Improved Search for Integral, Impossible Differential and Zero-Correlation Attacks

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FSE 2024 - Leuven, Belgium

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Motivation and Our Contributions

Motivation

- Providing a tool to find **complete** integral, and ID/ZC attacks

Contributions

- Improving the CP-based methods to find ID/ZC, and integral distinguishers.
- Introducing a CP model for the partial-sum technique for the first time.
- Improving distinguishers of Ascon, QARMAv2, and ForkSKINNY (**25** Dists.).
- Improving key recovery attacks of SKINNY, and ForkSKINNY (**24** Attacks).

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Part of Our Results Regarding Distinguishing Attacks

Cipher	#Rounds	Dist.	Data complexity	Ref.
QARMAv2-64	5	Integral	-	[Ava+23]
QARMAv2-64 ($\mathscr{T} = 1$)	**7 / 8 / 9**	Integral	$\mathbf{2^{8}}$ / $\mathbf{2^{16}}$ / $\mathbf{2^{44}}$	This work
QARMAv2-64 ($\mathscr{T} = 2$)	**8 / 9 / 10**	Integral	$\mathbf{2^{8}}$ / $\mathbf{2^{16}}$ / $\mathbf{2^{44}}$	This work
QARMAv2-128($\mathscr{T} = 2$)	**10 / 11 / 12**	Integral	$\mathbf{2^{16}}$ / $\mathbf{2^{44}}$ / 2^{96}	This work
ForkSKINNY-64-192	16	Integral	2^{72}	[Niu+21]
ForkSKINNY-64-192	**17**	Integral	$\mathbf{2^{60}}$	This work
ForkSKINNY-64-192	16	ID	-	[HSE23]
ForkSKINNY-64-192	**21**	ID	-	This work
ForkSKINNY-128-256	14	Integral	2^{56}	[HSE23]
ForkSKINNY-128-256	**15**	Integral	$\mathbf{2^{56}}$	This work

Part of Our Results Regarding Key Recovery Attacks

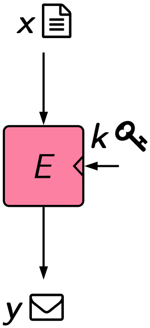
Cipher	#R	Time	Data	Mem.	Attack	Setting / Model	Ref.
SKINNY-64-64	17	2^{59}	$2^{58.79}$	2^{40}	ID	STK / CP	[HSE23]
	18	$\mathbf{2^{53.58}}$	$2^{53.58}$	2^{48}	Int	60,SK / CP,CT	This work
SKINNY-128-128	17	$2^{116.51}$	$2^{116.37}$	2^{80}	ID	STK / CP	[HSE23]
	18	$\mathbf{2^{105.58}}$	$2^{105.58}$	2^{96}	Int	120,SK / CP,CT	This work
SKINNY-128-384	26	2^{344}	2^{121}	2^{340}	Int	360,SK / CP,CT	[HSE23]
	26	$\mathbf{2^{331}}$	2^{122}	2^{328}	Int	360,SK / CP,CT	This work
ForkSKINNY-128-256	26	$2^{250.30}$	2^{127}	2^{160}	ID	256,RTK / CP	[BDL20]
	26	$\mathbf{2^{238.50}}$	$2^{128.60}$	$2^{175.60}$	ID	256,RTK / CP	This paper

Outline

Background and the Research Gap

Integral, ID, and ZC Distinguishers

- Integral attack [Lai94; DKR97]
- Impossible-differential attack [BBS99; Knu98]
- Zero-correlation attack [BR14]



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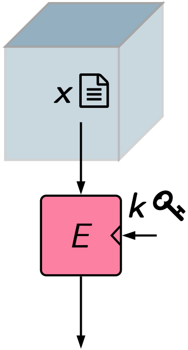

$\sum_{x \in \mathbb{X}} y = 0$

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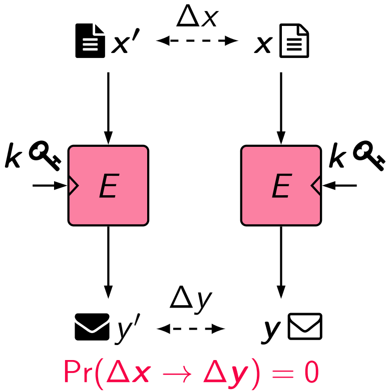

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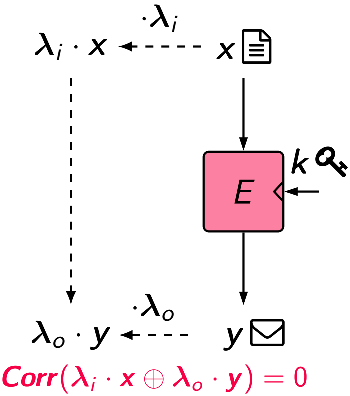
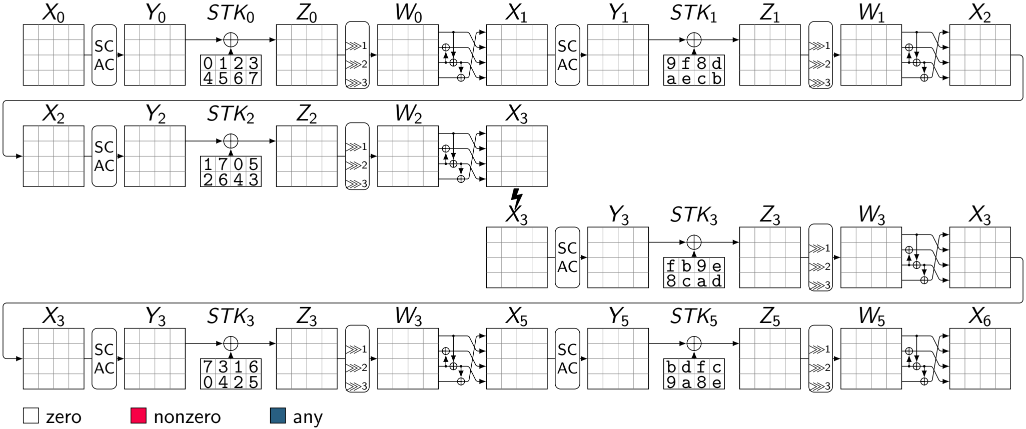


Miss-in-the-Middle Technique [BBS99]

- Find two differences (linear masks) that propagate forward and backward with probability one and contradict each other in the middle



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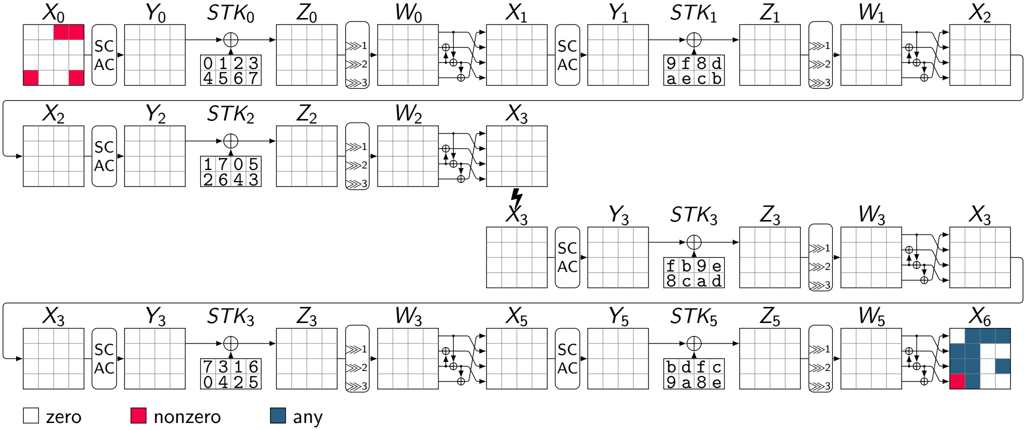

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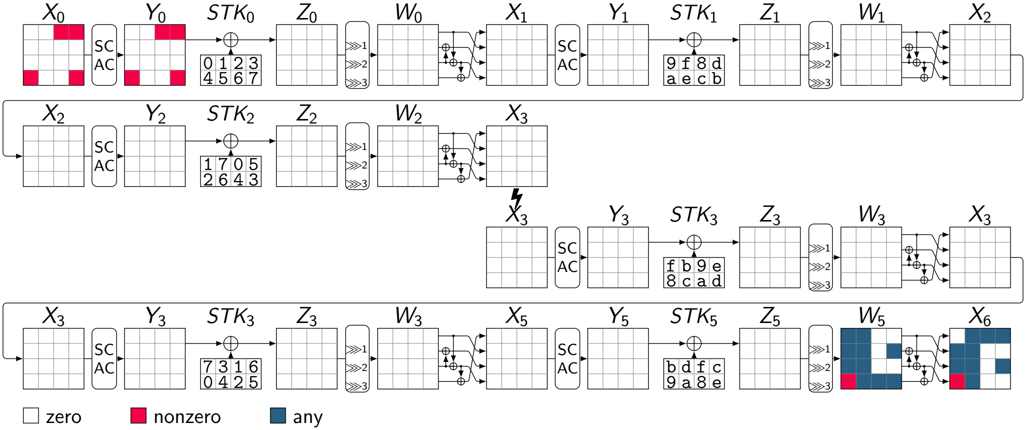

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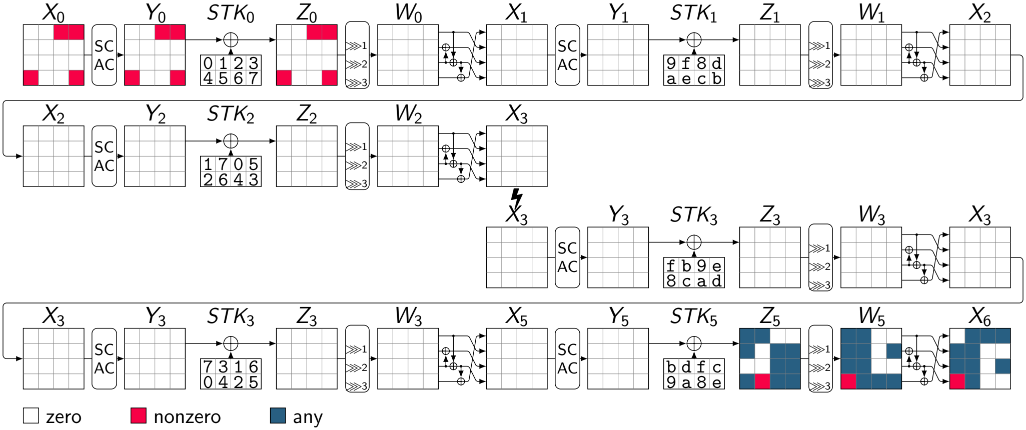

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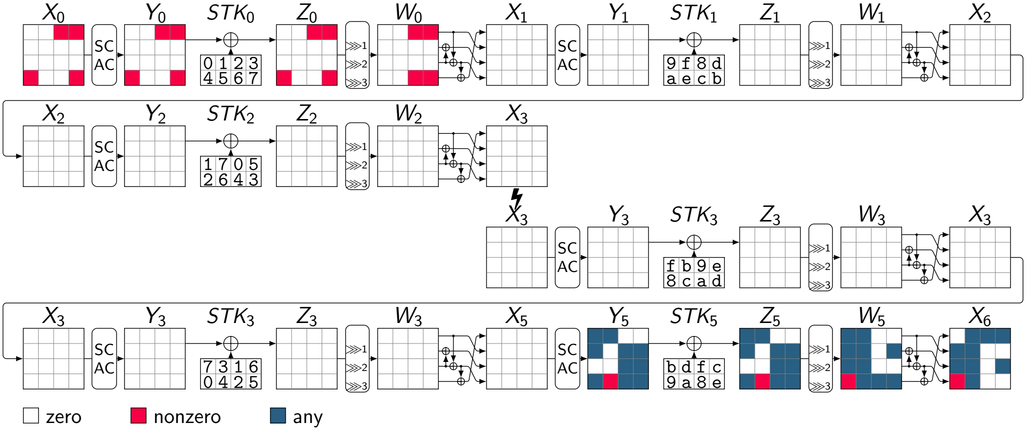

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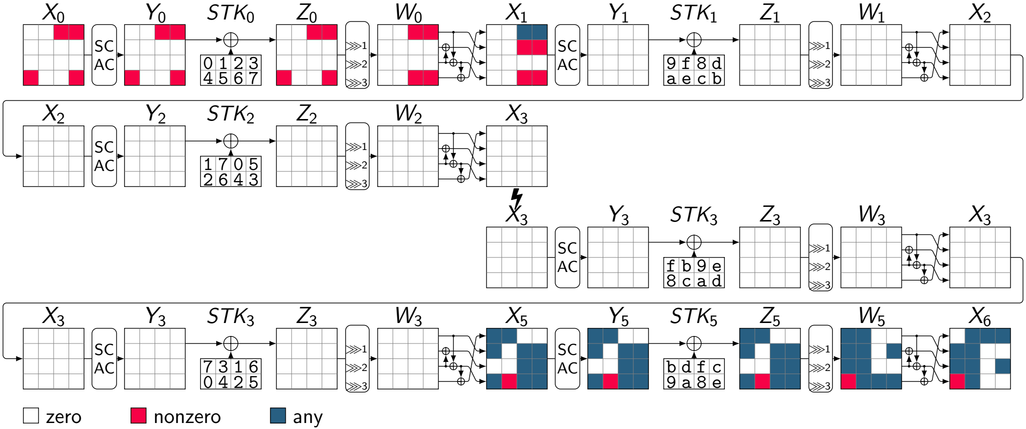

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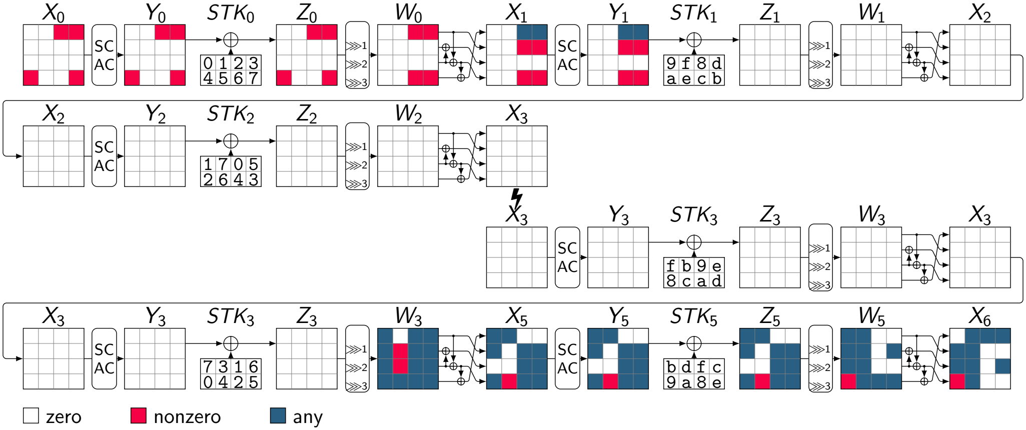

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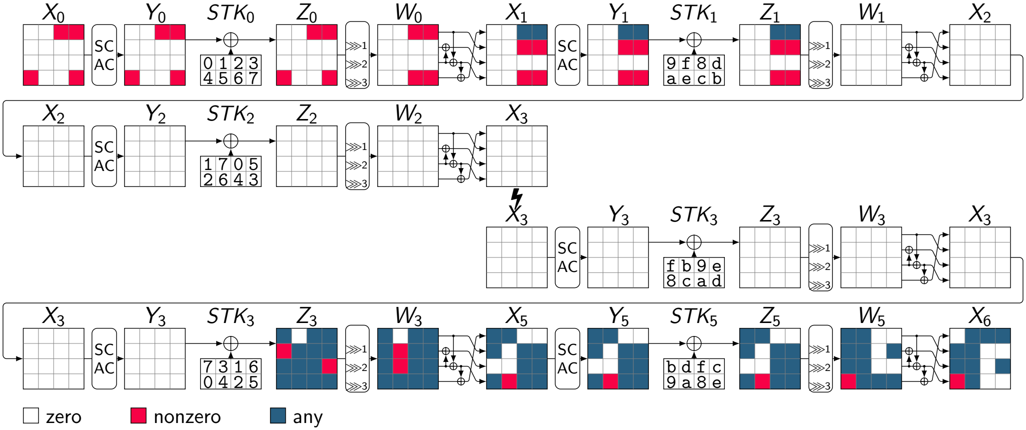

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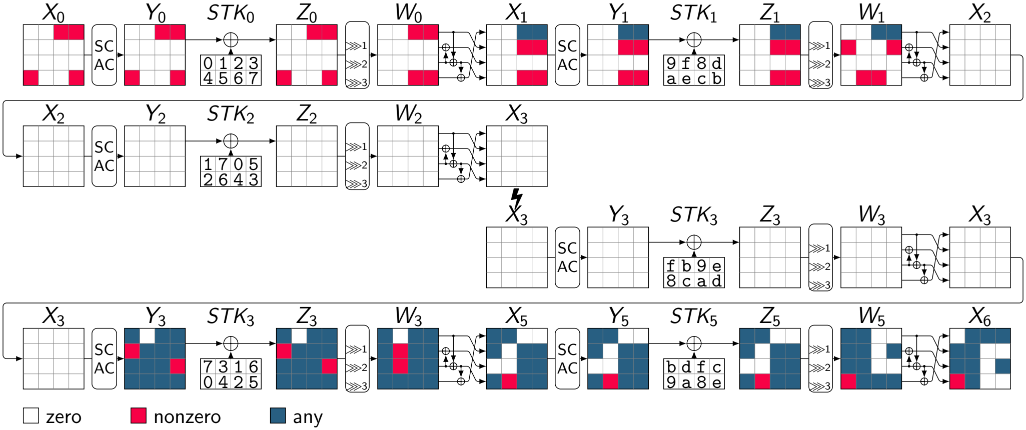

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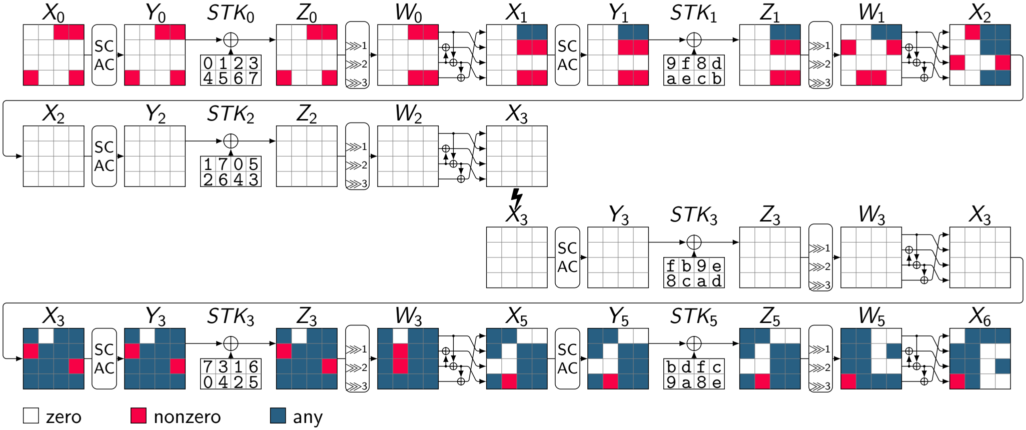

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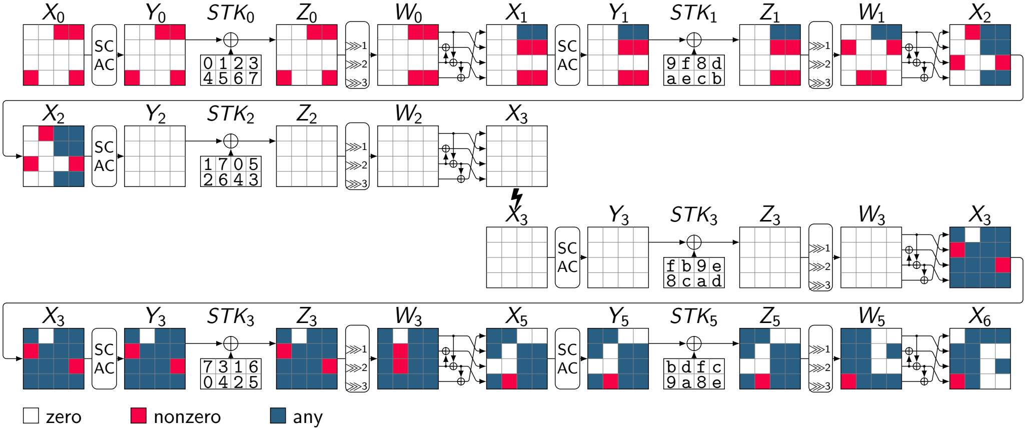

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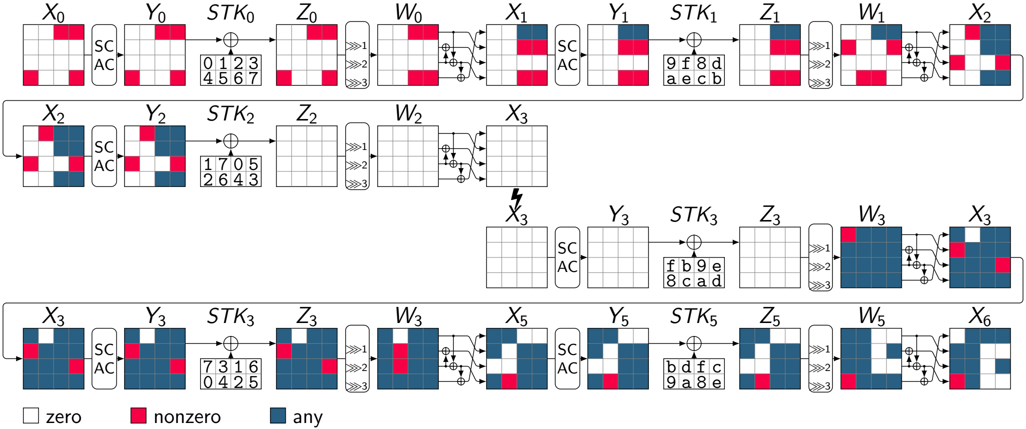

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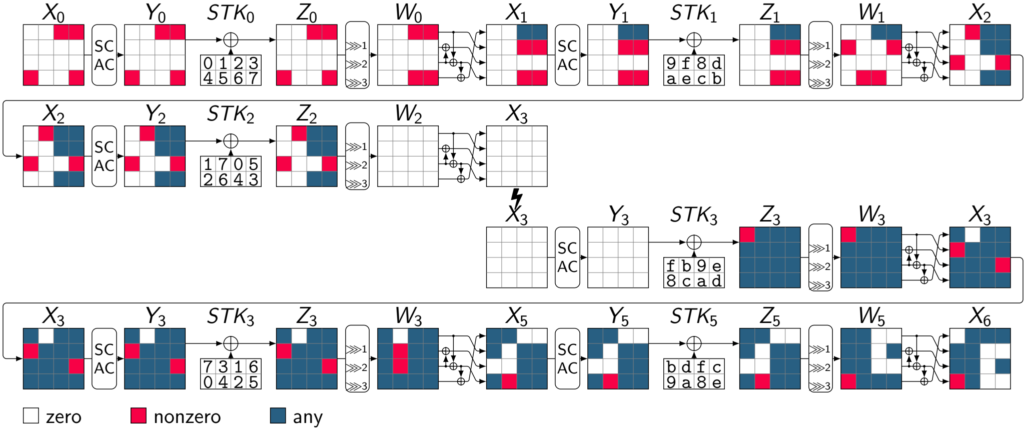

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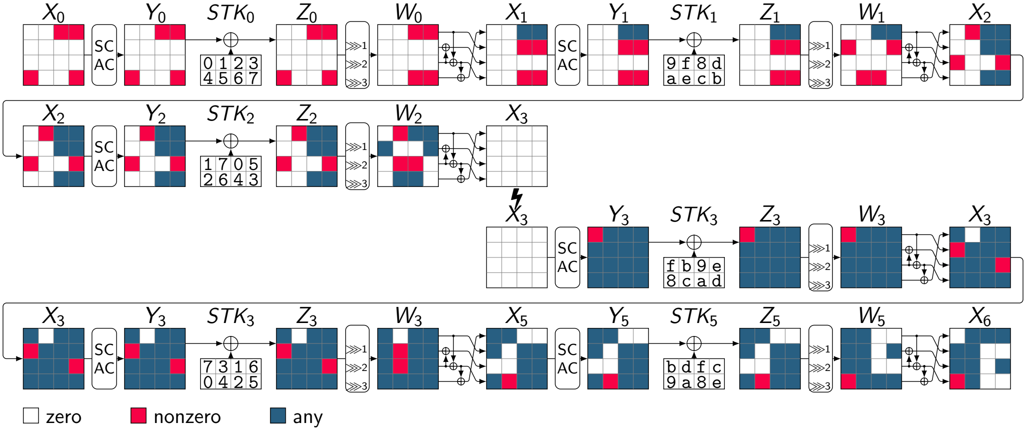

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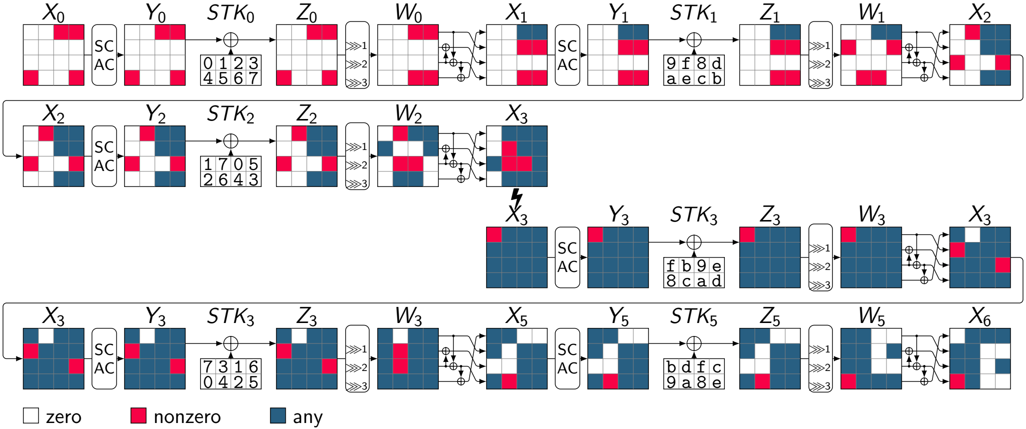

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Relation Between ZC and Integral Distinguishers

- Any ZC distinguisher can be converted to an integral distinguisher [Sun+15].

Link Between ZC and Integral Distinguishers [Sun+15]

Let $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ be a vectorial Boolean function. Assume A is a subspace of $\mathbb{F}_2^n$ and $\beta \in \mathbb{F}_2^n \setminus \{0\}$ such that (α, β) is a ZC approximation for any $\alpha \in A$. Then, for any $\lambda \in \mathbb{F}_2^n$, $\langle \beta, F(x + \lambda) \rangle$ is balanced over the set

$$A^{\perp} = \{x \in \mathbb{F}_2^n \mid \forall\, \alpha \in A : \langle \alpha, x \rangle = 0\}.$$

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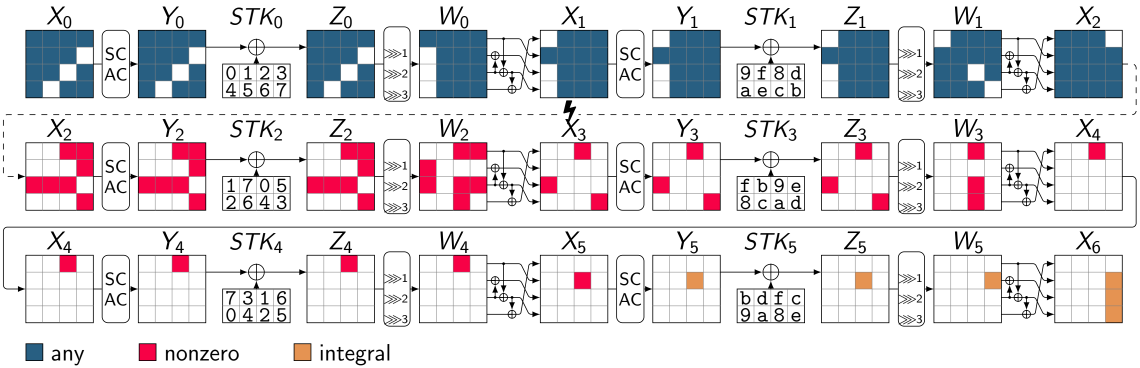
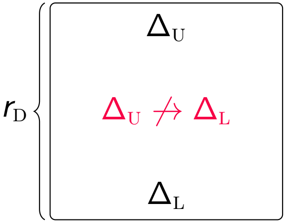
Example: Conversion of ZC Distinguisher to Integral Distinguisher



- $X_0[7, 10, 13]$ takes all possible values and the remaining cells take a fixed value
- $X_6[7] \oplus X_6[11] \oplus X_6[15]$ is balanced

ID, ZC, and Integral Key Recovery

- Common technique for ID key recovery:
  - Early abort technique [Lu+08]
- Common technique for ZC/Integral key recovery:
  - Partial-sum technique [Fer+00]



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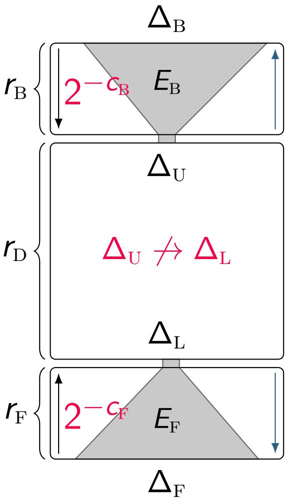

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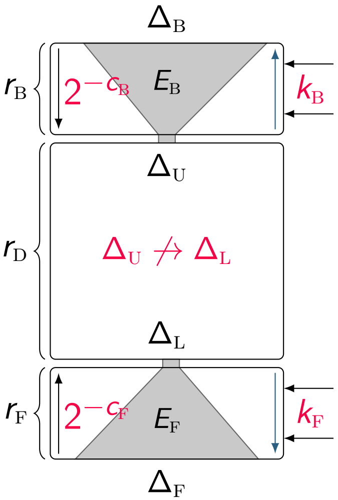

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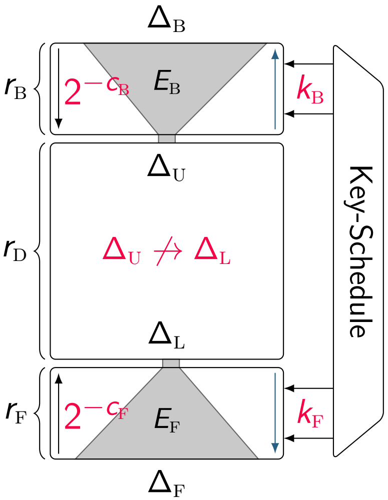

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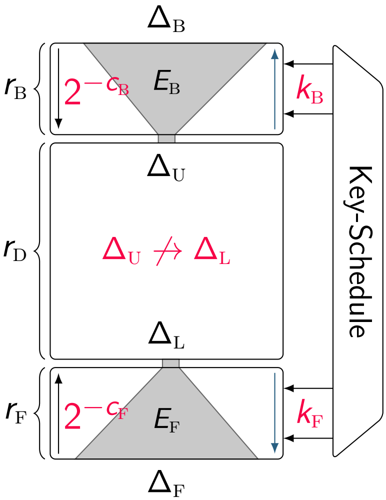
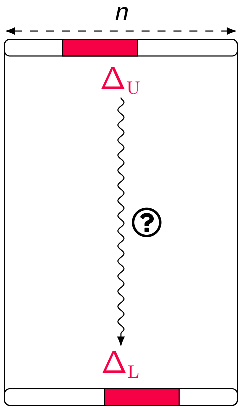


Previous Tools for ID/ZC, and Integral Attacks

- Tools based on dedicated algorithms:
  - CRYPTO 2016 ($\mathcal{DC}$-MITM, ID) [DF16]
- Tools based on general purpose solvers:
  - Eprint 2016 (ID) [Cui+16]
  - ASIACRYPT 2016 (Integral) [Xia+16]
  - EUROCRYPT 2017 (ID, ZC) [ST17]
  - ToSC 2017 (ID, ZC) [Sun+17]
  - ToSC 2020 (ID, ZC) [Sun+20]



Search for Distinguishers

Our Previous Method to Search Distinguishers [HSE23]

E

- $CSP_{\mathrm{U}}(\Delta_{\mathrm{U}}, \Delta'_{\mathrm{U}})$
- $CSP_{\mathrm{L}}(\Delta_{\mathrm{L}}, \Delta'_{\mathrm{L}})$
- $CSP_{\mathrm{M}}(\Delta'_{\mathrm{U}}, \Delta'_{\mathrm{L}})$

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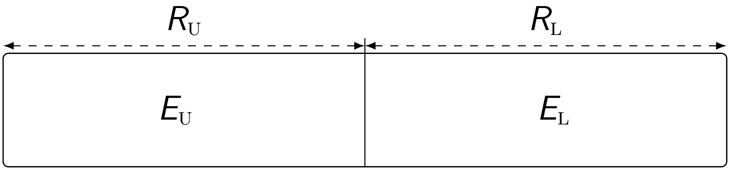

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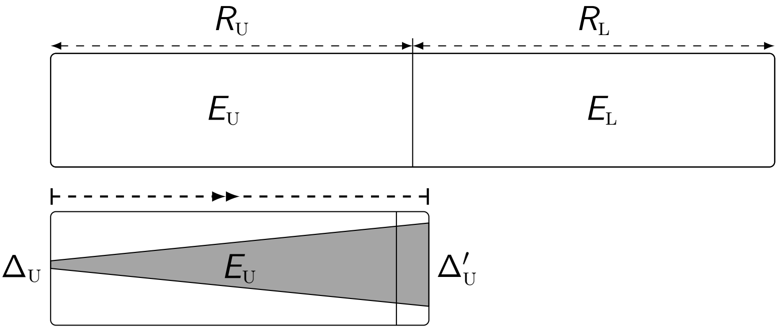

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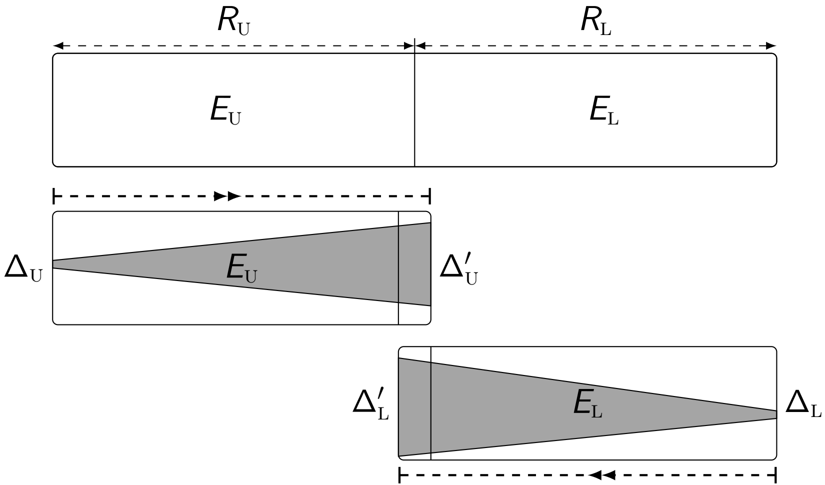

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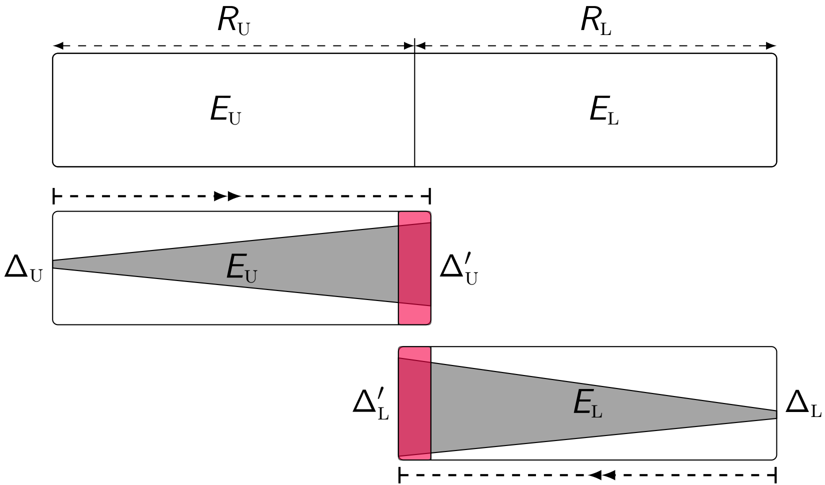

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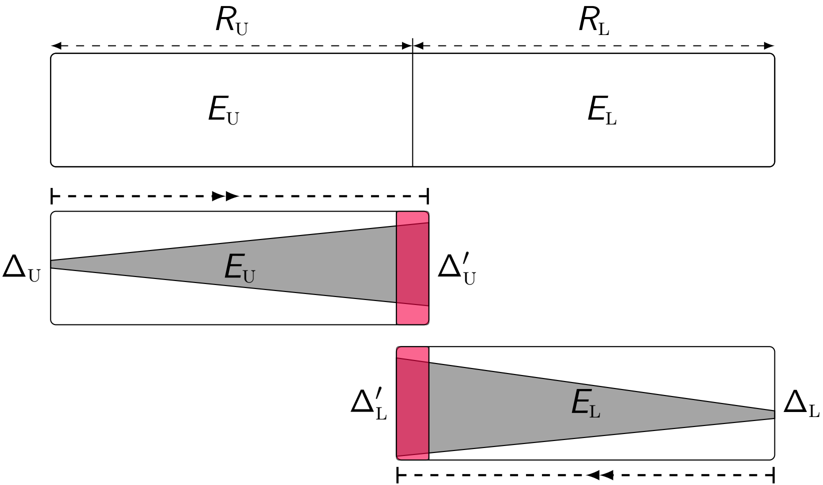
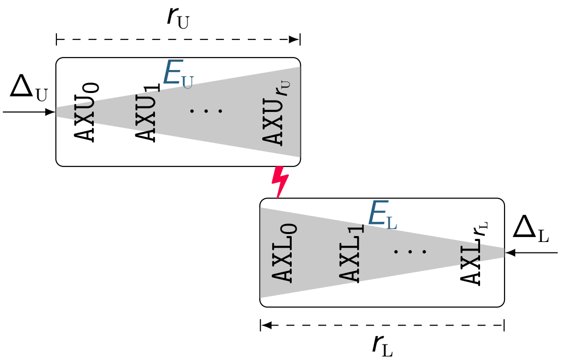


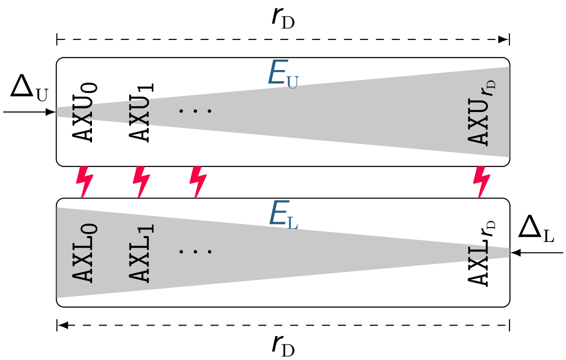
Our New Word-Wise Method for Finding Distinguishers

Relax the Limit of Fixing the Contradiction's Location

- Find ID distinguisher for $r_D (= r_U + r_L)$ rounds



Modeling the distinguishers in [HSE23].



Our modeling of the distinguishers.

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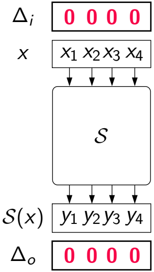
Our New Bit-Wise Method for Finding Distinguishers

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



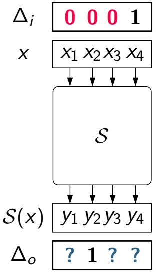
$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

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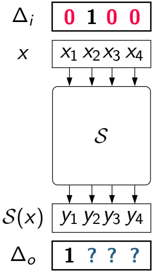
FSE 2024 - Leuven, Belgium

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

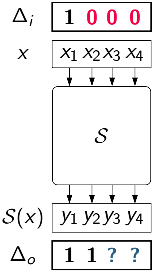
$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

Hosein Hadipour, Simon Gerhalter, Sadegh Sadeghi, Maria Eichlseder

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

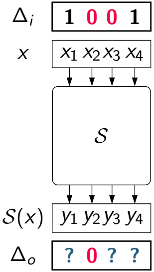
$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

$$\Delta_i = (1,0,0,0) \xrightarrow{S} \Delta_o = (1,1,?,?)$$

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

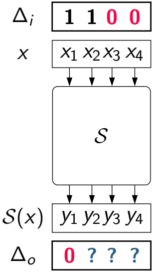
$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

$$\Delta_i = (1,0,0,0) \xrightarrow{S} \Delta_o = (1,1,?,?)$$

$$\Delta_i = (1,0,0,1) \xrightarrow{S} \Delta_o = (?,0,?,?)$$

Deterministic Bit-Wise Differential Trails (a.k.a. Undisturbed Bits [Tez14])

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\Delta_i \setminus \Delta_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	2	2	2	2	0	0	0	0	2	2	2	2
2	0	2	0	2	0	0	0	4	0	2	2	0	0	0	2	2
3	0	2	0	2	0	0	4	0	0	2	2	0	0	0	2	2
4	0	0	0	0	0	0	0	0	0	0	4	4	2	2	2	2
5	0	0	0	0	2	2	2	2	0	0	4	4	0	0	0	0
6	0	2	0	2	0	4	0	0	0	2	2	0	2	2	0	0
7	0	2	0	2	4	0	0	0	0	2	2	0	2	2	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	4	4	4	4
9	0	4	4	0	0	0	0	0	0	4	0	4	0	0	0	0
a	0	0	2	2	2	0	0	2	4	0	0	0	0	2	0	2
b	0	0	2	2	0	2	2	0	4	0	0	0	2	0	2	0
c	0	4	4	0	2	2	2	2	0	0	0	0	0	0	0	0
d	0	0	0	0	2	2	2	2	0	4	0	4	0	0	0	0
e	0	0	2	2	0	2	2	0	4	0	0	0	0	2	0	2
f	0	0	2	2	2	0	0	2	4	0	0	0	2	0	2	0

$$\Delta_i = (0,0,0,0) \xrightarrow{S} \Delta_o = (0,0,0,0)$$

$$\Delta_i \neq (0,0,0,0) \xrightarrow{S} \Delta_o \neq (0,0,0,0)$$

$$\Delta_i = (0,0,0,1) \xrightarrow{S} \Delta_o = (?,1,?,?)$$

$$\Delta_i = (0,1,0,0) \xrightarrow{S} \Delta_o = (1,?,?,?)$$

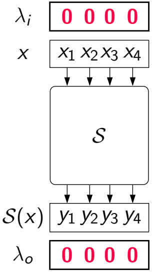
$$\Delta_i = (1,0,0,0) \xrightarrow{S} \Delta_o = (1,1,?,?)$$

$$\Delta_i = (1,0,0,1) \xrightarrow{S} \Delta_o = (?,0,?,?)$$

$$\Delta_i = (1,1,0,0) \xrightarrow{S} \Delta_o = (0,?,?,?)$$

Deterministic Bit-Wise Linear Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



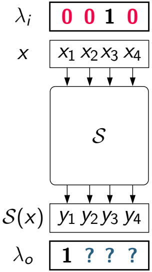
$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

$$\lambda_i = (0,0,0,0) \xrightarrow{S} \lambda_o = (0,0,0,0)$$

$$\lambda_i \neq (0,0,0,0) \xrightarrow{S} \lambda_o \neq (0,0,0,0)$$

Deterministic Bit-Wise Linear Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

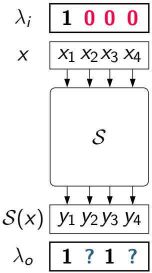
$$\lambda_i = (0,0,0,0) \xrightarrow{S} \lambda_o = (0,0,0,0)$$

$$\lambda_i \neq (0,0,0,0) \xrightarrow{S} \lambda_o \neq (0,0,0,0)$$

$$\lambda_i = (0,0,1,0) \xrightarrow{S} \lambda_o = (1,?,?,?)$$

Deterministic Bit-Wise Linear Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

$$\lambda_i = (0,0,0,0) \xrightarrow{S} \lambda_o = (0,0,0,0)$$

$$\lambda_i \neq (0,0,0,0) \xrightarrow{S} \lambda_o \neq (0,0,0,0)$$

$$\lambda_i = (0,0,1,0) \xrightarrow{S} \lambda_o = (1,?,?,?)$$

$$\lambda_i = (1,0,0,0) \xrightarrow{S} \lambda_o = (1,?,1,?)$$

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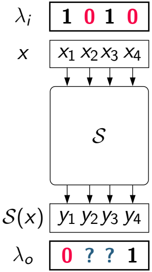
Deterministic Bit-Wise Linear Trails

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$\mathcal{S}(x)$	4	0	a	7	b	e	1	d	9	f	6	8	5	2	c	3



$\lambda_i \setminus \lambda_o$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	4	-4	0	-8	-4	-4	0	0	4	-4	-8	0	4	4
2	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8	0
3	0	-8	4	4	0	0	-4	4	0	0	-4	4	-8	0	-4	-4
4	0	4	0	4	0	4	8	-4	0	4	0	4	-8	-4	0	4
5	0	4	-4	-8	0	-4	-4	0	0	4	-4	8	0	-4	-4	0
6	0	-4	8	4	0	-4	0	-4	0	4	0	4	8	-4	0	4
7	0	4	4	0	0	-4	4	-8	0	-4	-4	0	0	4	-4	-8
8	0	0	0	0	0	0	0	0	0	0	8	8	0	0	8	-8
9	0	0	-4	4	8	0	-4	-4	0	0	4	-4	0	-8	-4	-4
a	0	8	0	8	0	-8	0	8	0	0	0	0	0	0	0	0
b	0	0	-4	4	-8	0	-4	-4	0	8	-4	-4	0	0	4	-4
c	0	4	0	4	0	4	-8	-4	8	-4	0	4	0	4	0	4
d	0	4	4	0	-8	4	-4	0	-8	-4	4	0	0	-4	-4	0
e	0	4	8	-4	0	4	0	4	8	4	0	-4	0	-4	0	-4
f	0	-4	-4	0	-8	-4	4	0	8	-4	4	0	0	-4	-4	0

$$\lambda_i = (0,0,0,0) \xrightarrow{S} \lambda_o = (0,0,0,0)$$

$$\lambda_i \neq (0,0,0,0) \xrightarrow{S} \lambda_o \neq (0,0,0,0)$$

$$\lambda_i = (0,0,1,0) \xrightarrow{S} \lambda_o = (1,?,?,?)$$

$$\lambda_i = (1,0,0,0) \xrightarrow{S} \lambda_o = (1,?,1,?)$$

$$\lambda_i = (1,0,1,0) \xrightarrow{S} \lambda_o = (0,?,?,1)$$

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CP Model for Deterministic Bit-Wise Trails - I

- For each bit position, we define an integer variable with domain $\{\mathbf{0}, \mathbf{1}, \mathbf{-1}\}$.
- Define CP constraints to model the propagation of deterministic bit-wise trails.

S-box

Assume that $x[i], y[i]$ are integer variables with domain $\{-1, 0, 1\}$ to encode the input and output differences at the *i*-th bit position, respectively. The valid deterministic differential transitions satisfy the following:

$$\begin{cases} \mathtt{if}(x[0] = 0 \land x[1] = 0 \land x[2] = 0 \land x[3] = 0)\ \mathtt{then}\ (y[0] = 0 \land y[1] = 0 \land y[2] = 0 \land y[3] = 0) \\ \mathtt{elseif}(x[0] = 0 \land x[1] = 0 \land x[2] = 0 \land x[3] = 1)\ \mathtt{then}\ (y[0] = -1 \land y[1] = 1 \land y[2] = -1 \land y[3] = -1) \\ \mathtt{elseif}(x[0] = 0 \land x[1] = 1 \land x[2] = 0 \land x[3] = 0)\ \mathtt{then}\ (y[0] = 1 \land y[1] = -1 \land y[2] = -1 \land y[3] = -1) \\ \mathtt{elseif}(x[0] = 1 \land x[1] = 0 \land x[2] = 0 \land x[3] = 0)\ \mathtt{then}\ (y[0] = 1 \land y[1] = 1 \land y[2] = -1 \land y[3] = -1) \\ \mathtt{elseif}(x[0] = 1 \land x[1] = 0 \land x[2] = 0 \land x[3] = 1)\ \mathtt{then}\ (y[0] = -1 \land y[1] = 0 \land y[2] = -1 \land y[3] = -1) \\ \mathtt{elseif}(x[0] = 1 \land x[1] = 1 \land x[2] = 0 \land x[3] = 0)\ \mathtt{then}\ (y[0] = 0 \land y[1] = -1 \land y[2] = -1 \land y[3] = -1) \\ \mathtt{else}(y[0] = -1 \land y[1] = -1 \land y[2] = -1 \land y[3] = -1)\ \mathtt{endif}; \end{cases}$$

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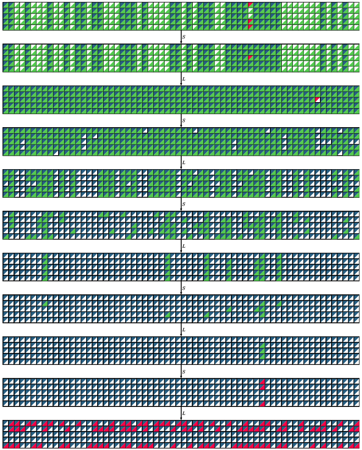
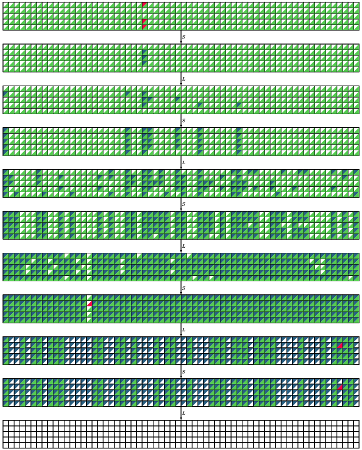
Example: ID/ZC Distinguishers for 5 Rounds of Ascon



2^{155} ZC Distinguishers (upper/lower nonzero: /)



2^{155} ID Distinguishers (upper/lower unknown: /)

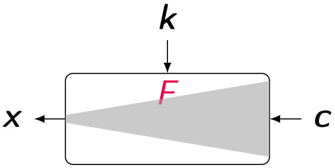
The Advantages of Our Method to Search for Distinguishers

- Based on satisfiability of the CP model
- Any feasible solutions of our CP model is a distinguisher
- We do not fix the input/output of distinguisher
- Extendable to a unified model for key-recovery
  - Enables us to find a distinguisher optimized for key-recovery
  - Enables us to consider key-recovery techniques:
    - MitM
    - Key bridging
    - *Partial-sum technique*

Our Unified CP Model for Partial-Sum Key-Recovery

Naive Approach v.s. Partial-Sum Technique

- Naive approach:
  - $x = F(\boldsymbol{k}, \boldsymbol{c})$
  - $T = N \cdot 2^{|\boldsymbol{k}|}$

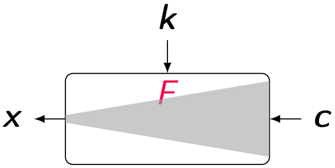


- Partial-sum technique:
  - $x_1 = f_1(k_1, x_0), x_2 = f_2(k_2, x_1), \cdots, x = f_n(k_n, x_{n-1})$
  - $x_0 = \boldsymbol{c}, N_0 = N, N_i < N$
  - $T = \sum_{i=1}^{n} \frac{N_{i-1}}{n} \cdot 2^{|k_1| + \cdots + |k_i|} < \sum_{i=1}^{n} \frac{N}{n} \cdot 2^{|\boldsymbol{k}|}$
  - $T < N \cdot 2^{|\boldsymbol{k}|}$

Naive Approach v.s. Partial-Sum Technique

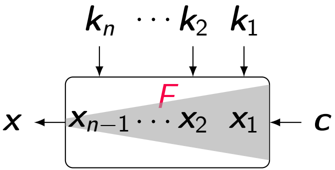
Naive approach:

- $x = F(\boldsymbol{k}, \boldsymbol{c})$
- $T = N \cdot 2^{|\boldsymbol{k}|}$



Partial-sum technique:

- $x_1 = f_1(\boldsymbol{k}_1, x_0), x_2 = f_2(\boldsymbol{k}_2, x_1), \cdots, x = f_n(\boldsymbol{k}_n, x_{n-1})$
- $x_0 = \boldsymbol{c}, N_0 = N, N_i < N$
- $T = \sum_{i=1}^{n} \frac{N_{i-1}}{n} \cdot 2^{|\boldsymbol{k}_1| + \cdots + |\boldsymbol{k}_i|} < \sum_{i=1}^{n} \frac{N}{n} \cdot 2^{|\boldsymbol{k}|}$
- $T < N \cdot 2^{|\boldsymbol{k}|}$



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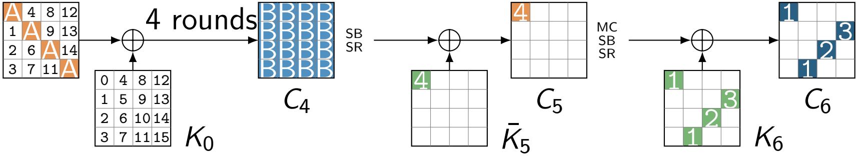
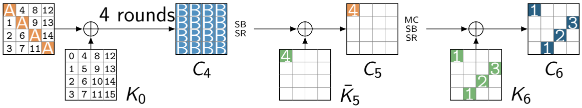
Example: Partial-Sum Integral Key Recovery for AES [Fer+00]



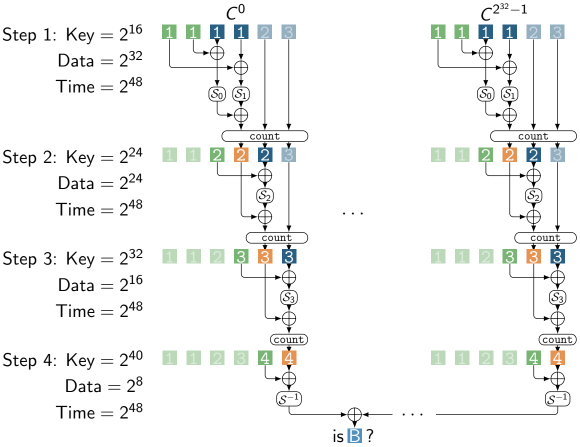
$$C_4[0] = \mathcal{S}^{-1}\Big(\bar{K}_5[0] \oplus \texttt{0E} \cdot \mathcal{S}^{-1}(C_6[0] \oplus K_6[0]) \oplus \texttt{09} \cdot \mathcal{S}^{-1}(C_6[7] \oplus K_6[7])$$
$$\oplus \texttt{0D} \cdot \mathcal{S}^{-1}(C_6[10] \oplus K_6[10]) \oplus \texttt{0B} \cdot \mathcal{S}^{-1}(C_6[13] \oplus K_6[13])\Big)$$

- Time complexity of naive key recovery: $6 \times 2^{32} \times 2^{40} \approx 2^{74.58}$

Partial-sum Technique for Integral Key Recovery [Fer+00]



- Guess $K_6[0, 7]$ and derive $\mathcal{S}_0\,(C_6[0] \oplus K_6[0]) \oplus \mathcal{S}_1\,(C_6[7] \oplus K_6[7])$
- Guess $K_6[10]$ and derive $\mathcal{S}_2\,(C_6[10] \oplus K_6[10])$
- Guess $K_6[13]$ and derive $\mathcal{S}_3\,(C_6[13] \oplus K_6[13])$
- Guess $\bar{K}_5[0]$ and derive $C_4[0]$
- Time complexity: $6 \times 4 \times 2^{48} \approx 2^{52}$ S-box lookups



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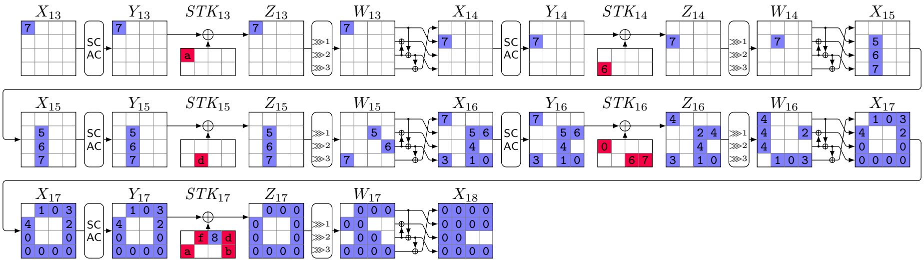
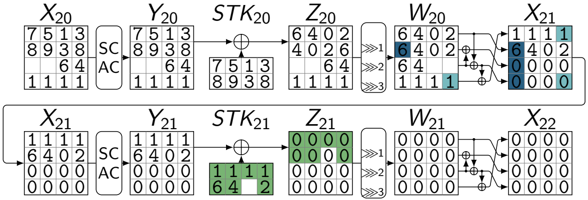
Our CP Model for Partial-Sum Technique - I



Step	Guessed	K × D = Mem	Time	Stored Texts
0	–	$2^0 \times 2^{40} = 2^{40}$	$2^{40-5.2}$	$Z_{17}[1,3,4,7]$; $X_{17}[8,11,12,13,15]$; $X_{16}[15]$
1	$STK_{17}[1]$	$2^4 \times 2^{36} = 2^{40}$	$2^{44-7.2}$	$Z_{17}[3,4,7]$; $X_{17}[8,11,12,15]$; $X_{16}[14,15]$
2	$STK_{17}[7]$	$2^8 \times 2^{32} = 2^{40}$	$2^{44-8.2}$	$Z_{17}[3,4]$; $X_{17}[8,12,15]$; $Z_{16}[6]$; $X_{16}[14,15]$
3	$STK_{17}[3]$	$2^{12} \times 2^{28} = 2^{40}$	$2^{44-7.2}$	$Z_{17}[4]$; $X_{17}[8,12]$; $Z_{16}[6]$; $X_{16}[12,14,15]$
4	$STK_{17}[4]$	$2^{16} \times 2^{28} = 2^{44}$	$2^{44-7.2}$	$Z_{16}[0,6,7]$; $X_{16}[10,12,14,15]$
5	$STK_{16}[6]$	$2^{20} \times 2^{20} = 2^{40}$	$2^{48-7.2}$	$Z_{16}[0,7]$; $X_{16}[12,15]$; $X_{15}[5]$
6	$STK_{16}[7]$	$2^{24} \times 2^{16} = 2^{40}$	$2^{44-7.2}$	$Z_{16}[0]$; $X_{16}[12]$; $X_{15}[5,9]$
7	$STK_{16}[0]$	$2^{28} \times 2^4 = 2^{32}$	$2^{44-6.2}$	$X_{13}[0]$
Σ		2^{44}	$2^{41.32}$	

Our CP Model for Partial-Sum Technique - II

- Assume that in each step we guess at least one cell of the involved keys.
- We define the number of steps s which is less than the number of involved key cells.
- For each cell we define an integer variable with domain $\{0, \cdots, s\}$.
- We define some constraints to compute the step number of deriving each cell.

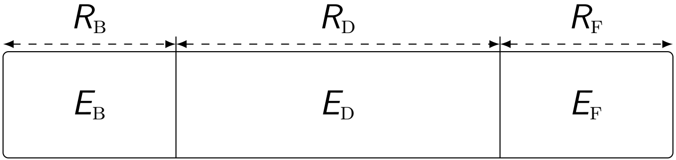


Our Unified Model for Finding Integral Attack

- Our CP model for finding complete integral attack includes the following modules:
  - Model the distinguisher part
  - Model the meet-in-the-middle technique
  - Model the involved cells in key recovery
  - Model the step assignment
  - Model the tweakey schedule (key-bridging)
  - Model the time/memory complexity evaluation
- Objective function: minimize the total time complexity

Usage of Our Tool

```
python3 attack.py -RB 1 -RD 12 -RF 5
```



- We use MiniZinc [Net+07] to create our CP models
- We use Gurobi [Gur22] and OrTools [PF] as the CP solvers
- Our tool can find the results in a few seconds running on a regular laptop

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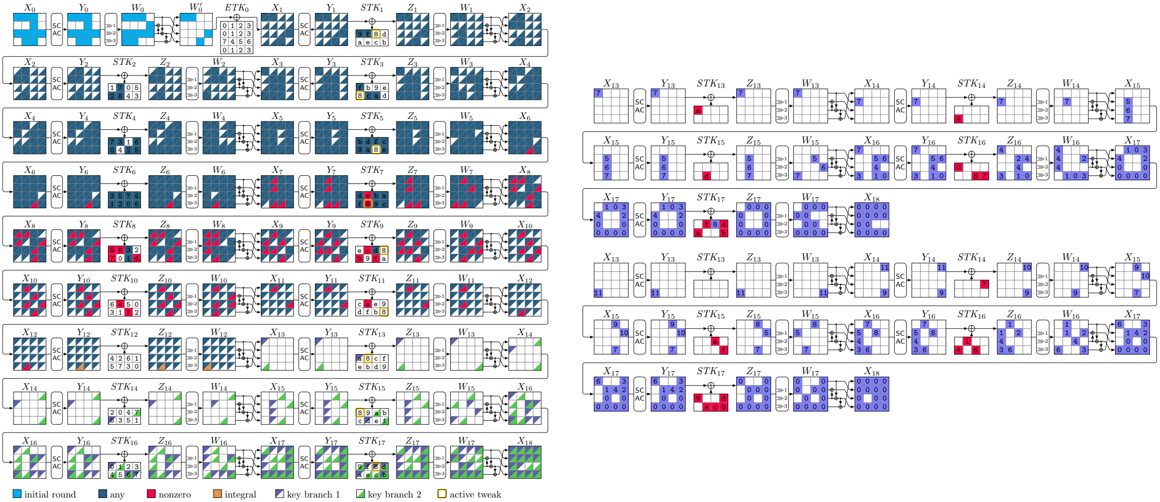
Example: 18-round Integral Attack on SKINNY-n-n



Contributions and Future Works

Contributions and Future Works

- Contributions
  - Improving unified models for finding complete ID/ZC/integral attacks
  - Introducing a CP model for the partial-sum technique for the first time
  - Found improved attacks for SKINNY, and ForskSKINNY, and QARMAv2
- Future works
  - Extending our distinguisher models for ID/ZC to find indirect contradictions
  - Extending our tools to AndRX and ARX ciphers, e.g., Simeck, and SPECK.
  - Extending our approach to division property or monomial prediction techniques
  - Improving the key-recovery part of our CP models for ZC attacks

: https://github.com/hadipourh/zeroplus

: https://ia.cr/2023/1701

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