CSE 5317/4305: Design and Construction of Compilers

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1 Introduction

1.1 What is a Compiler?

A *compiler* is a program that translates a source program written in some high-level programming language (such as Java) into machine code for some computer architecture (such as the Intel Pentium architecture). The generated machine code can be later executed many times against different data each time.

An *interpreter* reads an executable source program written in a high-level programming language as well as data for this program, and it runs the program against the data to produce some results. One example is the Unix shell interpreter, which runs operating system commands interactively.

Note that both interpreters and compilers (like any other program) are written in some high-level programming language (which may be different from the language they accept) and they are translated into machine code. For a example, a Java interpreter can be completely written in C, or even Java. The interpreter source program is machine independent since it does not generate machine code. (Note the difference between *generate* and *translated into* machine code.) An interpreter is generally slower than a compiler because it processes and interprets each statement in a program as many times as the number of the evaluations of this statement. For example, when a for-loop is interpreted, the statements inside the for-loop body will be analyzed and evaluated on every loop step. Some languages, such as Java and Lisp, come with both an interpreter and a compiler. Java source programs (Java classes with `.java` extension) are translated by the `javac` compiler into byte-code files (with `.class` extension). The Java interpreter, called the Java Virtual Machine (JVM), may actually interpret byte codes directly or may internally compile them to machine code and then execute that code (JIT: just-in-time compilation).

Compilers and interpreters are not the only examples of translators. Here are few more:

Source Language	**Translator**	**Target Language**
LaTeX	Text Formater	PostScript
SQL	database query processor	Query Evaluation Plan
Java	javac compiler	Java byte code
Java	cross-compiler	C++ code
English text	Natural Language Understanding	semantics (meaning)
Regular Expressions	JLex scanner generator	a scanner in Java
BNF of a language	CUP parser generator	a parser in Java

This course deals mainly with compilers for high-level programming languages, but the same techniques apply to interpreters or to any other compilation scheme.

1.2 What is the Challenge?

Many variations:

- many programming languages (eg, Java, C++, Scala)
- many programming paradigms (eg, object-oriented, functional, logic)

- many computer architectures (eg, x86, MIPS, ARM, alpha, PowerPC)
- many operating systems (eg, Linux, OS X, Microsoft Windows, Android, iOS)

Qualities of a compiler (in order of importance):

1. the compiler itself must be bug-free
2. it must generate correct machine code
3. the generated machine code must run fast
4. the compiler itself must run fast (compilation time must be proportional to program size)
5. the compiler must be portable (ie, modular, supporting separate compilation)
6. it must print good diagnostics and error messages
7. the generated code must work well with existing debuggers
8. must have consistent and predictable optimization.

Building a compiler requires knowledge of

- programming languages (parameter passing, variable scoping, memory allocation, etc)
- theory (automata, context-free languages, etc)
- algorithms and data structures (hash tables, graph algorithms, dynamic programming, etc)
- computer architecture (assembly programming)
- software engineering.

The course project (building a non-trivial compiler for a Pascal-like language) will give you a hands-on experience on system implementation that combines all this knowledge.

1.3 Compiler Architecture

A compiler can be viewed as a program that accepts a source code (such as a Java program) and generates machine code for some computer architecture. Suppose that you want to build compilers for n programming languages (eg, Scala, C, C++, Java, etc) and you want these compilers to run on m different architectures (eg, MIPS, x86, ARM, etc). If you do that naively, you need to write $n * m$ compilers, one for each language-architecture combination.

The holly grail of portability in compilers is to do the same thing by writing $n + m$ programs only. How? You use a universal *Intermediate Representation* (*IR*) and you make the compiler a two-phase compiler. An IR is typically a tree-like data structure that captures the basic features of most computer architectures. One example of an IR tree node is a representation of a 3-address instruction, such as $d \leftarrow s_1 + s_2$, that gets two source addresses,

s_1 and s_2, (ie. two IR trees) and produces one destination address, d. The first phase of this compilation scheme, called the *front-end*, maps the source code into IR, and the second phase, called the *back-end*, maps IR into machine code. That way, for each programming language you want to compile, you write one front-end only, and for each target computer architecture, you write one back-end. So, totally you have $n + m$ components.

But the above ideal separation of compilation into two phases does not work very well for existing programming languages and architectures. Ideally, you must encode all knowledge about the source programming language in the front end, you must handle all machine architecture features in the back end, and you must design your IRs in such a way that all language and machine features are captured properly.

A typical real-world compiler usually has multiple phases. This increases the compiler's portability and simplifies retargeting. The front end consists of the following phases:

1. *scanning*: a scanner groups input characters into tokens;

2. *parsing*: a parser recognizes sequences of tokens according to some grammar and generates *Abstract Syntax Trees* (ASTs);

3. *semantic analysis*: performs *type checking* (ie, checking whether the variables, functions etc in the source program are used consistently with their definitions and with the language semantics) and translates ASTs into IRs;

4. *optimization*: optimizes IRs.

The back end consists of the following phases:

1. *instruction selection*: maps IRs into assembly code;

2. *code optimization*: optimizes the assembly code using control-flow and data-flow analyses, register allocation, etc;

3. *code emission*: generates machine code from assembly code.

The generated machine code is written in an object file. This file is not executable since it may refer to external symbols (such as system calls). The operating system provides the following utilities to execute the code:

1. *linking*: A linker takes several object files and libraries as input and produces one executable object file. It retrieves from the input files (and puts them together in the executable object file) the code of all the referenced functions/procedures and it resolves all external references to real addresses. The libraries include the operating sytem libraries, the language-specific libraries, and, maybe, user-created libraries.

2. *loading*: A loader loads an executable object file into memory, initializes the registers, heap, data, etc and starts the execution of the program.

Relocatable shared libraries allow effective memory use when many different applications share the same code.

2 Lexical Analysis

A *scanner* groups input characters into tokens. For example, if the input is

```
x = x*(b+1);
```

then the scanner generates the following sequence of tokens:

```
id(x)
=
id(x)
*
(
id(b)
+
num(1)
)
;
```

where `id(x)` indicates the identifier with name `x` (a program variable in this case) and `num(1)` indicates the integer `1`. Each time the parser needs a token, it sends a request to the scanner. Then, the scanner reads as many characters from the input stream as it is necessary to construct a single token. The scanner may report an error during scanning (eg, when it finds an end-of-file in the middle of a string). Otherwise, when a single token is formed, the scanner is suspended and returns the token to the parser. The parser will repeatedly call the scanner to read all the tokens from the input stream or until an error is detected (such as a syntax error).

Tokens are typically represented by numbers. For example, the token `*` may be assigned the number 35. Some tokens require some extra information. For example, an identifier is a token (so it is represented by some number) but it is also associated with a string that holds the identifier name. For example, the token `id(x)` is associated with the string, `"x"`. Similarly, the token `num(1)` is associated with the number, 1.

Tokens are specified by patterns, called *regular expressions*. For example, the regular expression `[a-z][a-zA-Z0-9]*` recognizes all identifiers with at least one alphanumeric letter whose first letter is lower-case alphabetic. A typical scanner:

1. recognizes the *keywords* of the language (these are the reserved words that have a special meaning in the language, such as the word `class` in Java);

2. recognizes special characters, such as `(` and `)`, or groups of special characters, such as `:=` and `==`;

3. recognizes identifiers, integers, reals, decimals, strings, etc;

4. ignores whitespaces (tabs and blanks) and comments;

5. recognizes and processes special directives (such as the `#include "file"` directive in C) and macros.

A key issue is speed. One can always write a naive scanner that groups the input characters into lexical words (a lexical word can be either a sequence of alphanumeric characters without whitespaces or special characters, or just one special character), and then can try to associate a token (ie. number, keyword, identifier, etc) to this lexical word by performing a number of string comparisons. This becomes very expensive when there are many keywords and/or many special lexical patterns in the language. In this section you will learn how to build efficient scanners using regular expressions and finite automata. There are automated tools called *scanner generators*, such as *flex* for C and *JLex* for Java, which construct a fast scanner automatically according to specifications (regular expressions). You will first learn how to specify a scanner using regular expressions, then the underlying theory that scanner generators use to compile regular expressions into efficient programs (which are basically finite state machines), and then you will learn how to use a scanner generator for Java, called JLex.

2.1 Regular Expressions (REs)

Regular expressions are a very convenient form of representing (possibly infinite) sets of strings, called *regular sets*. For example, the RE $(a|b)*aa$ represents the infinite set $\{$"*aa*", "*aaa*", "*baa*", "*abaa*", ...$\}$, which is the set of all strings with characters a and b that end in aa. Formally, a RE is one of the following (along with the set of strings it designates):

name	RE	designation
epsilon	ε	$\{$""$\}$
symbol	a	$\{$"a"$\}$ for some character a
concatenation	AB	the set $\{\, rs \mid r \in \mathcal{A},\ s \in \mathcal{B}\,\}$, where rs is string concatenation, and $\mathcal{A}$ and $\mathcal{B}$ designate the REs A and B
alternation	$A\|B$	the set $\mathcal{A} \cup \mathcal{B}$, where $\mathcal{A}$ and $\mathcal{B}$ designate the REs A and B
repetition	A^*	the set $\epsilon\|A\|(AA)\|(AAA)\|\cdots$ (an infinite set)

Repetition is also called Kleen closure. For example, $(a|b)c$ designates the set $\{\, rs \mid r \in (\{\text{"}a\text{"}\} \cup \{\text{"}b\text{"}\}),\ s \in \{\text{"}c\text{"}\}\,\}$, which is equal to $\{$"*ac*", "*bc*"$\}$.

We will use the following notational conveniences:

$$\begin{array}{lcl} P^+ & = & PP^* \\ P? & = & P \mid \varepsilon \\ [a-z] & = & a|b|c|\cdots|z \end{array}$$

We can freely put parentheses around REs to denote the order of evaluation. For example, $(a|b)c$. To avoid using many parentheses, we use the following rules: concatenation and alternation are associative (ie, ABC means $(AB)C$ and is equivalent to $A(BC)$), alternation is commutative (ie, $A|B = B|A$), repetition is idempotent (ie, $A^{**} = A^*$), and concatenation distributes over alternation (eg, $(a|b)c = ac|bc$).

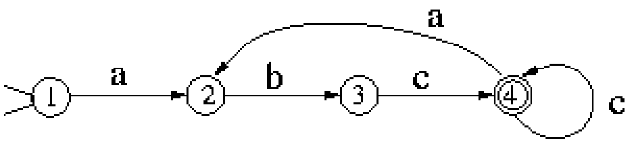
For convenience, we can give names to REs so we can refer to them by their name. For example:

$$\begin{aligned}
&\textit{for-keyword} &&= \quad for\\
&\textit{letter} &&= \quad [\,a-zA-Z\,]\\
&\textit{digit} &&= \quad [\,0-9\,]\\
&\textit{identifier} &&= \quad \textit{letter}\,(\textit{letter}\,|\,\textit{digit})^*\\
&\textit{sign} &&= \quad +|-|\,\varepsilon\\
&\textit{integer} &&= \quad \textit{sign}\,(0\,|\,[\,1-9\,]\,\textit{digit}^*)\\
&\textit{decimal} &&= \quad \textit{integer}\,.\,\textit{digit}^*\\
&\textit{real} &&= \quad (\textit{integer}\,|\,\textit{decimal})\;E\;\textit{sign}\,\textit{digit}^+
\end{aligned}$$

There is some ambiguity though: If the input includes the characters `for8`, then the first rule (for *for-keyword*) matches 3 characters (`for`), the fourth rule (for *identifier*) can match 1, 2, 3, or 4 characters, the longest being `for8`. To resolve this type of ambiguities, when there is a choice of rules, scanner generators choose the one that matches the maximum number of characters. In this case, the chosen rule is the one for *identifier* that matches 4 characters (`for8`). This disambiguation rule is called the *longest match rule*. If there are more than one rules that match the same maximum number of characters, the rule listed first is chosen. This is the *rule priority* disambiguation rule. For example, the lexical word `for` is taken as a *for-keyword* even though it uses the same number of characters as an identifier.

2.2 Deterministic Finite Automata (DFAs)

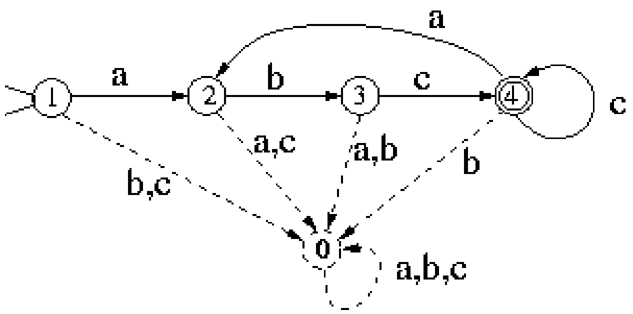
A DFA represents a finite state machine that recognizes a RE. For example, the following DFA:



recognizes $(abc^+)^+$. A finite automaton consists of a finite set of states, a set of transitions (moves), one start state, and a set of final states (accepting states). In addition, a DFA has a unique transition for every state-character combination. For example, the previous figure has 4 states, state 1 is the start state, and state 4 is the only final state.

A DFA accepts a string if starting from the start state and moving from state to state, each time following the arrow that corresponds the current input character, it reaches a final state when the entire input string is consumed. Otherwise, it rejects the string.

The previous figure represents a DFA even though it is not complete (ie, not all state-character transitions have been drawn). The complete DFA is:



but it is very common to ignore state 0 (called the *error state*) since it is implied. (The arrows with two or more characters indicate transitions in case of any of these characters.) The error state serves as a black hole, which doesn't let you escape.

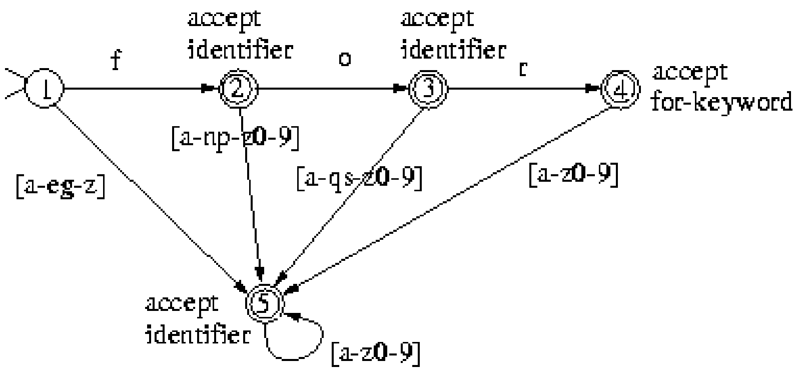
A DFA is represented by a *transition table* T, which gives the next state $T[s, c]$ for a state s and a character c. For example, the T for the DFA above is:

	a	b	c
0	0	0	0
1	2	0	0
2	0	3	0
3	0	0	4
4	2	0	4

Suppose that we want to build a scanner for the REs:

$$\begin{array}{lcl} \textit{for-keyword} & = & for \\ identifier & = & [a-z][a-z0-9]^* \end{array}$$

The corresponding DFA has 4 final states: one to accept the *for-keyword* and 3 to accept an identifier:



(the error state is omitted again). Notice that for each state and for each character, there is a single transition.

A scanner based on a DFA uses the DFA's transition table as follows:

```
state = initial_state;
current_character = get_next_character();
```

```
while ( true )
{   next_state = T[state,current_character];
    if (next_state == ERROR)
       break;
    state = next_state;
    current_character = get_next_character();
    if ( current_character == EOF )
       break;
};
if ( is_final_state(state) )
   ‘we have a valid token’
else ‘report an error’
```

This program does not explicitly take into account the longest match disambiguation rule since it ends at EOF. The following program is more general since it does not expect EOF at the end of token but still uses the longest match disambiguation rule.

```
state = initial_state;
final_state = ERROR;
current_character = get_next_character();
while ( true )
{   next_state = T[state,current_character];
    if (next_state == ERROR)
       break;
    state = next_state;
    if ( is_final_state(state) )
       final_state = state;
    current_character = get_next_character();
    if (current_character == EOF)
       break;
};
if ( final_state == ERROR )
   ‘report an error’
else if ( state != final_state )
   ‘we have a valid token but we need to backtrack
          (to put characters back into the input stream)’
else ‘we have a valid token’
```

Is there any better (more efficient) way to build a scanner out of a DFA? Yes! We can hardwire the state transition table into a program (with lots of gotos). You’ve learned in your programming language course never to use gotos. But here we are talking about a program generated automatically, which no one needs to look at. The idea is the following. Suppose that you have a transition from state s_1 to s_2 when the current character is c. Then you generate the program:

```
s1: current_character = get_next_character();
```

```
     ...
     if ( current_character == 'c' )
        goto s2;
     ...
s2: current_character = get_next_character();
     ...
```

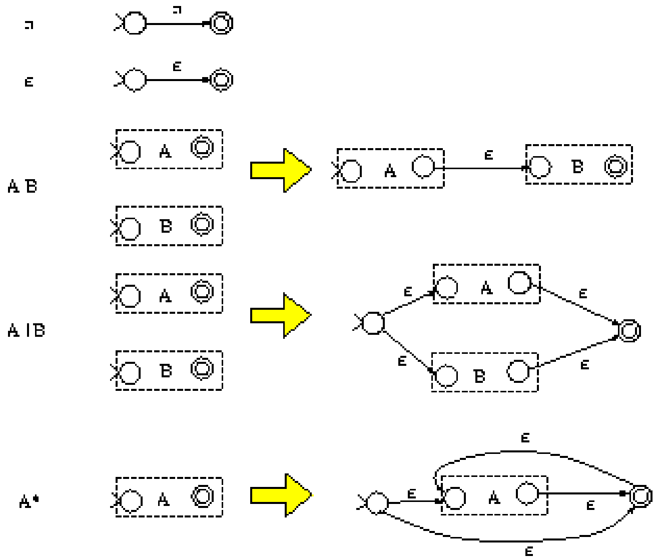
2.3 Converting a Regular Expression into a Deterministic Finite Automaton

The task of a scanner generator, such as JLex, is to generate the transition tables or to synthesize the scanner program given a scanner specification (in the form of a set of REs). So it needs to convert REs into a single DFA. This is accomplished in two steps: first it converts REs into a non-deterministic finite automaton (NFA) and then it converts the NFA into a DFA.

An NFA is similar to a DFA but it also permits multiple transitions over the same character and transitions over ε. In the case of multiple transitions from a state over the same character, when we are at this state and we read this character, we have more than one choice; the NFA succeeds if at least one of these choices succeeds. The ε transition doesn't consume any input characters, so you may jump to another state for free.

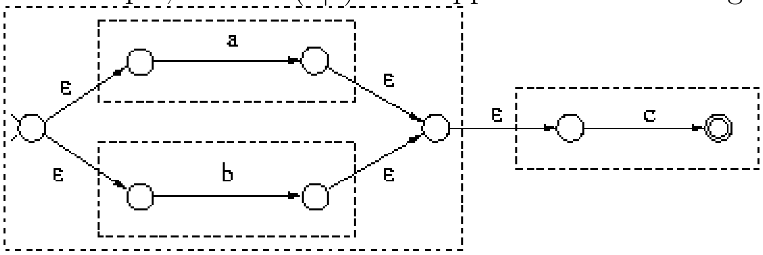
Clearly DFAs are a subset of NFAs. But it turns out that DFAs and NFAs have the same expressive power. The problem is that when converting a NFA to a DFA we may get an exponential blowup in the number of states.

We will first learn how to convert a RE into a NFA. This is the easy part. There are only 5 rules, one for each type of RE:



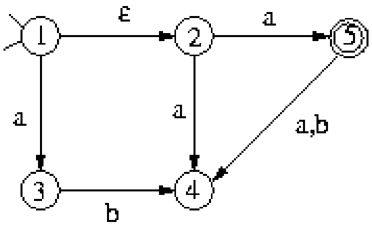
As it can been shown inductively, the above rules construct NFAs with only one final state. For example, the third rule indicates that, to construct the NFA for the RE AB, we construct the NFAs for A and B, which are represented as two boxes with one start state and one final state for each box. Then the NFA for AB is constructed by connecting the final state of A to the start state of B using an empty transition.

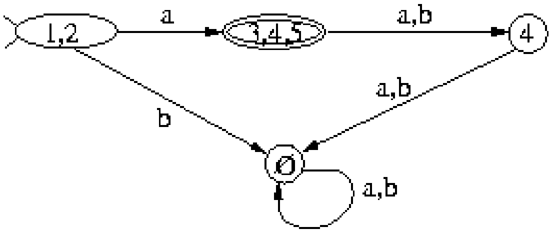
For example, the RE $(a|b)c$ is mapped to the following NFA:



The next step is to convert a NFA to a DFA (called *subset construction*). Suppose that you assign a number to each NFA state. The DFA states generated by subset construction have sets of numbers, instead of just one number. For example, a DFA state may have been assigned the set $\{5, 6, 8\}$. This indicates that arriving to the state labeled $\{5, 6, 8\}$ in the DFA is the same as arriving to the state 5, the state 6, or the state 8 in the NFA when parsing the same input. (Recall that a particular input sequence when parsed by a DFA, leads to a unique state, while when parsed by a NFA it may lead to multiple states.)

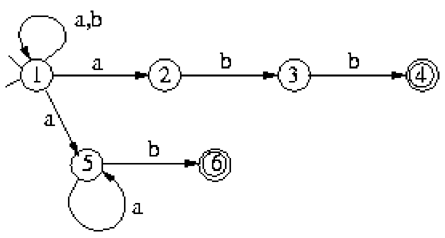
First we need to handle transitions that lead to other states for free (without consuming any input). These are the ε transitions. We define the *closure* of a NFA node as the set of all the nodes reachable by this node using zero, one, or more ε transitions. For example, The closure of node 1 in the left figure below

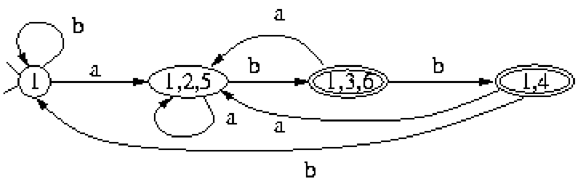




is the set $\{1, 2\}$. The start state of the constructed DFA is labeled by the closure of the NFA start state. For every DFA state labeled by some set $\{s_1, \ldots, s_n\}$ and for every character c in the language alphabet, you find all the states reachable by s_1, s_2, ..., or s_n using c arrows and you union together the closures of these nodes. If this set is not the label of any other node in the DFA constructed so far, you create a new DFA node with this label. For example, node $\{1, 2\}$ in the DFA above has an arrow to a $\{3, 4, 5\}$ for the character a since the NFA node 3 can be reached by 1 on a and nodes 4 and 5 can be reached by 2. The b arrow for node $\{1, 2\}$ goes to the error node which is associated with an empty set of NFA nodes.

The following NFA recognizes $(a|b)^*(abb\,|\,a^+b)$, even though it wasn't constructed with the above RE-to-NFA rules. It has the following DFA:





2.4 Case Study: The Calculator Scanner

As a running example for this and the subsequent sections, we will use a simple interpreter for a calculator written in Scala.

Scala requires the Java runtime engine. If you don't have Java on your PC, you need to download Java SE 7 JDK from `http://www.oracle.com/technetwork/java/javase/downloads/index.h` Then, you can install Scala from `http://www.scala-lang.org/download/`.

On a Unix or OS X system, you do:

1. download the file `http://lambda.uta.edu/cse5317/calc.tgz`
2. open a command shell and execute `tar xfz calc.tgz` to extract the files
3. execute: `cd calc; ./build`
4. run the calculator using `./run`

On Windows, you do:

1. download and unzip the file `http://lambda.uta.edu/cse5317/calc.zip`
2. on the Windows Command Prompt, go to the `calc` directory and execute `build` to compile the programs
3. execute `run` to run the calculator.

Using the calculator, you can calculate arithmetic expressions, such as `2*(3+8);`, you can assign values to variables, such as `x:=3+4;`, you can reference variables by name, such as `x+3;`, you can define recursive functions interactively, such as:

```
define f(n) = if n=0 then 1 else n*f(n-1);
```

and you can call them using `f(5);`, etc. You exit using `quit;`.

The source files of the calculator example are given in `http://lambda.uta.edu/cse5317/calc/src/edu` Each of these files will be explained in detail later, but now we are ready to look at the scanner only, given in `calc.lex`. The tokens, such as `sym.INT`, are imported from `calc.cup` and will be explained in the next section. For now, just assume that these are different constants (integers). The order of lines is important in some cases: if we put the rule for `{ID}` before the keywords, then the keywords will never be recognized (this is the consequence of the rule priority law). The lexical constructs that need to be skipped, such as comments and whitespaces, should never return (we must return only when we find a complete token).

3 Parsing

3.1 Context-free Grammars

Consider the following input string:

```
x+2*y
```

When scanned by a scanner, it produces the following stream of tokens:

```
id(x)
+
num(2)
*
id(y)
```

The goal is to parse this expression and construct a data structure (a parse tree) to represent it. One possible syntax for expressions is given by the following grammar G1:

```
E ::= E + T
    | E - T
    | T
T ::= T * F
    | T / F
    | F
F ::= num
    | id
```

where `E, T` and `F` stand for expression, term, and factor respectively. For example, the rule for `E` indicates that an expression `E` can take one of the following 3 forms: an expression followed by the token `+` followed by a term, or an expression followed by the token `-` followed by a term, or simply a term. The first rule for `E` is actually a shorthand of 3 productions:

```
E ::= E + T
E ::= E - T
E ::= T
```

G1 is an example of a context-free grammar (defined below); the symbols `E, T` and `F` are *nonterminals* and should be defined using production rules, while `+, -, *, /, num`, and `id` are *terminals* (ie, tokens) produced by the scanner. The nonterminal `E` is the *start symbol* of the grammar.

In general, a *context-free grammar* (CFG) has a finite set of terminals (tokens), a finite set of nonterminals from which one is the start symbol, and a finite set of *productions* of the form:

```
A ::= X1 X2 ... Xn
```

where A is a nonterminal and each Xi is either a terminal or nonterminal symbol.

Given two sequences of symbols a and b (can be any combination of terminals and nonterminals) and a production $A ::= X_1X_2...X_n$, the form $aAb => aX_1X_2...X_nb$ is called a *derivation*. That is, the nonterminal symbol A is replaced by the rhs (right-hand-side) of the production for A. For example,

```
T / F + 1 - x => T * F / F + 1 - x
```

is a derivation since we used the production T := T * F.

Top-down parsing starts from the start symbol of the grammar S and applies derivations until the entire input string is derived (ie, a sequence of terminals that matches the input tokens). For example,

```
E => E + T
  => E + T * F
  => T + T * F
  => T + F * F
  => T + num * F
  => F + num * F
  => id + num * F
  => id + num * id
```

which matches the input sequence id(x) + num(2) * id(y). Top down parsing occasionally requires backtracking. For example, suppose the we used the derivation E => E - T instead of the first derivation. Then, later we would have to backtrack because the derived symbols will not match the input tokens. This is true for all nonterminals that have more than one production since it indicates that there is a choice of which production to use. We will learn how to construct parsers for many types of CFGs that never backtrack. These parsers are based on a method called *predictive parsing*. One issue to consider is which nonterminal to replace when there is a choice. For example, in T + F * F we have 3 choices: we can use a derivation for T, for the first F, or for the second F. When we always replace the leftmost nonterminal, it is called *leftmost derivation*.

In contrast to top-down parsing, *bottom-up parsing* starts from the input string and uses derivations in the opposite directions (ie, by replacing the rhs sequence $X_1X_2...X_n$ of a production $A ::= X_1X_2...X_n$ with the nonterminal A. It stops when it derives the start symbol. For example,

```
   id(x) + num(2) * id(y)
=> id(x) + num(2) * F
=> id(x) + F * F
=> id(x) + T * F
=> id(x) + T
=> F + T
=> T + T
=> E + T
=> E
```

The *parse tree* of an input sequence according to a CFG is the tree of derivations. For example if we used a production $A ::= X_1X_2...X_n$ (in either top-down or bottom-up parsing) then we construct a tree with node A and children $X_1X_2...X_n$. For example, the parse tree of `id(x) + num(2) * id(y)` is:

```
        E
     /  |   \
   E    +     T
   |       /  |  \
   T       T  *   F
   |       |      |
   F       F      id
   |       |
   id     num
```

So a parse tree has non-terminals for internal nodes and terminals for leaves. As another example, consider the following grammar:

```
S ::= ( L )
    | a
L ::= L , S
    | S
```

Under this grammar, the parse tree of the sentence $(a, ((a, a), a))$ is:

```
         S
      /  |  \
     (   L   )
       / | \
      L  ,  S
      |   / | \
      S  (  L  )
      |   / | \
      a  L  ,  S
         |     |
         S     a
      /  |  \
     (   L   )
       / | \
      L  ,  S
      |     |
      S     a
      |
      a
```

Consider now the following grammar G2:

```
E ::= T + E
    | T - E
    | T
T ::= F * T
    | F / T
    | F
F ::= num
    | id
```

This is similar to our original grammar, but it is right associative when the leftmost derivation rules is used. That is, `x-y-z` is equivalent to `x-(y-z)` under G2, as we can see from its parse tree.

Consider now the following grammar G3:

```
E ::= E + E
    | E - E
    | E * E
    | E / E
    | num
    | id
```

Is this grammar equivalent to our original grammar G1? Well, it recognizes the same language, but it constructs the wrong parse trees. For example, the `x+y*z` is interpreted as `(x+y)*z` by this grammar (if we use leftmost derivations) and as `x+(y*z)` by G1 or G2. That is, both G1 and G2 grammar handle the operator precedence correctly (since $*$ has higher precedence than $+$), while the G3 grammar does not.

In general, to write a grammar that handles precedence properly, we can start with a grammar that does not handle precedence at all, such as our last grammar G3, and then we can refine it by creating more nonterminals, one for each group of operators that have the same precedence. For example, suppose we want to parse an expression E and we have 4 groups of operators: $\{not\}$, $\{*, /\}$, $\{+, -\}$, and $\{and, or\}$, in this order of precedence. Then we create 4 new nonterminals: N, T, F, and B and we split the derivations for E into 5 groups of derivations (the same way we split the rules for E in the last grammar into 3 groups in the first grammar).

A grammar is *ambiguous* if it has more than one parse tree for the same input sequence depending which derivations are applied each time. For example, the grammar G3 is ambiguous since it has two parse trees for `x-y-z` (one parses `x-y` first, while the other parses `y-z` first). Of course, the first one is the right interpretation since $-$ is left associative. Fortunately, if we always apply the leftmost derivation rule, we will never derive the second parse tree. So in this case the leftmost derivation rule removes the ambiguity.

3.2 Predictive Parsing

The goal of predictive parsing is to construct a top-down parser that never backtracks. To do so, we must transform a grammar in two ways:

1. eliminate left recursion, and

2. perform left factoring.

These rules eliminate most common causes for backtracking although they do not guarantee a completely backtrack-free parsing (called LL(1) as we will see later).

Consider this grammar:

```
A ::= A a
    | b
```

It recognizes the regular expression ba^*. The problem is that if we use the first production for top-down derivation, we will fall into an infinite derivation chain. This is called *left recursion*. But how else can you express ba^*? Here is an alternative way:

```
A ::= b A'
A' ::= a A'
     |
```

where the third production is an empty production (ie, it is `A' ::=`). That is, A' parses the RE a^*. Even though this CFG is recursive, it is not left recursive. In general, for each nonterminal X, we partition the productions for X into two groups: one that contains the left recursive productions, and the other with the rest. Suppose that the first group is:

```
X ::= X a1
...
X ::= X an
```

while the second group is:

```
X ::= b1
...
X ::= bm
```

where `a, b` are symbol sequences. Then we eliminate the left recursion by rewriting these rules into:

```
X ::= b1 X'
...
X ::= bm X'
X' ::= a1 X'
...
X' ::= an X'
X' ::=
```

For example, the CFG G1 is transformed into:

```
E ::= T E'
E' ::= + T E'
     | - T E'
     |
T ::= F T’
T’ ::= * F T’
     | / F T’
     |
F ::= num
    | id
```

Suppose now that we have a number of productions for X that have a common prefix in their rhs (but without any left recursion):

```
X ::= a b1
...
X ::= a bn
```

We factor out the common prefix as follows:

```
X ::= a X’
X’ ::= b1
...
X’ ::= bn
```

This is called *left factoring* and it helps predict which rule to use without backtracking. For example, the rule from our right associative grammar G2:

```
E ::= T + E
    | T - E
    | T
```

is translated into:

```
E ::= T E’
E’ ::= + E
     | - E
     |
```

As another example, let L be the language of all regular expressions over the alphabet $\Sigma = \{a, b\}$. That is, $L = \{$“ε”, “a”, “b”, “$a*$”, “$b*$”, “$a|b$”, “$(a|b)$”$, ...\}$. For example, the string “$a(a|b)*|b*$” is a member of L. There is no RE that captures the syntax of all REs. Consider for example the RE $((\cdots(a)\cdots))$, which is equivalent to the language $({}^n a)^n$ for all n. This represents a valid RE but there is no RE that can capture its syntax. A context-free grammar that recognizes L is:

$$\begin{array}{rcl} R & ::= & R\,R \\ & | & R\,\text{“}|\text{”}\,R \\ & | & R* \\ & | & (\,R\,) \\ & | & a \\ & | & b \\ & | & \text{“}\varepsilon\text{”} \end{array}$$

after elimination of left recursion, this grammar becomes:

$$\begin{array}{rcl} R & ::= & (\,R\,)\,R' \\ & | & a\,R' \\ & | & b\,R' \\ & | & \text{“}\varepsilon\text{”}\,R' \\ R' & ::= & R\,R' \\ & | & \text{“}|\text{”}\,R\,R' \\ & | & *\,R' \\ & | & \end{array}$$

3.2.1 Recursive Descent Parsing

Consider the transformed CFG G1 again:

```
E ::= T E’
E’ ::= + T E’
     | - T E’
     |
T ::= F T’
T’ ::= * F T’
     | / F T’
     |
F ::= num
    | id
```

Here is a Java program that parses this grammar:

```
static void E () { T(); Eprime(); }
static void Eprime () {
  if (current_token == PLUS)
     { read_next_token(); T(); Eprime(); }
  else if (current_token == MINUS)
     { read_next_token(); T(); Eprime(); };
}
static void T () { F(); Tprime(); }
static void Tprime() {
  if (current_token == TIMES)
     { read_next_token(); F(); Tprime(); }
  else if (current_token == DIV)
     { read_next_token(); F(); Tprime(); };
}
static void F () {
  if (current_token == NUM || current_token == ID)
     read_next_token();
  else error();
}
```

In general, for each nonterminal we write one procedure; For each nonterminal in the rhs of a rule, we call the nonterminal's procedure; For each terminal, we compare the current token with the expected terminal. If there are multiple productions for a nonterminal, we use an if-then-else statement to choose which rule to apply. If there was a left recursion in a production, we would have had an infinite recursion.

Please look at the following web page for a demo of the recursive descent algorithm using Java applets: `http://www.upb.de/cs/ag-kastens/uebi/parsdemo/`

3.2.2 Predictive Parsing Using Tables

Predictive parsing can also be accomplished using a *predictive parsing table* and a stack. It is sometimes called non-recursive predictive parsing. The idea is that we construct a table $M[X, \mathit{token}]$ which indicates which production to use if the top of the stack is a nonterminal X and the current token is equal to *token*; in that case we pop X from the stack and we push all the rhs symbols of the production $M[X, \mathit{token}]$ in reverse order. We use a special symbol `$` to denote the end of file. Let S be the start symbol. Here is the parsing algorithm:

```
push(S);
read_next_token();
repeat
  X = pop();
  if (X is a terminal or '$')
     if (X == current_token)
        read_next_token();
     else error();
  else if (M[X,current_token] == "X ::= Y1 Y2 ... Yk")
     {  push(Yk);
        ...
        push(Y1);
     }
  else error();
until X == '$';
```

Now the question is how to construct the parsing table. We need first to derive the $FIRST[a]$ for some symbol sequence a and the $FOLLOW[X]$ for some nonterminal X. In few words, $FIRST[a]$ is the set of terminals t that result after a number of derivations on a (ie, a => ... => tb for some b). For example, $FIRST[3+E] = \{3\}$ since 3 is the first terminal. To find $FIRST[E+T]$, we need to apply one or more derivations to E until we get a terminal at the beginning. If E can be reduced to the empty sequence, then the $FIRST[E+T]$ must also contain the $FIRST[+T] = \{+\}$. The $FOLLOW[X]$ is the set of all terminals that follow X in any legal derivation. To find the $FOLLOW[X]$, we need find all productions $Z ::= aXb$ in which X appears at the rhs. Then the $FIRST[b]$ must be part of the $FOLLOW[X]$. If b is empty, then the $FOLLOW[Z]$ must be part of the $FOLLOW[X]$.

Consider our CFG G1:

```
1)   E ::= T E’ $
2)   E’ ::= + T E’
3)        | - T E’
4)        |
5)   T ::= F T’
6)   T’ ::= * F T’
7)        | / F T’
8)        |
9)   F ::= num
10)      | id
```

`FIRST[F]` is of course `{num,id}`. This means that `FIRST[T]=FIRST[F]={num,id}`. In addition, `FIRST[E]=FIRST[T]={num,id}`. Similarly, `FIRST[T’]` is `{*,/}` and `FIRST[E’]` is `{+,-}`.

The FOLLOW of `E` is `{$}` since there is no production that has `E` at the rhs. For `E’`, rules 1, 2, and 3 have `E’` at the rhs. This means that the `FOLLOW[E’]` must contain both the `FOLLOW[E]` and the `FOLLOW[E’]`. The first one is `{$}` while the latter is ignored since we are trying to find `E’`. Similarly, to find `FOLLOW[T]`, we find the rules that have `T` at the rhs: rules 1, 2, and 3. Then `FOLLOW[T]` must include `FIRST[E’]` and, since E' can be reduced to the empty sequence, it must include `FOLLOW[E’]` too (ie, `{$}`). That is, `FOLLOW[T]={+,-,$}`. Similarly, `FOLLOW[T’]=FOLLOW[T]={+,-,$}` from rule 5. The `FOLLOW[F]` is equal to `FIRST[T’]={*,/}` plus `FOLLOW[T’]` and plus `FOLLOW[T]` from rules 5, 6, and 7, since `T’` can be reduced to the empty sequence.

To summarize, we have:

	FIRST	FOLLOW
E	{num,id}	{$}
E’	{+,-}	{$}
T	{num,id}	{+,-,$}
T’	{*,/}	{+,-,$}
F	{num,id}	{+,-,*,/,$}

Now, given the above table, we can easily construct the parsing table. For each $t \in FIRST[a]$, add $X ::= a$ to $M[X,t]$. If a can be reduced to the empty sequence, then for each $t \in FOLLOW[X]$, add $X ::= a$ to $M[X,t]$.

For example, the parsing table of the grammar G1 is:

	num	id	+	-	*	/	$
E	1	1					
E’			2	3			4
T	5	5					
T’			8	8	6	7	8
F	9	10					

where the numbers are production numbers. For example, consider the eighth production $T' ::=$. Since `FOLLOW[T’]={+,-,$}`, we add 8 to `M[T’,+]`, `M[T’,-]`, `M[T’,$]`.

A grammar is called LL(1) if each element of the parsing table of the grammar has at most one production element. (The first L in LL(1) means that we read the input from left to right, the second L means that it uses left-most derivations only, and the number 1 means that we need to look one token only ahead from the input.) Thus G1 is LL(1). If we have multiple entries in M, the grammar is not LL(1).

We will parse now the string `x-2*y$` using the above parse table:

```
Stack          current_token     Rule
---------------------------------------------------------------
E                   x          M[E,id] = 1   (using E ::= T E' $)
$ E' T              x          M[T,id] = 5   (using T ::= F T')
$ E' T' F           x          M[F,id] = 10  (using F ::= id)
$ E' T' id          x          read_next_token
$ E' T'             -          M[T',-] = 8   (using T' ::= )
$ E'                -          M[E',-] = 3   (using E' ::= - T E')
$ E' T -            -          read_next_token
$ E' T              2          M[T,num] = 5  (using T ::= F T')
$ E' T' F           2          M[F,num] = 9  (using F ::= num)
$ E' T' num         2          read_next_token
$ E' T'             *          M[T',*] = 6   (using T' ::= * F T')
$ E' T' F *         *          read_next_token
$ E' T' F           y          M[F,id] = 10  (using F ::= id)
$ E' T' id          y          read_next_token
$ E' T'             $          M[T',$] = 8   (using T' ::= )
$ E'                $          M[E',$] = 4   (using E' ::= )
$                   $          stop (accept)
```

As another example, consider the following grammar:

```
G ::= S $
S ::= ( L )
    | a
L ::= L , S
    | S
```

After left recursion elimination, it becomes

```
0)  G := S $
1)  S ::= ( L )
2)  S ::= a
3)  L ::= S L'
4)  L' ::= , S L'
5)  L' ::=
```

The first/follow tables are:

	FIRST	FOLLOW
G	(a	
S	(a	,) $
L	(a	)
L’	,	)

which are used to create the parsing table:

	(	)	a	,	$
G	0		0		
S	1		2		
L	3		3		
L’		5		4	

3.3 Bottom-up Parsing

The basic idea of a bottom-up parser is that we use grammar productions in the opposite way (from right to left). Like for predictive parsing with tables, here too we use a stack to push symbols. If the first few symbols at the top of the stack match the rhs of some rule, then we pop out these symbols from the stack and we push the lhs (left-hand-side) of the rule. This is called a *reduction*. For example, if the stack is `x * E + E` (where `x` is the bottom of stack) and there is a rule `E ::= E + E`, then we pop out `E + E` from the stack and we push `E`; ie, the stack becomes `x * E`. The sequence `E + E` in the stack is called a *handle*. But suppose that there is another rule `S ::= E`, then `E` is also a handle in the stack. Which one to choose? Also what happens if there is no handle? The latter question is easy to answer: we push one more terminal in the stack from the input stream and check again for a handle. This is called *shifting*. So another name for bottom-up parsers is shift-reduce parsers. There two actions only:

1. shift the current input token in the stack and read the next token, and
2. reduce by some production rule.

Consequently the problem is to recognize when to shift and when to reduce each time, and, if we reduce, by which rule. Thus we need a recognizer for handles so that by scanning the stack we can decide the proper action. The recognizer is actually a finite state machine exactly the same we used for REs. But here the language symbols include both terminals and nonterminal (so state transitions can be for any symbol) and the final states indicate either reduction by some rule or a final acceptance (success).

A DFA though can only be used if we always have one choice for each symbol. But this is not the case here, as it was apparent from the previous example: there is an ambiguity in recognizing handles in the stack. In the previous example, the handle can either be `E + E` or `E`. This ambiguity will hopefully be resolved later when we read more tokens. This implies that we have multiple choices and each choice denotes a valid potential for reduction. So instead of a DFA we must use a NFA, which in turn can be mapped into a DFA as we learned in Section 2.3. These two steps (extracting the NFA and map it to DFA) are done in one step using *item sets* (described below).

3.3.1 Bottom-up Parsing

Recall the original CFG G1:

```
0)  S :: = E $
1)  E ::= E + T
2)      | E - T
3)      | T
4)  T ::= T * F
5)      | T / F
6)      | F
7)  F ::= num
8)      | id
```

Here is a trace of the bottom-up parsing of the input `x-2*y$`:

```
      Stack          rest-of-the-input          Action
----------------------------------------------------------------
 1)                    id - num * id $      shift
 2)   id               - num * id $         reduce by rule 8
 3)   F                - num * id $         reduce by rule 6
 4)   T                - num * id $         reduce by rule 3
 5)   E                - num * id $         shift
 6)   E -              num * id $           shift
 7)   E - num          * id $               reduce by rule 7
 8)   E - F            * id $               reduce by rule 6
 9)   E - T            * id $               shift
10)   E - T *          id $                 shift
11)   E - T * id       $                    reduce by rule 8
12)   E - T * F        $                    reduce by rule 4
13)   E - T            $                    reduce by rule 2
14)   E $                                   shift
15)   S                                     accept (reduce by rule 0)
```

We start with an empty stack and we finish when the stack contains the non-terminal `S` only. Every time we shift, we push the current token into the stack and read the next token from the input sequence. When we reduce by a rule, the top symbols of the stack (the handle) must match the rhs of the rule *exactly*. Then we replace these top symbols with the lhs of the rule (a non-terminal). For example, in the line 12 above, we reduce by rule 4 (`T ::= T * F`). Thus we pop the symbols `F`, `*`, and `T` (in that order) and we push `T`. At each instance, the stack (from bottom to top) concatenated with the rest of the input (the unread input) forms a valid bottom-up derivation from the input sequence. That is, the derivation sequence here is:

```
 1)        id - num * id $
 3)   =>   F - num * id $
 4)   =>   T - num * id $
 5)  =>   E - num * id $
 8)  =>   E - F * id $
 9)  =>   E - T * id $
12)  =>   E - T * F $
13)  =>   E - T $
14)  =>   E $
15)  =>   S
```

The purpose of the above example is to understand how bottom-up parsing works. It involves lots of guessing. We will see that we don't actually push symbols in the stack. Rather, we push states that represent possibly more than one potentials for rule reductions.

As we learned in earlier sections, a DFA can be represented by transition tables. In this case we use two tables: ACTION and GOTO. ACTION is used for recognizing which action to perform and, if it is shifting, which state to go next. GOTO indicates which state to go after a reduction by a rule for a nonterminal symbol. We will first learn how to use the tables for parsing and then how to construct these tables.

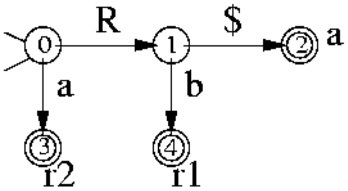
3.3.2 Shift-Reduce Parsing Using the ACTION/GOTO Tables

Consider the following grammar:

```
0) S ::= R $
1) R ::= R b
2) R ::= a
```

which parses the R.E. $ab^*\$$.

The DFA that recognizes the handles for this grammar is: (it will be explained later how it is constructed)



where 'r2' for example means reduce by rule 2 (ie, by `R :: = a`) and 'a' means accept. The transition from 0 to 3 is done when the current state is 0 and the current input token is 'a'. If we are in state 0 and have completed a reduction by some rule for R (either rule 1 or 2), then we jump to state 1.

The ACTION and GOTO tables that correspond to this DFA are:

state	action			goto	
	a	b	$	S	R
0	s3				1
1		s4	s2		
2	a	a	a		
3	r2	r2	r2		
4	r3	r3	r3		

where for example s3 means shift a token into the stack and go to state 3. That is, transitions over terminals become shifts in the ACTION table while transitions over non-terminals are used in the GOTO table.

Here is the shift-reduce parser:

```
push(0);
read_next_token();
for(;;)
{  s = top();    /* current state is taken from top of stack */
   if (ACTION[s,current_token] == 'si')   /* shift and go to state i */
   {  push(i);
      read_next_token();
   }
   else if (ACTION[s,current_token] == 'ri')
   /* reduce by rule i: X ::= A1...An */
   {  perform pop() n times;
      s = top();    /* restore state before reduction from top of stack */
      push(GOTO[s,X]);   /* state after reduction */
   }
   else if (ACTION[s,current_token] == 'a')
      success!!
   else error();
}
```

Note that the stack contains state numbers only, not symbols.

For example, for the input `abb$`, we have:

```
Stack     rest-of-input   Action
----------------------------------------------------------------------------
0         abb$            s3
0 3       bb$             r2  (pop(), push GOTO[0,R] since R ::= a)
0 1       bb$             s4
0 1 4     b$              r1  (pop(), pop(), push GOTO[0,R] since R ::= R b)
0 1       b$              s4
0 1 4     $               r1  (pop(), pop(), push GOTO[0,R] since R ::= R b)
0 1       $               s2
0 1 2                     accept
```

3.3.3 Table Construction

Now the only thing that remains to do is, given a CFG, to construct the finite automaton (DFA) that recognizes handles. After that, constructing the ACTION and GOTO tables will be straightforward.

The states of the finite state machine correspond to item sets. An *item* (or *configuration*) is a production with a dot (.) at the rhs that indicates how far we have progressed using this rule to parse the input. For example, the item `E ::= E + . E` indicates that we are using the rule `E ::= E + E` and that, using this rule, we have parsed `E`, we have seen a token `+`, and we are ready to parse another `E`. Now, why do we need a set (an item set) for each state in the state machine? Because many production rules may be applicable at this point; later when we will scan more input tokens we will be able tell exactly which production to use. This is the time when we are ready to reduce by the chosen production.

For example, say that we are in a state that corresponds to the item set with the following items:

```
S ::= id . := E
S ::= id . : S
```

This state indicates that we have already parsed an `id` from the input but we have 2 possibilities: if the next token is `:=` we will use the first rule and if it is `:` we will use the second.

Now we are ready to construct our automaton. Since we do not want to work with NFAs, we build a DFA directly. So it is important to consider closures (like we did when we transformed NFAs to DFAs). The closure of an item `X ::= a . t b` (ie, the dot appears before a terminal `t`) is the singleton set that contains the item `X ::= a . t b` only. The closure of an item `X ::= a . Y b` (ie, the dot appears before a nonterminal `Y`) is the set consisting of the item itself, plus all productions for `Y` with the dot at the left of the rhs, plus the closures of these items. For example, the closure of the item `E ::= E + . T` is the set:

```
E ::= E + . T
T ::= . T * F
T ::= . T / F
T ::= . F
F ::= . num
F ::= . id
```

The initial state of the DFA (state 0) is the closure of the item `S ::= . a $`, where `S ::= a $` is the first rule. In simple words, if there is an item `X ::= a . s b` in an item set, where `s` is a symbol (terminal or nonterminal), we have a transition labelled by `s` to an item set that contains `X ::= a s . b`. But it's a little bit more complex than that:

1. If we have more than one item with a dot before the same symbol `s`, say `X ::= a . s b` and `Y ::= c . s d`, then the new item set contains both `X ::= a s . b` and `Y ::= c s . d`.

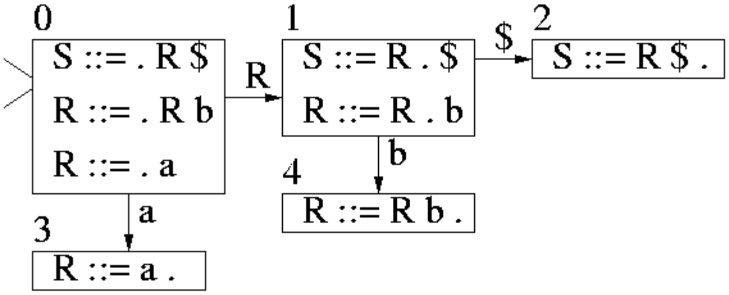
2. We need to get the closure of the new item set.

3. We have to check if this item set has been appeared before so that we don't create it again.

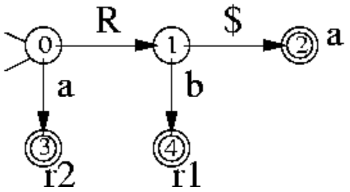
For example, our previous grammar which parses the R.E. $ab^*\$$:

```
0) S ::= R $
1) R ::= R b
2) R ::= a
```

has the following item sets:



which forms the following DFA:



We can understand now the R transition: If we read an entire R term (that is, after we reduce by a rule that parses R), and the previous state before the reduction was 0, we jump to state 1.

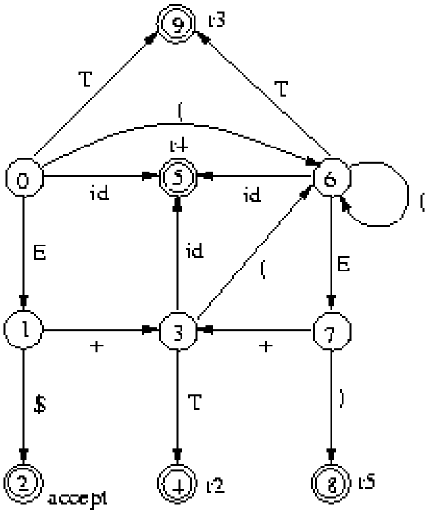
As another example, the CFG:

```
1)  S ::= E $
2)  E ::= E + T
3)      | T
4)  T ::= id
5)     | ( E )
```

has the following item sets:

```
I0:  S ::= . E $                I4:  E ::= E + T .
     E ::= . E + T
     E ::= . T                  I5:  T ::= id .
     T ::= . id
     T ::= . ( E )              I6:  T ::= ( . E )
                                     E ::= . E + T
I1:  S ::= E . $                     E ::= . T
     E ::= E . + T                   T ::= . id
                                     T ::= . ( E )
I2:  S ::= E $ .
                                I7:  T ::= ( E . )
I3:  E ::= E + . T                   E ::= E . + T
     T ::= . id
     T ::= . ( E )              I8:  T ::= ( E ) .

                                I9:  E ::= T .
```

that correspond to the DFA:



Explanation: The initial state `I0` is the closure of `S ::= . E $`. The two items `S ::= . E $` and `E ::= . E + T` in `I0` must have a transition by `E` since the dot appears before `E`. So we have a new item set `I1` and a transition from `I0` to `I1` by `E`. The `I1` item set must contain the closure of the items `S ::= . E $` and `E ::= . E + T` with the dot moved one place right. Since the dot in `I1` appears before terminals (`$` and `+`), the closure is the items themselves. Similarly, we have a transition from `I0` to `I9` by `T`, to `I5` by `id`, to `I6` by '(', etc, and we do the same thing for all item sets.

Now if we have an item set with only one item `S ::= E .`, where `S` is the start symbol, then this state is the *accepting state* (state `I2` in the example). If we have an item set with only one item `X ::= a .` (where the dot appears at the end of the rhs), then this state corresponds to a reduction by the production `X ::= a`. If we have a transition by a terminal symbol (such as from `I0` to `I5` by `id`), then this corresponds to a shifting.

The ACTION and GOTO tables that correspond to this DFA are:

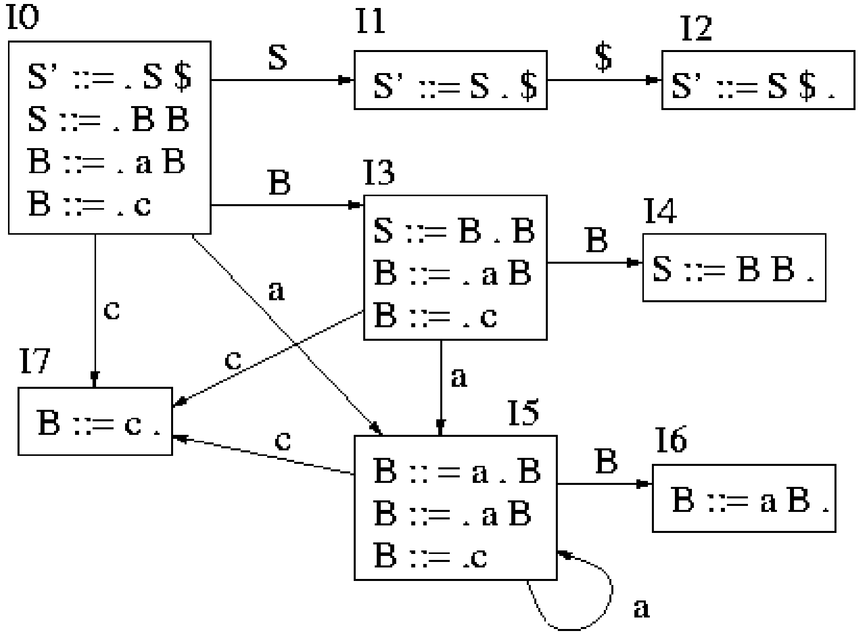
state	action					goto		
	id	(	)	+	$	S	E	T
0	s5	s6					1	9
1				s3	s2			
2	a	a	a	a	a			
3	s5	s6						4
4	r2	r2	r2	r2	r2			
5	r4	r4	r4	r4	r4			
6	s5	s6					7	9
7			s8	s3				
8	r5	r5	r5	r5	r5			
9	r3	r3	r3	r3	r3			

As another example, consider the following augmented grammar:

```
0) S' ::= S $
```

```
1) S  ::= B B
2) B  ::= a B
3) B  ::= c
```

The state diagram is:



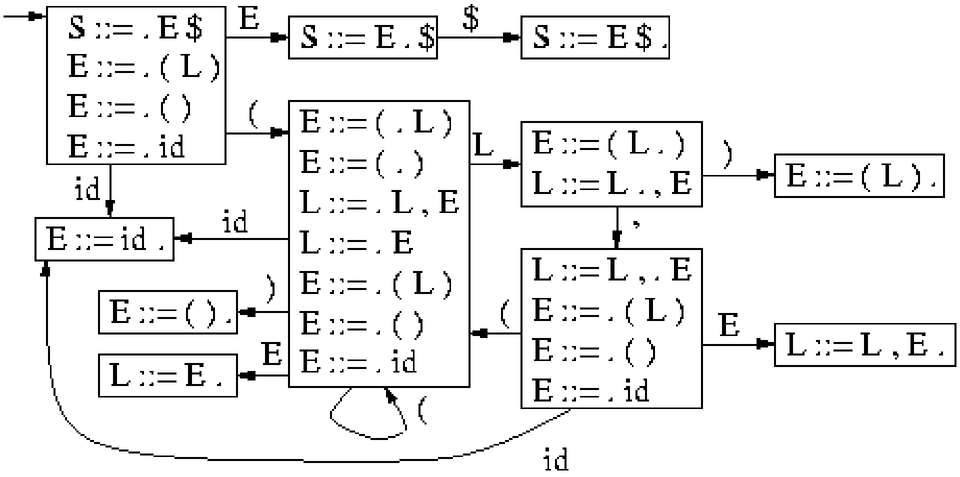
The ACTION and GOTO parsing tables are:

state	action			goto		
	a	c	$	S'	S	B
0	s5	s7			1	3
1			s2			
2	a	a	a			
3	s5	s7				4
4	r1	r1	r1			
5	s5	s7				6
6	r2	r2	r2			
7	r3	r3	r3			

As yet another example, the grammar:

```
S ::= E $
E ::= ( L )
E ::= ( )
E ::= id
L ::= L , E
L ::= E
```

has the following state diagram:



If we have an item set with more than one items with a dot at the end of their rhs, then this is an error, called a *reduce-reduce conflict*, since we can not choose which rule to use to reduce. This is a very important error and it should never happen. Another conflict is a *shift-reduce conflict* where we have one reduction and one or more shifts at the same item set. For example, when we find `b` in `a+b+c`, do we reduce `a+b` or do we shift the plus token and reduce `b+c` later? This type of ambiguities is usually resolved by assigning the proper precedences to operators (described later). The shift-reduce conflict is not as severe as the reduce-reduce conflict: a parser generator selects reduction against shifting but we may get a wrong behavior. If the grammar has no reduce-reduce and no shift-reduce conflicts, it is LR(0) (ie. we read left-to-right, we use right-most derivations, and we don't look ahead any tokens).

3.3.4 SLR(1), LR(1), and LALR(1) Grammars

Here is an example of a grammar that is not LR(0):

```
1)  S ::= E $
2)  E ::= E + T
3)      | T
4)  T ::= T * F
5)      | F
6)  F ::= id
7)      | ( E )
```

Let's focus on two item sets only:

```
I0:  S ::= . E $                I1:  E ::= T .
     E ::= . E + T                   T ::= T . * F
     E ::= . T
     T ::= . T * F
```

```
          T ::= . F
          F ::= . id
          F ::= . ( E )
```

State `I1` has a shift-reduce conflict. Another example is:

```
S ::= X
X ::= Y
    | id
Y ::= id
```

which includes the following two item sets:

```
I0:   S ::= . X $
      X ::= . Y              I1:   X ::= id .
      X ::= . id                   Y ::= id .
      Y ::= . id
```

State `I1` has a reduce-reduce conflict.

Can we find an easy fix for the reduce-reduce and the shift-reduce conflicts? Consider the state `I1` of the first example above. The FOLLOW of `E` is `{$,+,)}`. We can see that `*` is not in the `FOLLOW[E]`, which means that if we could see the next token (called the *lookahead token*) and this token is `*`, then we can use the item `T ::= T . * F` to do a shift; otherwise, if the lookahead is one of `{$,+,)}`, we reduce by `E ::= T`. The same trick can be used in the case of a reduce-reduce conflict. For example, if we have a state with the items `A ::= a .` and `B ::= b .`, then if `FOLLOW[A]` doesn't overlap with `FOLLOW[B]`, then we can deduce which production to reduce by inspecting the lookahead token. This grammar (where we can look one token ahead) is called SLR(1) and is more powerful than LR(0).

The previous trick is not always possible. Consider the grammar:

```
S ::= E $
E ::= L = R
    | R
L ::= * R
    | id
R ::= L
```

for C-style variable assignments. Consider the two states:

```
I0:   S ::= . E $
      E ::= . L = R              I1:   E ::= L . = R
      E ::= . R                        R ::= L .
      L ::= . * R
      L ::= . id
      R ::= . L
```

we have a shift-reduce conflict in I1. The FOLLOW of R is the union of the FOLLOW of E and the FOLLOW of L, ie, it is {$,=}. In this case, = is a member of the FOLLOW of R, so we can't decide shift or reduce using just one lookahead.

An even more powerful grammar is LR(1), described below. This grammar is not used in practice because of the large number of states it generates. A simplified version of this grammar, called LALR(1), has the same number of states as LR(0) but it is far more powerful than LR(0) (but less powerful than LR(1)). It is the most important grammar from all grammars we learned so far. CUP, Bison, and Yacc recognize LALR(1) grammars.

Both LR(1) and LALR(1) check one lookahead token (they read one token ahead from the input stream – in addition to the current token). An item used in LR(1) and LALR(1) is like an LR(0) item but with the addition of a set of expected lookaheads which indicate what lookahead tokens would make us perform a reduction when we are ready to reduce using this production rule. The expected lookahead symbols for a rule X ::= a are always a subset or equal to FOLLOW[X]. The idea is that an item in an itemset represents a potential for a reduction using the rule associated with the item. But each itemset (ie. state) can be reached after a number of transitions in the state diagram, which means that each itemset has an implicit context, which, in turn, implies that there are only few terminals permitted to appear in the input stream after reducing by this rule. In SLR(1), we made the assumption that the followup tokens after the reduction by X ::= a are exactly equal to FOLLOW[X]. But this is too conservative and may not help us resolve the conflicts. So the idea is to restrict this set of followup tokens by making a more careful analysis by taking into account the context in which the item appears. This will reduce the possibility of overlapping in sift/reduce or reduce/reduce conflict states. For example,

```
L ::= * . R, =$
```

indicates that we have the item L ::= * . R and that we have parsed * and will reduce by L ::= * R only when we parse R and the next lookahead token is either = or $ (end of file). It is very important to note that the FOLLOW of L (equal to {$,=}) is always a superset or equal to the expected lookaheads. The point of propagating expected lookaheads to the items is that we want to restrict the FOLLOW symbols so that only the relevant FOLLOW symbols would affect our decision when to shift or reduce in the case of a shift-reduce or reduce-reduce conflict. For example, if we have a state with two items A ::= a ., s1 and B ::= b ., s2, where s1 and s2 are sets of terminals, then if s1 and s2 are not overlapping, this is not a reduce-reduce error any more, since we can decide by inspecting the lookahead token.

So, when we construct the item sets, we also propagate the expected lookaheads. When we have a transition from A ::= a . s b by a symbol s, we just propagate the expected lookaheads. The tricky part is when we construct the closures of the items. Recall that when we have an item A ::= a . B b, where B is a nonterminal, then we add all rules B ::= . c in the item set. If A ::= a . B b has an expected lookahead t, then B ::= . c has all the elements of FIRST[bt] as expected lookaheads.

For example, consider the previous grammar:

```
S ::= E $
E ::= L = R
```

```
    | R
L ::= * R
    | id
R ::= L
```

Two of the LR(1) item sets are:

```
I0:  S ::= . E $     ?
     E ::= . L = R   $          I1:  E ::= L . = R   $
     E ::= . R       $               R ::= L .       $
     L ::= . * R     =$
     L ::= . id      =$
     R ::= . L       $
```

We always start with expected lookahead `?` for the rule `S ::= E $`, which basically indicates that we don't care what is after end-of-file. The closure of `L` will contain both `=` and `$` for expected lookaheads because in `E ::= . L = R` the first symbol after `L` is `=`, and in `R ::= . L` (the closure of `E ::= . R`) we use the `$` terminal for expected lookahead propagated from `E ::= . R` since there is no symbol after `L`. We can see that there is no shift reduce error in `I1`: if the lookahead token is `$` we reduce, otherwise we shift (for `=`).

In LR(1) parsing, an item `A ::= a, s1` is different from `A ::= a, s2` if `s1` is different from `s2`. This results to a large number of states since the combinations of expected lookahead symbols can be very large. To reduce the number of states, when we have two items like those two, instead of creating two states (one for each item), we combine the two states into one by creating an item `A := a, s3`, where `s3` is the union of `s1` and `s2`. Since we make the expected lookahead sets larger, there is a danger that some conflicts will have worse chances to be resolved. But the number of states we get is the same as that for LR(0). This grammar is called LALR(1).

There is an easier way to construct the LALR(1) item sets. Simply start by constructing the LR(0) item sets. Then we add the expected lookaheads as follows: whenever we find a new lookahead using the closure law given above we add it in the proper item; when we propagate lookaheads we propagate all of them. Sometimes when we insert a new lookahead we need to do all the lookahead propagations again for the new lookahead even in the case we have already constructed the new items. So we may have to loop through the items many times until no new lookaheads are generated. Sounds hard? Well that's why there are parser generators to do that automatically for you. This is how CUP work.

3.3.5 Practical Considerations for LALR(1) Grammars

Most parsers nowdays are specified using LALR(1) grammars fed to a parser generator, such as CUP. There are few tricks that we need to know to avoid reduce-reduce and shift-reduce conflicts.

When we learned about top-down predictive parsing, we were told that left recursion is bad. For LALR(1) the opposite is true: left recursion is good, right recursion is bad. Consider this:

```
L ::= id , L
    | id
```

which captures a list of ids separated by commas. If the FOLLOW of `L` (or to be more precise, the expected lookaheads for `L ::= id`) contains the comma token, we are in big trouble. We can use instead:

```
L ::= L , id
    | id
```

Left recursion uses less stack than right recursion and it also produces left associative trees (which is what we usually want).

Some shift-reduce conflicts are very difficult to eliminate. Consider the infamous if-then-else conflict:

```
S ::= if E then S else S
    | if E then S
    | ...
```

Read Section 3.3 in the textbook to see how this can be eliminated.

Most shift-reduce errors though are easy to remove by assigning precedence and associativity to operators. Consider the grammar:

```
S ::= E $
E ::= E + E
    | E * E
    | ( E )
    | id
    | num
```

and four of its LALR(1) states:

```
I0:  S ::= . E $     ?
     E ::= . E + E  +*$    I1:  S ::= E . $     ?      I2:  E ::= E * . E   +*$
     E ::= . E * E  +*$         E ::= E . + E  +*$          E ::= . E + E   +*$
     E ::= . ( E )  +*$         E ::= E . * E  +*$          E ::= . E * E   +*$
     E ::= . id     +*$                                     E ::= . ( E )   +*$
     E ::= . num    +*$    I3:  E ::= E * E .   +*$         E ::= . id      +*$
                                E ::= E . + E   +*$         E ::= . num     +*$
                                E ::= E . * E   +*$
```

Here we have a shift-reduce error. Consider the first two items in `I3`. If we have `a*b+c` and we parsed `a*b`, do we reduce using `E ::= E * E` or do we shift more symbols? In the former case we get a parse tree `(a*b)+c`; in the latter case we get `a*(b+c)`. To resolve this conflict, we can specify that `*` has higher precedence than `+`. The precedence of a grammar production is equal to the precedence of the rightmost token at the rhs of the production.

For example, the precedence of the production E ::= E * E is equal to the precedence of the operator *, the precedence of the production E ::= (E) is equal to the precedence of the token), and the precedence of the production E ::= if E then E else E is equal to the precedence of the token else. The idea is that if the lookahead has higher precedence than the production currently used, we shift. For example, if we are parsing E + E using the production rule E ::= E + E and the lookahead is *, we shift *. If the lookahead has the same precedence as that of the current production and is left associative, we reduce, otherwise we shift. The above grammar is valid if we define the precedence and associativity of all the operators. Thus, it is very important when you write a parser using CUP or any other LALR(1) parser generator to specify associativities and precedences for most tokens (especially for those used as operators). Note: you can explicitly define the precedence of a rule in CUP using the %prec directive:

```
 E ::= MINUS E   %prec UMINUS
```

where UMINUS is a pseudo-token that has higher precedence than TIMES, MINUS etc, so that -1*2 is equal to (-1)*2, not to -(1*2).

Another thing we can do when specifying an LALR(1) grammar for a parser generator is error recovery. All the entries in the ACTION and GOTO tables that have no content correspond to syntax errors. The simplest thing to do in case of error is to report it and stop the parsing. But we would like to continue parsing finding more errors. This is called *error recovery*. Consider the grammar:

```
S ::= L = E ;
    | { SL } ;
    | error ;
SL ::= S ;
     | SL S ;
```

The special token error indicates to the parser what to do in case of invalid syntax for S (an invalid statement). In this case, it reads all the tokens from the input stream until it finds the first semicolon. The way the parser handles this is to first push an error state in the stack. In case of an error, the parser pops out elements from the stack until it finds an error state where it can proceed. Then it discards tokens from the input until a restart is possible. Inserting error handling productions in the proper places in a grammar to do good error recovery is considered very hard.

3.4 Case Study: The Calculator Parser

The file http://lambda.uta.edu/cse5317/calc/simple_calc.cup contains the CUP grammar for the calculator parser (without semantics actions). Notice the section that specifies the precedence and associativity of the terminals symbols:

```
precedence nonassoc ELSE;
precedence right OR;
precedence right AND;
precedence nonassoc NOT;
precedence left EQ, LT, GT, LE, GE, NE;
precedence left PLUS, MINUS;
precedence left TIMES, DIV;
```

It lists the terminals in order of precedence, from lower to higher. Thus, TIMES and DIV have the highest precedence. For example, 1+3*4 is equivalent to 1+(3*4) since TIMES has higher precedence than PLUS. In addition, the keywords left, right, and nonassoc indicate that the operators have left, right, or no associativity at all, respectively. This means that 1+2+3 is equivalent to (1+2)+3, while (x and y and z) is equivalent to (x and (y and z)). This list is very important for helping CUP resolve shift-reduce conflicts. The reason that we set ELSE to the lowest precedence is to resolve the infamous if-then-else conflict: It basically means that we always shift in case of nested if-then-elses.

Another thing to notice is that, when there is a choice, left recursion is preferred from right recursion, such as in:

```
expl ::= expl COMMA exp
       | exp
```

4 Abstract Syntax

A grammar for a language specifies a recognizer for this language: if the input satisfies the grammar rules, it is accepted, otherwise it is rejected. But we usually want to perform semantic actions against some pieces of input recognized by the grammar. For example, if we are building a compiler, we want to generate machine code. The semantic actions are specified as pieces of code attached to production rules. For example, the CUP productions:

```
E ::= E:e1 + E:e2        {: RESULT = e1 + e2; :}
    | E:e1 - E:e2        {: RESULT = e1 - e2; :}
    | num:n              {: RESULT = n; :}
    ;
```

contain pieces of Java code (enclosed by {: :}) to be executed at a particular point of time. Here, each expression E is associated with an integer. The goal is to calculate the value of E stored in the variable RESULT. For example, the first rule indicates that we are reading one expression E and call its result e1, then we read a +, then we read another E and call its result e2, and then we are executing the Java code {: RESULT = e1 + e2; :}. The code can appear at any point of the rhs of a rule (even at the beginning of the rhs of the rule) and we may have more than one (or maybe zero) pieces of code in a rule. The code is executed only when all the symbols at the left of the code in the rhs of the rule have been processed.

It is very uncommon to build a full compiler by adding the appropriate actions to productions. It is highly preferable to build the compiler in stages. So a parser usually builds an Abstract Syntax Tree (AST) and then, at a later stage of the compiler, these ASTs are compiled into the target code. There is a fine distinction between parse trees and ASTs. A parse tree has a leaf for each terminal and an internal node for each nonterminal. For each production rule used, we create a node whose name is the lhs of the rule and whose children are the symbols of the rhs of the rule. An AST on the other hand is a compact data structure to represent the same syntax regardless of the form of the production rules used in building this AST.

4.1 Building Abstract Syntax Trees in Java

When building ASTs, it's a good idea to define multiple classes to capture various families of constructs. For example, we can have an `Exp` class to represent expressions, `Stmt` class to represent statements, and `Type` class to represent types. Here is an example of `Exp` in Java:

```
abstract class Exp {
}
class IntegerExp extends Exp {
   public int value;
   public IntegerExp ( int n ) { value=n; }
}
class StringExp extends Exp {
   public String value;
   public StringExp ( String s ) { value=s; }
}
class VariableExp extends Exp {
   public String name;
   public VariableExp ( String n ) { name=n; }
}
class BinaryExp extends Exp {
   public String operator;
   public Exp left;
   public Exp right;
   public BinaryExp ( String o, Exp l, Exp r ) { operator=o; left=l; right=r; }
}
class UnaryExp extends Exp {
   public String operator;
   public Exp operand;
   public UnaryExp ( String o, Exp e ) { operator=o; operand=e; }
}
class CallExp extends Exp {
   public String name;
   public List<Exp> arguments;
   public CallExp ( String nm, List<Exp> s ) { name=nm; arguments=s; }
}
class ProjectionExp extends Exp {
   public Exp record;
   public String attribute;
   public ProjectionExp ( Exp v, String a ) { record=v; attribute=a; }
}
class RecordElement {
   public String attribute;
   public Exp value;
   public RecordElement ( String a, Exp v ) { attribute=a; value=v; }
}
class RecordExp extends Exp {
    public List<RecordElement> elements;
    public RecordExp ( List<RecordElement> el ) { elements=el; }
}
```

For example,

```
new BinaryExp("+",new BinaryExp("-",new VariableExp("x"),new IntegerExp(2)),
              new IntegerExp(3))
```

constructs the AST for the input (x-2)+3.

4.2 Building Abstract Syntax Trees in Scala

The previous example of expression ASTs can be written as follows in Scala using case classes:

```
sealed abstract class Exp
case class IntegerExp ( value: Int ) extends Exp
case class StringExp ( value: String ) extends Exp
case class VariableExp ( name: String ) extends Exp
case class BinaryExp ( operator: String, left: Exp, right: Exp ) extends Exp
case class UnaryExp ( operator: String, operand: Exp ) extends Exp
case class CallExp ( name: String, arguments: List[Exp] ) extends Exp
case class ProjectionExp ( record: Exp, attribute: String ) extends Exp
case class RecordExp ( arguments: List[(String,Exp)] ) extends Exp
```

For example,

```
BinaryExp("+",BinaryExp("-",VariableExp("x"),IntegerExp(2)),IntegerExp(3))
```

constructs the AST for the input (x-2)+3.

4.3 Building Abstract Syntax Trees in C

The previous example of expression ASTs can be written as follows in C:

```
typedef struct Exp {
  enum { integer_exp, string_exp, variable_exp,
         binary_exp, unary_exp, call_exp,
         projection_exp, record_exp } tag;
  union { int                                      integerExp;
```

```
          string                                   stringExp;
          string                                   variableExp;
          struct { string          operator;
                   struct Exp*      left;
                   struct Exp*      right; }       binaryExp;
          struct { string          operator;
                   struct Exp*      operand; }     unaryExp;
          struct { string          name;
                   struct Exp_list* arguments; }   callExp;
          struct { struct Exp*  record;
                   string      attribute; }       projectionExp;
          struct rec { string       attribute;
                       struct Exp*  value;
                       struct rec*  next; }        recordExp;
      } op;
} ast;
```

where Exp_list is a list of ASTs:

```
typedef struct Exp_list {
  ast*             elem;
  struct Exp_list* next;
} ast_list;
```

It's a good idea to define a constructor for every kind of expression to simplify the task of constructing ASTs:

```
ast* make_binaryExp ( string operator, ast* left, ast* right ) {
  ast* e = (ast*) malloc(sizeof(ast));
  e->tag = binary_exp;
  e->op.binary.operator = operator;
  e->op.binary.left = left;
  e->op.binary.right = right;
  return e;
};
```

For example,

```
make_binaryExp("+",make_binaryExp("-",make_variableExp("x"),make_integerExp(2)),
               make_integerExp(3))
```

constructs the AST for the input `(x-2)+3`.

5 Semantic Actions

Let's consider now how actions are evaluated by different parsers. In recursive descent parsers, actions are pieces of code embedded in the recursive procedures. For the following grammar:

```
E ::= T E'
E' ::= + T E'
     | - T E'
     |
T ::= num
```

we have the following recursive descent parser:

```
int E () { return Eprime(T()); };
int Eprime ( int left ) {
  if (current_token=='+') {
     read_next_token();
     return Eprime(left + T());
  } else if (current_token=='-') {
     read_next_token();
     return Eprime(left - T());
  } else return left;
};
int T () {
  if (current_token=='num') {
     read_next_token();
     return num_value;
  } else error();
};
```

By passing `T()` as input to `Eprime`, we pass the left operand to `Eprime`.

Table-driven predictive parsers use the parse stack to push/pop actions (along with symbols) but they use a separate semantic stack to execute the actions. In that case, the parsing algorithm becomes:

```
push(S);
read_next_token();
repeat
  X = pop();
  if (X is a terminal or '$')
     if (X == current_token)
        read_next_token();
     else error();
  else if (X is an action)
     perform the action;
  else if (M[X,current_token] == "X ::= Y1 Y2 ... Yk")
     {  push(Yk);
        ...
        push(Y1);
     }
  else error();
until X == '$';
```

For example, suppose that pushV and popV are the functions to manipulate the semantic stack. The following is the grammar of an interpreter that uses the semantic stack to perform additions and subtractions:

```
E ::= T E' $ { print(popV()); }
E' ::= + T { pushV(popV() + popV()); } E'
     | - T { pushV(-popV() + popV()); } E'
     |
T ::= num { pushV(num); }
```

For example, for 1+5-2, we have the following sequence of actions:

```
pushV(1); pushV(5); pushV(popV()+popV()); pushV(3);
pushV(-popV()+popV()); print(popV());
```

Question: what would happen if we put the action of the second rule at the end of rhs?

In contrast to top-down parsers, bottom-up parsers can only perform an action after a reduction (ie, after the entire rhs of a rule has been processed). Why? because at a particular instance of time we may have a potential for multiple rules for reduction (this is the idea behind itemsets), which means that we may be in the middle of many rules at a time, but later only one rule will actually be used; so, we can't execute an action in the middle of a rule because we may have to undo it later if the rule is not used for reduction. This means that we can only have rules of the form

```
X ::= Y1 ... Yn { action }
```

where the action is always at the end of the rule. This action is evaluated after the rule `X ::= Y1 ... Yn` is reduced. To evaluate actions, in addition to state numbers, the parser pushes values into the parse stack: Both terminals and non-terminals are associated with typed values, which in the CUP parser generator are instances of the Object class (or of some subclass of the Object class). Since the Java Object class is a superclass of all classes, it doesn't carry any additional information, but the subclasses of Object, such as the class Integer, the class String, and the class Expr for ASTs, do. The value associated with a terminal is in most cases an Object, except for an identifier which is a String, for an integer which is an Integer, etc. The typical values associated with non-terminals in a compiler are ASTs, lists of ASTs, etc. In CUP, you can retrieve the value of a symbol s at the lhs of a rule by using the notation s:x, where @x@ is a variable name that hasn't appeared elsewhere in this rule. The value of the non-terminal defined by a rule is called RESULT and should always be assigned a value in the action. For example, if the non-terminal E is associated with an integer value (of type Integer), then the following rule:

```
E ::= E:n PLUS E:m    {: RESULT = n+m; :}
```

retrieves the value, n, of the left operand from the parse stack, retrieves the value, m, of the right operand from the parse stack, and pushes the value of RESULT on the parse stack, which has been set to n+m after the reduction of the rule. That is, the elements of the parser stack in CUP are pairs of a state-number (integer) and an Object. So when the above rule is reduced, the three top elements of the stack, which form the handle of the reduction, will contain three elements: the state reached when we reduced the rule for E to get the left operand, the state for shifting over PLUS, and the state reached when we reduced the rule for E to get the right operand (top of stack). Along with these states, there are three Objects: one bound to n, one ignored (since the terminal PLUS is associated with an empty Object, which is ignored), and one bound to m (top of stack). When we reduce by the above rule, we use the GOTO table to find which state to push, we pop the handle (three elements), and we push the pair of this state and the RESULT value on the parse stack.

If we want build an AST in CUP, we need to associate each non-terminal symbol with an AST type. For example, if we use the non-terminals exp and expl for expressions and list of expressions respectively, we can define their types as follows:

```
non terminal Expr exp;
non terminal List expl;
```

Then the production rules should have actions to build ASTs:

```
exp ::= exp:e1 PLUS exp:e2    {: RESULT = new BinaryExp("+",e1,e2); :}
    |   exp:e1 MINUS exp:e2   {: RESULT = new BinaryExp("-",e1,e2); :}
    |   id:nm LP expl:el RP   {: RESULT = new CallExp(nm,el); :}
    |   INT:n                 {: RESULT = new IntegerExp(n); :}
    ;
expl ::= expl:el COMMA exp:e  {: RESULT = el.append(e); :}
     |   exp:e                {: RESULT = nil.cons(e); :}
     ;
```

That is, for integer addition, we build an AST node that corresponds to a binary operation.

What if we want to put an action in the middle of the rhs of a rule in a bottom-up parser? In that case we use a dummy nonterminal, called a *marker*. For example,

```
X ::= a { action } b
```

is equivalent to

```
X ::= M b
M ::= a { action }
```

This is done automatically by the CUP parser generator (ie, we can actually put actions in the middle of a rhs of a rule and CUP will use the above trick to put it at the end of a rule). There is a danger though that the resulting rules may introduce new shift/reduce or reduce/reduce conflicts.

6 Semantic Analysis

6.1 Symbol Tables

After ASTs have been constructed, the compiler must check whether the input program is type-correct. This is called *type checking* and is part of the semantic analysis. During type checking, a compiler checks whether the use of names (such as variables, functions, type names) is consistent with their definition in the program. For example, if a variable x has been defined to be of type int, then x+1 is correct since it adds two integers while x[1] is wrong. When the type checker detects an inconsistency, it reports an error (such as "Error: an integer was expected"). Another example of an inconsistency is calling a function with fewer or more parameters or passing parameters of the wrong type.

Consequently, it is necessary to remember declarations so that we can detect inconsistencies and misuses during type checking. This is the task of a *symbol table*. Note that a symbol table is a compile-time data structure. It's not used during run time by statically typed languages. Formally, a symbol table maps names into declarations (called attributes), such as mapping the variable name x to its type int. More specifically, a symbol table stores:

- for each type name, its type definition (eg. for the C type declaration typedef int* mytype, it maps the name mytype to a data structure that represents the type int*).
- for each variable name, its type. If the variable is an array, it also stores dimension information. It may also store storage class, offset in activation record etc.
- for each constant name, its type and value.
- for each function and procedure, its formal parameter list and its output type. Each formal parameter must have name, type, type of passing (by-reference or by-value), etc.

One convenient data structure for symbol tables is a hash table. One organization of a hash table that resolves conflicts is chaining. The idea is that we have list elements of type:

```
class Symbol {
    public String key;
    public Object binding;
    public Symbol next;
    public Symbol ( String k, Object v, Symbol r ) { key=k; binding=v; next=r; }
}
```

to link key-binding mappings. Here a binding is any Object, but it can be more specific (eg, a Type class to represent a type or a Declaration class, as we will see below). The hash table is a vector Symbol[] of size SIZE, where SIZE is a prime number large enough to have good performance for medium size programs (eg, SIZE=109). The hash function must map any key (ie. any string) into a bucket (ie. into a number between 0 and SIZE-1). A typical hash function is, h(key) = num(key) mod SIZE, where num converts a key to an integer and mod is the modulo operator. In the beginning, every bucket is null. When we insert a new mapping (which is a pair of key-binding), we calculate the bucket location by applying the hash function over the key, we insert the key-binding pair into a Symbol object, and we insert the object at the beginning of the bucket list. When we want to find the binding of a name, we calculate the bucket location using the hash function, and we search the element list of the bucket from the beginning to its end until we find the first element with the requested name.

Consider the following Java program:

```
1) {
2)   int a;
3)   {
4)     int a;
5)     a = 1;
6)   };
7)   a = 2;
8) };
```

The statement `a = 1` refers to the second integer `a`, while the statement `a = 2` refers to the first. This is called a *nested scope* in programming languages. We need to modify the symbol table to handle structures and we need to implement the following operations for a symbol table:

- `insert ( String key, Object binding )`
- `Object lookup ( String key )`
- `begin_scope ()`
- `end_scope ()`

We have already seen `insert` and `lookup`. When we have a new block (ie, when we encounter the token `{`), we begin a new scope. When we exit a block (ie. when we encounter the token `}`) we remove the scope (this is the `end_scope`). When we remove a scope, we remove all declarations inside this scope. So basically, scopes behave like stacks. One way to implement these functions is to use a stack of numbers (from 0 to `SIZE`) that refer to bucket numbers. When we begin a new scope we push a special marker to the stack (eg, -1). When we insert a new declaration in the hash table using `insert`, we also push the bucket number to the stack. When we end a scope, we pop the stack until and including the first -1 marker. For each bucket number we pop out from the stack, we remove the head of the binding list of the indicated bucket number. For example, for the previous C program, we have the following sequence of commands for each line in the source program (we assume that the hash key for `a` is 12):

```
1) push(-1)
2) insert the binding from a to int into the beginning of the list table[12]
   push(12)
3) push(-1)
4) insert the binding from a to int into the beginning of the list table[12]
   push(12)
6) pop()
   remove the head of table[12]
   pop()
7) pop()
   remove the head of table[12]
   pop()
```

Recall that when we search for a declaration using lookup, we search the bucket list from the beginning to the end, so that if we have multiple declarations with the same name, the declaration in the innermost scope overrides the declaration in the outer scope.

The textbook makes an improvement to the above data structure for symbol tables by storing all keys (strings) into another data structure and by using pointers instead of strings for keys in the hash table. This new data structure implements a set of strings (ie. no string appears more than once). This data structure too can be implemented using a hash table. The symbol table itself uses the physical address of a string in memory as the hash key to locate the bucket of the binding. Also the key component of `element` is a pointer to the string. The idea is that not only we save space, since we store a name once regardless of the number of its occurrences in a program, but we can also calculate the bucket location very fast.

6.2 Types and Type Checking

A typechecker is a function that maps an AST that represents an expression into its type. For example, if variable `x` is an integer, it will map the AST that represents the expression `x+1` into the data structure that represents the type `int`. If there is a type error in the expression, such as in `x>"a"`, then it displays an error message (a type error). So before we describe the typechecker, we need to define the data structures for types. Suppose that we have five kinds of types in the language: integers, booleans, records, arrays, and named types (ie. type names that have been defined earlier by some typedef). Then one possible data structure for types is:

```
abstract class Type {
}
class IntegerType extends Type {
   public IntegerType () {}
}
class StringType extends Type {
   public StringType () {}
}
class NamedType extends Type {
   public String name;
   public NamedType ( String n ) { value=n; }
}
class ArrayType extends Type {
   public Type element;
   public ArrayType ( Type et ) { element=et; }
}
class RecordComponent {
   public String attribute;
   public Type type;
   public RecordComponent ( String a, Type t ) { attribute=a; type=t; }
}
```

```
class RecordType extends Type {
    public List<RecordComponent> elements;
    public RecordType ( List<RecordComponent> el ) { elements=el; }
}
```

that is, if the type is an integer or a boolean, there are no extra components. If it is a named type, we have the name of the type. If it is an array, we have the type of the array elements (assuming that the size of an array is unbound, otherwise we must include the array bounds). If it is a record, we have a list of attribute/types (the `RecordComponents` class) to capture the record components.

The symbol table must contain type declarations (ie. typedefs), variable declarations, constant declarations, and function signatures. That is, it should map strings (names) into `Declaration` objects:

```
abstract class Declaration {
}
class TypeDeclaration extends Declaration {
   public Type declaration;
   public TypeDeclaration ( Type t ) { declaration=t; }
}
class VariableDeclaration extends Declaration {
   public Type declaration;
   public VariableDeclaration ( Type t ) { declaration=t; }
}
class ConstantDeclaration extends Declaration {
   public Type declaration;
   public Exp value;
   public ConstantDeclaration ( Type t, Exp v ) { declaration=t; value=v; }
}
class FunctionDeclaration extends Declaration {
   public Type result;
   public List<Type> parameters;
   public FunctionDeclaration ( Type t, List<Type> tl ) {
      result = t;
      parameters = tl;
   }
}
```

If we use the hash table with chaining implementation, the symbol table `symbol_table` would look like this:

```
class Symbol {
    public String key;
    public Declaration binding;
    public Symbol next;
    public Symbol ( String k, Declaration v, Symbol r ) {
       key = k;
       binding = v;
       next = r;
    }
}
Symbol[] symbol_table = new Symbol[SIZE];
```

Recall that the symbol table should support the following operations:

```
insert ( String key, Declaration binding )
Declaration lookup ( String key )
begin_scope ()
end_scope ()
```

The typechecking function may have the following signature:

```
static Type typecheck ( Exp e );
```

The function typecheck must be recursive since the AST structure is recursive. In fact, this function is a tree traversals that checks each node of the AST tree recursively. The body of the typechecker may look like this:

```
static Type typecheck ( Exp e ) {
   if (e instanceof IntegerExp)
      return new IntegerType();
   else if (e instanceof StringExp)
      return new StringType();
   else if (e instanceof VariableExp) {
      VariableExp v = (VariableExp) e;
      Declaration decl = lookup(v.name);
      if (decl == null)
         error("undefined variable: "+v.name);
      else if (decl instanceof VariableDeclaration)
         return ((VariableDeclaration) decl).declaration;
      else error(v.name+" is not a variable name");
   } else if (e instanceof BinaryExp) {
      BinaryExp b = (BinaryExp) e;
      Type left = typecheck(b.left);
      Type right = typecheck(b.right);
      switch ( b.operator ) {
         case "+": if (left instanceof IntegerType
                       && right instanceof IntegerType)
                      return new IntegerType();
                   else error("expected integers in addition");
         ...
      }
   } else if (e instanceof CallExp) {
      CallExp c = (CallExp) e;
      Declaration decl = lookup(c.name);
      if (decl == null)
         error("undefined function");
      else if (!(decl instanceof FunctionDeclaration))
         error("this name is not a function name");
      FunctionDeclaration f = (FunctionDeclaration) decl;
      if (f.parameters.size() != c.arguments.size())
         error("wrong number of arguments");
      for ( int i = 0; i < f.parameters.size(); i++ )
          if (!equal_types(f.parameters.get(i),typecheck(c.arguments.get(i))))
             error("wrong type of the argument in function call");
      return f.result;
   }
   else ...
}
```

where `equal_types(x,y)` checks the types `x` and `y` for equality. We have two types of type equality: type equality based on type name equivalence, or based on structural equivalence. For example, if we have defined `T` to be a synonym for the type `int` and have declared the variable `x` to be of type `T`, then using the first type of equality, `x+1` will cause a type error (since `T` and `int` are different names), while using the second equality, it will be correct.

Note also that since most realistic languages support many binary and unary operators, it will be very tedious to hardwire their typechecking into the typechecker using code. Instead, we can use another symbol table to hold all operators (as well as all the system functions) along with their signatures. This also makes the typechecker easy to change.

6.3 Case Study: The Calculator Interpreter

The AST used in the calculator example is given in Scala as follows:

```
sealed abstract class Expr
case class BinOpExp ( op: String, left: Expr, right: Expr ) extends Expr
case class UnOpExp ( op: String, operand: Expr ) extends Expr
case class CallExp ( name: String, arguments: List[Expr] ) extends Expr
case class IfExp ( condition: Expr, then_expr: Expr, else_expr: Expr ) extends Expr
case class IntConst ( value: Int ) extends Expr
case class RealConst ( value: Float ) extends Expr
case class StringConst ( value: String ) extends Expr
case class Var ( name: String ) extends Expr
```

The calculator parser that constructs these ASTs is given at `http://lambda.uta.edu/cse5317/calc/src/` Most nonterminals and some terminals have a type:

```
terminal String         ID;
terminal Integer        INT;
terminal Float          REALN;
terminal String         STRINGT;
non terminal Expr exp;
non terminal List expl, ids;
non terminal item, prog;
```

For example, each production for the nonterminal `exp` constructs a value of type `Expr` when is reduced. For example, the following line in the calculator parser:

```
    |   exp:e1 PLUS exp:e2     {: RESULT = new BinOpExp("plus",e1,e2); :}
```

constructs a new `Expr` for `exp` when the rule is reduced. This `Expr`, which is assigned to the variable `RESULT`, is a binary operation "plus" with two children, `e1` and `e2`.

Recall that our calculator is an interpreter, rather than a compiler. The symbol table of an interpreter must bind each program variable to both its type and its value. For example, when the interpreter encounters the variable `x` in a program, it must assert the type of `x` (eg, an integer) so it can tell if `x` is used correctly in the program. It also needs the value of `x` because it needs to evaluate `x`. A compiler would only need the type of `x`. Fortunately, our calculator has only one type: double floating points. So there is no need for type information since everything is expected to be (or be able to be converted to) a floating point. The calculator symbol table is an instance of the class `SymbolTable` given in `http://lambda.uta.edu/cse5317/calc/src/edu/uta/calc/SymbolTable.scala`. It is a dictionary of type `List[List[(String,EnvDecl)]]` that maps a name to an `EnvDecl`. It's a list of lists because it is a stack of bindings that implements variable scoping, where each symbol table is a `List[(String,EnvDecl)]` (a list of bindings from names to `EnvDecl`). That way, `begin_scope` pushes an empty list to the stack, while `end_scope` pops the top list. The class `EnvDecl` is:

```
sealed abstract class EnvDecl
case class VarDec ( value: Double ) extends EnvDecl
case class FunDec ( body: Expr, params: List[String] ) extends EnvDecl
```

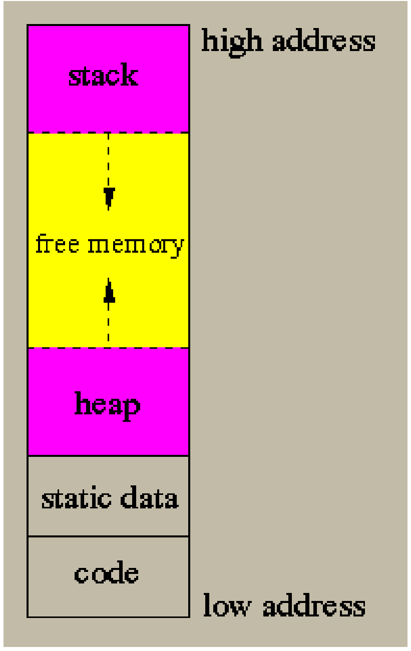
The binding `VarDec` binds a variable name to its actual value (a double floating point number). The binding `FunDec` binds a function name to its declaration, which consists of the function body (code) and the formal parameters.

The interpreter code is given in `http://lambda.uta.edu/cse5317/calc/src/edu/uta/calc/Eval.sca` Function `eval` evaluates the AST `e` using the symbol table `st` and returns a double floating point number. When a variable is encountered, its value is retrieved from the symbol table. A function call can be either a primitive operation, such as an integer addition, or a call to a defined function. In either case, the function arguments must be evaluated before the operation/call is performed. The function call is performed by creating a new scope, binding the functions' formal parameters to the evaluated call arguments, and evaluating the function body using the new scope. After the call, the function scope is popped out.

7 Activation Records

7.1 Run-Time Storage Organization

An executable program generated by a compiler will have the following organization in memory on a typical architecture (such as on MIPS):



This is the layout in memory of an executable program. Note that in a virtual memory architecture (which is the case for any modern operating system), some parts of the memory layout may in fact be located on disk blocks and they are retrieved in memory by demand (lazily).

The machine code of the program is typically located at the lowest part of the layout. Then, after the code, there is a section to keep all the fixed size static data in the program. The dynamically allocated data (ie. the data created using `malloc` in C) as well as the static data without a fixed size (such as arrays of variable size) are created and kept in the heap. The heap grows from low to high addresses. When you call `malloc` in C to create a dynamically allocated structure, the program tries to find an empty place in the heap with sufficient space to insert the new data; if it can't do that, it puts the data at the end of the heap and increases the heap size.

The focus of this section is the stack in the memory layout. It is called the *run-time stack*. The stack, in contrast to the heap, grows in the opposite direction (upside-down): from high to low addresses, which is a bit counterintuitive. The stack is not only used to push the return address when a function is called, but it is also used for allocating some of the local variables of a function during the function call, as well as for some bookkeeping.

Lets consider the lifetime of a function call. When you call a function you not only want to access its parameters, but you may also want to access the variables local to the function. Worse, in a nested scoped system where nested function definitions are allowed, you may want to access the local variables of an enclosing function. In addition, when a function calls another function, we must forget about the variables of the caller function and work with the variables of the callee function and when we return from the callee, we want to switch back to the caller variables. That is, function calls behave in a stack-like manner. Consider for example the following program:

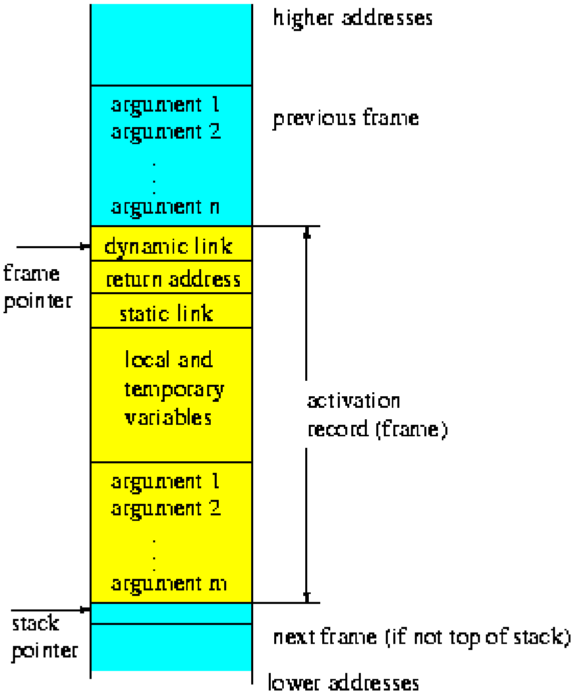
```
procedure P ( c: integer )
  x: integer;

  procedure Q ( a, b: integer )
  i, j: integer;
  begin
    x := x+a+j;
  end;

begin
  Q(x,c);
end;
```

At run-time, P will execute the statement `x := x+a+j` in Q. The variable a in this statement comes as a parameter to Q, while j is a local variable in Q and x is a local variable to P. How do we organize the runtime layout so that we will be able to access all these variables at run-time? The answer is to use the run-time stack.

When we call a function f, we push a new *activation record* (also called a *frame*) on the run-time stack, which is particular to the function f. Each frame can occupy many consecutive bytes in the stack and may not be of a fixed size. When the callee function returns to the caller, the activation record of the callee is popped out. For example, if the main program calls function P, P calls E, and E calls P, then at the time of the second call to P, there will be 4 frames in the stack in the following order: main, P, E, P

Note that a callee should not make any assumptions about who is the caller because there may be many different functions that call the same function. The frame layout in the stack should reflect this. When we execute a function, its frame is located on top of the stack. The function does not have any knowledge about what the previous frames contain. There two things that we need to do though when executing a function: the first is that we should be able to pop-out the current frame of the callee and restore the caller frame. This can be done with the help of a pointer in the current frame, called the *dynamic link*, that points to the previous frame (the caller's frame). Thus all frames are linked together in the stack using dynamic links. This is called the *dynamic chain*. When we pop the callee from the stack, we simply set the stack pointer to be the value of the dynamic link of the current frame. The second thing that we need to do, is if we allow nested functions, we need to be able to access the variables stored in previous activation records in the stack. This is done with the help of the *static link*. Frames linked together with static links form a *static chain*. The static link of a function f points to the latest frame in the stack of the function that statically contains f. If f is not lexically contained in any other function, its static link is null. For example, in the previous program, if P called Q then the static link of Q will point to the latest frame of P in the stack (which happened to be the previous frame in our example). Note that we may have multiple frames of P in the stack; Q will point to the latest. Also notice that there is no way to call Q if there is no P frame in the stack, since Q is hidden outside P in the program.

A typical organization of a frame is the following:



Before we create the frame for the callee, we need to allocate space for the callee's arguments. These arguments belong to the caller's frame, not to the callee's frame. There is a frame pointer (called FP) that points to the beginning of the frame. The stack pointer points to the first available byte in the stack immediately after the end of the current frame (the most recent frame). There are many variations to this theme. Some compilers use *displays* to store the static chain instead of using static links in the stack. A display is a register block where each pointer points to a consecutive frame in the static chain of the current frame. This allows a very fast access to the variables of deeply nested functions. Another important variation is to pass arguments to functions using registers.

When a function (the caller) calls another function (the callee), it executes the following code before (the pre-call) and after (the post-call) the function call:

- *pre-call:* allocate the callee frame on top of the stack; evaluate and store function parameters in registers or in the stack; store the return address to the caller in a register or in the stack.

- *post-call:* copy the return value; deallocate (pop-out) the callee frame; restore parameters if they passed by reference.

In addition, each function has the following code in the beginning (prologue) and at the end (epilogue) of its execution code:

- *prologue:* store frame pointer in the stack or in a display; set the frame pointer to be the top of the stack; store static link in the stack or in the display; initialize local variables.

- *epilogue:* store the return value in the stack; restore frame pointer; return to the caller.

We can classify the variables in a program into four categories:

- statically allocated data that reside in the static data part of the program; these are the global variables.

- dynamically allocated data that reside in the heap; these are the data created by malloc in C.

- register allocated variables that reside in the CPU registers; these can be function arguments, function return values, or local variables.

- frame-resident variables that reside in the run-time stack; these can be function arguments, function return values, or local variables.

Every frame-resident variable (ie. a local variable) can be viewed as a pair of (level,offset). The variable level indicates the lexical level in which this variable is defined. For example, the variables inside the top-level functions (which is the case for all functions in C) have level=1. If a function is nested inside a top-level function, then its variables have level=2, etc. The offset indicates the relative distance from the beginning of the frame that we can find the value of a variable local to this frame. For example, to access the nth argument of the frame in the above figure, we retrieve the stack value at the address `FP+1`, and to access the first argument, we retrieve the stack value at the address `FP+n` (assuming that each argument occupies one word). When we want to access a variable whose level is different from the level of the current frame (ie. a non-local variable), we subtract the level of the variable from the current level to find out how many times we need to follow the static link (ie. how deep we need to go in the static chain) to find the frame for this variable. Then, after we locate the frame of this variable, we use its offset to retrieve its value from the stack. For example, to retrieve to value of `x` in the `Q` frame, we follow the static link once (since the level of `x` is 1 and the level of `Q` is 2) and we retrieve `x` from the frame of `P`.

Another thing to consider is what exactly do we pass as arguments to the callee. There are two common ways of passing arguments: by value and by reference. When we pass an argument by reference, we actually pass the address of this argument. This address may be an address in the stack, if the argument is a frame-resident variable. A third type of passing arguments, which is not used anymore but it was used in Algol, is call by name. In that case we create a *thunk* inside the caller's code that behaves like a function and contains the code that evaluates the argument passed by name. Every time the callee wants to access the call-by-name argument, it calls the thunk to compute the value for the argument.

7.2 Case Study: Activation Records for the MIPS Architecture

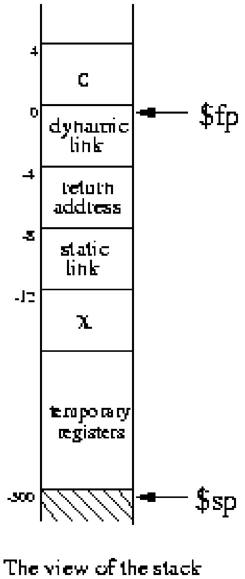
The following describes a function call abstraction for the MIPS architecture. This may be slightly different from the one you will use for the project.

MIPS uses the register `$sp` as the stack pointer and the register `$fp` as the frame pointer. In the following MIPS code we use both a dynamic link and a static link embedded in the activation records.

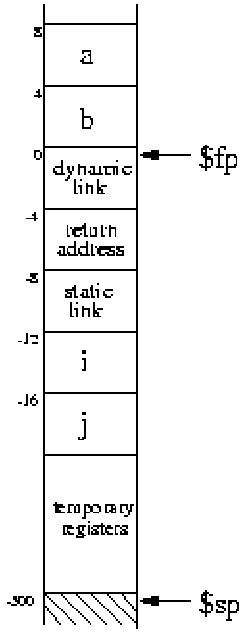
Consider the previous program:

```
procedure P ( c: integer )
  x: integer;

  procedure Q ( a, b: integer )
  i, j: integer;
  begin
    x := x+a+j;
  end;

begin
  Q(x,c);
end;
```

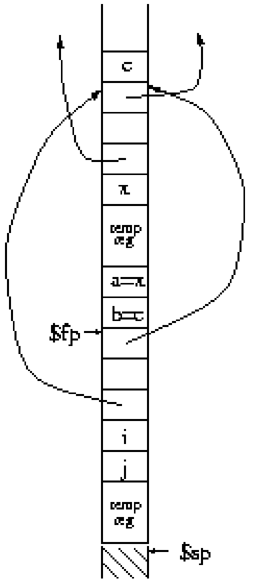
The activation record for `P` (as `P` sees it) is shown in the first figure below:



The view of the stack inside procedure P



The view of the stack inside procedure Q



Run-time stack at the point of x := x+a+j

The activation record for `Q` (as `Q` sees it) is shown in the second figure above. The third figure shows the structure of the run-time stack at the point where `x := x+a+j` is executed. This statement uses `x`, which is defined in `P`. We can't assume that `Q` called `P`, so we should not use the dynamic link to retrieve `x`; instead, we need to use the static link, which points to the most recent activation record of `P`. Thus, the value of variable `x` is computed by:

```
        lw        $t0, -8($fp)                # follow the static link of Q
        lw       $t1, -12($t0)              # x has offset=-12 inside P
```

Function/procedure arguments are pushed in the stack before the function call. If this is a function, then an empty placeholder (4 bytes) should be pushed in the stack before the function call; this will hold the result of the function.

Each procedure/function should begin with the following code (prologue):

```
        sw      $fp, ($sp)      # push old frame pointer (dynamic link)
        move    $fp, $sp        # frame pointer now points to the top of stack
        subu    $sp, $sp, 500   # allocate say 500 bytes in the stack
                                #   (for frame size = 500)
        sw      $ra, -4($fp)    # save return address in frame
        sw      $v0, -8($fp)    # save static link in frame
```

(where `$v0` is set by the caller – see below) and should end with the following code (epilogue):

```
        lw      $ra, -4($fp)    # restore return address
        move    $sp, $fp        # pop frame
        lw      $fp, ($fp)      # restore old frame pointer (follow dynamic link)
        jr      $ra             # return
```

For each procedure call, you need to push the arguments into the stack and set `$v0` to be the right static link (very often it is equal to the static link of the current procedure; otherwise, you need to follow the static link a number of times). For example, the call `Q(x,c)` in `P` is translated into:

```
        lw      $t0, -12($fp)
        sw      $t0, ($sp)        # push x
        subu    $sp, $sp, 4
        lw      $t0, 4($fp)
        sw      $t0, ($sp)        # push c
        subu    $sp, $sp, 4
        move    $v0, $fp          # load static link in $v0
        jal     Q                 # call procedure Q
        addu    $sp, $sp, 8       # pop stack
```

Note that there are two different cases for setting the static link before a procedure call. Lets say that caller_level and callee_level are the nesting levels of the caller and the callee procedures (recall that the nesting level of a top-level procedure is 0, while the nesting level of a nested procedure embedded inside another procedure with nesting level l, is $l+1$). When the callee is lexically inside the caller's body, that is, when callee_level=caller_level+1, we have:

```
        move    $v0, $fp
```

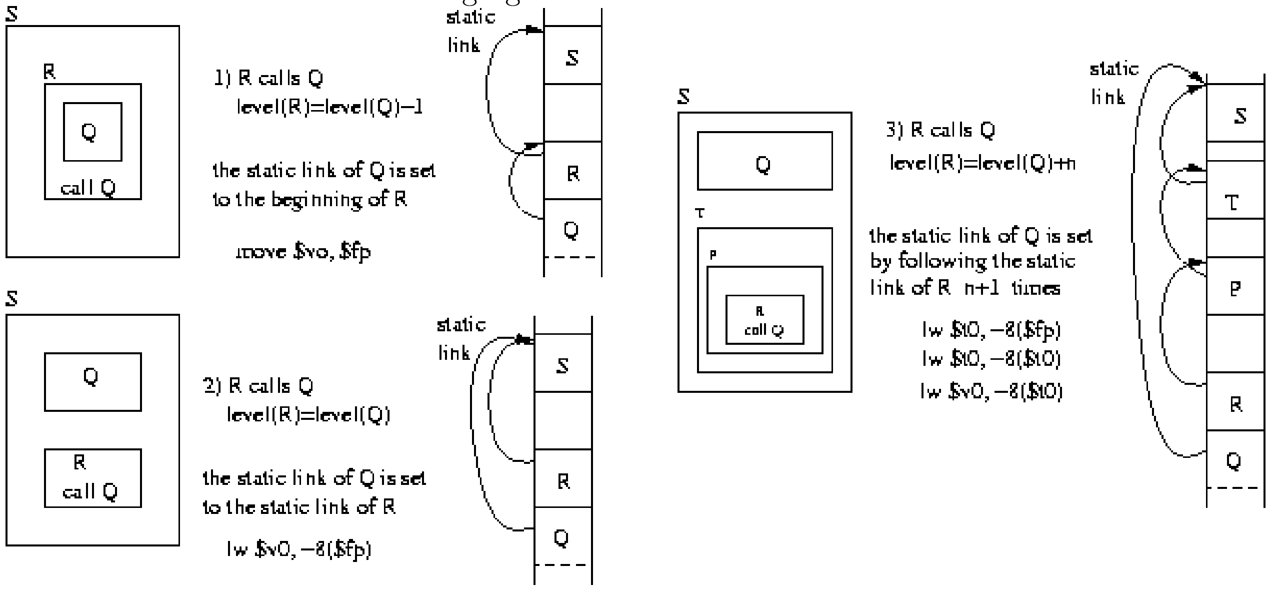
The call `Q(x,c)` in `P` is such a case because the nesting levels of `P` and `Q` are 0 and 1, respectively. Otherwise, we follow the static link of the caller $d + 1$ times, where d=caller_level-callee_level (the difference between the nesting level of the caller from that of the callee). For d=0, that is, when both caller and callee are at the same level, we have

```
        lw        $v0, -8($fp)
```

For d=2 we have

```
        lw        $t1, -8($fp)
        lw        $t1, -8($t1)
        lw        $v0, -8($t1)
```

These cases are shown in the following figure:



Note also that, if you have a call to a function, you need to allocate 4 more bytes in the stack to hold the result.

See `http://lambda.uta.edu/cse5317/spring98/note2.html` for a concrete example of a function expressed in MIPS code.

8 Intermediate Code

The semantic phase of a compiler first translates parse trees into an intermediate representation (IR), which is independent of the underlying computer architecture, and then generates machine code from the IRs. This makes the task of retargeting the compiler to another computer architecture easier to handle.

We will consider Tiger for a case study. For simplicity, all data are one word long (four bytes) in Tiger. For example, strings, arrays, and records, are stored in the heap and they are represented by one pointer (one word) that points to their heap location. We also assume that there is an infinite number of temporary registers (we will see later how to spill-out some temporary registers into the stack frames). The IR trees for Tiger are used for building expressions and statements. They are described in detail in Section 7.1 in the textbook. Here is a brief description of the meaning of the expression IRs:

- CONST(i): the integer constant i.

- MEM(e): if e is an expression that calculates a memory address, then this is the contents of the memory at address e (one word).
- NAME(n): the address that corresponds to the label n, eg. MEM(NAME(x)) returns the value stored at the location X.
- TEMP(t): if t is a temporary register, return the value of the register, eg.

  ```
  MEM(BINOP(PLUS,TEMP(fp),CONST(24)))
  ```

  fetches a word from the stack located 24 bytes above the frame pointer.
- BINOP(op,e1,e2): evaluate e1, then evaluate e2, and finally perform the binary operation op over the results of the evaluations of e1 and e2. op can be PLUS, AND, etc. We abbreviate BINOP(PLUS,e1,e2) as +(e1,e2).
- CALL(f,[e1,e2,...,en]): evaluate the expressions e1, e2, etc (in that order), and at the end call the function f over these n parameters. eg.

  ```
  CALL(NAME(g),ExpList(MEM(NAME(a)),ExpList(CONST(1),NULL)))
  ```

  represents the function call g(a,1).
- ESEQ(s,e): execute statement s and then evaluate and return the value of the expression e

And here is the description of the statement IRs:

- MOVE(TEMP(t),e): store the value of the expression e into the register t.
- MOVE(MEM(e1),e2): evaluate e1 to get an address, then evaluate e2, and then store the value of e2 in the address calculated from e1. eg,

  ```
  MOVE(MEM(+(NAME(x),CONST(16))),CONST(1))
  ```

  computes x[4] := 1 (since 4*4bytes = 16 bytes).
- EXP(e): evaluates e and discards the result.
- JUMP(L): Jump to the address L (which must be defined in the program by some LABEL(L)).
- CJUMP(o,e1,e2,t,f): evaluate e1, then e2. The binary relational operator o must be EQ, NE, LT etc. If the values of e1 and e2 are related by o, then jump to the address calculated by t, else jump the one for f.
- SEQ(s1,s2,...,sn): perform statement s1, s2, ... sn is sequence.
- LABEL(n): define the name n to be the address of this statement. You can retrieve this address using NAME(n).

In addition, when JUMP and CJUMP IRs are first constructed, the labels are not known in advance but they will be known later when they are defined. So the JUMP and CJUMP labels are first set to NULL and then later, when the labels are defined, the NULL values are changed to real addresses.

8.1 Translating Variables, Records, Arrays, and Strings

Local variables located in the stack are retrieved using an expression represented by the IR: MEM(+(TEMP(fp),CONST(offset))). If a variable is located in an outer static scope k levels lower than the current scope, the we retrieve the variable using the IR:

```
MEM(+(MEM(+(...MEM(+(MEM(+(TEMP(fp),CONST(static))),CONST(static))),...)),
      CONST(offset)))
```

where static is the offset of the static link. That is, we follow the static chain k times, and then we retrieve the variable using the offset of the variable.

An *l-value* is the result of an expression that can occur on the left of an assignment statement. eg, x[f(a,6)].y is an l-value. It denotes a location where we can store a value. It is basically constructed by deriving the IR of the value and then dropping the outermost MEM call. For example, if the value is MEM(+(TEMP(fp),CONST(offset))), then the l-value is +(TEMP(fp),CONST(offset)).

In Tiger (the language described in the textbook), vectors start from index 0 and each vector element is 4 bytes long (one word), which may represent an integer or a pointer to some value. To retrieve the ith element of an array a, we use MEM(+(A,*(I,4))), where A is the address of a (eg. A is +(TEMP(fp),CONST(34))) and I is the value of i (eg. MEM(+(TEMP(fp),CONST(26)))). But this is not sufficient. The IR should check whether $a < size(a)$: CJUMP(gt,I,CONST(size_of_A),MEM(+(A,*(I,4))),NAME(error_label)), that is, if i is out of bounds, we should raise an exception.

For records, we need to know the byte offset of each field (record attribute) in the base record. Since every value is 4 bytes long, the ith field of a structure a can be retrieved using MEM(+(A,CONST(i*4))), where A is the address of a. Here i is always a constant since we know the field name. For example, suppose that i is located in the local frame with offset 24 and a is located in the immediate outer scope and has offset 40. Then the statement `a[i+1].first := a[i].second+2` is translated into the IR:

```
MOVE(MEM(+(+(TEMP(fp),CONST(40)),
           *(+(MEM(+(TEMP(fp),CONST(24))),
              CONST(1)),
             CONST(4)))),
     +(MEM(+(+(+(TEMP(fp),CONST(40)),
                *(MEM(+(TEMP(fp),CONST(24))),
                  CONST(4))),
             CONST(4))),
       CONST(2)))
```

since the offset of first is 0 and the offset of second is 4.

In Tiger, strings of size n are allocated in the heap in $n+4$ consecutive bytes, where the first 4 bytes contain the size of the string. The string is simply a pointer to the first byte. String literals are statically allocated. For example, the MIPS code:

```
ls:     .word        14
        .ascii       "example string"
```

binds the static variable `ls` into a string with 14 bytes. Other languages, such as C, store a string of size n into a dynamically allocated storage in the heap of size $n+1$ bytes. The last byte has a null value to indicate the end of string. Then, you can allocate a string with address A of size n in the heap by adding $n+1$ to the global pointer ($gp in MIPS):

```
MOVE(A,ESEQ(MOVE(TEMP(gp),+(TEMP(gp),CONST(n+1))),TEMP(gp)))
```

8.2 Control Statements

Statements, such as for-loop and while-loop, are translated into IR statements that contain jumps and conditional jumps.

The while loop `while c do body;` is evaluated in the following way:

```
loop: if c goto cont else goto done
cont: body
      goto loop
done:
```

which corresponds to the following IR:

```
SEQ(LABEL(loop),
    CJUMP(EQ,c,1,NAME(done),NAME(cont)),
    LABEL(cont),
    s,
    JUMP(NAME(loop)),
    LABEL(done))
```

The for statement `for i:=lo to hi do body` is evaluated in the following way:

```
      i := lo
      j := hi
      if i>j goto done
loop: body
      i := i+1
      if i<=j goto loop
done:
```

The `break` statement is translated into a JUMP. The compiler keeps track which label to JUMP to on a “break” statement by maintaining a stack of labels that holds the “done:” labels of the for- or while-loop. When it compiles a loop, it pushes the label in the stack, and when it exits a loop, it pops the stack. The `break` statement is thus translated into a JUMP to the label at the top of the stack.

A function call `f(a1,...,an)` is translated into the IR `CALL(NAME(l_f),[sl,e1,...,en])`, where `l_r` is the label of the first statement of the `f` code, `sl` is the static link, and `ei` is the IR for `ai`. For example, if the difference between the static levels of the caller and callee is one, then `sl` is `MEM(+(TEMP(fp),CONST(static)))`, where `static` is the offset of the static link in a frame. That is, we follow the static link once. The code generated for the body of the function `f` is a sequence which contains the prolog, the code of `f`, and the epilogue, as it was discussed in Section 7.

For example, suppose that records and vectors are implemented as pointers (i.e. memory addresses) to dynamically allocated data in the heap. Consider the following declarations:

```
struct { X: int, Y: int, Z: int } S;      /* a record */
int i;
int V[10][10];                            /* a vector of vectors */
```

where the variables `S`, `i`, and `V` are stored in the current frame with offsets -16, -20, and -24 respectively. By using the following abbreviations:

```
S = MEM(+(TEMP(fp),CONST(-16)))
I = MEM(+(TEMP(fp),CONST(-20)))
V = MEM(+(TEMP(fp),CONST(-24)))
```

we have the following IR translations:

```
S.Z+S.X
  --> +(MEM(+(S,CONST(8))),MEM(S))

if (i<10) then S.Y := i else i := i-1
  --> SEQ(CJUMP(LT,I,CONST(10),trueL,falseL),
          LABEL(trueL),
          MOVE(MEM(+(S,CONST(4))),I),
          JUMP(exit),
          LABEL(falseL),
          MOVE(I,-(I,CONST(1))),
          JUMP(exit),
          LABEL(exit))

V[i][i+1] := V[i][i]+1
  --> MOVE(MEM(+(MEM(+(V,*(I,CONST(4)))),*(+(I,CONST(1)),CONST(4)))),
           MEM(+(MEM(+(V,*(I,CONST(4)))),*(I,CONST(4)))))

for i:=0 to 9 do V[0][i] := i
  --> SEQ(MOVE(I,CONST(0)),
          MOVE(TEMP(t1),CONST(9)),
          CJUMP(GT,I,TEMP(t1),done,loop),
          LABEL(loop),
          MOVE(MEM(+(MEM(V),*(I,CONST(4)))),I),
          MOVE(I,+(I,CONST(1))),
          CJUMP(LEQ,I,TEMP(t1),loop,done),
          LABEL(done))
```

9 Basic Blocks and Traces

Many computer architectures have instructions that do not exactly match the IR representations given in the previous sections. For example, they do not support two-way branching as in `CJUMP(op,e1,e2,l1,l2)`. In addition, nested calls, such as `CALL(f,[CALL(g,[1])])`, will cause interference between register arguments and returned results. The nested SEQs, such as `SEQ(SEQ(s1,s2),s3)`, impose an order of a evaluation, which restricts optimization (eg, if `s1` and `s2` do not interfere with each other, we would like to be able to switch `SEQ(s1,s2)` with `SEQ(s2,s1)` because it may result to a more efficient program.

The problems mentioned above can be eliminated in two phases:

1. transforming IR trees into a list of *canonical trees*, and
2. transforming unrestricted CJUMPs into CJUMPs that are followed by their false target label.

An IR is a canonical tree if it does not contain SEQ or ESEQ and the parent node of each CALL node is either an EXP or a MOVE(TEMP(t),...) node. The idea is that we transform an IR in such a way that all ESEQs are pulled up in the IR and become SEQs at the top of the tree. At the end, we are left with nested SEQs at the top of the tree, which are eliminated to form a list of statements. For example, the IR:

```
SEQ(MOVE(NAME(x),ESEQ(MOVE(TEMP(t),CONST(1)),TEMP(t))),
    JUMP(ESEQ(MOVE(NAME(z),NAME(L)),NAME(z))))
```

is translated into:

```
SEQ(SEQ(MOVE(TEMP(t),CONST(1)),
        MOVE(NAME(x),TEMP(t)))
    SEQ(MOVE(NAME(z),NAME(L)),
        JUMP(NAME(z))))
```

which corresponds to a list of statements (if we remove the top SEQs):

```
[ MOVE(TEMP(t),CONST(1)), MOVE(NAME(x),TEMP(t)),
  MOVE(NAME(z),NAME(L)), JUMP(NAME(z)) ]
```

Here are some rules to make canonical trees:

- ESEQ(s1,ESEQ(s2,e)) = ESEQ(SEQ(s1,s2),e)
- BINOP(op,ESEQ(s,e1),e2) = ESEQ(s,BINOP(op,e1,e2))

- MEM(ESEQ(s,e)) = ESEQ(s,MEM(e))
- JUMP(ESEQ(s,e)) = SEQ(s,JUMP(e))
- CJUMP(op,ESEQ(s,e1),e2,l1,l2) = SEQ(s,CJUMP(op.e1,e2,l1,l2))
- BINOP(op,e1,ESEQ(s,e2))
  = ESEQ(MOVE(temp(t),e1),ESEQ(s,BINOP(op,TEMP(t),e2)))
- CJUMP(op,e1,ESEQ(s,e2),l1,l2)
  = SEQ(MOVE(temp(t),e1),SEQ(s,CJUMP(op,TEMP(t),e2,l1,l2)))
- MOVE(ESEQ(s,e1),e2) = SEQ(s,MOVE(e1,e2))

To handle function calls, we store the function results into a new register:
CALL(f,a) = ESEQ(MOVE(TEMP(t),CALL(f,a)),TEMP(t))

That way expressions, such as `+(CALL(f,a),CALL(g,b))`, would not rewrite each others result register.

Now the next thing to do is to transform any CJUMP into a CJUMP whose false target label is the next instruction after CJUMP. This reflects the conditional JUMP found in most architectures (ie. "if condition then JUMP label"). To do so, we first form *basic blocks* from the IR tree. A basic block is a sequence of statements whose first statement is a LABEL, the last statement is a JUMP or CJUMP, and does not contain any other LABELs, JUMPs, or CJUMPs. That is, we can only enter at the beginning of a basic block and exit at the end. To solve the CJUMP problem, we first create the basic blocks for an IR tree and then we reorganize the basic blocks in such a way that every CJUMP at the end of a basic block is followed by a block the contains the CJUMP false target label. A secondary goal is to put the target of a JUMP immediately after the JUMP. That way, we can eliminate the JUMP (and maybe merge the two blocks). The algorithm is based on *traces*: You start a trace with an unmark block and you consider the target of the JUMP of this block or the false target block of its CJUMP. Then, if the new block is unmarked, you append the new block to the trace, you use it as your new start, and you apply the algorithm recursively; otherwise, you close this trace and you start a new trace by going back to a point where there was a CJUMP and you choose the true target this time. You continue until all blocks are marked. This is a greedy algorithm. At the end, there may be still some CJUMPs that have a false target that does not follow the CJUMP (this is the case where this false target label was the target of another JUMP or CJUMP found earlier in a trace). In that case:

- if we have a CJUMP followed by a true target, we negate the condition and switch the true and false targets;
- otherwise, we create a new block LABEL(L) followed by JUMP(F) and we replace CJUMP(op,a,b,T,F) with CJUMP(op,a,b,T,L).

Also, if there is a JUMP(L) followed by a LABEL(L), we remove the JUMP.

10 Instruction Selection

After IR trees have been put into a canonical form (described in the previous section), they are used in generating assembly code. The obvious way to do this is to macroexpand each IR tree node. For example, `MOVE(MEM(+(TEMP(fp),CONST(10))),CONST(3))` is macroexpanded into the pseudo-assembly code at the right column:

```
TEMP(fp)                               t1 := fp
CONST(10)                              t2 := 10
+(TEMP(fp),CONST(10))                  t3 := t1+t2
CONST(3)                               t4 := 3
MOVE(MEM(...),CONST(3))                M[t3] := t4
```

where ti stands for a temporary variable. This method generates very poor quality code. For example, the IR above can be done using only one instruction in most architectures: `M[fp+10] := 3`.

A method, called *maximum munch*, generates better code, especially for RISC machines. The idea is to use tree pattern matching to map a tree pattern (a fragment of an IR tree) into a list of assembly instructions. These tree patterns are called *tiles*. For RISC we always have one-to-one mapping (one tile to one assembly instruction). Note that for RISC machines the tiles are small (very few number of IR nodes) but for CISC machines the tiles are usually large since the CISC instructions are very complex.

The following is the mapping of some tiles (left) into MIPS code (right):

```
CONST(c)                               li   ’d0, c
+(e0,e1)                               add  ’d0, ’s0, ’s1
+(e0,CONST(c))                         add  ’d0, ’s0, c
*(e0,e1)                               mult ’d0, ’s0, ’s1
*(e0,CONST(2^k))                       sll  ’d0, ’s0, k
MEM(e0)                                lw   ’d0, (’s0)
MEM(+(e0,CONST(c)))                    lw   ’d0, c(’s0)
MOVE(MEM(e0),e1)                       sw   ’s1, (’s0)
MOVE(MEM(+(e0,CONST(c))),e1)           sw   ’s1, c(’s0)
JUMP(NAME(X))                          b    X
JUMP(e0)                               jr   ’s0
LABEL(X)                           X:  nop
```

Here `e0` and `e1` are subtrees of these IR tree nodes. Most of the assembly instructions above have one destination register `d0` and a number of source registers, `s0` and `s1`. So if a tree pattern, such as `MOVE(MEM(e0),e1)`, has two input subtrees `e0` and `e1`, then their values are taken from the source registers `s0` and `s1` (which are the destinations of `e0` and `e1`). The quote in the assembly instructions is used to indicate that temporary registers should be selected so that there is this match of source-destination names.

To translate an IR tree into assembly code, we perform a *tiling*: we cover the IR tree with non-overlapping tiles. We can see that there are many different tilings. Consider for example the tiger statement `a[i] := x`, where `i` is a temporary register, the `a` address is stored at the offset 20, and `x` at the offset 10. The IR form is:

```
MOVE(MEM(+(MEM(+(fp,CONST(20))),*(TEMP(i),CONST(4)))),MEM(+(fp,CONST(10))))
```

The following are two possible tilings of the IR:

```
lw   r1, 20($fp)                     add  r1, $fp, 20
sll  r2, r1, 2                       lw   r1, (r1)
add  r1, r1, r2                      sll  r2, r1, 2
lw   r2, 10($fp)                     add  r1, r1, r2
sw   r2, (r1)                        add  r2, $fp, x
                                     lw   r2, (r2)
                                     sw   r2, (r1)
```

The first tiling is obviously better since it can be executed faster.

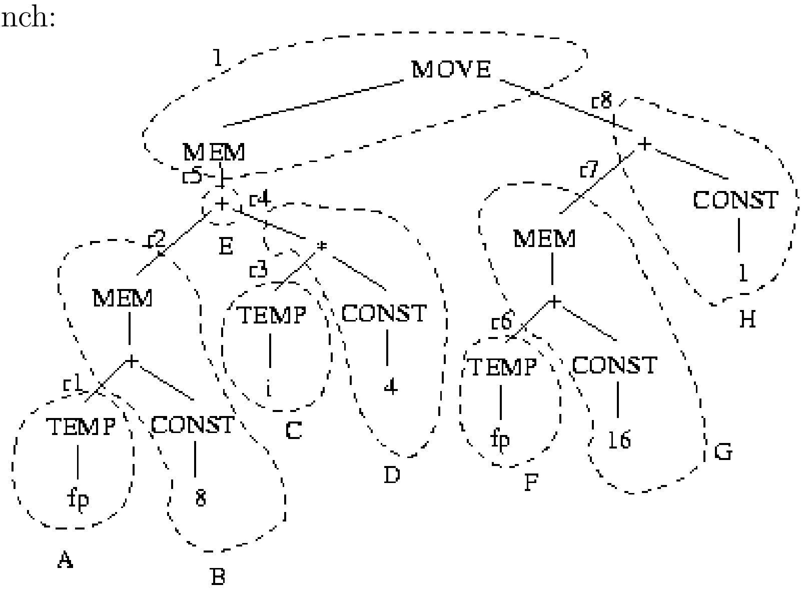
It's highly desirable to do *optimum tiling*, ie, to generate the shortest instruction sequence (or alternatively the sequence with the fewest machine cycles). But this is not always easy to achieve. So most compilers do an *optimal tiling* where no two adjacent tiles can be combined into a tile of lower cost. Optimum tiling is not always optimal but it's close for RISC machines.

There are two main ways of performing optimum tiling: using *maximal munch* (a greedy algorithm) or using dynamic programming. In maximal munch you start from the IR root and from all matching tiles you select the one with the maximum number of IR nodes (largest tile). Then you go to the children (subtrees) of this tile and apply the algorithm recursively until you reach the tree leaves. The dynamic programming works from the leaves to the root. It assigns a cost to every tree node by considering every tile that matches the node and calculating the minimum value of:

cost of a node = (number of nodes in the tile) + (total costs of all the tile children)

(the costs of the tile children are known since the algorithm is bottom-up).

For example, consider the following IR represented as a tree and its tiling using maximal munch:



The order of tiling is IEHBDGACF (ie, top-down). After we set the tiles, we use a different register `ri` for each connection between the tiles. Then we expand the tiles into MIPS code by following the tile mapping table. The order of tile expansion is ABCDEFGHI (ie, bottom-up). The MIPS code is:

```
A       lw   r1, fp
B       lw   r2, 8(r1)
C       lw   r3, i
D       sll  r4, r3, 2
E       add  r5, r2, r4
F       lw   r6, fp
G       lw   r7, 16(r6)
H       add  r8, r7, 1
I       sw   r8, (r5)
```

11 Liveness Analysis

When we generated IR trees, we assumed that we have a very large number of temporary variables stored in registers. Of course this is not true for real machines. In particular CISC machines have very few registers (Pentium has 6 general registers only). So it's highly desirable to use one machine register for multiple temporary variables (instead of using one register for one temporary variable). Consider this program:

```
1.      a := 0
2. L1: b := a+1
3.      c := c+b
4.      a := b*2
5.      if a<10 goto L1
6.      return c
```

Obviously we need a maximum of three registers, one for each variable `a`, `b`, and `c`. But we can do better if we use two registers: one for `c` and one for both `a` and `b`. This is possible because after `a+1` is computed in statement 2 and during statements 3 and 4, the value of `a` is not used, and these are the only places where `b` is used. We say that variables `a` and `b` do not *interfere* if they are not *live* during the same periods in the program. In that case, they can occupy the same register. A variable `x` is *live* at a particular point (statement) in a program, if it holds a value that may be needed in the future. That is, `x` is live at a particular point if there is a path (possibly following gotos) from this point to a statement that uses `x` and there is no assignment to `x` in any statement in the path (because if there was an assignment to `x`, the old value of `x` is discarded before it is used). For example, `a` is live in 2 (because it's the place where it is used), not live in 3 and 4 (because it is assigned a value in 4 before it is used in 5), and live again in 5. Variable `b` is live in 3 and 4 only, and variable `c` is live in 2, 3, 4, 5, and 6 (it's live in 2 because there is a path to 3 where it is used). In general, let's say that you have a use of a variable `v` in line `n`:

```
k.    v := ...
      ...
n.    x := ... v ..
```

(Here `v` is used in the source of an assignment but it may have also been used in a function call, return statement, etc.) We try to find an assignment `v := ...` such that there is path from this statement to line `n` and there is no other assignment to `v` along this path. In that case, we say that `v` is live along this path (immediately after the assignment until and including line `n`). We do that for every use of `v` and the union of all these regions in which `v` is live gives us the life of `v`. In our example, the life of variables are:

	a	b	c
1			X
2	X		X
3		X	X
4		X	X
5	X		X
6			X

Let's formalize the liveness of a variable. The first thing to do is to construct the *control flow graph* (CFG) of a program. The CFG nodes are individual statements (or maybe basic blocks) and the graph edges represent potential flow of control. The outgoing edges of a node n are $succ[n]$ and the ingoing edges are $pred[n]$. For example, $succ[5] = [6, 2]$ and $pred[5] = [4]$. For each CFG node n (a statement) we define $use[n]$ to be all the variables used (read) in this statement and $def[n]$ all the variables assigned a value (written) in this statement. For example, $use[3] = [b, c]$ and $def[3] = [c]$.

We say that variable v is live at a statement n if there is a path in the CFG from this statement to a statement m such that $v \in use[m]$ and for each $n \leq k < m : v \notin def[k]$. That is, there is no assignment to v in the path from n to m. For example, `c` is alive in 4 since it is used in 6 and there is no assignment to `c` in the path from 4 to 6.

The *liveness analysis* analyzes a CFG to determine which places variables are live or not. It is a data flow analysis since it flows around the edges of the CFG information about variables (in this case information about the liveness of variables). For each CFG node n we derive two sets: $in[n]$ (live-in) and $out[n]$ (live-out). The set $in[n]$ gives all variables that are live before the execution of statement n and $out[n]$ gives all variables that are live after the execution of statement n. So the goal here is to compute these sets from the sets $succ$, use and def. To do this, we consider the properties of in and out:

1. $v \in use[n] \Rightarrow v \in in[n]$. That is, if v is used in n then is live-in in n (regardless whether it is defined in n).

2. $v \in (out[n] - def[n]) \Rightarrow v \in in[n]$. That is, if v is live after the execution of n and is not defined in n, then v is live before the execution of n.

3. for each $s \in succ[n] : v \in in[s] \Rightarrow v \in out[n]$. This reflects the formal definition of the liveness of variable v.

The algorithm to compute *in* and *out* is the following:

```
**foreach** $n$ : $in[n] \leftarrow \emptyset; out[n] \leftarrow \emptyset$
**repeat**
   **foreach** $n$ :
         $in'[n] \leftarrow in[n]$
         $out'[n] \leftarrow out[n]$
         $in[n] \leftarrow use[n] \cup (out[n] - def[n])$
         $out[n] \leftarrow \bigcup_{s \in succ[n]} in[s]$
**until** $in' = in \wedge out' = out$
```

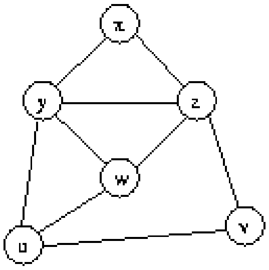
That is, we store the old values of *in* and *out* into in' and out' and we repeatedly execute the loop until we can't add any more elements. The algorithm converges very fast if we consider the CFG nodes in the reverse order (when is possible), ie, in the order 6, 5, 4, 3, 2, 1. See Table 10.6 in the textbook for an example.

The life of a variable can be directly derived from vector *in*: if $v \in in[n]$ then v is live on line n. Then, from the variable lives we can compute the *interference graph*. The nodes of this graph are the program variables and for each node v and w there is an interference edge if the lives of the variables v and w overlap in at least one program point (statement). For each CFG node n that assigns the value to the variable a (ie, $a \in def[n]$) we add the edges (a, b_1), $(a, b_2), \ldots, (a, b_m)$, where $out[n] = \{b_1, b_2, \ldots, b_m\}$. There is a special case when n is a move command: $a \leftarrow c$; in that case we do not add the edge (a, b_k) if $b_k = c$. For example, the previous program has an interference graph with three nodes: `a`, `b`, and `c`, and two edges: `a-c` and `b-c`.

The following is a larger example:

		x	y	z	w	u	v
1.	v := 1						
2.	z := v+1						X
3.	x := z * v			X			X
4.	y := x * 2	X		X			
5.	w := x+z * y	X	X	X			
6.	u := z+2		X	X	X		
7.	v := u+w+y		X		X	X	
8.	return v * u					X	X

For example, v is used in line 3 and the last assignment to v before this line was in line 1. So v is live on lines 2 and 3. Also v is used in line 8 and the last assignment to v before this line was in line 7. So v is also live on line 8. The interference graph is the following:



12 Register Allocation

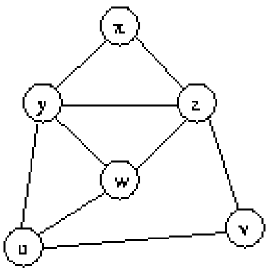
12.1 Register Allocation Using the Interference Graph

The interference graph is used for assigning registers to temporary variables. If two variables do not interfere (ie, there is no edge between these two variables in the interference graph) then we can use the same register for both of them, thus reducing the number of registers needed. On the other hand, if there is a graph edge between two variables, then we should not assign the same register to them since this register needs to hold the values of both variable at one point of time (because the lives of these variables overlap at one point of time – this is what interference means).

Suppose that we have n available registers: $r_1, r_2, \ldots, r_n$. If we view each register as a different color, then the register allocation problem for the interference graph is equivalent to the graph coloring problem where we try to assign one of the n different colors to graph nodes so that no two adjacent nodes have the same color. This algorithm is used in map drawing where we have countries or states on the map and we want to color them using a small fixed number of colors so that states that have a common border are not painted with the same color. The graph in this case has one node for each state and an edge between two states if they have common borders.

The register allocation algorithm uses a stack of graph nodes to insert all nodes of the interference graph one at a time. Each time it selects a node that has fewer than n neighbors (adjacent nodes). The idea is that if we can color the graph after we remove this node, then of course we can color the original graph (with the node included) because the neighbors of this node can have $n-1$ different colors in the worst case; so we can just assign the available nth color to the node. This is called *simplification* of the graph. Each selected node is pushed in the stack. Sometimes though we cannot simplify any further because all nodes have n or more neighbors. In that case we select one node (ie. variable) to be spilled into memory instead of assigning a register to it. This is called *spilling* and the spilled victim can be selected based on priorities (eg. which variable is used less frequently, is it outside a loop, etc). The spilled node is also pushed on the stack. When the graph is completely reduced (and all nodes have been pushed in the stack), we pop the stack one node at a time, we rebuild the interference graph and at the same time we assign a color to the popped-out node so that its color is different from the colors of its neighbors. This is called the *selection* phase. If we can't assign a color to a node, we spill out the node into memory (a node selected to be spilled out during the spill phase does not necessarily mean that it will actually spilled into memory at the end). If there are spilled nodes, we use a memory access for each spilled variable (eg. we use the frame location $fp-24 to store the spilled temporary variable and we replace all occurrences of this variable in the program with M[$fp-24]). Then we restart the whole process (the building of the interference graph, simplification, etc) from the beginning. Figures 11.1-11.3 in Section 11.1 in the textbook give an example of this method.

Consider the following interference graph from the previous section:



The nodes are pushed in the stack in the order of xvuzyw. Then, at the selection phase, xyzwuv variables are assigned the registers $R_0 R_2 R_1 R_0 R_1 R_0$.

If there is a move instruction in a program (ie. an assignment of the form `a:=b`) and there is no conflict between `a` and `b`, then it is a good idea to use the same register for both `a` and `b` so that you can remove the move instruction entirely from the program. In fact you can just merge the graph nodes for `a` and `b` in the graph into one node. That way nodes are now labeled by sets of variables, instead of just one variable. This is called *coalescing*. This is a good thing to do since it reduces the number of registers needed and it removes the move instructions, but it may be bad since it increases the number of neighbors of the merged nodes, which may lead to an irreducible graph and a potential spilling. To do this, we add another phase to the register allocation algorithm, called *coalescing*, that coalesces move related nodes. If we derive an irreducible graph at some point of time, there is another phase, called *freezing*, that de-coalesces one node (into two nodes) and continues the simplification.

A common criterion for coalescing is that we coalesce two nodes if the merged node has fewer than n neighbors of degree greater than or equal to n (where n is the number of available registers). This is called the Briggs criterion. A slight variation of this is the George criterion where we coalesce nodes if all the neighbors of one of the nodes with degree greater than or equal to n already interfere with the other node.

Coalescing is very useful when handling callee-save registers in a program. Recall that callee-save registers need to be saved by the callee procedure to maintain their values upon the exit of the procedure. Suppose that `r3` is a callee-save register. The procedure needs to save this register into a temporary variable at the beginning of the procedure (eg. `a := r3`) and to restore it at the end of the procedure (ie. `r3 := a`). That way, if `r3` is not used at all during the procedure body, it will be coalesced with `a` and the move instructions will be removed. But coalescing can happen in many other different situations as long as there is no interference, which makes this technique very general. Note that registers in a program are handled as temporary variables with a preassigned color (precolored nodes). This means that precolored nodes can only be coalesced with other nodes (they cannot be simplified or spilled). See the example in Section 11.3 in the textbook for a program with precolored nodes.

12.2 Code Generation for Trees

Suppose that you want generate assembly code for complex expression trees using the fewest number of registers to store intermediate results. Suppose also that we have two-address instructions of the form

```
  op   Ri, T
```

where op is an operation (add, sub, mult, etc), Ri is a register (R1, R2, R3, etc), and T is an address mode such as a memory reference, a register, indirect access, indexing etc. We also have a move instruction of the form:

```
  load   Ri, T
```

For example, for the expression `(A-B)+((C+D)+(E*F))`, which corresponds to the AST:

```
          +
        /   \
       /      \
      -        +
     / \     /   \
    A   B   +     *
           / \   / \
          C   D E   F
```

we want to generate the following assembly code:

```
  load  R2, C
  add   R2, D
  load  R1, E
  mult  R1, F
  add   R2, R1
  load  R1, A
  sub   R1, B
  add   R1, R2
```

That is, we used only two register.

There is an algorithm that generates code with the least number of registers. It is called the *Sethi-Ullman* algorithm. It consists of two phases: *numbering* and *code generation*. The numbering phase assigns a number to each tree node that indicates how many registers are needed to evaluate the subtree of this node. Suppose that for a tree node T, we need l registers to evaluate its left subtree and r registers to evaluate its right subtree. Then if one of these numbers is larger, say $l > r$, then we can evaluate the left subtree first and store its result into one of the registers, say R_k. Now we can evaluate the right subtree using the same registers we used for the left subtree, except of course R_k since we want to remember the result of the left subtree. But this is always possible since $l > r$. This means that we need l registers to evaluate T too. The same happens if $r > l$ but now we need to evaluate the right subtree first and store the result to a register. If $l = r$ we need an extra register R_{l+1} to remember the result of the left subtree. If T is a tree leaf, then the number of registers to evaluate T is either 1 or 0 depending whether T is a left or a right subtree (since an operation such as `add R1, A` can handle the right component `A` directly without storing it into a register). So the numbering algorithm starts from the tree leaves and assigns 1 or 0 as explained. Then for each node whose children are labeled l and r, if $l = r$ then the node number is $l + 1$ otherwise it is $max(l, r)$. For example, we have the following numbering for the previous example:

```
      2
    /   \
   /      \
  1         2
 / \      /   \
1   0   1      1
       / \   / \
      1   0 1   0
```

The code generation phase is as follows. We assume that for each node T has numbering $regs(T)$. We use a stack of available registers which initially contains all the registers in order (with the lower register at the top).

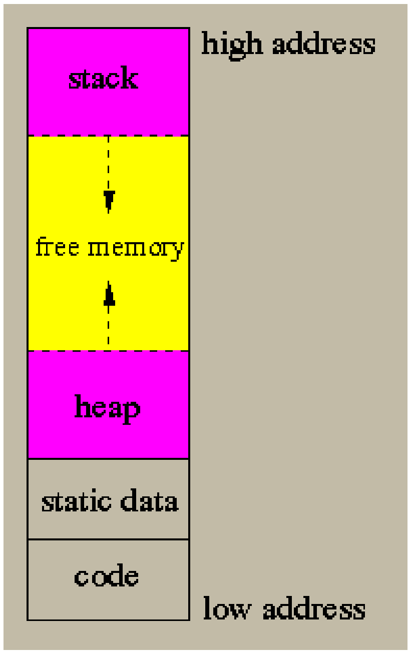
```
generate($T$) =
  if $T$ is a leaf write “load top(), $T$”
  if $T$ is an internal node with children $l$ and $r$ then
  if $regs(r) = 0$ then { generate($l$); write “op top(), $r$” }
  if $regs(l) >= regs(r)$
  then {  generate($l$)
          $R$ := pop()
          generate($r$)
          write “op $R$, top()”
          push($R$) }
  if $regs(l) < regs(r)$
  then {  swap the top two elements of the stack
          generate($r$)
          $R$ := pop()
          generate($l$)
          write “op top(), $R$”
          push($R$)
          swap the top two elements of the stack }
```

This assumes that the number of registers needed is no greater than the number of available registers. Otherwise, we need to spill the intermediate result of a node to memory. This algorithm also does not take advantage of the commutativity and associativity properties of operators to rearrange the expression tree.

13 Garbage Collection

13.1 Storage Allocation

Consider a typical storage organization of a program:



All dynamically allocated data are stored in the heap. These are the data created by `malloc` in C or `new` in C++. You can imagine the heap as a vector of bytes (characters) and `end_of_heap` a pointer to the first available byte in the heap:

```
char heap[heap_size];
int end_of_heap = 0;
```

A trivial implementation of `malloc` in C is:

```
void* malloc ( int size ) {
  void* loc = (void*) &heap[end_of_heap];
  end_of_heap += size;
  return loc;
};
```

If you always create dynamic structures but never delete them, you will eventually run out of heap space. In that case, `malloc` will request the operating system for more heap (this code is not shown). This is very expensive, because it may require to move the stack data to higher memory locations. Alternatively, you can recycle the dynamically allocated data that you don't use. This is done using `free` in C or `delete` in C++. To do so, you need to link all deallocated data in the heap into a list:

```
typedef struct Header { struct Header *next; int size; } Header;
Header* free_list;
```

Initially, the free list contains only one element that covers the entire heap (ie, it's `heap_size` bytes long):

```
void initialize_heap () {
  free_list = (Header*) heap;
  free_list->next = 0;
  free_list->size = heap_size;
};
```

(This is terrible C code, but this can only be done by coercing bytes to pointers.) Note that each cell in the heap has a `Header` component (ie, a `next` pointer and a `size`) followed by `size-sizeof(Header)` bytes (so that is `size` bytes long). `malloc` first searches the free list to find a cell large enough to fit the given number of bytes. If it finds one, it gets a chunk out of the cell leaving the rest untouched:

```
void* malloc ( int size ) {
  Header* prev = free_list;
  for(Header* r=free_list; r!=0; prev=r, r=r->next)
     if (r->size > size+sizeof(Header))
     {  Header* new_r = (Header*) (((char*) r)+size);
        new_r->next = r->next;
        new_r->size = r->size;
        if (prev==free_list)
           free_list = new_r;
        else prev->next = new_r;
        return (void*) r;
     };
  void* loc = (void*) &heap[end_of_heap];
  end_of_heap += size;
  return loc;
};
```

`free` simply puts the recycled cell at the beginning of the free list:

```
void free ( void* x ) {
  if ("size of *x" <= sizeof(Header))
     return;
  ((Header*) x)->next = free_list;
  ((Header*) x)->size = "size of *x";
  free_list = (Header*) x;
};
```

We can see that there is a lot of overhead in `malloc` since the free list may be very long. The most important problem though is fragmentation of the heap into tiny cells, so that even though the total free space in the free list is plenty, it is useless for large object allocation. The reason for fragmentation is that we unrestrictedly choose a cell from the free list to do allocation. It makes more sense to try to find a free cell that provides the requested number of bytes exactly. This can be done by maintaining a vector of free lists, so that the nth element of the vector is a free list that links all the free cells of size `n`.

The above malloc/free way of allocating dynamic cells is called a *manual memory allocation* since the programmer is responsible for allocating and deleting objects explicitly.

This is to be contrasted to the automatic memory management, in which the system manages the dynamic allocation automatically. Manual memory management is the source of the worst and hardest to find bugs. It's also the source of most of the mysterious program crashes. It takes the fun out of programming. It causes horrible memory problems due to "overflow", "fence past errors", "memory corruption", "step-on-others-toe" (hurting other variable's memory locations) or "memory leaks". The memory problems are extremely hard to debug and are very time consuming to fix and troubleshoot. Memory problems bring down the productivity of programmers. Memory related bugs are very tough to crack, and even experienced programmers take several days or weeks to debug memory related problems. Memory bugs may be hidden inside the code for several months and can cause unexpected program crashes. It is estimated that the memory bugs due to usage of char* and pointers in C/C++ is costing $2 billions every year in time lost due to debugging and downtime of programs. Now of course all modern languages (Java, SML, Haskell, etc) use automatic garbage collection. Despite that, C and C++ are still popular and programmers spend most of their time trying to do manual memory allocation (and then, after they remove most core dumps from their program, they port it to a different architecture and their program crashes!).

Why manual memory management is so hard to do correctly? Basically, you need to have a global view of how dynamic instances of a type are created and passed around. This destroys the good software engineering principle that programs should be developed in small independent components. The problem is to keep track of pointer assignments. If more than one objects point to a dynamically allocated object, then the latter object should be deleted only if all objects that point to it do not need it anymore. So basically, you need to keep track of how many pointers are pointing to each object. This is called *reference counting* and used to be popular for OO languages like C++. We do not call this method automatic garbage collection because the programmer again is responsible of putting counters to every object to count references. This method is easy to implement for languages like C++ where you can redefine the assignment operator `dest=source`, when both `dest` and `source` are pointers to data. Instead of using `C*` for a pointer to an object `C`, we use `Ref<C>`, where the template `Ref` provides reference counting to `C`:

```
template< class T >
class Ref {
private:
   int count;
   T* pointer;
   void MayDelete () { if (count==0) delete pointer; };
   void Copy ( const Ref &sp ) {
         ++sp.count;
         count = sp.count;
         pointer = sp.pointer;
   };
public:
   Ref ( T* ptr = 0 ) : count(1), pointer(ptr) {};
   Ref ( const Ref &sp ) { Copy(sp); };
   ~Ref () { MayDelete(); };
   T* operator-> () { return pointer; };
   Ref& operator= ( const Ref &sp ) {
      if (this != &sp) {
         count--;
         MayDelete();
         Copy(sp);
      };
      return *this;
   };
};
```

When an instance of Ref<C> is created, its counter is set to 1. When we delete the object, we just decrement the counter because there may be other objects pointing to it. But when the counter becomes zero, we delete it. The only way to move pointers around, is through an assignment from a C* object to a C* object. In that case, the object pointed by the source pointer will have one pointer more, while the object pointed by the destination pointer will have one pointer less. Reference counting avoids some misuses of the heap but it comes with a high cost: every assignment takes many cycles to be completed and some of the work may be unnecessary since you may pass a pointer around causing many unnecessary counter increments/decrements. In addition, we cannot get rid of cyclic objects (eg. when A points to B and B points to A) using reference counting. All objects that participate in a cyclic chain of references will always have their counters greater than zero, so they will never be deleted, even if they are garbage.

13.2 Automatic Garbage Collection

Heap-allocated records that are not reachable by any chain of pointers from program variables are considered garbage. All other data are ’live’. This is a very conservative approach but is always safe since you can never access unreachable data. On the other hand, it may keep objects that, even though reachable, they will never be accessed by the program in the future.

With automatic storage management, user programs do not reclaim memory manually. When the heap is full, the run-time system suspends the program and starts garbage collection. When the garbage collection is done, the user program resumes execution:

```
char heap[heap_size];
int end_of_heap = 0;
void* malloc ( int size ) {
    if (size+end_of_heap > heap_size)
         GC();            // garbage collection
    void* loc = (void*) &heap[end_of_heap];
    end_of_heap += size;
    return loc;
};
```

13.2.1 Mark-and-Sweep Garbage Collection

Here too we use the conservative approximation that if we can reach an object by following pointers from program variables, then the object is live (not garbage). These program variables are called *roots* and can be either frame-allocated or static. Therefore, the garbage collector needs to check the entire static section as well as all frames in the run-time stack for pointers to heap. It is common to use the following conservative approach: if a word has a value between the minimum and maximum address of the heap, then it is considered to be a pointer to the heap. An object is live if it is pointed by either a root or by a live object (this is a recursive definition). The garbage collector needs to start from each root and follow pointers recursively to mark all live objects.

A Mark-and-Sweep garbage collector has two phases:

1. Mark: starting from roots, mark all reachable objects by using a depth-first-search pointer traversal

2. Sweep: scan the heap from the beginning to the end and reclaim the unmarked objects (and unmark the marked objects).

The depth-first-search is done using the function

```
DFS ( p ) =
{ if *p record is unmarked
     then mark *p
  for each pointer p->fi of the record *p do DFS(p->fi)
}
```

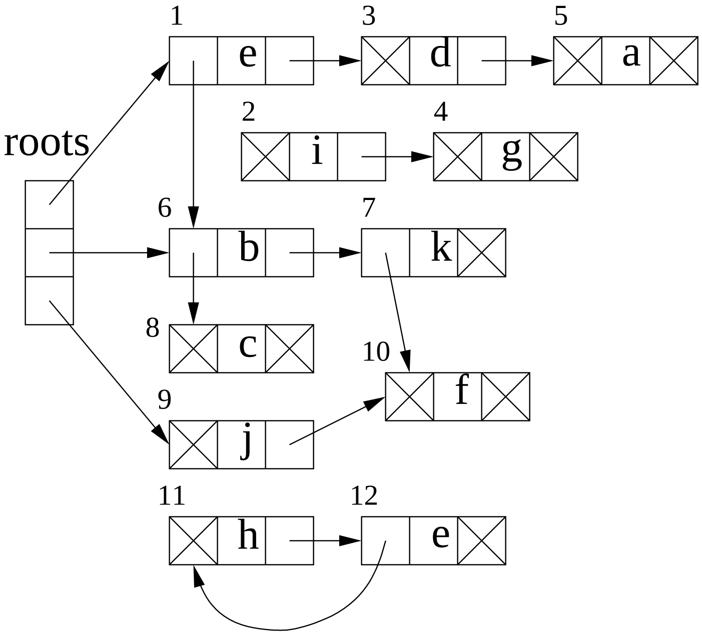
which is called from every root:

```
for each p in roots
   DFS(p)
```

The mark phase is:

```
p = 'first object in the heap'
while p is in the heap do
{  if *p is marked
      then unmark *p
   else insert *p into the free list
   p = p+(size of record *p)
}
```

For example, consider the following objects in memory at the time the garbage collector takes over:



The roots can be static or frame-allocated variables. Let's assume that these objects are represented as follows in memory (as consecutive bytes):

1	6	e	3
2	0	i	4
3	0	d	5
4	0	g	0
5	0	a	0
6	8	b	7
7	10	k	0
8	0	c	0
9	0	j	10
10	0	f	0
11	0	h	12
12	11	l	0

Here, 0 are null pointers while the other pointers are memory addresses. That is, the free list is 0 (empty) and the roots are 1, 6, and 9.

After the mark-and-sweep garbage collection, the memory layout becomes:

1	6	e	3
2	0		
3	0	d	5
4	2		
5	0	a	0
6	8	b	7
7	10	k	0
8	0	c	0
9	0	j	10
10	0	f	0
11	4		
12	11		

where the free list points to the cell 12 now.

13.2.2 Copying Garbage Collection

A copying garbage collector uses two heaps:

1. from-space: the current working heap

2. to-space: needs to be in memory during garbage collection only

At garbage collection time, the garbage collector creates an empty to-space heap in memory (of the same size as the from-space), copies the live objects from the from-space to the to-space (making sure that pointers are referring to the to-space), disposes the from-space, and finally uses the to-space as the new from-space.

Converting a pointer `p` that points to a from-space into a pointer that points to the to-space space, is called *forwarding a pointer*:

```
forward (p) {
   if p points t from-space
      then if p.f1 points to to-space
                 then return p.f1
      else for each field fi of p
                    next.fi := p.fi
             p.f1 := next
             next := next + (size of *p)
             return p.f1
   else return p
```

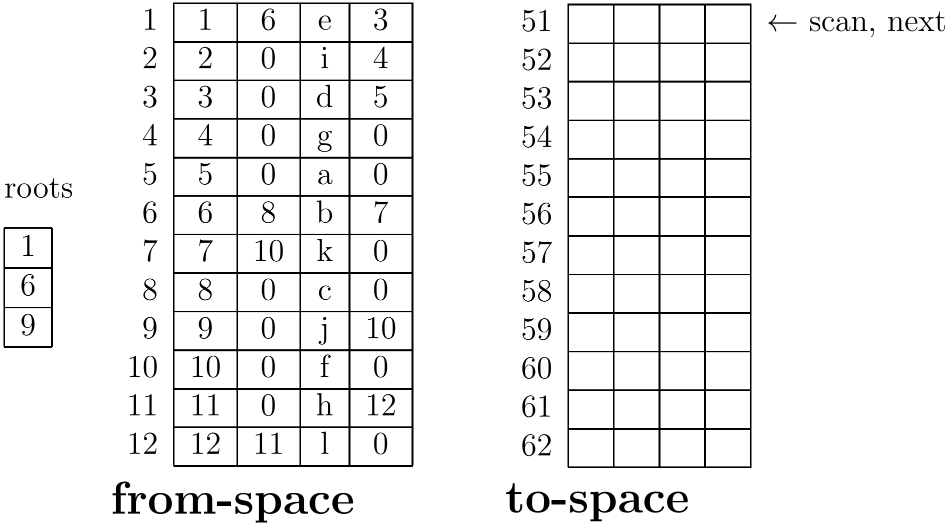
Forwarding pointers can be done in breadth-first-search, using Cheney's Algorithm. Compared to depth-first-search, breadth-first-search has better locality of reference:

```
scan := begin-of-to-space
next := scan
for each root r
    r := forward(r)
while scan < next
```

```
for each field fi of *scan
       scan.fi := forward(scan.fi)
scan := scan + (size of *scan)
```

Cheney's Algorithm uses a very simple allocation algorithm and has no need for stack (since is not recursive). More importantly, its run time depends on the number of live objects, not on the heap size. Furthermore, it does not suffer from data fragmentation, resulting into a more compact memory. In addition, it allows incremental (concurrent) garbage collection. On the other hand, it needs double the amount of memory and needs to recognize all pointers to heap.

For example,



roots

1
6
9

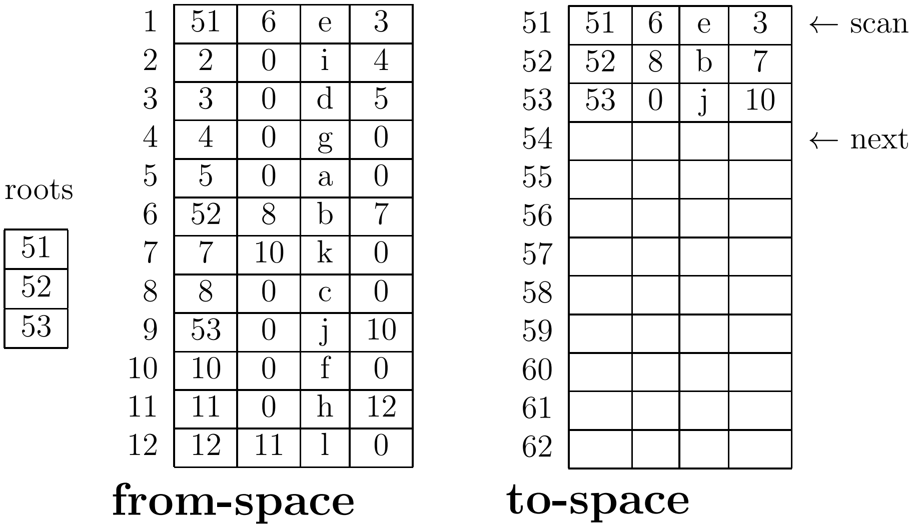
1	1	6	e	3
2	2	0	i	4
3	3	0	d	5
4	4	0	g	0
5	5	0	a	0
6	6	8	b	7
7	7	10	k	0
8	8	0	c	0
9	9	0	j	10
10	10	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51					← scan, next
52					
53					
54					
55					
56					
57					
58					
59					
60					
61					
62					

to-space

After we forward the roots from the from-space to the to-space, the to-space will contain the forwarded roots, and the roots and the forward pointers of the root elements in the from-space will point to the to-space, as shown below:



roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	3	0	d	5
4	4	0	g	0
5	5	0	a	0
6	52	8	b	7
7	7	10	k	0
8	8	0	c	0
9	53	0	j	10
10	10	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	6	e	3	← scan
52	52	8	b	7	
53	53	0	j	10	
54					← next
55					
56					
57					
58					
59					
60					
61					
62					

to-space

Then, we forward the pointers of the first element of the to-space pointed by the scan pointer (element 51). The first pointer 6 has already been forwarded to 52, as we can see from the element 6 in the from-space. The second pointer 3 is forwarded at the end of the to-space to 54:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	5	0	a	0
6	52	8	b	7
7	7	10	k	0
8	8	0	c	0
9	53	0	j	10
10	10	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	52	e	54	
52	52	8	b	7	← scan
53	53	0	j	10	
54	54	0	d	5	
55					← next
56					
57					
58					
59					
60					
61					
62					

to-space

Now we forward the pointer 8 of element 52:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	5	0	a	0
6	52	8	b	7
7	7	10	k	0
8	55	0	c	0
9	53	0	j	10
10	10	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	52	e	54	
52	52	55	b	7	← scan
53	53	0	j	10	
54	54	0	d	5	
55	55	0	c	0	
56					← next
57					
58					
59					
60					
61					
62					

to-space

and the pointer 7 of element 52:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	5	0	a	0
6	52	8	b	7
7	56	10	k	0
8	55	0	c	0
9	53	0	j	10
10	10	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	52	e	54	
52	52	55	b	56	
53	53	0	j	10	← scan
54	54	0	d	5	
55	55	0	c	0	
56	56	10	k	0	
57					← next
58					
59					
60					
61					
62					

to-space

Now we forward the pointer 10 of element 53:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	5	0	a	0
6	52	8	b	7
7	56	10	k	0
8	55	0	c	0
9	53	0	j	10
10	57	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	52	e	54	
52	52	55	b	56	
53	53	0	j	57	
54	54	0	d	5	← scan
55	55	0	c	0	
56	56	10	k	0	
57	57	0	f	0	
58					← next
59					
60					
61					
62					

to-space

Then we forward the pointer 5 of element 54:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	58	0	a	0
6	52	8	b	7
7	56	10	k	0
8	55	0	c	0
9	53	0	j	10
10	57	0	f	0
11	11	0	h	12
12	12	11	l	0

from-space

51	51	52	e	54	
52	52	55	b	56	
53	53	0	j	57	
54	54	0	d	58	
55	55	0	c	0	← scan
56	56	10	k	0	
57	57	0	f	0	
58	58	0	a	0	
59					← next
60					
61					
62					

to-space

The pointer 10 of element 56 has already been forwarded. Therefore, we get:

roots

51
52
53

1	51	6	e	3
2	2	0	i	4
3	54	0	d	5
4	4	0	g	0
5	58	0	a	0
6	52	8	b	7
7	56	10	k	0
8	55	0	c	0
9	53	0	j	10
10	57	0	f	0
11	11	0	h	12
12	12	11	l	0

51	51	52	e	54	
52	52	55	b	56	
53	53	0	j	57	
54	54	0	d	58	
55	55	0	c	0	
56	56	57	k	0	
57	57	0	f	0	
58	58	0	a	0	
59					← scan, next
60					
61					
62					

Finally, the to-space becomes the new heap and the from-space is deallocated from memory.