

Supplementary Material

In this material, we first provide our proof of **Theorem 1** in the paper, and then give some visualizations of intermediate CS reconstruction results and learned filters by our proposed ISTA-Net⁺. Finally, detailed results on Set11 and BSD68 are presented, including PSNR results of Set11 with different CS ratios, some visual comparisons of images in Set11 with different CS ratios and average PSNR performance comparison of various network-based algorithms on the BSD68 dataset with different ratios.

1 Proof of Theorem 1

Theorem 1 *Let X_1, \dots, X_n be independent normal random variables with common zero mean and variance σ^2 . If $\vec{X} = [X_1, \dots, X_n]^\top$ and given any matrices $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{B} \in \mathbb{R}^{s \times m}$, define a new random variable $\vec{Y} = \mathbf{BReLU}(\mathbf{A}\vec{X}) = \mathbf{B} \max(\mathbf{0}, \mathbf{A}\vec{X})$. Then, $\mathbb{E}[\|\vec{Y} - \mathbb{E}[\vec{Y}]\|_2^2]$ and $\mathbb{E}[\|\vec{X} - \mathbb{E}[\vec{X}]\|_2^2]$ are linearly related, i.e. $\mathbb{E}[\|\vec{Y} - \mathbb{E}[\vec{Y}]\|_2^2] = \alpha \mathbb{E}[\|\vec{X} - \mathbb{E}[\vec{X}]\|_2^2]$, where α is only a function of \mathbf{A} and \mathbf{B} .*

Proof: Let $\vec{W} = \mathbf{A}\vec{X} = [W_1, \dots, W_m]$, since $\vec{X} \sim \mathcal{N}(\mathbf{0}, \Sigma_{\vec{X}})$ then we have $\vec{W} \sim \mathcal{N}(\mathbf{0}, \Sigma_{\vec{W}})$, where $\Sigma_{\vec{W}} = \mathbf{A}\Sigma_{\vec{X}}\mathbf{A}^\top$. Note that $\Sigma_{\vec{X}} = \sigma^2\mathbf{I}$, and \mathbf{I} denotes the identity matrix. Let $\vec{Z} = \text{ReLU}(\vec{W}) = \max(\vec{W}, \mathbf{0}) = [Z_1, \dots, Z_m]$, we first discuss the relationship between \vec{W} and \vec{Z} .

Obviously, the variance of W_i can be expressed as $\text{var}(W_i) = (\eta_i\sigma)^2$, $i = 1, \dots, m$, η_i is related with \mathbf{A} , then the probability density function of W_i , denoted by $f_{W_i}(x)$, is expressed as

$$f_{W_i}(x) = \frac{1}{\sqrt{2\pi}\eta_i\sigma} e^{-\frac{x^2}{2(\eta_i\sigma)^2}}.$$

According to $Z_i = \max(W_i, 0)$, we have the mean and the variance of Z_i as below:

$$\begin{aligned} E(Z_i) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\eta_i\sigma} e^{-\frac{x^2}{2(\eta_i\sigma)^2}} \max(x, 0) dx = \int_0^{\infty} \frac{1}{\sqrt{2\pi}\eta_i\sigma} e^{-\frac{x^2}{2(\eta_i\sigma)^2}} x dx = \frac{\eta_i\sigma}{\sqrt{2\pi}}; \\ E(Z_i^2) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\eta_i\sigma} e^{-\frac{x^2}{2(\eta_i\sigma)^2}} \max(x, 0)^2 dx = \int_0^{\infty} \frac{1}{\sqrt{2\pi}\eta_i\sigma} e^{-\frac{x^2}{2(\eta_i\sigma)^2}} x^2 dx = \frac{(\eta_i\sigma)^2}{2}; \\ \text{var}(Z_i) &= E(Z_i^2) - E(Z_i)^2 = \frac{\pi - 1}{2\pi} (\eta_i\sigma)^2. \end{aligned}$$

Our purpose is to compute the covariance matrix of \vec{Z} , that is $\Sigma_{\vec{Z}}$. After computing $\text{var}(Z_i)$, which is on the diagonal position of $\Sigma_{\vec{Z}}$, we now calculate $\text{cov}(Z_i, Z_j)$, $i \neq j$, by

$$\text{cov}(Z_i, Z_j) = E(Z_i Z_j) - E(Z_i)E(Z_j).$$

Assume that the joint probability density function of W_i and W_j is written as

$$f_{W_i, W_j}(x, y) = \frac{1}{2\pi\eta_i\eta_j\sigma^2\sqrt{1 - \rho_{ij}^2}} e^{-\frac{1}{2(1 - \rho_{ij}^2)} \left(\frac{x^2}{(\eta_i\sigma)^2} - \frac{2\rho_{ij}xy}{\eta_i\eta_j\sigma^2} + \frac{y^2}{(\eta_j\sigma)^2} \right)},$$

where ρ_{ij} is the correlation coefficient between W_i and W_j . Then,

$$\begin{aligned} E(Z_i Z_j) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{W_i, W_j}(x, y) \max(x, 0) \max(y, 0) dx dy \\ &= \int_0^{\infty} \int_0^{\infty} f_{W_i, W_j}(x, y) xy dx dy \\ &= \int_0^{\infty} \int_0^{\infty} \frac{1}{2\pi\eta_i\eta_j\sigma^2\sqrt{1 - \rho_{ij}^2}} e^{-\frac{1}{2(1 - \rho_{ij}^2)} \left(\frac{x^2}{(\eta_i\sigma)^2} - \frac{2\rho_{ij}xy}{\eta_i\eta_j\sigma^2} + \frac{y^2}{(\eta_j\sigma)^2} \right)} xy dx dy. \end{aligned}$$

Furthermore, define $u = \frac{x}{\eta_i \sigma \sqrt{1-\rho_{ij}^2}}$, $v = \frac{y}{\eta_j \sigma \sqrt{1-\rho_{ij}^2}}$, the computation of $E(Z_i Z_j)$ is transformed to be

$$E(Z_i Z_j) = \frac{(\sqrt{1-\rho_{ij}^2})^3 \eta_i \eta_j \sigma^2}{2\pi} \int_0^\infty \int_0^\infty e^{-\frac{1}{2}(u^2 - 2\rho_{ij} uv + v^2)} u v d u d v = M_{ij} \sigma^2,$$

where $M_{ij} = \frac{(\sqrt{1-\rho_{ij}^2})^3 \eta_i \eta_j}{2\pi} \int_0^\infty \int_0^\infty e^{-\frac{1}{2}(u^2 - 2\rho_{ij} uv + v^2)} u v d u d v$.

Hence,

$$\text{cov}(Z_i, Z_j) = E(Z_i Z_j) - E(Z_i)E(Z_j) = M_{ij} \sigma^2 - \frac{\eta_i \eta_j \sigma^2}{2\pi} = (M_{ij} - \frac{\eta_i \eta_j}{2\pi}) \sigma^2.$$

According to the expressions of $\text{var}(Z_i)$ and $\text{cov}(Z_i, Z_j)$, it is clear to see that $\Sigma_{\bar{Z}}$ can be formulated as

$$\Sigma_{\bar{Z}} = \sigma^2 \hat{\Sigma}_{\bar{Z}},$$

where $\hat{\Sigma}_{\bar{Z}}$ is only determined by the matrix \mathbf{A} .

Therefore, the covariance matrix of $\vec{Y} = \vec{B}\vec{Z}$, i.e. $\Sigma_{\vec{Y}}$ is written as

$$\Sigma_{\vec{Y}} = \mathbf{B}\Sigma_{\bar{Z}}\mathbf{B}^\top = \sigma^2 \mathbf{B}\hat{\Sigma}_{\bar{Z}}\mathbf{B}^\top.$$

Due to $E[\|\vec{Y} - E[\vec{Y}]\|_2^2] = \text{tr}(\Sigma_{\vec{Y}}) = \sigma^2 \text{tr}(\mathbf{B}\hat{\Sigma}_{\bar{Z}}\mathbf{B}^\top)$ ($\text{tr}(\cdot)$ denotes the trace of a matrix) and $E[\|\vec{X} - E[\vec{X}]\|_2^2] = \text{tr}(\Sigma_{\vec{X}}) = n\sigma^2$, then we have

$$E[\|\vec{Y} - E[\vec{Y}]\|_2^2] = \alpha E[\|\vec{X} - E[\vec{X}]\|_2^2],$$

where $\alpha = \frac{\text{tr}(\mathbf{B}\hat{\Sigma}_{\bar{Z}}\mathbf{B}^\top)}{n}$. That means $E[\|\vec{Y} - E[\vec{Y}]\|_2^2]$ and $E[\|\vec{X} - E[\vec{X}]\|_2^2]$ are linearly related. ■

2 Visualization of Intermediate Results and Learned Filters

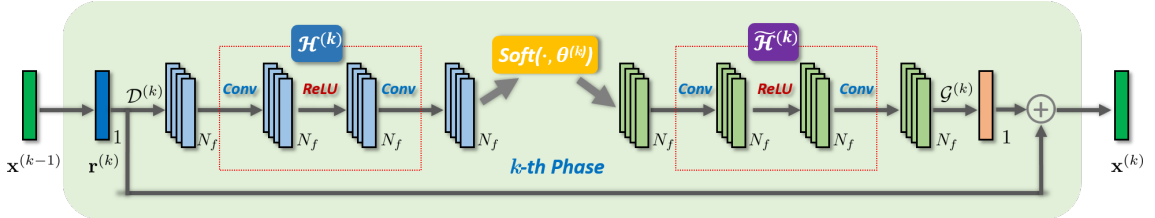


Figure 1: Illustrations of the k -th phase of the proposed ISTA-Net⁺. $\mathcal{D}^{(k)}$, $\mathcal{G}^{(k)}$, $\mathcal{H}^{(k)}$, $\tilde{\mathcal{H}}^{(k)}$ are learnable linear convolutional operators.

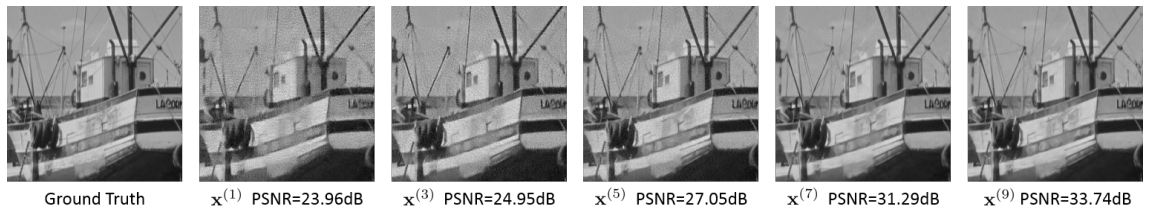


Figure 2: Visualization of intermediate CS reconstruction results for image *Boats* by the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%. As shown in Figure 1, $\mathbf{x}^{(k)}$ stands for the output of the k phase in ISTA-Net⁺. Here, the phase index set is $\{1, 3, 5, 7, 9\}$.

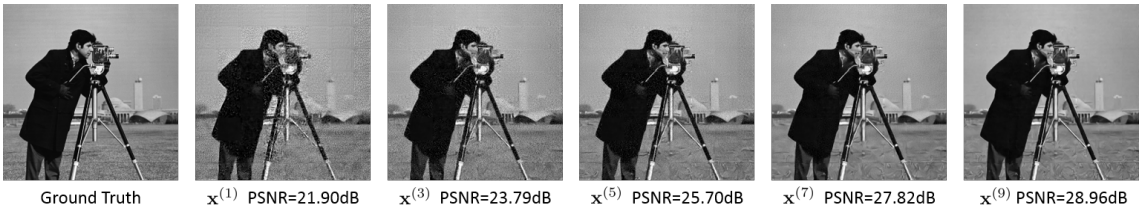


Figure 3: Visualization of intermediate CS reconstruction results for image *Cameraman* by the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%. As shown in Figure 1, $\mathbf{x}^{(k)}$ stands for the output of the k phase in ISTA-Net⁺. Here, the phase index set is $\{1, 3, 5, 7, 9\}$.

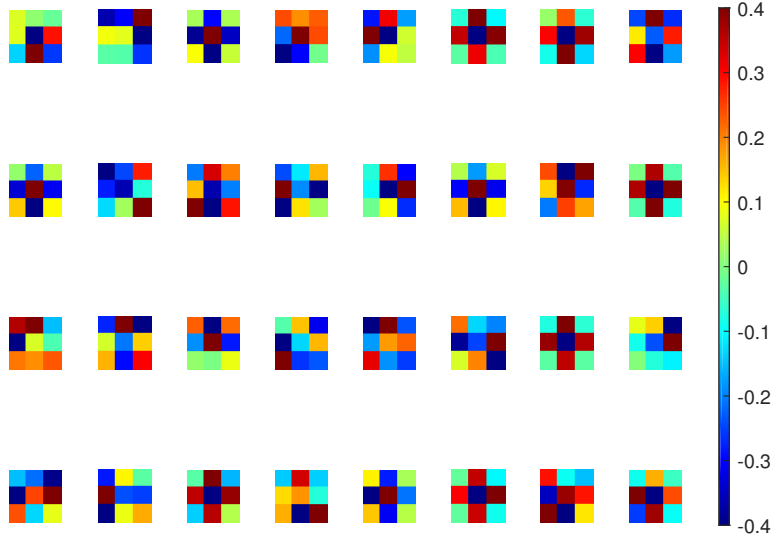


Figure 4: Visualization of $\mathcal{D}^{(1)}$ in the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%.

3 Detailed Results for Set11 and BSD68

References

- [1] Chengbo Li, Wotao Yin, Hong Jiang, and Yin Zhang. An efficient augmented lagrangian method with applications to total variation minimization. *Computational Optimization and Applications*, 56(3):507–530, 2013. 6, 7
- [2] Christopher A Metzler, Arian Maleki, and Richard G Baraniuk. From denoising to compressed sensing. *IEEE Transactions on Information Theory*, 62(9):5117–5144, 2016. 6, 7
- [3] Kai Zhang, Wangmeng Zuo, Shuhang Gu, and Lei Zhang. Learning deep CNN denoiser prior for image restoration. *CVPR*, 2017. 6, 7
- [4] Ali Mousavi, Ankit B Patel, and Richard G Baraniuk. A deep learning approach to structured signal recovery. In *Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 1336–1343. IEEE, 2015. 6, 7, 11

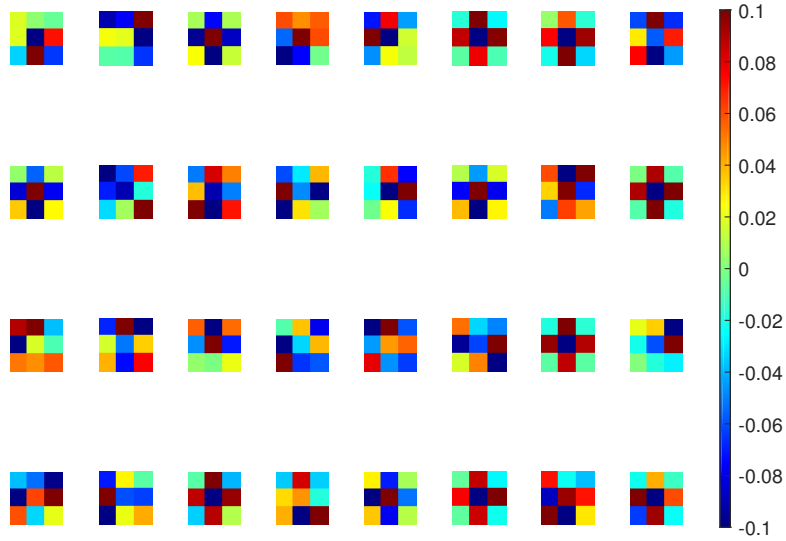


Figure 5: Visualization of $\mathcal{G}^{(1)}$ in the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%.

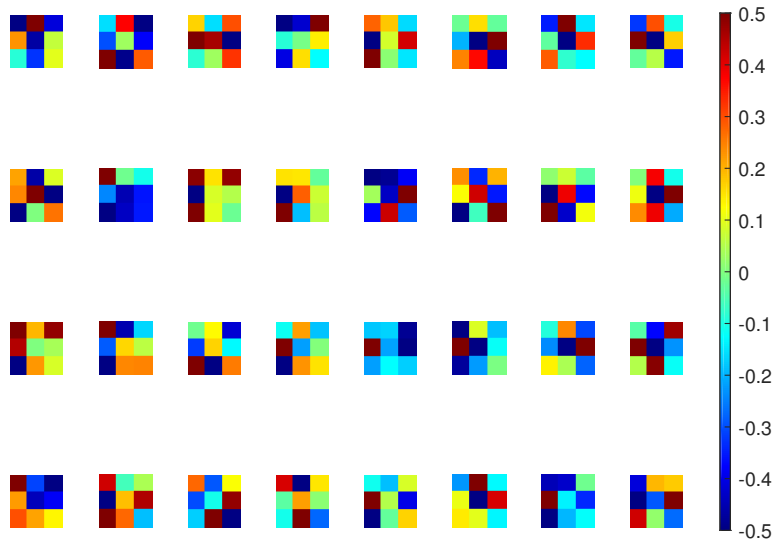


Figure 6: Visualization of the first 32 filters of $\mathcal{H}^{(1)}$ in the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%.

- [5] Kuldeep Kulkarni, Suhas Lohit, Pavan Turaga, Ronan Kerviche, and Amit Ashok. ReconNet: non-iterative reconstruction of images from compressively sensed measurements. In *CVPR*, pages 449–458, 2016. [6](#), [7](#), [11](#)

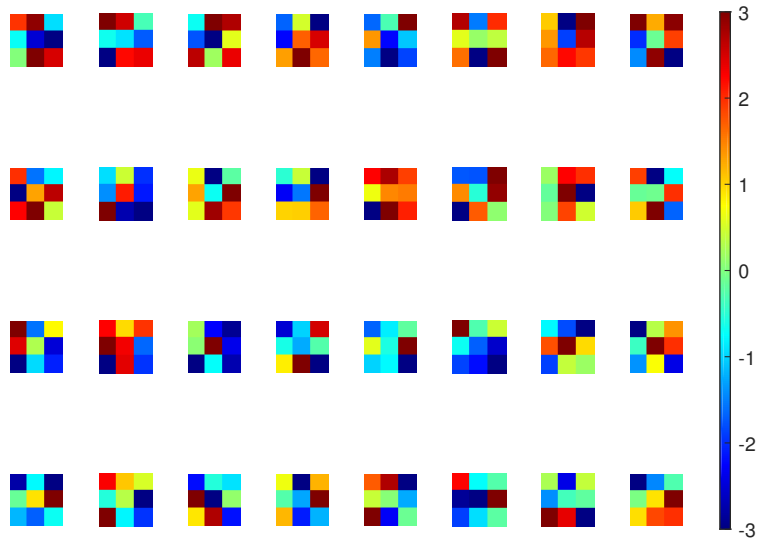


Figure 7: Visualization of the first 32 filters of $\tilde{\mathcal{H}}^{(1)}$ in the proposed ISTA-Net⁺ ($N_p = 9$ and $N_f = 32$) when CS ratio is 25%.



Figure 8: Eleven test images in Set11. Left to right and top to bottom: *Barbara*, *Boats*, *Cameraman*, *Fingerprint*, *Flintstones*, *Foreman*, *House*, *Lena*, *Monarch*, *Parrots* and *Pepper*.

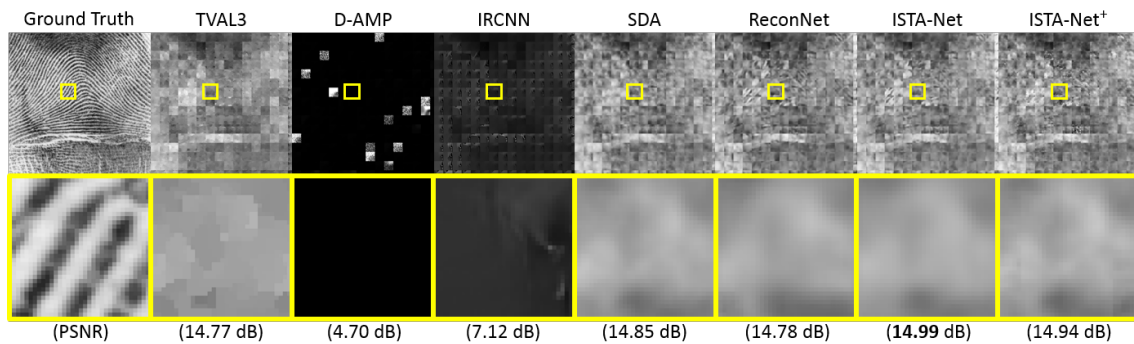


Figure 9: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Fingerprint* image in Set11 (CS ratio is 1%).

Table 1: PSNR (dB) performance for each image on Set11 with different CS ratios.

Image Name	Algorithm	CS Ratio						
		50%	40%	30%	25%	10%	4%	1%
<i>Barbara</i>	TVAL3 [1]	28.39	26.58	25.07	24.25	22.00	19.27	17.58
	D-AMP [2]	37.39	35.65	29.16	26.21	22.04	19.27	5.64
	IRCNN [3]	35.54	33.96	28.84	27.36	22.65	18.73	8.12
	SDA [4]	27.74	26.15	24.70	23.34	21.87	20.47	18.38
	ReconNet [5]	29.64	28.46	26.30	23.32	22.62	20.64	18.35
	ISTA-Net	35.59	33.04	29.27	28.16	23.29	20.98	18.43
	ISTA-Net ⁺	36.74	34.31	31.21	29.21	23.59	20.96	18.55
<i>Boats</i>	TVAL3 [1]	34.82	32.59	30.17	28.87	24.02	19.70	17.19
	D-AMP [2]	38.97	35.85	31.62	29.40	23.40	19.09	5.37
	IRCNN [3]	37.42	36.23	31.36	30.14	24.63	18.41	7.97
	SDA [4]	30.41	29.34	28.18	26.79	24.05	21.39	18.44
	ReconNet [5]	32.77	32.05	30.00	27.09	25.49	21.87	18.45
	ISTA-Net	39.15	36.89	34.24	32.74	26.94	22.28	18.39
	ISTA-Net ⁺	39.95	37.99	35.20	33.74	27.57	22.37	18.49
<i>Cameraman</i>	TVAL3 [1]	31.68	29.60	27.34	25.77	22.15	18.68	16.31
	D-AMP [2]	31.15	29.13	26.90	24.81	21.17	18.25	5.66
	IRCNN [3]	34.08	31.99	29.55	28.27	23.30	18.29	8.19
	SDA [4]	26.51	25.37	24.49	23.21	21.22	19.18	17.04
	ReconNet [5]	27.98	27.83	26.06	23.42	22.33	19.76	17.06
	ISTA-Net	34.17	31.73	29.54	28.25	23.62	20.18	17.07
	ISTA-Net ⁺	34.37	32.18	30.31	28.96	24.08	20.19	17.08
<i>Fingerprint</i>	TVAL3 [1]	27.82	25.91	23.93	22.75	18.71	16.24	14.77
	D-AMP [2]	33.35	31.42	27.53	25.41	18.21	15.30	4.70
	IRCNN [3]	33.44	32.00	27.62	26.40	20.77	15.80	7.12
	SDA [4]	29.38	28.34	26.89	25.33	20.92	17.16	14.85
	ReconNet [5]	31.34	29.76	27.72	25.44	21.51	17.26	14.78
	ISTA-Net	33.81	31.79	29.31	27.92	21.97	17.35	14.99
	ISTA-Net ⁺	34.48	32.11	29.72	28.19	22.61	17.45	14.93
<i>Flintstones</i>	TVAL3 [1]	30.20	27.91	25.44	24.10	19.03	15.27	13.20
	D-AMP [2]	31.75	30.37	27.79	25.51	17.71	14.45	4.37
	IRCNN [3]	32.65	28.21	29.73	28.87	20.37	13.82	6.64
	SDA [4]	26.39	25.37	24.04	22.59	19.10	16.36	13.81
	ReconNet [5]	28.92	28.28	26.45	22.93	20.96	16.81	13.78
	ISTA-Net	33.34	31.91	30.20	29.06	22.94	17.48	13.89
	ISTA-Net ⁺	33.69	32.33	30.77	30.00	23.74	17.47	13.91
<i>Foreman</i>	TVAL3 [1]	41.48	39.30	36.97	35.58	28.72	21.26	18.58
	D-AMP [2]	40.98	39.20	37.59	35.65	29.96	21.57	3.95
	IRCNN [3]	40.13	37.87	35.51	34.66	28.29	17.01	6.51
	SDA [4]	31.78	30.76	29.89	28.76	26.92	23.90	20.29
	ReconNet [5]	35.28	34.66	32.74	29.00	29.69	24.89	20.27
	ISTA-Net	43.24	41.37	39.21	37.54	32.10	25.94	20.41
	ISTA-Net ⁺	44.20	42.34	40.17	39.24	33.78	26.21	20.45
<i>House</i>	TVAL3 [1]	37.58	35.70	33.54	32.24	26.52	21.36	18.78
	D-AMP [2]	39.50	37.42	35.17	33.78	27.28	21.38	4.86
	IRCNN [3]	37.83	37.46	34.35	33.53	27.47	19.95	7.61
	SDA [4]	31.22	30.12	29.19	27.85	25.44	22.89	19.34
	ReconNet [5]	34.50	33.60	32.07	28.52	27.70	23.61	19.36
	ISTA-Net	39.60	38.12	36.51	35.32	29.68	24.48	19.42
	ISTA-Net ⁺	40.29	38.65	37.18	36.23	30.76	24.83	19.59
<i>Lena</i>	TVAL3 [1]	33.65	31.73	29.85	28.75	24.27	19.80	16.88
	D-AMP [2]	33.97	31.36	29.29	28.16	23.76	19.30	5.74
	IRCNN [3]	36.71	34.45	31.23	30.03	25.08	19.00	8.25
	SDA [4]	29.71	28.52	27.47	26.17	23.69	21.33	17.77
	ReconNet [5]	32.28	31.33	29.52	26.60	25.50	21.86	17.85
	ISTA-Net	37.97	35.80	33.43	32.04	26.87	22.72	17.94
	ISTA-Net ⁺	38.39	36.44	34.04	32.57	27.49	22.98	17.89
<i>Monarch</i>	TVAL3 [1]	35.13	32.51	29.48	27.83	21.40	16.89	14.67
	D-AMP [2]	35.54	32.01	28.75	26.80	19.85	15.98	6.15
	IRCNN [3]	37.09	33.84	31.01	29.94	23.31	16.43	8.54
	SDA [4]	28.32	27.12	25.71	24.53	21.26	18.15	15.11
	ReconNet [5]	31.11	30.26	28.10	24.53	22.93	18.55	15.09
	ISTA-Net	39.36	36.99	33.85	32.31	25.16	19.28	15.13
	ISTA-Net ⁺	40.18	37.77	34.83	33.52	25.91	19.27	15.15
<i>Parrots</i>	TVAL3 [1]	31.97	30.17	28.50	27.23	23.19	19.34	17.19
	D-AMP [2]	34.94	32.14	28.77	27.16	23.03	19.63	5.16
	IRCNN [3]	36.61	33.30	31.74	30.62	23.54	17.62	7.62
	SDA [4]	28.89	27.68	26.53	25.39	22.65	20.69	17.92
	ReconNet [5]	31.77	30.68	28.93	25.77	24.44	21.20	17.84
	ISTA-Net	37.04	34.59	32.34	31.34	25.31	21.90	17.97
	ISTA-Net ⁺	37.33	35.04	33.12	32.27	26.07	21.67	18.00
<i>Pepper</i>	TVAL3 [1]	36.34	34.13	31.21	29.73	22.88	18.46	15.56
	D-AMP [2]	37.57	34.65	31.72	30.14	22.58	18.13	5.74
	IRCNN [3]	37.14	35.39	32.04	30.99	24.80	18.12	8.09
	SDA [4]	28.07	26.91	25.85	24.75	22.08	19.80	16.71
	ReconNet [5]	30.92	29.43	28.28	24.95	23.98	20.21	16.72
	ISTA-Net	38.51	36.78	34.15	32.64	26.11	20.94	16.76
	ISTA-Net ⁺	39.13	37.54	35.46	34.35	27.40	20.97	16.76
Mean PSNR	TVAL3 [1]	33.55	31.46	29.23	27.92	22.99	18.75	16.43
	D-AMP [2]	35.92	33.56	30.39	28.46	22.64	18.40	5.21
	IRCNN [3]	36.23	34.06	31.18	30.07	24.02	17.56	7.70
	SDA [4]	28.95	27.79	26.63	25.34	22.65	20.12	17.29
	ReconNet [5]	31.50	30.58	28.74	25.60	24.28	20.63	17.27
	ISTA-Net	<u>37.43</u>	<u>35.36</u>	<u>32.91</u>	<u>31.53</u>	<u>25.80</u>	<u>21.23</u>	<u>17.30</u>
	ISTA-Net ⁺	38.07	36.06	33.82	32.57	26.64	21.31	17.34

Table 2: PSNR (dB) performance for each image on Set11 with different CS ratios.

Image Name	Algorithm	CS Ratio						
		50%	40%	30%	25%	10%	4%	1%
<i>Barbara</i>	TVAL3 [1]	28.39	26.58	25.07	24.25	22.00	19.27	17.58
	D-AMP [2]	37.39	35.65	29.16	26.21	22.04	19.27	5.64
	IRCNN [3]	35.54	33.96	28.84	27.36	22.65	18.73	8.12
	SDA [4]	27.74	26.15	24.70	23.34	21.87	20.47	18.38
	ReconNet [5]	29.64	28.46	26.30	23.32	22.62	20.64	18.35
	ISTA-Net	35.59	33.04	29.27	28.16	23.29	20.98	18.43
	ISTA-Net ⁺	37.33	35.30	31.79	30.32	24.08	21.51	18.23
<i>Boats</i>	TVAL3 [1]	34.82	32.59	30.17	28.87	24.02	19.70	17.19
	D-AMP [2]	38.97	35.85	31.62	29.40	23.40	19.09	5.37
	IRCNN [3]	37.42	36.23	31.36	30.14	24.63	18.41	7.97
	SDA [4]	30.41	29.34	28.18	26.79	24.05	21.39	18.44
	ReconNet [5]	32.77	32.05	30.00	27.09	25.49	21.87	18.45
	ISTA-Net	39.15	36.89	34.24	32.74	26.94	22.28	18.39
	ISTA-Net ⁺	40.22	38.14	35.71	34.40	28.04	23.28	18.65
<i>Cameraman</i>	TVAL3 [1]	31.68	29.60	27.34	25.77	22.15	18.68	16.31
	D-AMP [2]	31.15	29.13	26.90	24.81	21.17	18.25	5.66
	IRCNN [3]	34.08	31.99	29.55	28.27	23.30	18.29	8.19
	SDA [4]	26.51	25.37	24.49	23.21	21.22	19.18	17.04
	ReconNet [5]	27.98	27.83	26.06	23.42	22.33	19.76	17.06
	ISTA-Net	34.17	31.73	29.54	28.25	23.62	20.18	17.07
	ISTA-Net ⁺	35.10	33.37	31.32	29.88	25.01	20.70	17.22
<i>Fingerprint</i>	TVAL3 [1]	27.82	25.91	23.93	22.75	18.71	16.24	14.77
	D-AMP [2]	33.35	31.42	27.53	25.41	18.21	15.30	4.70
	IRCNN [3]	33.44	32.00	27.62	26.40	20.77	15.80	7.12
	SDA [4]	29.38	28.34	26.89	25.33	20.92	17.16	14.85
	ReconNet [5]	31.34	29.76	27.72	25.44	21.51	17.26	14.78
	ISTA-Net	33.81	31.79	29.31	27.92	21.97	17.35	14.99
	ISTA-Net ⁺	34.80	32.47	29.88	28.62	23.14	17.99	14.66
<i>Flintstones</i>	TVAL3 [1]	30.20	27.91	25.44	24.10	19.03	15.27	13.20
	D-AMP [2]	31.75	30.37	27.79	25.51	17.71	14.45	4.37
	IRCNN [3]	32.65	28.21	29.73	28.87	20.37	13.82	6.64
	SDA [4]	26.39	25.37	24.04	22.59	19.10	16.36	13.81
	ReconNet [5]	28.92	28.28	26.45	22.93	20.96	16.81	13.78
	ISTA-Net	33.34	31.91	30.20	29.06	22.94	17.48	13.89
	ISTA-Net ⁺	33.89	32.57	31.14	30.48	25.20	18.60	13.74
<i>Foreman</i>	TVAL3 [1]	41.48	39.30	36.97	35.58	28.72	21.26	18.58
	D-AMP [2]	40.98	39.20	37.59	35.65	29.96	21.57	3.95
	IRCNN [3]	40.13	37.87	35.51	34.66	28.29	17.01	6.51
	SDA [4]	31.78	30.76	29.89	28.76	26.92	23.90	20.29
	ReconNet [5]	35.28	34.66	32.74	29.00	29.69	24.89	20.27
	ISTA-Net	43.24	41.37	39.21	37.54	32.10	25.94	20.41
	ISTA-Net ⁺	44.45	42.72	40.81	40.06	35.10	28.82	20.74
<i>House</i>	TVAL3 [1]	37.58	35.70	33.54	32.24	26.52	21.36	18.78
	D-AMP [2]	39.50	37.42	35.17	33.78	27.28	21.38	4.86
	IRCNN [3]	37.83	37.46	34.35	33.53	27.47	19.95	7.61
	SDA [4]	31.22	30.12	29.19	27.85	25.44	22.89	19.34
	ReconNet [5]	34.50	33.60	32.07	28.52	27.70	23.61	19.36
	ISTA-Net	39.60	38.12	36.51	35.32	29.68	24.48	19.42
	ISTA-Net ⁺	40.46	39.08	37.50	36.93	32.33	26.29	19.45
<i>Lena</i>	TVAL3 [1]	33.65	31.73	29.85	28.75	24.27	19.80	16.88
	D-AMP [2]	33.97	31.36	29.29	28.16	23.76	19.30	5.74
	IRCNN [3]	36.71	34.45	31.23	30.03	25.08	19.00	8.25
	SDA [4]	29.71	28.52	27.47	26.17	23.69	21.33	17.77
	ReconNet [5]	32.28	31.33	29.52	26.60	25.50	21.86	17.85
	ISTA-Net	37.97	35.80	33.43	32.04	26.87	22.72	17.94
	ISTA-Net ⁺	38.72	36.63	34.26	33.16	28.31	24.19	18.28
<i>Monarch</i>	TVAL3 [1]	35.13	32.51	29.48	27.83	21.40	16.89	14.67
	D-AMP [2]	35.54	32.01	28.75	26.80	19.85	15.98	6.15
	IRCNN [3]	37.09	33.84	31.01	29.94	23.31	16.43	8.54
	SDA [4]	28.32	27.12	25.71	24.53	21.26	18.15	15.11
	ReconNet [5]	31.11	30.26	28.10	24.53	22.93	18.55	15.09
	ISTA-Net	39.36	36.99	33.85	32.31	25.16	19.28	15.13
	ISTA-Net ⁺	40.48	38.33	35.66	34.34	27.21	20.41	15.08
<i>Parrots</i>	TVAL3 [1]	31.97	30.17	28.50	27.23	23.19	19.34	17.19
	D-AMP [2]	34.94	32.14	28.77	27.16	23.03	19.63	5.16
	IRCNN [3]	36.61	33.30	31.74	30.62	23.54	17.62	7.62
	SDA [4]	28.89	27.68	26.53	25.39	22.65	20.69	17.92
	ReconNet [5]	31.77	30.68	28.93	25.77	24.44	21.20	17.84
	ISTA-Net	37.04	34.59	32.34	31.34	25.31	21.90	17.97
	ISTA-Net ⁺	38.08	36.21	34.02	32.56	27.36	22.96	18.02
<i>Pepper</i>	TVAL3 [1]	36.34	34.13	31.21	29.73	22.88	18.46	15.56
	D-AMP [2]	37.57	34.65	31.72	30.14	22.58	18.13	5.74
	IRCNN [3]	37.14	35.39	32.04	30.99	24.80	18.12	8.09
	SDA [4]	28.07	26.91	25.85	24.75	22.08	19.80	16.71
	ReconNet [5]	30.92	29.43	28.28	24.95	23.98	20.21	16.72
	ISTA-Net	38.51	36.78	34.15	32.64	26.11	20.94	16.76
	ISTA-Net ⁺	39.29	37.86	35.86	34.95	28.95	22.07	16.81
Mean PSNR	TVAL3 [1]	33.55	31.46	29.23	27.92	22.99	18.75	16.43
	D-AMP [2]	35.92	33.56	30.39	28.46	22.64	18.40	5.21
	IRCNN [3]	36.23	34.06	31.18	30.07	24.02	17.56	7.70
	SDA [4]	28.95	27.79	26.63	25.34	22.65	20.12	17.29
	ReconNet [5]	31.50	30.58	28.74	25.60	24.28	20.63	17.27
	ISTA-Net	37.43	<u>35.36</u>	<u>32.91</u>	<u>31.53</u>	<u>25.80</u>	<u>21.23</u>	<u>17.30</u>
	ISTA-Net ⁺	38.44	36.61	34.36	33.25	27.70	22.44	17.35

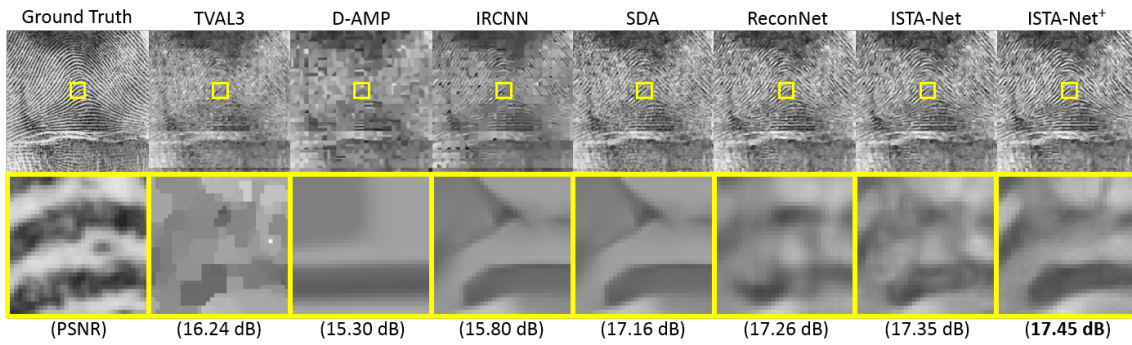


Figure 10: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Fingerprint* image in Set11 (CS ratio is 4%).

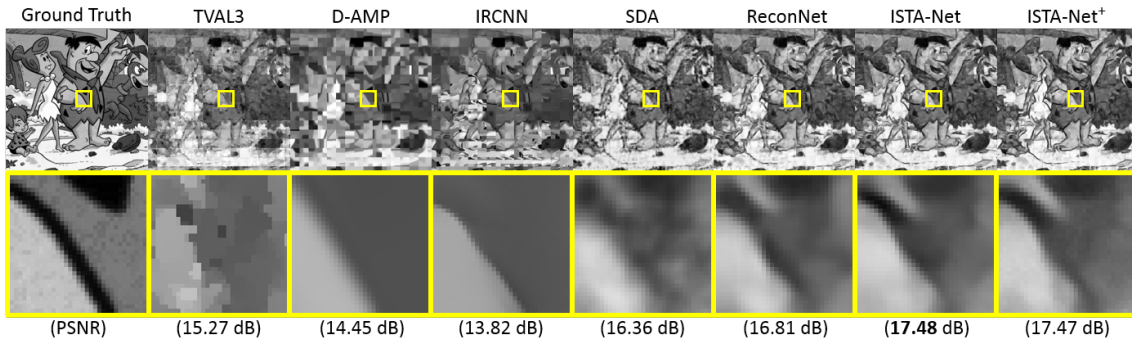


Figure 11: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Flintstones* image in Set11 (CS ratio is 4%).

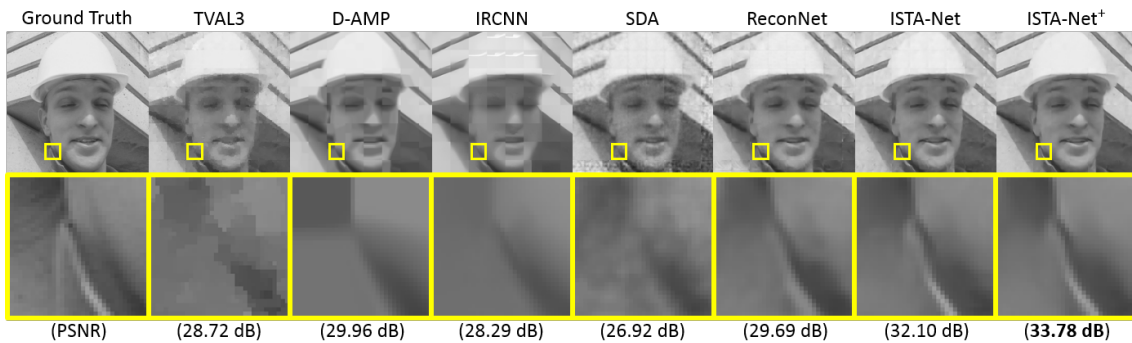


Figure 12: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Foreman* image in Set11 (CS ratio is 10%).

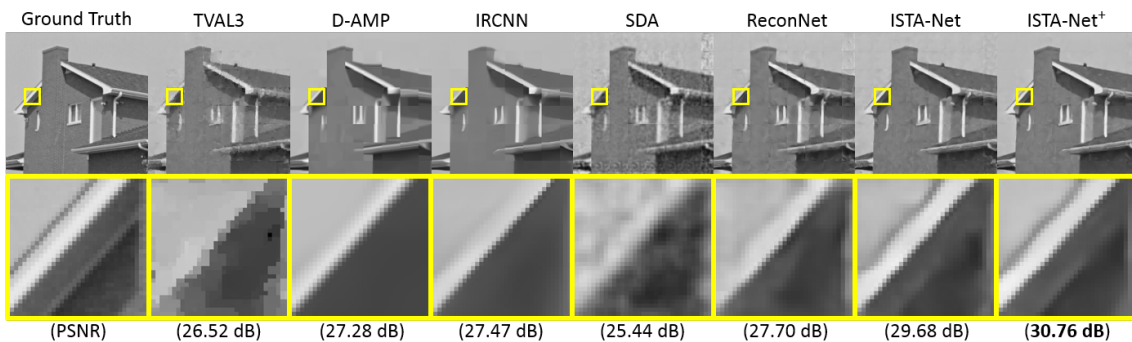


Figure 13: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *House* image in Set11 (CS ratio is 10%).

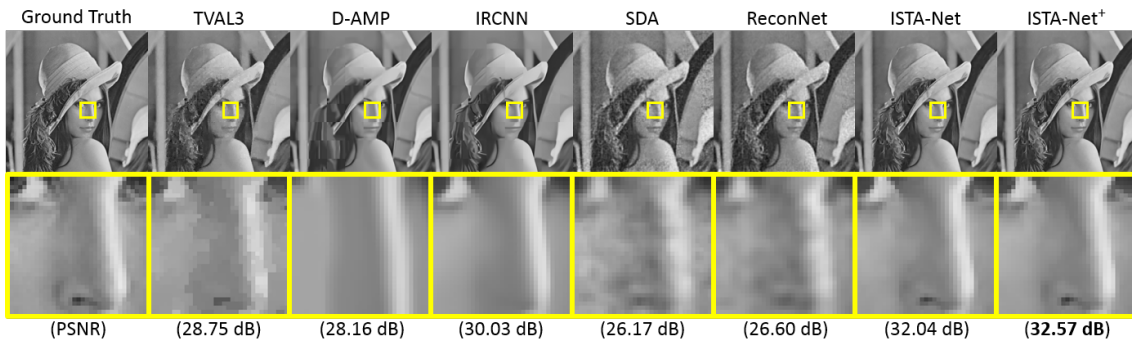


Figure 14: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Lena* image in Set11 (CS ratio is 25%).

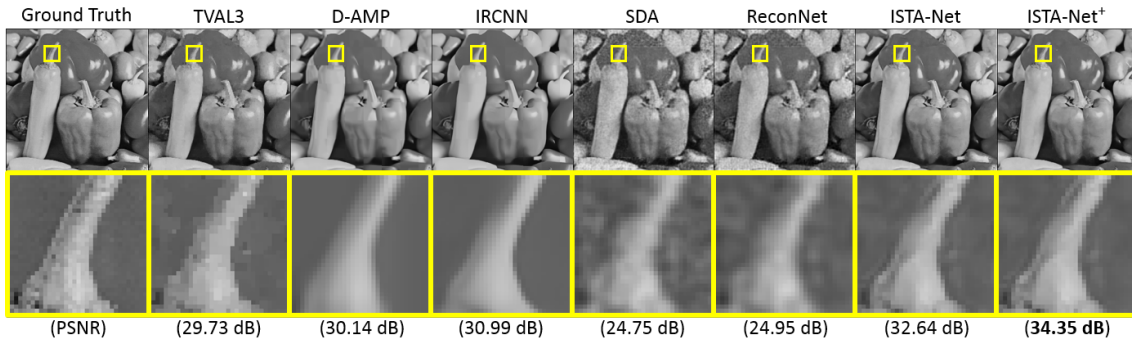


Figure 15: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Pepper* image in Set11 (CS ratio is 25%).

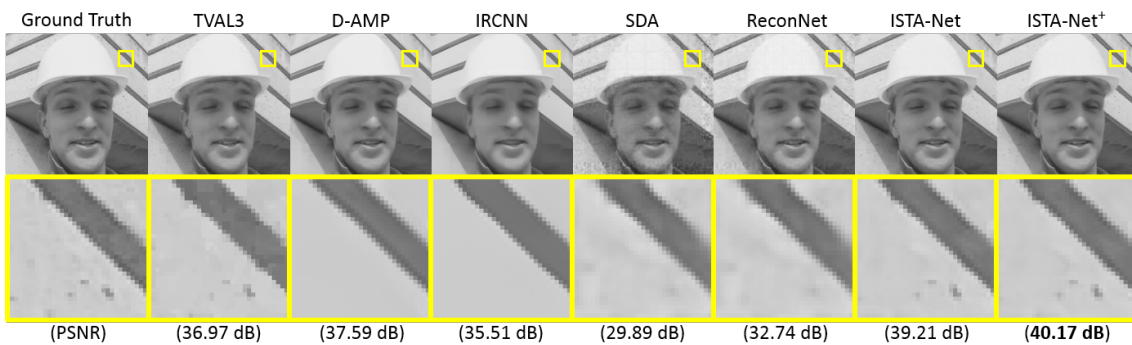


Figure 16: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Foreman* image in Set11 (CS ratio is 30%).

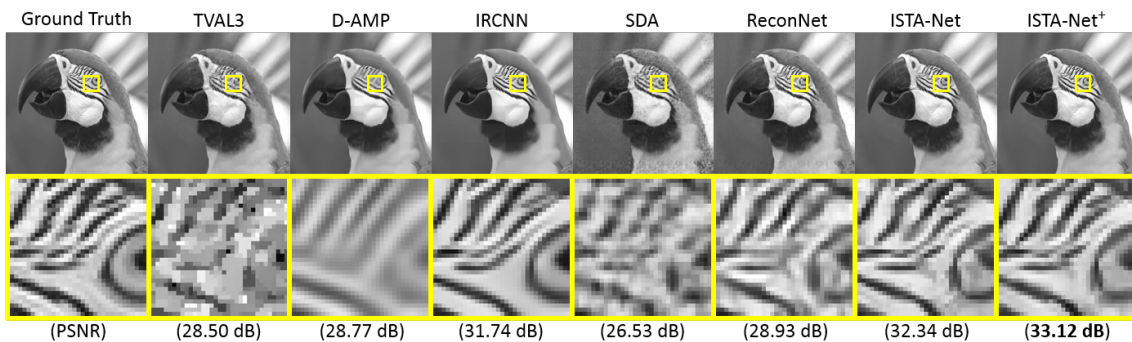


Figure 17: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Parrots* image in Set11 (CS ratio is 30%).

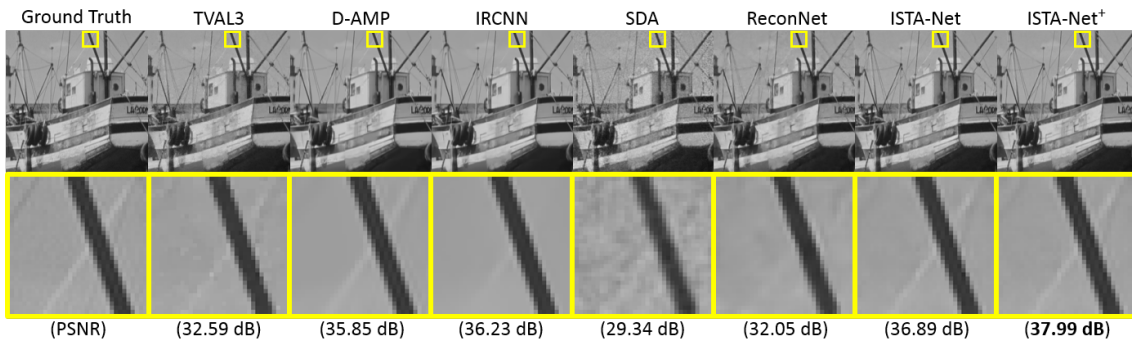


Figure 18: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Boats* image in Set11 (CS ratio is 40%).

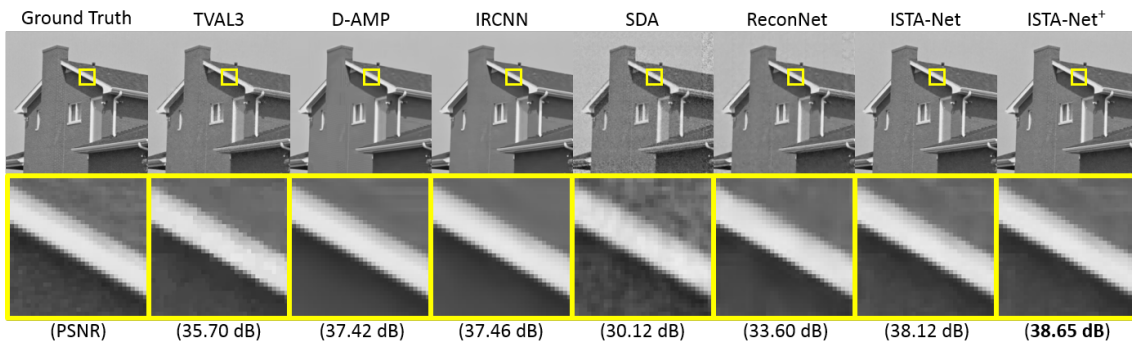


Figure 19: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *House* image in Set11 (CS ratio is 40%).

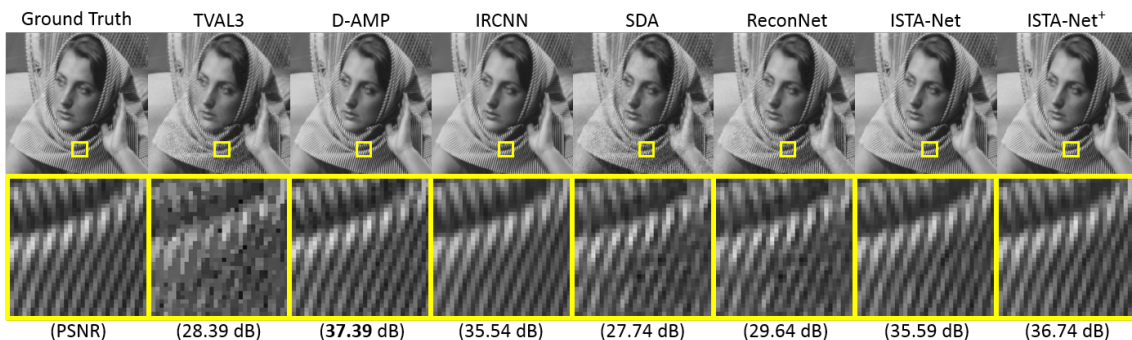


Figure 20: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Barbara* image in Set11 (CS ratio is 50%). Note that for image *Barbara* that is rich of repetitive textures, D-AMP achieves the best PSNR performance with higher CS ratios due to its utilization of non-local self-similarity.

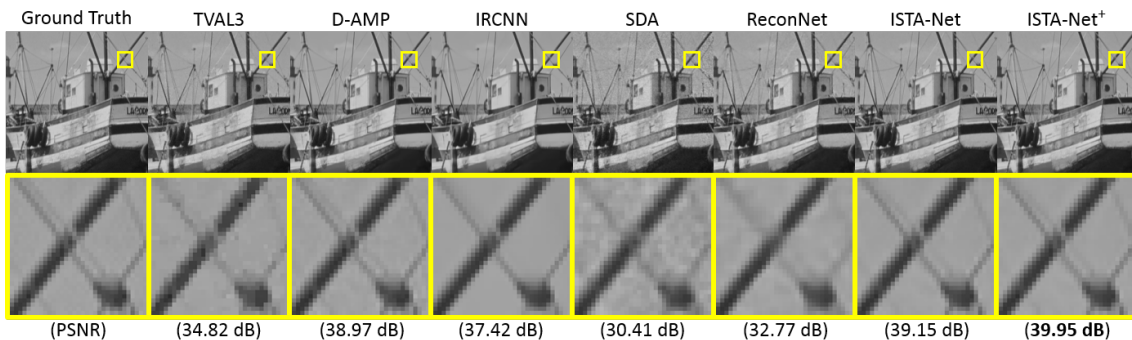


Figure 21: Comparison of seven CS reconstruction methods (including our ISTA-Net and ISTA-Net⁺), when applied to the *Boats* image in Set11 (CS ratio is 50%).



Figure 22: 68 test images in image BSD68 dataset.

Table 3: Average PSNR (dB) performance comparison of various network-based algorithms on the BSD68 dataset with different CS ratios.

Algorithm	CS Ratio						
	50%	40%	30%	25%	10%	4%	1%
SDA [4]	28.35	27.41	26.38	25.22	23.12	21.32	19.02
ReconNet [5]	29.86	29.08	27.53	25.46	24.15	21.66	19.09
ISTA-Net	33.60	31.85	29.93	28.91	25.02	22.12	19.19
ISTA-Net ⁺	34.01	32.21	30.34	29.37	25.33	22.17	19.27