Improving Mathematical Problem Solving in Grades 4 Through 8

Instructional Tips Based on the Educator's Practice Guide

Instructional tips for:

- Assisting Students in Monitoring and Reflecting on the Problem-Solving Process
- Teaching Students to Use Visual Representations to Solve Problems
- Helping Students Make Sense of Algebraic Notation

About the WWC Instructional Tips

Instructional tips help educators carry out recommendations contained in IES Educator's Practice Guides. The tips translate these recommendations into actionable approaches that educators can try in their classrooms. These tips are based on a practice guide authored by *Sybilla Beckmann, Mark Driscoll, Megan Franke, Patricia Herzig, Asha Jitendra, Kenneth R. Koedinger, Philip Ogbuehi*, and *John Woodward.* Each set of instructional tips highlights a key concept and presents relevant how-to steps in the practice guide that are actionable and supported by evidence.

What Works Clearinghouse™

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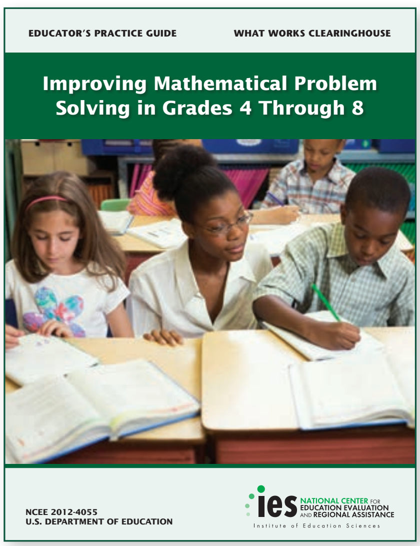


About the Evidence Supporting the Tips

These practices were identified by a panel of experts and are supported by research evidence that meets What Works Clearinghouse design standards. To learn more about this evidence base, read:

- *Summary of Evidence for Instructional Tips on Mathematical Problem Solving*
- *Improving Mathematical Problem Solving in Grades 4 Through 8*

Tips for Assisting Students in Monitoring and Reflecting on the Problem-Solving Process

Monitoring and Reflecting

Monitoring and **reflecting** during problem solving are processes that help students think about what they are doing, evaluate the steps they are taking to solve the problem, and connect new concepts to what they already know. Reflection on the problem-solving process improves mathematical reasoning and students' ability to apply this reasoning to new situations.

The practice guide ***Improving Mathematical Problem Solving in Grades 4 Through 8*** recommends providing students with a **list of prompts** to help them monitor and reflect during problem solving as well as **modeling** the process of monitoring and reflecting for students.

Tip: Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.

- **Provide** a list of prompts. For example:
  1. *Task lists* that help students complete steps in the problem-solving process
  2. *Questions* that students should ask themselves and answer as they solve problems
- **Select** a reasonable number of prompts (tasks or questions) rather than an exhaustive list, as too many prompts may slow down the problem-solving process.
- **List prompts** on the board or on index cards and include them on worksheets so students can easily access them.
- **Use prompts** that help students evaluate their work at each stage of the problem-solving process.
- **Encourage** students to explain and justify their response to each prompt either orally or in writing.

Sample Task List

Subject: Math

- Identify the information provided and goals of the problem.
- Identify the problem type.
- Recall similar problems to help solve the current problem.
- Use a visual to represent and solve the problem.
- Solve the problem.
- Check the solution.

Sample Questions

Subject: Math

- What is the problem asking?
- What do I know about the problem so far? What information is given to me? How can this help me?
- Which information in the problem is relevant?
- In what way is this problem similar to problems I have previously solved?
- What are the various ways I might approach the problem?
- Is my approach working? If I am stuck, is there another way I can think about solving this problem?
- Does the solution make sense? How can I verify the solution?
- Why did these steps work or not work?
- What would I do differently next time?

Source: Adapted from Hohn and Frey (2002).

Tip: Model how to monitor and reflect on the problem-solving process.

- **Model** how to monitor and reflect when introducing a problem-solving activity or new concept using one of the activities below.
  - o Say aloud not only the response to each prompt but also the reasons why each step was taken.
  - o For each step taken, state the step and ask students to explain why it would work.
- **Use a prompt** at each stage of the problem-solving process:
  - o When first reading the problem
  - o When attempting a strategy to solve the problem
  - o After solving the problem

EXAMPLE. Modeling how to monitor and reflect using questions

PROBLEM

Last year was unusually dry in Colorado. Denver usually gets 60 inches of snow per year. Vail, which is up in the mountains, usually gets 350 inches of snow. Both cities had 10 inches of snow less than the year before. Kara and Ramon live in Colorado and heard the weather report. Kara thinks the decline for Denver and Vail is the same. Ramon thinks that when you compare the two cities, the decline is different. Explain how both people are correct.

SOLUTION

What is this problem asking?

Teacher: First, I ask myself, **"What is this problem asking?"** I see that the problem has given me the usual amount of snowfall and the change in snowfall for each place, and that it talks about a decline in both cities. I know decline means: "a change that makes something less." Now I wonder how the decline in snowfall in each city can be the same for Kara and different for Ramon. I know that a decline of 10 inches in each city is the same, which is why Kara is correct. How is Ramon thinking about the problem?

Have I ever seen a problem like this before?

I ask myself, **"Have I ever seen a problem like this before?"** I remember seeing a problem that asked us to calculate the discount on a $20 item that was on sale for $15. I remember we had to determine the percent change. This could be a similar kind of problem. This might be the way Ramon is thinking about the problem.

What steps should I take to solve this problem?

Before I go on, I ask myself, **"What steps should I take to solve this problem?"** It looks like I need to divide the change amount by the original amount to find the percent change in snowfall for both Denver and Vail. Denver: $10 \div 60 = 0.166$ or 16.67% or 17% when we round it to the nearest whole number Vail: $10 \div 350 = 0.029$ or 2.9% or 3% when we round it to the nearest whole number.

So the percent decrease in snow for Denver (17%) was much greater than for Vail (3%). Now I see what Ramon is saying! It's different because the percent decrease for Vail is much smaller than it is for Denver.

Does this answer make sense when I reread the problem?

Finally, I ask myself, **"Does this answer make sense when I reread the problem?"** Kara's answer makes sense because both cities did have a decline of 10 inches of snow. Ramon is also right because the percent decrease for Vail is much smaller than it is for Denver.

Tips for Teaching Students to Use Visual Representations to Solve Problems

Visual Representations

Visual representations help students solve problems by linking the relationships between quantities within the problem with the mathematical operations to solve the problem.

Improving Mathematical Problem Solving in Grades 4 Through 8 recommends teaching students how to use visual representations by providing information about the appropriate type of visual representation, and using open discussion and guided questions to help students use visuals during problem solving.

Tip: Demonstrate for students how to select the appropriate visual representation for the problem they are solving.

- **Select** the visual representation that is best suited for the type of problem and that works well for the students. Use the same type of visual representation consistently for similar problems.
- **Explain** how to identify the type of problem based on mathematical ideas in the problem and explain why a certain visual representation is appropriate.
- **Demonstrate** how to represent the information in the problem visually by teaching students to identify what information is relevant to solving the problem.

EXAMPLE. Selecting Visual Representations for Problems

Problem

During a sale, prices were marked down by 20%. The sale price of an item was $84. What was the original price of the item before the discount?

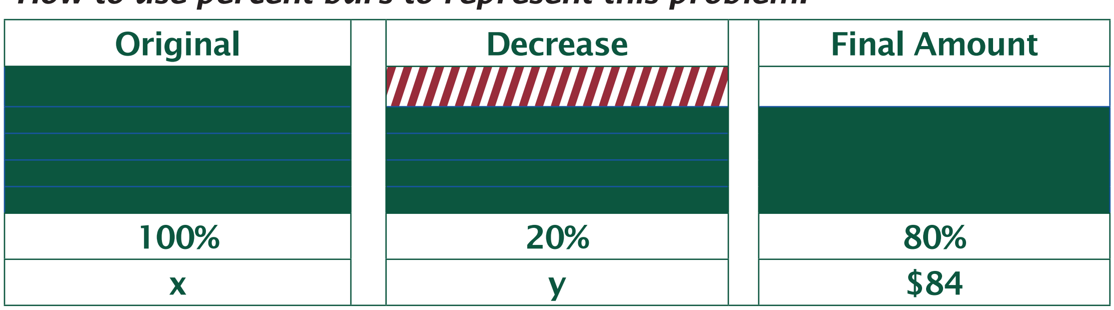
Type of Problem

Percent problem

Type of Visual Representation

Percent bars work well for percent problems

How to use percent bars to represent this problem.



Source: Adapted from Parker (2004).

EXAMPLE. Selecting Visual Representations for Problems

Problem

John recently participated in a 5-mile run. He usually runs 2 miles in 30 minutes. Because of an ankle injury, John had to take a 5-minute break after every mile. At each break, he drank 4 ounces of water. How much time did it take him to complete the 5-mile run?

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Type of Problem

Ratio or proportion problem

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Type of Visual Representation

Schematic diagrams work well with ratio or proportion problems

How to use a schematic diagram to represent this problem.

Problem
John recently participated in a 5-mile run. He usually runs 2 miles in 30 minutes. Because of an ankle injury, John had to take a 5-minute break after every mile. At each break, he drank 4 ounces of water. How much time did it take him to complete the 5-mile run?

Start time Finish time = 95 minutes

■Run ■Break

Tip: Use think-alouds and discussions to teach students how to represent problems visually.

- **Promote** discussion using guided questions.
- **Ask students how and why** they used a particular representation to solve a problem so they can model how to use visual representations for other students.

Guided Questions to Help Students Make Visual Representations

- What kind of problem is this?
  How do you know?
- What is the relevant information in this problem?
  Why is this information relevant?
- Which visual representation did you use when you solved this type of problem last time?
- What would you do next? Why?

Tips for Helping Students Make Sense of Algebraic Notation

Mathematical Notation

Mathematical notations provide students with familiar structures for organizing information in a problem. These notations help students recognize the mathematics present in the problem, extend their understanding to new problems, and explore various options when solving problems.

Understanding the symbolic notation used in algebra takes time. ***Improving Mathematical Problem Solving in Grades 4 Through 8*** recommends using familiar arithmetic problems linked to algebra and, as students gain comfort, asking students to link algebraic equations back to the problems.

Tip: Using familiar arithmetic problems, link arithmetic with algebra to help students gain comfort with algebraic symbols.

- **Provide** an intermediate arithmetic problem for students to solve. Intermediate arithmetic problems have the same mathematical representation as an algebra problem, but provide students with values for the unknown.
- **Discuss** how to apply algebraic notation to the problem so they can slowly integrate algebraic notation and symbols into the work and gain comfort with the new information.

EXAMPLE. Solve a problem arithmetically before solving it algebraically.

Problem

A plumbing company charges $42 an hour, plus $35 for the service call.

Teacher	Student
Teacher: How much would you pay for a 3-hour service call?	**Student:** $\$42 \times 3 + \$35 = \$161$ for a 3-hour service call.
Teacher: What will the bill be for 4.5 hours?	**Student:** $\$42 \times 4.5 + \$35 = \$224$ for 4.5 hours.
Teacher: Now, I'd like you to assign a variable for the number of hours the company works and write an expression for how much a service call costs.	**Student:** I'll choose h to represent the number of hours the company works. $42h + 35 = \$$ required
Teacher: What is the algebraic equation for the number of hours worked if the bill comes out to $140?	**Student:** $42h + 35 = \$140$

Source: Adapted from Koedinger and Anderson (1998).

Tip: Ask students to connect algebraic equations to details of a word problem.

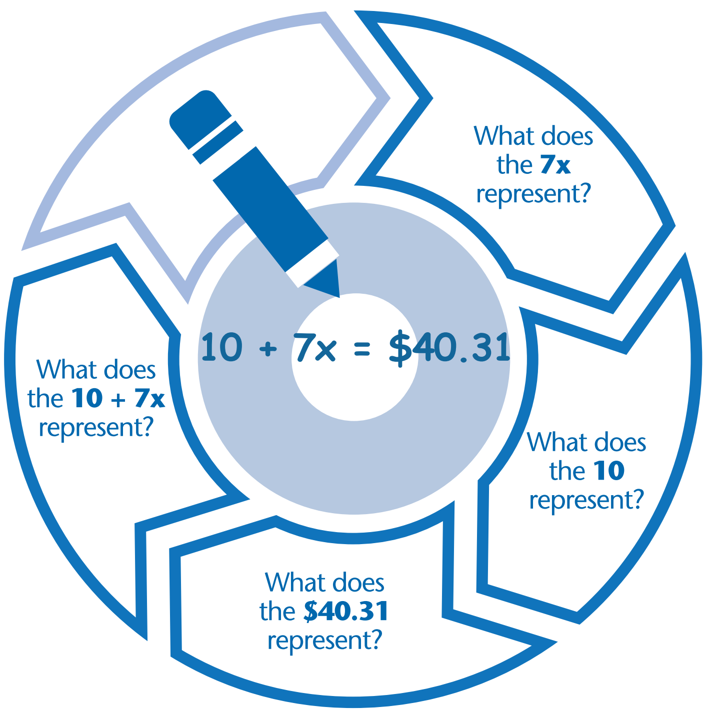
- **Ask students** to explain each component of an algebraic equation by having them link the equation back to the problem they are solving.
- **Use questions** to help students understand how components in the equation and elements of the problem correspond, what each component of the equation means, and how useful algebra is for solving the problem.

EXAMPLE. Link components of the equation to the problem

Problem
Joseph earned money for selling 7 DVDs and his old headphones. He sold the headphones for $10. He made $40.31. How much did he sell each DVD for?
Linking the Equation to the Problem

The teacher writes this equation: **10 + 7x = $40.31**

Teacher: If x represents the number of dollars he sold each DVD for…



Source: Adapted from Corbett et al. (2006).