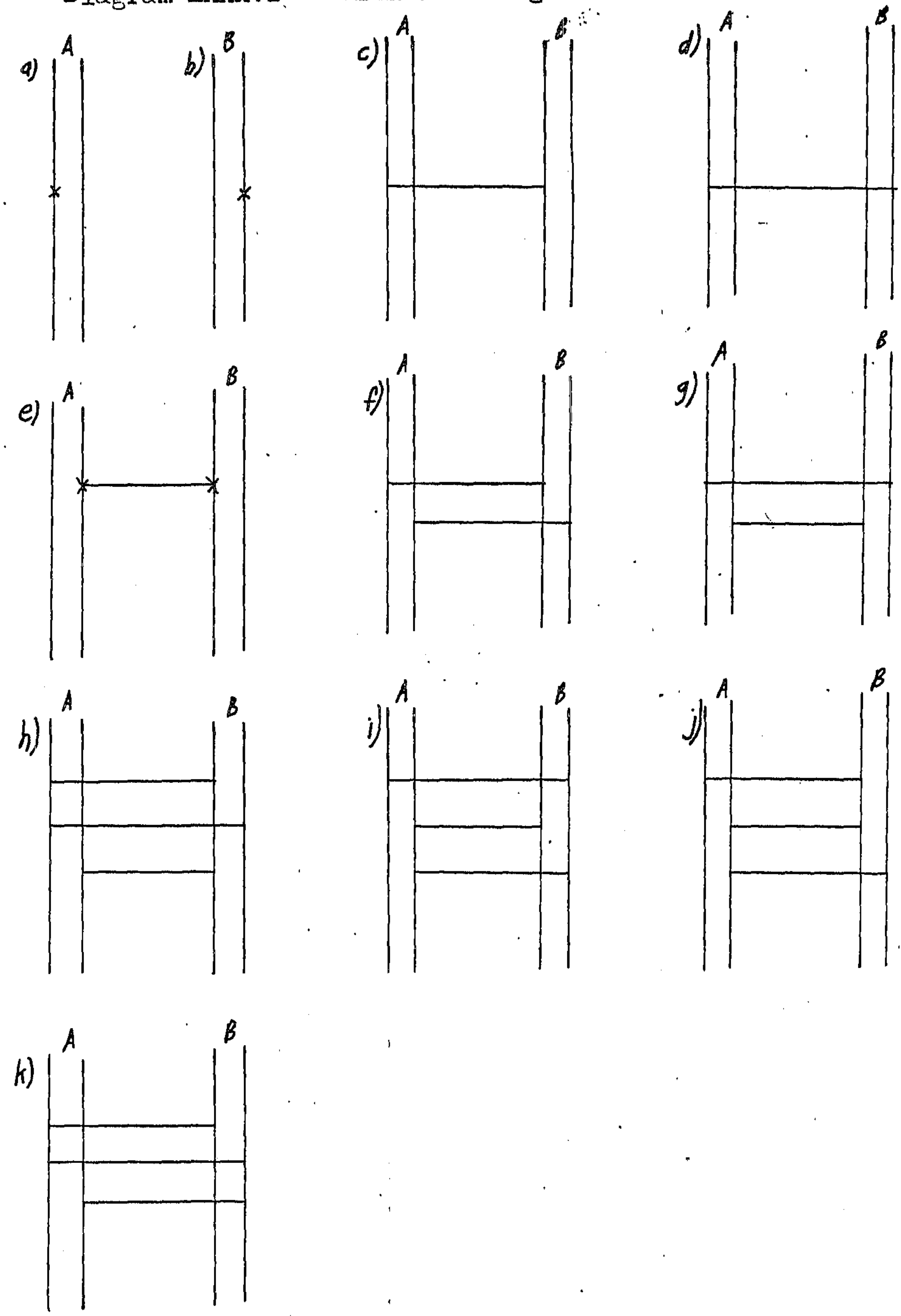
b)



a) Five term diagram

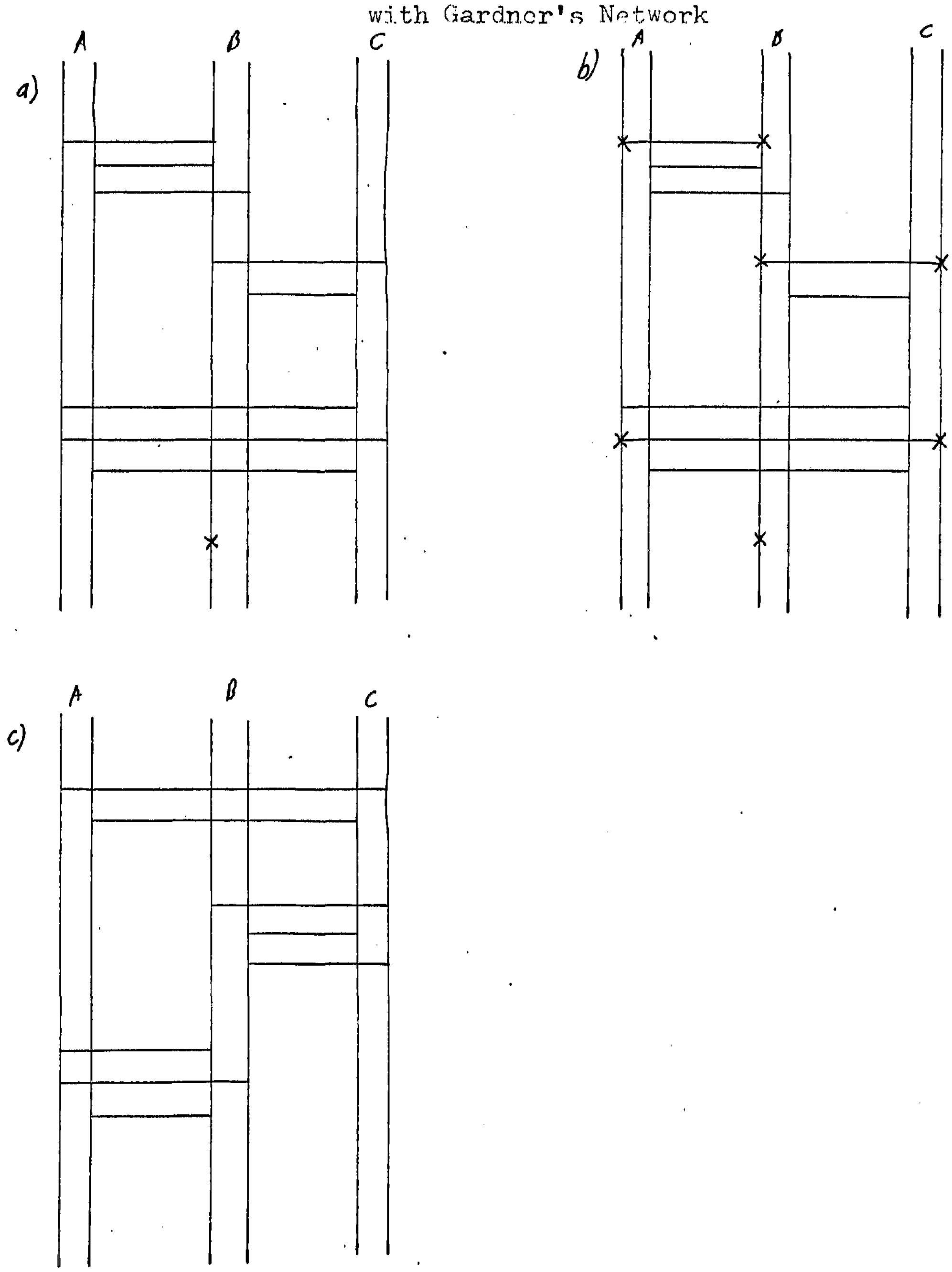
b) Diagram representing two term truth table

Diagram LXXXVI Gardner's Diagrams of Basic Propositions



a) A	b) ~B	c) A.B
d) A.~B	e) ~A.B	f) $A \equiv B$
g) $A \not\equiv B$	h) AvB	i) A|B
j) $A \supset B$	k) $B \supset A$	

Diagram LXXXVII   Solution of Simple Problems
with Gardner's Network

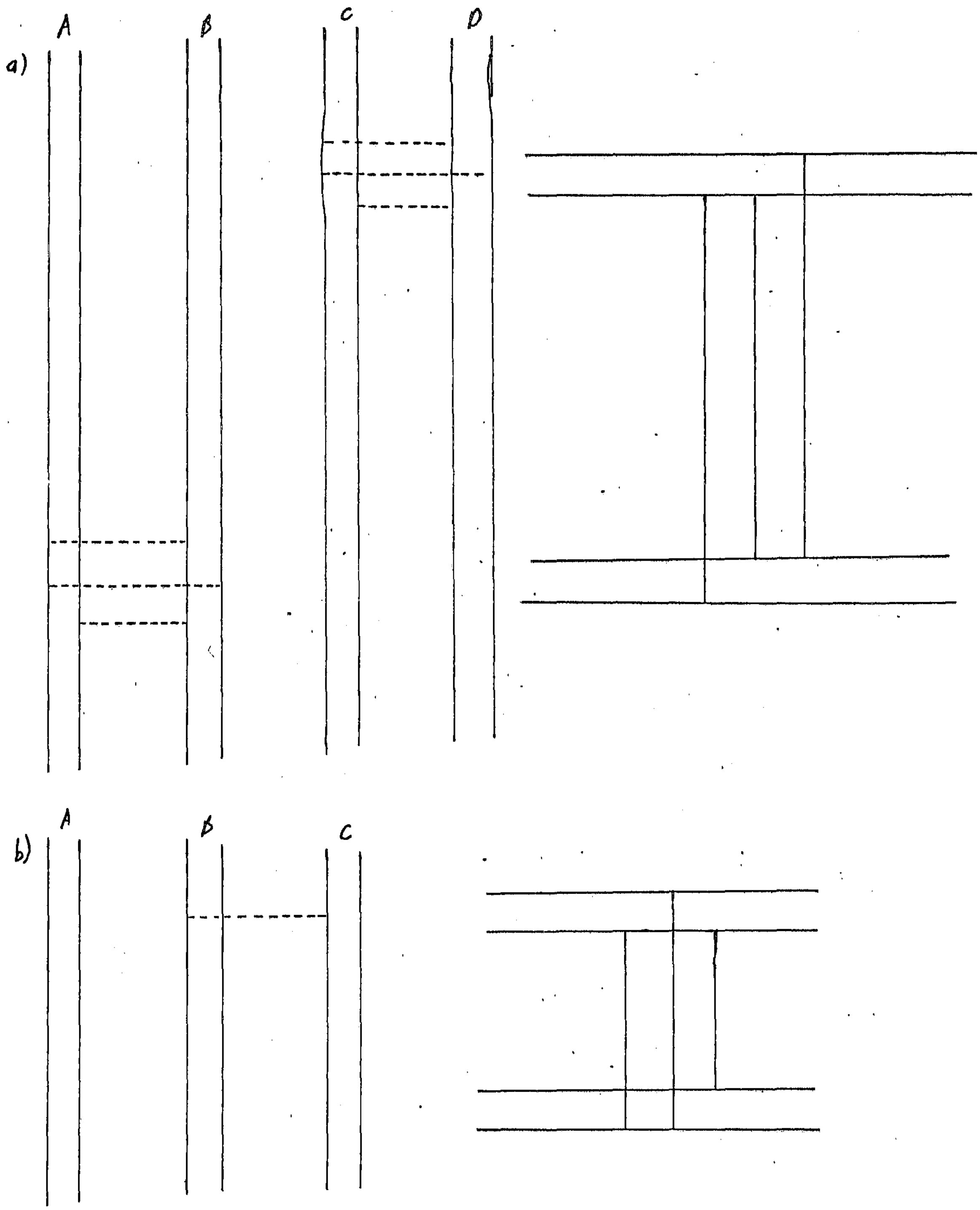


a) $A \supset B$
$B \not\equiv C$
$A \vee C$
B
Initial diagram

b) Same as a) depicting solution

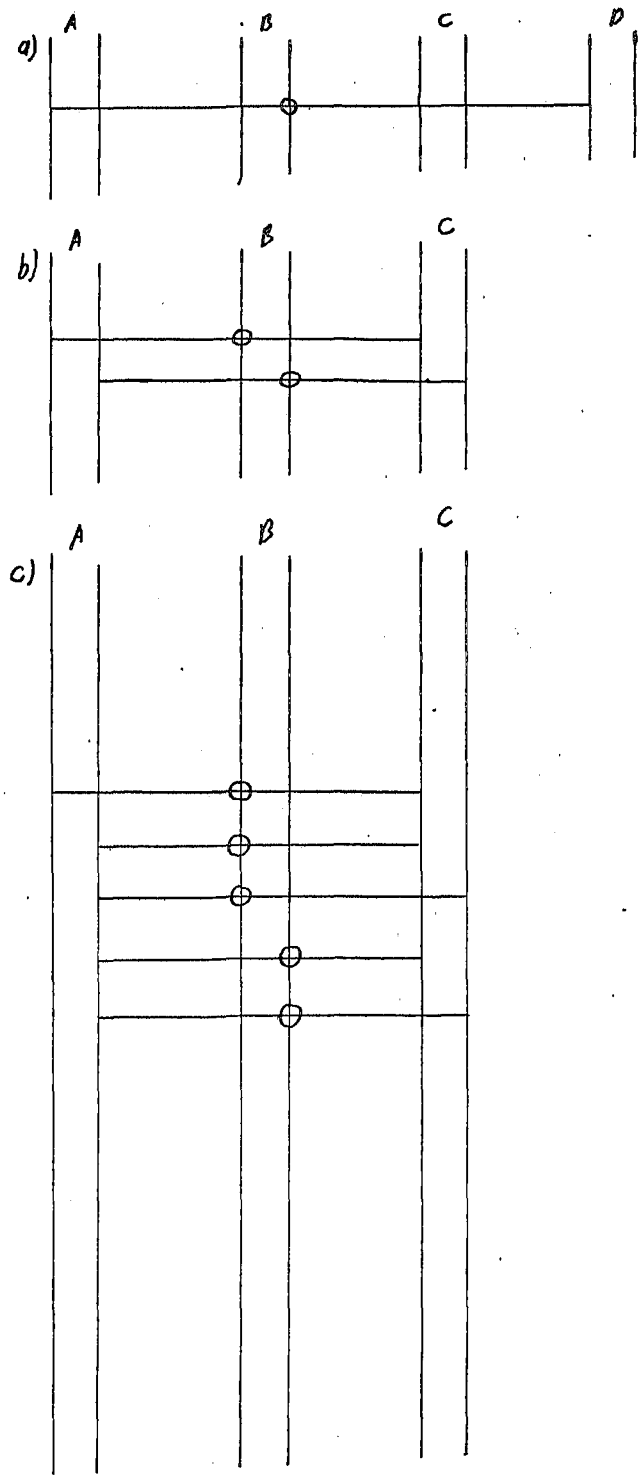
c) $A \not\equiv C$
$B | C$
$A \vee B$

Diagram LXXXVIII Gardner's Diagrams for Compound Statements Using Horizontal Truth-value Lines



a) (AvB)⊃(CvD) b) Av(B.C)

Diagram LXXXIX Gardner's Diagrams for Compound Statements Using Chains

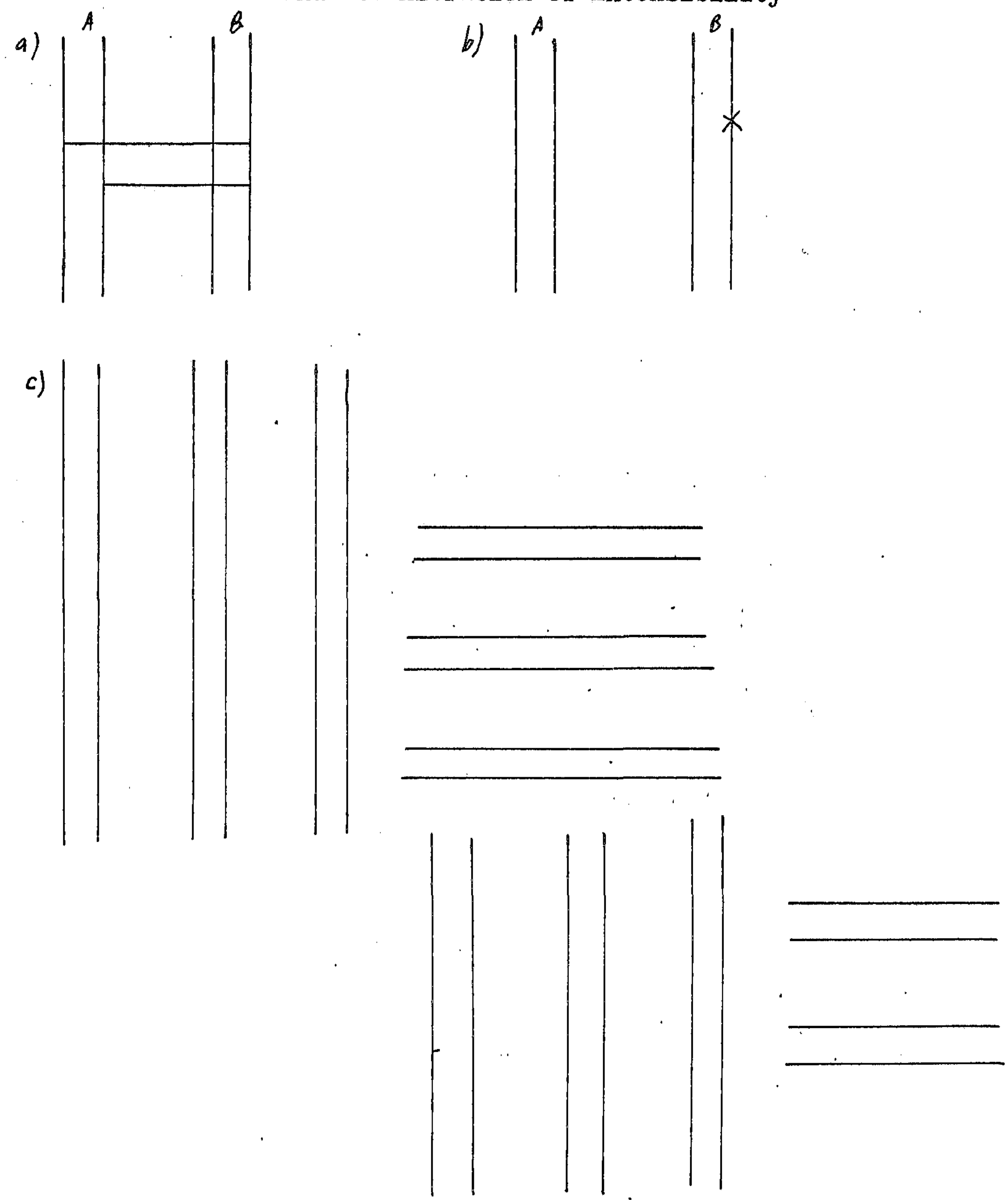


a) A.~B.D

b) A≡B≡C

c) A⊃(B.C)

Diagram XC Gardner's Method of Minimizing and Demonstration of Extensibility

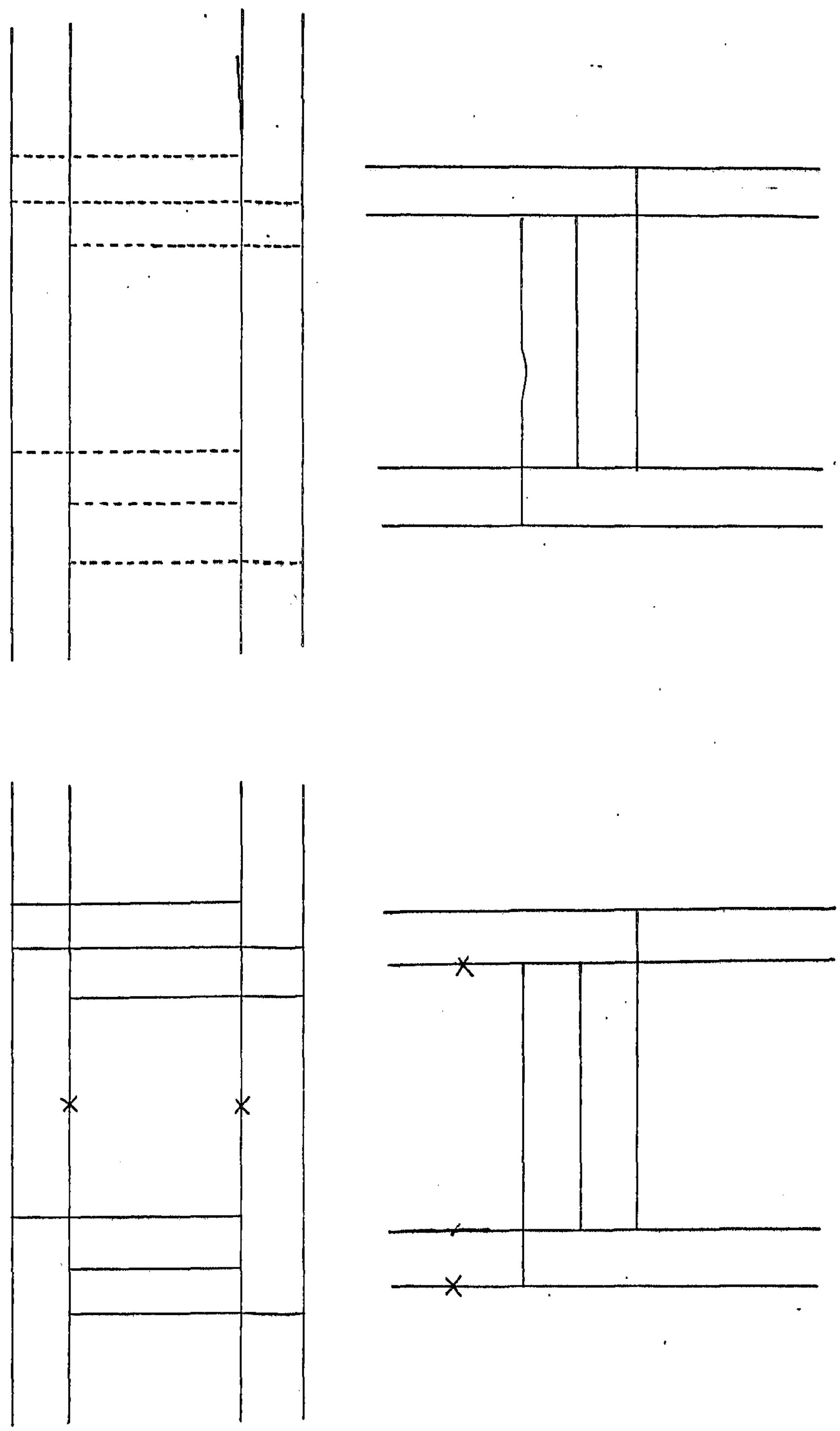


a) (A.~B)v(~A.~B)

b) ~B [i.e. a) reduced]

c) Method of extensibility

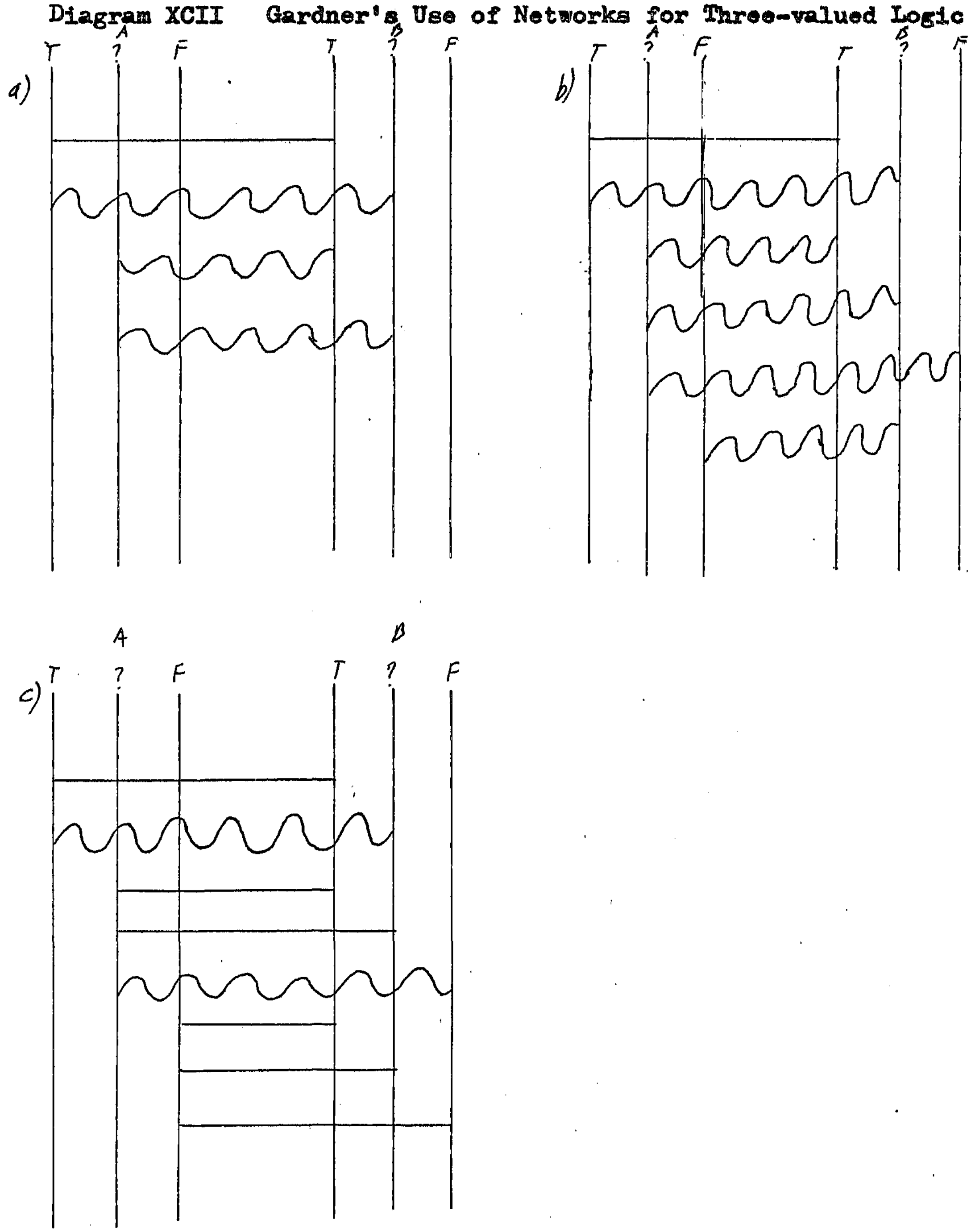
Diagram XCI Gardner's Solution of Complex Problems



a) (A⊃B)⊃(B⊃A)

b) Proof that a) is not a valid theorem

Diagram XCII   Gardner's Use of Networks for Three-valued Logic

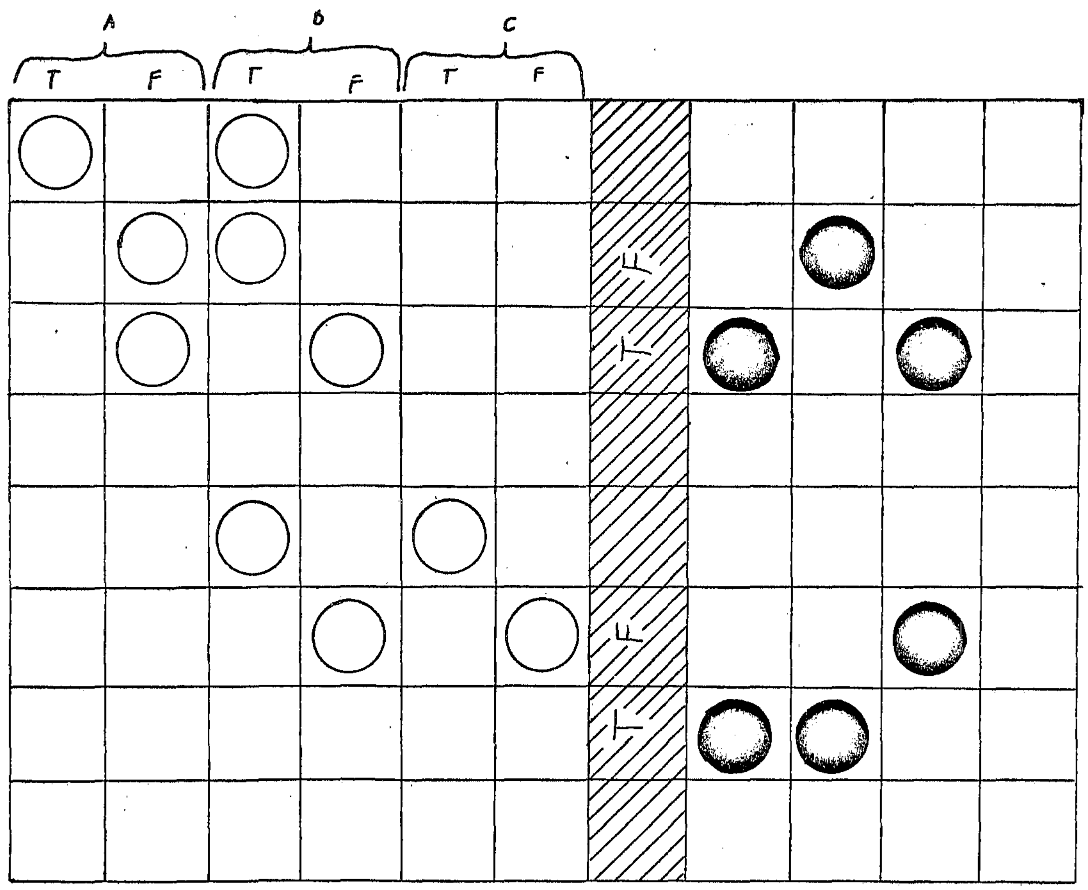


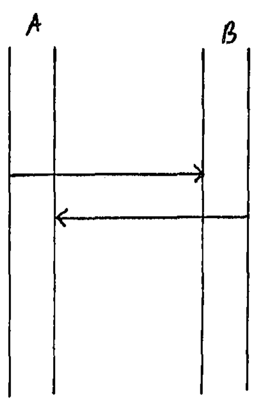
a) A.B in Lukasiewicz, Post and Rosser

b) A.B in Bochvar

c) A⊃B in Lukasiewicz and Tarski

Diagram XCIII Further Expansions of Gardner's System

a)


b)


a) (A⊃B)v(B≡C) using counters b) A⊃B using vectors

Diagram XCIV Hamilton's Scheme of the Two Quantities

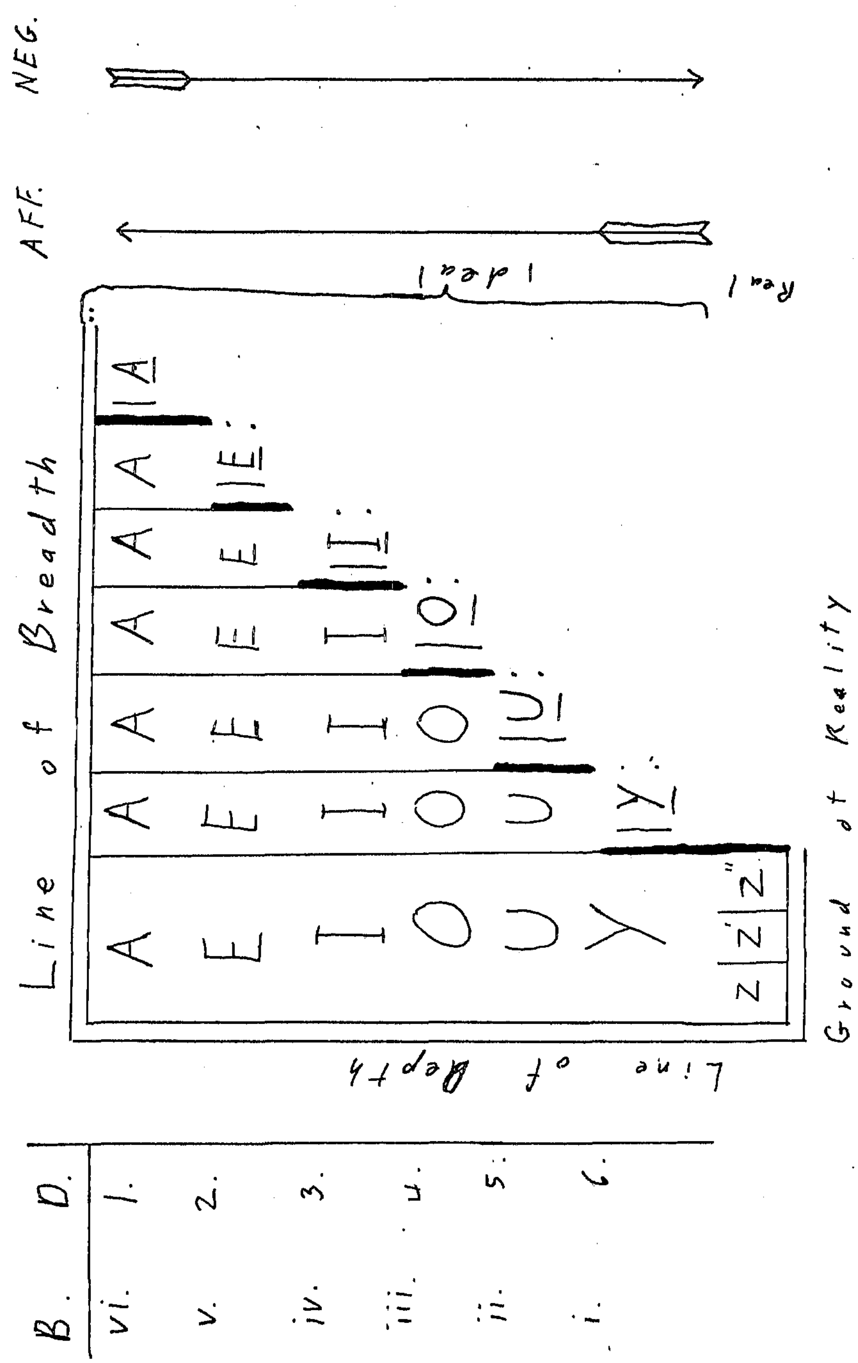
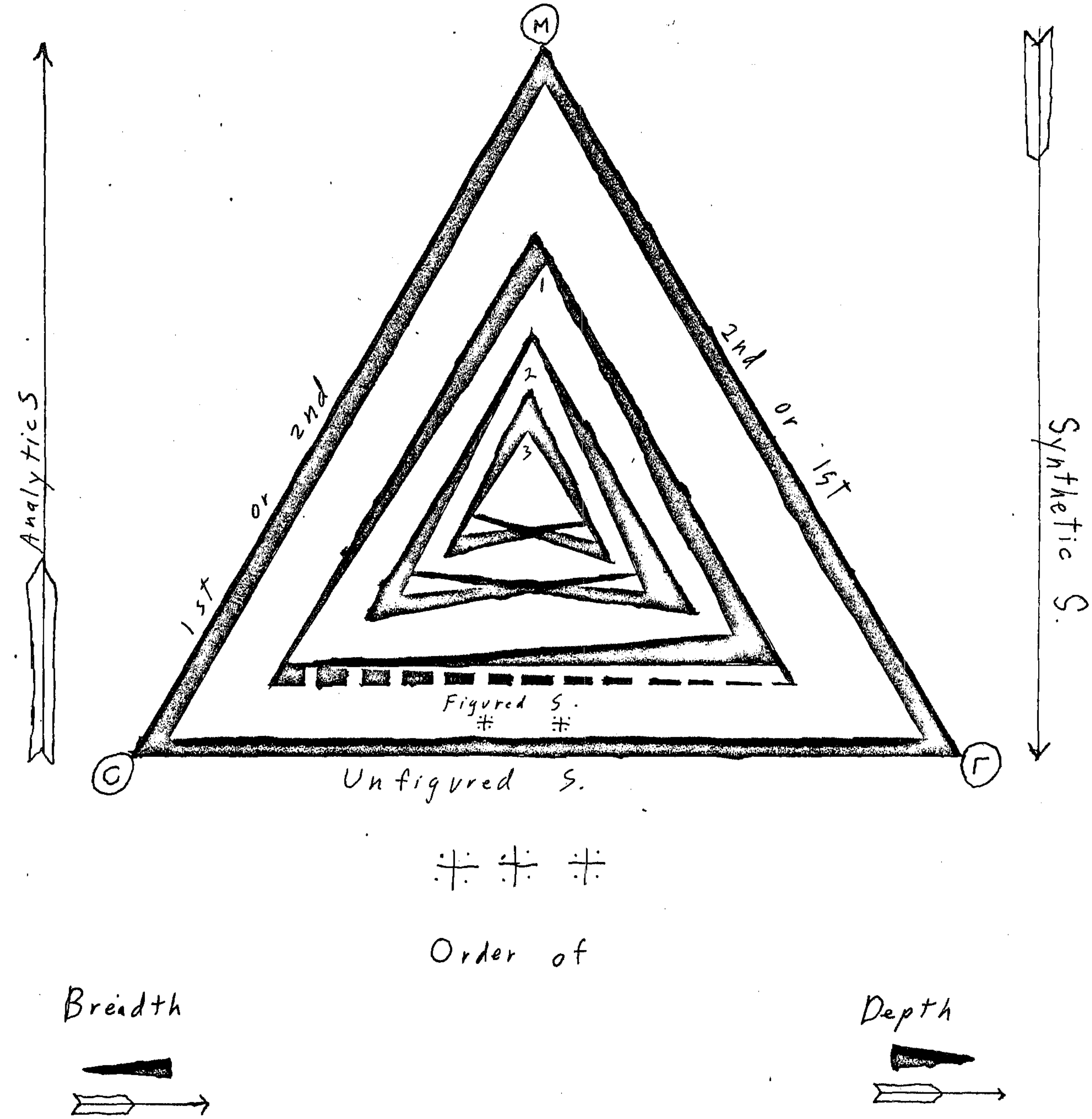


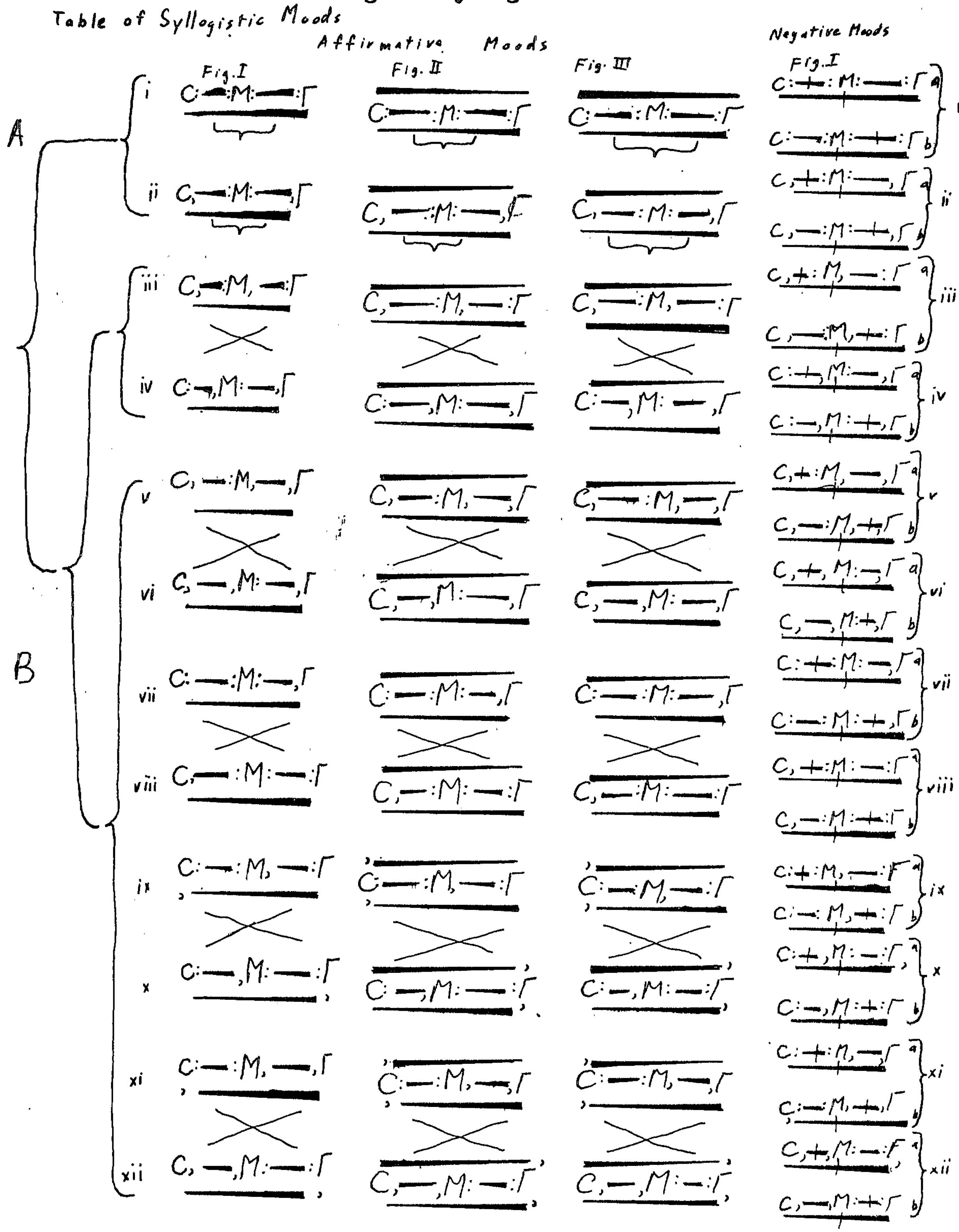
Table of the scheme of the two quantities

Diagram XCV **Condensed View of Hamilton's Notation**



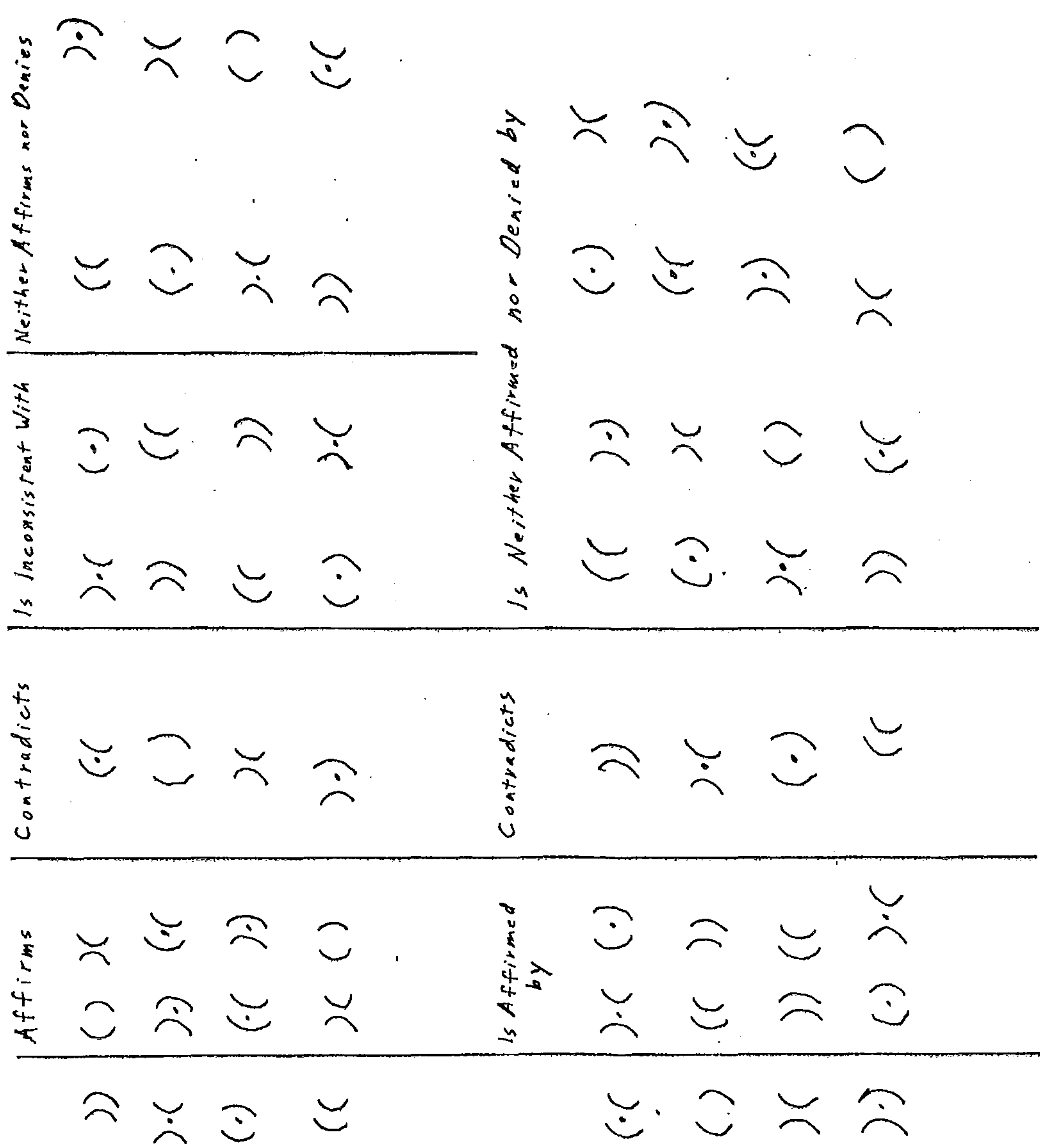
Condensation of Hamilton's Scheme of Notation

Diagram XCVI   Hamilton's Scheme of Notation – the Figured Syllogism



Hamilton's Final System

Diagram XCVII de Morgan's Chart of the Relationships of Propositions



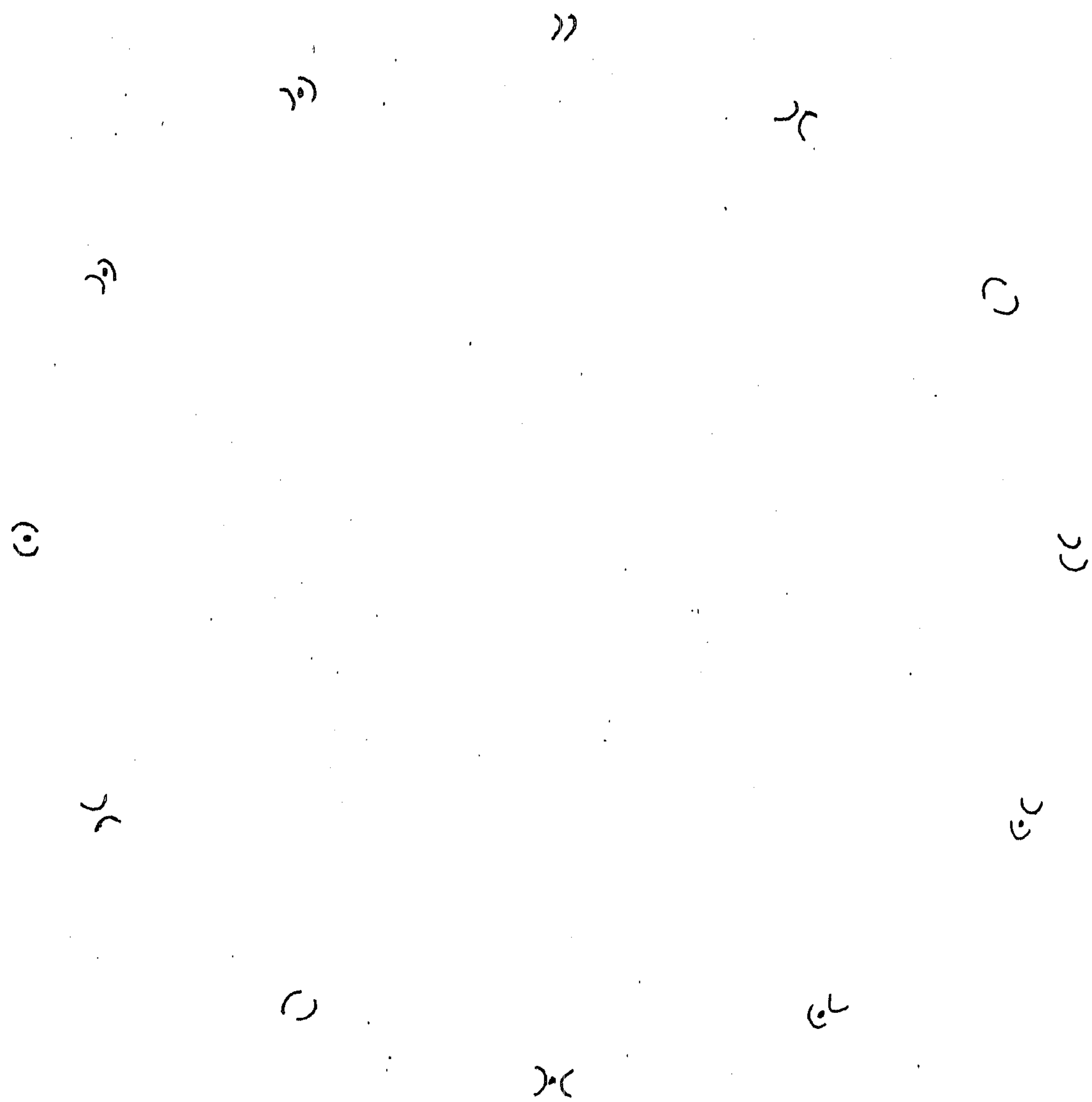
A chart of the relationships holding among propositions

De Morgan's table illustrating the 24 syllogisms

Premises	Strengthened Particular	Minor Major	Particular	Universal	Minor Major	Particular	Strengthened Particular
Affirmative	(( ))	( ) ((	)) ( )	)))) ((((	)))(	)(((	))))
Negative	(·) (·)	(·((·)	(·))·)	)·((·) (·))·(	)·( )·)	(·( )·(	)( )·(
Affirmative Minor	(( )·(	( ) ((	)·((·(	)))·((((·)	)))(	)·) (·)	)) (·)
Affirmative Major	(·) ((	(·((·)	(( )(	)·((((·)))	)·( )·)	( )))	)·( ))

Diagram XCVIII de Morgan's Table of Syllogisms

Diagram IC de Morgan's Logical Zodiac



The logical zodiac

Diagram C Chart of Significant Events in the History of the Logic Diagram Since 1675

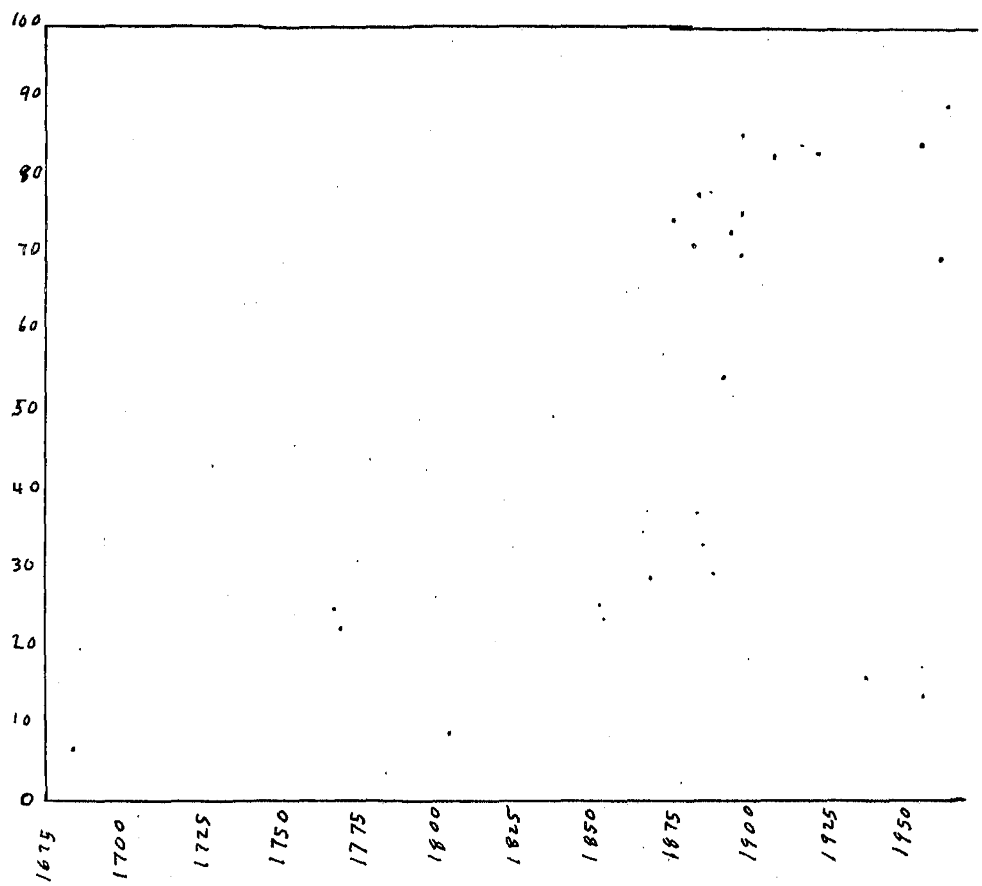
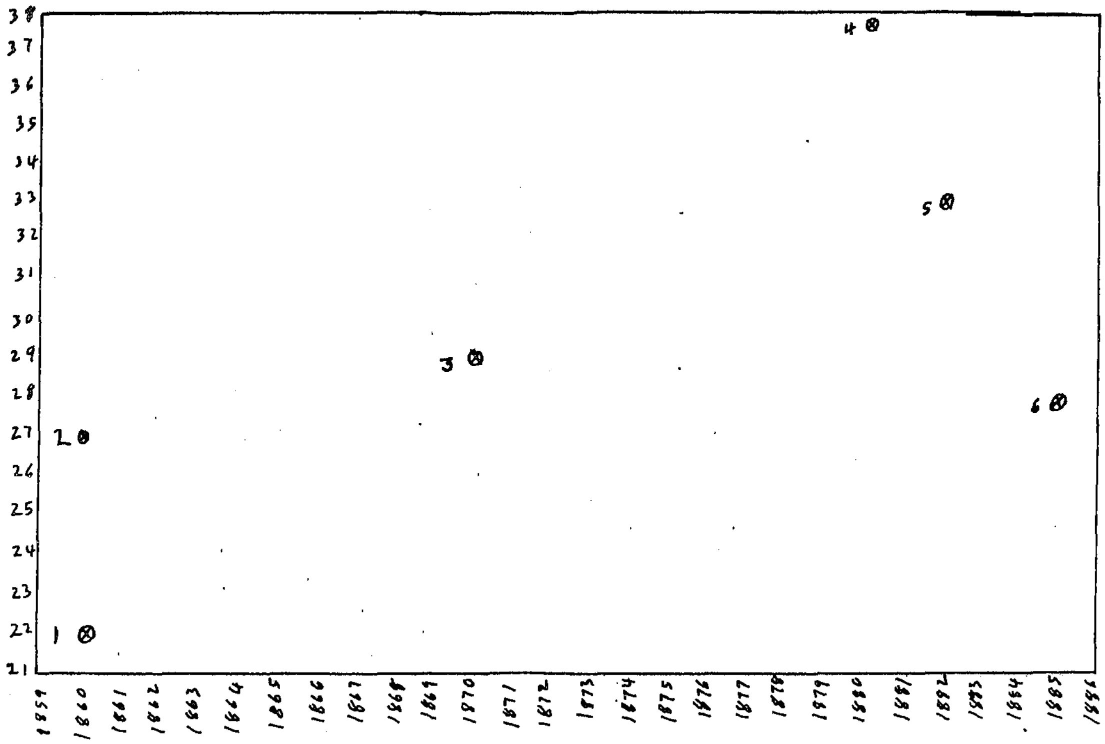


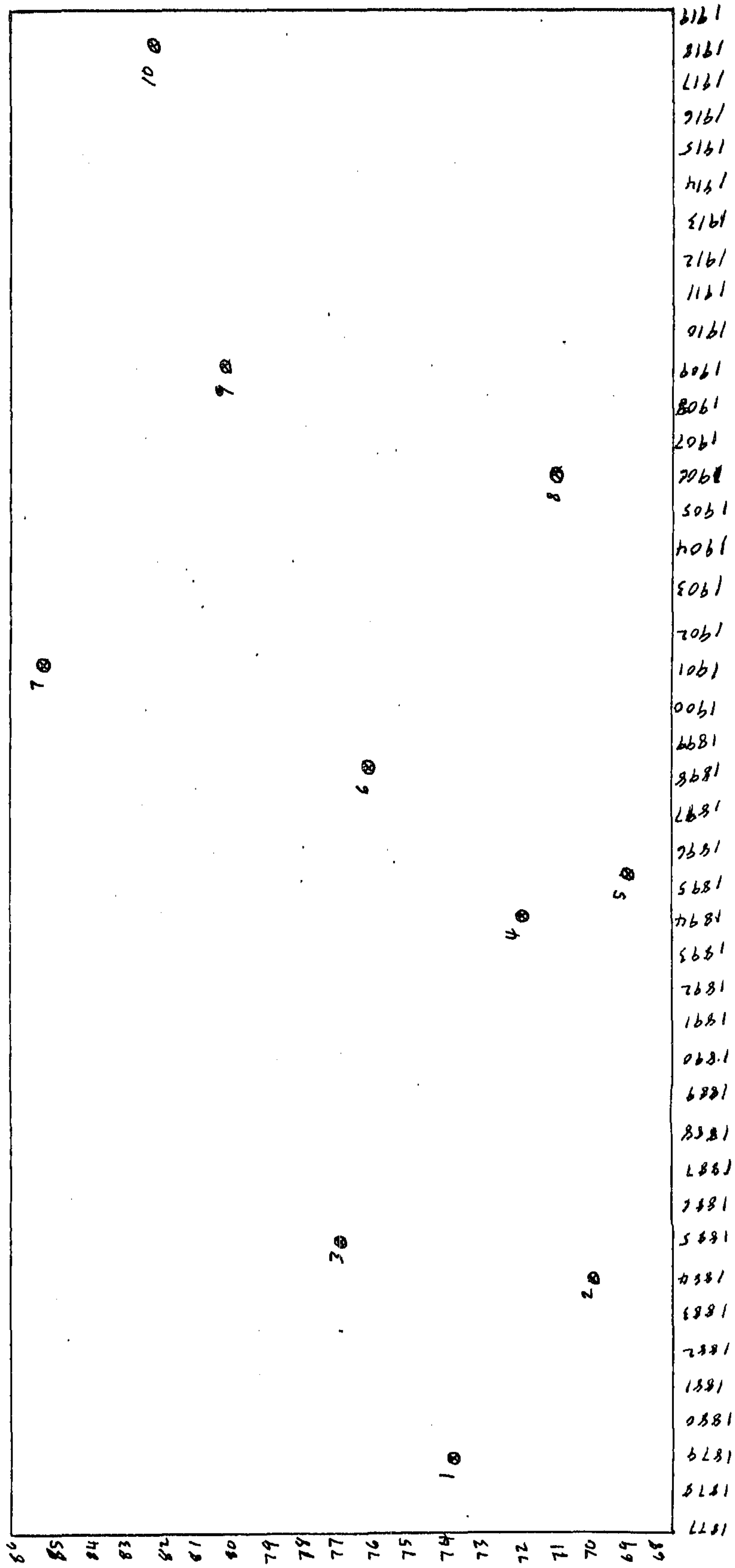
Diagram CI Chart of Significant Events in the History of the Logic Diagram Classed as of Low Value 1859-1887



Key

1. De Morgan
2. Hamilton
3. Jevons 1870
4. Jevons 1880
5. Peirce 1882
6. Carroll 1885

Diagram CII Chart of Significant Events in the History of the Logic Diagram Classed as of High Value 1877-1919



Key

1. Frege
2. Marquand
3. Macfarlane
4. Venn 1894
5. Carroll 1895
6. Peirce 1897
7. Peirce 1900
8. Newlin
9. Hocking
10. Lewis

APPENDIX II

INDEX*

*This index of persons mentioned in the text or footnotes excludes editors and translators.

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Only books and articles actually consulted by the author and mentioned in this paper are listed; books which are mentioned that were unavailable and books consulted but not mentioned are not listed.

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