

SYDE 556/750

**Simulating Neurobiological Systems
Lecture 8: Temporal Basis Functions**

Terry Stewart

October 27, 2021

- ▶ Slide design: Andreas Stöckel
- ▶ Content: Terry Stewart, Andreas Stöckel, Chris Eliasmith

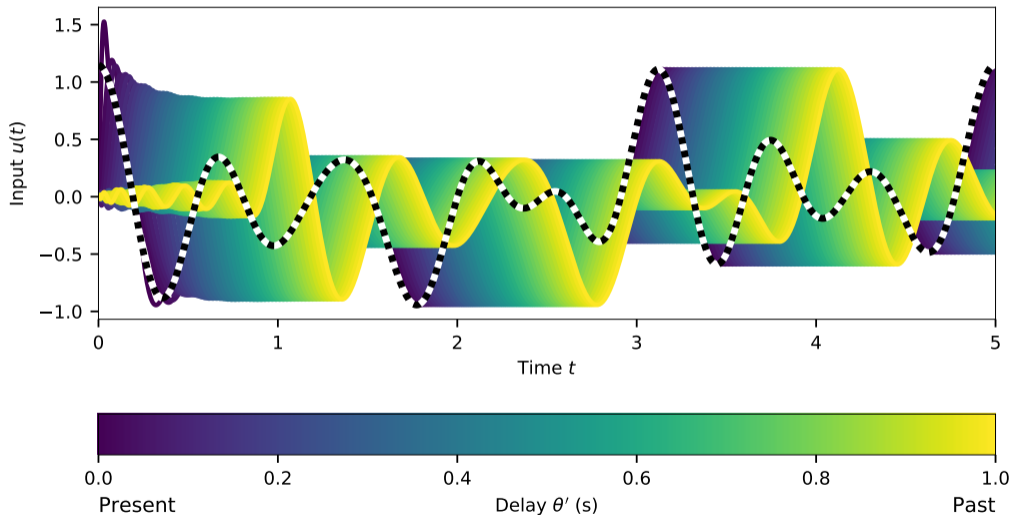


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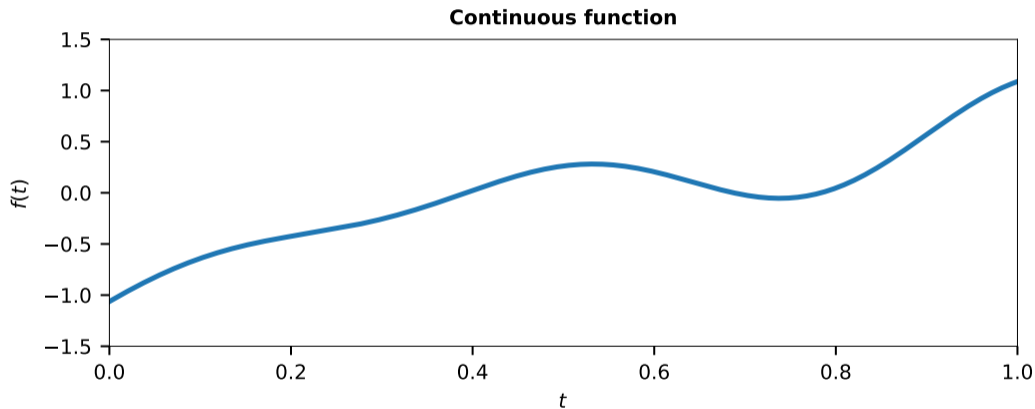
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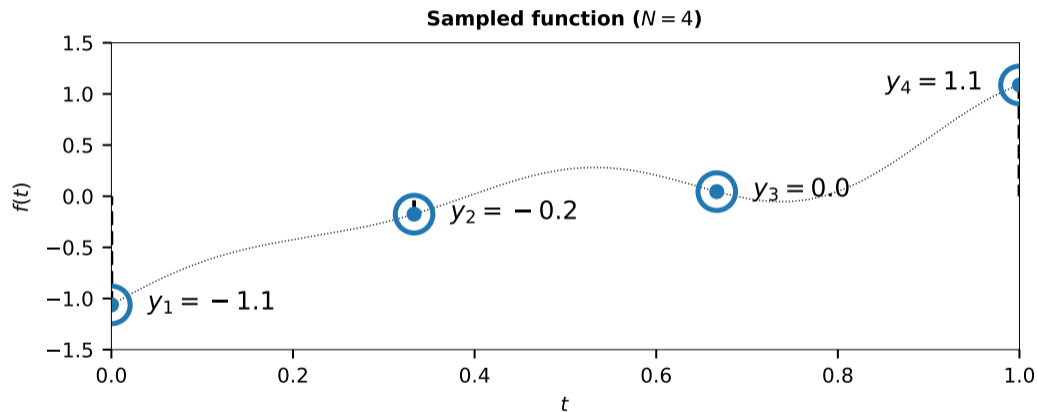
Representing Stimulus Histories



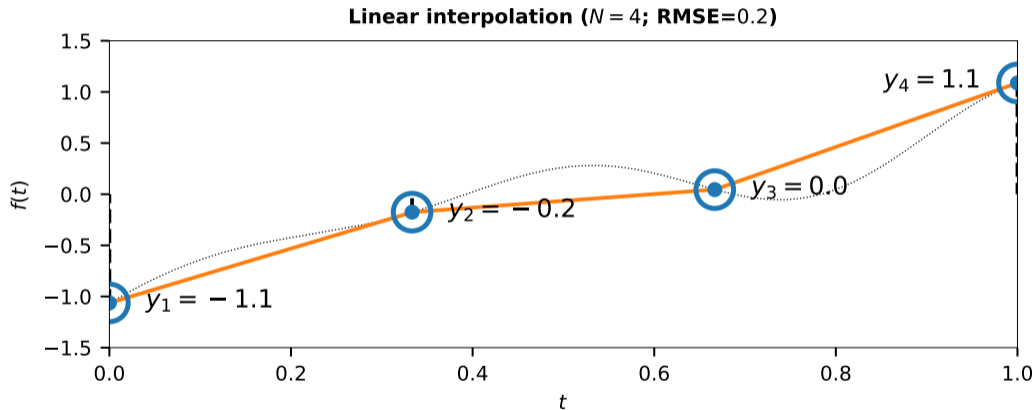
Representing Functions: Sampling



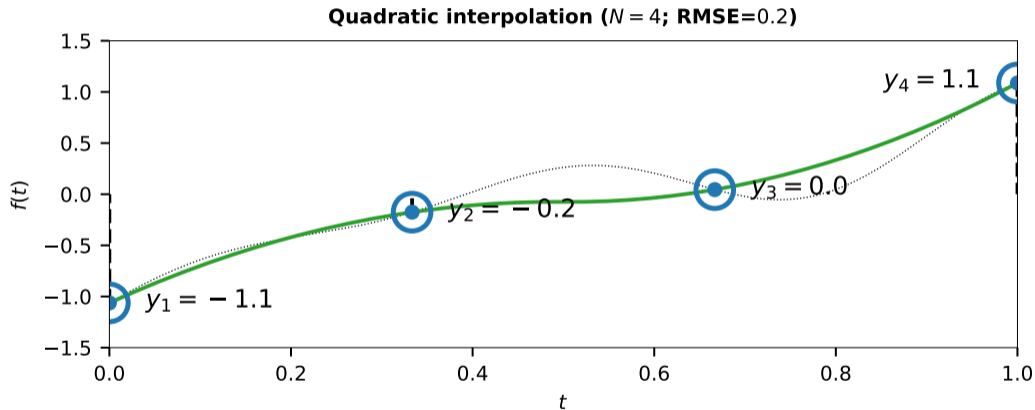
Representing Functions: Sampling



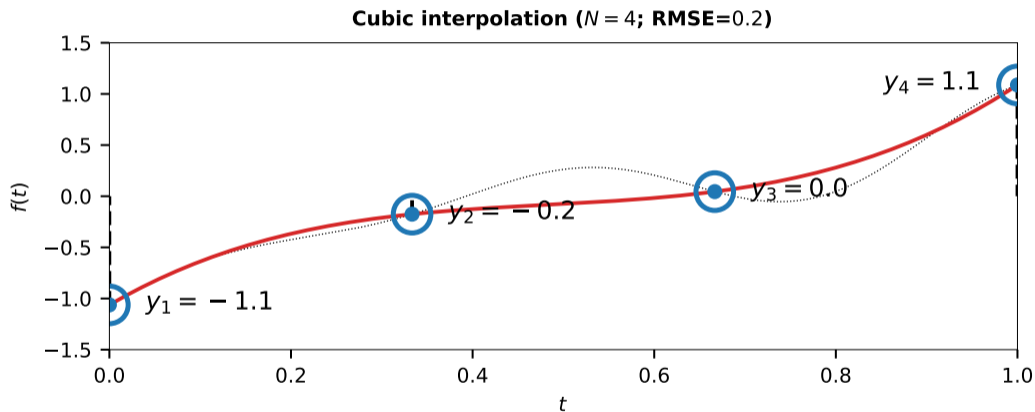
Representing Functions: Sampling



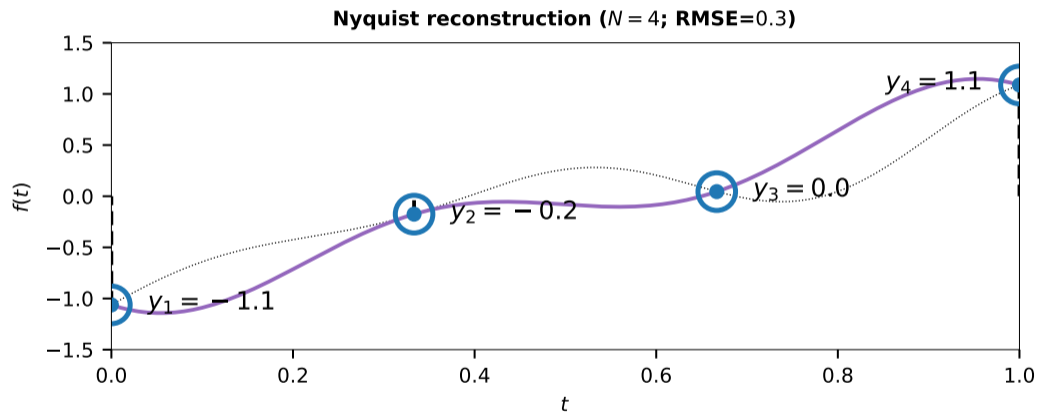
Representing Functions: Sampling



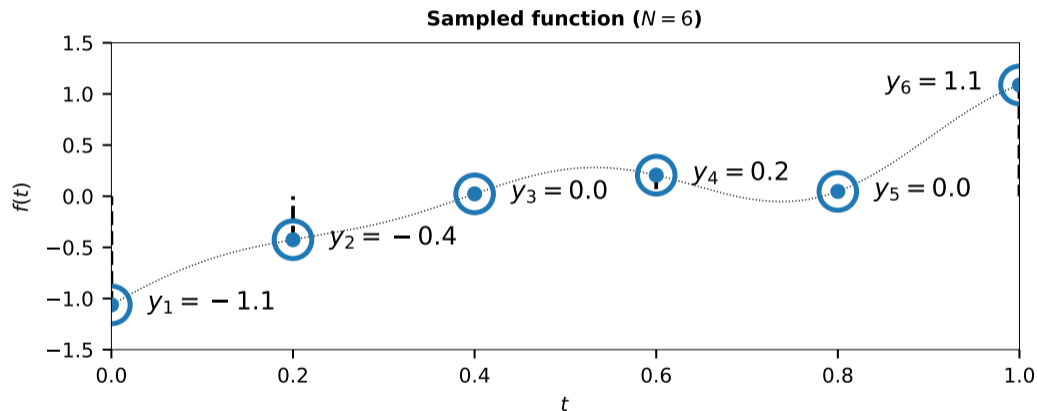
Representing Functions: Sampling



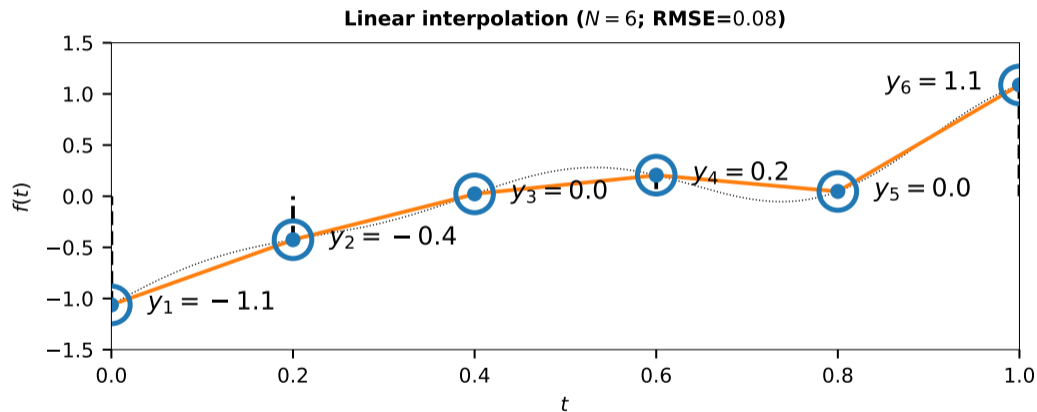
Representing Functions: Sampling



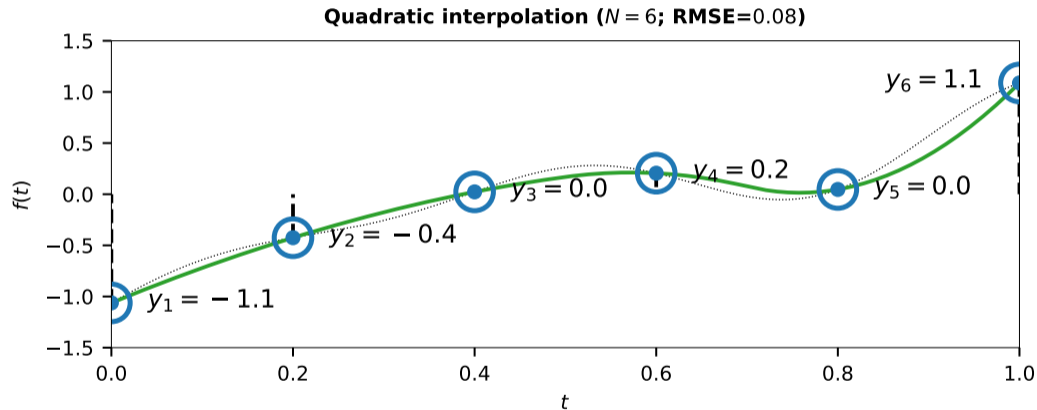
Representing Functions: Sampling



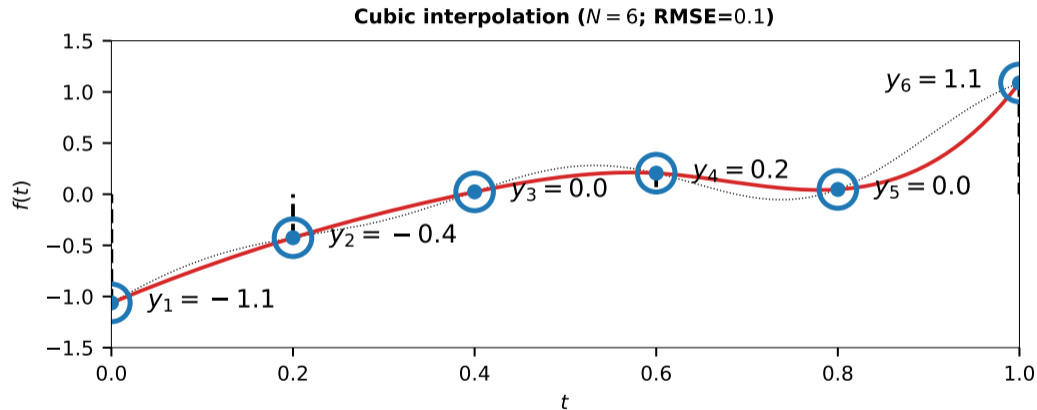
Representing Functions: Sampling



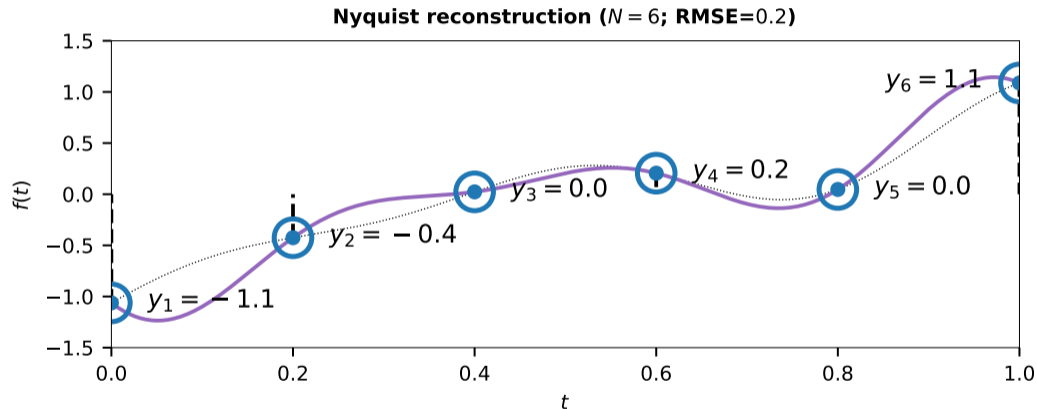
Representing Functions: Sampling



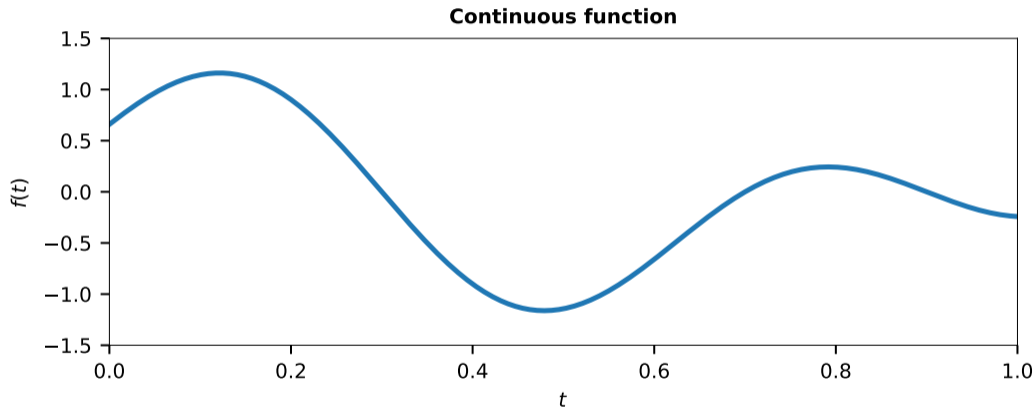
Representing Functions: Sampling



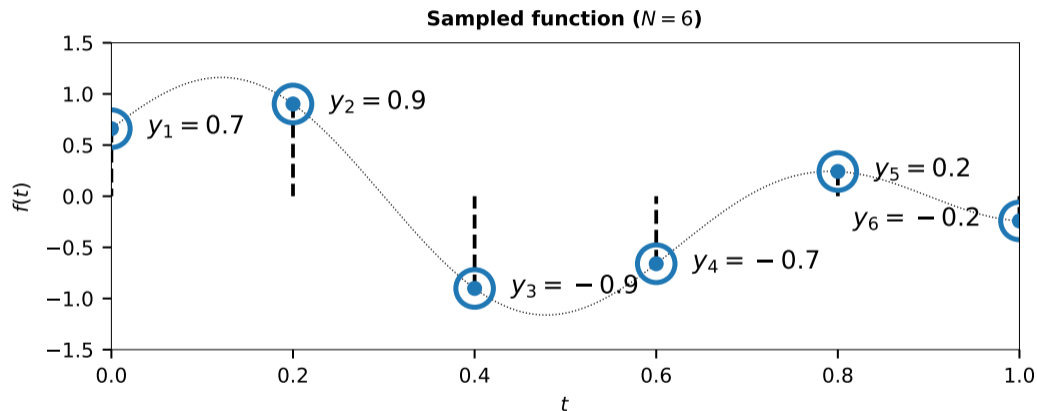
Representing Functions: Sampling



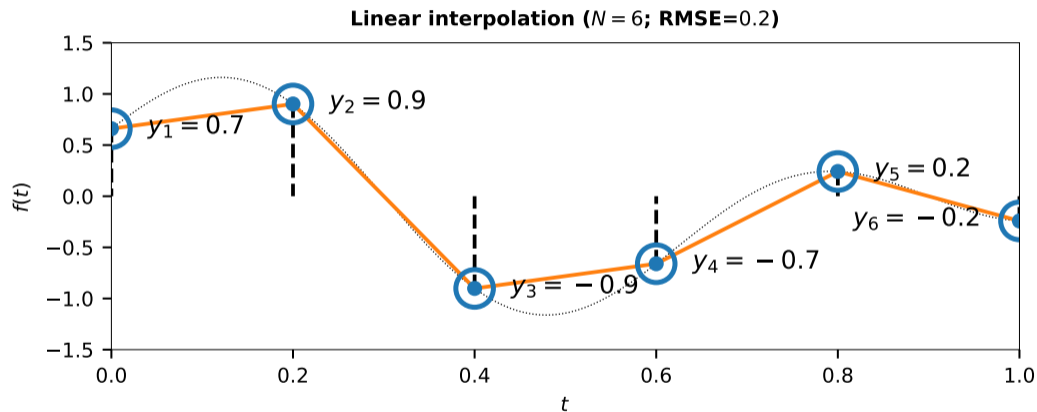
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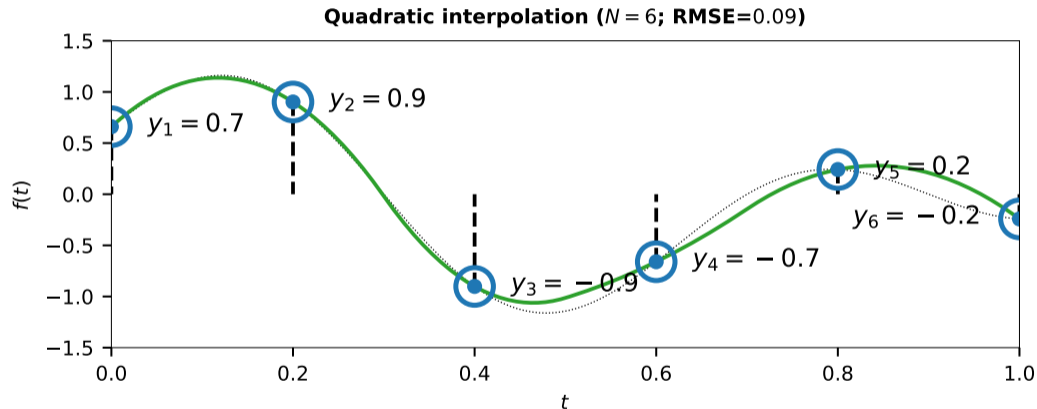
Representing Functions: Sampling



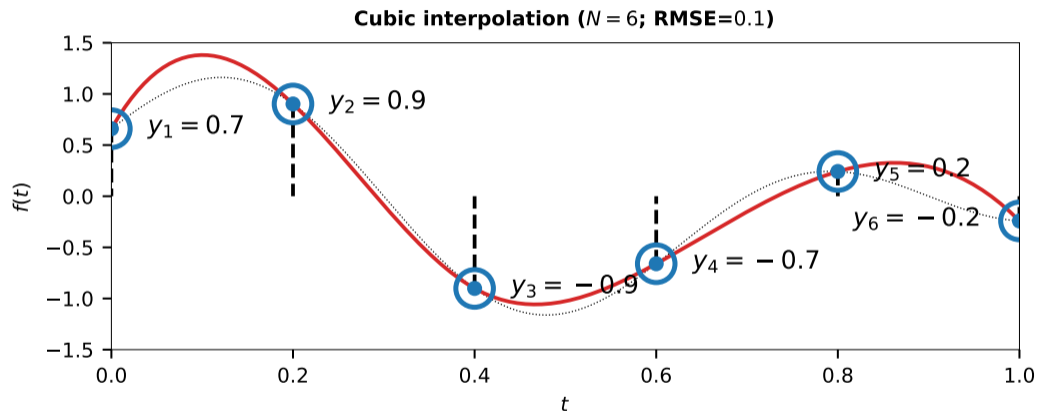
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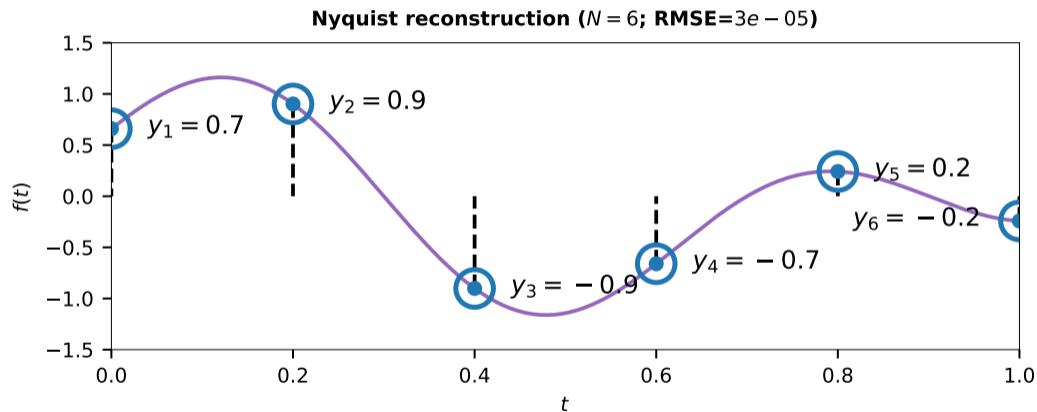
Representing Functions: Sampling



Representing Functions: Sampling

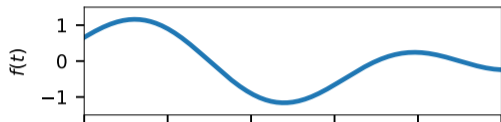


Representing Functions: Sampling

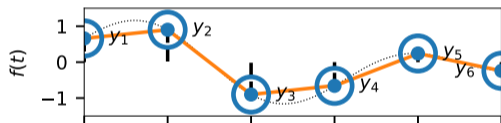


Representing Functions: Sampling

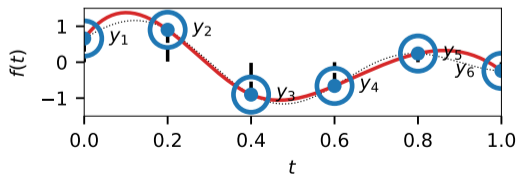
Continuous function



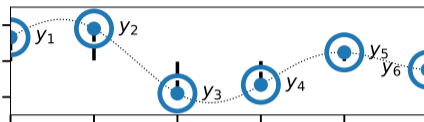
Linear interpolation (N = 6; RMSE=0.2)



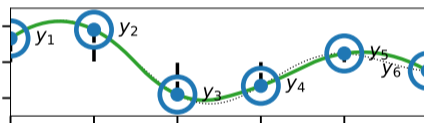
Cubic interpolation (N = 6; RMSE=0.1)



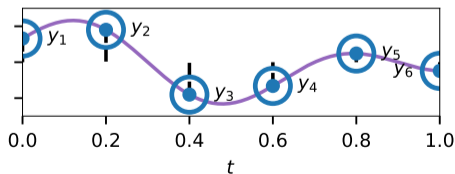
Sampled function (N = 6)



Quadratic interpolation (N = 6; RMSE=0.09)

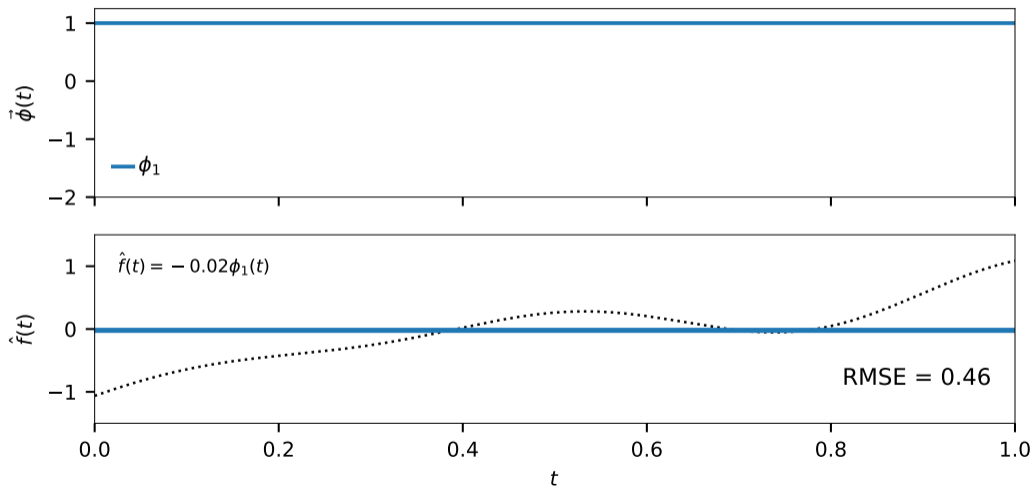


Nyquist reconstruction (N = 6; RMSE=3e-05)



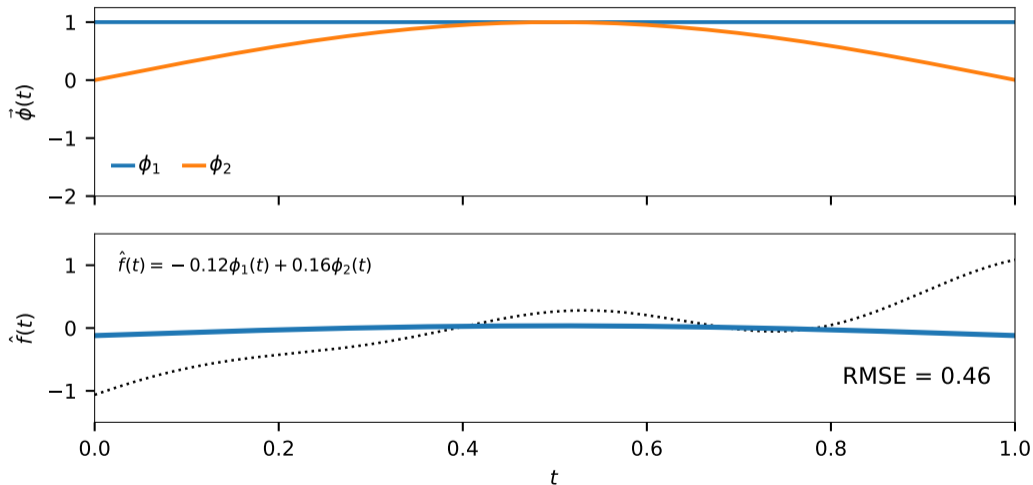
Representing Functions: Fourier Basis

Fourier Basis ($q = 1$)



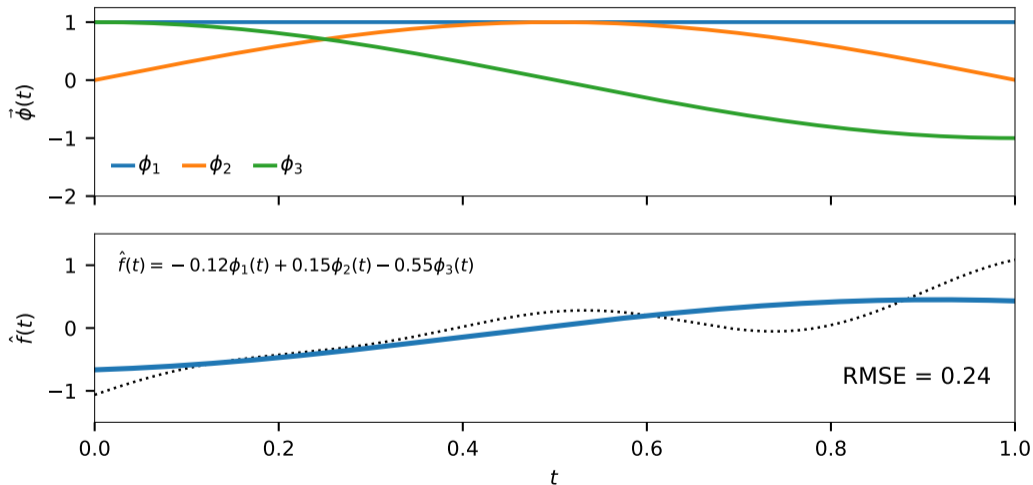
Representing Functions: Fourier Basis

Fourier Basis ($q = 2$)



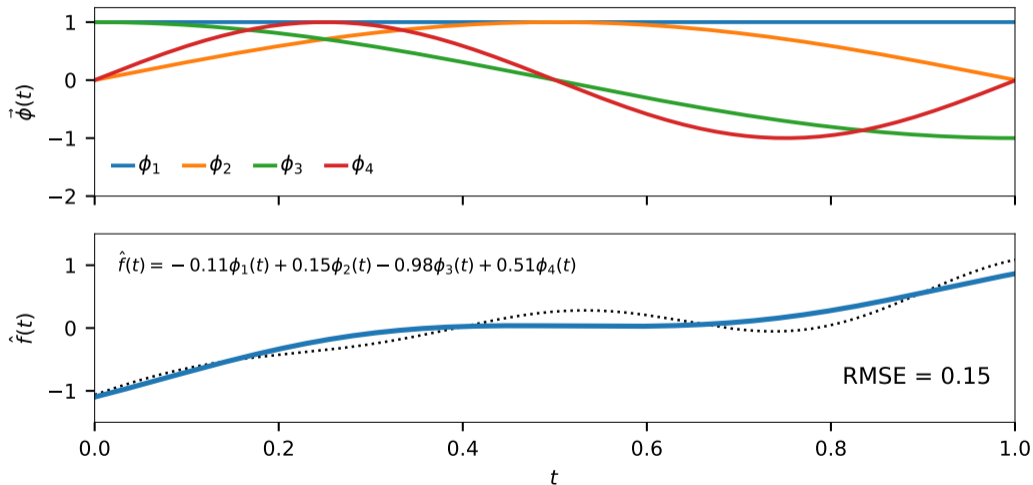
Representing Functions: Fourier Basis

Fourier Basis ($q = 3$)



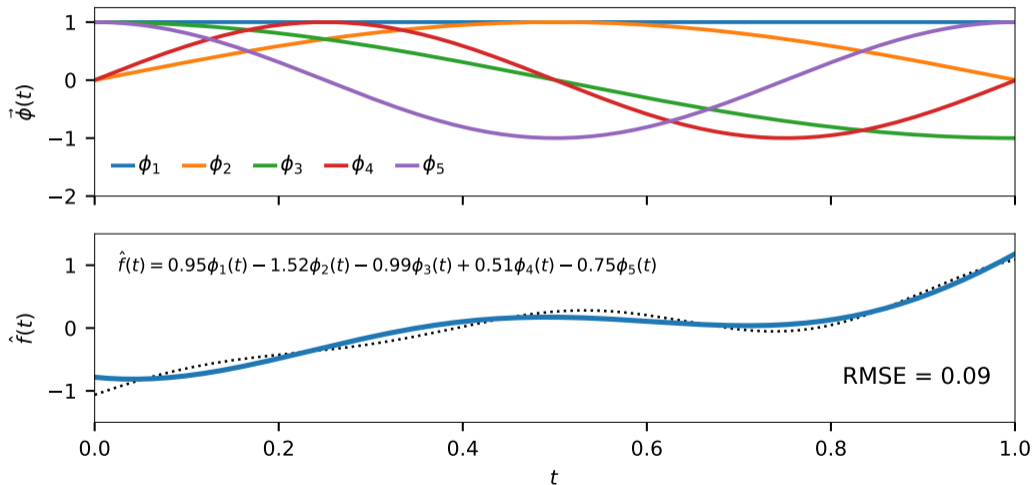
Representing Functions: Fourier Basis

Fourier Basis ($q = 4$)



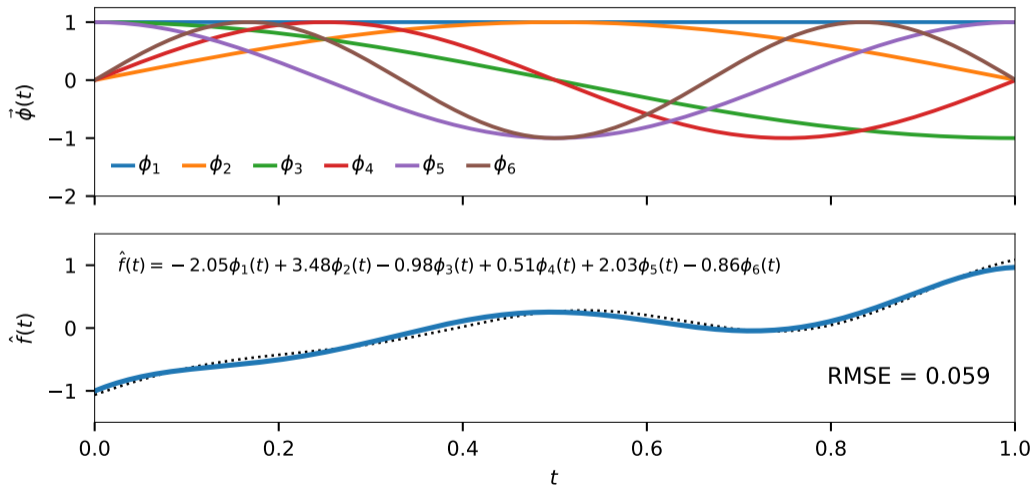
Representing Functions: Fourier Basis

Fourier Basis ($q = 5$)



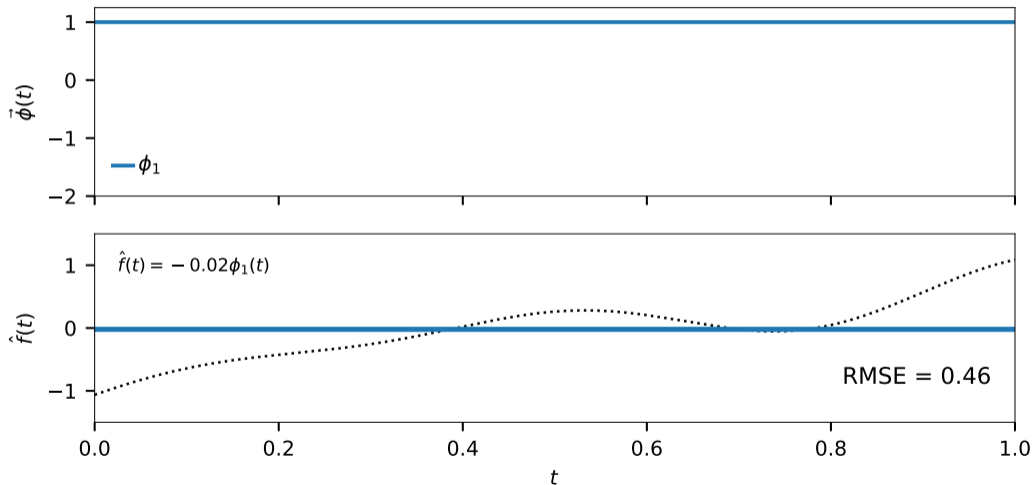
Representing Functions: Fourier Basis

Fourier Basis ($q = 6$)



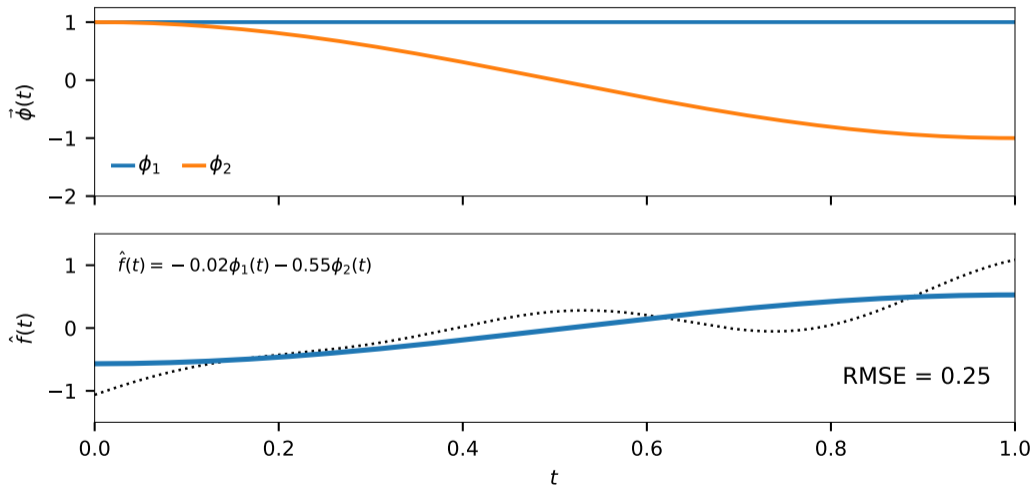
Representing Functions: Cosine Basis

Cosine Basis ($q = 1$)



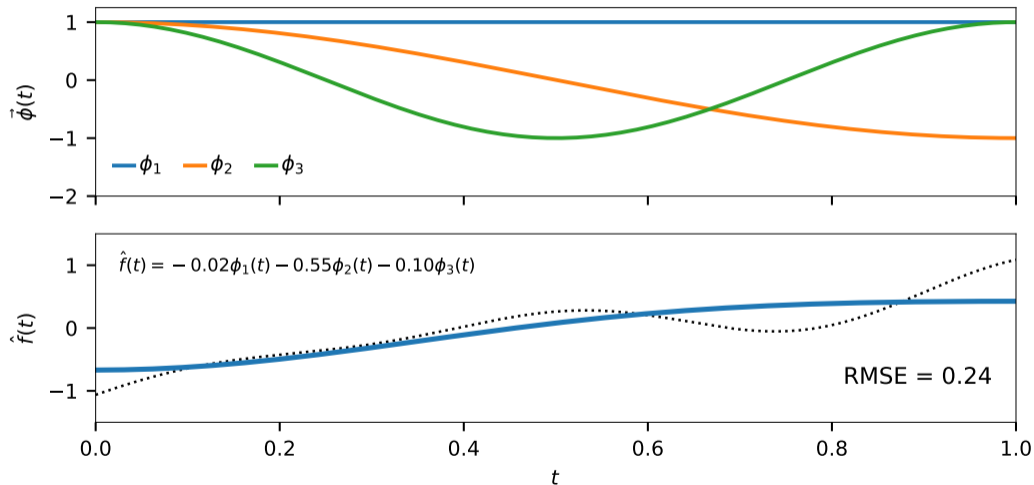
Representing Functions: Cosine Basis

Cosine Basis ($q = 2$)



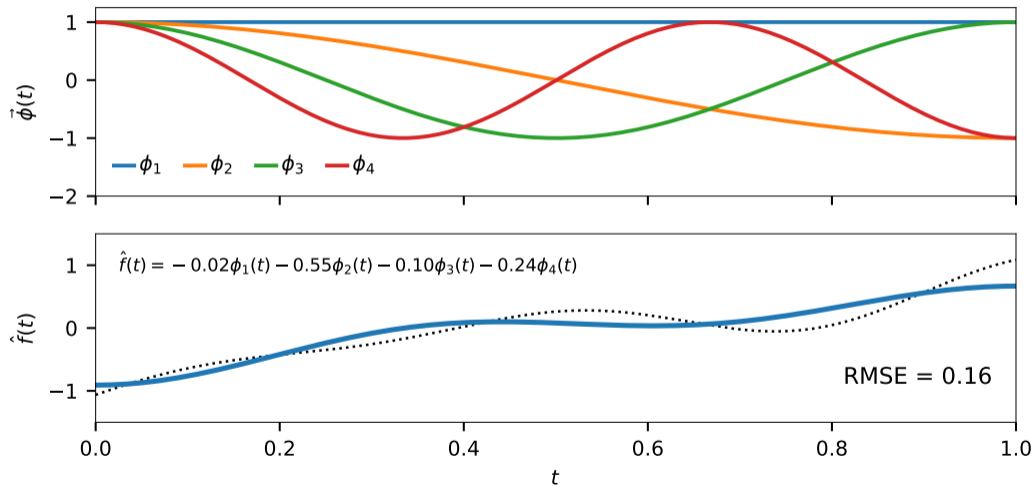
Representing Functions: Cosine Basis

Cosine Basis ($q = 3$)



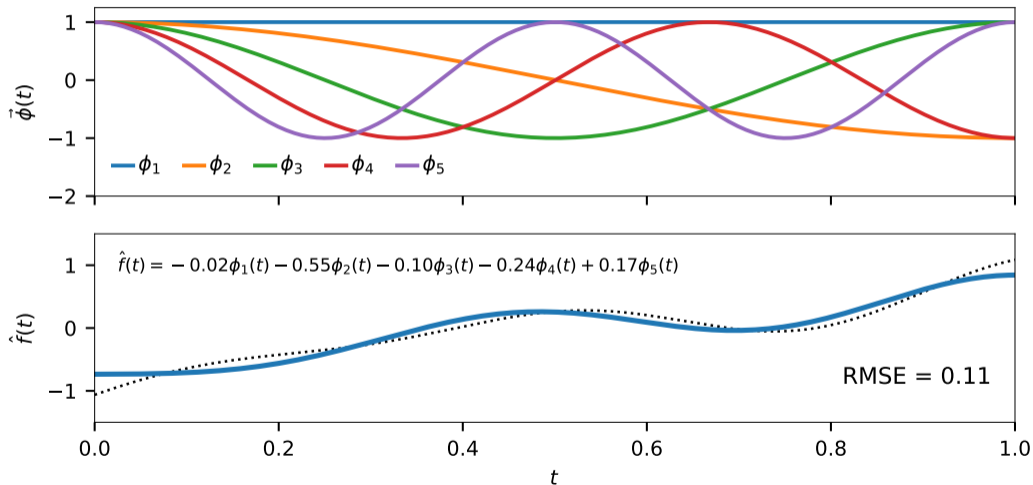
Representing Functions: Cosine Basis

Cosine Basis ($q = 4$)



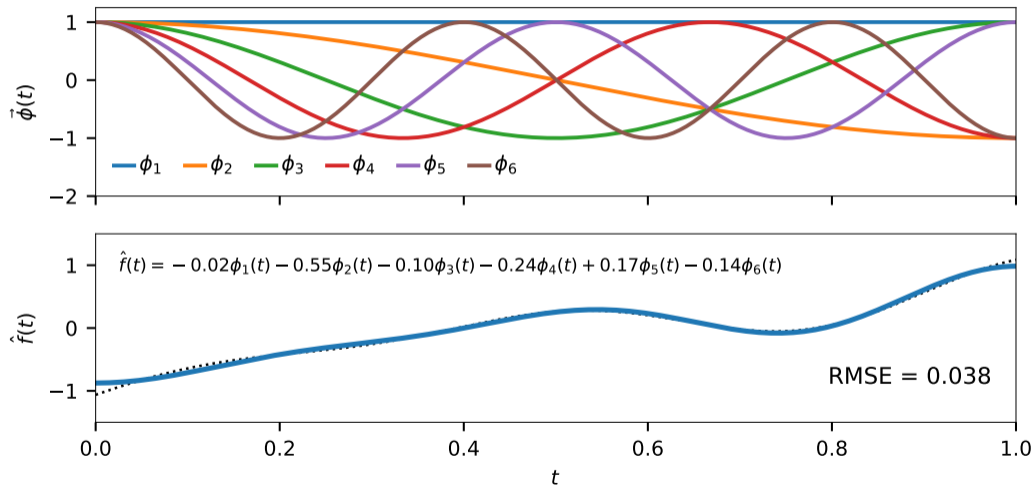
Representing Functions: Cosine Basis

Cosine Basis ($q = 5$)

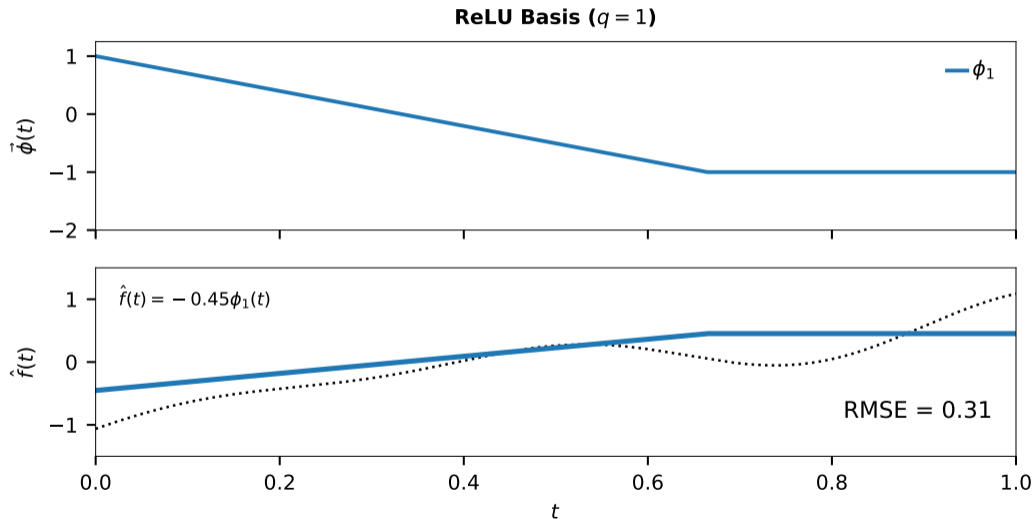


Representing Functions: Cosine Basis

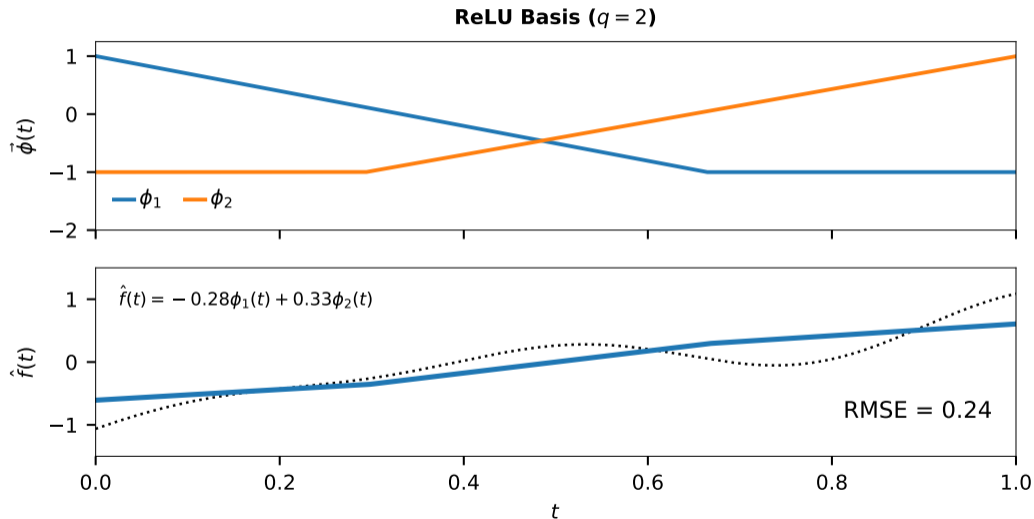
Cosine Basis ($q = 6$)



Representing Functions: ReLU Basis

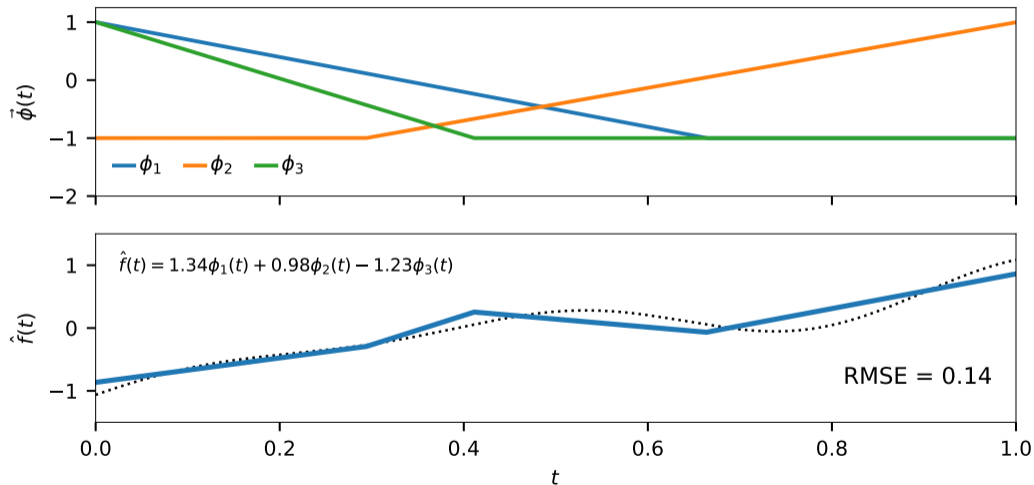


Representing Functions: ReLU Basis

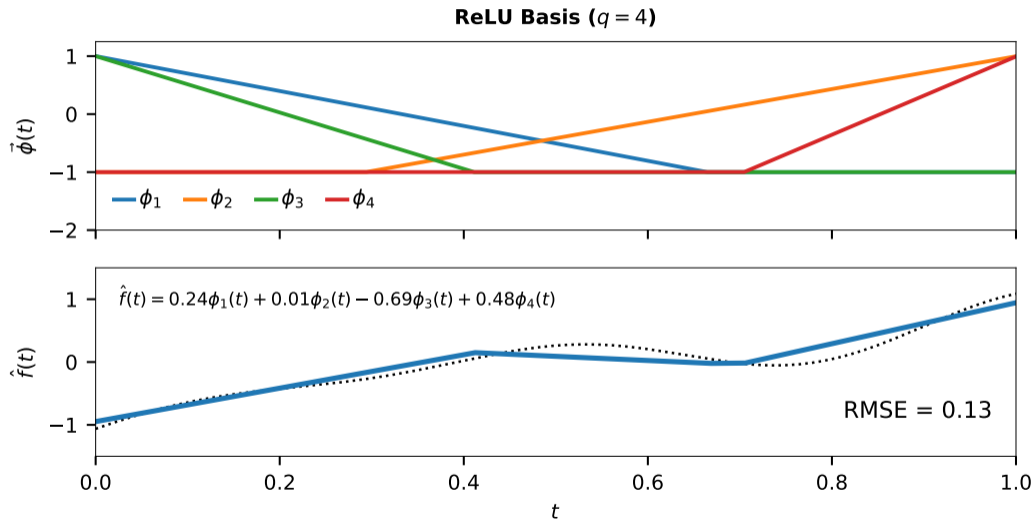


Representing Functions: ReLU Basis

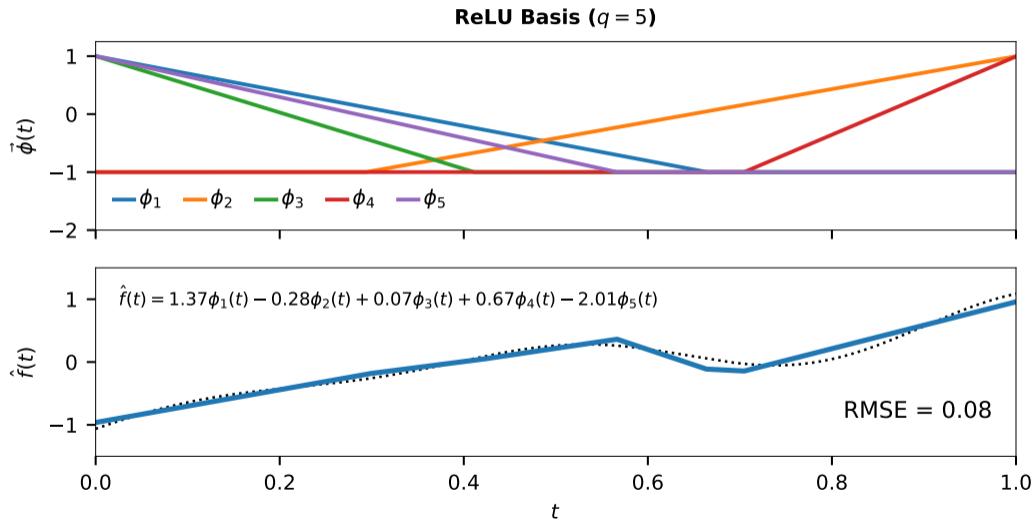
ReLU Basis ($q = 3$)



Representing Functions: ReLU Basis

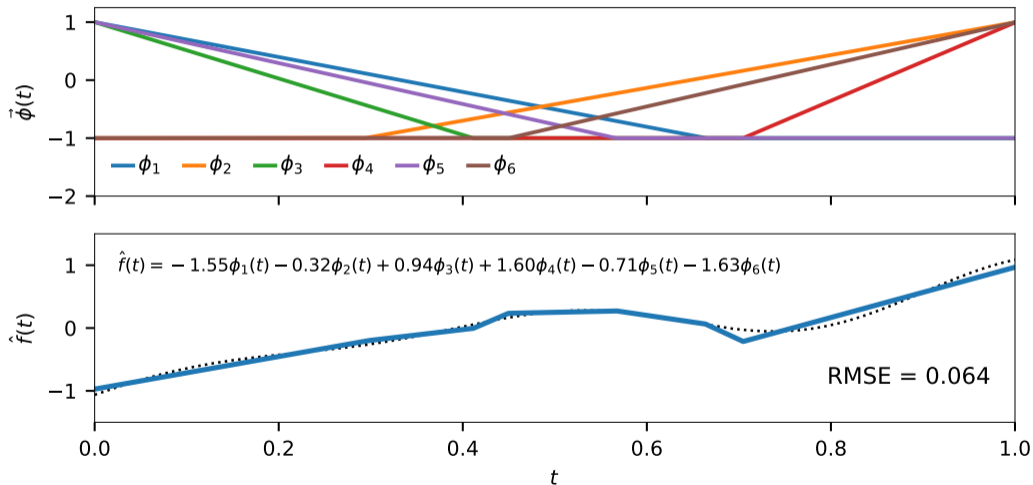


Representing Functions: ReLU Basis



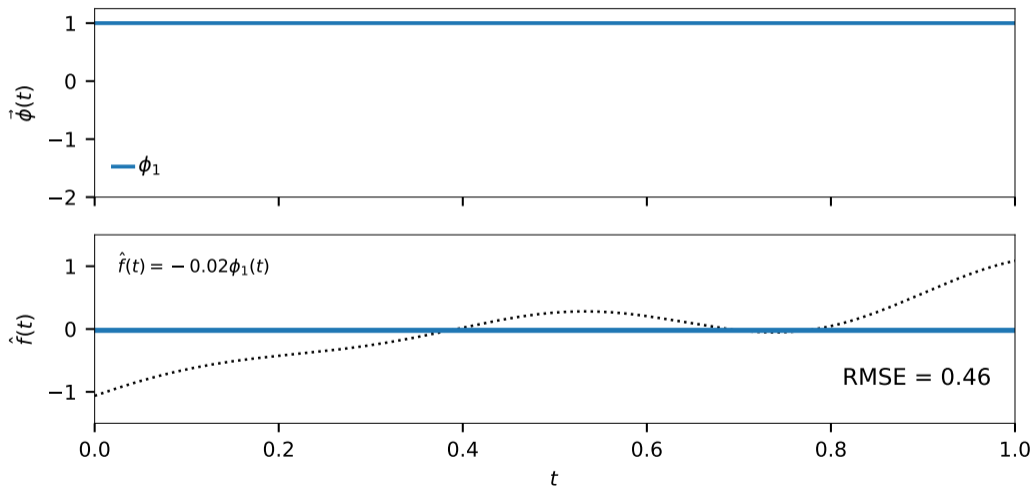
Representing Functions: ReLU Basis

ReLU Basis ($q = 6$)



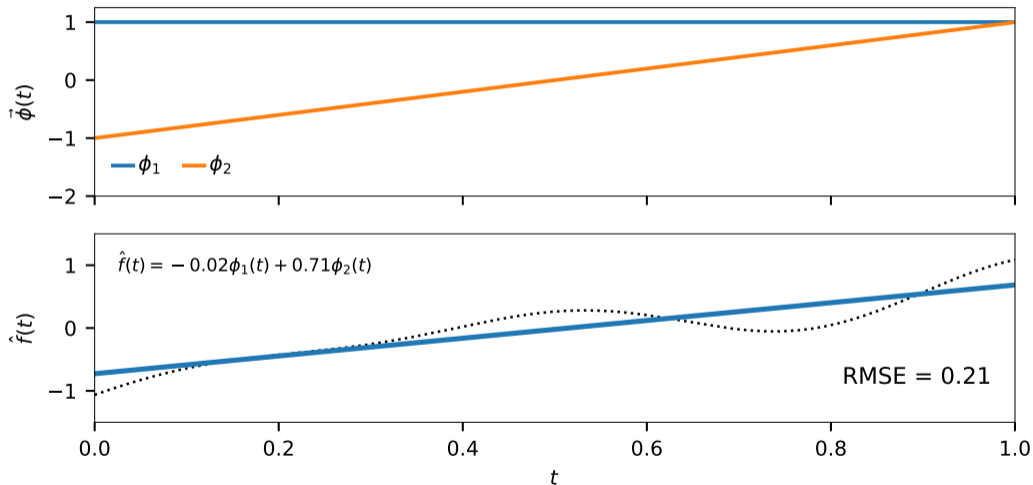
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 1$)



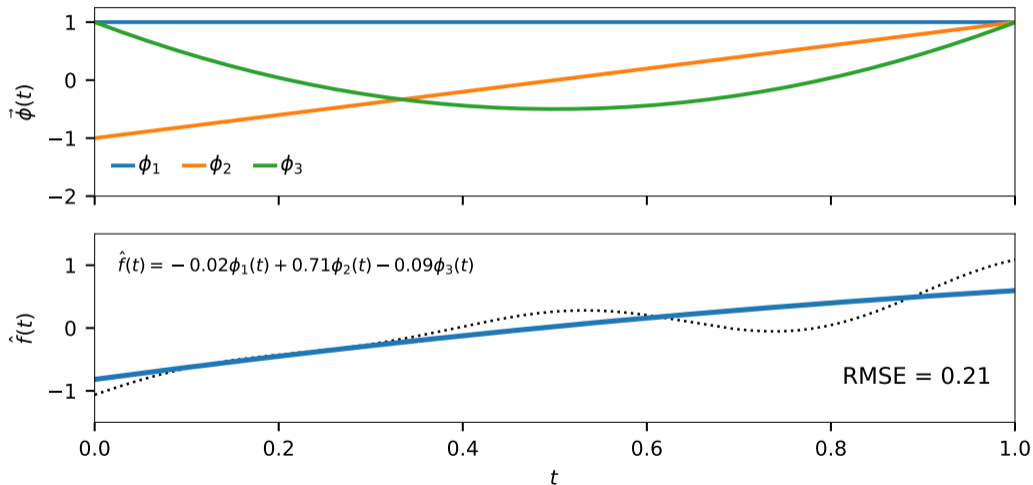
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 2$)



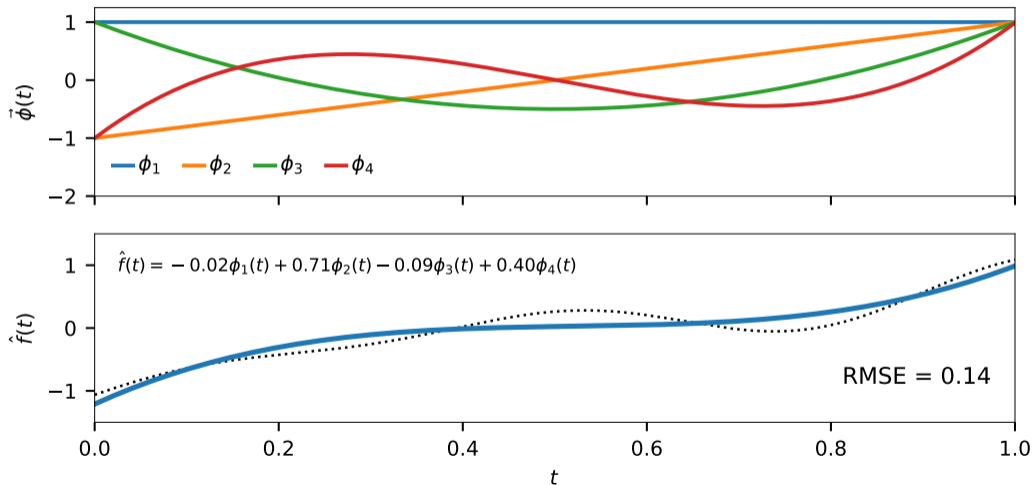
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 3$)



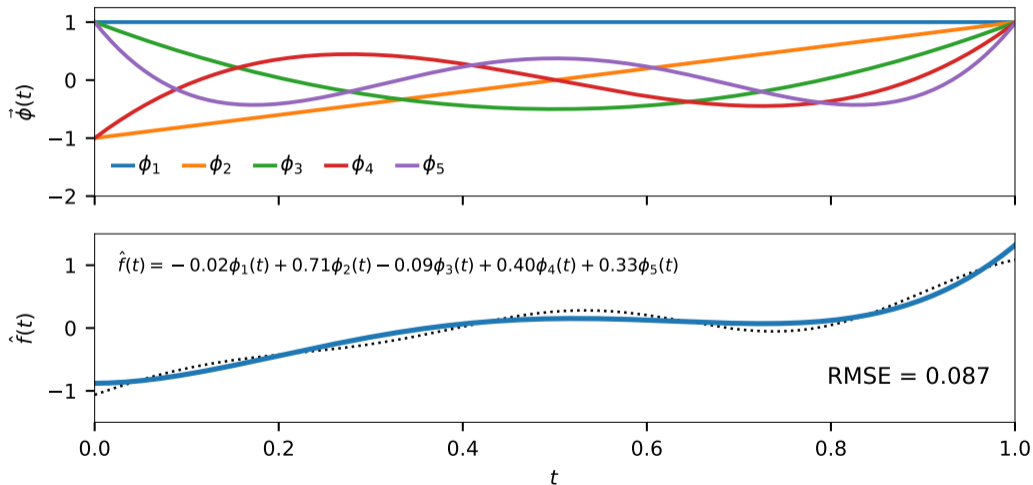
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 4$)



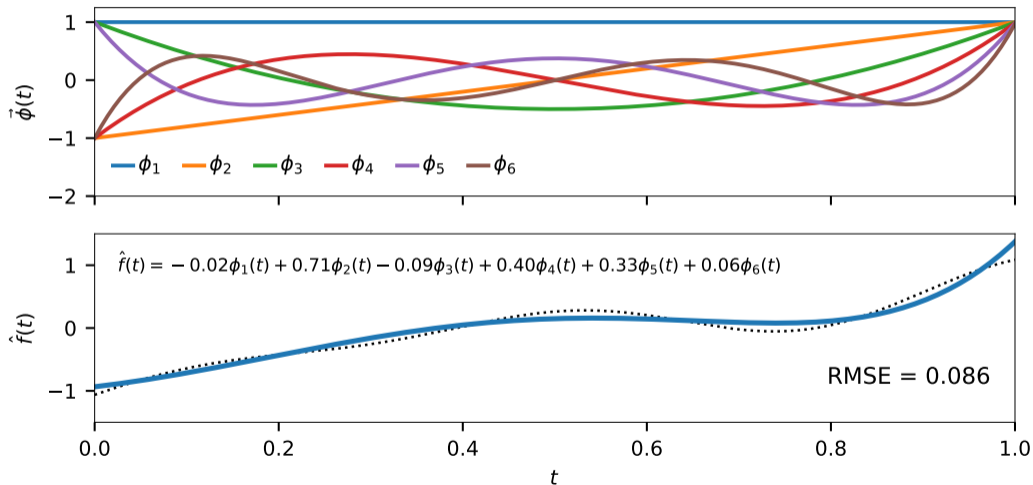
Representing Functions: Legendre Basis

Legendre Polynomials ($q = 5$)

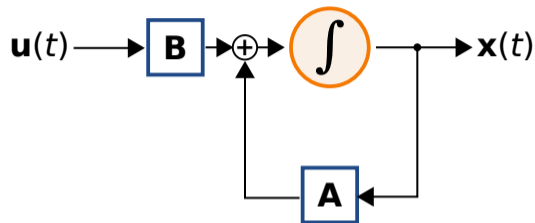


Representing Functions: Legendre Basis

Legendre Polynomials ($q = 6$)



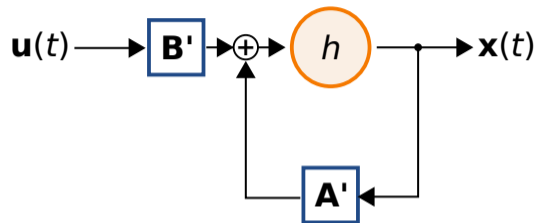
Implementing the Delay Network



$$\theta \mathbf{A} = a_{ij} \in \mathbb{R}^{q \times q},$$

$$\theta \mathbf{B} = b_i \in \mathbb{R}^q,$$

$$\mathbf{A}' = \tau \mathbf{A} + \mathbf{I}$$

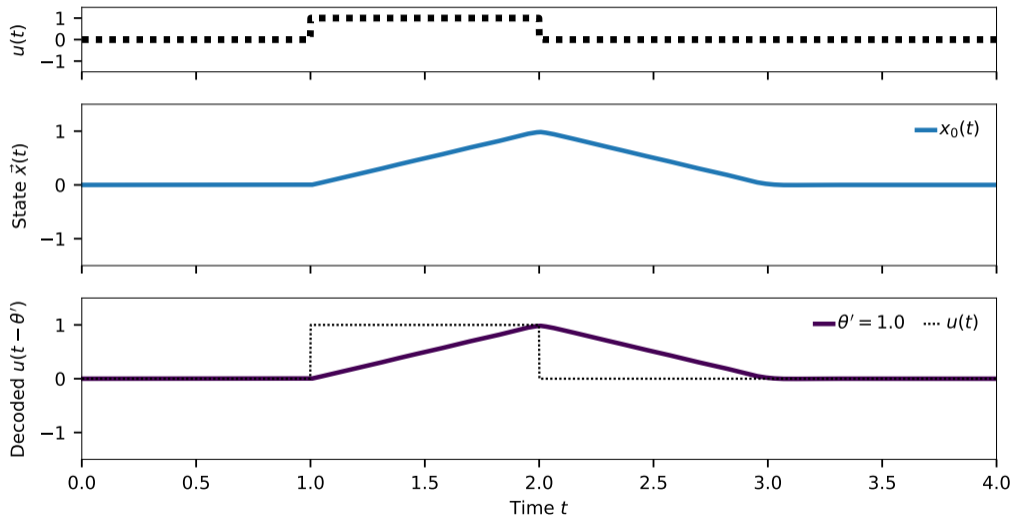


$$a_{ij} = \begin{cases} (2i+1)(-1) & i < j, \\ (2i+1)(-1)^{i-j+1} & i \geq j \end{cases}$$

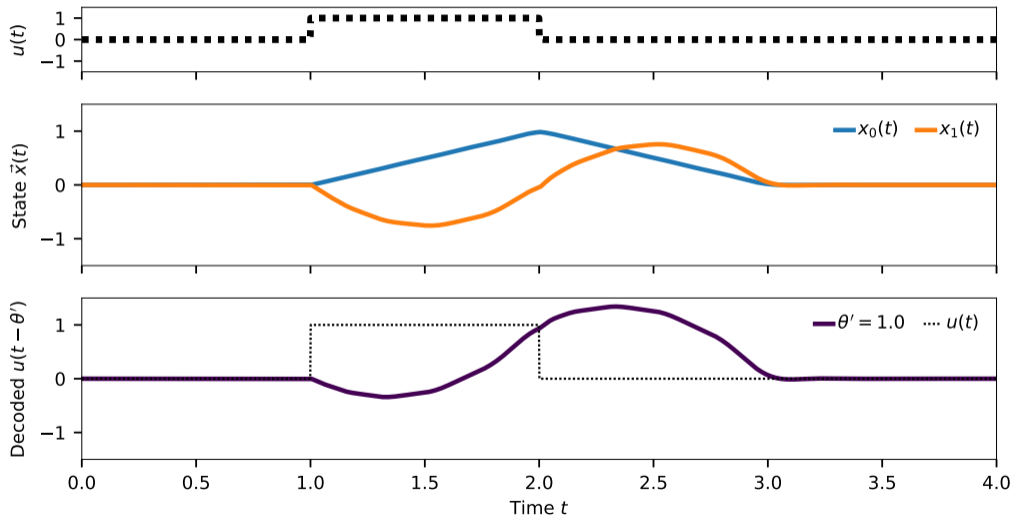
$$b_i = (2i+1)(-1)^i$$

$$\mathbf{B}' = \tau \mathbf{B}$$

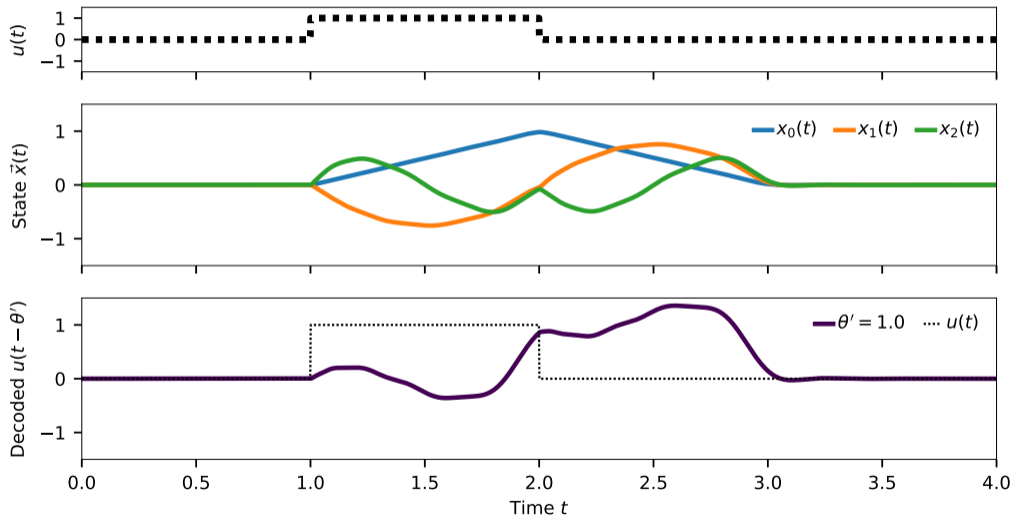
Delay Network: Step Function



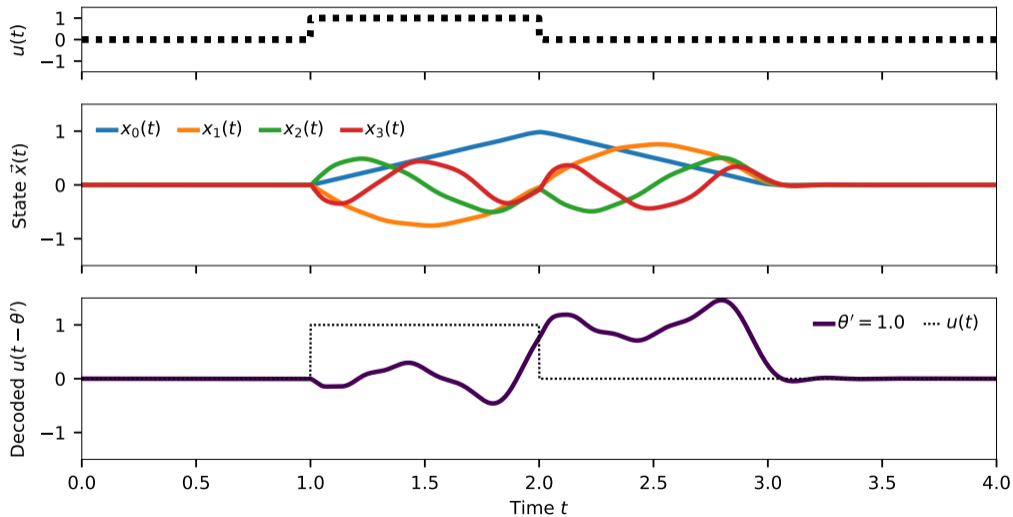
Delay Network: Step Function



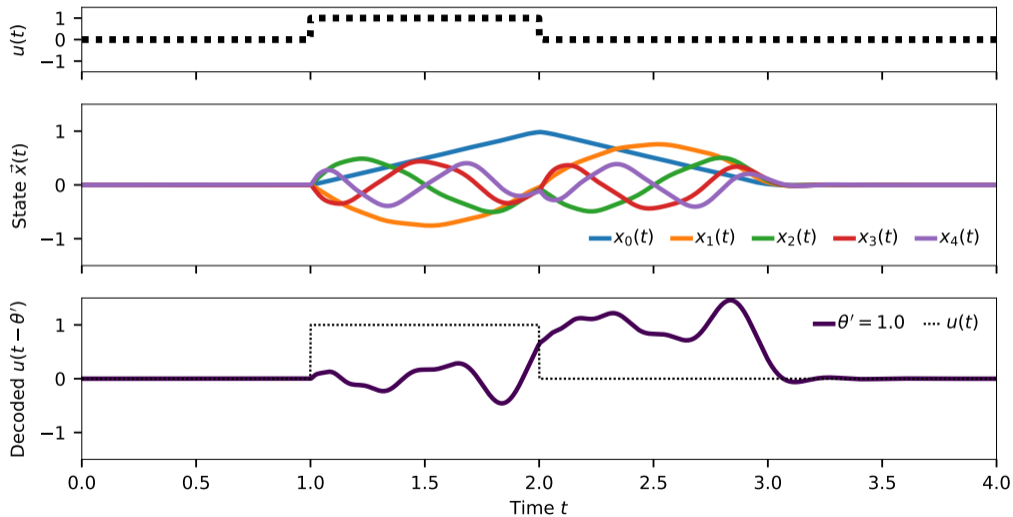
Delay Network: Step Function



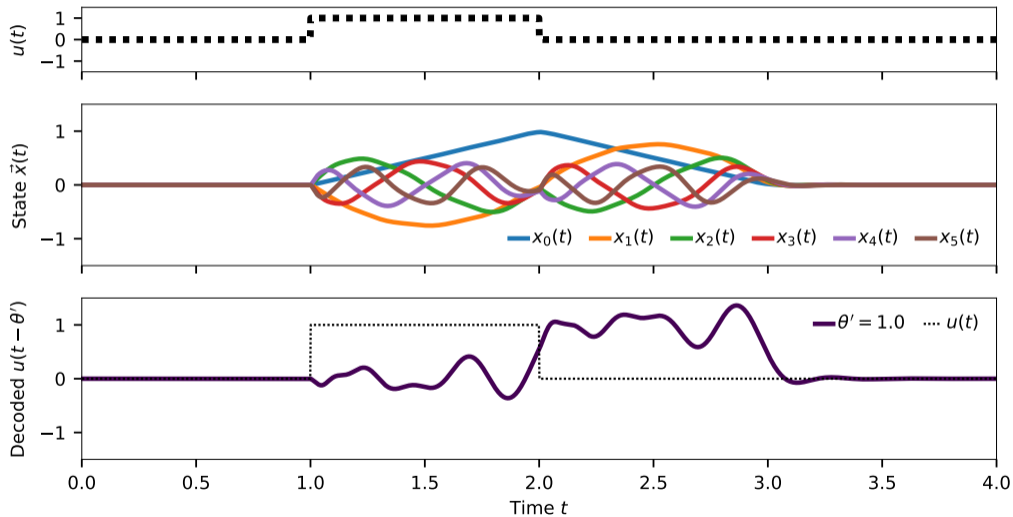
Delay Network: Step Function



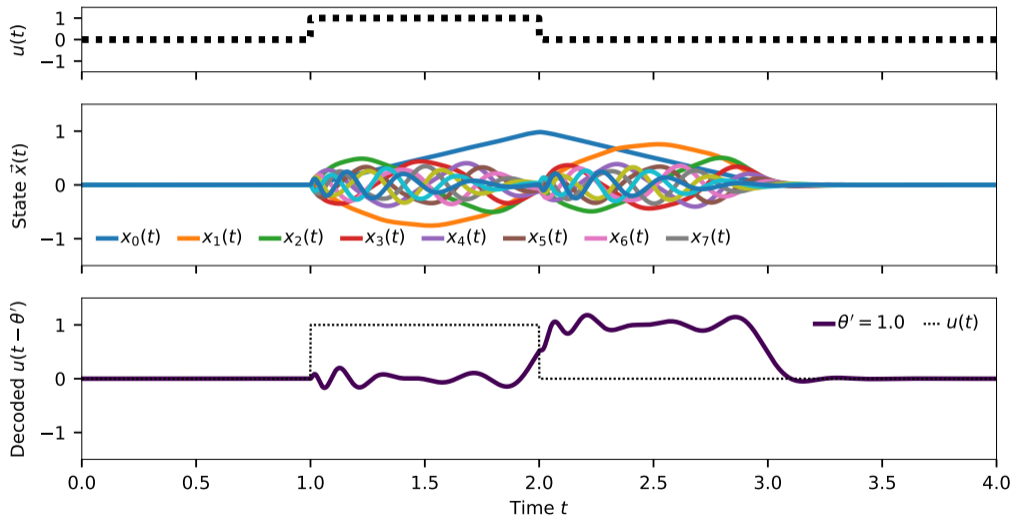
Delay Network: Step Function



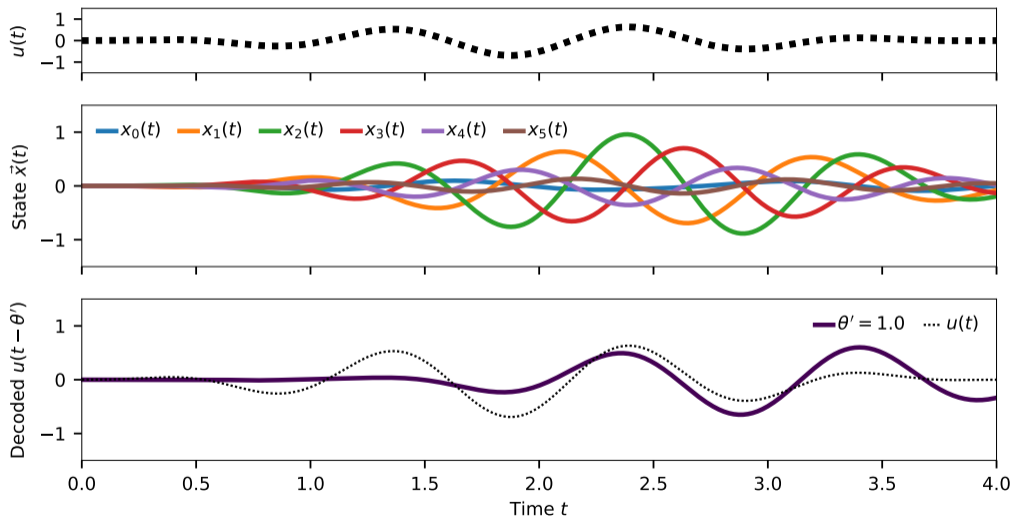
Delay Network: Step Function



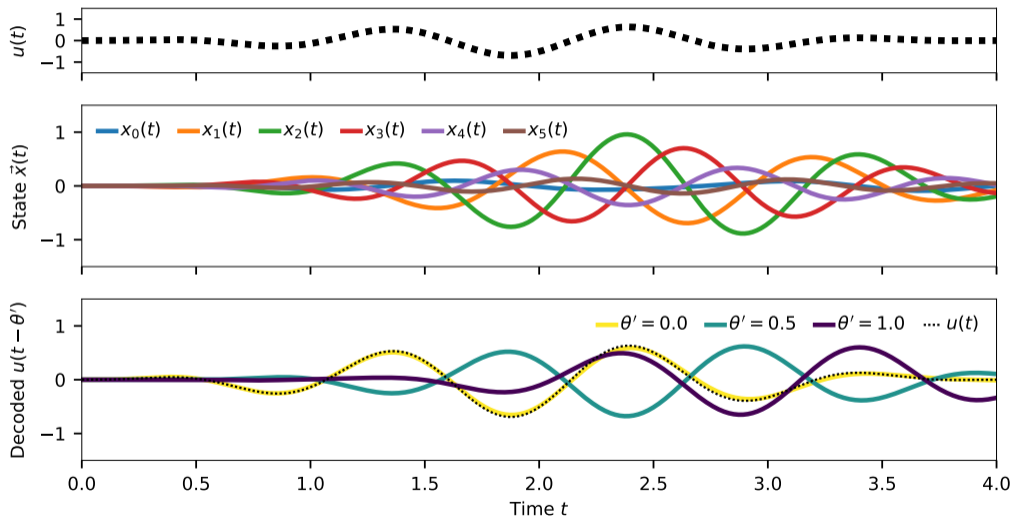
Delay Network: Step Function



Delay Network: Windowed Sine Function



Delay Network: Windowed Sine Function



Delay Network: Windowed Sine Function

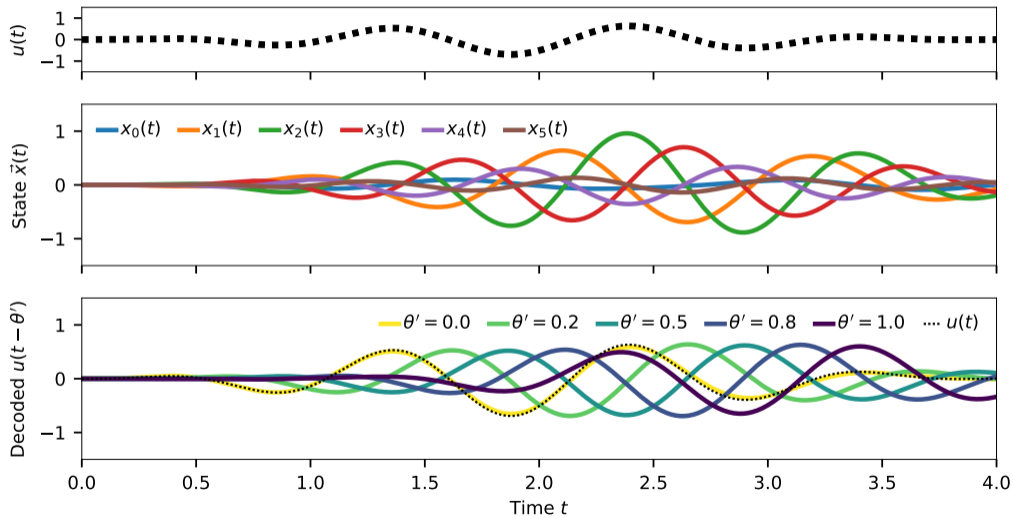


Image sources

Title slide

Infrared Photograph of a Sundial Near the Einstein Tower in Potsdam, Germany

Author: DrNRNowaczyk, 2007.

From Wikimedia.